Novel T_{cs} and $T_{c\bar{s}}$ candidates in a constituent-quark-model-based meson-meson coupled-channels calculation

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Using proton-proton collisions at center-of-mass energies 7, 8, and 13 TeV, with a total integrated luminosity of 9 fb^{-1}, the LHCb collaboration has performed amplitude analyses of the $B^+ \to D^+ D^- K^+$, $B^+ \to D^- D_s^+ \pi^+$, and $B^0 \to \bar{D}^0 D_s^+ \pi^-$ decays, observing that new T_{cs} and $T_{c\bar{s}}$ resonances are required in order to explain the experimental data. These signals could be the first observation of tetraquark candidates that do not contain a heavy quark-antiquark pair; in fact, they consist of four different flavors of quarks, one of which is a doubly charged open-charm state. We present herein an analysis of the T_{cs} and $T_{c\bar{s}}$ states, which is an extension of our recently published study of similar T_{cc}^+ exotic candidates. Our theoretical framework is a constituent-quark-model-based coupled-channels calculation of $qq'\bar{s}\,\bar{c}$ and $cq\bar{s}\bar{q}'$ tetraquark sectors for T_{cs} and $T_{c\bar{s}}$ structures, respectively. We explore the nature, and pole position, of the singularities that appear in the scattering matrix with spin-parity quantum numbers: $J^P = 0^\pm$, $1^\mp$, and $2^\pm$. The constituent quark model has been widely used in the heavy quark sector, and thus all model parameters are already constrained from previous works. This makes our predictions robust and parameter-free. We find many singularities in the solution of various scattering-matrix problems which are either virtual states or resonances, but not bound states. Some of them fit reasonably well with the experimental observations of the spin-parity, mass, and width of T_{cs} and $T_{c\bar{s}}$ candidates, and thus tentative assignments are made.

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I. INTRODUCTION

A very successful classification scheme for hadrons in terms of their valence quarks and antiquarks was proposed independently by Murray Gell-Mann [1] and George Zweig [2] in 1964. This classification, called the quark model, basically divides hadrons into two large families: mesons and baryons. These are quark-antiquark and three-quark bound-states, respectively.[1] However, QCD allows for the existence of more complex structures, generically called exotic hadrons or simply exotics [1]. These include tetra-, penta-, and even hexaquark systems, hadronic states with active gluonic degrees of freedom (hybrids), and even bound states consisting only of gluons (glueballs). This other class of hadrons, in addition to mesons and baryons, that can be observed tells us a lot about the nature of QCD.

Exotic hadrons have been systematically searched for since the 1960s in numerous experiments around the globe, without success until a remarkable discovery in 2003 when the Belle Collaboration found evidence of a narrow new particle at 3872 MeV [4], decaying to $J/\psi \to \pi^+\pi^-$ and $J/\psi \to \pi^+\pi^-\pi^0$ that behaves very unlike a pure $c\bar{c}$ state. This $X(3872)$ ["X" simply indicating "unknown"], is a "charmoniumlike" state (meaning that all of its known decays contain a $c\bar{c}$ pair), and is almost certainly a hadron of valence quark content $c\bar{c}q\bar{q}$. After this observation many new hadrons that do not exhibit the expected properties of ordinary mesons and baryons were discovered. These hadrons belong mostly to the heavy quark sector and are collectively known as XYZ states. An enormous effort devoted to unravel the nature of these exotic hadrons has been deployed using a wide variety of theoretical approaches. In fact, one can already find many comprehensive reviews on the subject in the literature [5–16].

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[1]For further details, the interested reader is referred to the Particle Data Group and its topical minireview on the subject [3].

At the end of 2020 the LHCb collaboration carried out an amplitude analysis of $B^+ \to D^+ D^- K^+$ decays using proton-proton collision data taken at $\sqrt{s} = 7$, 8, and 13 TeV, with an integrated luminosity of 9 fb^{-1} [17,18]. In order to obtain good agreement with the experimental data, it turned out to be necessary to include new spin-0 and spin-1 T_{cs} resonances in the $D^- K^+$ channel. These signals may constitute the first observation of exotic hadrons not containing a heavy quark-antiquark pair and, moreover, they could be the first experimental detection of four-quark candidates with four different flavors of quarks $ud\bar{s}\,\bar{c}$.

The Breit-Wigner parameters of these resonances are

$$T_{cs0}(2900)^0\colon\; M = (2866 \pm 7 \pm 2)\ \mathrm{MeV/c^2}, \quad \Gamma = (57 \pm 12 \pm 4)\ \mathrm{MeV}, \tag{1}$$

$$T_{cs1}(2900)^0\colon\; M = (2904 \pm 5 \pm 1)\ \mathrm{MeV/c^2}, \quad \Gamma = (110 \pm 11 \pm 4)\ \mathrm{MeV}, \tag{2}$$

where the first uncertainties are statistical and the second systematic.

In December 2022, the LHCb collaboration reported in Refs. [19,20] a combined amplitude analysis for the decays $B^0 \to \bar{D}^0 D_s^+ \pi^-$ and $B^+ \to D^- D_s^+ \pi^+$, based on proton-proton collision data at centre-of-mass energies of 7, 8, and 13 TeV, with an integrated luminosity of 9 fb^{-1}. The enhancement in the $D_s^+\pi^+$ invariant mass of the may indicate the first observation of a $T_{c\bar{s}}$ doubly charged open-charm tetraquark state with minimal quark content $c\bar{s}u\bar{d}$; whereas the one observed in the $D_s^+\pi^-$ channel is interpreted as the $T_{c\bar{s}}$ neutral partner of an isospin triplet. Both $T_{c\bar{s}}$ candidates are found to have isospin 1 and spin-parity $J^P = 0^+$. The Breit-Wigner mass and width of the new resonant states are:

$$T^a_{c\bar{s}0}(2900)^0\colon\; M = (2892 \pm 14 \pm 15)\ \mathrm{MeV/c^2}, \quad \Gamma = (119 \pm 26 \pm 13)\ \mathrm{MeV}, \tag{3}$$

$$T^a_{c\bar{s}0}(2900)^{++}\colon\; M = (2921 \pm 17 \pm 20)\ \mathrm{MeV/c^2}, \quad \Gamma = (137 \pm 32 \pm 17)\ \mathrm{MeV}, \tag{4}$$

Interestingly, the $cq\bar{s}\bar{q}'$ sector was one of the first to show evidence of exotic structures, with the discovery of the $D^*_{s0}(2317)$ and the $D_{s1}(2460)$ in 2003 that did not fit the quark model expectations. These states are instead isoscalars and were tackled by our group considering them an effect of the coupling of conventional $c\bar{s}$ states with nearby DK and D^*K channels [21]. Since the recent $T^a_{c\bar{s}}$ states are isospin-1, there is no chance of coupling with naive meson structures, and a pure coupled-channels calculation has to be assumed.

Assuming the neutral $D_s^+\pi^-$ resonance and the doubly charged $D_s^+\pi^+$ resonance belong to the same isospin triplet, the common mass and width are determined to be [20]

$$M = (2908 \pm 11 \pm 20)\ \mathrm{MeV/c^2}, \quad \Gamma = (136 \pm 23 \pm 13)\ \mathrm{MeV}, \tag{5}$$

The above announcements made by the LHCb collaboration triggered a lot of theoretical work using a wide variety of approaches. Regarding the T_{cs} candidates, one can mention interpretations of these states using QCD sum rules [22–26], nonrelativistic and (extended) relativized quark models with different types of quark–(anti)quark interactions [27–37], and effective field theories [38–40]. They can be also interpreted as triangle singularities [41,42]. In addition, their decay and production properties have been studied in the literature [43–45]. With respect to $T_{c\bar{s}}$ signals, the literature is more limited; to mention only a few, the reader is referred to Refs. [46–53].

An important question about the nature of the exotic hadrons is whether they are expected to be compact objects like ordinary hadrons or whether they behave like hadron-hadron molecular states. Certainly, the difference between the two possibilities should lie in the dynamics of the quarks, however, on general grounds, when the state is near a hadron-hadron threshold, and therefore its binding energy B is small or, more precisely, when $R = \sqrt{\hbar/2MB}$ is large, the multiquark system should look more like a hadron-hadron molecule. Although its multiquark nature should not be forgotten.

In this manuscript we analyse the nature of the recently discovered $T_{c\bar{s}}$ and T_{cs} states in a multichannel calculation using the resonating group method and the constituent quark model (CQM) of Refs. [54,55]. Quarks and antiquarks are suppose to form two meson clusters which interacts through an effective cluster-cluster interaction which emerges from the underlying quark dynamics. The model has been widely used in the heavy quark sector, by studying their spectra [56–60], their electromagnetic, weak and strong decays and reactions [61–65], their possible compact multiquark components [66–70] and also their potential coupling to meson-meson channels [71–75]. The advantage of using an approach with a relatively long history is that all model parameters are already constrained by previous works. Consequently, from this perspective, we present a parameter-free calculation of the $T_{c\bar{s}}$ and T_{cs} states, which is also an extension of our recently published analysis of similar T^+_{cc} exotic candidates [76].

The manuscript is structured as follows. After this introduction, the theoretical framework is briefly presented in Sec. II. Section III is mainly devoted to the analysis and discussion of our theoretical results. Finally, we summarize and draw some conclusions in Sec. IV.

II. THEORETICAL FORMALISM

A. Naive quark model

Dynamical chiral symmetry breaking of the QCD Lagrangian together with the perturbative one-gluon exchange (OGE) and the nonperturbative confining interactions are the main pieces of potential models. Using this idea, Vijande *et al.* [54] developed a model of the quark-(anti)quark interaction which is able to describe meson phenomenology from the light to the heavy quark sectors. We briefly explain the model below. Further details can be found in Refs. [54,55,58].

One consequence of the dynamical chiral symmetry breaking is that the nearly massless *current* light quarks acquire a dynamical, momentum-dependent mass $M(p)$ with $M(0) \approx 300$ MeV for the u and d quarks, namely, the constituent mass. To preserve chiral invariance of the QCD Lagrangian new interaction terms, given by Goldstone boson exchanges, should appear between constituent quarks.

A simple Lagrangian invariant under chiral transformations can be derived as [77]

$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - MU^{\gamma_5})\psi, \tag{6}$$

where $U^{\gamma_5} = \exp(i\pi^a\lambda^a\gamma_5/f_\pi)$, π^a denotes the pseudoscalar fields $(\vec{\pi}, K, \eta_8)$ and M is the constituent quark mass. The momentum-dependent mass acts as a natural cutoff of the theory. The chiral quark-(anti)quark interaction can be written as

$$V_{qq}(\vec{r}_{ij}) = V_{qq}^{\mathrm{C}}(\vec{r}_{ij}) + V_{qq}^{\mathrm{T}}(\vec{r}_{ij}) + V_{qq}^{\mathrm{SO}}(\vec{r}_{ij}), \tag{7}$$

where C, T, and SO stand for central, tensor, and spin-orbit potentials. The central part presents four different contributions,

$$V_{qq}^{\mathrm{C}}(\vec{r}_{ij}) = V_{\pi}^{\mathrm{C}}(\vec{r}_{ij}) + V_{\sigma}^{\mathrm{C}}(\vec{r}_{ij}) + V_{K}^{\mathrm{C}}(\vec{r}_{ij}) + V_{\eta}^{\mathrm{C}}(\vec{r}_{ij}), \tag{8}$$

given by

$$\begin{aligned}
V_{\pi}^{\mathrm{C}}(\vec{r}_{ij}) &= \frac{g_{ch}^2}{4\pi}\frac{m_\pi^2}{12m_im_j}\frac{\Lambda_\pi^2}{\Lambda_\pi^2 - m_\pi^2}m_\pi\left[Y(m_\pi r_{ij}) - \frac{\Lambda_\pi^3}{m_\pi^3}Y(\Lambda_\pi r_{ij})\right](\vec{\sigma}_i\cdot\vec{\sigma}_j)\sum_{a=1}^{3}(\lambda_i^a\cdot\lambda_j^a),\\
V_{\sigma}^{\mathrm{C}}(\vec{r}_{ij}) &= -\frac{g_{ch}^2}{4\pi}\frac{\Lambda_\sigma^2}{\Lambda_\sigma^2 - m_\sigma^2}m_\sigma\left[Y(m_\sigma r_{ij}) - \frac{\Lambda_\sigma}{m_\sigma}Y(\Lambda_\sigma r_{ij})\right],\\
V_{K}^{\mathrm{C}}(\vec{r}_{ij}) &= \frac{g_{ch}^2}{4\pi}\frac{m_K^2}{12m_im_j}\frac{\Lambda_K^2}{\Lambda_K^2 - m_K^2}m_K\left[Y(m_K r_{ij}) - \frac{\Lambda_K^3}{m_K^3}Y(\Lambda_K r_{ij})\right](\vec{\sigma}_i\cdot\vec{\sigma}_j)\sum_{a=4}^{7}(\lambda_i^a\cdot\lambda_j^a),\\
V_{\eta}^{\mathrm{C}}(\vec{r}_{ij}) &= \frac{g_{ch}^2}{4\pi}\frac{m_\eta^2}{12m_im_j}\frac{\Lambda_\eta^2}{\Lambda_\eta^2 - m_\eta^2}m_\eta\left[Y(m_\eta r_{ij}) - \frac{\Lambda_\eta^3}{m_\eta^3}Y(\Lambda_\eta r_{ij})\right](\vec{\sigma}_i\cdot\vec{\sigma}_j)[\cos\theta_p(\lambda_i^8\cdot\lambda_j^8) - \sin\theta_p],
\end{aligned} \tag{9}$$

where $Y(x)$ is the standard Yukawa function defined by $Y(x) = e^{-x}/x$. We consider the physical η meson instead of the octet one and so we introduce the angle θ_p. The λ^a are the SU(3) flavor Gell-Mann matrices, m_i is the quark mass and m_π, m_K, and m_η are the masses of the SU(3) Goldstone bosons, taken from experimental values. The value of m_σ used herein is given by the partially conserved axial current (PCAC) relation $m_\sigma^2 \simeq m_\pi^2 + 4m_{u,d}^2$ [78]. Note, however, that better determinations of the mass of the σ-meson have been reported since then, see the relatively recent review [79]; one should simply consider the value used herein as a model parameter. Finally, the chiral coupling constant, g_{ch}, is determined from the πNN coupling constant through

$$\frac{g_{ch}^2}{4\pi} = \frac{9}{25}\frac{g_{\pi NN}^2}{4\pi}\frac{m_{u,d}^2}{m_N^2}, \tag{10}$$

which assumes that flavor SU(3) is an exact symmetry only broken by the different mass of the strange quark.

There are three different contributions to the tensor potential

$$V_{qq}^{\mathrm{T}}(\vec{r}_{ij}) = V_{\pi}^{\mathrm{T}}(\vec{r}_{ij}) + V_{K}^{\mathrm{T}}(\vec{r}_{ij}) + V_{\eta}^{\mathrm{T}}(\vec{r}_{ij}), \tag{11}$$

given by

$$\begin{aligned}
V_{\pi}^{\mathrm{T}}(\vec{r}_{ij}) &= \frac{g_{ch}^2}{4\pi}\frac{m_\pi^2}{12m_im_j}\frac{\Lambda_\pi^2}{\Lambda_\pi^2 - m_\pi^2}m_\pi\left[H(m_\pi r_{ij}) - \frac{\Lambda_\pi^3}{m_\pi^3}H(\Lambda_\pi r_{ij})\right]S_{ij}\sum_{a=1}^{3}(\lambda_i^a\cdot\lambda_j^a),\\
V_{K}^{\mathrm{T}}(\vec{r}_{ij}) &= \frac{g_{ch}^2}{4\pi}\frac{m_K^2}{12m_im_j}\frac{\Lambda_K^2}{\Lambda_K^2 - m_K^2}m_K\left[H(m_K r_{ij}) - \frac{\Lambda_K^3}{m_K^3}H(\Lambda_K r_{ij})\right]S_{ij}\sum_{a=4}^{7}(\lambda_i^a\cdot\lambda_j^a),\\
V_{\eta}^{\mathrm{T}}(\vec{r}_{ij}) &= \frac{g_{ch}^2}{4\pi}\frac{m_\eta^2}{12m_im_j}\frac{\Lambda_\eta^2}{\Lambda_\eta^2 - m_\eta^2}m_\eta\left[H(m_\eta r_{ij}) - \frac{\Lambda_\eta^3}{m_\eta^3}H(\Lambda_\eta r_{ij})\right]S_{ij}[\cos\theta_p(\lambda_i^8\cdot\lambda_j^8) - \sin\theta_p].
\end{aligned} \tag{12}$$

$S_{ij} = 3(\vec{\sigma}_i \cdot \hat{r}_{ij})(\vec{\sigma}_j \cdot \hat{r}_{ij}) - \vec{\sigma}_i \cdot \vec{\sigma}_j$ is the quark tensor operator and $H(x) = (1 + 3/x + 3/x^2)Y(x)$.

Finally, the spin-orbit potential only presents a contribution coming from the scalar part of the interaction

$$V^{\rm SO}_{qq}(\vec{r}_{ij}) = V^{\rm SO}_{\sigma}(\vec{r}_{ij}) = -\frac{g^2_{ch}}{4\pi}\frac{m^3_\sigma}{2m_i m_j}\frac{\Lambda^2_\sigma}{\Lambda^2_\sigma - m^2_\sigma} \times \left[G(m_\sigma r_{ij}) - \frac{\Lambda^3_\sigma}{m^3_\sigma}G(\Lambda_\sigma r_{ij})\right](\vec{L}\cdot\vec{S}). \quad (13)$$

In the last equation $G(x)$ is the function $(1 + 1/x)Y(x)/x$.

Beyond the chiral symmetry breaking scale one expects the dynamics to be governed by QCD perturbative effects. In this way one-gluon fluctuations around the instanton vacuum are taken into account through the qqg coupling

$$\mathcal{L}_{qqg} = i\sqrt{4\pi\alpha_s}\,\overline{\psi}\gamma_\mu G^\mu_c \lambda^c \psi, \quad (14)$$

with λ^c being the $SU(3)$ color matrices and G^μ_c the gluon field.

The different terms of the potential derived from the Lagrangian contain central, tensor, and spin-orbit contributions and are given by

$$\begin{aligned}
V^{\rm C}_{\rm OGE}(\vec{r}_{ij}) &= \frac{1}{4}\alpha_s(\vec{\lambda}^c_i\cdot\vec{\lambda}^c_j)\left[\frac{1}{r_{ij}} - \frac{1}{6m_i m_j}(\vec{\sigma}_i\cdot\vec{\sigma}_j)\frac{e^{-r_{ij}/r_0(\mu)}}{r_{ij}r^2_0(\mu)}\right],\\
V^{\rm T}_{\rm OGE}(\vec{r}_{ij}) &= -\frac{1}{16}\frac{\alpha_s}{m_i m_j}(\vec{\lambda}^c_i\cdot\vec{\lambda}^c_j)\left[\frac{1}{r^3_{ij}} - \frac{e^{-r_{ij}/r_g(\mu)}}{r_{ij}}\left(\frac{1}{r^2_{ij}} + \frac{1}{3r^2_g(\mu)} + \frac{1}{r_{ij}r_g(\mu)}\right)\right]S_{ij},\\
V^{\rm SO}_{\rm OGE}(\vec{r}_{ij}) &= -\frac{1}{16}\frac{\alpha_s}{m^2_i m^2_j}(\vec{\lambda}^c_i\cdot\vec{\lambda}^c_j)\left[\frac{1}{r^3_{ij}} - \frac{e^{-r_{ij}/r_g(\mu)}}{r^3_{ij}}\left(1 + \frac{r_{ij}}{r_g(\mu)}\right)\right]\\
&\quad\times[((m_i + m_j)^2 + 2m_i m_j)(\vec{S}_+\cdot\vec{L}) + (m^2_j - m^2_i)(\vec{S}_-\cdot\vec{L})], \quad (15)
\end{aligned}$$

where $\vec{S}_\pm = \frac{1}{2}(\vec{\sigma}_i \pm \vec{\sigma}_j)$. Besides, $r_0(\mu) = \hat{r}_0\frac{\mu_{nn}}{\mu_{ij}}$ and $r_g(\mu) = \hat{r}_g\frac{\mu_{nn}}{\mu_{ij}}$ are regulators which depend on μ_{ij}, the reduced mass of the $q\bar{q}$ pair. The contact term of the central potential has been regularized as

$$\delta(\vec{r}_{ij}) \sim \frac{1}{4\pi r^2_0}\frac{e^{-r_{ij}/r_0}}{r_{ij}} \quad (16)$$

The wide energy range needed to provide a consistent description of light, strange, and heavy mesons requires an effective scale-dependent strong coupling constant. We use the frozen coupling constant of Ref. [54]

$$\alpha_s(\mu) = \frac{\alpha_0}{\ln\left(\frac{\mu^2+\mu^2_0}{\Lambda^2_0}\right)}, \quad (17)$$

in which μ is the reduced mass of the $q\bar{q}$ pair and α_0, μ_0 and Λ_0 are parameters of the model determined by a global fit to the meson spectra.

Confinement is one of the crucial aspects of QCD. Color charges are confined inside hadrons. It is well known that multigluon exchanges produce an attractive linearly rising potential proportional to the distance between quarks. This idea has been confirmed, but not rigorously proved, by quenched lattice gauge Wilson loop calculations for heavy valence quark systems. However, sea quarks are also important ingredients of the strong interaction dynamics. When included in the lattice calculations they contribute to the screening of the rising potential at low momenta and eventually to the breaking of the quark-antiquark binding string. This fact, which has been observed in $n_f = 2$ lattice QCD [80], has been taken into account in our model by including the terms

$$\begin{aligned}
V^{\rm C}_{\rm CON}(\vec{r}_{ij}) &= [-a_c(1 - e^{-\mu_c r_{ij}}) + \Delta](\vec{\lambda}^c_i\cdot\vec{\lambda}^c_j),\\
V^{\rm SO}_{\rm CON}(\vec{r}_{ij}) &= -(\vec{\lambda}^c_i\cdot\vec{\lambda}^c_j)\frac{a_c\mu_c e^{-\mu_c r_{ij}}}{4m^2_i m^2_j r_{ij}}[((m^2_i + m^2_j)(1 - 2a_s) + 4m_i m_j(1 - a_s))(\vec{S}_+\cdot\vec{L}) + (m^2_j - m^2_i)(1 - 2a_s)(\vec{S}_-\cdot\vec{L})], \quad (18)
\end{aligned}$$

where a_s controls the mixture between the scalar and vector Lorentz structures of the confinement. At short distances this potential presents a linear behavior with an effective confinement strength $\sigma = -a_c\mu_c(\vec{\lambda}^c_i\cdot\vec{\lambda}^c_j)$ and becomes constant at large distances with a threshold defined by

$$V_{\rm thr} = \{-a_c + \Delta\}(\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c). \quad (19)$$

No $q\bar{q}$ bound states can be found for energies higher than this threshold. The system suffers a transition from a color string configuration between two static color sources into a pair of static mesons due to the breaking of the color string and the most favored decay into hadrons.

Among the different methods to solve the Schrödinger equation in order to find the quark-antiquark bound states, we use the Gaussian expansion method [81] because it provides sufficient accuracy and simplifies the subsequent evaluation of the required matrix elements. This procedure yields the radial wave function solution of the Schrödinger equation as an expansion in terms of basis functions

$$R_\alpha(r) = \sum_{n=1}^{n_{\rm max}} c_n^\alpha \phi_{nl}^G(r), \quad (20)$$

where α refers to the channel quantum numbers. The coefficients, c_n^α, and the eigenvalue, E, are determined from the Rayleigh-Ritz variational principle

$$\sum_{n=1}^{n_{\rm max}} \left[(T_{n'n}^\alpha - EN_{n'n}^\alpha)c_n^\alpha + \sum_{\alpha'} V_{n'n}^{\alpha\alpha'} c_n^{\alpha'} = 0\right], \quad (21)$$

where $T_{n'n}^\alpha$, $N_{n'n}^\alpha$ and $V_{n'n}^{\alpha\alpha'}$ are the matrix elements of the kinetic energy, the normalization and the potential, respectively. $T_{n'n}^\alpha$ and $N_{n'n}^\alpha$ are diagonal whereas the mixing between different channels is given by $V_{n'n}^{\alpha\alpha'}$.

Following Ref. [81], we employ Gaussian trial functions with ranges in geometric progression. This enables the optimization of ranges employing a small number of free parameters. Moreover, the geometric progression is dense at short distances, so that it allows the description of the dynamics mediated by short range potentials. The fast damping of the Gaussian tail is not a problem, since we can choose the maximal range much longer than the hadronic size.

Table I shows the model parameters fitted over all meson spectra [54], updated in Ref. [56]. We would like to point out here that the interaction terms between light-light, light-heavy, and heavy-heavy quarks are not the same in our formalism, i.e., while Goldstone-boson exchanges are considered when the two quarks are light, they do not appear in the other two configurations: light-heavy and heavy-heavy; however, the one-gluon exchange and confinement potentials are blinded in flavor and so they affect all the cases.

TABLE I. Quark model parameters.

Quark masses	m_n (MeV)	313
	m_s (MeV)	555
	m_c (MeV)	1763
	m_b (MeV)	5110
Goldstone Bosons	m_π (fm^{-1})	0.70
	m_σ (fm^{-1})	3.42
	m_K (fm^{-1})	2.51
	m_η (fm^{-1})	2.77
	Λ_π (fm^{-1})	4.20
	Λ_σ (fm^{-1})	4.20
	Λ_K (fm^{-1})	4.21
	Λ_η (fm^{-1})	5.20
	$g_{ch}^2/4\pi$	0.54
	θ_p (°)	−15
OGE	α_0	2.118
	Λ_0 (fm^{-1})	0.113
	μ_0 (MeV)	36.976
	$\hat{r}_0$ (fm)	0.181
	$\hat{r}_g$ (fm)	0.259
Confinement	a_c (MeV)	507.4
	μ_c (fm^{-1})	0.576
	Δ (MeV)	184.432
	a_s	0.81

B. Resonating group method

The aforementioned CQM specifies the microscopic interaction between the constituent quarks and antiquarks. To describe the interaction at the meson level, we use the resonating group method (RGM) [82,83], where mesons are considered as quark-antiquark clusters and an effective cluster-cluster interaction emerges from the underlying quark(antiquark) dynamics (see, e.g., Refs. [84,85] for details). The main idea behind the RGM is that the degrees of freedom of the particles within a cluster are frozen, resulting in a fixed wave function for the internal degrees of freedom. Consequently, the interactions solely contribute to the dynamics of relative degrees of freedom between clusters.

Traditionally, the RGM has been formulated in coordinate space. However, the introduction of antisymmetry leads to non-localities in the potentials between clusters, thereby resulting in a final RGM equation that becomes an integro-differential equation, making its solution more complex. Nevertheless, an alternative formulation in momentum space is also feasible, where the treatment of local or non-local interactions becomes entirely equivalent, yielding an integral equation. Moreover, it is worth noting that in momentum space, the coupling between different channels can be readily implemented, whereas it is considerably more intricate in coordinate space.

We assume that the wave function of a system composed of two mesons A and B can be written as[2]

$$\langle \vec{p}_A \vec{p}_B \vec{P} \vec{P}_{\rm c.m.} | \psi \rangle = \mathcal{A}[\phi_A(\vec{p}_A)\phi_B(\vec{p}_B)\chi_\alpha(\vec{P})], \quad (22)$$

[2]Note that, for the simplicity of the discussion presented here, we have omitted the spin-isospin wave function, the product of the two color singlets and the wave function describing the center-of-mass motion.

where $\mathcal{A}$ is the full antisymmetric operator, $\phi_C(\vec{p}_C)$ is the wave function of a general meson C calculated in the naive quark model, and $\vec{p}_C$ is the relative momentum between the quark and antiquark of the meson C. The wave function taking into account the relative motion of the two mesons is $\chi_\alpha(\vec{P})$, where α denotes the set of quantum numbers needed to uniquely define a particular partial wave.

For the $T_{c\bar{s}}$, with a minimum quark content of $cq\bar{s}\bar{q}'$, there are no indistinguishable quarks among clusters, so $\mathcal{A}=1$. In the case of the T_{cs}, with a minimum quark content of $qq'\bar{c}\,\bar{s}$, we have two identical quarks, and the antisymmetrizer can be written as $\mathcal{A}=\frac{1}{2}(1-P_q)$, with P_q the operator that exchanges the light quark between mesons.

The dynamics of the system is governed by the Hamiltonian

$$\mathcal{H}=\sum_{i=1}^{N}\frac{\vec{p}_i^{\,2}}{2m_i}+\sum_{i<j}V_{ij}-T_{\rm CM} \tag{23}$$

where we have removed the kinetic energy of the center-of-mass $T_{\rm CM}$, m_i is the (constituent) mass of quark i and V_{ij} is the interactions between quarks i and j. Using this Hamiltonian, we can build the projected Schrödinger equation as a variational equation,

$$(\mathcal{H}-E_T)|\psi\rangle=0\Rightarrow\langle\delta\psi|(\mathcal{H}-E_T)|\psi\rangle=0. \tag{24}$$

Under the assumption that the internal wave function of the mesons remains fixed, the variations are solely applied to the relative wave function. Consequently, all possible internal degrees of freedom are integrated out.

The projected Schrödinger equation for the relative wave function can be written as follows:

$$\left(\frac{\vec{P}'^2}{2\mu}-E\right)\chi_\alpha(\vec{P}')+\sum_{\alpha'}\int[{}^{\rm RGM}V_D^{\alpha\alpha'}(\vec{P}',\vec{P}_i)+{}^{\rm RGM}K^{\alpha\alpha'}(\vec{P}',\vec{P}_i)]\chi_{\alpha'}(\vec{P}_i)d\vec{P}_i=0, \tag{25}$$

where $E=E_T-E_{M_1}-E_{M_2}$ is the relative energy between clusters, with E_T the total energy of the system, $\vec{P}_i$ is a continuous parameter and ${}^{\rm RGM}V_D^{\alpha\alpha'}(\vec{P}',\vec{P}_i)$ and ${}^{\rm RGM}K^{\alpha\alpha'}(\vec{P}',\vec{P}_i)$ are the direct and exchange RGM kernels, respectively.

The direct potential ${}^{\rm RGM}V_D^{\alpha\alpha'}(\vec{P}',\vec{P}_i)$, from the factor 1 in $\mathcal{A}$, can be written as

$$\begin{aligned}{}^{\rm RGM}V_D^{\alpha\alpha'}(\vec{P}',\vec{P}_i)&=\sum_{i\in A,j\in B}\int d\vec{p}_{A'}d\vec{p}_{B'}d\vec{p}_{A}d\vec{p}_{B}\\&\times\phi_{A'}^*(\vec{p}_{A'})\phi_{B'}^*(\vec{p}_{B'})V_{ij}^{\alpha\alpha'}(\vec{P}',\vec{P}_i)\phi_A(\vec{p}_A)\phi_B(\vec{p}_B).\end{aligned} \tag{26}$$

where $V_{ij}^{\alpha\alpha'}$ is the CQM potential between the quark i and the quark j of the mesons A and B, respectively.

The exchange kernel ${}^{\rm RGM}K$ models the quark rearrangement between mesons. For T_{cs}, the exchange kernel comes from the term P_q in $\mathcal{A}$, and it is expressed in terms of overlap integrals involving the internal wave functions when quarks are exchanged between different mesons. Consequently, they are more important at short distances. The kernel is a nonlocal and energy-dependent term which can be separated in a potential term plus a normalization term, given by

$${}^{\rm RGM}K(\vec{P}',\vec{P}_i)={}^{\rm RGM}H_E(\vec{P}',\vec{P}_i)-E_T\,{}^{\rm RGM}N_E(\vec{P}',\vec{P}_i) \tag{27}$$

where

$$\begin{aligned}{}^{\rm RGM}H_E(\vec{P}',\vec{P}_i)&=\int d\vec{p}_{A'}d\vec{p}_{B'}d\vec{p}_{A}d\vec{p}_{B}d\vec{P}\phi_{A'}^*(\vec{p}_{A'})\\&\times\phi_{B'}^*(\vec{p}_{B'})\mathcal{H}(\vec{P}',\vec{P})P_q[\phi_A(\vec{p}_A)\phi_B(\vec{p}_B)\delta^{(3)}(\vec{P}-\vec{P}_i)],\end{aligned} \tag{28a}$$

$$\begin{aligned}{}^{\rm RGM}N_E(\vec{P}',\vec{P}_i)&=\int d\vec{p}_{A'}d\vec{p}_{B'}d\vec{p}_{A}d\vec{p}_{B}d\vec{P}\phi_{A'}^*(\vec{p}_{A'})\\&\times\phi_{B'}^*(\vec{p}_{B'})P_q[\phi_A(\vec{p}_A)\phi_B(\vec{p}_B)\delta^{(3)}(\vec{P}-\vec{P}_i)],\end{aligned} \tag{28b}$$

For $T_{c\bar{s}}$ the antisymmetrizer is $\mathcal{A}=1$, so we only have direct interaction between DK. Nevertheless, the exchange diagrams represents a natural way to connect meson-meson channels with the same quark content, such as $DK\to D_s\pi$ channels. In that case, the exchange kernel is reduced to a quark rearrangement potential ${}^{\rm RGM}V_R(\vec{P}',\vec{P}_i)$, given by

$$\begin{aligned}{}^{\rm RGM}V_R(\vec{P}',\vec{P}_i)&=\sum_{i\in A,j\in B}\int d\vec{p}_{A'}d\vec{p}_{B'}d\vec{p}_{A}d\vec{p}_{B}d\vec{P}\phi_{A'}^*(\vec{p}_{A'})\\&\times\phi_{B'}^*(\vec{p}_{B'})V_{ij}(\vec{P}',\vec{P})P_{mn}[\phi_A(\vec{p}_A)\phi_B(\vec{p}_B)\delta^{(3)}(\vec{P}-\vec{P}_i)],\end{aligned} \tag{29}$$

where P_{mn} is the operator that exchanges the quark m of A with the quark n of B.

From Eq. (25), we derive a set of coupled Lippmann-Schwinger equations of the form

$$T_\alpha^{\alpha'}(E;p',p)=V_\alpha^{\alpha'}(p',p)+\sum_{\alpha''}\int dp''p''^2V_{\alpha''}^{\alpha'}(p',p'')\times\frac{1}{E-\mathcal{E}_{\alpha''}(p'')}T_\alpha^{\alpha''}(E;p'',p), \tag{30}$$

where $V_{\alpha}^{\alpha'}(p',p)$ is the projected potential containing the direct and rearrangement kernels, and $\mathcal{E}_{\alpha''}(p'')$ is the energy corresponding to a momentum p'', written in the nonrelativistic case as

$$\mathcal{E}_{\alpha}(p) = \frac{p^2}{2\mu_{\alpha}} + \Delta M_{\alpha}. \tag{31}$$

Here, μ_{α} is the reduced mass of the (AB)-system corresponding to the channel α, and ΔM_{α} is the difference between the threshold of the (AB)-system and the one we use as a reference.

We solve the coupled Lippmann-Schwinger equations using the matrix-inversion method proposed in Ref. [86], but generalized to include channels with different thresholds. Once the T-matrix is computed, we determine the on-shell part which is directly related to the scattering matrix. In the case of nonrelativistic kinematics, it can be written as

$$S_{\alpha}^{\alpha'} = 1 - 2\pi i\sqrt{\mu_{\alpha}\mu_{\alpha'}k_{\alpha}k_{\alpha'}}T_{\alpha}^{\alpha'}(E+i0^{+};k_{\alpha'},k_{\alpha}), \tag{32}$$

where k_{α} is the on-shell momentum for channel α, defined by,

$$k_{\alpha}^2 = 2\mu_{\alpha}(E - \Delta M_{\alpha}) \tag{33}$$

Our aim is to explore the existence of states above and below thresholds within the same formalism. Thus, we have to continue analytically all the potentials and kernels for complex momenta in order to find the poles of the T-matrix in any possible Riemann sheet.

For each channel, we can define two Riemann sheets. The first Riemann sheet is defined as $0 \leq \arg(k_{\alpha}) < \pi$, whereas the second Riemann sheet is defined as $\pi \leq \arg(k_{\alpha}) < 2\pi$. Poles of the T-matrix on the first Riemann sheet on the real axis below threshold are interpreted as bound states. Poles on the second Riemann sheet below threshold are identified as virtual states, while those above threshold are interpreted as resonances.

III. RESULTS

A detailed discussion of the peculiarities of our coupled-channels calculation will be given in the following subsections. However, two comments are in order here. The first one refers to the theoretical uncertainties. There are two types of theoretical uncertainties in our results: one is intrinsic to the numerical algorithm and the other is related to the way the model parameters are fixed. The numerical error is negligible and, as mentioned above, the model parameters are adjusted to reproduce a certain number of hadron observables within a determinate range of agreement with experiment. It is therefore difficult to assign an error to these parameters and consequently to the quantities calculated using them. In order to analyze the uncertainty of the calculations presented in this manuscript, we will estimate the error of the pole properties by varying the strength of our potentials by $\pm 10\%$.

The second comment has to do with the fact that the experimental resonance parameters are obtained by a Breit-Wigner parametrization and one should be caution to compare these values with the pole positions.

A. Nature of T_{cs} states

We perform a coupled-channels calculation in charged basis[3] of the $J^P = 0^+$, 1^- and 2^+ $qq'\bar{s}\,\bar{c}$ sectors in which the D^-K^+ discovery-channel can be measured. We include the following meson-meson channels in the calculation[4]: $\bar{D}^0K^0$ (2362.45), D^-K^+ (2363.34), $D^{*-}K^{*+}$ (2901.92), and $\bar{D}^{*0}K^{*0}$ (2902.40). Besides the direct interaction between $\bar{D}^{(*)}K^{(*)}$ pairs, we have to consider exchange diagrams to deal with indistinguishable quarks from different mesons in the molecule. Moreover, the decay width of the strange vector meson is large enough to be included in the calculation, i.e. it is taken into account in Eq. (31) in such a way that the corresponding real valued mass, M, is replaced by the complex expression $M - i\Gamma/2$, with the mass M and total decay width Γ of the particular meson taken from the Particle Listings of the Review of Particle Physics collected by the Particle Data Group [3]. The experimental values of the widths reported in Ref. [3] for the neutral and charged partners, respectively, are $\Gamma_{K^{*0}} = 47.3$ MeV and $\Gamma_{K^{*\pm}} = 51.4$ MeV.

With all the above, our calculation yields the information shown in Table II. The first observation is that many poles appear in the scattering matrix; they are either virtual or resonance states but we do not find bound states. A tentative assignment of the $T_{cs0}(2900)^0$ experimental signal would be the second state with quantum numbers $J^P = 0^+$. Its mass and width are 2902 MeV/c^2 and 51 MeV, respectively; both compare well with the corresponding experimental values shown in Eq. (1). It is worth noting that a very similar state appears in the $J^P = 2^+$ channel; however, there is a resonance close (at 2.922 GeV/c^2) to it which has a large decay width that could interfere with the experimental signal. We find a possible candidate of the $T_{cs1}(2900)^0$ signal in the $J^P = 1^-$ channel with pole parameters 2888 MeV/c^2 and 190 MeV, and whose nature seems to be virtual. Notice that the 1^- $\bar{D}^*K^*$ molecule is in a relative P-wave, whereas it is in a

[3]The charged basis is selected in this case because there is no experimental evidence of the isospin content of the T_{cs} states.
[4]In parenthesis the mass of the threshold in MeV/c^2.

TABLE II. Coupled-channels calculation of the $J^P = 0^+$, 1^- and 2^+ $qq'\bar{s}\,\bar{c}$ sectors (T_{cs} states) in which the D^-K^+ discovery-channel can be measured. We include the following meson-meson channels in the calculation (in parenthesis the threshold's mass in $\mathrm{MeV/c^2}$): $\bar{D}^0K^0$ (2362.45), D^-K^+ (2363.34), $D^{*-}K^{*+}$ (2901.92) and $\bar{D}^{*0}K^{*0}$ (2902.40). Errors are estimated by varying the strength of the potential by $\pm 10\%$. 1st column: Pole's quantum numbers; 2nd column: Pole's mass in $\mathrm{MeV/c^2}$; 3rd column: Pole's width in MeV; 4th column: Refers to $\bar{D}^0K^0$, D^-K^+, $D^{*-}K^{*+}$ and $\bar{D}^{*0}K^{*0}$ Riemann sheets, respectively, with F meaning first and S second; 5th-6th columns: Isospin channel probabilities in %; 7th-10th columns: Channel probabilities in %; 11th–14th columns: Branching ratios in %.

J^P	$M_{\rm pole}$	$\Gamma_{\rm pole}$	RS	$\mathcal{P}_{I=0}$	$\mathcal{P}_{I=1}$	$\mathcal{P}_{\bar{D}^0K^0}$	$\mathcal{P}_{D^-K^+}$	$\mathcal{P}_{D^{*-}K^{*+}}$	$\mathcal{P}_{\bar{D}^{*0}K^{*0}}$	$\mathcal{B}_{\bar{D}^0K^0}$	$\mathcal{B}_{D^-K^+}$	$\mathcal{B}_{D^{*-}K^{*+}}$	$\mathcal{B}_{\bar{D}^{*0}K^{*0}}$
0^+	2340.0 ± 0.6	$28.8^{+0.5}_{-0.7}$	(S, S, F, F)	96^{+2}_{-3}	4^{+3}_{-2}	57^{+5}_{-4}	33^{+5}_{-6}	5.4 ± 0.6	$4.3^{+0.2}_{-0.3}$	0 ± 0	0 ± 0	0 ± 0	0 ± 0
	$2901.9^{+0.5}_{-0.8}$	51^{+0}_{-1}	(S, S, S, S)	22^{+7}_{-13}	78^{+13}_{-7}	$0.5^{+0.2}_{-0}$	$0.1^{+0.1}_{-0}$	54^{+1}_{-34}	45^{+34}_{-1}	72^{+2}_{-12}	28^{+12}_{-14}	0^{+7}_{-0}	0^{+4}_{-0}
1^-	$2887.7^{+0.3}_{-0.4}$	$189.5^{+0.4}_{-0.6}$	(S, S, S, S)	10^{+4}_{-3}	90^{+3}_{-4}	$0.7^{+0.6}_{-0.4}$	9.2 ± 0.8	$51.0^{+0.9}_{-0.5}$	$39.1^{+0.1}_{-0.7}$	12 ± 5	88 ± 5	0 ± 0	0 ± 0
	3010 ± 2	257^{+23}_{-20}	(S, S, S, S)	100 ± 0	0 ± 0	2.44 ± 0.06	2.51 ± 0.06	$48.5^{+0}_{-0.1}$	$46.6^{+0.1}_{-0}$	$6.0^{+0.5}_{-0.4}$	$6.2^{+0.4}_{-0.5}$	$44.1^{+0.5}_{-0.6}$	$43.7^{+0.5}_{-0.4}$
2^+	2902^{+1}_{-2}	$49.4^{+0.2}_{-0.3}$	(S, S, S, S)	14^{+30}_{-12}	86^{+12}_{-30}	$0.05^{+0.03}_{-0.01}$	$0.04^{+0.03}_{-0.02}$	56^{+5}_{-3}	44^{+3}_{-5}	48^{+2}_{-46}	50^{+2}_{-48}	0^{+55}_{-0}	0^{+40}_{-0}
	2922 ± 2	287^{+8}_{-7}	(S, S, S, S)	$99.98^{+0}_{-0.01}$	$0.02^{+0.01}_{-0}$	$5.26^{+0.02}_{-0.01}$	$5.28^{+0.02}_{-0.01}$	$45.45^{+0.04}_{-0.03}$	$44.01^{+0.05}_{-0.08}$	11.8 ± 0.5	$11.9^{+0.6}_{-0.4}$	38.6 ± 0.5	$37.6^{+0.6}_{-0.5}$
	3160 ± 1	522^{+39}_{-33}	(S, S, S, S)	100 ± 0	0 ± 0	$2.1^{+0.3}_{-0.1}$	$2.1^{+0.3}_{-0.1}$	$48.4^{+0.1}_{-0.3}$	$47.4^{+0.1}_{-0.3}$	3.4 ± 0.2	3.5 ± 0.2	$46.7^{+0.2}_{-0.3}$	$46.5^{+0.2}_{-0.3}$

TABLE III. Coupled-channels calculation of the of the $J^P = 0^-$, 1^+ and 2^- $qq'\bar{s}\,\bar{c}$ sectors (T_{cs} states) including the following meson-meson channels in the calculation (in parenthesis the threshold's mass in $\mathrm{MeV/c^2}$): $D^{*-}K^+$ (2503.94), $\bar{D}^{*0}K^0$ (2504.46), $\bar{D}^0K^{*0}$ (2760.39), D^-K^{*+} (2761.32), $D^{*-}K^{*+}$ (2901.92) and $\bar{D}^{*0}K^{*0}$ (2902.40). Errors are estimated by varying the strength of the potential by $\pm 10\%$. 1st column: Pole's quantum numbers; 2nd column: Pole's mass in $\mathrm{MeV/c^2}$; 3rd column: Pole's width in MeV; 4th column: Refers to $D^{*-}K^+$, $\bar{D}^{*0}K^0$, $\bar{D}^0K^{*0}$, D^-K^{*+}, $D^{*-}K^{*+}$ and $\bar{D}^{*0}K^{*0}$ Riemann sheets, respectively, with F meaning first and S second; 5th-6th columns: Isospin channel probabilities in %; 7th-12th columns: Channel probabilities in %; 13th-18th columns: Branching ratios in %.

J^P	$M_{\rm pole}$	$\Gamma_{\rm pole}$	RS	$\mathcal{P}_{I=0}$	$\mathcal{P}_{I=1}$	$\mathcal{P}_{D^{*-}K^+}$	$\mathcal{P}_{\bar{D}^{*0}K^0}$	$\mathcal{P}_{\bar{D}^0K^{*0}}$	$\mathcal{P}_{D^-K^{*+}}$	$\mathcal{P}_{D^{*-}K^{*+}}$	$\mathcal{P}_{\bar{D}^{*0}K^{*0}}$	$\mathcal{B}_{D^{*-}K^+}$	$\mathcal{B}_{\bar{D}^{*0}K^0}$	$\mathcal{B}_{\bar{D}^0K^{*0}}$	$\mathcal{B}_{D^-K^{*+}}$	$\mathcal{B}_{D^{*-}K^{*+}}$	$\mathcal{B}_{\bar{D}^{*0}K^{*0}}$
0^-	$2744.6^{+0.1}_{-0.2}$	77.45 ± 0.03	(S, S, S, S, F, F)	74.7 ± 0.7	25.3 ± 0.7	$37.8^{+0.6}_{-0.7}$	8.3 ± 0.3	19.6 ± 0.3	$1.5^{+0.2}_{-0.1}$	26.7 ± 0.4	$5.97^{+0.07}_{-0.08}$	90.0 ± 0.8	10.0 ± 0.8	0 ± 0	0 ± 0	0 ± 0	0 ± 0
	3018^{+7}_{-11}	378 ± 3	(S, S, S, S, S, S)	$99.97^{+0.02}_{-0.03}$	$0.03^{+0.03}_{-0.01}$	15^{+1}_{-3}	15^{+1}_{-2}	10^{+0}_{-1}	10^{+1}_{-0}	26 ± 2	25^{+2}_{-3}	18^{+2}_{-3}	17^{+3}_{-2}	$13.1^{+0.2}_{-0.3}$	$13.0^{+0.7}_{-0.6}$	20^{+1}_{-2}	19 ± 2
1^+	2488 ± 3	0.04 ± 0.01	(S, S, F, F, F, F)	1^{+3}_{-0}	99^{+0}_{-3}	57^{+10}_{-3}	42^{+3}_{-11}	$0.3^{+0.4}_{-0.1}$	$0.13^{+0.01}_{-0.07}$	$0.3^{+0}_{-0.1}$	$0.6^{+0.4}_{-0.2}$	0 ± 0	0 ± 0	0 ± 0	0 ± 0	0 ± 0	0 ± 0
	2757 ± 2	50.3 ± 0.2	(S, S, S, S, F, F)	1^{+2}_{-0}	99^{+0}_{-2}	$1.0^{+0.4}_{-0.3}$	$2.4^{+0.6}_{-0.5}$	50^{+2}_{-1}	$46.6^{+0.2}_{-0.9}$	$0.12^{+0.04}_{-0.03}$	$0.10^{+0.04}_{-0.03}$	33^{+3}_{-4}	67^{+4}_{-3}	0 ± 0	0 ± 0	0 ± 0	0 ± 0
	2896^{+3}_{-9}	57^{+1}_{-28}	(S, S, S, S, S, S)	5^{+82}_{-2}	95^{+2}_{-82}	3 ± 1	3^{+11}_{-1}	9^{+7}_{-5}	12^{+3}_{-6}	40^{+14}_{-8}	31^{+1}_{-10}	36^{+3}_{-31}	38^{+28}_{-3}	12^{+0}_{-2}	13^{+5}_{-0}	0 ± 0	0 ± 0
	2903^{+2}_{-6}	34^{+10}_{-12}	(S, S, S, S, F, F)	99^{+1}_{-7}	1^{+7}_{-1}	10^{+4}_{-6}	10^{+6}_{-7}	3 ± 2	3 ± 2	29^{+4}_{-0}	41^{+22}_{-10}	32^{+1}_{-0}	$34.3^{+0.1}_{-0.4}$	$16.8^{+0.4}_{-0.7}$	16.6 ± 0.1	0 ± 0	0 ± 0
2^-	2744.4 ± 0.2	236.0 ± 0.2	(S, S, S, S, F, F)	52^{+4}_{-3}	48^{+3}_{-4}	6^{+2}_{-1}	$17.6^{+0.4}_{-0.2}$	22 ± 2	33 ± 4	$4.4^{+0.8}_{-0.6}$	$16.8^{+0.4}_{-0.3}$	26 ± 5	74 ± 5	0 ± 0	0 ± 0	0 ± 0	0 ± 0
	2936^{+3}_{-11}	330^{+24}_{-41}	(S, S, S, S, S, S)	$100.0^{+0}_{-0.2}$	$0.0^{+0.2}_{-0}$	10^{+0}_{-2}	9^{+0}_{-1}	15 ± 2	16^{+1}_{-3}	25^{+2}_{-1}	26 ± 1	$9.8^{+0.5}_{-0.4}$	$9.0^{+0.8}_{-0.5}$	25^{+2}_{-4}	26^{+2}_{-4}	15^{+3}_{-2}	16^{+3}_{-2}

TABLE IV. Coupled-channels calculation of the isospin-1 $J^P = 0^+$, 1^- and 2^+ $cq\bar{s}\bar{q}'$ sectors ($T_{c\bar{s}}$ states), in which the $D_s\pi$ discovery-channel can be naturally measured. We include the following meson-meson channels in the calculation (in parenthesis the threshold's mass in MeV/c^2): $D_s^+\pi^-$ (2107.92), D^0K^0 (2362.45), $D_s^{*+}\rho^-$ (2887.46) and $D^{*0}K^{*0}$ (2902.40). Errors are estimated by varying the strength of the potential by $\pm 10\%$. 1st column: Pole's quantum numbers; 2nd column: Pole's mass in MeV/c^2; 3rd column: Pole's width in MeV; 4th column: Refers to $D_s^+\pi^-$, D^0K^0, $D_s^{*+}\rho^-$ and $D^{*0}K^{*0}$ Riemann sheets, respectively, with F meaning first and S second; 5th-8th columns: Channel probabilities in %; 9th-12th columns: Branching ratios in %.

J^P	M_{pole}	Γ_{pole}	RS	$\mathcal{P}_{D_s^+\pi^-}$	$\mathcal{P}_{D^0K^0}$	$\mathcal{P}_{D_s^{*+}\rho^-}$	$\mathcal{P}_{D^{*0}K^{*0}}$	$\mathcal{B}_{D_s^+\pi^-}$	$\mathcal{B}_{D^0K^0}$	$\mathcal{B}_{D_s^{*+}\rho^-}$	$\mathcal{B}_{D^{*0}K^{*0}}$
0^+	2892^{+4}_{-3}	156^{+60}_{-7}	(S, S, S, F)	0^{+4}_{-0}	1^{+9}_{-1}	90^{+10}_{-33}	10^{+20}_{-10}	43^{+4}_{-38}	22^{+2}_{-5}	35^{+35}_{-1}	0 ± 0
	2954^{+8}_{-9}	129^{+13}_{-11}	(S, S, S, S)	$0.5^{+0}_{-0.3}$	$0.8^{+0.3}_{-0.5}$	81^{+13}_{-8}	18^{+8}_{-12}	$6.2^{+1.0}_{-0.8}$	3 ± 0	54 ± 1	36 ± 2
1^-	2889^{+2}_{-1}	248^{+0}_{-1}	(S, S, S, S)	$0.65^{+0.04}_{-0.07}$	$1.6^{+0.1}_{-0.2}$	55^{+4}_{-3}	42 ± 4	25^{+6}_{-5}	8^{+2}_{-1}	67^{+6}_{-8}	0 ± 0
2^+	2888^{+2}_{-4}	155^{+3}_{-5}	(S, S, S, F)	0 ± 0	$0.08^{+0.08}_{-0.06}$	92^{+6}_{-8}	8^{+8}_{-6}	0^{+3}_{-0}	7^{+90}_{-3}	93^{+3}_{-93}	0 ± 0

relative S-wave for the 0^+ and 2^+ sectors. The theoretical width is 42% larger than the experimental value, see Eq. (2). This could be due by multiple reasons such as the complexity of the theoretical calculation which increases the model uncertainties usually assigned to be of the order of 25% or the fact that experimentalists generally perform cross section fits and do not derive the pole structure which produces the bumps in experiments, deriving in error determinations of pole parameters. It may also happen that the model, fitted to many aspects of hadron phenomenology in the past, is simply not able to better predict the mass and width of the $T_{cs1}(2900)^0$. This last case shall be explore in the future with the determination of the same observables measured in experiments.

For completeness, we perform a coupled-channels calculation of the $J^P = 0^-$, 1^+ and 2^- $qq'\bar{s}\,\bar{c}$ sectors; in this case, however, the meson-meson channels to be included are $D^{*-}K^+$ (2503.94), $\bar{D}^{*0}K^0$ (2504.46), $\bar{D}^0K^{*0}$ (2760.39), D^-K^{*+} (2761.32), $D^{*-}K^{*+}$ (2901.92), and $\bar{D}^{*0}K^{*0}$ (2902.40). This means that the final state $\bar{D}K$, through which the $T_{cs0}(2900)^0$ and $T_{cs1}(2900)^0$ resonances were found, is not reached by the tetraquark channels we are now considering. Our results are shown in Table III. Again, many poles are found in the complex energy plane, they are all virtual states or resonances close to $\bar{D}^{(*)}K^{(*)}$ thresholds, with decay widths of the order of tens to hundreds of MeV. There is no evidence of bound states. The 1^+ emerges as a promising sector for new T_{cs1} states. In this case, the $\bar{D}^{(*)}K^{(*)}$ are in relative S-waves, so the formation of molecules is favored. Potential detection channels are the lower $\bar{D}^*K$ channels.

B. Nature of $T_{c\bar{s}}$ states

Hence, we perform a coupled-channels calculation of the isospin-1 $J^P = 0^+$, 1^- and 2^+ $cq\bar{s}\bar{q}'$ sectors, in which the $D_s\pi$ discovery-channel can naturally be measured. We include the following meson-meson channels in the calculation[5]: $D_s^+\pi^-$ (2107.92), D^0K^0 (2362.45), $D_s^{*+}\rho^-$ (2887.46) and $D^{*0}K^{*0}$ (2902.40). In this case, all the quarks involved are distinguishable, but the exchange diagrams are taken into account to deal with the connection between the DK- and $D_s\pi$-type channels. In addition, the decay widths of the light and strange vector mesons are large enough to be taken into account. The experimental value reported in Ref. [3] for the ρ-meson is $\Gamma_\rho = 149.5$ MeV, and the widths of the neutral and charged kaon partners are $\Gamma_{K^{*0}} = 47.3$ MeV and $\Gamma_{K^{*\pm}} = 51.4$ MeV, respectively.

Table IV shows our results. Again when we are so close to meson-meson thresholds, it is evident that we predict a resonance with quantum number $J^P = 0^+$ whose mass, 2892 MeV/c^2, and width, 156 MeV, are perfectly compatible with the experimental measurements, Eqs. (3) and (4). On top of that, another resonance in the same channel is found to be close to the first one; moreover, its mass and width are also compatible with the experimental measurements. It would be interesting to see if the LHCb experiment signals to one 0^+ $T_{c\bar{s}}$ state or, actually, two independent resonances. There is also a singularity in each of the channels $J^P = 1^-$ and 2^+ of the scattering problem. In the first case we have a virtual state, while a resonance is found in the $J^P = 2^+$ channel, though it does not seem to decay into the $D_s^+\pi^-$ final state. Both have masses close to 2.9 GeV/c^2 but have total decay widths larger than those of the resonances found in the 0^+ case.

Finally, we perform a coupled-channels calculation of the isospin-1 $J^P = 0^-$, 1^+, and 2^- $cq\bar{s}\bar{q}'$ sectors; in this case, however, the meson-meson channels to be included are $D_s^{*+}\pi^-$ (2251.77), $D^{*0}K^0$ (2504.46), $D_s^+\rho^-$ (2743.61), D^0K^{*0} (2760.39), $D_s^{*+}\rho^-$ (2887.46), and $D^{*0}K^{*0}$ (2902.40). That is to say, the final state $D_s\pi$, through which the $T_{c\bar{s}0}(2900)^0$ and $T_{c\bar{s}0}(2900)^{++}$ resonances have been found, is not reached by the tetraquark channels we are now considering, but could be detected in the $D_s^*\pi$

[5]In parenthesis the mass of the threshold in MeV/c^2.

TABLE V. Coupled-channels calculation of the isospin-1 $J^P = 0^-$, 1^+ and 2^- $cq\bar{s}\bar{q}'$ sectors ($T_{c\bar{s}}$ states) including the following meson-meson channels in the calculation (in parenthesis the threshold's mass in MeV/c^2): $D_s^{*+}\pi^-$ (2251.77), $D^{*0}K^0$ (2504.46), $D_s^+\rho^-$ (2743.61), D^0K^{*0} (2760.39), $D_s^{*+}\rho^-$ (2887.46) and $D^{*0}K^{*0}$ (2902.40). Errors are estimated by varying the strength of the potential by $\pm 10\%$. 1st column: Pole's quantum numbers; 2nd column: Pole's mass in MeV/c^2; 3rd column: Pole's width in MeV; 4th column: Refers to $D_s^{*+}\pi^-$, $D^{*0}K^0$, $D_s^+\rho^-$, D^0K^{*0}, $D_s^{*+}\rho^-$ and $D^{*0}K^{*0}$ Riemann sheets, respectively, with F meaning first and S second; 5th-10th columns: Channel probabilities in %; 11th-16th columns: Branching ratios in %.

J^P	M_{pole}	Γ_{pole}	RS	$\mathcal{P}_{D_s^{*+}\pi^-}$	$\mathcal{P}_{D^{*0}K^0}$	$\mathcal{P}_{D_s^+\rho^-}$	$\mathcal{P}_{D^0K^{*0}}$	$\mathcal{P}_{D_s^{*+}\rho^-}$	$\mathcal{P}_{D^{*0}K^{*0}}$	$\mathcal{B}_{D_s^{*+}\pi^-}$	$\mathcal{B}_{D^{*0}K^0}$	$\mathcal{B}_{D_s^+\rho^-}$	$\mathcal{B}_{D^0K^{*0}}$	$\mathcal{B}_{D_s^{*+}\rho^-}$	$\mathcal{B}_{D^{*0}K^{*0}}$
0^-	2479.6 ± 0.3	134.95 ± 0.08	(S, S, F, F, F, F)	21.99 ± 0.04	$55.96^{+0.08}_{-0.09}$	4.14 ± 0.01	13.4 ± 0.1	1.24 ± 0.02	3.24 ± 0.04	100 ± 0	0 ± 0	0 ± 0	0 ± 0	0 ± 0	0 ± 0
	2718.8 ± 0.5	$371.8^{+0.2}_{-0.3}$	(S, S, S, S, F, F)	0.05 ± 0.01	12.2 ± 0.2	56.06 ± 0.10	22.4 ± 0.1	0.06 ± 0.01	9.23 ± 0.03	$0.9^{+0.1}_{-0.2}$	$99.1^{+0.2}_{-0.1}$	0 ± 0	0 ± 0	0 ± 0	0 ± 0
	2892^{+4}_{-3}	251^{+33}_{-4}	(S,S,S,S,S,S)	$1.1^{+0.1}_{-0.6}$	$1.2^{+0.4}_{-0.1}$	20^{+1}_{-12}	7^{+0}_{-1}	44^{+4}_{-2}	28^{+8}_{-2}	23^{+2}_{-3}	$2.8^{+0.3}_{-0.2}$	33^{+2}_{-4}	3^{+3}_{-0}	38 ± 7	0 ± 0
1^+	2480^{+1}_{-0}	59^{+0}_{-3}	(S, S, F, F, F, F)	8^{+53}_{-0}	14^{+23}_{-0}	2^{+0}_{-2}	58^{+0}_{-58}	$0.4^{+0}_{-0.1}$	17^{+0}_{-17}	100 ± 0	0 ± 0	0 ± 0	0 ± 0	0 ± 0	0 ± 0
	$2482.8^{+0.3}_{-0.4}$	26 ± 1	(S, S, F, F, F, F)	$68.5^{+0.5}_{-0.3}$	$30.6^{+0.3}_{-0.5}$	0.22 ± 0.01	$0.292^{+0.002}_{-0.001}$	0 ± 0	0.133 ± 0.001	100 ± 0	0 ± 0	0 ± 0	0 ± 0	0 ± 0	0 ± 0
	2742^{+2}_{-4}	155^{+1}_{-2}	(S, S, S, F, F, F)	0.02 ± 0.01	$1.1^{+0.8}_{-0.6}$	88 ± 7	10 ± 6	$0.09^{+0.02}_{-0.04}$	$0.8^{+0.6}_{-0.5}$	5^{+0}_{-3}	95^{+0}_{-65}	0^{+69}_{-0}	0 ± 0	0 ± 0	0 ± 0
	2777 ± 7	115^{+8}_{-7}	(S, S, S, S, F, F)	$0.51^{+0}_{-0.06}$	1.2 ± 0.4	74^{+8}_{-6}	24^{+6}_{-7}	$0.32^{+0.01}_{-0.04}$	$0.8^{+0.1}_{-0.2}$	$5.7^{+0.6}_{-0.2}$	$3.3^{+0.5}_{-0.4}$	59^{+1}_{-0}	32^{+0}_{-2}	0 ± 0	0 ± 0
	2878^{+3}_{-4}	142 ± 1	(S, S, S, S, S, F)	$0.04^{+0}_{-0.01}$	1^{+1}_{-0}	70^{+3}_{-14}	5^{+4}_{-0}	21^{+5}_{-4}	3^{+2}_{-0}	$2.3^{+0.8}_{-0.7}$	28^{+3}_{-2}	13 ± 2	$56.5^{+0.1}_{-0.6}$	0 ± 0	0 ± 0
	2901 ± 4	83 ± 4	(S, S, S, S, S, S)	1.2 ± 0.1	$5.1^{+0.3}_{-0.6}$	$8.6^{+0.8}_{-0.9}$	29^{+4}_{-2}	24 ± 4	32^{+1}_{-3}	18^{+2}_{-3}	$9.4^{+0.2}_{-0.7}$	13^{+0}_{-2}	24^{+0}_{-2}	33^{+2}_{-1}	0^{+12}_{-0}
2^-	$2758.8^{+0}_{-0.2}$	338^{+3}_{-2}	(S, S, S, S, F, F)	0.19 ± 0.01	$2.62^{+0.03}_{-0.04}$	13^{+2}_{-1}	77^{+1}_{-2}	$0.44^{+0.07}_{-0.06}$	$6.51^{+0.01}_{-0.02}$	$0.8^{+0.2}_{-0.1}$	$4.1^{+0.2}_{-0.1}$	95.0 ± 0.3	0 ± 0	0 ± 0	0 ± 0
	2886.2 ± 0.3	$296.6^{+1.0}_{-0.7}$	(S, S, S, S, S, S)	0 ± 0	$0.56^{+0.02}_{-0.01}$	$1.49^{+0.04}_{-0.07}$	$2.60^{+0.01}_{-0}$	$47.3^{+0.3}_{-0.5}$	$48.1^{+0.4}_{-0.3}$	$13.1^{+0.4}_{-0.2}$	$2.8^{+0.7}_{-0.6}$	47 ± 1	37 ± 2	0 ± 0	0 ± 0

or $D_s\rho$ channels. Our results are shown in Table V. Many poles are found in the complex energy plane. Except for one resonance state found in the $J^P = 1^+$ channel with mass 2777 MeV/c^2 and width 115 MeV, all the others are virtual states mostly located near a certain $D^{(*)}K^{(*)}$ threshold, and with decay widths of the order of tens to hundreds of MeV. Again, as in the T_{cs} case, the 1^+ sector allows the $D^{(*)}K^{(*)}$ states to be in a relative S-wave and, thus, shows a rich spectroscopy to be worth exploring.

C. Scattering lengths and effective ranges

We have computed the scattering lengths and effective ranges of all S-wave coupled meson-meson channels taking into account the relation between the T-matrix and phase shifts δ_n, for channel n, as

$$e^{2i\delta_n} = 1 - i2\pi\mu_n k_n T_{nn}, \tag{34}$$

where k_n is the on-shell momentum and μ_n the reduced mass of the meson-meson channel n. The usual effective range expansion is given by

$$k_n \text{cotan}(\delta_n) = \frac{1}{a_n} + \frac{1}{2} r_n k_n^2, \tag{35}$$

in such a way that, from Eqs. (34) and (35), at $k_n \to 0$, the scattering length has the expression

$$a_n = -\pi\mu_n T_{nn}(E_{\text{th}}), \tag{36}$$

with T_{nn} evaluated at threshold (e.g., $E_{\text{th}} = m_A + m_B$ for AB-channel, with A and B stable mesons).

If one considers unstable mesons, as it is the case for either K^* or ρ whose large widths prevent us to consider them as stable particles, the effect of the width can be studied by using a complex mass in the propagator of the Lippmann-Schwinger equation, e.g., one rewrites $m_{K^*} \to m_{K^*} - i\Gamma_{K^*}/2$. Therefore, the on-shell momentum is replaced by [87–89]:

$$k_n = \sqrt{2\mu_n(E - M_{\text{th}} + i\Gamma_{\text{th}}/2)}, \tag{37}$$

where, for instance, the T-matrix is evaluated at $E_{\text{th}} = m_D + m_{K^*} - i\Gamma_{K^*}/2$ in Eq. (36) for DK^*-channel.

In order to estimate the effect of having unstable mesons, we compare the value of the scattering lengths at complex threshold, a_{sc}, with the ones evaluated at real threshold: $-\pi\mu_n T_{nn}(M_{\text{real}})$, which are clearly different from the complex two-body threshold branch point. These values, together with the effective ranges, can be found in Tables VI and VII for T_{cs} and $T_{c\bar{s}}$ sectors, respectively.

TABLE VI. Scattering lengths and effective ranges for the $J^P = 0^+$, 1^+, and 2^+ $qq'\bar{s}\,\bar{c}$ sectors (T_{cs} states).

J^P	Channel	$-\pi\mu T(M_{\rm real})$ [fm]	$a_{\rm sc}$ [fm]	$r_{\rm eff}$ [fm]
0^+	$\bar{D}^0K^0$	$-1.1^{+0.4}_{-1.4}+\mathrm{i}\,0.02^{+0.26}_{-0.02}$	$-1.1^{+0.4}_{-1.4}+\mathrm{i}\,0.02^{+0.26}_{-0.02}$	$1.3^{+0.7}_{-1.5}+\mathrm{i}\,0.06^{+0.16}_{-0.04}$
	D^-K^+	$-1.1^{+0.4}_{-1.3}+\mathrm{i}\,0.07^{+0.77}_{-0.05}$	$-1.1^{+0.4}_{-1.6}+\mathrm{i}\,0.02^{+0.34}_{-0.02}$	$3.7^{+1.5}_{-0.3}+\mathrm{i}\,1.7^{+1.8}_{-0.7}$
	$D^{*-}K^{*+}$	$0.2^{+0.1}_{-0.1}+\mathrm{i}\,0.80^{+0.02}_{-0.03}$	$-2.8^{+0.9}_{-1.2}+\mathrm{i}\,1.4^{+2.3}_{-0.7}$	$0.87^{+0.02}_{-0.13}-\mathrm{i}\,0.10^{+0.08}_{-0.09}$
	$\bar{D}^{*0}K^{*0}$	$0.63^{+0.07}_{-0.27}+\mathrm{i}\,1.1^{+0.3}_{-0.2}$	$-2.0^{+0.4}_{-1.4}+\mathrm{i}\,1.4^{+1.1}_{-0.4}$	$1.2^{+3.1}_{-2.7}-\mathrm{i}\,4.8^{+2.3}_{-0.9}$
1^+	$D^{*-}K^+$	$-1.5^{+0.5}_{-1.2}+\mathrm{i}\,0.05^{+0.12}_{-0.03}$	$-1.4^{+0.5}_{-1.1}+\mathrm{i}\,0.00^{+0.02}_{-0.00}$	$1.1^{+0.8}_{-1.5}+\mathrm{i}\,0.05^{+0.09}_{-0.03}$
	$\bar{D}^{*0}K^0$	$-1.5^{+0.5}_{-1.2}+\mathrm{i}\,0.03^{+0.22}_{-0.02}$	$-1.4^{+0.5}_{-1.2}+\mathrm{i}\,0.03^{+0.22}_{-0.02}$	$2.32^{+0.69}_{-0.03}+\mathrm{i}\,1.0^{+1.5}_{-0.6}$
	$\bar{D}^0K^{*0}$	$-0.06^{+0.11}_{-0.10}+\mathrm{i}\,0.77^{+0.03}_{-0.07}$	$-1.4^{+0.3}_{-0.4}+\mathrm{i}\,0.06^{+0.03}_{-0.02}$	$1.8^{+0.2}_{-0.2}-\mathrm{i}\,0.30^{+0.07}_{-0.09}$
	D^-K^{*+}	$-0.09^{+0.06}_{-0.05}+\mathrm{i}\,0.63^{+0.05}_{-0.05}$	$-1.5^{+0.4}_{-0.5}-\mathrm{i}\,0.10^{+0.07}_{-0.20}$	$3.7^{+0.4}_{-0.2}+\mathrm{i}\,0.01^{+0.30}_{-0.14}$
	$D^{*-}K^{*+}$	$0.51^{+0.07}_{-0.12}+\mathrm{i}\,0.8^{+0.1}_{-0.1}$	$-0.6^{+1.8}_{-1.7}+\mathrm{i}\,2.9^{+1.0}_{-0.8}$	$1.1^{+0.6}_{-0.3}+\mathrm{i}\,0.9^{+0.3}_{-0.4}$
	$\bar{D}^{*0}K^{*0}$	$0.4^{+0.1}_{-0.2}+\mathrm{i}\,0.85^{+0.02}_{-0.07}$	$-0.03^{+0.08}_{-0.16}+\mathrm{i}\,1.27^{+0.09}_{-0.01}$	$-0.4^{+1.8}_{-1.1}-\mathrm{i}\,2.52^{+1.06}_{-0.01}$
2^+	$D^{*-}K^{*+}$	$0.01^{+0.09}_{-0.09}+\mathrm{i}\,0.68^{+0.03}_{-0.06}$	$-2.3^{+0.7}_{-1.4}+\mathrm{i}\,0.3^{+0.7}_{-0.2}$	$1.6^{+0.5}_{-0.4}+\mathrm{i}\,1.6^{+0.2}_{-0.2}$
	$\bar{D}^{*0}K^{*0}$	$0.12^{+0.02}_{-0.03}+\mathrm{i}\,0.42^{+0.00}_{-0.00}$	$-1.9^{+1.4}_{-0.4}+\mathrm{i}\,2.1^{+1.5}_{-1.4}$	$5.6^{+0.5}_{-0.3}+\mathrm{i}\,1.2^{+0.4}_{-0.7}$

TABLE VII. Scattering lengths and effective ranges for the $J^P = 0^+$, 1^+ and 2^+ $cq\bar{s}\bar{q}'$ sectors ($T_{c\bar{s}}$ states).

J^P	Channel	$-\pi\mu T(M_{\rm real})$[fm]	$a_{\rm sc}$ [fm]	$r_{\rm eff}$ [fm]
0^+	$D_s^+\pi^-$	$-0.16^{+0.02}_{-0.02}+\mathrm{i}\,0$	$-0.16^{+0.02}_{-0.02}+\mathrm{i}\,0$	$4.5^{+0.6}_{-0.5}+\mathrm{i}\,0$
	D^0K^0	$-0.6^{+0.1}_{-0.1}+\mathrm{i}\,0.10^{+0.04}_{-0.03}$	$-0.6^{+0.1}_{-0.1}+\mathrm{i}\,0$	$2.4^{+0.3}_{-0.2}+\mathrm{i}\,0.32^{+0.00}_{-0.00}$
	$D_s^{*+}\rho^-$	$0.25^{+0.02}_{-0.04}+\mathrm{i}\,0.37^{+0.03}_{-0.03}$	$0.6^{+1.1}_{-1.8}+\mathrm{i}\,2.7^{+0.6}_{-0.4}$	$0.96^{+0.11}_{-0.08}+\mathrm{i}\,0.45^{+0.05}_{-0.05}$
	$D^{*0}K^{*0}$	$0.20^{+0.00}_{-0.01}+\mathrm{i}\,0.21^{+0.02}_{-0.02}$	$0.51^{+0.01}_{-0.02}+\mathrm{i}\,0.30^{+0.07}_{-0.06}$	$-0.02^{+1.14}_{-1.14}+\mathrm{i}\,4.2^{+0.2}_{-0.3}$
1^+	$D_s^{*+}\pi^-$	$-0.16^{+0.02}_{-0.02}+\mathrm{i}\,0$	$-0.16^{+0.02}_{-0.02}+\mathrm{i}\,0$	$4.8^{+0.7}_{-0.6}-\mathrm{i}\,0.02^{+0.00}_{-0.00}$
	$D^{*0}K^0$	$-0.59^{+0.09}_{-0.10}+\mathrm{i}\,0.07^{+0.03}_{-0.02}$	$-0.6^{+0.1}_{-0.1}+\mathrm{i}\,0$	$2.6^{+0.3}_{-0.2}+\mathrm{i}\,0.23^{+0.00}_{-0.00}$
	$D_s^+\rho^-$	$0.16^{+0.05}_{-0.06}+\mathrm{i}\,0.42^{+0.00}_{-0.01}$	$-2.1^{+0.4}_{-0.3}+\mathrm{i}\,1.6^{+2.4}_{-1.0}$	$1.15^{+0.15}_{-0.12}+\mathrm{i}\,0.40^{+0.01}_{-0.01}$
	D^0K^{*0}	$0.12^{+0.01}_{-0.02}+\mathrm{i}\,0.33^{+0.01}_{-0.01}$	$0.05^{+0.05}_{-0.09}+\mathrm{i}\,0.59^{+0.03}_{-0.05}$	$1.2^{+0.2}_{-0.2}+\mathrm{i}\,0.5^{+0.1}_{-0.1}$
	$D_s^{*+}\rho^-$	$0.07^{+0.03}_{-0.03}+\mathrm{i}\,0.33^{+0.01}_{-0.01}$	$-1.2^{+0.2}_{-0.3}-\mathrm{i}\,0.3^{+0.1}_{-0.2}$	$1.8^{+0.2}_{-0.1}+\mathrm{i}\,0.08^{+0.03}_{-0.03}$
	$D^{*0}K^{*0}$	$0.00^{+0.04}_{-0.05}+\mathrm{i}\,0.55^{+0.02}_{-0.03}$	$-0.62^{+0.03}_{-0.02}+\mathrm{i}\,0.55^{+0.09}_{-0.09}$	$0.80^{+0.00}_{-0.03}-\mathrm{i}\,1.76^{+0.06}_{-0.05}$
2^+	$D_s^{*+}\rho^-$	$0.19^{+0.04}_{-0.06}+\mathrm{i}\,0.42^{+0.01}_{-0.02}$	$-1.67^{+1.84}_{-0.05}+\mathrm{i}\,2.8^{+1.2}_{-1.5}$	$1.03^{+0.14}_{-0.11}+\mathrm{i}\,0.42^{+0.04}_{-0.04}$
	$D^{*0}K^{*0}$	$0.19^{+0.01}_{-0.02}+\mathrm{i}\,0.27^{+0.01}_{-0.01}$	$0.16^{+0.02}_{-0.05}+\mathrm{i}\,0.48^{+0.05}_{-0.05}$	$-2.4^{+0.7}_{-0.8}-\mathrm{i}\,0.1^{+1.0}_{-1.1}$

IV. SUMMARY

The scientific community has witnessed two decades of continuous exciting discoveries of exotic hadrons through systematic searches in numerous experiments around the world. These hadrons belong mostly to the heavy quark sector and are collectively known as *XYZ* states. An enormous theoretical effort has been devoted to unraveling their nature, employing a wide variety of theoretical approaches. However, due to the complexity of the problem, many of our theoretical expectations in exotic heavy hadrons are still based on phenomenological potential models.

Using data from proton-proton collisions at center-of-mass energies of 7, 8, and 13 TeV, with an integrated luminosity of 9 fb^{-1}, the LHCb collaboration has very recently performed amplitude analyses of the $B^+ \to D^+D^-K^+$, $B^+ \to D^-D_s^+\pi^+$, and $B^0 \to \bar{D}^0D_s^+\pi^-$ decays. For the $B^+ \to D^+D^-K^+$, it is necessary to include new spin-0 and spin-1 T_{cs} resonances in the D^-K^+ channel to obtain good agreement with the experimental data. The enhancements observed in $B^+ \to D^-D_s^+\pi^+$ and $B^0 \to \bar{D}^0D_s^+\pi^-$ decays are interpreted as two $J^P = 0^+$ $T_{c\bar{s}}$ states, partners of the same isospin triplet.

We have analyzed the T_{cs} and $T_{c\bar{s}}$ states using as a theoretical framework a constituent-quark-model-based coupled-channels calculation of $qq'\bar{s}\,\bar{c}$ and $cq\bar{s}\bar{q}'$ tetraquark sectors. We have explored the nature and pole position of the singularities in the scattering matrix with spin-parity quantum numbers: $J^P = 0^\pm$, $1^\mp$, and $2^\pm$. The constituent quark model has been widely used in heavy quark sectors and thus all model parameters are thoroughly constrained.

We find 15 T_{cs} poles in the energy range from 2.3 to 3.2 $\mathrm{GeV/c^2}$ and a further 15 $T_{c\bar{s}}$ poles in the energy interval 2.1 to 3.0 $\mathrm{GeV/c^2}$. A tentative assignment of the $T_{cs0}(2900)^0$ experimental signal has been made. Our virtual state has quantum numbers $J^P = 0^+$, its mass and width are $2901.9^{+0.5}_{-0.8}$ $\mathrm{MeV/c^2}$ and 51^{+0}_{-1} MeV, respectively. We find a possible virtual-state candidate of the $T_{cs1}(2900)^0$ signal in the $J^P = 1^-$ channel with pole parameters $2887.7^{+0.3}_{-0.4}$ $\mathrm{MeV/c^2}$ and $189.5^{+0.4}_{-0.6}$ MeV (this width is nevertheless 42% larger than the experimental measurement). With respect to $T_{c\bar{s}}$ candidates, we have predicted a resonance with quantum numbers $J^P = 0^+$ and whose mass, 2892^{+4}_{-3} $\mathrm{MeV/c^2}$, and width, 156^{+60}_{-7} MeV, are perfectly compatible with the experimental measurements. Finally, we encourage experimentalists to search for either more T_{cs} states in the $\bar{D}^*K$ and $\bar{D}K^*$ channels or $T_{c\bar{s}}$ signals in the $D_s^*\pi$ and $D_s\rho$ final states.

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