Pion gravitational form factors at large momentum transfer in the instant-form relativistic impulse approximation approach

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We extend our relativistic theory of gravitational structure of composite hadrons to obtain the pion gravitational form factors at large momentum transfers. The approach was used in the case of intermediate region of the variable in our preceding works [Phys. Rev. D **103**, 014029 (2021)] and [Phys. Rev. D **106**, 054013 (2022)]. The calculation is carried out in the framework of a relativistic composite-particle model complemented by the special relativistic form of impulse approximation. It is found that in the limit of massless and pointlike quarks, the obtained asymptotic expansion coincides with the predictions of perturbative QCD for gravitational pion form factors. The principal contribution to the asymptotics, coinciding with the predictions of QCD, is given by the relativistic effect of spin rotation. In particular, the asymptotics of the D form factor is completely determined by this kinematic effect. Several restrictions on the allowed form of gravitational form factors of quarks are derived.

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I. INTRODUCTION

The theory of the gravitational structure of hadrons is the focus of investigation during the last decades. Its current status is reviewed in [1] and for earlier reviews see, e.g., [2–6]. The basic mathematical object in this theory is the operator of the energy-momentum tensor (EMT) of the particle (see, e.g., [7]), and, respectively, its Lorentz-covariant decomposition in terms of the gravitational form factors (GFFs). GFFs encode key information including the mass and spin of a particle, the less well-known but equally fundamental D-term (D stands for the German word *Druck* meaning pressure), as well as the information about distributions of energy, angular momentum, and various mechanical properties such as, e.g., internal forces inside the system.

Since the gravitational interaction is very weak, the *direct* measurement of GFFs cannot be carried out in experiments today, nor in the foreseeable future. However, as it turns out, information about the EMT can be extracted *indirectly* (see useful discussion in [8]). At present, one obtains the information about the GFF mainly from the hard-exclusive processes described in terms of unpolarized generalized parton distribution (GPDs) or in terms of generalized distribution amplitudes (GDAs) [1,4,8–10]. Several indirect measurements are underway and some are planned (see, e.g., [11–15]). In these experiments, kinematic ranges are extended and the scale of momentum transfers is significantly enlarged ($Q^2 = -t = q^2$, $q-$ momentum transfer). In this connection, it is interesting to study GFFs at large momentum transfer, up to the asymptotic region $Q^2 \to \infty$.

Another interesting point is the possible comparison of our purely relativistic model results with the results of [16–18] where strict predictions of perturbative quantum chromodynamics (QCD) are given for the asymptotic behavior of pion GFFs.

At $Q^2 \to \infty$, QCD gives a trustable description of hadron physics. Results obtained from the first principles within the framework of this generally accepted fundamental theory of strong interactions should be considered as some additional constraint on the GFFs calculated in other approaches. In particular, this is the case for various formulations of the quark model [19–25]. It seems that an analog of the correspondence principle should be required, i.e., in the theories of the gravitational structure operating with quark and gluon degress of freedom, there must be a limiting transition, giving for the GFFs the behavior coinciding with the predictions of the perturbative QCD.

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Since the asymptotics of the GFFs of composite hadrons depend on the asymptotic behavior of the GFFs of the constituent quarks, then the correspondence condition imposes restrictions on quark form factors, the determination of which is also a relevant task (see, e.g., [26]).

To complete the motivation, it is important to mention two more points of different generality of significance. First, the calculation of the mean-square mechanical radius requires information about the behavior of the form factor D in the entire region of momentum transfer, including asymptotic [4]. Second, our calculation can shed light on the position of the boundary of a perturbative regime for GFFs.

The presented paper is devoted to the calculation of the asymptotic behavior of the pion GFF. In our calculations we use a particular variant of the instant-form (IF) Dirac [27] relativistic quantum mechanics (RQM) (see also [28–32]), extended for composite systems (see, e.g., [33–36]). The approach was successfully used to describe the pion electromagnetic form factor. Recently we have shown [24,25] that the pion GFFs can be derived in the same formalism using the same approximations and the same model parameters, adding only one new parameter fixed by fitting the slope at zero of the normalized to pion D-term form factor D of pion. The present paper, in fact, generalizes this approach to a larger region of momentum transfer. We will frequently refer to the results of these two articles.

The basic features of our approach are the following (see, e.g., Ref. [35]).

(1) The main differences between our version and the conventional IF RQM are, first, the construction of the matrix elements of local operators, which are based on the analog of the Wigner-Eckart theorem for the Poincaré group [36,37], and, second, the interpretation of the corresponding reduced matrix elements, that is form factors, as generalized functions.

(2) We include the interaction in the composite system by adding the interaction operator to the operator of the mass of the free constituent system by analogy with the conventional IF of RQM.

(3) Note that it is possible to include the interaction in our approach by the solutions of the Muskhelishvili-Omnès-type equations [38]. These solutions represent wave functions of constituent quarks.

(4) It is important to notice that the approach we use differs from the IF *per se*; it is rather fruitfully complemented by a modified impulse approximation, MIA, constructed by making use of a dispersion-relation approach in terms of the reduced matrix elements. So, the Lorentz-covariance condition and the current conservation law are satisfied automatically. The difference between MIA and conventional IA is detailed in our paper [33].

In the context of the present paper, the following point is worth noting. An important advantage of the approach we use is matching with the QCD predictions in the ultraviolet limit, when quark masses are switched off, as expected at high energies [39–41]. The model reproduces correctly not only the functional form of the QCD asymptotics, but also the numerical coefficient. The analogous result holds also for the kaon [42].

Integral representations for electromagnetic and gravitational form factors of composite systems in our approach are given by double integrals of a special form, which are analogs of dispersion integrals over the composite-system mass [38] (see also [43]). We have proved relevant theorems and formulas for asymptotic expansion of such integrals at $Q^2 \to \infty$ in [44].

In the present work we calculate the GFFs asymptotics using a modified impulse approximation (MIA) that we formulated and successfully exploited earlier. MIA is relativistic by construction *a priori*. As we realize our study in IF RQM, it is natural to name our approximation as instant form relativistic impulse approximation as it was done by the authors of [1].

Now (using actually [24,25,44]) we calculate the asymptotics of the pion form factors A and D at $Q^2 \to \infty$. Since the pion GFFs depend on the choice of the model two-quark wave function only weakly [25], we use the wave functions of the harmonic oscillator (the Gaussian one) for simplicity. The pion GFFs obtained in such way decrease exponentially in Q with power–law corrections in $1/Q$. It is shown that if in the asymptotic expansion we go to massless point constituents, i.e., in the limit of zero mass and zero mean-square mass radius of quarks, then our asymptotics for the form factor A coincides with that predicted by the perturbative QCD [16–18]. This result is obtained with the same quark form factors with logarithmic decay as we used previously for pion GFFs at finite Q^2 in the works [24,25], and on the pion electroweak structure [33,34,39–41,45].

However, if we want to prioritize the principle of correspondence, and to obtain, for low quark mass, a power-law asymptotics of pion form factor D, then it is necessary to modify the gravitational form factor D of quarks, namely to go from logarithmic decreasing with increasing momentum transfer to a power-law one. This modification leads to nonzero rms of the mechanical radius of the quark, in contrast to the logarithmic dependence that gives a zero value for this quantity. Thus, in the domain where perturbative QCD is applicable, its predictions can be obtained as a limiting case of our formulation of the relativistic composite-particle model. Note that the correspondence principle is satisfied here actually due to the importance and universality of the kinematical relativistic effect of spin rotation [46].

The structure of the paper is as follows. Section II is devoted to a brief description of the instant-form relativistic impulse approximation and calculation within its framework of the pion gravitational form factors. In Sec. III the procedure of asymptotic expansion of the pion gravitational form factors is described and the corresponding asymptotic formulas are given. In Sec. IV we discuss physical consequences of the obtained asymptotic expansion, in particular, the role of the relativistic spin effect, limitations on the used gravitational form factors of constituent quarks and the possibility of obtaining in our approach the asymptotics of the pion GFF which coincides with QCD prediction. Section V contains the main conclusions of the work. The Appendices contain the formulas for the Clebsch-Gordan coefficients of Poincaré group and for the so-called free two-particle gravitational form factors used in the modified impulse approximation.

II. INSTANT-FORM RELATIVISTIC IMPULSE APPROXIMATION AND THE CALCULATION OF THE PION GFFs

In the present section, for convenience of the reader, we remind the main stages of our approach used for the calculation of GFFs in immediately preceding papers [24,25]. The approach is based on the instant form relativistic quantum mechanics with a fixed number of particles (IF RQM) (see [27–31]).

The difference between RQM and its nonrelativistic analog comes down to the difference between the algebra of the Poincaré group and the algebra of the Galilean group at their realizations on the set of dynamic observables of a composite system. A special feature of the Poincaré algebra in comparison with the algebra of the Galilean group is the fact that at additive inclusion of the constituent-interaction operator into the zero component of the total momentum (into the operator of the total energy), to preserve the corresponding algebraic structure it is necessary to make operators of some other observables interaction-dependent, too. Different ways of incorporating interaction into algebra lead to various forms of relativistic (Dirac) dynamics. Notice, that to preserve the algebra of the Galilean group at additive inclusion of the interaction to the zero component of the total 4-momenta does not require modification of other group generators, and this leads to the only nonrelativistic dynamics—the dynamics of the Schrödinger equation. Relativistic dynamics can be classified into so-called kinematic subgroups, i.e., subgroups of observables independent of the interaction. The IF RQM has as kinematic subgroup the group of motions of three-dimensional Euclidean space, i.e., rotations and translations. One might say that RQM occupies an intermediate position between the local quantum field theory and nonrelativistic quantum mechanics. Thus, the constituents of the composite system are assumed to lie on the mass shell, and the wave function of interacting particles is defined as an eigenfunction of the complete set, which in IF RQM consists of the following operators:

$$\hat{M}_I^2\,(\text{or}\,\hat{M}_I),\quad \hat{J}^2,\quad \hat{J}_3,\quad \hat{\vec{P}}, \tag{1}$$

where $\hat{M}_I$ is the mass operator for the system with interaction, $\hat{J}^2$ is the operator of the square of the total angular momentum, $\hat{J}_3$ is the operator of the projection of the total angular momentum on the z axis, and $\hat{\vec{P}}$ is the operator of the total momentum.

In the IF RQM, the operators $\hat{J}^2, \hat{J}_3, \hat{\vec{P}}$ coincide with corresponding operators for the composite system without interaction, and only the term $\hat{M}_I^2(\hat{M}_I)$ is interaction dependent.

To solve the problem on eigenfunctions of the set (1) it is necessary to choose a suitable basis in the Hilbert state space of the composite system (see details in [24]). In the case of a system of two constituent quarks one can use, first, the basis of individual spins and momenta:

$$|\vec{p}_1, m_1; \vec{p}_2, m_2\rangle = |\vec{p}_1, m_1\rangle \otimes |\vec{p}_2, m_2\rangle, \tag{2}$$

where $\vec{p}_1, \vec{p}_2$ are the 3-momenta of particles, m_1, m_2 are the projections of spins to the z axis.

Second, it is possible to use the basis in which the motion of the center of mass of two particles is separated:

$$|\vec{P}, \sqrt{s}, J, l, S, m_J\rangle, \tag{3}$$

where $P_\mu = (p_1 + p_2)_\mu$, $P_\mu^2 = s$, $\sqrt{s}$ is the invariant mass of the system of two particles, l is the orbital momentum in the center-of-mass of the system (c.m.s.), $\vec{S}^2 = (\vec{S}_1 + \vec{S}_2)^2 = S(S+1)$, S is the total spin in c.m.s., J is the total angular momentum, m_J is the projection of the total angular momentum.

The bases (2) and (3) are linked by the Clebsch-Gordan decomposition of a direct product (2) of two irreducible representations of the Poincaré group into irreducible representations (3) [24,31]. The formulas for the corresponding Clebsch-Gordan coefficients are given in Appendix A.

In the basis (3) only the operator $\hat{M}_I$ in the complete set (1) is nondiagonal. So, the two-quark wave function in pion in the basis (3) has the following form:

$$\langle\vec{P}, \sqrt{s}|\vec{p}_\pi\rangle = N_C\delta(\vec{P} - \vec{p}_\pi)\varphi(s), \tag{4}$$

where $\vec{p}_\pi$ is the 3-momentum of the pion. Explicit form of the normalization constant N_C is given in the paper [24] and is not used here. In the notations of basis vectors (3), the quantum numbers of the pion are omitted.

The wave function of the intrinsic motion is the eigenfunction of the operator $\hat{M}_I^2(\hat{M}_I)$ and in the case of two particles of equal masses is (see, e.g., [33])

$$\varphi(s(k)) = \sqrt[4]{s}\, k\, u(k), \qquad s = 4(k^2 + M^2),$$
$$\int u^2(k) k^2 dk = 1, \tag{5}$$

where $u(k)$ is a model quark-antiquark wave function of the pion and M is the mass of the constituents.

Let us now construct the pion EMT in the IF RQM using the general method of the relativistically invariant parametrization of matrix elements of local operators [47] (see also [6]). For convenience and to preserve the generality of the results of [24], here we are dealing with the same notations. The relation between our GFF and generally accepted form factors A and D [4,48] is given below in Sec. III. For the pion EMT matrix element we obtained in [24]:

$$\langle \vec{p}_\pi | T^{(\pi)}_{\mu\nu}(0) | \vec{p}\,'_\pi \rangle = \frac{1}{2} G^{(\pi)}_{10}(Q^2) K'_\mu K'_\nu - G^{(\pi)}_{60}(Q^2)[Q^2 g_{\mu\nu} + K_\mu K_\nu], \tag{6}$$

where $G^{(\pi)}_{10}, G^{(\pi)}_{60}$ are the gravitational form factors of the pion, $g_{\mu\nu}$ is the metric tensor and

$$K_\mu = (p_\pi - p'_\pi)_\mu, \qquad K'_\mu = (p_\pi + p'_\pi)_\mu,$$
$$Q^2 = -t = -K^2_\mu$$

We present the decomposition of the left-hand side (lhs) of (6) over the basis (3) as a superposition of the same tensors as in the right-hand side (rhs) of (6), and so we obtain the pion GFFs in the following form of the functionals defined on two-quark wave functions (4) and (5):

$$G^{(\pi)}_{i0}(Q^2) = \int d\sqrt{s} d\sqrt{s'} \varphi(s) \tilde{G}_{i0}(s, Q^2, s') \varphi(s'), \quad i = 1, 6. \tag{7}$$

Here $\tilde{G}_{i0}(s, Q^2, s'), i = 1,\ 6$ are the Lorentz-invariant regular distributions.

To calculate the invariant distributions on the rhs of (7), we use a version of impulse approximation. The generally accepted impulse approximation (IA) is formulated in the language of operators which means that EMT of the composite system is assumed to be equal to the sum of single-particle EMT of the components:

$$T \approx \sum_k T^{(k)}. \tag{8}$$

To construct the pion GFFs we use a modified impulse approximation (MIA) that we first formulated earlier (see, e.g., Refs. [33,34] and the review [31]) In contrast to the baseline impulse approximation, MIA is formulated in terms of the reduced matrix elements, that is form factors, and not in terms of the operators itself. So, in MIA there appears important objects—the free gravitational form factors describing the gravitational characteristics of systems without interaction.

Consider the system of two free constituent quarks [24]. Note that in the work [49] it was shown that the form factor D is zero in the case of pointlike free fermions. In contrast, our constituent quarks have all properties of realistic particles with internal structure that is described by a set of form factors including form factor D.

Using the general method of parametrization of local-operators matrix elements [47] we write the matrix element of the EMT for system of noninteracting fermions with total quantum numbers of pion $J = l = S = 0$ in the following form [24]:

$$\langle P, \sqrt{s} | T^{(0)}_{\mu\nu}(0) | P', \sqrt{s'} \rangle = \frac{1}{2} G^{(0)}_{10}(s, Q^2, s') A'_\mu A'_\nu - G^{(0)}_{60}(s, Q^2, s')[Q^2 g_{\mu\nu} + A_\mu A_\nu], \tag{9}$$

where $G^{(0)}_{i0}(s, Q^2, s'), i = 1,\ 6$ are free two-particle GFFs, In the lhs zero discrete quantum numbers in state vectors are ignored.

$$A_\mu = (P - P')_\mu, \qquad A^2 = t = -Q^2,$$
$$A'_\mu = \frac{1}{Q^2}[(s - s' + Q^2)P_\mu + (s' - s + Q^2)P'_\mu].$$

The corresponding expression, based on the Clebsch-Gordan decomposition, that connects the bases (2) and (3), for the EMT of a system of noninteracting fermions with total quantum numbers of pion $J = l = S = 0$ in terms of the one-particle EMTs has the following form [24]:

$$\begin{aligned} &\langle P, \sqrt{s} | T^{(0)}_{\mu\nu}(0) | P', \sqrt{s'} \rangle \\ &= \sum \int \frac{d\vec{p}_1}{2p_{10}} \frac{d\vec{p}_2}{2p_{20}} \frac{d\vec{p}\,'_1}{2p'_{10}} \frac{d\vec{p}\,'_2}{2p'_{20}} \langle P, \sqrt{s} | \vec{p}_1, m_1; \vec{p}_2, m_2 \rangle \\ &\quad \times [\langle \vec{p}_1, m_1 | \vec{p}\,'_1, m'_1 \rangle \langle \vec{p}_2, m_2 | T^{(u)}_{\mu\nu}(0) | \vec{p}\,'_2, m'_2 \rangle \\ &\quad + \langle \vec{p}_2, m_2 | \vec{p}\,'_2, m'_2 \rangle \langle \vec{p}_1, m_1 | T^{(\bar{d})}_{\mu\nu}(0) | \vec{p}\,'_1, m'_1 \rangle] \\ &\quad \times \langle \vec{p}\,'_1, m'_1; \vec{p}\,'_2, m'_2 | P', \sqrt{s'} \rangle, \end{aligned} \tag{10}$$

where $\langle P, \sqrt{s} | \vec{p}_1, m_1; \vec{p}_2, m_2 \rangle$ is the Clebsch-Gordan coefficient (see Appendix A), the sums are over the variables m_1, m_2, m'_1, m'_2.

The method gives for the one-particle matrix elements in the rhs of (10) the form:

$$\langle \vec{p}, m|T^{(q)}_{\mu\nu}(0)|\vec{p}', m'\rangle = \sum_{m'} \langle m|D^{1/2}_w(p, p')|m'\rangle \times \langle m''|(1/2)g^{(q)}_{10}(Q^2)K'_\mu K'_\nu + ig^{(q)}_{40}(Q^2)[K'_\mu R_\nu + R_\mu K'_\nu] - g^{(q)}_{60}(Q^2)[Q^2 g_{\mu\nu} + K_\mu K_\nu]|m'\rangle, \tag{11}$$

$q = u, \bar{d}$, $D^j_w(p, p')$ is the transformation operator from the small group, the matrix of three-dimensional rotation, $g^{(u,\bar{d})}_{i0}$, $i = 1$, 4, 6 are the constituent-quark GFFs, the relation of which with conventional notations is given below in Sec. III.

$$K_\mu = (p - p')_\mu, \qquad K'_\mu = (p + p')_\mu,$$
$$R_\mu = \epsilon_{\mu\nu\lambda\rho}p^\nu\, p'^\lambda\, \Gamma^\rho(p'). \tag{12}$$

Here $\Gamma^\rho(p')$ is the well known 4-vector of spin (see, e.g., [24,31,34,47]), $\epsilon_{\mu\nu\lambda\rho}$ is the absolutely antisymmetric pseudotensor of rank 4, $\epsilon_{0123} = -1$.

Using the coefficients of the Clebsch-Gordan decomposition from Appendix A, we obtain free two-particle form factors in (9) in terms of gravitational form factors of constituents (11). The corresponding formulas are given in Appendix B.

Now let us exploit MIA in dealing with the obtained system of equations for the free two-particle form factors. MIA consists in replacing the invariant distribution in rhs of (7) by free two-particle form factors from Eq. (9). The physical meaning of MIA is equivalent to that of the universally accepted IA (8), because the free two-particle form factors are given in terms of one-particle currents.

Thus, we get the expressions for the pion GFFs in MIA, which for convenience we present in the form given in [25]:

$$G^{(\pi)}_{10}(Q^2) = \frac{1}{2}[g^{(u)}_{10}(Q^2) + g^{(\bar{d})}_{10}(Q^2)]G^{(\pi)}_{110}(Q^2) + [g^{(u)}_{40}(Q^2) + g^{(\bar{d})}_{40}(Q^2)]G^{(\pi)}_{140}(Q^2), \tag{13}$$

$$G^{(\pi)}_{60}(Q^2) = \frac{1}{2}[g^{(u)}_{10}(Q^2) + g^{(\bar{d})}_{10}(Q^2)]\, G^{(\pi)}_{610}(Q^2) + [g^{(u)}_{40}(Q^2) + g^{(\bar{d})}_{40}(Q^2)]\, G^{(\pi)}_{640}(Q^2) + [g^{(u)}_{60}(Q^2) + g^{(\bar{d})}_{60}(Q^2)]\, G^{(\pi)}_{660}(Q^2), \tag{14}$$

where $g^{(q)}_{i0}(Q^2), q = u, \bar{d}, i = 1$, 4, 6 are the GFFs of the constituent quarks, also defined previously in (11).

The form factors in the rhs of the Eqs. (13) and (14) are given now in terms of the integrals [24]:

$$G^{(\pi)}_{ij0}(Q^2) = \int d\sqrt{s}d\sqrt{s'}\varphi(s)G^{(0)}_{ij0}(s, Q^2, s')\varphi(s'), \tag{15}$$

where $i = 1$, 6; at $i = 1$ $j = 1$, 4; at $i = 6$ $j = 1$, 4, 6; $G^{(0)}_{1i0}(s, Q^2, s')$, $G^{(0)}_{6k0}(s, Q^2, s')$ are components of the free GFFs that describe the system of two free particles with total quantum numbers of pion, given in Appendix B, $\varphi(s)$ is the pion wave function in the sense of RQM (5), s', s are the invariant masses of the free two-particle system in the initial and final states, respectively.

Recall that the form factors $G^{(0)}_{k0}(s, Q^2, s')$ from (15) describe gravitational features of a system of two particles without interaction. Free two-particle form factors are regular generalized functions (distributions) given by the corresponding functionals, defined on the space of test functions depending on the variables (s, s'). The functionals, in turn, are functions of the variable $Q^2 = -t$, a square of momentum transfer. This variable is to be considered as a parameter.

In the frameworks of MIA, the pion GFFs are functionals (13)–(15), generated by the free two-particle GFFs of (9) on test functions which are the products of the two-quark wave functions, see (13)–(15).

III. ASYMPTOTIC EXPANSION OF PION GRAVITATIONAL FORM FACTORS

In this section we calculate pion GFFs at large momentum transfer using MIA in the IF RQM, that is in the instant form relativistic impulse approximation. The conventional pion GFFs are connected with the matrix elements given above by the equations:

$$A^{(\pi)}(Q^2) = G^{(\pi)}_{10}(Q^2), \qquad D^{(\pi)}(Q^2) = -2G^{(\pi)}_{60}(Q^2), \tag{16}$$

where $A^{(\pi)}$ and $D^{(\pi)}$ are commonly used pion GFFs (see, e.g., [4,48]), and $G^{(\pi)}_{10}$, $G^{(\pi)}_{60}$ are given by the equalities (13), (14), $t = (p_\pi - p'_\pi)^2 = -Q^2$, and p'_π, p_π are the pion 4-momenta in the initial and the final states, respectively.

To obtain the asymptotic expansion of the pion form factors given by (15) at $Q^2 \to \infty$, we use, for the quark-antiquark model wave function (5), the wave function of the ground state of a harmonic oscillator, widely used in composite quark models:

$$u(k) = 2(1/(\sqrt{\pi}b^3))^{1/2}\exp\left(-k^2/(2b^2)\right). \tag{17}$$

We use the parameter b in (17) fixed previously in the works [45,50] on the electroweak. properties of the pion. It is important to note, that the actual choice of the model wave function is not crucial in our approach.

As can be seen from the formulas for the pion form factors (15), the asymptotics of the pion GFFs depends on the behavior of quark GFFs at $Q^2 \to \infty$. Our GFFs of quarks, $g^{(q)}_{i0}(Q^2), q = u, \bar{d}, i = 1$, 4, 6 of (11), are related to generally accepted GFFs of a particle of spin 1/2 (see, e.g., [4]) as follows [24]:

$$g_{10}^{(q)}(Q^2) = \frac{1}{\sqrt{1+Q^2/4M^2}}\left[\left(1+\frac{Q^2}{4M^2}\right)A^{(q)}(Q^2) - 2\frac{Q^2}{4M^2}J^{(q)}(Q^2)\right], \quad (18)$$

$$g_{40}^{(q)}(Q^2) = -\frac{1}{M^2}\frac{J^{(q)}(Q^2)}{\sqrt{1+Q^2/4M^2}}, \quad (19)$$

$$g_{60}^{(q)}(Q^2) = -\frac{1}{2}\sqrt{1+\frac{Q^2}{4M^2}}D^{(q)}(Q^2), \quad (20)$$

where $A^{(q)}, J^{(q)}, D^{(q)}$ are the conventional GFFs of particles with spin 1/2 [4,48]. We assume that the GFFs of u- and $\bar{d}$-quarks are equal: $g_{i0}^{(u)}(Q^2) = g_{i0}^{(\bar{d})}(Q^2), i = 1, 4, 6$. However, for more generality, now we relax the condition of one and the same dependence of both D and A form factors of quark on momentum transfer (as it was assumed in [25]):

$$A^{(q)}(Q^2) = f_q^A(Q^2), \qquad J^{(q)}(Q^2) = \frac{1}{2}f_q^J(Q^2),$$
$$D^{(q)}(Q^2) = D_q f_q^D(Q^2), \quad (21)$$

where D_q is the quark D-term. The functions in rhs must ensure the standard static limits (see, e.g., [4]):

$$A^{(q)}(0) = 1, \qquad J^{(q)}(0) = \frac{1}{2}, \qquad D^{(q)}(0) = D_q. \quad (22)$$

The choice of functions in the right-hand sides of (21) will be discussed in more detail below.

Let us make one more remark about the actual use of MIA for calculation of the form factor D of the pion (15) and (16). In our work [25], it was found that in MIA the D-form factor has a singularity at $Q^2 = 0$. So, to make the form factor regular at zero, we were forced to abandon MIA in its pure form and use some of its minimal generalization. In the present work we are dealing with larger momentum transfer and therefore we do not need to go beyond MIA.

The pion GFFs (13)–(15), are expressed in our approach in terms of double integrals of some special kind. In particular, the boundary of the integration region (see the cutoff function $\vartheta(s, Q^2, s')$ in Appendix B) depends on the parameter of expansion, Q^2. The theorems defining the form of asymptotic expansions of such integrals, and the corresponding formulas were derived in our paper [44]. For the integrals in (13)–(15) with wave function (17) we have (see [44], Eqs. (55)–(58) at $l = l' = 0$):

$$G_{ij0}^{(\pi)}(Q^2) \sim A_{ij}\exp\left(-\frac{1}{2b^2}\left(\frac{M}{2}\sqrt{Q^2+4M^2} - M^2\right)\right) \times [g_{j0}^{(u)}(Q^2) + g_{j0}^{(\bar{d})}(Q^2)]\sum_{k=0}^{\infty}\sum_{m=0}^{\infty} h_{ij0}^{km}, \quad (23)$$

$$h_{ij0}^{km} = \sum_{p=0}^{p_m}\frac{b^{2m+2k-2p}}{Q^{2k+3m-5p-1/2}}\frac{2^{2m+5k/2-7p+5/2}}{M^{3p-m-1/2}}C_{2m}^{4p}\frac{(4p)!}{p!m!} \times \frac{\partial^{2m-4p}}{\partial t^{2m-4p}}\left[G_{ij0}^{(0)(k)}(t, Q^2, \phi(t)) \times \frac{\sqrt{(s-4M^2)(s'-4M^2)}}{\sqrt[4]{ss'}}\right]\Bigg|_{t=0}, \quad (24)$$

$$G_{ij0}^{(0)(k)}(t, Q^2, t') = \frac{\partial^k}{\partial t'^k}G_{ij0}^{(0)}(t, Q^2, t'), \quad (25)$$

where $p_m = m/2 + [(-1)^m - 1]/4$, the constant $A_{11} = A_{61} = 1/2$, all other constants $A_{ij} = 1$, C_{2m}^{4p} is binomial coefficient. The variables t, t' are linked to the integration variables in (15) by the relations:

$$s = \frac{Q^2}{\sqrt{2}}(t'+t) + 2M^2 + M\sqrt{Q^2+4M^2},$$
$$s' = \frac{Q^2}{\sqrt{2}}(t'-t) + 2M^2 + M\sqrt{Q^2+4M^2}. \quad (26)$$

The function $t' = \phi(t)$ defines in variables t, t' a part of the boundary of the integration region in (15) containing the point $(t, t') = (0, 0)$:

$$t' = \phi(t) = \sqrt{1+\frac{4M^2}{Q^2}}\left(-\frac{\sqrt{2}M}{Q} + \sqrt{\frac{2M^2}{Q^2} + t^2}\right). \quad (27)$$

The neighborhood of this point gives the main contribution to the asymptotic expansion of the integrals. The total boundary of the region of integration in (15) in variables (s, s') is given by the cutoff function $\vartheta(s, Q^2, s')$ given in Appendix B.

We derive the asymptotic expansion of pion GFFs (16) by means of general formulas (23)–(27), taking into account the expressions (13)–(16) and (18)–(21). Thus, in each term containing the functions $f_q^i(Q^2), i = A, J, D$ (21), we leave only two main terms of the $1/Q$ expansion, but the order of the retained terms must not exceed $1/Q^2$. So, the form factor A of the pion at $Q^2 \to \infty$ is

$$A^{(\pi)}(Q^2) = G_{10}^{(\pi)}(Q^2) \sim \exp\left(-\frac{MQ}{4b^2}\right)\exp\left(\frac{M^2}{2b^2}\right)\left[-2\sqrt{2}f_q^A(Q^2)\left(-4\frac{M}{Q}+20\frac{M^2}{Q^2}+24\frac{b^2}{Q^2}+2\frac{M^4}{b^2Q^2}\right)\right.$$
$$\left.-2\sqrt{2}(f_q^A(Q^2)-f_q^J(Q^2))\frac{Q^2}{4M^2}\left(-4\frac{M}{Q}+20\frac{M^2}{Q^2}+24\frac{b^2}{Q^2}+2\frac{M^4}{b^2Q^2}\right)+96\sqrt{2}f_q^J(Q^2)\frac{b^2}{Q^2}\right]. \tag{28}$$

The analogous expansion of the form factor D is of the form:

$$D^{(\pi)}(Q^2) = -2G_{60}^{(\pi)}(Q^2) \sim \exp\left(-\frac{MQ}{4b^2}\right)\exp\left(\frac{M^2}{2b^2}\right)\left[4\sqrt{2}D_q\sqrt{1+\frac{Q^2}{4M^2}}f_q^D(Q^2)\left(1-\frac{M}{Q}-\frac{M^3}{2b^2Q}\right)-32\sqrt{2}f_q^J(Q^2)\frac{b^2}{Q^2}\right]. \tag{29}$$

In the form-factor D expansion the term proportional to $f_q^A(Q^2)$ [see Eqs. (14), (18), and (21)] is of order $\sim 1/Q^4$ and therefore is not included in (29).

The consequences of the Eqs. (28) and (29) will be discussed in the next section.

IV. DISCUSSION OF RESULTS

From the expansions (28) and (29) it can be seen that the pion GFFs in our relativistic approach and for the Gaussian type model wave functions (17) show at $Q^2 \to \infty$ the exponential decay in the parameter Q. It is interesting to compare this result with that of the nonrelativistic case. Note, that in the nonrelativistic limit the integrals (15) with u from (17) can be calculated analytically. So, the corresponding nonrelativistic asymptotics, for example, of the form factor A is of the form:

$$A_{NR}^{(\pi)}(Q^2) = G_{10NR}^{(\pi)}(Q^2) \sim f_q^A(Q^2)\exp\left(-\frac{Q^2}{16b^2}\right). \tag{30}$$

So, in nonrelativistic limit of our approach we obtain the Gaussian (30) decay in parameter Q at large momentum transfer. Thus, the exponential decay in Eqs. (28) and (29) is a strictly relativistic effect. It is a consequence of our essentially relativistic nonperturbative approach.

Decay in (28) and (29) is rather fast and provides, in particular, a finite value of the rms mechanical radius of pion $\langle r^2\rangle_{\text{mech}}$, defined as follows (see, e.g., [4]):

$$\langle r^2\rangle_{\text{mech}} = 6\frac{D(0)}{\int_0^\infty D(Q^2)dQ^2}, \tag{31}$$

The exponential decay of pion form factor D (29) ensures the convergence of the integral at the upper limit in (31) and, thus, its finite value. The obtained decreasing of GFFs (28) and (29) differs fundamentally from the power-law form in perturbative QCD [16–18] $\sim 1/Q^2$. In this latter case the integral in the denominator of (31) diverges, and the rms mechanical radius is zero. However, it can be shown that in our approach there exists a limiting transition to the results coinciding with the predictions of QCD, i.e., a kind of correspondence principle is satisfied. We formulate it as follows. At large momentum transfer, where perturbative QCD is applicable, its predictions can be obtained as certain limiting case of our nonperturbative asymptotics (28) and (29).

In this connection, let us first discuss the asymptotic expansion of the form factor A of the pion (28). To ensure a correct transition in the Eq. (28) to small masses of constituents $M \to 0$, we require the equality: $f_q^A(Q^2) = f_q^J(Q^2)$. Then at $M \to 0$ the Eq. (28) takes the form:

$$A^{(\pi)}(Q^2) = G_{10}^{(\pi)}(Q^2) \sim -48\sqrt{2}f_q^A(Q^2)\frac{b^2}{Q^2} + 96\sqrt{2}f_q^J(Q^2)\frac{b^2}{Q^2}. \tag{32}$$

In the expansion (32) there is no exponent, the presence of which in (28) is due to the type of two-quark wave function (17), i.e., to the elastic coupling between quarks. Thus, the transition to zero mass in the asymptotic region $Q^2 \to \infty$ effectively eliminates any manifestation of quark-antiquark interactions. Note that the second term in the sum (32) has a purely relativistic origin and is a consequence of the relativistic effect of spin rotation [46]. At neglecting this effect this summand vanishes [see (15) and Eq. (B2) in Appendix B].

Let us now discuss the constraints on the functions $f_q^A(Q^2)$ and $f_q^J(Q^2)$ which can be derived using the asymptotic expansion (32). In our previous work [25] we calculated the pion GFFs at finite momentum transfer assuming that these functions are equal to one another, that is to the same function $f_q(Q^2)$:

$$f_q^A(Q^2) = f_q^J(Q^2) = f_q(Q^2) = \frac{1}{1+\ln(1+\langle r_q^2\rangle Q^2/6)}, \tag{33}$$

where $\langle r_q^2\rangle$ is the rms mass radius of the constituent quark, which we choose to be equal to its charge radius.

The function (33) has been used successfully in our works on the electroweak properties of the pion to describe electric and magnetic form factors of constituent quarks.

This function was derived from asymptotics of the charge form factor of the pion at $Q^2 \to \infty$ [45]. We note that at large momentum transfer the function (33) decreases as the inverse of the logarithm of Q^2. Because of this slow decreasing of quark form factors, such a choice corresponds to quark which is close to the point quark.

The choice of the form (33) causes the appearence of logarithmic multipliers in the expansion without changing the actual powers of $1/Q^2$ in (32):

$$A^{(\pi)}(Q^2) = G_{10}^{(\pi)}(Q^2) \sim 48\sqrt{2}\frac{b^2}{Q^2} f_q(Q^2). \qquad (34)$$

In (34) we have for the form factor A of pion, with the acceptable accuracy, up to logarithmic multiplier, the same power-law decrease that is obtained in the perturbative QCD [17]. To calculate GFFs at finite momentum transfer [24,25] we fixed the parameter $\langle r_q^2\rangle$, entering (33), basing on the works [51–54]: $\langle r_q^2\rangle \simeq 0.3/M^2$. However, this relation is valid only for finite masses of constituents in hadrons. In our papers [24,25,33,34,45], we have fixed this parameter at the value of the mass of constituents $M = 0.22$ GeV, which has long ago become generally accepted in relativistic calculations (see, e.g., [55,56]). In the present work in our asymptotic calculations we consider the mean-square mass radius of the constituents as a free parameter.

If we go to point quarks, i.e., when $\langle r_q^2\rangle = 0$ in (33), simultaneously with the limiting transition to the zero mass of constituents, that is, in fact, from constituent quarks to current quarks, we get the asymptotics, coinciding with the predictions of the perturbative QCD [17]:

$$A^{(\pi)}(Q^2) = G_{10}^{(\pi)}(Q^2) \sim 48\sqrt{2}\frac{b^2}{Q^2}. \qquad (35)$$

Note that this result coincides, up to a numerical multiplier, with the result obtained in our approach for the asymptotics of the charge form factor of the pion [45]. The correct (positive) sign of (35) is due to the presence of the second term of the sum in (32), i.e., to the relativistic effect of spin rotation. It is interesting that the asymptotics of (35) in the pointlike limit for quarks $\langle r_q^2\rangle \to 0$ takes place for any choice of decreasing function in (33), provided that this function has the form of the product $\langle r_q^2\rangle Q^2$.

Let us consider now the asymptotic expansion of the pion D-form factor and, in particular, discuss the constraints, which can be derived, concerning the possible form of the quark form factor D (21). If we take the function $f_q^D(Q^2)$ entering the D-form factor of the quark (21) in the form with logarithm (33), used at finite Q^2 [25], then from the asymptotic expansion (29) at $Q^2 \to \infty$ and in the limit of small M we obtain an increasing, i.e., unphysical behavior of the form factor D of the pion. Thus, for this choice of the quark form factor D there is no limit transition from the constituent quarks to current quarks. So, to satisfy the correspondence principle it is necessary to choose some other form for this quark form factor.

There is one more argument in favor of changing the form of function $f_q^D(Q^2)$ in (21) and (29), a rather simple one. Indeed, when we choose $f_q^D(Q^2)$ in the form (33) the rms mechanical radius of the constituent quark $\langle r_q^2\rangle_{\rm mech}$ is zero, as can be seen from the formula analogous to (31): the integral in the denominator diverges at upper limit.

On the other hand, we consider a constituent quark as a quasiparticle, that has all the properties of a real particle, in particular, a nonzero mean-square mass radius $\langle r_q^2\rangle$ (33). To obtain a nonzero value also of the rms mechanical radius of the quark it is necessary to have for its form factor D a function with power-law decreasing in $1/Q^2$ with the power higher than one. To satisfy the condition of decreasing of the pion form factor D at $Q^2 \to \infty$ in the (29) in the limit of small quark masses simultaneously with the condition of finiteness of the rms of the mechanical radius of the constituent quark allows, in particular, for the following choice of the function $f_q^D(Q^2)$:

$$f_q^D(Q^2) = \frac{1}{\sqrt{1+Q^2/4M^2}}\frac{1}{1+\langle r_q^2\rangle Q^2/6}. \qquad (36)$$

Note that the power-law dependence of the electromagnetic form factors of constituent quarks, similar to (36), was considered in [54].

Taking into account the explicit form of the function (36), the main terms of the expansion (29) in the limit of almost zero masses of constituents, we obtain:

$$D^{(\pi)}(Q^2) = -2G_{60}^{(\pi)}(Q^2) \sim 4\sqrt{2}\frac{D_q}{1+\langle r_q^2\rangle Q^2/6} - 32\sqrt{2} f_q^J(Q^2)\frac{b^2}{Q^2}. \qquad (37)$$

It can be seen that when $Q^2 \to \infty$, the expression (37) means that the form factor D of the pion behaves as $\sim 1/Q^2$, coinciding with the predictions of QCD. The second summand in (37), due to the choice of the function $f_q^J(Q^2)$ in the form (33), contains a logarithmically decreasing multiplier. The second term in the sum in (37), as well as in (32) is due to the relativistic effect of spin rotation [see formulas (15) and (B4) in Appendix B]. This term has a decisive meaning in both cases.

Let us now move on in our discussion of (37) to point quarks, i.e., to current quarks. There is a general result for the D-term of the pointlike noninteracting fermion with spin 1/2 [4,49], namely, it was shown that the D-term of such a fermion is zero. If appeal to the concept of asymptotic freedom at $Q^2 \to \infty$, then the transition from constituent to current quark means $D_q = 0$ and $\langle r_q^2\rangle = 0$ in expressions (21) and (37). Thus, when going to the point quarks, the expansion (37) takes the form:

$$D^{(\pi)}(Q^2) = -2G_{60}^{(\pi)}(Q^2) \sim -32\sqrt{2}\frac{b^2}{Q^2}. \quad (38)$$

The resulting formula for the asymtotics of the form factor D of the pion coincides with predicted by QCD and differs by a numerical multiplier from the asymptotics of the A-form factor (35), as takes place in QCD as well [17]. So, a kind of correspondence principle is valid for the pion form factor D, too. Note that analogous correspondence was obtained in our works on electroweak properties of scalar mesons, where we described the electromagnetic form factors of pion and kaon at large momentum transfer [41,42]. In the limit of zero-mass point quarks, our approach again appears to be common to problems of electroweak and gravitational structures of the pion. In the electroweak case the coincidence with perturbative QCD predictions was obtained not only in calculations with Gaussian functions of harmonic oscillator (17), but also with other functions [39], in particular with rational functions [57]. We expect the same for the gravitational case.

The asymptotics (38), which coincides with the QCD predictions, in the limit of the point quarks ($D_q = 0, \langle r_q^2\rangle = 0$), can be obtained for rather arbitrary choice of second multiplier in the quark form factor D (36), for example, any power of the multiplier can be used instead.

As can be seen from (35) and (38), the multiplier before the asymptotics depends on the parameter of our model b, which determines the actual scale of the confinement. In calculations of the pion GFFs at finite momentum transfer [24,25] in the model (17), as well as in our previous successful works on the electroweak properties of the pion [33,34,45], we have used value $b = 0.35$ GeV. This value gives good results also in other forms of RQM, for example, in calculation of electroweak decays of pions in the point-form dynamics [56].

Note that the asymptotics of the form factor D of the pion in the limit of point quarks remains to be completely determined by the second summand in (37), i.e., completely is due to the relativistic effect of spin rotation. Thus, in the asymptotic expansions (35) and (38), obtained in our approach and coinciding with the predictions of the perturbative QCD, it is the kinematical relativistic effect of spin rotation that plays a determining role.

Let us discuss another consequence of choosing the function $f_q^D(Q^2)$ in the form (36). In this case we obtain the rms mechanical radius of the quark (31) in the following form:

$$\langle r_q^2\rangle_{\mathrm{mech}} = \frac{\sqrt{6\langle r_q^2\rangle}}{4M}\sqrt{1 - 2M^2\langle r_q^2\rangle/3} \times \frac{1}{\arccos\left(\sqrt{2M^2\langle r_q^2\rangle/3}\right)}. \quad (39)$$

It can be seen from (39), that the mechanical radius of the constituent quark is zero in the limit of zero mass radius. This expression also can be used in the calculation of gravitational properties of pions and other composite hadrons at finite momentum transfer and finite masses of constituents.

V. CONCLUSIONS

In this paper, the asymptotic expansions of A and D pion GFFs are obtained at large momentum transfer, $Q^2 \to \infty$. We used a version of the instant-form of Dirac relativistic quantum mechanics with fixed number of particles (IF RQM) complemented by an essentially relativistic variant of impulse approximation, formulated previously in our papers on the electroweak structure of composite particles. The calculation was carried out in the model of interaction of constituent quarks with quadratic confinement, namely with two-quark wave functions of the ground state of the harmonic oscillator, i.e., with Gaussians. The asymptotic decreasing of the form factors obtained in the paper is exponential in the parameter Q, multiplied by a polynomial in $1/Q$. In the nonrelativistic case the analog of our model approach with Gaussian wave function gives the Gaussian decreasing of the GFFs for increasing Q. So, the asymptotic exponential decrease of the GFFs in this interaction model is a consequence of our fundamentally essentially relativistic study of the problem.

It is shown that for the obtained asymptotic relativistic expansion there exists a limit transition from constituent to current quarks, i.e., the transition to pointlike quarks of almost zero mass. This limit obtained in our principally nonperturbative approach, gives the asymptotics of gravitational form factors, coinciding with the predictions of the perturbative QCD ($\sim 1/Q^2$) [16–18]. Thus, an analog of the correspondence principle is satisfied in the following terms: in the region where the perturbative QCD is applicable, its predictions can be obtained as a limit case of the fundamentally nonperturbative relativistic model of composite particles. Note that this correspondence principle is fulfilled also in the case of electromagnetic structure of the pion and kaon.

From the coincidence principle we obtained certain constraints on the allowed form of quarks GFFs and proposed simple formulas for them. Such kind of constraints are currently in demand (see, for example, [26]) because of the existing arbitrariness in calculations of hadron form factors in composite models. To obtain the quark GFFs we use not only constraints on the piom GFFs asymptotics, but also the property of the finiteness of the rms mechanical radius of quark.

An equation is also derived connecting the rms mechanical and mass radii of constituent quarks, (39), which can be used in the calculations of gravitational properties of composite systems.

In this work we find that the determining contribution to the asymptotics of pion GFFs, in the limit of massless and pointlike quarks, coinciding with the predictions of the perturbative QCD, comes in our approach from the relativistic kinematical effect of rotation of quark spins (the Wigner spin rotation). In particular, the correct, i.e., coinciding with predictions of the QCD asymptotics of the pion form factor D is completely determined by this kinematic effect.

To summarize, in this paper we have increased the scope of our research of our previous papers [24,25] to the range of large momentum transfer and, by the way, obtained several results that can be used elsewhere.

APPENDIX A: THE CLEBSCH-GORDAN COEFFICIENT

The Clebsch-Gordan coefficient of the Poincaré group (10):

$$\begin{aligned}&\langle\vec{p}_1, m_1; \vec{p}_2, m_2|\vec{P}, \sqrt{s}, J, l, S, m_J\rangle\\&= \sqrt{2s}[\lambda(s, M^2, M^2)]^{-1/2} 2P_0\delta(P - p_1 - p_2)\\&\quad\times\sum\langle m_1|D_w^{1/2}(p_1, P)|\tilde{m}_1\rangle\langle m_2|D_w^{1/2}(p_2, P)|\tilde{m}_2\rangle\\&\quad\times\langle 1/2 1/2\tilde{m}_1\tilde{m}_2|Sm_S\rangle Y_{lm_l}(\vartheta, \varphi)\\&\quad\times\langle Slm_Sm_l|Jm_J\rangle,\end{aligned}\qquad\text{(A1)}$$

where $\vec{p} = (\vec{p}_1 - \vec{p}_2)/2,\ p = |\vec{p}|,\ \vartheta, \varphi$ are the spherical angles of the vector $\vec{p}$ in c.m.s., Y_{lm_l} is the spherical function, $\langle Sm_S|1/2\,1/2\tilde{m}_1\tilde{m}_2\rangle$ and $\langle Jm_J|Slm_Sm_l\rangle$ are the Clebsh-Gordan coefficients of the group $SU(2)$, $\langle\tilde{m}|D_w^{1/2}(P, p)|m\rangle$ is the matrix of the three-dimensional spin rotation, that is necessary for the relativistic invariant summation of the particle spins, the sum being over $\tilde{m}_1, \tilde{m}_2, m_l, m_S$,

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc),\qquad\text{(A2)}$$

M is the mass of constituent quarks.

We use in present paper the Clebsch-Gordan coefficient with pion quantum numbers $J = l = S = 0$.

APPENDIX B: FREE TWO-PARTICLE GRAVITATIONAL FORM FACTORS

GFF of two noninteracting fermion with spin 1/2 (15):

$$\begin{aligned}G^{(0)}_{110}(s, Q^2, s') = &-\frac{R(s, Q^2, s')Q^2}{\lambda(s, -Q^2, s')}\\&\times[(4M^2 + Q^2)\lambda(s, -Q^2, s')\\&-3Q^2(s + s' + Q^2)^2]\cos(\omega_1 + \omega_2),\end{aligned}\qquad\text{(B1)}$$

$$\begin{aligned}G^{(0)}_{140}(s, Q^2, s') = &-3M\frac{R(s, Q^2, s')Q^4}{\lambda(s, -Q^2, s')}\\&\times\xi(s, Q^2, s')(s + s' + Q^2)\sin(\omega_1 + \omega_2),\end{aligned}\qquad\text{(B2)}$$

$$\begin{aligned}G^{(0)}_{610}(s, Q^2, s') = &\frac{1}{2}R(s, Q^2, s')[(s + s' + Q^2)^2\\&-(4M^2 + Q^2)\lambda(s, -Q^2, s')/Q^2]\\&\times\cos(\omega_1 + \omega_2),\end{aligned}\qquad\text{(B3)}$$

$$\begin{aligned}G^{(0)}_{640}(s, Q^2, s') = &-\frac{M}{2}R(s, Q^2, s')\,\xi(s, Q^2, s')(s + s' + Q^2)\\&\times\sin(\omega_1 + \omega_2),\end{aligned}\qquad\text{(B4)}$$

$$G^{(0)}_{660}(s, Q^2, s') = R(s, Q^2, s')\lambda(s, -Q^2, s')\cos(\omega_1 + \omega_2),\qquad\text{(B5)}$$

where

$$\begin{aligned}R(s, Q^2, s') &= \frac{(s + s' + Q^2)}{2\sqrt{(s - 4M^2)(s' - 4M^2)}}\\&\quad\times\frac{\vartheta(s, Q^2, s')}{[\lambda(s, -Q^2, s')]^{3/2}},\\\xi(s, Q^2, s') &= \sqrt{-(M^2\lambda(s, -Q^2, s') - ss'Q^2)},\end{aligned}$$

ω_1 and ω_2 are the Wigner spin-rotation parameters:

$$\begin{aligned}\omega_1 &= \arctan\frac{\xi(s, Q^2, s')}{M[(\sqrt{s} + \sqrt{s'})^2 + Q^2] + \sqrt{ss'}(\sqrt{s} + \sqrt{s'})},\\\omega_2 &= \arctan\frac{\alpha(s, s')\xi(s, Q^2, s')}{M(s + s' + Q^2)\alpha(s, s') + \sqrt{ss'}(4M^2 + Q^2)},\end{aligned}$$

$\alpha(s, s') = 2M + \sqrt{s} + \sqrt{s'}$, $\vartheta(s, Q^2, s') = \theta(s' - s_1) - \theta(s' - s_2)$, θ is the Heaviside function.

$$\begin{aligned}s_{1,2} &= 2M^2 + \frac{1}{2M^2}(2M^2 + Q^2)(s - 2M^2)\\&\quad\mp\frac{1}{2M^2}\sqrt{Q^2(4M^2 + Q^2)s(s - 4M^2)},\end{aligned}$$

$\lambda(a, b, c)$ and M are determined in (A2).

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