Neutrino mass in nonsupersymmetric $SO(10)$ GUT

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We study a prediction on neutrino observables in a nonsupersymmetric renormalizable $SO(10)$ grand unified theory model that contains a **10** complex scalar field and a **126** scalar field whose Yukawa couplings with **16** matter fields provide the quark and charged lepton Yukawa couplings, neutrino Dirac Yukawa coupling and Majorana mass for the singlet neutrinos. The $SO(10)$ breaking is achieved in two steps by a $\mathcal{O}(10^{15})$ GeV vacuum expectation value (VEV) of a **54** real scalar field and a $\mathcal{O}(10^{14})$ GeV VEV of the **126** field. First, we analyze the gauge coupling unification conditions and determine the VEV of the **126** field. Next, we constrain the Yukawa couplings of the **10** and **126** fields at the scale of the **126** field's VEV from experimental data on quark and charged lepton masses and quark flavor mixings. Then we express the active neutrino mass with the above Yukawa couplings and the **126** field's VEV based on the Type-1 seesaw mechanism, and fit neutrino oscillation data, thereby deriving a prediction on poorly or not measured neutrino observables. What distinguishes our work from previous studies is that we do not assign Peccei-Quinn charges on visible sector fields so that the **10** scalar field and its complex conjugate both have Yukawa couplings with **16** matter fields. From the fitting of neutrino oscillation data, we find that not only the normal neutrino mass hierarchy, but also the inverted hierarchy can be realized. We also reveal that in the normal hierarchy case, the Dirac CP phase of the neutrino mixing matrix δ_{CP} is likely in the ranges of $-2.4 < \delta_{CP} < -1.2$ and $1.2 < \delta_{CP} < 2.4$, and not in the region with $\delta_{CP} \sim \pi$, and that in the normal hierarchy case, θ_{23} is likely in the upper octant and in the range of $0.50 \lesssim \sin^2\theta_{23} \lesssim 0.55$.

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I. INTRODUCTION

The $SO(10)$ grand unified theory (GUT) [1,2] is a viable candidate for physics beyond the Standard Model (SM), because it accounts for the origin of the hypercharge quantization, and it is automatically equipped with the seesaw mechanism that naturally explains the tiny neutrino mass [3–8]. There are four classes of $SO(10)$ GUT models which are either supersymmetric or nonsupersymmetric and either renormalizable or nonrenormalizable. In supersymmetric models, the gauge coupling unification is achieved without intermediate scale, whereas in nonsupersymmetric models, one or more intermediate scales are necessary for the successful unification. In renormalizable models [9,10], one introduces **10** and $\mathbf{126} + \overline{\mathbf{126}}$ (and optionally **120**) representation fields from which the electroweak symmetry-breaking Higgs field originates, and the renormalizable couplings of **10** and $\overline{\mathbf{126}}$ with **16** matter fields give rise to realistic SM Yukawa couplings and neutrino Dirac Yukawa coupling. Additionally, the renormalizable coupling of $\overline{\mathbf{126}}$ and its vacuum expectation value (VEV) generate Majorana mass for the singlet neutrinos. In nonrenormalizable models, one introduces **10** and $\mathbf{16} + \overline{\mathbf{16}}$ fields (let us denote the latter by $\mathbf{16}_H + \overline{\mathbf{16}}_H$ to avoid confusion), and the renormalizable coupling of **10** with **16** matter fields and the nonrenormalizable couplings of two $\overline{\mathbf{16}}_H$'s with **16** matter fields, combined with the VEV of $\overline{\mathbf{16}}_H$, generate realistic SM Yukawa couplings, neutrino Dirac Yukawa coupling and Majorana mass for the singlet neutrinos. The renormalizable models are attractive because the flavor structures of the neutrino Dirac Yukawa coupling and Majorana mass term can be constrained from experimental data on quark and charged lepton masses and quark flavor mixings, so that one can make a restrictive prediction on the neutrino mass and mixings. However, the supersymmetric renormalizable models confront a serious trouble that the $SO(10)$ gauge coupling becomes nonperturbative near the unification scale because the Dynkin index of **126** representation is large and both scalar and fermionic components contribute to the renormalization group (RG) evolution of the gauge coupling. For the above reasons, the nonsupersymmetric renormalizable models are worth for scrutiny.

In this paper, we study a nonsupersymmetric renormalizable $SO(10)$ GUT model with emphasis on its prediction on the neutrino mass and mixings. Specifically, we consider a nonsupersymmetric $SO(10)$ GUT model containing a **10** complex scalar field and a **126** scalar field that couple with **16** matter fields. The GUT breaking is achieved by a high-scale ($\mathcal{O}(10^{15})$ GeV) VEV of a **54** real scalar field and an intermediate scale ($\mathcal{O}(10^{14})$ GeV) VEV of the **126** field [11–15]. Our analysis proceeds as follows: First, we analyze the gauge coupling unification conditions and evaluate the intermediate-scale VEV. Next, we constrain the Yukawa couplings of the **10** and $\mathbf{126}^*$ fields at the intermediate scale from experimental data on the quark and charged lepton masses and quark flavor mixings. Then we express the neutrino Dirac Yukawa coupling and Majorana mass for the singlet neutrinos with these Yukawa couplings and the intermediate-scale VEV and calculate the neutrino mass matrix based on the Type-1 seesaw model. Finally, we fit experimental data on the neutrino mixing angles and mass differences, and make a prediction on poorly or not measured neutrino observables.

Previously, nonsupersymmetric renormalizable $SO(10)$ GUT models and their implication on neutrino physics have been studied in a number of papers [11–34]. Our work differs from them in that we consider a more general model where the **10** complex scalar field and its complex conjugate both have Yukawa couplings with **16** matter fields, as Peccei-Quinn charges [35] are not assigned to these fields. As a result, the fitting of neutrino data becomes easier and we obtain an interesting finding that the $SO(10)$ GUT model can be consistent with not only the normal hierarchy but also the inverted hierarchy of the neutrino mass.[1] Another feature of our study is that we include information on the VEV of the **126** field in the fitting of neutrino data, which allows us to determine the portions of the SM Higgs field in the **126** field [c_3, c_4 defined in Eq. (8)].

This paper is organized as follows: In Sec. II, we describe the nonsupersymmetric renormalizable $SO(10)$ GUT model we consider. In Sec. III, we analyze the gauge coupling unification conditions and evaluate the intermediate-scale VEV of the **126** scalar field. In Sec. IV, we constrain the Yukawa couplings of the **10** and **126** scalar fields at the intermediate scale from experimental data on the quark and charged lepton masses and quark flavor mixings, express the neutrino mass matrix with these Yukawa couplings and the intermediate-scale VEV, and fit neutrino oscillation data to derive a prediction on neutrino observables. In Sec. V, we inspect the validity of approximations made in Secs. III and IV. Section VI summarizes the paper.

II. NONSUPERSYMMETRIC RENORMALIZABLE $SO(10)$ GUT MODEL

The model is a nonsupersymmetric $SO(10)$ gauge theory with the following field content. Three generations of left-handed Weyl spinors in **16** representation, denoted by $\mathbf{16}^i$ with i the flavor index, a complex scalar field in **10** denoted by $\mathbf{10}_H$, a complex scalar field in **126** denoted by $\mathbf{126}_H$, and a real scalar field in **54**, denoted by $\mathbf{54}_H$. The Yukawa couplings are given by

$$-\mathcal{L}_{\text{Yukawa}} = (Y_{10})_{ij}\mathbf{16}^i\mathbf{10}_H\mathbf{16}^j + (Z_{10})_{ij}\mathbf{16}^i\mathbf{10}_H^*\mathbf{16}^j + (Y_{126})_{ij}\mathbf{16}^i\mathbf{126}_H^*\mathbf{16}^j + \text{H.c.} \tag{1}$$

The gauge symmetry breaking proceeds as follows: The component of $\mathbf{54}_H$ with charge $(\mathbf{1},\mathbf{1},\mathbf{1})$ in the $SU(4)\times SU(2)_L\times SU(2)_R$ subgroup develops a VEV and breaks $SO(10)$. We write the $SO(10)$-breaking scale as $\mu=\mu_{\text{GUT}}$. The effective theory below scale $\mu=\mu_{\text{GUT}}$ is a Pati-Salam model with $SU(4)\times SU(2)_L\times SU(2)_R$ gauge group [36]. We assume that the $(\mathbf{1},\mathbf{2},\mathbf{2})$ component of $\mathbf{10}_H$ and the $(\overline{\mathbf{10}},\mathbf{1},\mathbf{3})+(\mathbf{10},\mathbf{3},\mathbf{1})+(\mathbf{15},\mathbf{2},\mathbf{2})$ components of $\mathbf{126}_H$ have mass much smaller than μ_{GUT} and remain in the effective Pati-Salam model. The $(\overline{\mathbf{10}},\mathbf{1},\mathbf{3})$ and $(\mathbf{10},\mathbf{3},\mathbf{1})$ components have the same mass as a consequence of D-parity [37,38]. We write the fields of these components as $(\mathbf{1},\mathbf{2},\mathbf{2})_H, (\overline{\mathbf{10}},\mathbf{1},\mathbf{3})_H, (\mathbf{10},\mathbf{3},\mathbf{1})_H, (\mathbf{15},\mathbf{2},\mathbf{2})_H$, respectively, and write the fields of the $(\mathbf{4},\mathbf{2},\mathbf{1})+(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$ components of $\mathbf{16}^i$ fermions as $(\mathbf{4},\mathbf{2},\mathbf{1})^i+(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})^i$. The effective Pati-Salam model contains the following Yukawa couplings:

$$-\mathcal{L}_{\text{Yukawa}} \supset (Y_1)_{ij}(\mathbf{4},\mathbf{2},\mathbf{1})^i(\mathbf{1},\mathbf{2},\mathbf{2})_H(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})^j + (Z_1)_{ij}(\mathbf{4},\mathbf{2},\mathbf{1})^i(\mathbf{1},\mathbf{2},\mathbf{2})_H^*(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})^j + (Y_{15})_{ij}(\mathbf{4},\mathbf{2},\mathbf{1})^i(\mathbf{15},\mathbf{2},\mathbf{2})_H^*(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})^j + \frac{1}{2}(Y_N)_{ij}(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})^i(\overline{\mathbf{10}},\mathbf{1},\mathbf{3})_H^*(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})^j + \frac{1}{2}(Y_N)_{ij}(\mathbf{4},\mathbf{2},\mathbf{1})^i(\mathbf{10},\mathbf{3},\mathbf{1})_H^*(\mathbf{4},\mathbf{2},\mathbf{1})^j + \text{H.c.}, \tag{2}$$

where the Yukawa couplings satisfy the following matching conditions at scale $\mu=\mu_{\text{GUT}}$ at tree level [39]:

$$Y_1 = -2\sqrt{2}Y_{10}, \tag{3}$$

$$Z_1 = -2\sqrt{2}Z_{10}, \tag{4}$$

$$Y_{15} = 8\sqrt{2}Y_{126}, \tag{5}$$

[1] There is a recent report [34] that a model with a **10** and a **120** real scalar fields and a **126** complex scalar field can also fit the inverted neutrino mass hierarchy.

$$Y_N = 8Y_{126}. \tag{6}$$

Next, the component of the $(\overline{\mathbf{10}},\mathbf{1},\mathbf{3})_H$ field with charge $(\mathbf{1},\ \mathbf{1},\ \mathbf{0})$ in the $SU(3)_C\times SU(2)_L\times U(1)_Y$ subgroup develops a VEV, $v_{\rm PS}$, as

$$\langle(\overline{\mathbf{10}},\mathbf{1},\mathbf{3})_H\rangle = v_{\rm PS} \tag{7}$$

and breaks $SU(4)\times SU(2)_R$. We write the $SU(4)\times SU(2)_R$-breaking scale as $\mu=\mu_{\rm PS}$. The effective theory below scale $\mu=\mu_{\rm PS}$ is the SM with $SU(3)_C\times SU(2)_L\times U(1)_Y$ gauge group. The pair of $(\mathbf{1},\mathbf{2},\pm\frac{1}{2})$ components in the $(\mathbf{1},\mathbf{2},\mathbf{2})_H$ field, denoted by H_u, H_d, and the pair of $(\mathbf{1},\mathbf{2},\pm\frac{1}{2})$ components in the $(\mathbf{15},\mathbf{2},\mathbf{2})_H$ field, denoted by Φ_u, Φ_d, gain a mass matrix and the $H_u, \epsilon H_d^*, \Phi_u, \epsilon\Phi_d^*$ fields mix with each other [ϵ denotes the antisymmetric tensor in $SU(2)_L$ space]. This mass matrix is assumed to have one negative eigenvalue at the electroweak scale and three positive eigenvalues at $\mathcal{O}(v_{\rm PS}^2)$. The eigenstate belonging to the negative eigenvalue is identified with the SM Higgs field, denoted by H. We express the H component of each field as

$$\begin{aligned} H_u &= c_1 H + \cdots, \\ \epsilon H_d^* &= c_2 H + \cdots., \\ \Phi_u &= c_3 H + \cdots, \\ \epsilon\Phi_d^* &= c_4 H + \cdots., \end{aligned} \tag{8}$$

where c_1, c_2, c_3, c_4 are numbers satisfying $|c_1|^2+|c_2|^2+|c_3|^2+|c_4|^2=1$, and "$\cdots$" is an abbreviation for other mass eigenstates. The $(\mathbf{4},\mathbf{2},\mathbf{1})^i$ and $(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})^i$ fermions are decomposed into the isospin-doublet quarks, isospin-singlet up-type quarks, isospin-singlet down-type quarks, isospin-doublet leptons, isospin-singlet charged leptons and singlet neutrinos, denoted by q^i, u^{ci}, d^{ci}, ℓ^i, e^{ci}, ν^{ci}, respectively. This effective theory contains the following Yukawa couplings:

$$\begin{aligned} -\mathcal{L}_{\rm Yukawa} \supset\, & (Y_u)_{ij} q^i \epsilon H u^{cj} + (Y_d)_{ij} q^i H^* d^{cj} \\ & + (Y_e)_{ij} \ell^i H^* e^{cj} + (Y_D)_{ij} \ell^i \epsilon H \nu^{cj} + \text{H.c.}, \end{aligned} \tag{9}$$

where the Yukawa couplings satisfy the following matching conditions at scale $\mu=\mu_{\rm PS}$ at tree level:

$$Y_u = c_1 Y_1 - c_2 Z_1 - \frac{1}{2\sqrt{3}} c_4 Y_{15}, \tag{10}$$

$$Y_d = -c_2^* Y_1 + c_1^* Z_1 + \frac{1}{2\sqrt{3}} c_3^* Y_{15}, \tag{11}$$

$$Y_e = -c_2^* Y_1 + c_1^* Z_1 - \frac{\sqrt{3}}{2} c_3^* Y_{15}, \tag{12}$$

$$Y_D = c_1 Y_1 - c_2 Z_1 + \frac{\sqrt{3}}{2} c_4 Y_{15}. \tag{13}$$

The singlet neutrinos gain Majorana mass term below,

$$-\mathcal{L}_{\rm Yukawa} \supset \frac{1}{2\sqrt{2}} v_{\rm PS} (Y_N)_{ij} \nu^{ci}\nu^{cj} + \text{H.c.}, \tag{14}$$

and are integrated out at scale $\mu=\mu_{\rm PS}$. Then the Weinberg operator is derived as

$$-\mathcal{L}_{\rm eff} = \frac{1}{2}(C_\nu)_{ij} \ell^i \epsilon H \ell^j \epsilon H + \text{H.c.}, \tag{15}$$

where C_ν satisfies at scale $\mu=\mu_{\rm PS}$ at tree level,

$$C_\nu = -\frac{\sqrt{2}}{v_{\rm PS}} Y_D Y_N^{-1} Y_D^T. \tag{16}$$

The Yukawa coupling $\frac{1}{2}(Y_N)_{ij}(\mathbf{4},\mathbf{2},\mathbf{1})^i(\mathbf{10},\mathbf{3},\mathbf{1})_H^*(\mathbf{4},\mathbf{2},\mathbf{1})^j$ and the quartic couplings involving $(\overline{\mathbf{10}},\mathbf{1},\mathbf{3})_H$, $(\mathbf{10},\mathbf{3},\mathbf{1})_H$ and $(\mathbf{1},\mathbf{2},\mathbf{2})_H$ or $(\mathbf{15},\mathbf{2},\mathbf{2})_H$ in the effective Pati-Salam model also generate the Weinberg operator, after $(\overline{\mathbf{10}},\mathbf{1},\mathbf{3})_H$ develops VEV $v_{\rm PS}$ and after $(\mathbf{10},\mathbf{3},\mathbf{1})_H$ field is integrated out. Namely, there also are Type-2 seesaw contributions to the tiny neutrino mass. In the current study, we assume that such quartic couplings are sufficiently small that the Type-2 seesaw contributions are negligible compared to the Type-1 seesaw ones, for the sake of predictive power of the model.

We comment that in the present model, the $H_u, \epsilon H_d^*$ fields and the $\Phi_u, \epsilon\Phi_d^*$ fields are allowed to have mixing terms because quartic terms in the $SO(10)$ gauge theory below,

$$-\mathcal{L} \supset \lambda \mathbf{126}_H^* \mathbf{126}_H^2 \mathbf{10}_H + \lambda' \mathbf{126}_H \mathbf{126}_H^{*2} \mathbf{10}_H + \text{H.c.}, \tag{17}$$

give rise to quartic terms in the effective Pati-Salam model below,

$$\begin{aligned} & \tilde{\lambda}(\overline{\mathbf{10}},\mathbf{1},\mathbf{3})_H^*(\overline{\mathbf{10}},\mathbf{1},\mathbf{3})_H(\mathbf{15},\mathbf{2},\mathbf{2})_H(\mathbf{1},\mathbf{2},\mathbf{2})_H \\ & \quad + \tilde{\lambda}'(\overline{\mathbf{10}},\mathbf{1},\mathbf{3})_H^*(\overline{\mathbf{10}},\mathbf{1},\mathbf{3})_H(\mathbf{15},\mathbf{2},\mathbf{2})_H^*(\mathbf{1},\mathbf{2},\mathbf{2})_H \\ & \quad + \text{H.c.} \end{aligned} \tag{18}$$

When the $(\mathbf{1},\ \mathbf{1},\ 0)$ component of the $(\overline{\mathbf{10}},\mathbf{1},\mathbf{3})_H$ field develops the VEV $v_{\rm PS}$, Eq. (18) yields mixing terms for the $H_u, \epsilon H_d^*$ fields and the $\Phi_u, \epsilon\Phi_d^*$ fields. Remarkably, these mixing terms are $\mathcal{O}(v_{\rm PS}^2)$ and of the same order as the mass terms of the $(\mathbf{1},\mathbf{2},\mathbf{2})_H$ and $(\mathbf{15},\mathbf{2},\mathbf{2})_H$ fields. As a result, large mixings of $H_u, \epsilon H_d^*$ and $\Phi_u, \epsilon\Phi_d^*$ can be realized.

We comment on the number of parameters of the current $SO(10)$ GUT model. The parameters relevant to the current study are as follows: The three Yukawa coupling matrices

Y_{10}, Z_{10}, Y_{126} in Eq. (1). The mass terms for the $\mathbf{10}_H, \mathbf{126}_H$ fields, $m_{10}^2, \mu^2, m_{126}^2$, defined as

$$-\mathcal{L} \supset m_{10}^2 \mathbf{10}_H^* \mathbf{10}_H + \mu^2 \mathbf{10}_H^2 + \mu^{*2} \mathbf{10}_H^{*2} + m_{126}^2 \mathbf{126}_H^* \mathbf{126}_H. \tag{19}$$

The coupling constants for the $\mathbf{10}_H, \mathbf{126}_H$ fields λ, λ' in Eq. (17). The VEV of the (**1**, **1**, 0) component of the $(\overline{\mathbf{10}}, \mathbf{1}, \mathbf{3})_H$ field in the $\mathbf{126}_H$ field, $v_{\rm PS}$. The number of real degrees of freedom of the above parameters is counted as follows: By fixing the phase of the $\mathbf{10}_H$ field such that μ^2 is real and that of the $\mathbf{126}_H$ field such that $v_{\rm PS}$ is real, $m_{10}^2, \mu^2, m_{126}^2, \lambda, \lambda', v_{\rm PS}$ respectively have 1, 1, 1, 2, 2, 1 real degrees of freedom. By further fixing the flavor basis of the $\mathbf{16}^i$ fields such that Y_{10} is diagonal and real, Y_{10} has 3 real degrees of freedom, and Z_{10}, Y_{126} each has 12 real degrees of freedom. In total, there are 35 real degrees of freedom for the parameters relevant to the current study.

Prior to the fitting analysis of Sec. IV, v_{PS} is fixed by the gauge coupling unification conditions. Also, in the fitting analysis, only specific combinations of $m_{10}^2, \mu^2, m_{126}^2, \lambda, \lambda'$ are used for the fitting. These combinations are defined as follows. Parameters $m_{10}^2, \mu^2, m_{126}^2, \lambda, \lambda'$ determine the mass matrix of H_u, H_d, Φ_u, Φ_d. After diagonalizing this mass matrix, we find, by assumption, one negative eigenvalue. The eigenstate belonging to this eigenvalue H is contained in H_u, H_d, Φ_u, Φ_d as Eq. (8), and among the coefficients in Eq. (8), only $|c_3|$ and c_4/c_3^* are used for the fitting. In total, the three Yukawa couplings Y_{10}, Z_{10}, Y_{126} and $|c_3|, c_4/c_3^*$ are used in the fitting analysis of Sec. IV. The real degrees of freedom of $Y_{10}, Z_{10}, Y_{126}, |c_3|, c_4/c_3^*$ are respectively 3, 12, 12, 1, 2, which sum to 30. Hence, we use 30 real degrees of freedom to fit the six quark masses, three charged lepton masses, three Cabibbo-Kobayashi-Maskawa (CKM) mixing angles, one Kobayashi-Maskawa phase, three neutrino mixing angles, and two neutrino mass-squared differences. It should be noted that although the number of free parameters is larger than the number of observables to be fit, the fitting is nontrivial because the observables depend on the free parameters nonlinearly, in a way involving singular value decompositions.

(The quartic couplings involving four $\mathbf{126}_H$ fields or two $\mathbf{10}_H$ and two $\mathbf{126}_H$ fields contribute to the mass matrix of H_u, H_d, Φ_u, Φ_d after the $\mathbf{126}_H$ field develops VEV $v_{\rm PS}$. However, these contributions can be absorbed by a redefinition of $m_{10}^2, \mu^2, m_{126}^2$, and so we do not regard these quartic couplings as independent free parameters.)

We comment on the strong CP problem and dark matter in the present model. Unlike previous models of non-supersymmetric $SO(10)$ GUT, we do not assign $U(1)$ Peccei-Quinn charges [35] to the $\mathbf{16}^i$ matter fields and the $\mathbf{10}_H, \mathbf{126}_H$ scalar fields. Nevertheless, we can implement the Peccei-Quinn mechanism by introducing new **16** and $\overline{\mathbf{16}}$ Weyl fermions and **1** complex scalar and assigning $U(1)$ Peccei-Quinn charge $+1$ to $\mathbf{16}, \overline{\mathbf{16}}$ and -2 to **1**. Then a Kim-Shifman-Vainshtein-Zakharov axion [40,41] emerges and can solve the strong CP problem. It can also be a dark matter candidate.

III. GAUGE COUPLING UNIFICATION

In this section and the next section, we adopt the following experimental values of the gauge coupling constants, quark and charged lepton masses and quark flavor mixings. The QCD and QED gauge coupling constants in 5-quark-flavor QCD $\times$ QED theory are fixed as $\alpha_s^{(5)}(M_Z) = 0.1181$ and $\alpha^{(5)}(M_Z) = 1/127.95$. The lepton pole masses and W, Z, Higgs boson pole masses are taken from Particle Data Group [42]. We use the results of lattice calculations of the individual up and down quark masses, the strange quark mass, the charm quark mass, and the bottom quark mass in $\overline{\rm MS}$ scheme reviewed in Ref. [43], which read $m_u(2\ {\rm GeV}) = 2.14(8)$ MeV, $m_d(2\ {\rm GeV}) = 4.70(5)$ MeV [44,45], $m_s(2\ {\rm GeV}) = 93.40(57)$ MeV [44,46–48], $m_c(3\ {\rm GeV}) = 0.988(11)$ GeV [44,46,48–50], $m_b(m_b) = 4.203(11)$ GeV [44,48,51–54]. We use the top quark pole mass measured by CMS in Ref. [55], which reads $M_t = 170.5(8)$ GeV. We calculate the CKM mixing angles and CP phase from the Wolfenstein parameters in Ref. [56]. The above data are translated into the values of the quark and lepton Yukawa coupling matrices and the gauge coupling constants at scale $\mu = M_Z$ in $\overline{\rm MS}$ scheme with the help of the code [57] based on Refs. [58–62].

We analyze the gauge coupling unification conditions and evaluate the $SU(4) \times SU(2)_R$-breaking VEV $v_{\rm PS}$. To this end, we solve the two-loop renormalization group (RG) equations [63–65] of SM, and match the theory with the $SU(4) \times SU(2)_L \times SU(2)_R$ gauge theory containing three generations of Weyl fermions $(\mathbf{4}, \mathbf{2}, \mathbf{1})^i, (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})^i$ and complex scalars $(\mathbf{1}, \mathbf{2}, \mathbf{2})_H, (\overline{\mathbf{10}}, \mathbf{1}, \mathbf{3})_H, (\mathbf{10}, \mathbf{3}, \mathbf{1})_H, (\mathbf{15}, \mathbf{2}, \mathbf{2})_H$ at scale $\mu = \mu_{\rm PS}$. Then we calculate the two-loop RG equations of the $SU(4) \times SU(2)_L \times SU(2)_R$ gauge theory, and match the theory with the $SO(10)$ gauge theory at scale $\mu = \mu_{\rm GUT}$. From the above matching conditions, we evaluate $v_{\rm PS}$. Additionally, we evaluate the mass of the $SO(10)$ gauge boson that gains mass along the breaking of $SO(10)$ to $SU(4) \times SU(2)_L \times SU(2)_R$.

We make two approximations. First, we approximate that the scalar particles decoupled at scale $\mu = \mu_{\rm PS}$ have a common mass $M_{H_{\rm PS}}$. This has little impact on the evaluation of $v_{\rm PS}$ because the power of $M_{H_{\rm PS}}$ in the equation determining $v_{\rm PS}$ Eq. (27) is relatively small. Second, when solving the two-loop RG equations of the gauge couplings of the $SU(4) \times SU(2)_L \times SU(2)_R$ gauge theory, we omit two-loop contributions involving the Yukawa couplings. Later in Sec. V we will check that this approximation does not affect the result.

Given the approximation on the scalar particle masses, the matching conditions around scale $\mu \sim \mu_{\rm PS}$ in $\overline{\rm MS}$ scheme are given by

$$\frac{1}{g_{2R}^2(\mu)} - \frac{2}{48\pi^2} = \frac{5}{3}\left(\frac{1}{g_1^2(\mu)} - \frac{1}{8\pi^2}\frac{28}{5}\ln\frac{M_{G(\mathbf{3},\mathbf{1},\frac{2}{3})}}{\mu} - \frac{1}{8\pi^2}\frac{21}{5}\ln\frac{M_{G(\mathbf{1},\mathbf{1},1)}}{\mu} + \frac{1}{8\pi^2}\frac{67}{6}\ln\frac{M_{H_{\rm PS}}}{\mu}\right)$$
$$- \frac{2}{3}\left(\frac{1}{g_3^2(\mu)} - \frac{1}{8\pi^2}\frac{7}{2}\ln\frac{M_{G(\mathbf{3},\mathbf{1},\frac{2}{3})}}{\mu} + \frac{1}{8\pi^2}\frac{67}{6}\ln\frac{M_{H_{\rm PS}}}{\mu} - \frac{3}{48\pi^2}\right), \tag{20}$$

$$\frac{1}{g_{2L}^2(\mu)} - \frac{2}{48\pi^2} = \frac{1}{g_2^2(\mu)} + \frac{1}{8\pi^2}\frac{71}{6}\ln\frac{M_{H_{\rm PS}}}{\mu} - \frac{2}{48\pi^2}, \tag{21}$$

$$\frac{1}{g_4^2(\mu)} - \frac{4}{48\pi^2} = \frac{1}{g_3^2(\mu)} - \frac{1}{8\pi^2}\frac{7}{2}\ln\frac{M_{G(\mathbf{3},\mathbf{1},\frac{2}{3})}}{\mu} + \frac{1}{8\pi^2}\frac{67}{6}\ln\frac{M_{H_{\rm PS}}}{\mu} - \frac{3}{48\pi^2}, \tag{22}$$

and those around scale $\mu \sim \mu_{\rm GUT}$ in $\overline{\rm MS}$ scheme are given by

$$\frac{1}{g_{10}^2(\mu)} - \frac{8}{48\pi^2} = \frac{1}{g_{2R}^2(\mu)} - \frac{1}{8\pi^2}21\ln\frac{M_{G(\mathbf{6},\mathbf{2},\mathbf{2})}}{\mu} + \frac{1}{8\pi^2}\ln\frac{M_{H(\mathbf{1},\mathbf{3},\mathbf{3})}}{\mu} - \frac{2}{48\pi^2}, \tag{23}$$

$$\frac{1}{g_{10}^2(\mu)} - \frac{8}{48\pi^2} = \frac{1}{g_{2L}^2(\mu)} - \frac{1}{8\pi^2}21\ln\frac{M_{G(\mathbf{6},\mathbf{2},\mathbf{2})}}{\mu} + \frac{1}{8\pi^2}\ln\frac{M_{H(\mathbf{1},\mathbf{3},\mathbf{3})}}{\mu} - \frac{2}{48\pi^2}, \tag{24}$$

$$\frac{1}{g_{10}^2(\mu)} - \frac{8}{48\pi^2} = \frac{1}{g_4^2(\mu)} - \frac{1}{8\pi^2}14\ln\frac{M_{G(\mathbf{6},\mathbf{2},\mathbf{2})}}{\mu} + \frac{1}{8\pi^2}\frac{1}{3}\ln\frac{M_{H(\mathbf{6},\mathbf{1},\mathbf{1})}^2 M_{H(\mathbf{20}',\mathbf{1},\mathbf{1})}^4}{\mu^6} - \frac{4}{48\pi^2}, \tag{25}$$

where g_3, g_2, g_1 denote the gauge couplings of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory (g_1 is in the GUT normalization), g_4, g_{2L}, g_{2R} denote those of the $SU(4) \times SU(2)_L \times SU(2)_R$ gauge theory, and g_{10} denotes that of the $SO(10)$ gauge theory. $M_{G(\mathbf{3},\mathbf{1},\frac{2}{3})}$, $M_{G(\mathbf{1},\mathbf{1},1)}$ denote the masses of the gauge bosons that become massive along the $SU(4) \times SU(2)_R$ breaking (subscripts display the charges in $SU(3)_C \times SU(2)_L \times U(1)_Y$), and $M_{G(\mathbf{6},\mathbf{2},\mathbf{2})}$ denotes the mass of the gauge boson that becomes massive along the $SO(10)$ breaking into $SU(4) \times SU(2)_L \times SU(2)_R$. $M_{H_{\rm PS}}$ denotes the common mass of the scalar particles decoupled at scale $\mu = \mu_{\rm PS}$, and $M_{H(\mathbf{6},\mathbf{1},\mathbf{1})}$, $M_{H(\mathbf{20}',\mathbf{1},\mathbf{1})}$, $M_{H(\mathbf{1},\mathbf{3},\mathbf{3})}$ denote the masses of the scalar particles decoupled at scale $\mu = \mu_{\rm GUT}$ [subscripts display the charges in $SU(4) \times SU(2)_L \times SU(2)_R$]. There are two complex scalar particles with the same charge $(\mathbf{6}, \mathbf{1}, \mathbf{1})$, and $M_{H(\mathbf{6},\mathbf{1},\mathbf{1})}$ should be regarded as the geometric mean of their masses. The scalar particles with charge $(\mathbf{20}', \mathbf{1}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{3}, \mathbf{3})$ are real.

From Eqs. (23) and (24) and the particle content of the $SU(4) \times SU(2)_L \times SU(2)_R$ gauge theory, we see that $g_{2R} = g_{2L}$ holds at any scale, even if the Yukawa couplings are not neglected when solving the RG equations of the gauge couplings of the $SU(4) \times SU(2)_L \times SU(2)_R$ gauge theory. This is in accord with the fact that D-parity is unbroken when the VEV of $\mathbf{54}_H$ breaks $SO(10)$. From Eqs. (20) and (21), and the fact that $g_{2R} = g_{2L}$, we obtain the one-loop relation

$$\frac{M_{G(\mathbf{3},\mathbf{1},\frac{2}{3})}^{21} M_{G(\mathbf{1},\mathbf{1},1)}^{21}}{\mu^{42}} \frac{M_{H_{\rm PS}}^2}{\mu^2} = \exp\left[8\pi^2\left(\frac{5}{g_1^2(\mu)} - \frac{3}{g_2^2(\mu)} - \frac{2}{g_3^2(\mu)}\right) + 2\right]. \tag{26}$$

We solve the two-loop RG equations of SM and insert the result into Eq. (26), thereby obtaining

$$M_{G(\mathbf{3},\mathbf{1},\frac{2}{3})}^{21} M_{G(\mathbf{1},\mathbf{1},1)}^{21} M_{H_{\rm PS}}^2 = e^2(10^{13.70}\ \text{GeV})^{44}. \tag{27}$$

From the above relation, we evaluate $v_{\rm PS}$. We note $M_{G(\mathbf{3},\mathbf{1},\frac{2}{3})}^2 = g_4^2 v_{\rm PS}^2$, $M_{G(\mathbf{1},\mathbf{1},1)}^2 = g_{2R}^2 v_{\rm PS}^2$. The value of $M_{H_{\rm PS}}$ has little impact on the evaluation of $v_{\rm PS}$ because of its comparably small power of 2. If $M_{H_{\rm PS}}$ lies in a natural range of $0.3 v_{\rm PS} > M_{H_{\rm PS}} > 0.03 v_{\rm PS}$, we get

$$v_{\rm PS} = 10^{14.0}\ \text{GeV}. \tag{28}$$

From Eqs. (24) and (25), and the fact that $g_{2R} = g_{2L}$, we obtain the one-loop relation

$$\frac{\mu}{M_{H(\mathbf{1},\mathbf{3},\mathbf{3})}}\frac{M^{2/3}_{H(\mathbf{6},\mathbf{1},\mathbf{1})}M^{4/3}_{H(\mathbf{20}',\mathbf{1},\mathbf{1})}}{\mu^2}\frac{M^7_{G(\mathbf{6},\mathbf{2},\mathbf{2})}}{\mu^7} = \exp\left[8\pi^2\left(\frac{1}{g^2_{2L}(\mu)}-\frac{1}{g^2_4(\mu)}\right)+\frac{1}{3}\right]. \tag{29}$$

We solve the two-loop RG equations of the $SU(4)\times SU(2)_L\times SU(2)_R$ gauge theory and insert the result into Eq. (29), thereby obtaining

$$\frac{M^{2/3}_{H(\mathbf{6},\mathbf{1},\mathbf{1})}M^{4/3}_{H(\mathbf{20}',\mathbf{1},\mathbf{1})}}{M_{H(\mathbf{1},\mathbf{3},\mathbf{3})}}M^7_{G(\mathbf{6},\mathbf{2},\mathbf{2})} = e^{1/3}(10^{15.04}\ \mathrm{GeV})^8. \tag{30}$$

If we assume a mild hierarchy among the scalar particle masses as

$$M_{H(\mathbf{6},\mathbf{1},\mathbf{1})} = M_{H(\mathbf{20}',\mathbf{1},\mathbf{1})} \simeq 10^{13.5}\ \mathrm{GeV}, \tag{31}$$

$$M_{H(\mathbf{1},\mathbf{3},\mathbf{3})} \simeq 10^{16.5}\ \mathrm{GeV}, \tag{32}$$

then we get $M_{G(\mathbf{6},\mathbf{2},\mathbf{2})}\simeq 6\times 10^{15}$ GeV and the current bound on the $p\to e^+\pi^0$ partial lifetime as well as those of other nucleon decay modes are satisfied.

IV. FITTING OF NEUTRINO DATA

We constrain the Yukawa couplings of the $SU(4)\times SU(2)_L\times SU(2)_R$ gauge theory Y_1, Z_1, Y_{15}, Y_N at scale $\mu=\mu_{\mathrm{PS}}$, from experimental data on the quark and charged lepton masses and quark flavor mixings. Then we express the neutrino mass matrix with Y_1, Z_1, Y_{15}, Y_N, and v_{PS} in Eq. (28) based on the Type-1 seesaw mechanism. Finally, we fit experimental data on the neutrino mixing angles and mass differences with Y_1, Z_1, Y_{15}, Y_N under the above constraints.

First, we calculate the up-type quark, down-type quark and charged lepton Yukawa couplings in SM at scale $\mu=\mu_{\mathrm{PS}}$ by solving the SM two-loop RG equations. We take $\mu_{\mathrm{PS}}=10^{13.7}$ GeV, in accordance with Eq. (27). The result is presented in Table I in the form of the singular values of the Yukawa coupling matrices and the parameters of the CKM matrix at scale $\mu=\mu_{\mathrm{PS}}$.

Due to D-parity, Y_1, Z_1, Y_{15} at scale $\mu=\mu_{\mathrm{PS}}$ have symmetric flavor indices. Therefore, in the flavor basis where the isospin-doublet down-type quarks have a diagonal Yukawa coupling, the up-type quark, down-type quark and charged lepton Yukawa coupling matrices Y_u, Y_d, Y_e at scale $\mu=\mu_{\mathrm{PS}}$ can be written as

$$Y_u = V^T_{\mathrm{CKM}}\begin{pmatrix} y_u & 0 & 0\\ 0 & y_c e^{2id_2} & 0\\ 0 & 0 & y_t e^{2id_3}\end{pmatrix}V_{\mathrm{CKM}}, \tag{33}$$

TABLE I. The singular values of the Yukawa coupling matrices and the CKM mixing angles and CP phase in SM at scale $\mu=\mu_{\mathrm{PS}}=10^{13.7}$ GeV. Also shown are the errors of the quark Yukawa couplings, propagated from the experimental errors of the corresponding masses, and maximal errors of the CKM parameters, obtained by assuming maximal correlation of experimental errors of the Wolfenstein parameters.

	Value
y_u	$2.98(11)\times 10^{-6}$
y_c	0.001519(17)
y_t	0.4458(42)
y_d	$6.729(72)\times 10^{-6}$
y_s	0.00013369(82)
y_b	0.006402(20)
y_e	2.732×10^{-6}
y_μ	0.0005767
y_τ	0.009803
$\cos\theta^{\mathrm{ckm}}_{13}\sin\theta^{\mathrm{ckm}}_{12}$	0.22503(24)
$\cos\theta^{\mathrm{ckm}}_{13}\sin\theta^{\mathrm{ckm}}_{23}$	0.04576(77)
$\sin\theta^{\mathrm{ckm}}_{13}$	0.00403(22)
δ_{km} (rad)	1.148(33)

$$Y_d = \begin{pmatrix} y_d & 0 & 0\\ 0 & y_s & 0\\ 0 & 0 & y_b\end{pmatrix}, \tag{34}$$

$$Y_e = U^T_e\begin{pmatrix} y_e & 0 & 0\\ 0 & y_\mu & 0\\ 0 & 0 & y_\tau\end{pmatrix}U_e, \tag{35}$$

where $y_u, y_c, y_t, y_d, y_s, y_b, y_e, y_\mu, y_\tau$ are given in Table I, V_{CKM} is the CKM matrix whose parameters are given in Table I, d_2, d_3 are unknown phases and U_e is an unknown unitary matrix. d_2, d_3, U_e are not constrained experimentally. Using Eqs. (10)–(12), we can write Y_1, Z_1, Y_{15} at scale $\mu=\mu_{\mathrm{PS}}$ as

$$Y_1 = \frac{1}{4c^*_3(|c_1|^2-|c_2|^2)}\{4c^*_1c^*_3Y_u + (c^*_1c_4+3c_2c^*_3)Y_d + (-c^*_1c_4+c_2c^*_3)Y_e\}, \tag{36}$$

$$Z_1 = \frac{1}{4c^*_3(|c_1|^2-|c_2|^2)}\{4c^*_2c^*_3Y_u + (c^*_2c_4+3c_1c^*_3)Y_d + (-c^*_2c_4+c_1c^*_3)Y_e\}, \tag{37}$$

$$Y_{15} = \frac{\sqrt{3}}{2}\frac{1}{c^*_3}(Y_d-Y_e) \tag{38}$$

with Y_u, Y_d, Y_e given by Eqs. (33)–(35). Y_N is related to Y_d-Y_e in the following way: Eqs. (5) and (6) give that

$Y_N = Y_{15}/\sqrt{2}$ at scale $\mu = \mu_{\rm GUT}$. Then Y_N and Y_{15} evolve from $\mu = \mu_{\rm GUT}$ to lower-energy scales through different RG equations in the $SU(4) \times SU(2)_L \times SU(2)_R$ gauge theory. At scale $\mu = \mu_{\rm PS}$, Y_{15} is proportional to $Y_d - Y_e$ as Eq. (38). Hence, to relate Y_N to $Y_d - Y_e$, we have to solve the RG equations of Y_N and Y_{15} in the $SU(4) \times SU(2)_L \times SU(2)_R$ gauge theory. Unfortunately, this is not possible because the RG equations depend on the Yukawa couplings Y_1, Z_1, Y_{15} that are undetermined before the fitting analysis is finished. Therefore, we approximate $Y_N = Y_{15}/\sqrt{2}$ at scale $\mu = \mu_{\rm PS}$. Later in Sec. V we will assess the impact of this approximation after Y_1, Z_1, Y_{15} are determined. Given the above approximation, we can express the coefficient of the Weinberg operator at scale $\mu = \mu_{\rm PS}$ using Eqs. (36)–(38) and Eqs. (13), (16) as

$$C_\nu = \frac{2\sqrt{2}}{\sqrt{3}} c_3^* \frac{\sqrt{2}}{v_{\rm PS}} \left(Y_u + \frac{c_4}{c_3^*}(Y_d - Y_e) \right) (Y_d - Y_e)^{-1} \times \left(Y_u + \frac{c_4}{c_3^*}(Y_d - Y_e) \right) \quad \text{at } \mu = \mu_{\rm PS}. \tag{39}$$

The fitting analysis is performed as follows. First, we evaluate Eq. (39) with the central values in Table I and the estimate of $v_{\rm PS} = 10^{14.0}$ GeV in Eq. (28). At this stage, phases d_2, d_3, unitary matrix U_e and complex numbers c_3, c_4 are free parameters except that the latter two satisfy

$$|c_3|^2 + |c_4|^2 < 1. \tag{40}$$

Next, we solve the one-loop RG equation for the Wilson coefficient of the Weinberg operator C_ν from scale $\mu = \mu_{\rm PS}$ to $\mu = M_Z$, and evaluate the neutrino mass matrix as

$$M_\nu = \frac{v^2}{2} C_\nu(M_Z) \tag{41}$$

with $v = 246$ GeV. From M_ν above, we derive the three neutrino mixing angles and the two neutrino mass differences. Finally, we fit the neutrino oscillation data in NuFIT5.1 (with Super-Kamiokande atmospheric data) [66,67] with the free parameters $d_2, d_3, U_e, |c_3|, c_4/c_3^*$. We consider both the normal hierarchy and the inverted hierarchy of the neutrino mass. We perform the fitting repeatedly and collect multiple fitting results in which two mixing angles $\sin^2\theta_{12}, \sin^2\theta_{13}$ and the ratio of the neutrino mass differences $\Delta m^2_{21}/|\Delta m^2_{31}|$ are within the 2σ ranges and mixing angle $\sin^2\theta_{23}$ is within the 3σ range of the NuFIT5.1 data.

We plot the fitting results on the plane of $|c_3|$ versus $|c_4|$ in Fig. 1. Recall that c_3, c_4 quantify the portions of $(\mathbf{1}, \mathbf{2}, \pm\frac{1}{2})$ components of $(\mathbf{15}, \mathbf{2}, \mathbf{2})_H$ field in the SM Higgs field as defined in Eq. (8). The left panel is for the case of the normal neutrino mass hierarchy and the right panel is for the case of the inverted hierarchy.

We see from Fig. 1 that $|c_3|$ is $\mathcal{O}(0.01)$ in both normal and inverted hierarchy cases. $|c_4|$ is $\mathcal{O}(10)$ times larger than $|c_3|$ in the normal hierarchy case, while it is on the same order or smaller than $|c_3|$ in the inverted hierarchy case. This implies that from the point of view of naturalness of the mass matrix of $(\mathbf{1}, \mathbf{2}, \pm\frac{1}{2})$ components of $(\mathbf{15}, \mathbf{2}, \mathbf{2})_H$ field, the inverted hierarchy is favored because $|c_3|$ and $|c_4|$ can be on the same order. Also, Fig. 1 and Eq. (39) indicate that the normal hierarchy is realized when the neutrino Dirac Yukawa coupling is dominated by the term proportional to $Y_d - Y_e$, whereas the inverted hierarchy is realized

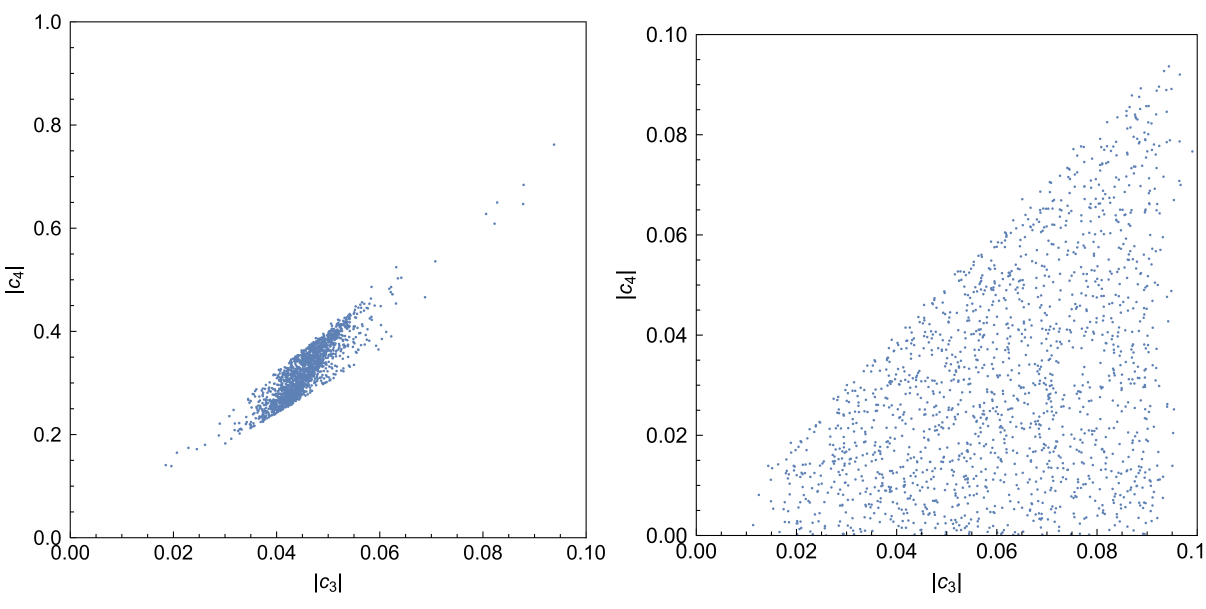

FIG. 1. Fitting results on the plane of $|c_3|$ versus $|c_4|$, where c_3, c_4 quantify the portions of $(\mathbf{1}, \mathbf{2}, \pm\frac{1}{2})$ components of $(\mathbf{15}, \mathbf{2}, \mathbf{2})_H$ field in the SM Higgs field as defined in Eq. (8). The left panel is for the case of the normal neutrino mass hierarchy and the right panel is for the case of the inverted hierarchy.

when the term proportional to Y_u is dominant or comparable to that proportional to $Y_d - Y_e$.

We examine the prediction of the model on neutrino observables, by plotting the fitting results on the planes of neutrino mixing angle $\sin^2\theta_{23}$ versus the Dirac CP phase of the neutrino mixing matrix δ_{CP}, the effective neutrino mass for neutrinoless double beta decay $|m_{ee}|$, and the neutrino mass sum $\sum_{i=1}^{3} m_i$ in Fig. 2. The left-side panels are for the case of the normal neutrino mass hierarchy and the right-side panels are for the case of the inverted hierarchy. The reason that we focus on $\sin^2\theta_{23}$ is that it still has large uncertainty and further improvement of its measurement is anticipated.

An interesting finding in Fig. 2 is that in the normal hierarchy case, the Dirac CP phase δ_{CP} is mostly in the ranges of $-2.4 < \delta_{CP} < -1.2$ and $1.2 < \delta_{CP} < 2.4$, and it is unlikely that $\delta_{CP} \sim \pi$. This is in clear contrast with the inverted hierarchy case, where δ_{CP} is equally distributed in the whole range. Also, in the normal hierarchy case, mixing angle θ_{23} is likely in the upper octant and in a narrow range of $0.50 \lesssim \sin^2\theta_{23} \lesssim 0.55$. The predictions on the effective neutrino mass for neutrinoless double beta decay $|m_{ee}|$ and

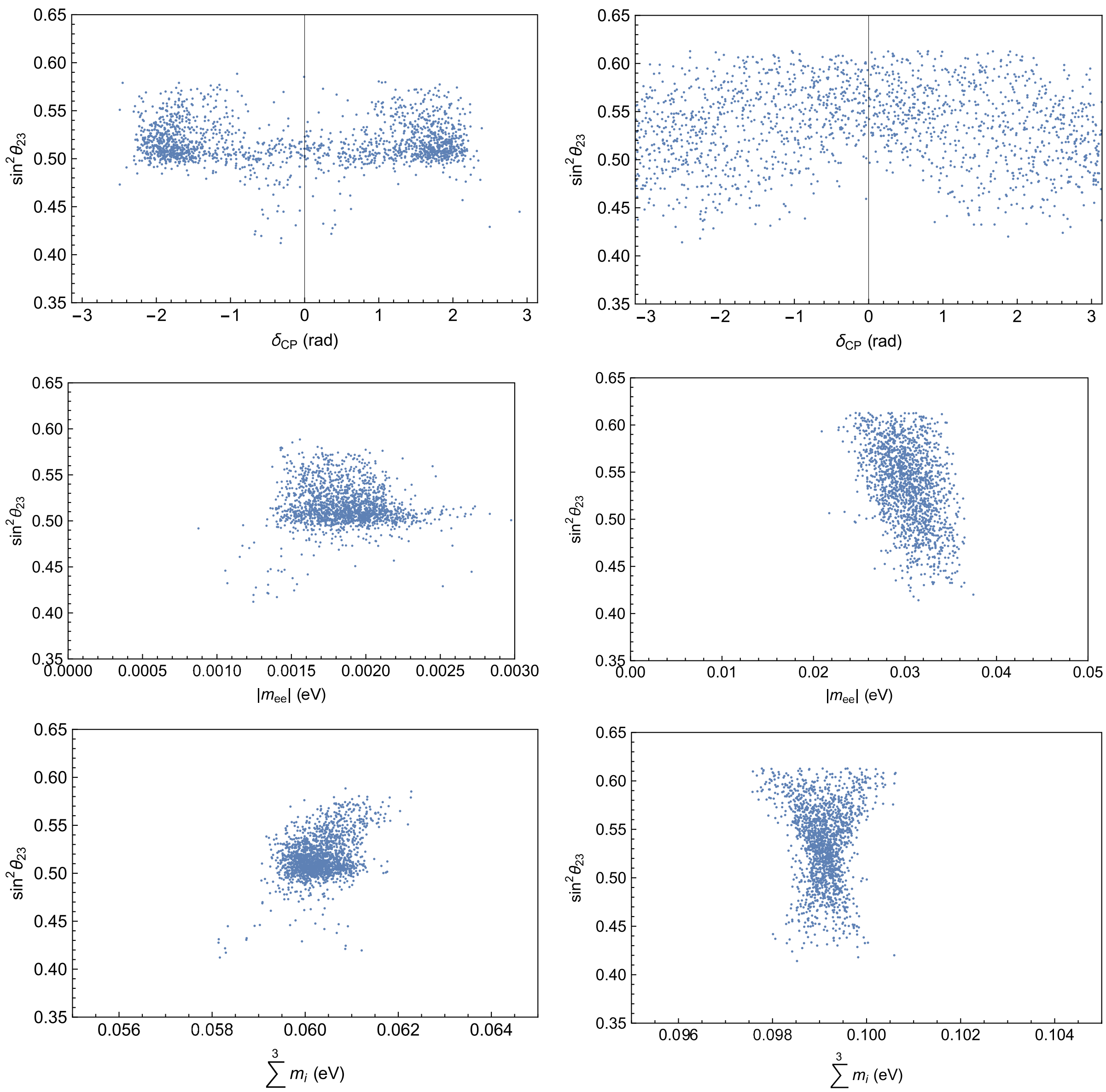

FIG. 2. Panels in the first row: Fitting results on the plane of neutrino mixing angle $\sin^2\theta_{23}$ versus the Dirac CP phase of the neutrino mixing matrix δ_{CP}. Panels in the second row: Those on the plane of $\sin^2\theta_{23}$ versus the effective neutrino mass for neutrinoless double beta decay $|m_{ee}|$. Panels in the third row: Those on the plane of $\sin^2\theta_{23}$ versus the neutrino mass sum $\sum_{i=1}^{3} m_i$. The left-side panels are for the case of the normal neutrino mass hierarchy and the right-side panels are for the case of the inverted hierarchy.

the neutrino mass sum $\sum_{i=1}^{3} m_i$ are less interesting, since these predictions simply correspond to the situation with small lightest neutrino mass ($m_1 \ll \Delta m_{21}^2$ in the normal hierarchy case and $m_3 \ll \Delta m_{21}^2$ in the inverted hierarchy case).

We further study the correlations between the predictions, by plotting the fitting results on the planes of δ_{CP} versus $|m_{ee}|$; δ_{CP} versus $\sum_{i=1}^{3} m_i$; $|m_{ee}|$ versus $\sum_{i=1}^{3} m_i$ in Fig. 3 in the Appendix. Unfortunately, we do not observe clear correlations among the predictions on δ_{CP}, $|m_{ee}|$ and $\sum_{i=1}^{3} m_i$.

We comment that if $\mathbf{10}_H$ field has only one coupling to $\mathbf{16}^i$ matter fields, i.e., if $Z_{10} = 0$, it is not possible to write Y_1 with Y_u, Y_d, Y_e like Eq. (36) and thus the fitting becomes more difficult. In fact, we have numerically found that the fitting is impossible if $Z_{10} = 0$ and if the tiny neutrino mass is generated solely from the Type-1 seesaw mechanism. The fitting analysis with $Z_{10} = 0$ and in the presence of Type-2 seesaw contributions is left for future work.

V. VALIDITY OF THE APPROXIMATIONS

We have made two approximations: First, in Sec. III, we have neglected the contribution of the Yukawa couplings to the two-loop RG equations of the gauge couplings in the $SU(4) \times SU(2)_L \times SU(2)_R$ gauge theory. Second, in Sec. IV, we have neglected the RG evolutions of Yukawa couplings Y_{15}, Y_N in the $SU(4) \times SU(2)_L \times SU(2)_R$ gauge theory from scale $\mu = \mu_{\rm GUT}$ to $\mu = \mu_{\rm PS}$.

Now we inspect the validity of the above approximations. For this purpose, we solve the full RG equations of the $SU(4) \times SU(2)_L \times SU(2)_R$ gauge theory including the Yukawa couplings Y_1, Z_1, Y_{15}, Y_N. Here the initial conditions of Y_1, Z_1, Y_{15} at scale $\mu = \mu_{\rm PS}$ are given by Eqs. (36)–(38) and Eqs. (33)–(35) with the fitting results of Sec. IV inserted into d_2, d_3, U_e, c_3, c_4. Parameters c_1, c_2 have not been determined in the analysis of Sec. IV and so we take $c_1 = \sqrt{1 - |c_3|^2 - |c_4|^2}$ and $c_2 = 0$ to minimize the magnitudes of Y_1, Z_1. We approximate $Y_N = Y_{15}/\sqrt{2}$ at scale $\mu = \mu_{\rm PS}$ (which should hold exactly at scale $\mu = \mu_{\rm GUT}$, not at $\mu = \mu_{\rm PS}$) and study the relation between Y_N and $Y_{15}/\sqrt{2}$ at scale $\mu = \mu_{\rm GUT}$ under the above approximation.

By solving the full RG equations of the $SU(4) \times SU(2)_L \times SU(2)_R$ gauge theory, we find that the estimate of $10^{15.04}$ GeV in Eq. (30) is valid even if the two-loop contributions of the Yukawa couplings are included in the gauge coupling running. We also find that the components of $Y_{15}/\sqrt{2}$ and Y_N at scale $\mu = \mu_{\rm GUT}$ differ by at most 0.1%, which implies that the approximation of taking $Y_N = Y_{15}/\sqrt{2}$ at scale $\mu = \mu_{\rm PS}$ (instead of at scale $\mu = \mu_{\rm GUT}$) has negligible impact compared to errors of the NuFIT5.1 data used in the fitting analysis.

VI. SUMMARY

We have studied a prediction on neutrino observables in a nonsupersymmetric renormalizable $SO(10)$ GUT model that contains a **10** complex scalar field and a **126** scalar field. The **10** field and its complex conjugate and the complex conjugate of the **126** field have Yukawa couplings with the **16** matter fields Y_{10}, Z_{10}, Y_{126}, which give rise to the SM Yukawa couplings and neutrino Dirac Yukawa coupling. The $SO(10)$ is broken into $SU(4) \times SU(2)_L \times SU(2)_R$ by the VEV of a **54** real scalar field, and it is broken into the SM gauge groups by the VEV of the **126** field. The latter VEV and Y_{126} generate Majorana mass for the singlet neutrinos. For the above model, we have determined the VEV of the **126** field from the gauge coupling unification conditions. We have constrained the Yukawa couplings of the **10** and **126** fields at the scale of the **126** VEV from experimental data on the quark and charged lepton masses and quark flavor mixings, expressed the neutrino mass matrix with the above Yukawa couplings and the **126** VEV based on the Type-1 seesaw mechanism, fit the neutrino oscillation data, and derived a prediction on neutrino observables.

We have found that both the normal hierarchy and the inverted hierarchy of the neutrino mass can be fit and that the inverted hierarchy is favored from the point of view of naturalness of the mass matrix of $(\mathbf{1}, \mathbf{2}, \pm\frac{1}{2})$ components of $(\mathbf{15}, \mathbf{2}, \mathbf{2})_H$ field. Also, in the normal hierarchy case, the Dirac CP phase of the neutrino mixing matrix δ_{CP} is predicted to be likely in the ranges of $-2.4 < \delta_{CP} < -1.2$ and $1.2 < \delta_{CP} < 2.4$, and not in the range of $\delta_{CP} \sim \pi$. In the normal hierarchy case, mixing angle θ_{23} is predicted to be likely in the upper octant and in a narrow range of $0.50 \lesssim \sin^2\theta_{23} \lesssim 0.55$.

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APPENDIX

To study the correlations between the predictions on neutrino observables, we plot the fitting results of Sec. IV on the planes of δ_{CP} versus $|m_{ee}|$, δ_{CP} versus $\sum_{i=1}^{3} m_i$, $|m_{ee}|$ versus $\sum_{i=1}^{3} m_i$ in Fig. 3.

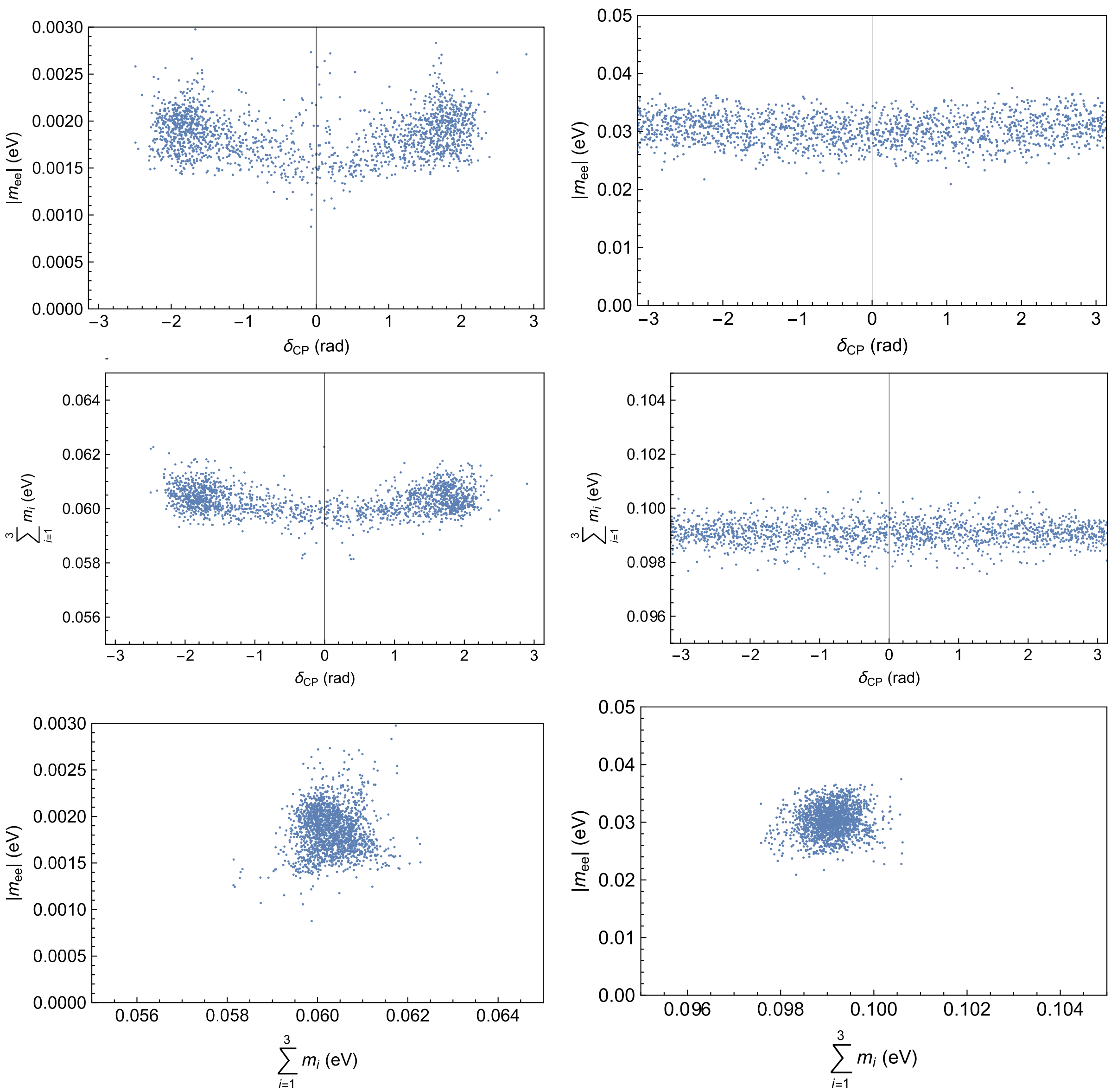

FIG. 3. Panels in the first row: Fitting results on the plane of δ_{CP} versus $|m_{ee}|$. Panels in the second row: Those on the plane of δ_{CP} versus $\sum_{i=1}^{3} m_i$. Panels in the third row: Those on the plane of $|m_{ee}|$ versus $\sum_{i=1}^{3} m_i$. The left-side panels are for the case of the normal neutrino mass hierarchy, and the right-side panels are for the case of the inverted hierarchy.

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