Role of strange quarks in the *D*-term and cosmological constant term of the proton

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We investigate the mechanics of the proton by examining the flavor-decomposed proton cosmological constants and generalized vector form factors. The interplay of up, down, and strange quarks within the proton is explored, shedding light on its internal structure. The contributions of strange quarks play a crucial role in the *D*-term and cosmological constants. We find that the flavor blindness of the isovector *D*-term form factor is only valid in flavor SU(3) symmetry.

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I. INTRODUCTION

The proton, a fundamental building block of matter, possesses a set of fundamental observables, including its electric charge, magnetic dipole moment, mass, spin, and the *D*-term. The *D*-term, akin to these observables, plays a crucial role in unraveling the mechanical properties of the proton and shedding light on how it achieves stability through the intricate interplay of quarks and gluons. Understanding the distribution of mass, spin, pressure, and shear force within the proton is facilitated by its gravitational form factors (GFFs) [1] in a manner similar to how the electromagnetic form factors reveal charge and magnetic distributions. Specifically, the pressure and shear-force distributions are intimately linked to the *D*-term form factor (see the recent review [2] and references therein).

Direct measurement of the proton GFFs necessitates the interaction of gravitons with protons, which is experimentally impractical due to the exceedingly weak gravitational coupling strength of the proton. However, a promising avenue emerges through the generalized parton distributions (GPDs), which offer indirect access to the mechanical properties of the proton. The GFFs can be regarded as the second Mellin moments of the vector GPDs [3–5] (see also the reviews [6,7]), providing insight into the proton's mechanical structure [2]. Deeply virtual Compton scattering (DVCS) serves as an effective means to access the GPDs, enabling the extraction of valuable information about the GFFs [8–10].

Recent advancements by Burkert *et al.* [11] have witnessed the experimental extraction of the quark component of the proton *D*-term form factor, marking a significant breakthrough. By leveraging experimental data on the beam-spin asymmetry and unpolarized cross section for DVCS on the proton, they successfully obtained valuable insights into the *D*-term. However, their analysis assumed the large-N_c limit, thereby considering the up-quark contribution (d_1^u) to be approximately equal to the down-quark contribution (d_1^d) at the leading order. This assumption leads to an almost null result for the leading isovector *D*-term d_1^{u-d} [12]. Hence, it becomes imperative to critically examine the validity of this assumption. DVCS provides an effective way to access the GPDs.

Furthermore, Burkert *et al.* [11] also assumed flavor SU(2) symmetry, neglecting the contribution of strange quarks. However, as we establish in this study, the inclusion of strange quarks becomes indispensable for accurately describing the *D*-term. While the strange quark's contribution to the nucleon mass and spin may be marginal, it assumes a pivotal role in characterizing the *D*-term, indicating that the nucleon's stability can only be comprehensively understood by considering the degrees of freedom associated with up, down, and strange quarks. Note that the gluon GPDs are only accessible at higher orders in α_s, so they are expected to be smaller than the quark GPDs.

In this paper, our objective is to elucidate the significance of strange quarks in unraveling the mechanical structure of the proton within the framework of the chiral quark-soliton model (χQSM) [13–15], The χQSM provides a suitable

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relativistic quantum-field-theoretic framework for our analysis. It is noteworthy that previous studies employing the χQSM [16] have yielded results in excellent agreement with experimental data on the pressure and shear-force distributions of the proton, as reported by Burkert *et al.* [11]. In a recent publication by Won *et al.* [17], the proton D-term and proton cosmological constant (PCC) were further explored by decomposing them into up- and down-quark flavor components through the computation of generalized isovector vector form factors. The magnitude of the down-quark component was found to be larger than that of the up-quark component, leading to a nonzero value of d_1^{u-d} (see also Ref. [18]). However, in order to comprehensively understand the proton's mechanical structure, it becomes imperative to consider flavor SU(3) symmetry and incorporate the generalized triplet and octet vector form factors alongside the GFFs. This extension enables us to decompose the GFFs into their up-, down-, and strange-quark components, thereby gaining a deeper understanding of the internal mechanics of the proton.

Notably, a recent lattice calculation has provided insights into the flavor decomposition of the proton's spin and momentum [19]. However, it is important to emphasize that the flavor-decomposed PCCs were not considered in this analysis. In our current work, we aim to fill this gap by investigating the role of flavor-decomposed PCCs in examining the mechanical structure of the proton. By incorporating these crucial factors, we can refine our understanding of the proton's intricate mechanics and shed further light on its fundamental properties.

II. GENERAL VECTOR FORM FACTORS

The GFFs can be related to the matrix element of the flavored (q) symmetric energy-momentum tensor (EMT) current defined as $\hat{T}_q^{\mu\nu} = \bar{q}\frac{i}{4}\overleftrightarrow{D}^{\{\mu}\gamma^{\nu\}}q$ with the covariant derivative $\overleftrightarrow{D}^\mu = \overleftrightarrow{\partial}^\mu - 2igA^\mu$ and $\overleftrightarrow{\partial}^\mu = \overrightarrow{\partial}^\mu - \overleftarrow{\partial}^\mu$, and $a^{\{\mu}b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$, which can be parametrized in terms of the four GFFs A_q, J_q, D_q, and $\bar{c}_q$:

$$\begin{aligned}&\langle p'|\hat{T}_q^{\mu\nu}(0)|p\rangle\\&= \bar{u}(p')\left[A_q(t)\frac{P^\mu P^\nu}{M_N} + J_q(t)\frac{i(P^\mu\sigma^{\nu\rho} + P^\nu\sigma^{\mu\rho})\Delta_\rho}{2M_N}\right.\\&\quad\left. + D_q(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M_N} + \bar{c}_q(t)M_N g^{\mu\nu}\right]u(p), \qquad (1)\end{aligned}$$

where $P = (p' + p)/2$, $\Delta = p' - p$, and $\Delta^2 = -\mathbf{\Delta}^2 = t$. A_q, J_q, and D_q are related to the second moments of the vector GPDs defined in Ref. [4],

$$\begin{aligned}&A_q(t) = A_{20,q}(t), \qquad 2J_q(t) = A_{20,q}(t) + B_{20,q}(t),\\&D_q(t) = 4C_{20,q}(t), \qquad (2)\end{aligned}$$

where the subscript stands for the quark flavor. In the forward limit ($t \to 0$) [20,21], Eq. (1) reduces to

$$\langle p|\hat{T}_q^{\mu\nu}|p\rangle = \bar{u}(p)\left[A_q(0)\frac{p^\mu p^\nu}{M_N} + \bar{c}_q(0)M_N g^{\mu\nu}\right]u(p). \qquad (3)$$

In the χQSM, the gluon degrees of freedom are integrated out through the instanton vacuum, and their effects are absorbed into the dynamical quark mass M, which was originally momentum dependent [22,23]. Thus, the quark part of the EMT current is conserved within the framework of the χQSM,

$$\sum_q \partial_\mu \hat{T}_q^{\mu\nu} = 0. \qquad (4)$$

This implies that the PCC form factor $\bar{c} = \sum_q \bar{c}_q$ vanishes in the whole range of the momentum transfer. At $t = 0$, the mass, spin, and cosmological constant of the proton are normalized, respectively, as $A = 1$, $J = \frac{1}{2}$, and $\bar{c} = 0$. However, note that there are no constraints on the generalized triplet and octet vector form factors.

III. CHIRAL QUARK-SOLITON MODEL: PION MEAN-FIELD APPROACH

The χQSM has been successful in describing not only the well-known baryonic observables [14,24] but also the first data on the D-term form factor [11,12] and other GFFs [16,25,26]. The formalism is well known already [26]; we briefly explain the essential feature of the model. The detailed expressions can be found in Ref. [27]. We start from the low-energy QCD effective partition function in Euclidean space [13,14,22,23],

$$\mathcal{Z}_{\mathrm{eff}} = \int \mathcal{D}\pi^a \exp\left[-S_{\mathrm{eff}}(\pi^a)\right], \qquad (5)$$

where π^a are the SU(3) pseudo-Nambu-Goldstone (pNG) boson fields with the superscript $a = 1, \cdots 8$ and S_{eff} represents the effective chiral action expressed as

$$S_{\mathrm{eff}} = -N_c \mathrm{Tr}\log\left[i\partial\!\!\!/ + iMU^{\gamma_5} + i\hat{m}\right]. \qquad (6)$$

N_c denotes the number of colors. The chiral field U^{γ_5} is defined by $U^{\gamma_5} \coloneqq e^{i\gamma_5\pi^a\lambda^a} = P_L U + P_R U^\dagger$, where $P_{L(R)} \coloneqq (1 \mp \gamma_5)/2$ and $U \coloneqq e^{i\pi^a\lambda^a}$. M stands for the dynamical quark mass, which arises from the spontaneous breakdown of chiral symmetry [22,23]. Though M is originally momentum dependent and its value at zero virtuality is determined by the saddle-point equation from the instanton vacuum [22,23], we use it as the only free parameter in the current work. However, $M = 420$ MeV is known to be the best value for describing various baryonic observables [14,24]. $\hat{m}$ is the current-quark mass matrix

$\mathrm{diag}(m_{\mathrm{u}} m_{\mathrm{d}}, m_{\mathrm{s}})$. We consider the flavor SU(3) symmetry ($m_{\mathrm{u}} = m_{\mathrm{d}} = m_{\mathrm{s}}$) in the current work.

The first three components of the pNG fields can be coupled to the three-dimensional coordinates, which is called the hedgehog ansatz $\pi^a = P(r)n^a$ with the unit basis vectors $n^a = x^a/r$. It is the minimal generalization that allows to incorporate the pion fields. $P(r)$ is called the profile function for the classical soliton. It can be determined by solving the classical equation of motion self-consistently. In flavor SU(3) symmetry, we employ Witten's embedding to preserve the hedgehog symmetry,

$$U = e^{i\pi^a\lambda^a} = \begin{pmatrix} e^{i\boldsymbol{n}\cdot\boldsymbol{\tau}P(r)} & 0 \\ 0 & 1 \end{pmatrix}. \tag{7}$$

Note that the zero-mode quantization with this embedding correctly yields the spectrum of the lowest-lying SU(3) baryons such as the baryon octet and decuplet. The zero-mode quantization can be performed by the functional integration over rotational and translational zero modes of the U field. Including the external tensor source field, we can evaluate the matrix element of the EMT current. The zero-mode quantization naturally furnishes the rotational $1/N_c$ corrections. While they do not contribute to the GFFs except for $J_q(t)$, they provide substantial effects on the generalized triplet and octet vector form factors.

The matrix element of the effective EMT operator is calculated as follows:

$$\begin{aligned}
&\langle N(p', J_3')|\hat{T}^{\mu\nu,\chi}_{\mathrm{eff}}(0)|N(p, J_3)\rangle \\
&= \lim_{T\to\infty} \frac{2M_N}{Z_{\mathrm{eff}}} e^{ip_4\frac{T}{2} - ip_4'\frac{T}{2}} \int d^3\boldsymbol{x}\, d^3\boldsymbol{y}\, e^{(-i\boldsymbol{p}'\cdot\boldsymbol{y} + i\boldsymbol{p}\cdot\boldsymbol{x})} \\
&\quad \times \int \mathcal{D}U \int \mathcal{D}\psi\mathcal{D}\psi^\dagger J_N(\boldsymbol{y}, T/2)\hat{T}^{\mu\nu,\chi}_{\mathrm{eff,E}}(0) J_N^\dagger(\boldsymbol{x}, -T/2) \\
&\quad \times \exp\,[-S_{\mathrm{eff}}],
\end{aligned} \tag{8}$$

where J_N represents the Ioffe-type current consisting of the N_c valence quarks [28] and $\hat{T}^{\mu\nu,\chi}_{\mathrm{eff,E}}(0)$ denotes the symmetrized EMT current in Euclidean space, which can be derived from Noether's theorem. Note that the nucleon state in flavor SU(3) implicitly carries the spin, isospin, and hypercharge quantum numbers $N = \{J, J_3, Y_R = N_c/3, T, T_3, Y\}$.

The EMT current in Minkowski space can be expressed as

$$\hat{T}^{\mu\nu,\chi}_{\mathrm{eff}} = \frac{i}{4}\bar{\psi}(\gamma^\mu \overrightarrow{\partial}^\nu + \gamma^\nu \overrightarrow{\partial}^\mu - \gamma^\mu \overleftarrow{\partial}^\nu - \gamma^\nu \overleftarrow{\partial}^\mu)\lambda^\chi\psi, \tag{9}$$

where we introduce the superscripts $\chi = 0, 3, 8$ to represent the flavor-singlet $\chi = 0 = (u + d + s)$, the flavor-triplet $\chi = 3 = (u - d)$, and flavor-octet $\chi = 8 = \frac{1}{\sqrt{3}}(u + d - 2s)$ components of the EMT current.

By performing the three-dimensional Fourier transform, the matrix element of the EMT current can be interpreted as the static EMT distributions [1,29],

$$T^{\mu\nu,\chi}_{\mathrm{eff}}(\boldsymbol{r}) = \int \frac{d^3\Delta}{2M_N(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \times \langle N(p', J_3')|\hat{T}^{\mu\nu,\chi}_{\mathrm{eff}}|N(p, J_3)\rangle. \tag{10}$$

We then arrive at the final expressions for the GFFs with flavor q:

$$\begin{aligned}
&\left[A_q(t) + \bar{c}_q(t) - \frac{t}{4M_N^2}(D_q(t) - 2J_q(t))\right] \\
&= \frac{4\pi}{M_N}\int dr r^2 j_0(kr)\varepsilon_q(r), \\
&\left[\bar{c}_q(t) - \frac{t}{6M_N^2} D_q\right] = -\frac{4\pi}{M_N}\int dr r^2 j_0(kr) p_q(r), \\
&D_q(t) = 16\pi M_N \int dr r^2 \frac{j_2(kr)}{t} s_q(r), \\
&J_q(t) = 12\pi \int dr r^2 \frac{j_1(kr)}{kr}\rho^J_q(r),
\end{aligned} \tag{11}$$

with $k = \sqrt{-t}$ and $J_3' = J_3 = 1/2$. For detailed expressions for the distributions, we refer to Ref. [27].

IV. RESULTS AND DISCUSSION

In the current work, we ignore the effects of the flavor SU(3) symmetry breaking. Figure 1 illustrates the results obtained for the GFFs. The dashed curves represent the contributions from valence quarks, while the short-dashed curves depict the contributions from sea quarks. Remarkably, the sea-quark contributions are found to dominate over the valence-quark contributions, as demonstrated in the first panel of Fig. 1. This observation aligns with the classical nucleon mass values, given by $M_N = N_c E_{\mathrm{val}} + E_{\mathrm{sea}} = 611.1\ \mathrm{MeV} + 645.3\ \mathrm{MeV} = 1256\ \mathrm{MeV}$.[1] It is noteworthy that sea quarks contribute approximately 51.4% to the proton's mass.

Regarding the average momentum fraction, a recent lattice calculation yielded a value of $\langle x\rangle_p = 0.497(12)(5)|_{\mathrm{conn}} + 0.307(121)(95)|_{\mathrm{disc}} + 0.267(12)(10)|_{\mathrm{gluon}} = 1.07(12)(10)$ [19], which can be identified as the mass form factor $A(t)$ in the forward limit. By assuming that the sea-quark contributions implicitly incorporate the effects of integrated-out gluon degrees of freedom within the instanton vacuum, our current findings are in good agreement with the lattice

[1] In order to avoid confusion, we want to mention the distinction between the terminology "valence" and "sea" in the χQSM versus their counterparts in QCD. Within the framework of the χQSM, the term "valence quark" pertains to level quarks, while "sea quarks" specifically denotes quarks present on the negative Dirac continuum for the classical nucleon.

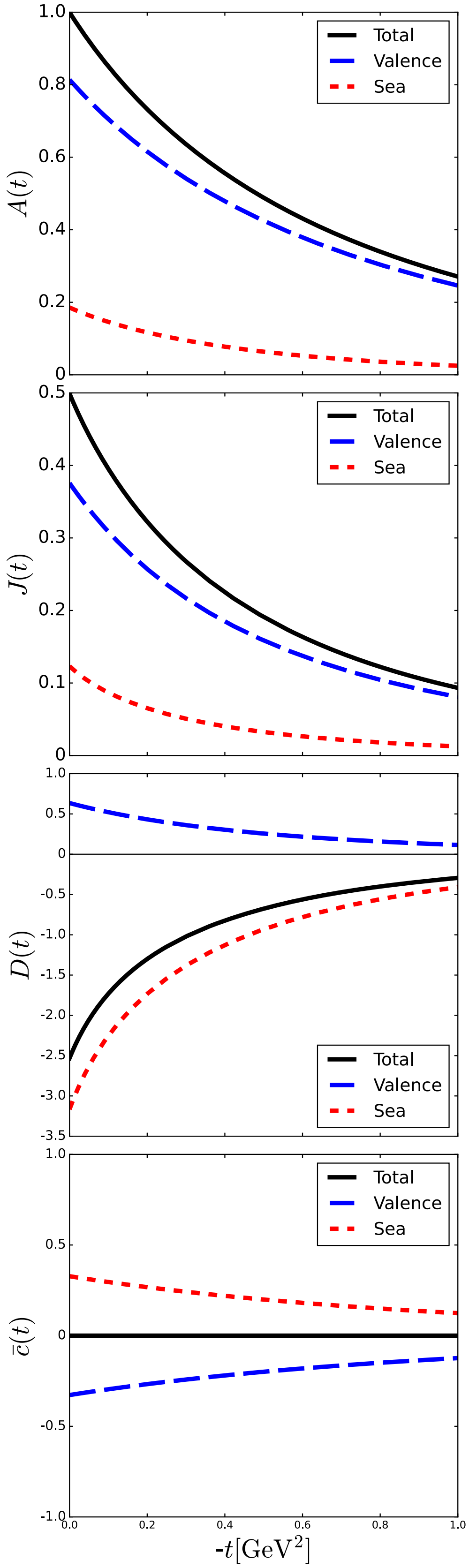

FIG. 1. Results for the gravitational form factors of the proton. The dashed, short-dashed, and solid curves depict the valence-quark, sea-quark, and total contributions, respectively.

data. In contrast to the proton mass, the sea quarks contribute approximately 24% to the proton spin, as depicted in the second panel of Fig. 1.

In the third panel of Fig. 1, it is evident that the sea-quark contribution surpasses the valence-quark contribution. This observation aligns with the nature of the proton D-term form factor, which exhibits a quadrupole structure due to the rank-2 tensor characteristics of the EMT, as described in Eq. (11). Remarkably, similar behavior can be observed in the electric quadrupole (E2) form factors of spin-3/2 baryons, as discussed in Refs. [30–33].

The PCC form factor $\bar{c}(t)$ is expected to vanish, as a consequence of EMT current conservation, as shown in Eq. (4). Interestingly, the valence-quark contribution is exactly canceled by the sea-quark contribution. These results, depicted in Fig. 1, underscore the importance of employing relativistic quantum-field-theoretic approaches to comprehend the mechanical structure of the proton. Such methodologies are crucial for unraveling the intricate dynamics within the proton and gaining deeper insights into its mechanical properties.

Once we have evaluated the GFFs, we can decompose them as follows:

$$F_u = F^0/3 + F^3/2 + F^8/2\sqrt{3},$$
$$F_d = F^0/3 - F^3/2 + F^8/2\sqrt{3},$$
$$F_s = F^0/3 - F^8/\sqrt{3}, \qquad (12)$$

where F^0, F^3, and F^8 denote the generic GFFs and generalized triplet and octet form factors, respectively. The flavor decomposition of the proton mass form factor $A(t)$ is presented in the upper panel of Fig. 2. As emphasized in the Introduction, considering the PCCs is crucial for a proper understanding of the proton mass decomposition [20,21]. The contribution of strange quarks to the mass form factor is found to be negligible. Again, the up-quark contribution dominates over the contributions from down and strange quarks. This dominance of up quarks is also reflected in the proton spin, with up quarks accounting for the majority inside a proton, as depicted in the second panel of Fig. 2. In contrast, the spin of the neutron is primarily attributed to down quarks, as evidenced in Table I.

The flavor decomposition results for the D-term form factor are displayed in the third panel of Fig. 2. Notably, the up- and down-quark contributions exhibit remarkable similarity, while the strange-quark contribution comprises approximately 25% of their combined effect. This finding exhibits profound physical implications. The result for the flavor-decomposed D-term shown in the third panel of Fig. 2 apparently yields an almost negligible value of D^{u-d}. However, one should keep in mind that in flavor SU(2), D^{u-d} is non-negligible [17]. This implies that a certain amount of the d-quark contribution is taken over by the strange quark in flavor SU(3). Thus, the blindness of

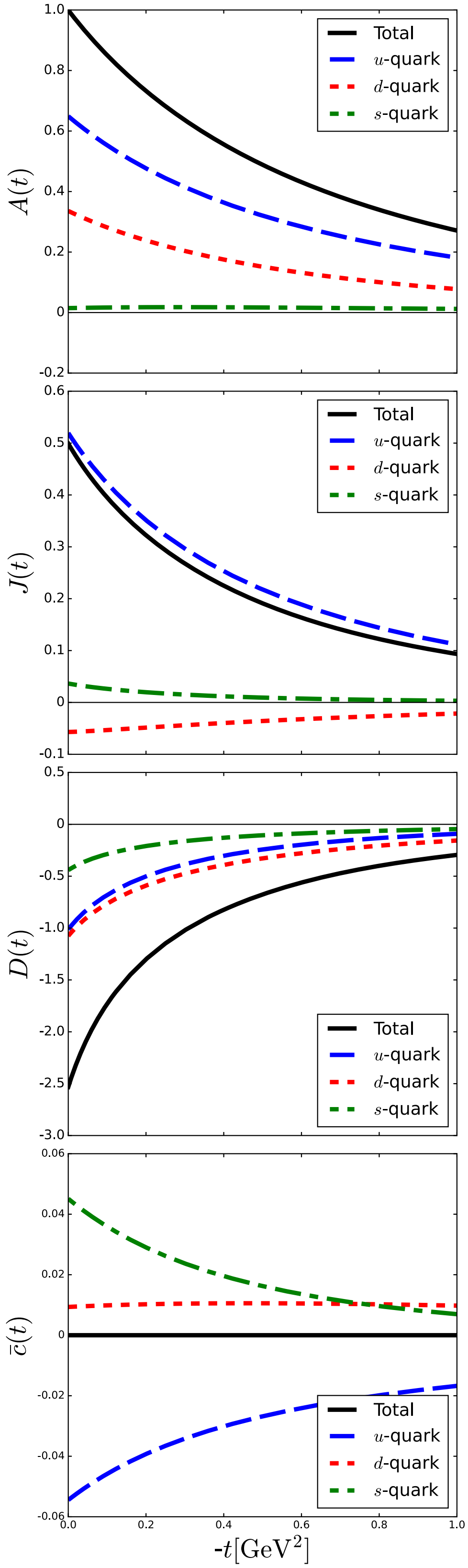

FIG. 2. Results for the gravitational form factors of the proton. The long-dashed, short-dashed, dot-dashed, and solid curves depict the up-quark, down-quark, strange-quark, and total contributions, respectively.

$D^{u-d} \sim 0$ assumed in Ref. [11] is only valid in the flavor-SU(3)-symmetric case. The flavor blindness in flavor SU(3) was also advocated in a recent lattice calculation [34].

In the final panel of Fig. 2, the flavor decomposition of the PCC form factor is depicted. Interestingly, the up- and strange-quark contributions dominate the PCC form factor, while the down-quark contribution is relatively small. The PCC form factor $\bar{c}^0$ vanishes due to the conservation of the EMT current, whereas $\bar{c}^3$ and $\bar{c}^8$ can have finite values. Notably, both $\bar{c}^3$ and $\bar{c}^8$ exhibit negative numerical values, resulting in a small magnitude for $\bar{c}_d$ according to Eq. (12). On the other hand, the up and strange components are defined, respectively, as $(\bar{c}^3 + \bar{c}^8/\sqrt{3})/2$ and $-\bar{c}^8/\sqrt{3}$. Although the sign of $\bar{c}_s$ is opposite to that of $\bar{c}_u$, its magnitude is comparable to that of the up-quark component. This observation has significant implications. While the PCC form factor itself vanishes, its flavor-decomposed components remain finite and the strange quark plays a crucial role. As stated in Eq. (11), the PCC is linked to the pressure distribution, indicating that the strange quark should be considered in understanding the internal mechanical structure of the proton. Furthermore, it suggests that when the *D*-term form factor is extracted in future experimental data, one should carefully consider the contribution from strange quarks.

When the GFFs are understood as the second Mellin moments of the vector GPDs as expressed in Eq. (2), the PCC does not appear from the leading-twist GPDs. The form factor A_q in the forward limit is identified as the momentum fraction of the proton, denoted as $\langle x \rangle_q$. However, if we specifically consider the temporal component of Eq. (3), we derive the general decomposition of the proton mass in the rest frame as [20,21]

$$M_p = \sum_q (A_q(0) + \bar{c}_q(0)) M_p, \tag{13}$$

which leads to $\sum_q (A_q(0) + \bar{c}_q(0)) = 1$.

Given the conservation of the EMT current, the total PCC is expected to vanish, implying that $\sum_q A_q = 1$. However, it does not necessarily mean that each flavor component $\bar{c}_q$ is zero. Consequently, the decomposed momentum fraction, denoted as $\langle x \rangle_q$, may not be equivalent to the decomposed proton mass expressed as $M_p^q = (A_q(0) + \bar{c}_q(0)) M_p$. Hence, a compelling comparison arises between M_p^q and $\langle x \rangle_q$:

$$\begin{aligned} M_p^{\mathrm{u}}/M_p &= 59.5\% < \langle x \rangle_u = 64.9\%, \\ M_p^{\mathrm{d}}/M_p &= 34.5\% > \langle x \rangle_d = 33.6\%, \\ M_p^{\mathrm{s}}/M_p &= 6.0\% > \langle x \rangle_s = 1.5\%. \end{aligned} \tag{14}$$

These results indicate a remarkable feature of $\bar{c}_q$ in describing the proton mass. One can generalize the above

TABLE I. Results for the flavor-decomposed GFFs of the proton at $t = 0$.

N	$A_u(0)$	$A_d(0)$	$A_s(0)$	$J_u(0)$	$J_d(0)$	$J_s(0)$	$D_u(0)$	$D_d(0)$	$D_s(0)$	$\bar{c}_u(0)$	$\bar{c}_d(0)$	$\bar{c}_s(0)$
p	0.649	0.336	0.015	0.520	−0.057	0.036	−1.014	−1.076	−0.441	−0.054	0.009	0.045
n	0.336	0.649	0.015	−0.057	0.520	0.036	−1.076	−1.014	−0.441	0.009	−0.054	0.045

findings as follows: if the $\bar{c}_q$ is positive (negative), then the flavor-decomposed proton M_p^q/M_p is larger (smaller) than the nucleon momentum fraction carried by quarks $\langle x\rangle_q$,

$$\bar{c}_q(0) > 0 \rightarrow M_p^q/M_p > \langle x\rangle_q,$$
$$\bar{c}_q(0) < 0 \rightarrow M_p^q/M_p < \langle x\rangle_q. \tag{15}$$

If the $\bar{c}_q(0)$ is zero, then we obtain the trivial relation, i.e., $M_p^q/M_p = \langle x\rangle_q$.

V. SUMMARY AND CONCLUSIONS

We have conducted an investigation into the flavor decomposition of the GFFs of the proton. Here are the key findings of our study:

(1) The dominant effects on the mass, D-term, and cosmological constant of the proton stem from the sea quarks rather than the valence quarks. This emphasizes the necessity of employing a relativistically quantum-field-theoretic approach to accurately describe these quantities.

(2) The contributions of strange quarks play a particularly significant role in the D-term and cosmological constants. Therefore, when extracting these contributions from experimental data, it is essential to take into account the influence of strange quarks.

(3) While the mass form factor $A(t)$ is associated with the average momentum fraction, the decomposition of the nucleon mass reveals a noteworthy contribution from the cosmological constants. This indicates a profound connection between the mass distribution of the proton and its mechanical structure.

(4) Burkert *et al.* [11] assumed the flavor blindness of the isovector D-term ($D^{u-d} \sim 0$). We showed in the current work that it is only valid in the flavor-SU(3)-symmetric case, since the strange quark takes off a certain amount of the down-quark contribution.

In conclusion, our investigation emphasizes the importance of considering sea quarks, especially strange quarks, in understanding the GFFs of the proton. It highlights the role of the cosmological constant in the nucleon mass decomposition and its implications for the mechanical properties of the proton.

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