Electroweak mass difference of mesons

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We consider electroweak gauge boson corrections to the masses of pseudoscalar mesons to next to leading order in α_s and $1/N_C$. The pion mass shift induced by the Z boson is shown to be $m_{\pi^\pm} - m_{\pi^0} = -0.00201(12)$ MeV. While being small compared to the electromagnetic mass shift, the prediction lies about a factor of ~4 above the precision of the current experimental measurement and a factor $O(10)$ below the precision of current lattice calculations. This motivates future implementations of these electroweak gauge boson effects on the lattice. Finally, we consider beyond standard model contributions to the pion mass difference.

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I. INTRODUCTION

At very low energies, the strong interaction of mesons is successfully described by the chiral Lagrangian, a perturbative expansion in derivatives of the Goldstone fields, and light quark masses. The effective action is entirely determined by the symmetries, and, once the parameters of the theory are fixed by observation of several meson quantities, a highly predictive theory emerges, chiral perturbation theory [1–3].

In QCD with three light flavors, the global symmetry is $SU(3)_L \times SU(3)_R$, giving eight Goldstone bosons after spontaneous symmetry breaking by the formation of quark condensates. Turning on quark masses, $M_q = \mathrm{diag}(m_u, m_d, m_s)$, explicitly breaks the flavor symmetry, and the meson fields get a mass. The effective action does not allow one to obtain the meson masses purely as a function of quark masses, but it is possible to find relations that connect ratios of the meson masses to (renormalization-scheme-independent) ratios of quark masses, one example being the renowned Gell-Mann-Oakes-Renner relation $\frac{m^2_{K^\pm} - m^2_{K^0}}{m^2_\pi} = \frac{m_u - m_d}{m_u + m_d}$.

The process of gauging part of the global symmetries also breaks the chiral flavor symmetry, generating masses for the pseudoscalar mesons. This is well known for the case of electromagnetism, which breaks the shift symmetries of the charged mesons, thereby generating the pion and kaon mass shifts: $\delta m_\pi = m_{\pi^\pm} - m_{\pi^0}$. This quantity has been computed using current algebra [4] and in chiral perturbation theory with explicit resonance fields [5], giving δm_π compatible with the experimental result [6]:

$$\delta m_\pi|_{\exp} = m_{\pi^\pm} - m_{\pi^0} = 4.5936 \pm 0.0005 \text{ MeV}. \quad (1)$$

The pion mass shift is a quantity that can also be computed on the lattice. This direction was initiated in [7] and currently has reached a level of considerable accuracy [8,9]. The most precise lattice result [8]

$$\delta m_\pi = m_{\pi^\pm} - m^0_\pi = 4.534 \pm 0.042 \pm 0.043 \text{ MeV} \quad (2)$$

is compatible with the experimental measurement in Eq. (1). While the error on the lattice still has to be substantially reduced to reach the experimental precision, given the rate of improvement of lattice precision in recent years it is not unreasonable to think that in the near future the size of both errors might be comparable.

In this paper, we show that heavy electroweak (EW) gauge bosons induce small but possibly *observable* mass shifts between the neutral and charged mesons for both the pion and the kaon. Because of the chiral structure of the weak interaction, to leading order (LO) in G_F, only the Z boson contributes to the mass shifts. Similar results to LO in α_s were noted in [10].

By doing a calculation at next to leading order (NLO) in both α_s and $1/N_c$, our results will show that the expected mass shift induced by the Z lies well above the uncertainty

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of the current experimental measurement and slightly below the lattice uncertainties. This implies that future lattice simulations should be sensitive to the effects of the EW gauge bosons, reflecting the need for an implementation on the lattice. This direction is particularly interesting to learn about flavor symmetry breaking by the weak interaction in the chiral limit. Finally, we discuss future directions including effects of new physics on the mass differences of mesons.

II. ELECTROWEAK INTERACTION AND THE PION MASS DIFFERENCE

QCD with three light flavors has a $SU(3)_L \times SU(3)_R$ global flavor symmetry. Starting at the order of $O(p^2)$ and neglecting momentarily quark masses, the effective Lagrangian below the chiral symmetry breaking scale is of the form

$$\mathcal{L}_2 = \frac{F^2}{4}\mathrm{Tr}(D^\mu U (D_\mu U)^\dagger), \tag{3}$$

where F is the chiral coupling constant and the $SU(3)$ matrix $U = \exp[i\frac{\sqrt{2}}{F}\Phi]$ incorporates the pseudoscalar Goldstone octet

$$\Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta^0 \end{pmatrix}. \tag{4}$$

In the SM, the $SU(2) \times U(1)$ subgroup of this flavor symmetry is gauged. In general, gauging a subgroup of $SU(3)_L \times SU(3)_R$ by gauge bosons L and R is done by introducing a covariant derivative of the form

$$D_\mu U = \partial_\mu U - iQ_L \ell_\mu U + iU r_\mu Q_R. \tag{5}$$

For the SM gauge bosons, this amounts to introducing

$$D_\mu U = \partial_\mu U - i\frac{g}{\sqrt{2}}(W_\mu^+ T_W^- + W_\mu^- T_W^+)U - ie(A_\mu - \tan\theta_W Z_\mu)[Q_{\mathrm{em}}, U] - i\frac{g}{\cos\theta_W} Z_\mu T_{3L} U, \tag{6}$$

where we have explicitly included the photon and the EW gauge bosons with the generators

$$T_W^- = (T_W^+)^\dagger = \begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{7}$$

and the diagonal matrices $T_{3L} = \mathrm{diag}(1/2, -1/2, -1/2)$ and $Q_{\mathrm{em}} = \mathrm{diag}(2/3, -1/3, -1/3)$. The heavy EW gauge bosons are introduced as spurions in order to track the pattern of explicit symmetry breaking. However, since these particles lie well above the cutoff of the effective theory, usually taken to be $\Lambda_{\chi\mathrm{SB}} \sim 4\pi F$, special care has to be taken in deriving explicit results from this Lagrangian. We shall return to this issue momentarily.

Expanding Eq. (3) to quadratic order in Φ, we can see that nonzero Goldstone masses are generated by terms of the form

$$-\frac{F^2}{2}\mathrm{Tr}(Q_L U Q_R U^\dagger) \doteq \frac{1}{2}\mathrm{Tr}([Q_L, \Phi][\Phi, Q_R]), \tag{8}$$

where Q_L and Q_R are spurion matrices representing the action of gauge fields.

One notices that not all of these terms are breaking the shift symmetries in the chiral limit, because meson self-energies are generated by loop diagrams with no external gauge bosons. Consequently, terms involving different gauge bosons do not contribute at LO to the meson masses. Since the $W^\pm$ couplings are purely left-handed, they cannot contribute to Q_R and, therefore, do not generate any meson mass shift.

The only contribution to Q_R comes from the spurion Q_{em}, which as seen from Eq. (6) occurs for both the photon and the Z, and acts as

$$[Q_{\mathrm{em}}, \Phi] = \begin{pmatrix} 0 & \pi^+ & K^+ \\ -\pi^- & 0 & 0 \\ -K^- & 0 & 0 \end{pmatrix}. \tag{9}$$

This implies that only charged mesons can get a mass, and this occurs through the interaction with neutral gauge bosons, which contribute as

$$\frac{eg}{2\cos\theta_W}\mathrm{Tr}([T_{3L}, \Phi][\Phi, Q_{\mathrm{em}}])(A_\mu - \tan\theta_W Z_\mu)Z^\mu \tag{10}$$

and

$$\frac{e^2}{2}\mathrm{Tr}([Q_{\mathrm{em}}, \Phi][\Phi, Q_{\mathrm{em}}])(A_\mu - \tan\theta_W Z_\mu)(A^\mu - \tan\theta_W Z^\mu). \tag{11}$$

Again, the term involving $A_\mu Z^\mu$ cannot contribute to meson masses. Combining Eqs. (10) and (11) and retaining only the relevant terms involving $A_\mu A^\mu$ and $Z_\mu Z^\mu$, the interaction reads

$$e^2(\pi^+\pi^- + K^+K^-)(A_\mu A^\mu - Z_\mu Z^\mu). \tag{12}$$

An order of magnitude estimate can be given at this point for the Z-boson-induced mass shift using naive dimensional analysis:

$$\Delta m_\pi^2 = \frac{e^2}{4\pi^2 M_Z^2}\Lambda_{\chi\mathrm{SB}}^4 \to \delta m_\pi \sim 0.002\ \mathrm{MeV}. \quad (13)$$

The fact that this estimate lies above the current experimental uncertainty and is comparable to the lattice precision motivates us to perform a more careful analysis.

As in the electromagnetic (EM) contribution [5], we capture the effects of both A_μ and Z_μ by adding the following local operators involving the spurion matrices Q_{em} and $Q^Z_{L,R} \equiv \frac{g}{\cos\theta_W}\mathcal{Q}_{L,R}$:

$$\mathcal{L}_2^C = e^2 C_{\mathrm{em}}\langle Q_{\mathrm{em}} U Q_{\mathrm{em}} U^\dagger\rangle + 4\sqrt{2}G_F C_Z \langle \mathcal{Q}_L U \mathcal{Q}_R U^\dagger\rangle, \quad (14)$$

with $4\sqrt{2}G_F$ the low-energy coupling of the Z boson,

$$\mathcal{Q}_L = \begin{pmatrix} \frac{1}{2}-\frac{2}{3}x & 0 & 0 \\ 0 & -\frac{1}{2}+\frac{1}{3}x & 0 \\ 0 & 0 & -\frac{1}{2}+\frac{1}{3}x \end{pmatrix}, \quad (15)$$

$$\mathcal{Q}_R = \begin{pmatrix} -\frac{2}{3}x & 0 & 0 \\ 0 & \frac{1}{3}x & 0 \\ 0 & 0 & \frac{1}{3}x \end{pmatrix}, \quad (16)$$

and $x = \sin^2\theta_W$. The determination of C_Z to NLO in α_s and $1/N_c$ is the goal of this paper.

The coefficients C_{em} and C_Z are low-energy constants determined from the high-energy theory and determine the electromagnetic and electroweak meson mass differences $\Delta m_P^2 \equiv m_{P^\pm}^2 - m_{P^0}^2$ of pions and kaons in the chiral limit:

$$\Delta m_\pi^2 = \Delta m_K^2 = \frac{2e^2}{F^2}\left(C_{\mathrm{em}} - \frac{C_Z}{M_Z^2}\right). \quad (17)$$

In [5] it was shown that the EM mass shift from resonance exchange saturates the constant C_{em} and is given in terms of the resonance parameters F_V and M_V by

$$\Delta m_\pi^2|_{\mathrm{em}} = \frac{3\alpha_{\mathrm{em}}}{4\pi F^2}F_V^2 M_V^2 \ln\frac{F_V^2}{F_V^2 - F^2}. \quad (18)$$

A corresponding resonance loop calculation including the Z boson in order to determine C_Z is subtle. The reason is that the parameter M_Z lies well above the cutoff, $\Lambda_{\chi\mathrm{SB}}$, and the Z therefore must be integrated out.

The resulting EFT is QCD with four-fermion operators that encode all the information of the chiral symmetry breaking by the EW bosons. Using the renormalization group (RG) to run the Wilson coefficients of these operators down to a scale $\mu \sim 1$ GeV allows matching to the operators in Eq. (14) of the chiral Lagrangian and thereby a determination of C_Z.

A. Z-induced left-right four-quark operators

Integrating out the Z boson introduces four-fermion operators that break the chiral $SU(3)_L \times SU(3)_R$ symmetry. The relevant left-right (LR) operators are

$$[Q_1^{LR}]_{ijk\ell} = (\bar{q}_{Li}\gamma^\mu q_{Lj})(\bar{q}_{Rk}\gamma^\mu q_{R\ell}), \quad (19)$$

$$[Q_2^{LR}]_{ijk\ell} = (\bar{q}_{Li} q_{Rk})(\bar{q}_{R\ell} q_{Lj}), \quad (20)$$

with i, j, k, and ℓ being light-quark flavor indices. While Q_1^{LR} is generated by a Z exchange at tree level, Q_2^{LR} is obtained after applying a Fierz identity on the gluon corrections to Q_1^{LR}.

The effective Lagrangian below M_Z reads

$$\mathcal{L}_{\mathrm{eff}} = -4\sqrt{2}G_F \sum_{ijk\ell}(\mathcal{Q}_L)_{ij}(\mathcal{Q}_R)_{k\ell}[C_1 Q_1^{LR} + C_2 Q_2^{LR}]_{ijk\ell}, \quad (21)$$

with $C_{1,2}$ being the Wilson coefficients.

When QCD effects are taken into account, the renormalized Wilson coefficients at the M_Z scale become [11]

$$C_1 = 1 + \frac{\alpha_s}{4\pi}\frac{3}{N_c}\left[-\ln\frac{M_Z^2}{\mu^2} - \frac{1}{6}\right], \quad (22)$$

$$C_2 = \frac{\alpha_s}{4\pi}\left[-6\ln\frac{M_Z^2}{\mu^2} - 1\right], \quad (23)$$

where the nonlogarithmic corrections are scheme dependent. The operators above will mix under RG flow, and their evolution down to the scale of interest (~1 GeV) can be calculated by standard procedures [12], using their anomalous dimension matrices:

$$\frac{d\vec{C}}{d\ln\mu} = \gamma^T \vec{C}. \quad (24)$$

Up to the order of $O(\alpha_s^2)$, this matrix can be expanded as

$$\gamma = \frac{\alpha_s}{4\pi}\gamma^0 + \left(\frac{\alpha_s}{4\pi}\right)^2 \gamma^1 + O(\alpha_s^3), \quad (25)$$

with γ^0 and γ^1 given by [13]

$$\gamma^0 = \begin{pmatrix} \frac{6}{N_c} & 12 \\ 0 & -6N_c + \frac{6}{N_c} \end{pmatrix}, \qquad \gamma^1 = \begin{pmatrix} \frac{137}{6} + \frac{15}{2N_c^2} - \frac{22}{3N_c}f & \frac{200}{3}N_c - \frac{6}{N_c} - \frac{44}{3}f \\ \frac{71}{4}N_c + \frac{9}{N} - 2f & -\frac{203}{6}N_c^2 + \frac{479}{6} + \frac{15}{2N_c^2} + \frac{10}{3}N_c f - \frac{22}{3N_c}f \end{pmatrix}. \quad (26)$$

Solving Eq. (24) yields the evolution:

$$\vec{C}(\mu) = T \exp\left[\int_{\alpha_s(M_Z)}^{\alpha_s(\mu)} d\alpha_s \frac{\gamma^T}{\beta(\alpha_s)}\right]\vec{C}(M_Z), \tag{27}$$

where we have introduced the QCD β function as

$$\beta = -2\alpha_s\left[\beta_0 \frac{\alpha_s}{4\pi} + \beta_1\left(\frac{\alpha_s}{4\pi}\right)^2 + O(\alpha_s^3)\right]. \tag{28}$$

The coefficients used are given by $\beta_0 = \frac{11N_c - 2f}{3}$ and $\beta_1 = \frac{34}{3}N_c^2 - \frac{10}{3}N_c f - \frac{N_c^2-1}{N_c} f$ [14], where f is the number of active flavors.

To NLO and after integrating out the b and c quarks, the Wilson coefficients at the scale $\mu \sim 1$ GeV are

$$C_1 = 0.92, \qquad C_2 = -2.45. \tag{29}$$

Similar enhancements of C_2 are noticed in [15].

B. Matching to the chiral Lagrangian at large N_c

We proceed to match the resulting EFT to the chiral Lagrangian. We do so by calculating the expectation value of the matrix elements of the four-fermion operators in the large-N_c limit in which products of color-singlet currents factorize.

In this limit, the operator Q_1^{LR} reduces to the product of a left and a right current:

$$[Q_1^{LR}]_{ijk\ell} = \mathcal{J}^{\mu}_{L,ji}\mathcal{J}^{R}_{\mu,\ell k}. \tag{30}$$

Since the low-energy representation of these currents starts at $O(p)$ in the chiral-perturbation-theory expansion, the large-N_C expression of Q_1^{LR} is of $O(p^2)$ and, therefore, does not contribute to the $O(p^0)$ operator in Eq. (14). Owing to its different scalar-pseudoscalar structure, the operator Q_2^{LR} does contribute at $O(p^0)$, receiving a chiral enhancement of the form

$$[Q_2^{LR}]_{ijk\ell} = \langle \bar{q}_L^i q_R^k\rangle\langle \bar{q}_R^\ell q_L^j\rangle\left\{1 + O\left(\frac{1}{N_c}\right)\right\} \tag{31}$$

$$= \frac{1}{4}B_0^2 F^4 U_{ki}U_{j\ell}^\dagger\left\{1 + O\left(\frac{1}{N_c}\right)\right\} + O(p^2), \tag{32}$$

with $B_0 = -\langle \bar{q}q\rangle/F^2 = m_{\pi^\pm}^2/(m_u + m_d)$.

Matching the contribution of Q_2^{LR} to the effective theory, a LO estimate in N_c can be given for C_Z:

$$C_Z = -\frac{1}{4}B_0^2(\mu)F^4 C_2(\mu). \tag{33}$$

One can easily check that, in the large-N_c limit, the μ dependence of $C_2(\mu)$ is exactly canceled by the quark-mass factors in $B_0^2(\mu)$, as it should.

C. $1/N_c$ corrections to Q_1^{LR}

As shown in [10], the low-energy constants in Eq. (14) can be related to the two-point correlation function of a left and a right QCD current, $\Pi_{LR}(Q^2)$, which converges nicely in the UV. This fact allows one to evaluate the leading nonzero $O(p^0)$ contributions of Q_1^{LR}, originating from loops of Goldstone bosons and vector and axial-vector resonance fields, which are NLO corrections in $1/N_c$. The full details of the calculation are given in the Appendix. Integrating only the low-energy region $0 \le Q^2 \le \mu^2$ (contributions from $Q^2 > \mu^2$ are already included in the Wilson coefficients), one finds

$$\Delta C_Z|_{Q_1^{LR}} = \frac{3}{32\pi^2}\left\{\sum_A F_{A_i}^2 M_{A_i}^4 \log\left(1 + \frac{\mu^2}{M_{A_i}}\right) - \sum_V F_{V_i}^2 M_{V_i}^4 \log\left(1 + \frac{\mu^2}{M_{V_i}}\right)\right\} C_1(\mu). \tag{34}$$

Since we are interested in the matrix element of the operator Q_1^{LR} at around the $\mu \sim 1$ GeV scale, we work in the lightest-resonance approximation with their couplings fixed through the Weinberg conditions [16,17]:

$$F_V^2 = \frac{M_A^2}{M_A^2 - M_V^2}F^2, \qquad F_A^2 = \frac{M_V^2}{M_A^2 - M_V^2}F^2. \tag{35}$$

Within the single-resonance approximation that we have adopted, $M_A = \sqrt{2}M_V$ [17]. For the numerical evaluation we will take $M_V = M_\rho = 775.26 \pm 0.23$ MeV and $F = F_\pi = 92.1 \pm 0.8$ MeV [14]. As expected from its loop suppression, $\Delta C_Z|_{Q_1^{LR}}$ is of $O(F^2) \sim O(N_c)$ and, therefore, is a NLO correction in $1/N_c$ of about $O(10\%)$ with respect to the leading $O(F^4) \sim O(N_c^2)$ contribution from Q_2^{LR} in Eq. (33).

D. EW contribution to the pion mass difference

Using Eq. (17) and the results above in Eqs. (33)–(35), the pion mass shift induced by the Z reads

$$\Delta m_\pi^2|_Z = \frac{e^2}{M_Z^2}\left\{\frac{F^2}{2}B_0^2(\mu)C_2(\mu) + \frac{3}{16\pi^2}C_1(\mu)\frac{M_A^2 M_V^2}{M_A^2 - M_V^2} \times\left[M_V^2\log\left(1 + \frac{\mu^2}{M_V^2}\right) - M_A^2\log\left(1 + \frac{\mu^2}{M_A^2}\right)\right]\right\}. \tag{36}$$

This translates into a Z-induced pion mass difference:

$$\delta m_\pi|_Z \approx \frac{\Delta m_\pi^2|_Z}{2m_\pi} = -0.00201(7)(2)(10)\ \text{MeV}, \tag{37}$$

where we have used $m_\pi = 134.9768 \pm 0.0005$ MeV [14] and $(m_u + m_d)/2 = 3.381 \pm 0.040$ MeV [18]. The first

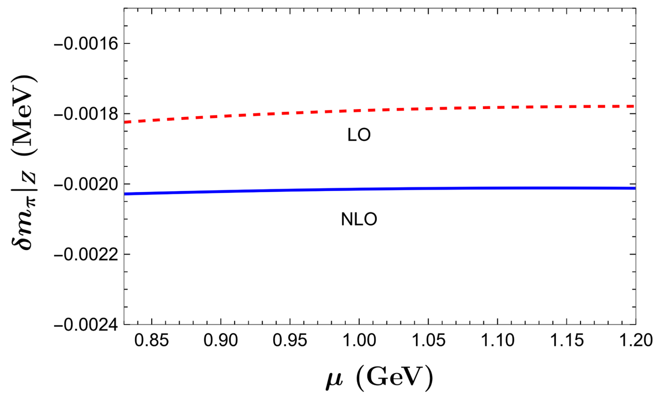

FIG. 1. Renormalization scale dependence of the pion mass shift induced by the Z boson.

error displays the parametric uncertainty induced by the different inputs. The second uncertainty accounts for the renormalization-scale dependence in the interval $\mu \in [0.8, 1.2]$ GeV which, as shown in the figure, is tiny. We have added half the difference between the LO and NLO results as an estimate of unknown higher-order effects (third error).

We notice that the Z-boson contribution is about a factor of ~ 4 larger than the experimental error in Eq. (1) and $\sim O(10)$ smaller than the current lattice precision in Eq. (2), reinforcing the motivation to incorporate these effects on the lattice. The renormalization scale dependence of this result for energies in the range [0.8, 1.2] GeV is plotted in Fig. 1.

III. DISCUSSION

Before closing, we comment on several points that deserve mention.

(i) The estimate in Eq. (37) is based on a NLO evaluation of the Wilson coefficients $C_{1,2}(\mu)$, which depends on the precise values of the strong coupling at M_Z, $\alpha_s(M_Z) = 0.1184 \pm 0.0008$ [18], and at the different matching scales (known to percent level or better).

(ii) Our result $\delta m_\pi|_Z$ appears to be of the same order as the two-loop EM effect, which naively one expects to be

$$\delta m_\pi|^{(2)}_{\rm em} \approx \left(\frac{\alpha_{\rm em}}{2\pi}\right) \delta m_\pi|^{(1)}_{\rm em}. \qquad (38)$$

A full understanding of these additional $\mathcal{O}(\alpha_{\rm em}^2)$ electromagnetic corrections to the pion mass shift, including both the leptonic and hadronic contributions, would feature considerable challenges and is beyond the scope of this paper.

(iii) Theories beyond the standard model that generate four-quark LR operators at energies below the new physics scale, $\Lambda_{\rm NP} \gg \Lambda_{\chi{\rm SB}}$, will induce similar pion mass shifts. This is the case, for example, of the Z' models studied in [11] and similar SM extensions. Since the QCD corrections dominate near the GeV scale, a reasonable estimate is just the rescaling:

$$\delta m_\pi|_{\rm NP} = \frac{g_{\rm NP}^2}{\Lambda_{\rm NP}^2} \frac{\delta m_\pi|_Z}{4\sqrt{2} G_F}. \qquad (39)$$

If new physics is instead light, as proposed in [19–21], one should appropriately modify the EM resonance calculation [5].

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APPENDIX

In the large-N_C limit, the strong interaction reduces to tree-level hadronic diagrams. Keeping only those terms that are relevant for our calculation, the effective Lagrangian describing the mesonic world contains the LO Goldstone term $\mathcal{L}_2$ and the vector and axial-vector couplings (kinetic terms are omitted) [17]:

$$\mathcal{L}_{V,A} = \sum_{V_i} \frac{F_{V_i}}{2\sqrt{2}} \langle V_i^{\mu\nu} f_{+\mu\nu} \rangle + \sum_{A_i} \frac{F_{A_i}}{2\sqrt{2}} \langle A_i^{\mu\nu} f_{-\mu\nu} \rangle, \qquad (A1)$$

where $f_\pm^{\mu\nu} = u^\dagger F_L^{\mu\nu} u \pm u F_R^{\mu\nu} u^\dagger$ with $U = u^2$ the Goldstone $SU(3)$ matrix and $F_{L,R}^{\mu\nu}$ the left (ℓ^μ) and right (r^μ) field strengths. The spin-1 resonances are described through the antisymmetric tensors $V_i^{\mu\nu}$ and $A_i^{\mu\nu}$ [5,22].

The left and right QCD currents are easily computed, taking derivatives with respect to the external ℓ^μ and r^μ fields:

$$\mathcal{J}_L^\mu = i \frac{F^2}{2} D^\mu U U^\dagger + \sum_{V_i} \frac{F_{V_i}}{\sqrt{2}} \partial_\nu (u V_i^{\mu\nu} u^\dagger) + \sum_{A_i} \frac{F_{A_i}}{\sqrt{2}} \partial_\nu (u A_i^{\mu\nu} u^\dagger) + \cdots, \qquad (A2)$$

while $\mathcal{J}_R^\mu$ is obtained from this expression exchanging $u \leftrightarrow u^\dagger$ and putting a negative sign in the axial contributions.

The bosonization of $[Q_1^{LR}]_{ijk\ell}$ is formally given by [23]

$$\langle [Q_1^{LR}(x)]_{ijkl} \rangle_G = \frac{\partial \Gamma}{\partial \ell_\mu^{ij}(x)} \frac{\partial \Gamma}{\partial r^{\mu,kl}(x)} - i \frac{\partial^2 \Gamma}{\partial \ell_\mu^{ij}(x) \partial r^{\mu,kl}(x)} \qquad (A3)$$

with $\Gamma[\ell, r]$ the effective theory generating functional. The first term is just the product of the two currents and receives $O(p^0)$ contributions from loop diagrams with vector and axial-vector internal propagators. The second term (the derivative of $\mathcal{J}_L^\mu$ with respect to r^μ) generates an additional $O(p^0)$ contribution through Goldstone loops. The combined result can be written in the form

$$\sum_{ijkl} \mathcal{Q}_L^{ij} \mathcal{Q}_R^{kl} [Q_1^{LR}]_{ijkl} = \frac{3}{32\pi^2} \langle \mathcal{Q}_L U \mathcal{Q}_R U^\dagger \rangle \int_0^\infty dQ^2 \times \left\{ \sum_V \frac{F_{V_i}^2 M_{V_i}^4}{M_{V_i}^2 + Q^2} - \sum_A \frac{F_{A_i}^2 M_{A_i}^4}{M_{A_i}^2 + Q^2} \right\}, \tag{A4}$$

where the Weinberg conditions [16]

$$\sum_i (F_{V_i}^2 - F_{A_i}^2) = F^2, \qquad \sum_i (M_{V_i}^2 F_{V_i}^2 - M_{A_i}^2 F_{A_i}^2) = 0 \tag{A5}$$

have been used in order to simplify the final expression. Equation (A4) agrees with the result obtained in [10], using the alternative Proca description of spin-1 fields. Performing the integration in the low-energy region $0 \leq Q^2 \leq \mu^2$, one obtains the result for $\Delta C_Z|_{Q_1^{LR}}$ in Eq. (34).

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