Quantum hair during gravitational collapse

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We consider quantum-gravitational corrections to the Oppenheimer-Snyder metric describing time-dependent dust ball collapse. The interior metric also describes Friedmann-Lemaître-Robertson-Walker cosmology and our results are interpreted in that context. The exterior corrections are an example of quantum hair and are shown to persist throughout the collapse. Our results show that the quantum hair survives throughout the horizon formation and the internal state of the resulting black hole is accessible to outside observers.

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I. INTRODUCTION

The unique quantum-gravitational effective action program allows model-independent calculations in quantum gravity [1–9]. This approach has been used to study quantum-gravitational corrections to a variety of cosmological [10–16] and astrophysical models [17–24]. A study of quantum-gravitational corrections to a static dust ball used to model a star [22,25] (see also [26] for earlier work) revealed the existence of quantum hair. It was found that the quantum-gravitational potential of a star depends on the composition of the star at second order in the curvature expansion of the effective action. In [25], we suggested that the quantum hair would also apply to a collapsing star model and thus to a black hole. The term quantum hair has been used for some time [27]. Specifically, by *quantum hair* we mean quantum corrections to classical solutions in general relativity describing the exterior metric. These quantum corrections can carry information about the interior quantum state which would otherwise be forbidden by the no-hair theorem, and hence play an important role in the black hole information paradox [25,28,29].

The aim of the paper is to extend our previous work on quantum hair. We first present a very generic result that is independent of the chosen energy-momentum tensor, proving that the quantum hair must exist for any energy-momentum tensor $T_{\mu\nu}$. This result is fully model independent: it does not depend on the matter model (i.e., $T_{\mu\nu}$) or the high-energy completion of the effective action.

We then study a specific model for the gravitational collapse of a dust ball, namely, the Oppenheimer-Snyder model of gravitational collapse [30], and demonstrate that quantum hair is present in this dynamical model, calculable from first principles. The corrections in r^{-3} and r^{-5} are identical to those identified in [25] in the static case. Moreover, the quantum hair persists throughout the gravitational collapse of the star. Our work demonstrates that the resulting black hole has quantum hair. These results are also relevant to Friedmann-Lemaître-Robertson-Walker (FLRW) cosmology, as the inside of the collapsing object is described by the FLRW metric. We calculate for the first time the complete leading-order quantum-gravitational correction to FLRW, and comment on previous works on FLRW quantum cosmology.

This paper is organized as follows. In Sec. II we present a model-independent proof that quantum hair exists for any energy-momentum tensor. In Sec. III we review the Oppenheimer-Snyder model. In Sec. IV we compute the leading quantum-gravitational corrections to the interior and exterior metrics of this model. In the conclusions, we discuss some of the implications of our work for black hole information and long-wavelength quantum gravity.

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II. QUANTUM HAIR AND GENERIC MATTER DISTRIBUTION

In this section, we use the results presented in [31] and argue that there is quantum hair for any matter distribution. Quantum hair can manifest itself as quantum corrections to classical solutions in general relativity describing the exterior metric of an astrophysical object. In the case of black holes, these quantum corrections can carry information about the interior quantum state, whereas the classical no-hair theorem would forbid this. Hence, the existence of quantum hair bears relevance to the black hole information paradox.

The quantum corrections to classical solutions of general relativity are reliably calculable using quantum-corrected field equations obtained from the variation of the Vilkovisky-DeWitt unique effective action of quantum gravity as long as curvature invariants remain weak. At second order in curvature, the effective action is given by [2–6]

$$\Gamma_{\rm QG} = \Gamma_{\rm L} + \Gamma_{\rm NL}, \tag{1}$$

with a local part

$$\Gamma_{\rm L} = \int d^4x\sqrt{|g|}\left[\frac{M_P^2}{2}(\mathcal{R}-2\Lambda)+c_1(\mu)\mathcal{R}^2+c_2(\mu)\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + c_3(\mu)\mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}+c_4(\mu)\Box\mathcal{R}+\mathcal{O}(M_P^{-2})\right], \tag{2}$$

where $M_P = \sqrt{\hbar/G_{\rm N}}$ denotes the Planck mass, and a nonlocal part

$$\Gamma_{\rm NL} = -\int d^4x\sqrt{|g|}\left[\alpha\mathcal{R}\ln\left(\frac{\Box}{\mu^2}\right)\mathcal{R}+\beta\mathcal{R}_{\mu\nu}\ln\left(\frac{\Box}{\mu^2}\right)\mathcal{R}^{\mu\nu} + \gamma\mathcal{R}_{\mu\nu\rho\sigma}\ln\left(\frac{\Box}{\mu^2}\right)\mathcal{R}^{\mu\nu\rho\sigma}+\mathcal{O}(M_P^{-2})\right]. \tag{3}$$

For simplicity, we set the cosmological constant to zero. In addition, we ignore the boundary term associated with c_4, as it does not contribute to the field equations. Then, after applying the local and nonlocal Gauss-Bonnet identities [9], we obtain

$$\Gamma_{\rm QG} = \int d^4x\sqrt{|g|}\left[\frac{M_P^2}{2}\mathcal{R}+\tilde{c}_1(\mu)\mathcal{R}^2+\tilde{c}_2(\mu)\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \tilde{\alpha}\mathcal{R}\ln\left(\frac{\Box}{\mu^2}\right)\mathcal{R}+\tilde{\beta}\mathcal{R}_{\mu\nu}\ln\left(\frac{\Box}{\mu^2}\right)\mathcal{R}^{\mu\nu}+\mathcal{O}(M_P^{-2})\right], \tag{4}$$

with $\tilde{c}_1 = c_1 - c_3$, $\tilde{c}_2 = c_2 + 4c_3$, $\tilde{\alpha} = \alpha - \gamma$, and $\tilde{\beta} = \beta + 4\gamma$.

The renormalization scale μ in the effective action is a free parameter: it is the energy scale at which the effective action is matched to a UV-complete theory of quantum gravity. We take μ to be of the order of the Planck scale. We note that, as always in quantum field theory, physical observables should not depend on the renormalization scale, even though the action depends on it.

The coefficients of the local part of the effective action depend on the renormalization scale as

$$c_1(\mu) = c_1(\mu_*) - \alpha\ln\left(\frac{\mu^2}{\mu_*^2}\right), \tag{5}$$

$$c_2(\mu) = c_2(\mu_*) - \beta\ln\left(\frac{\mu^2}{\mu_*^2}\right), \tag{6}$$

$$c_3(\mu) = c_3(\mu_*) - \gamma\ln\left(\frac{\mu^2}{\mu_*^2}\right), \tag{7}$$

where μ_* is the scale at which the effective action is matched to the UV-complete theory of quantum gravity [6]; see also [32]. We give the values of the Wilson coefficients of the nonlocal part of the action in Table I.

The quantum-gravitational field equations to second order in curvature can be derived from this action, and are given by

$$\mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R}g_{\mu\nu} - 16\pi G_{\rm N}(H^{\rm L}_{\mu\nu} + H^{\rm NL}_{\mu\nu}) = 8\pi G_{\rm N}T_{\mu\nu}, \tag{8}$$

where $G_{\rm N}$ is Newton's constant, $T_{\mu\nu}$ is the energy-momentum tensor,

$$H^{\rm L}_{\mu\nu} = \bar{c}_1\left(2\mathcal{R}\mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R}^2 + 2g_{\mu\nu}\Box\mathcal{R} - 2\nabla_\mu\nabla_\nu\mathcal{R}\right) + \bar{c}_2\left(2\mathcal{R}^\alpha{}_\mu\mathcal{R}_{\nu\alpha} - \frac{1}{2}g_{\mu\nu}\mathcal{R}_{\alpha\beta}\mathcal{R}^{\alpha\beta} + \Box\mathcal{R}_{\mu\nu} + \frac{1}{2}g_{\mu\nu}\Box\mathcal{R} - \nabla_\alpha\nabla_\mu\mathcal{R}^\alpha{}_\nu - \nabla_\alpha\nabla_\nu\mathcal{R}^\alpha{}_\mu\right), \tag{9}$$

and

TABLE I. Nonlocal Wilson coefficients for different fields. All numbers should be divided by $11520\pi^2$. Here, ξ denotes the value of the nonminimal coupling for a scalar theory [6].

	α	β	γ
Scalar	$5(6\xi-1)^2$	−2	2
Fermion	−5	8	7
Vector	−50	176	−26
Graviton	250	−244	424

$$H^{\mathrm{NL}}_{\mu\nu} = -2\alpha\left(\mathcal{R}_{\mu\nu} - \frac{1}{4}g_{\mu\nu}\mathcal{R} + g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu\right)\ln\left(\frac{\Box}{\mu^2}\right)\mathcal{R} \\ -\beta\left(2\delta^\alpha_{(\mu}\mathcal{R}_{\nu)\beta} - \frac{1}{2}g_{\mu\nu}\mathcal{R}^\alpha{}_\beta + \delta^\alpha_\mu g_{\nu\beta}\Box + g_{\mu\nu}\nabla^\alpha\nabla_\beta \right. \\ \left. -\delta^\alpha_\mu\nabla_\beta\nabla_\nu - \delta^\alpha_\nu\nabla_\beta\nabla_\mu\right)\ln\left(\frac{\Box}{\mu^2}\right)\mathcal{R}^\beta{}_\alpha \\ -2\gamma\left(\delta^\alpha_{(\mu}\mathcal{R}_{\nu)}{}^\beta{}_{\sigma\tau} - \frac{1}{4}g_{\mu\nu}\mathcal{R}^{\alpha\beta}{}_{\sigma\tau} + (\delta^\alpha_\mu g_{\nu\sigma} + \delta^\alpha_\nu g_{\mu\sigma})\nabla^\beta\nabla_\tau\right) \\ \times\ln\left(\frac{\Box}{\mu^2}\right)\mathcal{R}_{\alpha\beta}{}^{\sigma\tau}. \tag{10}$$

In the background of weak curvature invariants, perturbation theory can be applied to solve these complicated coupled partial differential equations. We obtain a controlled approximation by expanding in curvature. We thus set $\tilde{g}_{\mu\nu} = g_{\mu\nu} + g^{\mathrm{q}}_{\mu\nu}$, where $g_{\mu\nu}$ is the classical solution and $g^{\mathrm{q}}_{\mu\nu}$ is the quantum solution one is solving Eq. (8) for. The $\log \Box\mathcal{R}^{\mu\ldots\nu}_{\alpha\ldots\beta}$ terms correspond to kernels that are integrated over curvature terms, which are functions of the energy-momentum tensor (see, e.g., the Appendix).

A generic astrophysical body has, relative to its surface or horizon, an interior $T^{\mu\nu}$ tensor and an exterior one. It is clear that the quantum corrections of the outside metric due to the $H^{\mathrm{NL}}_{\mu\nu}$ terms must be dependent on $T^{\mu\nu}$ and on the higher-curvature terms in the effective action. This fact is independent of the specific type of matter distribution, and thus applies to, e.g., a static star, a collapsing star, or a real astrophysical black hole (i.e., not a static vacuum solution) [33]. Hence, the exterior metric will keep a memory [35] of the interior of the matter distribution, which implies the presence of quantum hair for any gravitational body and, in particular, for realistic black holes. This hair is expressed in terms of deviation from the $1/r$ Newtonian potential. These deviations are due to quantum-gravitational corrections to Newton's law.

This is an explicit realization of the observation that the asymptotic graviton state of an energy eigenstate source is determined at leading order by the energy eigenvalue and that the quantum-gravitational fluctuations (i.e., graviton loops) produce corrections to the long-range potential whose coefficients depend on the internal state of the source [29]. We consider an explicit application of this result to the Oppenheimer-Snyder gravitational collapse model and calculate the leading-order quantum hair correction to that classical solution.

III. OPPENHEIMER-SNYDER MODEL: CLASSICAL SOLUTION

We now consider the Oppenheimer-Snyder model of gravitational collapse [30]. In the exterior region, the metric is defined by the line element [36]

$$ds^2 = f(R)dt^2 - g(R)^{-1}dR^2 - R^2 d\Omega^2, \tag{11}$$

with

$$f(R) = g(R) = \left(1 - \frac{2G_{\mathrm{N}}M}{R}\right), \tag{12}$$

where M is the total Arnowitt-Deser-Misner mass of the ball [37] and R is the areal radius, with $R \in [R_s(t), \infty)$. The energy-momentum tensor vanishes: $T_{\mu\nu} = 0$. In the interior region, the metric is defined by the line element

$$ds^2 = d\tau^2 - a(\tau)^2(dr^2 + r^2 d\Omega^2), \tag{13}$$

with $r \in [0, r_s]$, where the scale factor is given by

$$a(\tau) = \left(1 - \frac{\tau}{\tau_s}\right)^{2/3}, \tag{14}$$

with $\tau < \tau_s$ and τ_s is the time at which the ball collapses to a singularity. This time can be calculated and is given by

$$\tau_s = \sqrt{\frac{2R_s(0)^3}{9G_{\mathrm{N}}M}}. \tag{15}$$

The scale factor corresponds to a Hubble scale

$$H(\tau) = \frac{\dot{a}(\tau)}{a(\tau)} = \frac{2}{3(\tau - \tau_s)}. \tag{16}$$

Furthermore, the energy-momentum tensor is that of a perfect fluid:

$$T_\mu{}^\nu = \mathrm{diag}(-\rho, p, p, p). \tag{17}$$

The dust ball model assumes $p = w\rho$, with $w = 0$.

IV. QUANTUM CORRECTIONS

Our goal here is to describe the gravitational collapse of a star, and to show that we can compute, in a controlled approximation, the quantum gravity corrections to the metric during the formation of a black hole. Note that the initial formation of an astrophysical black hole (i.e., nonquantum black hole) does not require large curvatures anywhere in the dust ball. Specifically, this means that we can use the flat-space kernel function (see the Appendix) to compute quantum corrections arising from the effective action. The only region in the spacetime, sourced by the dust ball collapse, with large curvature is the future black hole singularity. In particular, the singularity does not affect the result: the history that is integrated over by an outside observer does not contain any regions with Planckian curvature (for astronomical black holes), such that the quantum corrections remain under control [38].

Thus, we work again with the Vilkovisky-DeWitt unique effective action of quantum gravity at second order in curvature. However, to calculate the quantum correction to the interior metric, it is easiest to use the Weyl basis, in which case one has

$$\Gamma_{\rm QG} = \int d^4x\sqrt{|g|}\left[\frac{M_P^2}{2}\mathcal{R} + \hat{c}_1(\mu)\mathcal{R}^2 + \hat{c}_2(\mu)\mathcal{C}_{\mu\nu\rho\sigma}\mathcal{C}^{\mu\nu\rho\sigma} + \hat{\alpha}\mathcal{R}\ln\left(\frac{\Box}{\mu^2}\right)\mathcal{R} + \hat{\beta}\mathcal{C}_{\mu\nu\rho\sigma}\ln\left(\frac{\Box}{\mu^2}\right)\mathcal{C}^{\mu\nu\rho\sigma} + \mathcal{O}(M_P^{-2})\right], \quad (18)$$

with $\hat{c}_1 = \tilde{c}_1 + \frac{1}{3}\tilde{c}_2$, $\hat{c}_2 = \frac{1}{2}\tilde{c}_2$, $\hat{\alpha} = \tilde{\alpha} + \frac{1}{3}\tilde{\beta}$, and $\hat{\beta} = \frac{1}{2}\tilde{\beta}$.

A. Interior corrections

In the interior we have an FLRW metric. For this metric the Weyl tensor vanishes, and it is thus convenient to work in the Weyl basis. Quantum corrections to this metric have been studied before in the literature. Corrections due to the $\mathcal{R}^2$ term have, for example, been studied in [11,40,41], while nonlocal corrections due to the $\mathcal{R}\log(\Box)\mathcal{R}$ terms have been studied in [8,10–13]. However, none of the previous studies considered the local and nonlocal corrections together. We will explain that this is crucial for obtaining a consistent result. By considering the local corrections to the action, one obtains the modified Friedmann equation [11]

$$H^2 + \frac{96\pi t_P^2 c_1(\mu)}{M_P^2}(2H\ddot{H} + 6H^2\dot{H} - \dot{H}^2) = \frac{8\pi G_{\rm N}}{3}\rho, \quad (19)$$

where t_P is the Planck time. We solve this perturbatively, i.e., we set $H = H_c + H_{\rm L}$, where $H_{\rm L} = \mathcal{O}(t_P^2)$ and $H_{\rm c}$ solves the classical Friedman equation. It is thus given by (16) and

$$H_{\rm c}^2 = \frac{8\pi G_{\rm N}}{3}\rho. \quad (20)$$

Solving for $H_{\rm L}$ then yields

$$H_{\rm L}(\tau) = \frac{32\pi t_P^2 \hat{c}_1(\mu)}{(\tau - \tau_s)^3} + \mathcal{O}(t_P^4). \quad (21)$$

To obtain the nonlocal corrections, one must evaluate the $\log\Box\mathcal{R}$ term in the field equations, which is discussed in the Appendix. This leads to the modified Friedmann equation [11] given by

$$H^2(\tau) - \frac{256\pi t_P^2\hat{\alpha}}{3(\tau_s - \tau)^4}\left[\ln(\mu\tau) + \ln\left(1 - \frac{\tau}{\tau_s}\right) - \frac{2\tau}{3\tau_s}\right] = \frac{8\pi G_{\rm N}}{3}\rho(\tau). \quad (22)$$

Solving this perturbatively yields

$$H_{\rm NL}(t) = \frac{64\pi t_P^2\hat{\alpha}}{(\tau - \tau_s)^3}\left[-\frac{2\tau}{3\tau_s} + \ln\left(1 - \frac{\tau}{\tau_s}\right) + \ln(\mu\tau)\right] + \mathcal{O}(t_P^4). \quad (23)$$

Gathering the local and nonlocal corrections, we find

$$H(\tau) = \frac{2}{3(\tau - \tau_s)}\left\{1 + \frac{48\pi t_P^2}{(\tau - \tau_s)^2}\left[\hat{c}_1(\mu) + 2\hat{\alpha}\left(\ln\left[\mu\tau\left(1 - \frac{\tau}{\tau_s}\right)\right] - \frac{2\tau}{3\tau_s}\right)\right] + \mathcal{O}(t_P^4)\right\}. \quad (24)$$

This equation is renormalization group invariant because we have considered the local and nonlocal corrections together. This had not been done previously in the literature, implying that previous studies of FLRW cosmology within this framework were flawed. We can now solve for $a(\tau)$. We find

$$a(\tau) = \exp\left(\int_0^\tau H(s)ds\right) = \left(1 - \frac{\tau}{\tau_s}\right)^{2/3}\left(1 + \frac{16\pi t_P^2}{(\tau_s - \tau)^2}\left\{\hat{c}_1(\mu)\frac{\tau}{\tau_s}\left(\frac{\tau}{\tau_s} - 2\right) + 2\hat{\alpha}\left[\frac{\tau^2}{6\tau_s^2} + \frac{\tau}{\tau_s}\left(\frac{\tau}{\tau_s} - 2\right)\ln(\mu\tau) - \left(\frac{\tau^2}{\tau_s^2} - \frac{2\tau}{\tau_s} + 2\right)\ln\left(1 - \frac{\tau}{\tau_s}\right)\right]\right\} + \mathcal{O}(t_P^4)\right), \quad (25)$$

where $\tau \in [0, \tau_s)$. This is the quantum correction to the interior metric. Note that it depends on both the nonlocal and local Wilson coefficients. The latter are not calculable within the effective theory approach that we have used, as it depends on the UV completion of the effective action.

Because we have considered the full effective action to second order in curvature, our result differs from previous studies of quantum cosmology within this framework, such as that in [10] where only the nonlocal contributions were considered. The phenomenology clearly needs to be considered again and the question of a big bounce should be investigated anew. We now turn our attention to the exterior solution and its quantum-gravitational corrections.

B. Exterior corrections

We discuss the kernel in spherically symmetric coordinates in the Appendix. The calculation follows the methodology of that presented in [22] with the notable complication that we now have a spacetime-dependent problem. In the Appendix we show that $\ln(\Box)\mathcal{R}$ can be approximated by Eq. (A15),

$$\ln\left(\frac{\Box}{\mu^2}\right)\mathcal{R}(x) = -\frac{2G_{\mathrm{N}}M}{3R_s(0)^3}\left\{\frac{2R_s(t_r)}{r} + \ln\left[\frac{r-R_s(t_r)}{r+R_s(t_r)}\right]\right\} + \mathcal{O}(\dot{R}_s),$$

where

$$\dot{R}_s(t_r) = \frac{dR_s(t_r)}{dt_r} \tag{26}$$

denotes a derivative with respect to the retarded time coordinate as measured by a distant observer. As this derivative remains small during the formation of the black hole at $t \to \infty$, the corrections encapsulated in the term $\mathcal{O}(\dot{R}_s)$ remain subleading throughout the collapse. We note that, at this order in the derivative expansion, one can easily obtain similar expressions for the corrections due to $\ln(\Box)\mathcal{R}_{\mu\nu}$ and $\ln(\Box)\mathcal{C}_{\mu\nu\rho\sigma}$, cf., e.g., [22].

Using these results, the quantum-corrected Einstein equations (8) can be solved perturbatively. This yields a correction to the functions $f(R)$ and $g(R)$ defined in Eq. (12). At leading order, these corrections are given by

$$\delta f(t_r,R) = (\alpha+\beta+3\gamma)\frac{192\pi l_P^2 G_{\mathrm{N}}M}{R_s(0)^3} \times\left\{\frac{2R_{\mathrm{s}}(t_r)}{R} + \ln\left[\frac{R-R_s(t_r)}{R+R_s(t_r)}\right]\right\} + \mathcal{O}(\dot{R}_s), \tag{27}$$

$$\delta g(t_r,R) = (\alpha-\gamma)\frac{384\pi l_P^2 G_{\mathrm{N}}M}{R_s(0)^3}\frac{R_s(t_r)^3}{R[R^2-R_s(t_r)^2]} + \mathcal{O}(\dot{R}_s), \tag{28}$$

which coincides with the results for the static dust ball given in Ref. [25] after making the replacement $R_s \to R_s(t_r)$. In the above, $l_P = \sqrt{\hbar G_{\mathrm{N}}}$ is the Planck length.

Let us focus on the tt component of the metric and expand the result for $R \gg R_s$ making the different expansion parameters explicit. We obtain

$$f(t_r,R) = 1 - \frac{2G_{\mathrm{N}}M}{R} - 128\pi^2(\alpha+\beta+3\gamma)\frac{l_P^2}{R^2}\left\{\frac{G_{\mathrm{N}}M}{R}\frac{R_s(t_r)^3}{R_s(0)^3} \times\left[1+\frac{3R_s(t_r)^2}{5R^2}+\mathcal{O}\left(\frac{R_s(t_r)}{R}\right)^4\right]+\mathcal{O}(\dot{R}_s)\right\} + \mathcal{O}\left(\frac{l_P}{R}\right)^4. \tag{29}$$

The expansion in l_P/R reflects the truncation of the effective action at second order in curvature.

Taking these expansion parameters into account, we have calculated the leading-order corrections to the metric. Our main result is as follows: we have computed the coefficient of a (fully quantum-mechanical) correction to the exterior metric which behaves as R^{-5} asymptotically far from the black hole. The R^{-5} correction can be seen explicitly by expanding the brackets. At this order, the coefficient of this term depends on the density distribution of the dust ball from which the black hole was formed. Corrections to this result are suppressed by factors of order l_P/R_s and $\dot{R}_s$. A distant observer could in principle measure these deviations from the classical gravitational potential.

Our result implies that the quantum hair identified in [25] is present in the collapse (i.e., spacetime-dependent) model considered here. This correction survives throughout the gravitational collapse. Our result is further evidence that black holes have quantum hair.

Our calculations show explicitly that the quantum hair persists throughout the history of the system. As we are considering large black holes (i.e., black holes with masses much larger than the Planck mass), the only region of large curvature is near the singularity of the hole, at the center.

V. CONCLUSIONS

In this paper, we have revisited the question of quantum hair and quantum memory in quantum gravity. Within the context of the unique effective action, we have shown that quantum hair and quantum memory are very generic features of quantum gravity and that any nonzero energy-momentum tensor will produce quantum hair in the form of quantum corrections to the classical spacetime resulting from the Einstein equations. While the general proof is model independent, we have illustrated this result with a direct calculation in the case of the Oppenheimer-Snyder collapse model. We have shown by explicit calculation that the quantum corrections to the Oppenheimer-Snyder classical solution are sensitive to the density of the matter distribution. The outside gravitational field contains information about the collapse process that is stored in the quantum hair. In principle, a distant observer could measure the deviation from the Newton potential. This work is a further demonstration that all classical solutions in general relativity, including black holes, are hairy in quantum gravity.

This manuscript has no associated data. Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

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APPENDIX

We are interested in evaluating the expression

$$\ln\left(\frac{\Box}{\mu^2}\right)\mathcal{R}(x), \tag{A1}$$

which we can write as

$$\int d^4x' \sqrt{|g(x')|} L(x-x')\mathcal{R}(x'). \tag{A2}$$

As we are working at second order in curvature, we approximate the kernel L by its flat-space kernel $L^{\rm flat}$ that is given in Eq. (55) of Ref. [17],

$$L^{\rm flat}(x-x') = \lim_{\delta\to 0}\left[\frac{i}{\pi^2}\left(\frac{\Theta(\Delta t)\Theta((x-x')^2)}{[(\Delta t+i\delta)^2-(\vec{x}-\vec{x}')^2]^2} - \frac{\Theta(\Delta t)\Theta((x-x')^2)}{[(\Delta t-i\delta)^2-(\vec{x}-\vec{x}')^2]^2}\right) - 2\delta^{(4)}(x-x')\ln(\delta\mu)\right], \tag{A3}$$

with $\Delta t \coloneqq t-t'$. We are interested in evaluating this kernel in two regimes:

(1) the interior of the collapsing star described by the FLRW metric (13);

(2) the exterior of the collapsing star described by the Schwarzschild metric (11).

1. Interior

In the interior coordinate system (τ, r, θ, ϕ), the Ricci scalar is given by

$$\mathcal{R}(x) = -\frac{4\Theta(r_s-r)}{3\tau_s^2 a(\tau)^3}. \tag{A4}$$

Using this expression, we can perform the spatial integrals in Eq. (A2). This yields

$$\ln\left(\frac{\Box}{\mu^2}\right)\mathcal{R}(x) = \frac{4}{3\tau_s^2}\int_{\tau-(r_s+r)}^{\tau-(r_s-r)}\left(\frac{1}{\tau-\tau'}-\frac{1}{r}\right)d\tau' + \frac{8}{3\tau_s^2}\lim_{\delta\to 0}\left(\int_{t-(r_s-r)}^{t-\delta}\frac{1}{\tau-\tau'}d\tau' + \ln(\delta\mu)\right), \tag{A5}$$

which reduces to the results obtained in Refs. [10,11] in the limit $r_s \to \infty$. In deriving this result we have approximated the function $\ln(\Box)$ by its kernel in flat spacetime (A3), which is a valid approximation at second order in curvature.

2. Exterior

In the exterior coordinate system (t, R, θ, ϕ), the Ricci scalar (A4) can be written as

$$\mathcal{R}(x) = -\frac{6G_{\rm N}M\Theta(R_s(t)-R)}{R_s(0)^3 a(t)^3}. \tag{A6}$$

Using this expression, we can perform the spatial integrals in Eq. (A2). This yields

$$\ln\left(\frac{\Box}{\mu^2}\right)\mathcal{R}(x) = \frac{6G_{\rm N}M}{R_s(0)^3}\int_{-\infty}^{t}\Theta[R_s(t')+t'-t+R] \times\Theta[R_s(t')-t'+t-R]\left(\frac{1}{t-t'}-\frac{1}{R}\right)dt'. \tag{A7}$$

As was the case for the interior calculation, this result relies on approximating the function $\ln(\Box)$ by its flat-space kernel (A3), which is valid in the coordinate frame of a distant observer, where the collapse remains slow. Corrections to the result appear at $\mathcal{O}(\dot{R}_s^2, \ddot{R}_s R_s)$, where the derivatives are taken with respect to the retarded time coordinate t_r.

We can further evaluate the time integral in Eq. (A7). The domain of integration of this expression in t' is determined by the two Heaviside functions, which require

$$t-R-R_s(t') \le t' \le t-R+R_s(t'). \tag{A8}$$

We note that $R_s(t) = r_s a(\tau)$, where $\tau = \tau(t)$ and $r_s = R_s(0)$. However, at leading order in the derivative expansion the task becomes much simpler. In general, the integration domain (A8) will be given by

$$T_-(t_r; R_s(t_r), \dot{R}_s(t_r)) \le t' \le T_+(t_r; R_s(t_r), \dot{R}_s(t_r)) < t, \tag{A9}$$

where we explicitly showed that the end points $T_\pm$ depend on both the retarded time t_r and the radius of the star evaluated at the retarded time $R_s(t_r)$. We note that the integration bounds may also depend on higher derivatives, but these terms are suppressed, as they appear at higher order in the derivative expansion.

At this order in the derivative expansion, we can Taylor expand $R_s(t')$, which yields

$$R_s(t') = R_s(t_r) + \dot{R}_s(t_r)(t' - t_r) + \mathcal{O}(\ddot{R}_s). \tag{A10}$$

Using this expansion, we find that the range of integration in t' is explicitly determined by

$$T_- = \frac{t_r - R_s(t_r) + t_r\dot{R}_s(t_r)}{1 + \dot{R}_s(t_r)} \lesssim t' \lesssim \frac{t_r + R_s(t_r) - t_r\dot{R}_s(t_r)}{1 - \dot{R}_s(t_r)} = T_+. \tag{A11}$$

Hence, up to first-order time derivatives, we obtain

$$t_r - R_s(t_r)[1 - \dot{R}_s(t_r)] \lesssim t' \lesssim t_r + R_s(t_r)[1 + \dot{R}_s(t_r)]. \tag{A12}$$

Therefore, Eq. (A7) simplifies to

$$\ln\left(\frac{\Box}{\mu^2}\right)\mathcal{R}(x) = \frac{6G_\mathrm{N}M}{R_s(0)^3}\int_{t_r - R_s(t_r)[1-\dot{R}_s(t_r)]}^{t_r + R_s(t_r)[1+\dot{R}_s(t_r)]}\left(\frac{1}{t-t'} - \frac{1}{R}\right)dt'. \tag{A13}$$

Evaluating this integral, we find our result,

$$\ln\left(\frac{\Box}{\mu^2}\right)\mathcal{R}(x) = -\frac{6G_\mathrm{N}M}{R_s(0)^3}\left\{\frac{2R_s(t_r)}{R} + \ln\left[\frac{R - R_s(t_r)[1+\dot{R}_s(t_r)]}{R + R_s(t_r)[1-\dot{R}_s(t_r)]}\right]\right\} + \mathcal{O}(\dot{R}_s^2, \ddot{R}_s R_s), \tag{A14}$$

which reduces to the result obtained in Ref. [22] for a constant radius $R_s(t_r) = R_s$. We can further approximate this result by

$$\ln\left(\frac{\Box}{\mu^2}\right)\mathcal{R}(x) = -\frac{6G_\mathrm{N}M}{R_s(0)^3}\left\{\frac{2R_s(t_r)}{R} + \ln\left[\frac{R - R_s(t_r)}{R + R_s(t_r)}\right]\right\} + \mathcal{O}(\dot{R}_s), \tag{A15}$$

which is the result applied in Sec. IV B.

Let us emphasize that all expressions hold throughout the formation of a black hole at $t \to \infty$. In this regime, $|\dot{R}_s| \lesssim \frac{1}{6}$, implying that the perturbative expansion is under control. Further corrections can be calculated reliably, but the calculations are rather complicated and the results are difficult to display.

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