Synchrotron radiation in an *P*-odd (2 + 1)-dimensional electrodynamics

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In this paper we develop the theory of synchrotron radiation in P-odd two-dimensional electrodynamics based on the quantum theory of radiation and the solution of the Dirac equation in a constant magnetic field in $(2+1)$-dimensional space-time. Exact analytical expressions for the partial probabilities of the process obtained for the first time both in the case of a massive and massless charged fermion. Asymptotic formulas obtained for the spectral distribution and total radiation power of a relativistic massive fermion in the quasiquantum and ultraquantum limits. The spectral distribution and total power of the synchrotron radiation of a massless charged fermion calculated in the limiting case of large values of the principal quantum number of the initial and final states of the fermion.

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I. INTRODUCTION

Synchrotron radiation in $(3+1)$-dimensional space-time is one of the fundamental processes accompanying the propagation of charged fermions in an external magnetic field [1,2].

Recently, the study of the classical and quantum theory of *SR* in various models of $(2+1)$-dimensional electrodynamics has been of great interest. Note that the theory of *SR* in QED_{2+1} has not fully developed for both massive and massless charged fermions [3–11].

In $(2+1)$-dimensional electrodynamics the Huygens principle is violated, which is mathematically expressed in the fact that the retarded Green's function of the d'Alembert equation has support everywhere inside the future light cone, and not only on the surface of the characteristic cone, as in the $(3+1)$-dimensional case.

The retarded Green's function of the d'Alembert equation in $(2+1)$-dimensional space-time is determined by the expression [12,13]

$$G^{\mathrm{ret}}_{2+1}(X) = \frac{\Theta(X_0)\Theta(X^2)}{2\pi\sqrt{X^2}}, \qquad X^\mu = x^\mu - x'^\mu,$$

where the Heaviside step function $\Theta(X)$ is introduced, while in the $(3+1)$-dimensional space-time

$$G^{\mathrm{ret}}_{3+1}(X) = \frac{1}{4\pi R}\delta(t - t' - R), \qquad R = |\vec{r} - \vec{r}'|,$$

where $\delta(t - t' - R)$ is the Dirac δ function.

As a result, in an odd-dimensional space-time, the gradient of the retarded potential of the field of a point charge in the wave zone determined by the integral over proper time of the sum of two terms, and the integrals of each of these terms diverge separately. These integrals regularized in [3], and it is shown that the well-known methods of classical electrodynamics can also be used to calculate the *SR* power of a point charge in an odd-dimensional space-time.

It is essential that in the case of a circular motion of a point charge with an ultrarelativistic velocity, the main contribution to the integral over proper time made by a small interval preceding the retarded one [3].

Note that before the appearance of the work [3], the calculation of the *SR* power in $(2+1)$-dimensional electrodynamics carried out by a method based on the classical Dirac-Lorentz equation, which describes the motion of an electron, taking into account radiative friction [4].

In [5,6], based on the calculation of the radiative shift of the electron energy and the optical theorem, the *SR* process in the P-odd QED_{2+1} model investigated in the limit of zero charged fermion mass. The quantum theory of *SR* developed in a P-even QED_{2+1} model with a doubled fermionic representation and in a reduced QED_{3+1} [14,15] in papers [7,8].

Massless quantum electrodynamics in an external magnetic field is of great interest in connection with the prediction of magnetic catalysis of chiral symmetry breaking, in solid-state physics and low-dimensional systems, as well as in cosmology [3,16–20]. Induced bremsstrahlung, as indicated in [21], may be the main mechanism of perpendicular light reflection in the case when the incident monochromatic wave is perpendicular to the graphene layer. The *SR* theory of a massless charged particle has

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only recently been constructed by Galtsov [11] within the framework of quantum theory in $(3+1)$-dimensional scalar and spinor quantum electrodynamics by the operator Schwinger method.

The aim of this work is to construct a quantum theory of *SR* in *P*-odd QED_{2+1}.

In contrast to the method used in the works [5–8,11], to solve the problem, we use the quantum theory of radiation and the exact solutions of the Dirac equation in a constant magnetic field in *P*-odd QED_{2+1}. We consider the case when the Dirac equation contains only the parity-violating mass term. In the presence of both parity-preserving and parity-violating mass terms, the solution of the Dirac equation in a magnetic field in both the $(2+1)$- and $(3+1)$-dimensional case obtained in Ref. [22].

Section II presents the calculation of the exact formula for the partial probability of a spontaneous transition of a massive electron from the initial to any low-lying state with the emission of a photon in a magnetic field with an arbitrary strength.

In Sec. III, analytical results are obtained that describe the dependence of the spectral distribution and total radiation power on the dynamic parameter of synchrotron radiation in the quasiquantum approximation, taking into account the first quantum correction to the radiation power, as well as in the ultra-quantum approximation.

In Sec. IV, we obtain an exact formula for the partial probabilities of the *SR* of a massless charged fermion. It shown that the transition probabilities depend only on the principal quantum numbers of the initial and final states of the fermion and do not contain an explicit dependence on the magnetic field strength. The spectrum and total power of the *SR* of a massless fermion investigated in the limiting case of values of the principal quantum number of the initial and final states of the fermion that are large compared to unity.

In Sec. V, we discuss the results of the work.

II. SPECTRAL DISTRIBUTION OF PROCESS PROBABILITY

In $(2+1)$-dimensional space-time, the Dirac matrix algebra described using Dirac matrices in two nonequivalent ways [16,23–26]. In this work, we will use the representations [27]

$$\gamma^0 = \xi\sigma_3, \qquad \gamma^1 = i\xi\sigma_1, \qquad \gamma^2 = i\xi\sigma_2, \tag{2.1}$$

where $\xi = \pm 1$, $\sigma_k (k = 1,2,3)$ are the Pauli matrices.

As a result, there are two different Dirac equations for describing spinor fields

$$[\gamma^\mu (p_\mu + eA_\mu) - mI]\Psi(x) \quad \mu = 0,1,2. \tag{2.2}$$

Here $\Psi(x)$ is a two-component spinor, $\hat{p}_x$ and $\hat{p}_y$ the projections of the momentum operator, and m is the mass of an electron with the charge $-e < 0$, $A^\mu(x)$ potential of the external field.

Unlike QED_{3+1}, a massive electron in $(2+1)$-dimensional space-time has only one spin state, i.e. the electron spin is not a pseudovector but a pseudoscalar with respect to Lorentz transformations, and the mass term in Eq. (2.2) violates *P* and *T* parity. Note also that the magnetic field strength in QED_{2+1} is also a pseudoscalar.

Thus, in QED_{2+1} we have two different and odd Dirac equations, each of which can be used to describe an electron or positron with one spin degree of freedom.

In a constant magnetic field given in the Landau gauge by the potential

$$A^\mu = (0,0,xH), \tag{2.3}$$

the energy levels and the wave function of the stationary states of a two-dimensional electron determined by the following formulas:

$$\Psi(t,x,y) = \frac{1}{\sqrt{2E_n}} \begin{pmatrix} u_{n-1}(\eta)\sqrt{E_n + \xi m} \\ u_n(\eta)\sqrt{E_n - \xi m} \end{pmatrix} e^{-iE_n t + iyp_y}, \tag{2.4}$$

$$E_n = \sqrt{m^2 + p_\perp^2} = \sqrt{m^2 + 2eHn}, \tag{2.5}$$

where $n = 0,1,2,\ldots$ is the principle quantum number and p_y is the electron momentum projection.

The Hermite function in the formula (2.4) expressed in terms of the Hermite polynomials by the formula

$$u_n(\eta) = \frac{(eH)^{\frac{1}{4}}}{\left(2^n n!\sqrt{\pi}\right)^{1/2}} e^{-\frac{\eta^2}{2}} H_n(\eta),$$
$$H_n(\eta) = (-1)^n e^{\eta^2} \frac{d^n}{d\eta^n}(e^{-\eta^2}), \tag{2.6}$$

and the argument of these functions

$$\eta = \sqrt{eH}\left(x + \frac{p_y}{eH}\right). \tag{2.7}$$

In the Furry's representation, the probability of transition in a unit of time from the initial state of a fermion (n, p_y) to a state (n', p'_y) with radiation a photon with a 3-momentum $k^\mu = (\omega, k_x, k_y)$ is determined by the formula

$$w(n, p_y \to n', p'_y) = \frac{g^2}{4\pi} \int \frac{d\vec{k}}{\omega} \delta(E_n - E_{n'} - \omega) |\vec{e}^* \cdot \vec{j}|^2, \tag{2.8}$$

$$\vec{j} = \int \Psi^+_{n',p'_y}(\vec{x}) \vec{\alpha} e^{-i\vec{k}\vec{r}} \Psi_{n,p_y}(\vec{x}) d^2x. \tag{2.9}$$

Here $\vec{\alpha} = \gamma^0\vec{\gamma}$, the Dirac δ function expresses the energy conservation law, the quantity g^2 in QED_{2+1} has the dimension of mass, and of the two independent 4-vectors of linear polarization in QED_{3+1}, on the mass shell in $(2+1)$-dimensional QED the only possible choice for the linear-polarization vector is [28,29]

$$e^{\mu} = (0, -\sin\phi, \cos\phi), \tag{2.10}$$

where ϕ is the polar angle of the photon momentum. The matrix elements in the formula (2.9) with wave functions (2.4) calculated using the integral [30]

$$\int_{-\infty}^{+\infty} dx e^{-ixk_x} u_n(\gamma^{\frac{1}{2}}x + \gamma^{-\frac{1}{2}}p_y) u_{n'}(\gamma^{\frac{1}{2}}x + \gamma^{-\frac{1}{2}}p_{y'}) = e^{[i\mu + i(n-n')\Phi]} I_{nn'}(\rho), \tag{2.11}$$

where the following notations are adopted:

$$\gamma = eH, \qquad \mu = \frac{k_1}{2\gamma}(p_y + p_{y'}),$$
$$\rho = \frac{(E_n - E_{n'})^2}{2\gamma}, \qquad k_2 - ik_1 = |\vec{k}|e^{i\Phi}. \tag{2.12}$$

The Laguerre function defined by the formula

$$I_{nn'}(\rho) = \frac{1}{\sqrt{nn'}} e^{-\frac{\rho}{2}} \rho^{\frac{n-n'}{2}} Q_{n'}^{n-n'}(\rho), \tag{2.13}$$

and Laguerre polynomials

$$Q_{n'}^{n-n'}(\rho) = e^{\rho}\rho^{-(n-n')} \frac{d^{n'}}{d\rho^{n'}}(\rho^n e^{-\rho}). \tag{2.14}$$

As a result, we get

$$(\vec{e}\cdot\vec{j}) = i\frac{e^{i\mu + i(n-n')\Phi}}{2\sqrt{E_n E_{n'}}}\Big\{I_{n-1,n'}(\rho)\sqrt{E_n + \xi m}\sqrt{E_{n'} - \xi m} - I_{n,n'-1}(\rho)\sqrt{E_n - \xi m}\sqrt{E_{n'} + \xi m}\Big\}\delta_{p_y', p_y - k_y}.$$

where $\delta_{p_y', p_y - k_y}$ is the Kronecker symbol.

After integrating over the variables ω and p_y' we arrive at the following expression for the partial probability of the process per unit time and per unit volume:

$$w(n \to n') = \frac{g^2}{8E_n E_{n'}}\Big[I_{n-1,n'}(\rho)\sqrt{E_n + \xi m}\sqrt{E_{n'} - \xi m} - I_{n,n'-1}(\rho)\sqrt{E_n - \xi m}\sqrt{E_{n'} + \xi m}\Big]^2. \tag{2.15}$$

Thus, in contrast to the standard QED_{3+1}, for the partial probability of the *SR* it is possible to obtain the exact result in two-dimensional quantum electrodynamics described by the formula (2.15).

Let us find, for example, the probability of *SR* for the transition of a massive charged fermion from excited states $(n \geq 1, \xi = -1)$ to the ground state $(n' = 0)$:

$$w(n \to n' = 0) = \frac{g^2}{4}\left(1 - \frac{m}{E_n}\right)\frac{1}{(n-1)!}e^{-\rho}\rho^{n-1},$$
$$\rho = \frac{(E_n - m)^2}{2eH}. \tag{2.16}$$

In the limiting case of a relativistic initial electron $(p_\perp \gg m, n \gg 1, \rho \simeq n)$, using Stirling's formula

$$n! \simeq \left(\frac{n}{e}\right)^n \sqrt{2\pi n}, \quad n \gg 1, \tag{2.17}$$

from (2.16) we get the following result:

$$w(n \gg 1 \to n' = 0) = \frac{g^2}{4\sqrt{2\pi n}}. \tag{2.18}$$

III. *SR* POWER OF A MASSIVE RELATIVISTIC FERMION

Let us find the spectral distribution and total *SR* power of a massive relativistic fermion in a relatively weak magnetic field when the following conditions are satisfied:

$$\frac{e}{m^2}\left[\frac{1}{2}(F_{\mu\nu}F^{\mu\nu})^2\right]^{\frac{1}{2}} = \frac{H}{H_0} \ll 1,$$
$$\frac{m}{p_\perp} \ll 1, \qquad \frac{m}{p_\perp'} \ll 1, \qquad n - n' \gg 1. \tag{3.1}$$

Dynamic parameter of synchrotron radiation

$$\chi = \frac{e}{m^3}\sqrt{-(F_{\mu\nu}p^{\nu})^2} = \frac{H \cdot p_\perp}{H_0 \cdot m}, \tag{3.2}$$

where $H_0 = \frac{m^2}{e}$ is the Schwinger value of the magnetic field strength for the considered fermion. Let us also introduce the invariant spectral variable

$$u = \frac{\chi - \chi'}{\chi'} = \frac{p_\perp - p_\perp'}{p_\perp'} = \frac{\sqrt{n} - \sqrt{n'}}{\sqrt{n'}}, \qquad n' \neq 0, \tag{3.3}$$

which does not depend on the mass of the electron.

Under conditions (3.1), the argument of the Laguerre functions $\rho \to \rho_0 - 0$, where $\rho_0 = (\sqrt{n} - \sqrt{n'})^2$:

$$1 - \frac{\rho}{\rho_0} \simeq \frac{m^2}{p_\perp p_\perp'} \ll 1. \tag{3.4}$$

Note that inequality (3.4) remains valid in a magnetic field of any finite strength starting from some sufficiently small value of the mass of a hypothetical charged fermion, and the limiting case of a massless fermion corresponds to the case when $\rho = \rho_0$.

When condition (3.4) are satisfied, the following asymptotic formulas are valid for the Laguerre functions in terms of the Macdonald functions [1]

$$I_{n,n'-1}(\rho) \simeq 2N\sqrt{\varepsilon}[K_{\frac{1}{3}}(x) - (u+1)\sqrt{\varepsilon}K_{\frac{2}{3}}(x)],$$
$$I_{n-1,n'}(\rho) \simeq 2N\sqrt{\varepsilon}[K_{\frac{1}{3}}(x) + \sqrt{\varepsilon}K_{\frac{2}{3}}(x)], \tag{3.5}$$

where the notations are accepted

$$N = \frac{(u+1)^{\frac{1}{2}}}{2\pi\sqrt{3}}, \quad \varepsilon = \left(\frac{m}{p_\perp}\right)^2, \quad u = \sqrt{\frac{n}{n'}} - 1, \quad x = \frac{u}{3\chi}. \tag{3.6}$$

The SR power obtained by multiplying the probability by the photon energy and replacing the summation over the quantum number with integration over the spectral variable u. As a result, from the formulas (2.15), (3.5), and (3.6) for the spectral power distribution SR in odd QED_{2+1} we get the following representation:

$$\frac{dP}{du} = \frac{mg^2u^3}{24\pi^2\chi(1+u)^3}\left[K_{\frac{1}{3}}(x) - \xi\frac{u+2}{u}K_{\frac{2}{3}}(x)\right]^2, \quad x = \frac{u}{3\chi}. \tag{3.7}$$

Let us find the asymptotics of the total radiation power in the quasiquantum approximation. To do this, we first transform the formula (3.7) to the form

$$dP = \frac{3mg^2\chi t dt}{8\pi^2(1+3\chi t)^3}[9\chi^2t^2K^2_{\frac{1}{3}}(t) - 6\xi\chi t(2+3\chi t)K_{\frac{1}{3}}(t)K_{\frac{2}{3}}(t) + (2+3\chi t)^2K^2_{\frac{2}{3}}(t)]. \tag{3.8}$$

In the classical approximation, everywhere in the formula (3.8), except for the common factor proportional to the parameter χ, we assume $\chi = 0$, and integration over the variable t carried out using the value of the integral [1]

$$\int_0^\infty t^{\mu-1}K^2_\rho(t)dt = \frac{2^{\mu-3}}{\Gamma(\mu)}\Gamma^2\left(\frac{\mu}{2}\right)\Gamma\left(\frac{\mu}{2}+\rho\right)\Gamma\left(\frac{\mu}{2}-\rho\right), \quad \mu > 2\rho, \tag{3.9}$$

where $\Gamma(x)$ is the Euler gamma function.

Then for the radiation power, we obtain the formula of the classical approximation

$$P_{cl} = \frac{mg^2\chi}{\pi\sqrt{3}}, \tag{3.10}$$

which agrees with the results of the work [3,4,8] up to a numerical factor. Note that the classical radiation power is the same for two possible types of charged fermions in odd two-dimensional electrodynamics.

Next, we calculate the first quantum correction to the classical radiation power. Keeping in formula (3.8) the next term of the expansion with respect to the parameter χ and using (3.9), as well as the integral

$$\int_0^\infty t^\lambda K_\mu(t)K_\nu(t)dt = \frac{2^{\lambda-2}}{\Gamma(\lambda+1)}\Gamma\left(\frac{1+\lambda+\mu+\nu}{2}\right) \times\Gamma\left(\frac{1+\lambda-\mu+\nu}{2}\right)\Gamma\left(\frac{1+\lambda+\mu-\nu}{2}\right) \times\Gamma\left(\frac{1+\lambda-\mu-\nu}{2}\right), \quad \lambda > \mu+\nu+1, \tag{3.11}$$

we find

$$P^1_q = -\frac{mg^2\chi^2}{\sqrt{3}\pi}\left[\xi + \frac{7\pi\sqrt{3}}{16}\right]. \tag{3.12}$$

As in QED_{3+1}, in the ultraquantum, case ($\chi \gg 1$) the main contribution to the radiation power in the formula (3.7) given by the area in which the argument of the Macdonald function $x \ll 1$ and the asymptotic formula is valid

$$K_\mu(x) \simeq \frac{2^{\mu-1}\Gamma(\mu)}{x^\mu}, \quad x \ll 1. \tag{3.13}$$

Using further the value of the integral

$$\int_0^\infty \frac{x^\mu}{(1+3\chi x)^\nu}dx = \frac{\Gamma(\mu+1)\Gamma(\nu-\mu-1)(3\chi)^{-\nu-1}}{\Gamma(\nu)}, \tag{3.14}$$

we get

$$P_{uq} \simeq \frac{17mg^2}{36\pi\sqrt{3}}2^{-\frac{2}{3}}\Gamma^2\left(\frac{2}{3}\right)(3\chi)^{\frac{1}{3}}, \tag{3.15}$$

where $\chi \sim m^{-3}$. Thus, so as in the standard QED_{3+1}, in the ultraquantum case the leading term of the radiation power asymptotics does not depend on the parity-violating electron mass.

IV. *SR* OF A MASSLESS CHARGED FERMION IN QED_{2+1}

For a massless charged fermion, the partial probabilities of the process determined by the exact formula (2.15) at $m=0$:

$$w(n\to n') = \frac{g^2}{8}[I_{n,n'-1}(\rho_0) - I_{n-1,n'}(\rho_0)]^2, \qquad (4.1)$$

where the argument of the Laguerre functions is

$$\rho_0 = (\sqrt{n} - \sqrt{n'})^2. \qquad (4.2)$$

The result (4.1) and (4.2) is exact and does not explicitly depend on the magnetic field strength. The partial probabilities of spontaneous transitions of a massless charged fermion depend only on the principal quantum numbers of the initial and final states of the fermion.

The recurrence relations for the Laguerre functions with shifted indices and with an argument equal to the transition point transformed to the form

$$I_{n,n'-1}(\rho_0) = I_{n,n'}(\rho_0) - \sqrt{\frac{\rho_0}{n'}} I'_{n,n'}(\rho_0),$$
$$I_{n-1,n'}(\rho_0) = I_{n,n'}(\rho_0) + \sqrt{\frac{\rho_0}{n}} I'_{n,n'}(\rho_0). \qquad (4.3)$$

Then formula (4.1) takes the following final form:

$$w(n\to n') = \frac{g^2}{8}\frac{(n-n')^2}{nn'}(I'_{n,n'}(\rho_0))^2. \qquad (4.4)$$

Further, we find the total *SR* power of a massless fermion when the quantum numbers n and n' satisfy the conditions

$$n \gg 1, \qquad n' \gg 1, \qquad n - n' \gg 1 \qquad (4.5)$$

for any magnetic field strength.

To do this, we note that under conditions (4.5) for the derivative of the Laguerre function at $\rho \to \rho_0 - 0$ the following asymptotic formula is valid in terms of the Macdonald function [1]

$$I'_{n,n'}(\rho) \simeq \frac{\sqrt[4]{nn'}}{\pi\sqrt{3\rho_0}}\left(1 - \frac{\rho}{\rho_0}\right) K_{\frac{2}{3}}(z), \qquad (4.6)$$

where the argument of the Macdonald function is defined by the formula

$$z = \frac{2}{3}\sqrt[4]{\rho_0^2 nn'}\left(1 - \frac{\rho}{\rho_0}\right)^{\frac{3}{2}}. \qquad (4.7)$$

Therefore, taking into account also the formula (3.13), for any values n and n' satisfying the conditions (4.5), we find

$$I'_{n,n'}(\rho_0) = \lim_{\varepsilon\to 0} I'_{n,n'}(\rho_0 - \varepsilon)$$
$$= \frac{\sqrt[4]{nn'}}{2\pi\sqrt{3\rho_0}} 3^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)\frac{1}{(\rho_0^2 nn')^{\frac{1}{6}}}. \qquad (4.8)$$

Thus, for the partial probabilities of the process in the limiting case (4.5) we obtain the following result:

$$w(n\to n') \simeq \frac{g^2 3^{\frac{1}{3}}\Gamma^2\left(\frac{2}{3}\right)\left(\sqrt{n}+\sqrt{n'}\right)^2}{32\pi^2\sqrt{nn'}[(\sqrt{n}-\sqrt{n'})^4 nn']^{\frac{1}{3}}}. \qquad (4.9)$$

The summation over the quantum number n' in the formula approximately replaced by integration over the spectral variable u defined by the formula (3.3) and for the spectral distribution of the *SR* intensity we find the expression

$$\frac{dP}{du} = \frac{g^2 3^{\frac{1}{3}}\Gamma^2\left(\frac{2}{3}\right)\sqrt{2eHn}\,u^{-\frac{1}{3}}(u+2)^2}{16\pi^2 n^{\frac{1}{3}}(u+1)^3}. \qquad (4.10)$$

Integrals over a variable u calculated using the formula

$$\int_0^\infty \frac{x^{p-1}}{(1+x)^{p+r}}dx = \frac{\Gamma(p)\Gamma(r)}{\Gamma(p+r)}. \qquad (4.11)$$

As a result, for the *SR* intensity of a massless charged fermion in *QED* we obtain under the conditions (4.5) the following asymptotic expression:

$$P = \frac{17g^2}{36\pi\sqrt{3}} 2^{-\frac{2}{3}} 3^{\frac{1}{3}}\Gamma^2\left(\frac{2}{3}\right)(eHp_\perp)^{\frac{1}{3}}. \qquad (4.12)$$

V. CONCLUSION

In this work, the process of synchrotron radiation in an odd QED_{2+1} studied for the first time based on the quantum theory of radiation using the exact solution of the Dirac equation in a constant magnetic field.

Analytical formulas obtained that describe the dependences of the partial probabilities of the process of spontaneous transition of a charged fermion, both massive and massless, from any initial state to an arbitrary final state with photon emission. In contrast to similar formulas of standard QED_{3+1}, our results (2.15) and (4.4) are valid for any magnetic field strength.

The dependence of the spectral distribution and total radiation power of a massive relativistic fermion on the *SR* dynamic parameter in a relatively weak magnetic field is studied. In the classical approximation, the result (3.3) of this work for the radiation power of a relativistic fermion is consistent with the results obtained in the works [3,4] by different methods in the framework of classical electrodynamics, as well as with the corresponding result of the work [8]. It is shown that the first quantum correction to the classical *SR* power contains two types of terms, one of which is proportional to the parameter ξ, i.e., depends on the choice of one of the two irreducible representations of the Dirac matrix algebra in QED_{2+1}, and the second term is the same for both types of fermions $n' \gg 1, n - n' \gg 1$. We assume that, if one introduce two types of mass terms in the Dirac equation [22], then the most favorable conditions for the manifestation of vodd effects in the synchrotron radiation by the relativistic electron realized in the quasi-quantum approximation.

The spectral distribution and total power of the *SR* of a massless charged fermion calculated in the case of relatively large values of the principal quantum number of the initial and final states of the fermion, when the conditions $n \gg 1, n' \gg 1, n - n' \gg 1$ are satisfied.

The calculation performed shows that, in the considered approximation, the total radiation power of a massless charged fermion [Eq. (4.12)] coincides with the main term of the asymptotic expansion of the radiation power of a massive two-dimensional electron in the ultraquantum limit of large values of the *SR* dynamic parameter, which is described by Eq. (3.15).

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