Dalitz decays $D_{sJ}^{(*)} \to D_s^{(*)} \ell^+ \ell^-$

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The Dalitz decays of the positive-parity $D_{sJ}^{(*)}$ charmed mesons, $D_{sJ}^{(*)} \to D_s^{(*)} \ell^+ \ell^-$ with $J = 0$, 1, 2 and $\ell = e$, μ, are important processes to investigate the nature of the $D_{sJ}^{(*)}$ states. We analyze the full set of decays, considering the four lightest $D_{sJ}^{(*)}$ mesons as belonging to the heavy quark spin doublets $s_\ell^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$, with s_ℓ^P the spin parity of the light degrees of freedom in mesons. The description implies relations among the observables in various modes. We study the decay distributions in the dilepton invariant mass squared and the distributions in the angle between the charged lepton momentum and the momentum of the produced meson, which are expressed in terms of universal form factors and of effective strong couplings. Such measurements are feasible at present facilities.

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I. INTRODUCTION

The Dalitz decays of the positive-parity mesons with open charm and strangeness, $D_{sJ}^{(*)} \to D_s^{(*)} \ell^+ \ell^-$ with $J = 0$, 1, 2 and $\ell = e$, μ, are of interest, since they can shed new light on the nature of the $D_{sJ}^{(*)}$ states. $D_{s0}^*(2317)$ and $D_{s1}'(2460)$ show puzzling features, already emerged at their first observation by the *BABAR* and CLEO Collaborations [1,2]. In particular, their mass is below the DK and D^*K thresholds [3]. Together with $D_{s1}(2536)$ and $D_{s2}^*(2573)$, they are the lightest positive-parity mesons with charm and strangeness [4]. The classification of the four states in two heavy quark spin doublets with $s_\ell^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$, with s_ℓ^P the spin parity of the light degrees of freedom in the mesons, implies relations among various observables, namely, among mass parameters and widths [5,6]. Relations can also be established among different Dalitz modes, thanks to the hadronic parameters common to the various decay amplitudes. Such relations can be experimentally verified, and this provides us with a new way to probe the nature of the states, discriminating ordinary quark-antiquark mesons from hadrons with large molecular or multiquark components.[1] In that respect, the $D_{sJ}^{(*)}$ Dalitz modes complement the information from the electric dipole radiative decays [7]. We have to say that, at present, the description of the full spectrum of mesons with a single charm or beauty quark in terms of heavy quark spin doublets naturally emerges from data [8].

The Dalitz amplitudes involve terms with the virtual photon coupled to the charm quark and to the light $\bar{s}$ quark. For such terms, we use the effective QCD theory based on the heavy quark expansion and on the hidden gauge symmetry, together with the vector meson dominance (VMD) [9] for the contribution related to the light (anti)quark. The features of the effective theory are described in Sec. II. In Sec. III, we collect the expressions of the decay amplitudes, which involve two form factors parametrizing the matrix elements of the charm vector current with positive- and negative-parity mesons organized in spin doublets. The effective strong couplings of the charmed mesons with $\phi(1020)$ mesons also appear in the amplitudes. The expressions of double- and single-decay distributions are given in Sec. IV, with some functions collected in the Appendix. In Sec. V, we discuss examples of decay distributions useful for a comparison with measurement. Then we conclude.

II. DYNAMICS OF MESONS WITH A SINGLE HEAVY QUARK

In the heavy quark limit $m_Q \to \infty$, mesons comprising a single heavy quark can be classified in doublets of s_ℓ^P, the spin parity of the light degrees of freedom (light quark and gluons). Indeed, in that limit, the spin of the heavy quark decouples from the strong dynamics, and the QCD Lagrangian displays heavy quark symmetry, invariance

[1]A molecular structure of $D_{s0}^*(2317)$ is advocated in Prelovsek's talk given at Hadron 2023, Genoa.

under heavy quark spin transformations, and heavy quark flavor transformations.[2]

The four positive-parity mesons corresponding to the P-wave states in the constituent quark model can be collected in two doublets with $s_\ell^P = \frac{1}{2}^+$ ($J^P = 0^+, 1^+$) and $s_\ell^P = \frac{3}{2}^+$ ($J^P = 1^+, 2^+$). The pseudoscalar and vector mesons belong to the $s_\ell^P = \frac{1}{2}^-$ doublet. In the case of charm, the mesons in the $s_\ell^P = \frac{1}{2}^-$ doublet are (D_a, D_a^*), $a = u, d, s$ being a light flavor index, and the two positive-parity doublets comprise (D_{a0}^*, D'_{a1}) and (D_{a1}, D_{a2}^*). Each doublet is described by a 4×4 matrix:

$$H_a = \frac{1+\slashed{v}}{2}\left[P_{a\mu}^*\gamma^\mu - P_a\gamma_5\right] \quad \left(s_\ell^P = \frac{1^-}{2}\right), \quad (2.1)$$

$$S_a = \frac{1+\slashed{v}}{2}\left[P_{1a}^{\prime\mu}\gamma_\mu\gamma_5 - P_{0a}^*\right] \quad \left(s_\ell^P = \frac{1^+}{2}\right), \quad (2.2)$$

$$T_a^\mu = \frac{1+\slashed{v}}{2}\left\{P_{2a}^{\mu\nu}\gamma_\nu - P_{1a\nu}\sqrt{\frac{3}{2}}\gamma_5\left[g^{\mu\nu} - \frac{1}{3}\gamma^\nu(\gamma^\mu - v^\mu)\right]\right\} \quad \left(s_\ell^P = \frac{3^+}{2}\right). \quad (2.3)$$

In (2.1)–(2.3), the fields $P_{Ja}^{(*)}$ are normalized including a factor $\sqrt{m_{D_a^{(*)}}}$ and have dimension 3/2. v is the heavy meson four-velocity, which is conserved in strong interaction processes.

The low-energy Lagrangian describing the strong interactions of the heavy mesons and the light pseudoscalar and vector mesons can be constructed on the basis of the heavy quark symmetry, the chiral symmetry, and the principle of hidden gauge invariance [12–17]. The octet of light pseudoscalar mesons is introduced defining $\xi = e^{\frac{i\mathcal{M}}{f_\pi}}$ and $\Sigma = \xi^2$. The matrix $\mathcal{M}$ comprises the π, K, and $\eta^{(8)}$ fields (the normalization corresponds to $f_\pi = 132$ MeV):

$$\mathcal{M} = \begin{pmatrix} \sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta^{(8)} & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta^{(8)} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta^{(8)} \end{pmatrix}. \quad (2.4)$$

The fields Σ and ξ transform under the chiral group $SU(3)_L \times SU(3)_R$ as

$$\Sigma \to L\Sigma R^\dagger, \qquad \xi(x) \to L\xi U^\dagger(x) = U(x)\xi R^\dagger, \quad (2.5)$$

with $L(R)$ belonging to $SU(3)_{L(R)}$ and $U(x)$ to the unbroken subgroup $SU(3)_V$. Vector and axial-vector currents can be constructed in terms of ξ:

$$\mathcal{V}_\mu = \frac{1}{2}\left(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\right), \quad (2.6)$$

$$\mathcal{A}_\mu = \frac{i}{2}\left(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger\right) \quad (2.7)$$

with transformation properties

$$\mathcal{A}_\mu \to U\mathcal{A}_\mu U^\dagger, \quad (2.8)$$

$$\mathcal{V}_\mu \to U\mathcal{V}_\mu U^\dagger + U\partial_\mu U^\dagger. \quad (2.9)$$

The interactions of the heavy mesons with the light vector mesons can be constructed using the principle of hidden gauge symmetry [16,17]. The fields ρ_μ are introduced:

$$\rho_\mu = i\frac{g_V}{\sqrt{2}}\hat{\rho}_\mu \quad (2.10)$$

in terms of the Hermitian fields of vector mesons

$$\hat{\rho}_\mu = \begin{pmatrix} \sqrt{\frac{1}{2}}\rho^0 + \sqrt{\frac{1}{6}}\phi^{(8)} & \rho^+ & K^{*+} \\ \rho^- & -\sqrt{\frac{1}{2}}\rho^0 + \sqrt{\frac{1}{6}}\phi^{(8)} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\sqrt{\frac{2}{3}}\phi^{(8)} \end{pmatrix}_\mu. \quad (2.11)$$

The value $g_V = 5.8$ is chosen to satisfy the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin relations [18,19]. The mesons ω and ϕ correspond to the mixing between the flavor octet $\phi^{(8)}$ in (2.11) and the flavor singlet component $\phi^{(0)}$:

$$\phi = \sin\theta_V\phi^{(0)} - \cos\theta_V\phi^{(8)}, \qquad \omega = \cos\theta_V\phi^{(0)} + \sin\theta_V\phi^{(8)}. \quad (2.12)$$

Flavor eigenstates $\phi_q = \frac{\bar{u}u+\bar{d}d}{\sqrt{2}}$ and $\phi_s = \bar{s}s$ are obtained for $\theta_V = \arctan\frac{1}{\sqrt{2}}$, and the observed ω and ϕ are identified with $\omega = \phi_q$ and $\phi = \phi_s$. Replacing $\frac{1}{\sqrt{3}}\phi^{(8)} = \sin\theta_V\phi^{(8)} \to \phi_q$ and $-\frac{2}{\sqrt{3}}\phi^{(8)} = -\cos\theta_V\phi^{(8)} \to \phi_s$, the ideal $\omega-\phi$ mixing (which is exact in the large N_c limit [16]), we have

$$\hat{\rho}_\mu = \begin{pmatrix} \sqrt{\frac{1}{2}}\rho^0 + \sqrt{\frac{1}{2}}\omega & \rho^+ & K^{*+} \\ \rho^- & -\sqrt{\frac{1}{2}}\rho^0 + \sqrt{\frac{1}{2}}\omega & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}_\mu. \quad (2.13)$$

[2]For reviews, see [10,11].

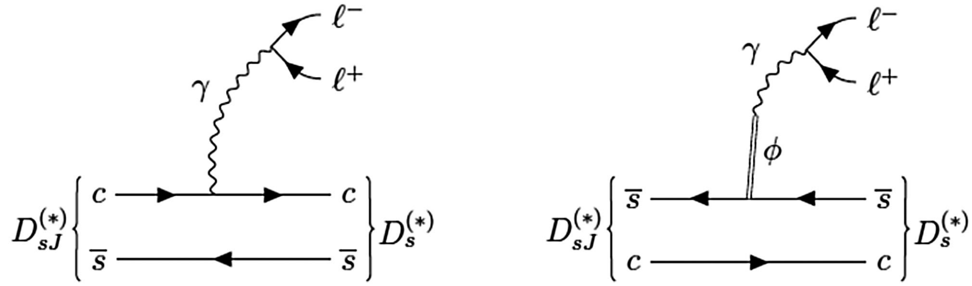

FIG. 1. $D_{sJ}^{(*)} \to D_s^{(*)}\ell^+\ell^-$ amplitude: photon coupled to the charm quark (left) and vector meson dominance contribution for the photon coupled to the light $\bar{s}$ (right).

The interactions of the positive- and negative-parity heavy quark doublets with the light vector mesons, of interest for our analysis, are described by the effective Lagrangian terms [17,20]

$$\mathcal{L}_1^S = -g_1^S \mathrm{Tr}[\bar{H} S \gamma^\alpha (\mathcal{V}_\alpha - \rho_\alpha)] + \mathrm{H.c.}, \tag{2.14}$$

$$\mathcal{L}_2^S = g_2^S \frac{1}{\Lambda} Tr[\bar{H} S \sigma^{\alpha\beta} \mathcal{F}_{\alpha\beta}] + \mathrm{H.c.} \tag{2.15}$$

(with $\bar{H} = \gamma^0 H^\dagger \gamma^0$), and

$$\mathcal{L}_2^T = i h^T \frac{1}{\Lambda^2} Tr[\bar{H} T_\mu \sigma^{\alpha\beta} \mathcal{D}^\mu \mathcal{F}_{\alpha\beta}] + \mathrm{H.c.} \tag{2.16}$$

The field strength $\mathcal{F}_{\alpha\beta}$ and the covariant derivative $\mathcal{D}_\mu$ are defined, respectively, as

$$\begin{aligned} \mathcal{F}_{\mu\nu} &= \partial_\mu \rho_\nu - \partial_\nu \rho_\mu + [\rho_\mu, \rho_\nu], \\ \mathcal{D}_\mu &= \partial_\mu + \mathcal{V}_\mu. \end{aligned} \tag{2.17}$$

$g_{1,2}^S$ and h^T are dimensionless low-energy constants, and Λ is a low-energy scale set to $\Lambda = 1$ GeV.

We compute the $D_{sJ}^{(*)}$ Dalitz amplitudes using this formalism.

III. $D_{sJ}^{(*)}$ DALITZ AMPLITUDES

The low-energy theory has been applied to the heavy vector meson magnetic dipole transitions $D_a^* \to D_a \gamma$ [21,22]. Here, we focus on the electric dipole transitions of positive-parity heavy mesons, which contribute to the Dalitz amplitudes. The $D_{sJ}^{(*)} \to D_s^{(*)}\ell^+\ell^-$ amplitude reads

$$\begin{aligned} &\mathcal{A}(D_{sJ}^{(*)}(p') \to D_s^{(*)}(p)\ell^-(p_1)\ell^+(p_2)) \\ &= \langle D_s^{(*)}(p,\epsilon)| i J_\mu^{\mathrm{em}} | D_{sJ}^{(*)}(p',\epsilon')\rangle \\ &\quad \times \frac{-i g^{\mu\nu}}{q^2} (-ie) \bar{u}(p_1)\gamma_\nu v(p_2), \end{aligned} \tag{3.1}$$

where $q = p_1 + p_2 = p' - p$ is the dilepton momentum. ϵ' is the polarization vector of $D_{s1}^{(\prime)}$ or the polarization tensor of D_{s2}^*, and ϵ is the polarization vector of D_s^*. The relevant term of the electromagnetic current is

$$J_\mu^{\mathrm{em}} = e(e_c \bar{c}\gamma_\mu c + e_s \bar{s}\gamma_\mu s), \tag{3.2}$$

with e_c and e_s the charm and strange quark electric charge, respectively (in units of e). The amplitude comprises two contributions depicted in Fig. 1, with the photon coupled to the charm quark and to the light antiquark $\bar{s}$.

For the photon coupled to the charm quark, the $\bar{c}\Gamma c$ current (with Γ a generic Dirac matrix) matrix elements of the S, H and T, H doublets can be computed in the effective theory using the trace formalism [23]. They involve the $\tau_{1/2}(w)$ (for $S \to H$) and $\tau_{3/2}(w)$ (for $T \to H$) universal functions [24]:

$$\begin{aligned} \langle H(v)|\bar{c}\Gamma c|S(v')\rangle &= -\tau_{1/2}(w)\mathrm{Tr}[\bar{H}(v)\Gamma S(v')], \\ \langle H(v)|\bar{c}\Gamma c|T(v')\rangle &= -\tau_{3/2}(w)\mathrm{Tr}[\bar{H}(v)\Gamma v_\mu T^\mu(v')], \end{aligned} \tag{3.3}$$

with w the product of meson four-velocities $w = v' \cdot v = \frac{m^2_{D_{sJ}^{(*)}} + m^2_{D_s^{(*)}} - q^2}{2 m_{D_{sJ}^{(*)}} m_{D_s^{(*)}}}$. The $D_{sJ}^{(*)} \to D_s^{(*)}$ matrix elements of the charm vector current read

$$\begin{aligned} &\langle D_s^*(m_{D_s^*} v, \epsilon)|\bar{c}\gamma_\mu c|D_{s0}^*(m_{D_{s0}^*} v')\rangle \\ &\quad = \tau_{1/2}(w)\sqrt{m_{D_{s0}^*} m_{D_s^*}}[-(w-1)\epsilon_\mu^* + (\epsilon^* \cdot v')v_\mu], \end{aligned} \tag{3.4}$$

$$\begin{aligned} &\langle D_s(m_{D_s} v)|\bar{c}\gamma_\mu c|D_{s1}'(m_{D_{s1}'} v', \epsilon')\rangle \\ &\quad = \tau_{1/2}(w)\sqrt{m_{D_{s1}'} m_{D_s}}[(w-1)\epsilon_\mu' - (\epsilon' \cdot v)v_\mu'], \end{aligned} \tag{3.5}$$

$$\begin{aligned} &\langle D_s^*(m_{D_s^*} v, \epsilon)|\bar{c}\gamma_\mu c|D_{s1}'(m_{D_{s1}'} v', \epsilon')\rangle \\ &\quad = \tau_{1/2}(w)\sqrt{m_{D_{s1}'} m_{D_s^*}}\, i\left(\epsilon_{\alpha\beta\tau\mu}\epsilon'^\alpha \epsilon^{*\beta} v^\tau - \epsilon_{\alpha\beta\tau\mu}\epsilon'^\alpha \epsilon^{*\beta} v'^\tau\right), \end{aligned} \tag{3.6}$$

$$\begin{aligned} &\langle D_s(m_{D_s} v)|\bar{c}\gamma_\mu c|D_{s1}(m_{D_{s1}} v', \epsilon')\rangle \\ &\quad = \tau_{3/2}(w)\sqrt{m_{D_{s1}} m_{D_s}} \frac{1}{\sqrt{6}}\left\{(w^2-1)\epsilon_\mu' + (\epsilon' \cdot v) \right. \\ &\qquad \left. \times (3v_\mu - (w-2)v_\mu')\right\}, \end{aligned} \tag{3.7}$$

TABLE I. Input parameters in the numerical analysis.

$\alpha(M_Z) = 1/127.9$	[4]	$m_{D_s} = 1968.35 \pm 0.07$ MeV	[4]
$m_\mu = 105.66$ MeV	[4]	$m_{D_s^*} = 2112.2 \pm 0.4$ MeV	[4]
		$m_{D_{s0}^*} = 2317.8 \pm 0.5$ MeV	[4]
$g_V = 5.8$	[17]	$m_{D'_{s1}} = 2459.5 \pm 0.6$ MeV	[4]
$\Lambda = 1$ GeV	[17]	$m_{D_{s1}} = 2535.11 \pm 0.06$ MeV	[4]
$m_\phi = 1019.461 \pm 0.016$ MeV	[4]	$\Gamma(D_{s1}) = 0.92 \pm 0.05$ MeV	[4]
$\Gamma_\phi = 4.249 \pm 0.013$ MeV	[4]	$m_{D_{s2}^*} = 2569.1 \pm 0.8$ MeV	[4]
$\mathcal{B}(\phi \to \mu^+\mu^-) = (2.85 \pm 0.19) \times 10^{-4}$	[4]	$\Gamma(D_{s2}^*) = 16.9 \pm 0.7$ MeV	[4]
$f_\phi = 208 \pm 7$ MeV	[4]		
		$\tau_{1/2}(1) = 0.70 \pm 0.21$	[27]
$\lvert g_1^S\rvert = 0.10$	[17]	$\hat{\rho}_{1/2}^2 = -0.2 \pm 1.4$	[27]
$\lvert g_2^S\rvert = 0.29 \pm 0.03$	[7]	$\tau_{3/2}(1) = 0.70 \pm 0.07$	[27]
$\lvert h_T\rvert = 0.23 \pm 0.09$	[20]	$\hat{\rho}_{3/2}^2 = 1.6 \pm 0.2$	[27]

$$\begin{aligned}
&\langle D_s^*(m_{D_s^*}v,\epsilon)|\bar{c}\gamma_\mu c|D_{s1}(m_{D_{s1}}v',\epsilon')\rangle \\
&\quad = \tau_{3/2}(w)\left(-\frac{i}{\sqrt{6}}\right)\sqrt{m_{D_{s1}}m_{D_s^*}}\{(w-1)(\epsilon_{\alpha\beta\tau\mu}\epsilon'^\alpha\epsilon^{*\beta}v^\tau \\
&\qquad + \epsilon_{\alpha\beta\tau\mu}\epsilon'^\alpha\epsilon^{*\beta}v'^\tau) + (\epsilon'\cdot v)\epsilon_{\alpha\beta\tau\mu}\epsilon^{*\alpha}v^\beta v'^\tau \\
&\qquad - 2\epsilon_{\alpha\beta\tau\sigma}\epsilon'^\alpha\epsilon^{*\beta}v^\tau v'^\sigma v_\mu\},
\end{aligned} \tag{3.8}$$

$$\begin{aligned}
&\langle D_s(m_{D_s}v)|\bar{c}\gamma_\mu c|D_{s2}^*(m_{D_{s2}^*}v',\epsilon')\rangle \\
&\quad = \tau_{3/2}(w)i\sqrt{m_{D_{s2}^*}m_{D_s}}\epsilon_{\mu\nu\alpha\beta}\epsilon'^{\tau\nu}v_\tau v^\alpha v'^\beta,
\end{aligned} \tag{3.9}$$

$$\begin{aligned}
&\langle D_s^*(m_{D_s^*}v,\epsilon)|\bar{c}\gamma_\mu c|D_{s2}^*(m_{D_{s2}^*}v',\epsilon')\rangle \\
&\quad = \tau_{3/2}(w)\sqrt{m_{D_{s2}^*}m_{D_s^*}}\{(\epsilon^*\cdot v')\epsilon'_{\tau\mu}v^\tau + \epsilon'_{\tau\sigma}v^\tau v^\sigma\epsilon^*_\mu \\
&\qquad - \epsilon'_{\tau\sigma}v^\tau\epsilon^{*\sigma}(v_\mu + v'_\mu)\}.
\end{aligned} \tag{3.10}$$

$\langle D_s|\bar{c}\gamma_\mu c|D_{s0}^*\rangle$ vanishes.

The term with the photon coupled to $\bar{s}$ involves a long-distance contribution that can be computed using the vector meson dominance [25,26]. The matrix elements of the strange quark vector current are written, neglecting the ϕ width, as

$$\begin{aligned}
&\langle D_s^{(*)}(p)|\bar{s}\gamma_\mu s|D_{sJ}^{(*)}(p')\rangle \\
&\quad = \langle D_s^{(*)}(p)\phi(q,\eta)|D_{sJ}^{(*)}(p')\rangle\frac{i}{q^2-m_\phi^2}\langle 0|\bar{s}\gamma_\mu s|\phi(q,\eta)\rangle
\end{aligned} \tag{3.11}$$

with

$$\langle 0|\bar{s}\gamma_\mu s|\phi(q,\eta)\rangle = m_\phi f_\phi \eta_\mu. \tag{3.12}$$

The decay constant f_ϕ, set by the measurement of $\Gamma(\phi\to\mu^+\mu^-)$, is quoted in Table I. The strong matrix elements of positive- and negative-parity charmed states with $\phi(1020)$,

$$\langle D_s^{(*)}(p)\phi(q,\eta)|D_{sJ}^{(*)}(p')\rangle = \mathcal{A}_\alpha^{D_{sJ}^{(*)}D_s^{(*)}\phi}\eta^{*\alpha}, \tag{3.13}$$

can be obtained from the low-energy Lagrangians (2.14)–(2.16) [17,20]. Hence, the elements in (3.11) are written as

$$\begin{aligned}
\langle D_s^{(*)}(p)|\bar{s}\gamma_\mu s|D_{sJ}^{(*)}(p')\rangle &= \mathcal{A}_\alpha^{D_{sJ}^{(*)}D_s^{(*)}\phi}\frac{i}{q^2-m_\phi^2}m_\phi f_\phi \\
&\quad \times\left(-g_\mu^\alpha + \frac{q^\alpha q_\mu}{q^2}\right),
\end{aligned} \tag{3.14}$$

with the following amplitudes $\mathcal{A}_\alpha^{D_{sJ}^{(*)}D_s^{(*)}\phi}$:

$$\begin{aligned}
\mathcal{A}_\alpha^{D_{s0}^*D_s^*\phi} &= \frac{ig_V}{\sqrt{2}}\sqrt{\frac{m_{D_{s0}^*}}{m_{D_s^*}}}\Bigg\{m_{D_{s0}^*}\left(g_1^S + 2(m_{D_{s0}^*}+m_{D_s^*})\frac{g_2^S}{\Lambda}\right) \\
&\quad \times(\epsilon^*\cdot v')v'_\alpha - m_{D_s^*}(w+1) \\
&\quad \times\left(g_1^S + 2(m_{D_{s0}^*}-m_{D_s^*})\frac{g_2^S}{\Lambda}\right)\epsilon^*_\alpha\Bigg\},
\end{aligned} \tag{3.15}$$

$$\begin{aligned}
\mathcal{A}_\alpha^{D'_{s1}D_s\phi} &= \frac{ig_V}{\sqrt{2}}\sqrt{m_{D'_{s1}}m_{D_s}}\Bigg\{\left(g_1^S - 2(m_{D'_{s1}}+m_{D_s})\frac{g_2^S}{\Lambda}\right) \\
&\quad \times(\epsilon'\cdot v)v'_\alpha - (w+1) \\
&\quad \times\left(g_1^S + 2(m_{D'_{s1}}-m_{D_s})\frac{g_2^S}{\Lambda}\right)\epsilon'_\alpha\Bigg\},
\end{aligned} \tag{3.16}$$

$$\begin{aligned}
\mathcal{A}_\alpha^{D'_{s1}D_s^*\phi} &= \frac{g_V}{\sqrt{2}}\sqrt{m_{D'_{s1}}m_{D_s^*}}\Bigg\{\Bigg[g_1^S + 2\frac{g_2^S}{\Lambda} \\
&\quad \times\left(m_{D'_{s1}} - m_{D_s^*}(1+2w)\right)\Bigg]\epsilon_{\alpha\beta\sigma\tau}\epsilon^{*\beta}\epsilon'^\sigma v^\tau \\
&\quad + \left[g_1^S + 2\frac{g_2^S}{\Lambda}(m_{D'_{s1}}+m_{D_s^*})\right]\epsilon_{\alpha\beta\sigma\tau}\epsilon^{*\beta}\epsilon'^\sigma v'^\tau \\
&\quad - 4\frac{g_2^S}{\Lambda}m_{D_s^*}(\epsilon'\cdot v)\epsilon_{\alpha\beta\sigma\tau}\epsilon^{*\beta}v^\sigma v'^\tau\Bigg\},
\end{aligned} \tag{3.17}$$

$$\mathcal{A}_\alpha^{D_{s1}D_s\phi} = \frac{i g_V h_T}{\sqrt{3}\Lambda^2}\sqrt{m_{D_{s1}} m_{D_s}}\{-m_{D_s}(m_{D_{s1}} + m_{D_s}) \times (w^2-1)\epsilon'_\alpha + [m_{D_{s1}}^2 + m_{D_s}^2(w+2) - m_{D_s} m_{D_{s1}}(1+3w)](\epsilon'\cdot v)v'_\alpha\}, \quad (3.18)$$

$$\mathcal{A}_\alpha^{D_{s1}D_s^*\phi} = \frac{g_V h_T}{\sqrt{3}\Lambda^2}\sqrt{m_{D_{s1}} m_{D_s^*}} m_{D_s^*}\{(m_{D_{s1}} - m_{D_s^*}) \times (w+1)\epsilon_{\alpha\beta\sigma\tau}\epsilon^{*\beta}\epsilon'^{\sigma}v'^{\tau} + (m_{D_{s1}} + (2w+3)m_{D_s^*}) \times (\epsilon'\cdot v)\epsilon_{\alpha\beta\sigma\tau}\epsilon^{*\beta}v^{\sigma}v'^{\tau} - (m_{D_{s1}} - (2w-1)m_{D_s^*}) \times (w+1)\epsilon_{\alpha\beta\sigma\tau}\epsilon^{*\beta}\epsilon'^{\sigma}v^{\tau}\}, \quad (3.19)$$

$$\mathcal{A}_\alpha^{D_{s2}^*D_s\phi} = \frac{\sqrt{2} g_V h_T}{\Lambda^2}\sqrt{m_{D_{s2}^*} m_{D_s}} m_{D_s} \times \left(m_{D_{s2}^*} + m_{D_s}\right)\epsilon_{\alpha\beta\sigma\tau}v^{\beta}v'^{\sigma}\epsilon'^{\nu\tau}v_{\nu}, \quad (3.20)$$

$$\mathcal{A}_\alpha^{D_{s2}^*D_s^*\phi} = \frac{i\sqrt{2} g_V h_T}{\Lambda^2}\sqrt{m_{D_{s2}^*} m_{D_s^*}}\{(m_{D_{s2}^*} + m_{D_s^*})m_{D_s^*} \times (\epsilon^*\cdot v')\epsilon'^{\tau\alpha}v_\tau - \left(m_{D_{s2}^*}^2 + m_{D_s^*}^2 - 2w m_{D_{s2}^*} m_{D_s^*}\right)\epsilon'^{\tau\sigma}v_\tau\epsilon^{*\sigma}v'_\alpha + m_{D_s^*}\epsilon'^{\tau\sigma}v_\tau v_\sigma \times \left[(m_{D_{s2}^*} + m_{D_s^*})\epsilon^*_\alpha - 2m_{D_{s2}^*}(\epsilon^*\cdot v')v'_\alpha\right]\}. \quad (3.21)$$

IV. DECAY DISTRIBUTIONS

The previous amplitudes allow us to compute the various decay distributions. We define the functions appearing in the distributions:

$$g(q^2) = \frac{\alpha^2}{12\pi q^6}|\vec{p}|\sqrt{1 - \frac{4m_\ell^2}{q^2}}, \qquad f(q^2) = \frac{f_\phi g_V m_\phi}{q^2 - m_\phi^2}, \quad (4.1)$$

with $|\vec{p}| = \frac{\lambda^{1/2}(m^2_{D_{sJ}^{(*)}}, m^2_{D_s^{(*)}}, q^2)}{2m_{D_{sJ}^{(*)}}}$ the modulus of the $D_s^{(*)}$ three-momentum in the decaying particle rest frame.

A. $D_{sJ}^{(*)} \to D_s\ell^+\ell^-$ distributions

For $D_{sJ}^{(*)} \to D_s\ell^+\ell^-$, the double differential distribution in q^2 and $\cos\theta$, θ being the angle between the charged lepton momentum and the D_s momentum, is written as

$$\frac{d^2\Gamma}{dq^2 d\cos\theta} = \left(\frac{d^2\Gamma}{dq^2 d\cos\theta}\right)_c + \left(\frac{d^2\Gamma}{dq^2 d\cos\theta}\right)_s + \left(\frac{d^2\Gamma}{dq^2 d\cos\theta}\right)_{\text{int}}. \quad (4.2)$$

The subscripts c and s refer to the photon coupled to the charm and strange quark, respectively, and the last term is the interference. Each term in (4.2) has the form

$$\left(\frac{d^2\Gamma}{dq^2 d\cos\theta}\right)_X = A_X(q^2) + B_X(q^2)\cos^2\theta \quad (4.3)$$

with $X = \{c, s, \text{int}\}$. The q^2 distribution is obtained integrating over $\cos\theta$:

$$\frac{d\Gamma}{dq^2} = \left(\frac{d\Gamma}{dq^2}\right)_c + \left(\frac{d\Gamma}{dq^2}\right)_s + \left(\frac{d\Gamma}{dq^2}\right)_{\text{int}}. \quad (4.4)$$

For the various processes, the expressions of the functions $A_X(q^2)$ and $B_X(q^2)$ in (4.3) are in the Appendix. The distributions in the lepton pair invariant mass squared are listed below.

1. $D'_{s1} \to D_s\ell^+\ell^-$

$$\left(\frac{d\Gamma}{dq^2}\right)_c = g(q^2)[\tau_{1/2}(q^2)]^2\frac{2[(m_{D'_{s1}} - m_{D_s})^2 - q^2]^2}{27m_{D'_{s1}}^3 m_{D_s}} \times [(m_{D'_{s1}} + m_{D_s})^2 + 2q^2](2m_\ell^2 + q^2), \quad (4.5)$$

$$\left(\frac{d\Gamma}{dq^2}\right)_s = \frac{g(q^2)f^2(q^2)}{108m_{D'_{s1}}^3 m_{D_s}}[(m_{D'_{s1}} + m_{D_s})^2 - q^2]^2(2m_\ell^2 + q^2) \times \left\{(g_1^S)^2[(m_{D'_{s1}} - m_{D_s})^2 + 2q^2] + \frac{12g_1^S g_2^S q^2}{\Lambda} \times (m_{D'_{s1}} - m_{D_s}) + \frac{4(g_2^S)^2 q^2}{\Lambda^2} \times [2(m_{D'_{s1}} - m_{D_s})^2 + q^2]\right\}, \quad (4.6)$$

$$\left(\frac{d\Gamma}{dq^2}\right)_{\text{int}} = \frac{g(q^2)f(q^2)[\tau_{1/2}(q^2)]}{27m_{D'_{s1}}^3 m_{D_s}}\sqrt{2}\lambda\left(m_{D'_{s1}}^2, m_{D_s}^2, q^2\right) \times (2m_\ell^2 + q^2)\left\{g_1^S\left[m_{D'_{s1}}^2 - m_{D_s}^2 + 2q^2\right] + 2\frac{g_2^S q^2}{\Lambda}\left[3m_{D'_{s1}} - m_{D_s}\right]\right\}. \quad (4.7)$$

2. $D_{s1} \to D_s\ell^+\ell^-$

$$\left(\frac{d\Gamma}{dq^2}\right)_c = g(q^2)[\tau_{3/2}(q^2)]^2\frac{\lambda^2(m_{D_{s1}}^2, m_{D_s}^2, q^2)}{162m_{D_{s1}}^5 m_{D_s}^3} \times \left[2(m_{D_{s1}} + m_{D_s})^2 + q^2\right](2m_\ell^2 + q^2), \quad (4.8)$$

$$\left(\frac{d\Gamma}{dq^2}\right)_s = g(q^2)f^2(q^2)\frac{h_T^2 q^2}{\Lambda^4 324 m_{D_{s1}}^5 m_{D_s}}\lambda^2(m_{D_{s1}}^2, m_{D_s}^2, q^2) \times \left[(m_{D_{s1}} + m_{D_s})^2 + 2q^2\right](2m_\ell^2 + q^2), \quad (4.9)$$

$$\left(\frac{d\Gamma}{dq^2}\right)_{\rm int} = -g(q^2)f(q^2)\tau_{3/2}(q^2)\frac{h_T q^2}{\Lambda^2 81\sqrt{2}m_{D_{s1}}^5 m_{D_s}^2} \times \lambda^2(m_{D_{s1}}^2, m_{D_s}^2, q^2)(2m_\ell^2+q^2)(m_{D_{s1}}+m_{D_s}). \tag{4.10}$$

3. $D_{s2}^* \to D_s\ell^+\ell^-$

$$\left(\frac{d\Gamma}{dq^2}\right)_c = g(q^2)[\tau_{3/2}(q^2)]^2\frac{\lambda^2(m_{D_{s2}^*}^2, m_{D_s}^2, q^2)}{90m_{D_{s2}^*}^5 m_{D_s}^3} \times q^2(2m_\ell^2+q^2), \tag{4.11}$$

$$\left(\frac{d\Gamma}{dq^2}\right)_s = g(q^2)f^2(q^2)\frac{h_T^2 q^2}{\Lambda^4 180 m_{D_{s2}^*}^5 m_{D_s}}\lambda^2(m_{D_{s2}^*}^2, m_{D_s}^2, q^2) \times (m_{D_{s2}^*}+m_{D_s})^2(2m_\ell^2+q^2), \tag{4.12}$$

$$\left(\frac{d\Gamma}{dq^2}\right)_{\rm int} = g(q^2)f(q^2)\tau_{3/2}(q^2)\frac{h_T q^2}{\Lambda^2 45\sqrt{2}m_{D_{s2}^*}^5 m_{D_s}^2} \times \lambda^2(m_{D_{s2}^*}^2, m_{D_s}^2, q^2)(2m_\ell^2+q^2)(m_{D_{s2}^*}+m_{D_s}). \tag{4.13}$$

B. $D_{sJ}^{(*)} \to D_s^*\ell^+\ell^-$ distributions

For $D_{sJ}^{(*)} \to D_s^*\ell^+\ell^-$, we consider the double distributions in q^2 and in the angle θ between the momentum of the charged lepton and the momentum of D_s^*. The functions $A_X(q^2)$ and $B_X(q^2)$ are in the Appendix, and the distributions in the invariant mass squared of the lepton pair are given below.

1. $D_{s0}^* \to D_s^*\ell^+\ell^-$

This process is kinematically allowed only for $\ell = e$. The terms in the q^2 distribution (4.4) read

$$\left(\frac{d\Gamma}{dq^2}\right)_c = g(q^2)\tau_{1/2}(q^2)\frac{2[(m_{D_{s0}^*}-m_{D_s^*})^2-q^2]^2}{9m_{D_{s0}^*}^3 m_{D_s^*}} \times [(m_{D_{s0}^*}+m_{D_s^*})^2+2q^2](2m_\ell^2+q^2), \tag{4.14}$$

$$\left(\frac{d\Gamma}{dq^2}\right)_s = \frac{g(q^2)f^2(q^2)}{36m_{D_{s0}^*}^3 m_{D_s^*}}[(m_{D_{s0}^*}+m_{D_s^*})^2-q^2]^2(2m_\ell^2+q^2) \times \left\{(g_1^S)^2[(m_{D_{s0}^*}-m_{D_s^*})^2+2q^2]+\frac{12g_1^S g_2^S q^2}{\Lambda} \times (m_{D_{s0}^*}-m_{D_s^*})+\frac{4(g_2^S)^2q^2}{\Lambda^2} \times [2(m_{D_{s0}^*}-m_{D_s^*})^2+q^2]\right\}, \tag{4.15}$$

$$\left(\frac{d\Gamma}{dq^2}\right)_{\rm int} = \frac{g(q^2)f(q^2)[\tau_{1/2}(q^2)]}{9m_{D_{s0}^*}^3 m_{D_s^*}}\sqrt{2}\lambda(m_{D_{s0}^*}^2, m_{D_s^*}^2, q^2) \times (2m_\ell^2+q^2)\left\{g_1^S[m_{D_{s0}^*}^2-m_{D_s^*}^2-2q^2] - 2\frac{g_2^S q^2}{\Lambda}[m_{D_{s0}^*}-3m_{D_s^*}]\right\}. \tag{4.16}$$

2. $D_{s1}' \to D_s^*\ell^+\ell^-$

$$\left(\frac{d\Gamma}{dq^2}\right)_c = g(q^2)[\tau_{1/2}(q^2)]^2\frac{4[(m_{D_{s1}'}-m_{D_s^*})^2-q^2]^2}{27m_{D_{s1}'}^3 m_{D_s^*}}[(m_{D_{s1}'}+m_{D_s^*})^2+2q^2](2m_\ell^2+q^2),$$

$$\left(\frac{d\Gamma}{dq^2}\right)_s = \frac{g(q^2)f^2(q^2)}{54m_{D_{s1}'}^3 m_{D_s^*}}[(m_{D_{s1}'}+m_{D_s^*})^2-q^2]^2(2m_\ell^2+q^2)\left\{(g_1^S)^2[(m_{D_{s1}'}-m_{D_s^*})^2+2q^2] + \frac{12g_1^S g_2^S q^2}{\Lambda}(m_{D_{s1}'}-m_{D_s^*})+\frac{4(g_2^S)^2q^2}{\Lambda^2}[2(m_{D_{s1}'}-m_{D_s^*})^2+q^2]\right\}, \tag{4.17}$$

$$\left(\frac{d\Gamma}{dq^2}\right)_{\rm int} = \frac{g(q^2)f(q^2)\tau_{1/2}(q^2)}{27m_{D_{s1}'}^3 m_{D_s^*}}2\sqrt{2}\lambda(m_{D_{s1}'}^2, m_{D_s^*}^2, q^2) \times (2m_\ell^2+q^2)(m_{D_{s1}'}+m_{D_s^*}) \times \left\{g_1^S(m_{D_{s1}'}-m_{D_s^*})+2\frac{g_2^S q^2}{\Lambda}\right\}. \tag{4.18}$$

3. $D_{s1} \to D_s^*\ell^+\ell^-$

$$\left(\frac{d\Gamma}{dq^2}\right)_c = g(q^2)[\tau_{3/2}(q^2)]^2\frac{\lambda^2(m_{D_{s1}}^2, m_{D_s^*}^2, q^2)}{162m_{D_{s1}}^5 m_{D_s^*}^3} \times [(m_{D_{s1}}+m_{D_s}^*)^2+5q^2] \times (2m_\ell^2+q^2), \tag{4.19}$$

$$\left(\frac{d\Gamma}{dq^2}\right)_s = g(q^2) f^2(q^2) \frac{h_T^2 q^2}{\Lambda^4 324 m_{D_{s1}}^5 m_{D_s^*}} \times [5(m_{D_{s1}} + m_{D_s^*})^2 + q^2] \lambda^2(m_{D_{s1}}^2, m_{D_s^*}^2, q^2) \times (2m_\ell^2 + q^2), \tag{4.20}$$

$$\left(\frac{d\Gamma}{dq^2}\right)_{\rm int} = g(q^2) f(q^2) \tau_{3/2}(q^2) \frac{\sqrt{2} h_T q^2}{\Lambda^2 81 m_{D_{s1}}^5 m_{D_s^*}^2} \times (m_{D_{s1}} + m_{D_s^*}) \lambda^2(m_{D_{s1}}^2, m_{D_s^*}^2, q^2)(2m_\ell^2 + q^2). \tag{4.21}$$

4. $D_{s2}^* \to D_s^* \ell^+ \ell^-$

$$\left(\frac{d\Gamma}{dq^2}\right)_c = g(q^2) [\tau_{3/2}(q^2)]^2 \frac{\lambda^2(m_{D_{s2}^*}^2, m_{D_s^*}^2, q^2)}{270 m_{D_{s2}^*}^5 m_{D_s^*}^3} \times [5(m_{D_{s2}^*} + m_{D_s}^*)^2 + 7q^2](2m_\ell^2 + q^2), \tag{4.22}$$

$$\left(\frac{d\Gamma}{dq^2}\right)_s = g(q^2) f^2(q^2) \frac{h_T^2 q^2}{\Lambda^4 540 m_{D_{s2}^*}^5 m_{D_s^*}} \times [7(m_{D_{s2}^*} + m_{D_s^*})^2 + 5q^2] \lambda^2(m_{D_{s2}^*}^2, m_{D_s^*}^2, q^2) \times (2m_\ell^2 + q^2), \tag{4.23}$$

$$\left(\frac{d\Gamma}{dq^2}\right)_{\rm int} = -g(q^2) f(q^2) \tau_{3/2}(q^2) \frac{2\sqrt{2} h_T q^2}{\Lambda^2 45 m_{D_{s2}^*}^5 m_{D_s^*}^2} \times (m_{D_{s2}^*} + m_{D_s^*}) \lambda^2(m_{D_{s2}^*}^2, m_{D_s^*}^2, q^2)(2m_\ell^2 + q^2). \tag{4.24}$$

V. NUMERICS

The expressions of the decay distributions involve the universal form factors $\tau_{1/2}(w)$ and $\tau_{3/2}(w)$ and the low-energy couplings in (2.14)–(2.16). Numerical results can be given using determinations of such quantities available in the literature.

For the form factors $\tau_{1/2}$ and $\tau_{3/2}$, the parametrization

$$\tau_i(w) = \tau_i(1)[1 - (w - 1)\rho_i^2] \tag{5.1}$$

allows us to encode the uncertainties in the value at the zero-recoil point $w = 1$ and in the slope. We use the values in Table I, obtained from data on semileptonic B decays to positive-parity charmed mesons [27], which for $\tau_{1/2}$ agree with the computation in [28]. The value of g_1^S in Table I is an estimate obtained using $D \to K^*$ semileptonic form factors [17]. The value of g_2^S comes from a light-cone QCD sum rule computation of the decay amplitude of the positive-parity mesons to real photons [7], together with VMD. The value of h_T is obtained from the strong decay widths of the excited charmed mesons [20].

The relative phases of the two amplitudes describing the photon coupled to c and $\bar{s}$ depend on the relative phases of the functions τ_i and the strong couplings. Since such phases are not fixed, we consider the extreme cases: case A, where the product of the functions τ_i and the strong couplings is positive (a relative phase between g_1^S and g_2^S is ignored), and case B, where the product is negative. In the experimental analyses, the hadronic quantities are parameters to be determined from data.

Table II contains the decay widths corresponding to the maximal interference cases. We also display the branching fractions for D_{s1} and D_{s2}^*, for which the full widths are measured. The effect of the interference is large in $D'_{s1} \to D_s\mu^+\mu^-$ and $D_{s2}^* \to D_s\mu^+\mu^-$; for the other modes, the interference contribution is small. Such effects can be better observed in the decay distributions. The dilepton invariant mass distributions for the modes $D'_{s1} \to D_s\mu^+\mu^-$ and $D'_{s1} \to D_s^*\mu^+\mu^-$ is depicted in Fig. 2 for the maximal interferences. The interference term is sizable in the former mode. The amplitude corresponding to the photon coupled to the light quark gives the largest contribution. The distributions for $D_{s1} \to D_s^{(*)}\mu^+\mu^-$ and $D_{s2}^* \to D_s^{(*)}\mu^+\mu^-$ are shown in Figs. 3 and 4, with a sizable interference term visible in $D_{s2}^* \to D_s\mu^+\mu^-$.

The angular distributions are displayed in Fig. 5 in the cases of extremal interferences. From the plots, one finds

TABLE II. Decay widths of Dalitz modes for the two cases of maximal interference. Branching fractions are displayed only for D_{s1} and D_{s2}^* for which the full widths are measured.

	Width $\times 10^8$ (GeV) (case A)	Width $\times 10^8$ (GeV) (case B)	BR $\times 10^6$ (case A)	BR $\times 10^6$ (case B)
$D'_{s1} \to D_s\mu^+\mu^-$	4.7 ± 0.8	8.1 ± 1.3		
$D'_{s1} \to D_s^*\mu^+\mu^-$	2.1 ± 0.3	2.7 ± 0.4		
$D_{s1} \to D_s\mu^+\mu^-$	2.9 ± 0.6	2.8 ± 0.6	31.4 ± 6.4	30.7 ± 6.3
$D_{s1} \to D_s^*\mu^+\mu^-$	0.18 ± 0.04	0.19 ± 0.04	1.95 ± 0.4	2.1 ± 0.4
$D_{s2}^* \to D_s\mu^+\mu^-$	0.05 ± 0.06	0.22 ± 0.13	0.03 ± 0.04	0.13 ± 0.08
$D_{s2}^* \to D_s^*\mu^+\mu^-$	0.96 ± 0.19	0.87 ± 0.17	0.6 ± 0.1	0.5 ± 0.1

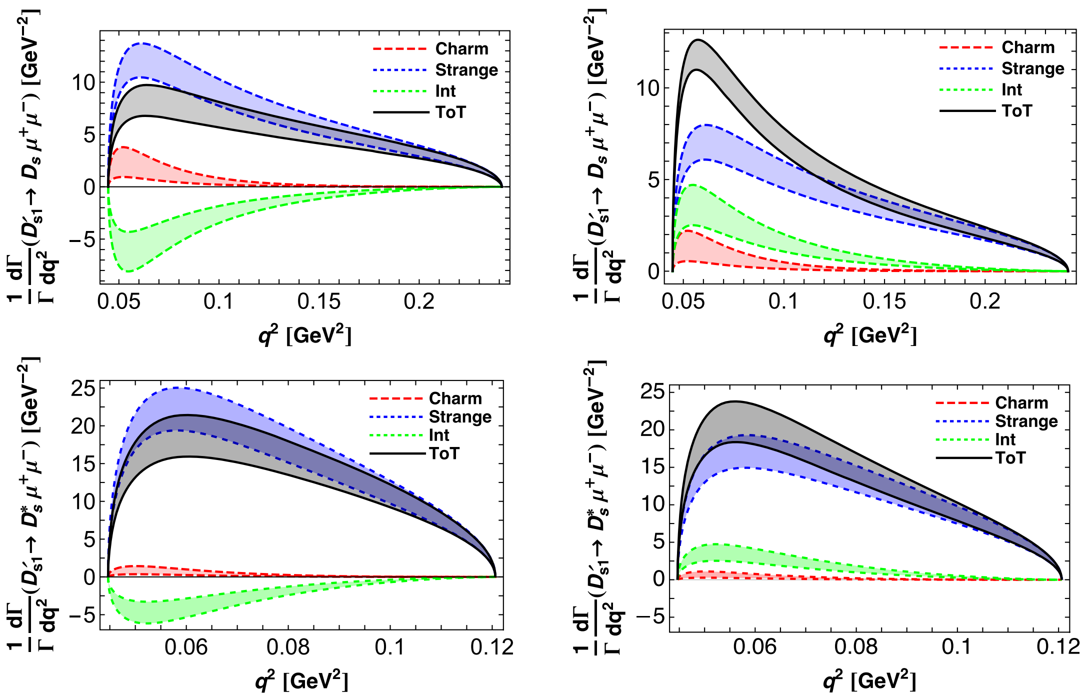

FIG. 2. Distribution $\frac{1}{\Gamma}\frac{d\Gamma}{dq^2}$ for the modes $D'_{s1} \to D_s\mu^+\mu^-$ (top) and $D'_{s1} \to D_s^*\mu^+\mu^-$ (bottom panels) for the two signs of the interference between the amplitudes with the photon coupled to the charm and $\bar{s}$ quark (case A, left panels; case B, right panels). The distributions for the photon coupled to quarks and the interference term are separately shown, together with their sum.

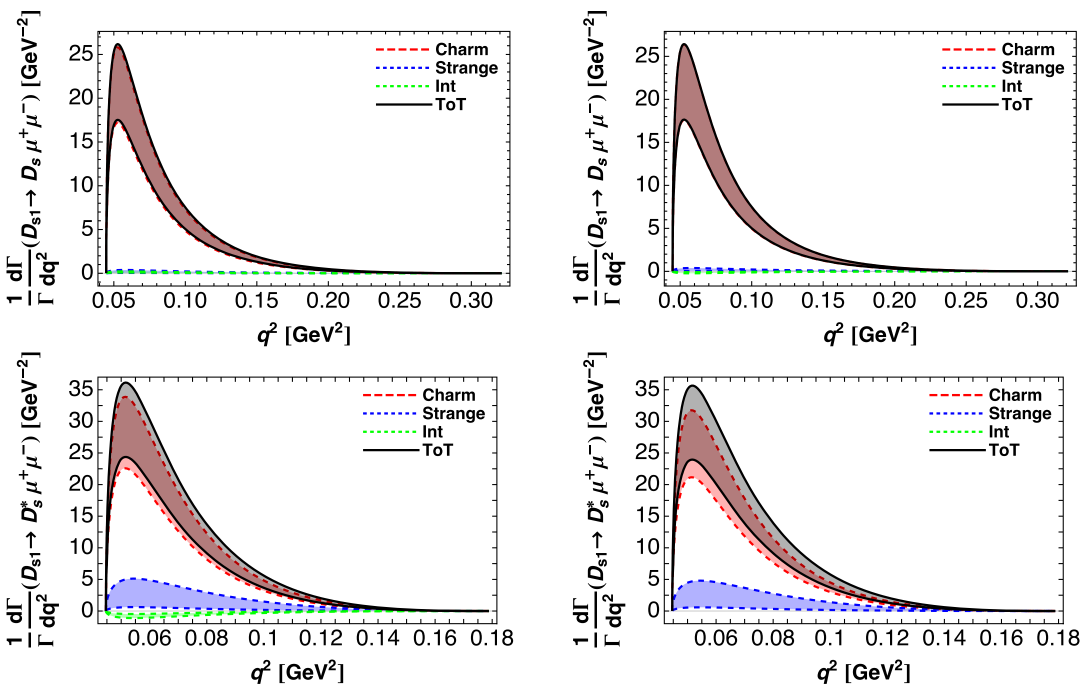

FIG. 3. Distribution $\frac{1}{\Gamma}\frac{d\Gamma}{dq^2}$ for $D_{s1} \to D_s\mu^+\mu^-$ (top) and $D_{s1} \to D_s^*\mu^+\mu^-$ (bottom panels). Notations as in Fig. 2.

that the $D_{s2}^* \to D_s\mu^+\mu^-$ mode is affected by the largest uncertainty.

Since the amplitudes of different modes involve the same hadronic parameters, the decay rates and other observables are correlated, as shown in Fig. 6 for the decay widths. Varying the parameters of the ranges in Table I, a positive correlation is found between $\Gamma(D'_{s1} \to D_s\mu^+\mu^-)$ and $\Gamma(D'_{s1} \to D_s^*\mu^+\mu^-)$ for both cases of interference, as well

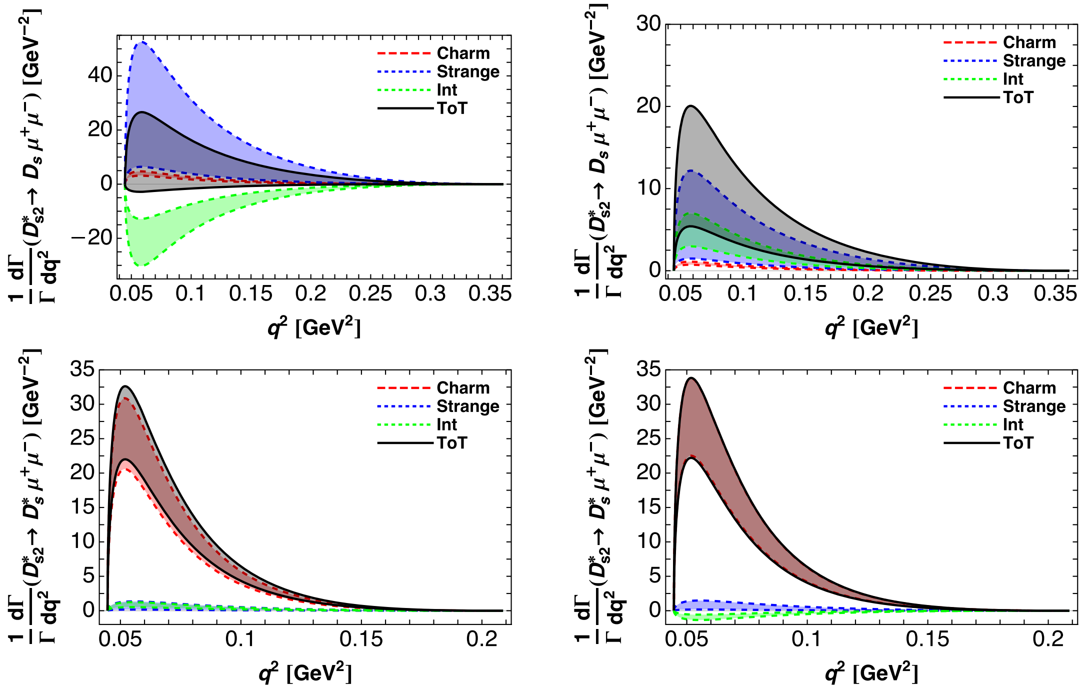

FIG. 4. Distribution $\frac{1}{\Gamma}\frac{d\Gamma}{dq^2}$ for $D_{s2}^* \to D_s\mu^+\mu^-$ (top) and $D_{s2}^* \to D_s^*\mu^+\mu^-$ (bottom panels). Notations as in Fig. 2.

as for the widths of processes with $s_\ell^P = \frac{3}{2}^+$ particles. The experimental confirmation of such a behavior would further support the classification of $D_{sJ}^{(*)}$ in heavy quark spin doublets.

The requests for the experimental analyses are the optimized reconstruction of D_s and D_s^* and of their momentum and the precise measurement of the lepton momenta to access q^2 and angular distributions. Both the requirements are well satisfied by a hadron facility as the CERN LHC, in particular, by the LHCb experiment, which can exploit the large charm production rate needed for signals with branching fractions as in Table II. Measurements are also feasible at a lepton facility such as the Belle II experiment at KEK, in particular, when all the statistics are available.

In the experimental analyses, the expressions for the amplitudes and the distributions can be parametrically used, leaving the strong couplings $g_{1,2}^S$ and h_T and the parameters of the universal functions τ_i in Table I as quantities to be measured. This will allow an interesting comparison with the determinations from different observables (strong decay widths of positive parity charmed mesons and semileptonic widths of $B_{q,s}$ decays to $D_{q,sJ}^{(*)}$).

From a general viewpoint, we emphasize that the expressions given in the previous sections and used in the examples presented here have been worked out under the assumption that the four lightest $D_{sJ}^{(*)}$ belong to heavy quark spin doublet. In such a case, the formulas involve the heavy quark effective theory of QCD together with the VMD model to describe the coupling of the virtual photon to the strange quark. All such features can be probed using the wealth of observables that can be constructed: for single modes, with rates and distributions in dilepton mass and angles, and comparing different modes, in pairs or all together. This justifies the great interest for the processes we have discussed.

VI. CONCLUSIONS

We have presented an analysis of the Dalitz decays of the positive-parity $D_{sJ}^{(*)}$ charmed mesons, $D_{sJ}^{(*)} \to D_s^{(*)}\ell^+\ell^-$, with $J = 0, 1, 2$ and $\ell = e, \mu$. The study is based on the classification of the heavy mesons in spin doublets. The amplitudes are expressed in terms of universal form factors and of effective strong couplings and can be computed using information from different processes. We have discussed how correlations can be established among different observables: Their experimental confirmation would further support our classification scheme for the $D_{sJ}^{(*)}$ charmed mesons.

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APPENDIX: FUNCTIONS IN THE ANGULAR DISTRIBUTION EQ. (4.3)

For the various Dalitz processes, the expressions of the functions $A_X(q^2)$ and $B_X(q^2)$ in the angular distribution in Eq. (4.3) are given below.

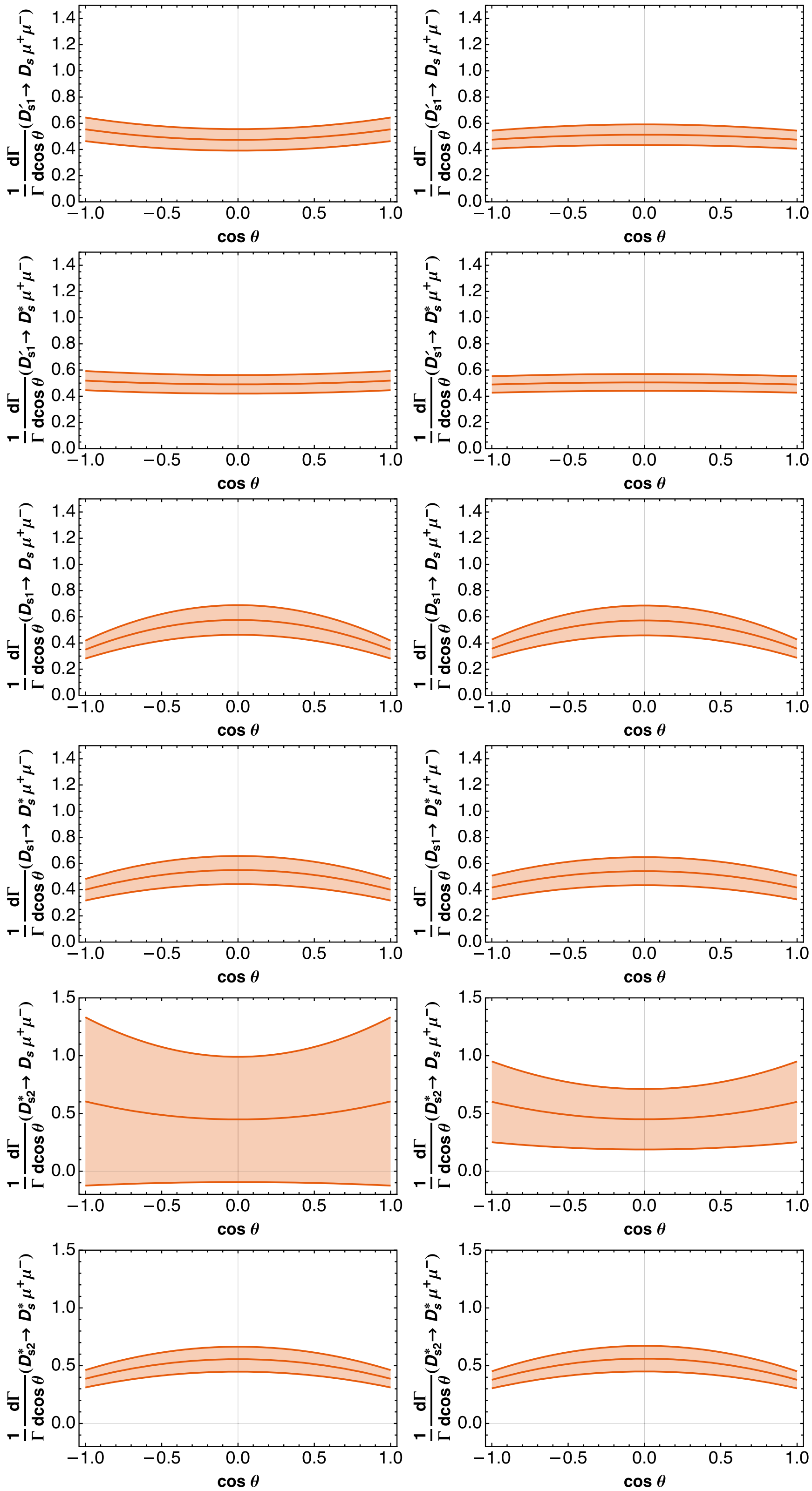

FIG. 5. Angular distribution $\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta}$ for the processes (from top to bottom) $D'_{s1}\to D_s\mu^+\mu^-$, $D'_{s1}\to D_s^*\mu^+\mu^-$, $D_{s1}\to D_s\mu^+\mu^-$, $D_{s1}\to D_s^*\mu^+\mu^-$, $D_{s2}^*\to D_s\mu^+\mu^-$, and $D_{s2}^*\to D_s^*\mu^+\mu^-$. Left panels correspond to case A, right panels to case B for the maximal interference.

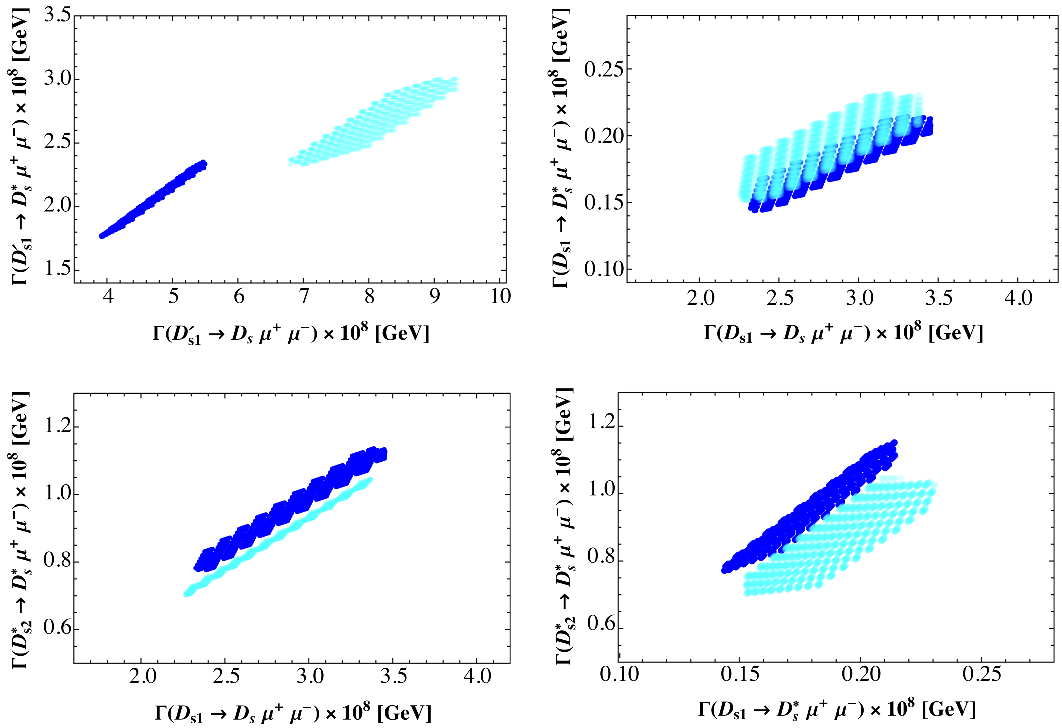

FIG. 6. Correlations between decay widths for different Dalitz decays. The dark and light regions correspond to the maximal interference cases A and B, respectively.

1. $D'_{s1} \to D_s \ell^+ \ell^-$

$$A_c(q^2) = g(q^2)[\tau_{1/2}(q^2)]^2 \frac{[(m_{D'_{s1}} - m_{D_s})^2 - q^2]^2}{18 m^3_{D'_{s1}} m_{D_s}} q^2 \times [(m_{D'_{s1}} + m_{D_s})^2 + q^2 + 4m_\ell^2], \tag{A1}$$

$$B_c(q^2) = -g(q^2)[\tau_{1/2}(q^2)]^2 \frac{[(m_{D'_{s1}} - m_{D_s})^2 - q^2]^2}{18 m^3_{D'_{s1}} m_{D_s}} \times [(m_{D'_{s1}} + m_{D_s})^2 - q^2](q^2 - 4m_\ell^2), \tag{A2}$$

$$A_s(q^2) = g(q^2) f^2(q^2) \frac{[(m_{D'_{s1}} + m_{D_s})^2 - q^2]^2}{144 m^3_{D'_{s1}} m_{D_s}} q^2 \times \left\{ (g_1^S)^2 [(m_{D'_{s1}} - m_{D_s})^2 + q^2 + 4m_\ell^2] + \frac{8 g_1^S g_2^S}{\Lambda} \times (m_{D'_{s1}} - m_{D_s})(q^2 + 2m_\ell^2) + \frac{4(g_2^S)^2}{\Lambda^2} \times [q^4 + q^2 (m_{D'_{s1}} - m_{D_s})^2 + 4m_\ell^2 (m_{D'_{s1}} - m_{D_s})^2] \right\}, \tag{A3}$$

$$B_s(q^2) = -g(q^2) f^2(q^2) \frac{[(m_{D'_{s1}} + m_{D_s})^2 - q^2]^2}{144 m^3_{D'_{s1}} m_{D_s}} \times [(m_{D'_{s1}} - m_{D_s})^2 - q^2](q^2 - 4m_\ell^2) \times \left\{ (g_1^S)^2 - \frac{4(g_2^S)^2}{\Lambda^2} q^2 \right\}, \tag{A4}$$

$$A_{\rm int}(q^2) = g(q^2) f(q^2) \tau_{1/2}(q^2) \frac{q^2 \lambda(m^2_{D'_{s1}}, m^2_{D_s}, q^2)}{18\sqrt{2} m^3_{D'_{s1}} m_{D_s}} \times \left\{ g_1^S [m^2_{D'_{s1}} - m^2_{D_s} + q^2 + 4m_\ell^2] + \frac{4 g_2^S}{\Lambda} [q^2 m_{D'_{s1}} + 2m_\ell^2 (m_{D'_{s1}} - m_{D_s})] \right\}, \tag{A5}$$

$$B_{\rm int}(q^2) = -g(q^2) f(q^2) \tau_{1/2}(q^2) \frac{\lambda(m^2_{D'_{s1}}, m^2_{D_s}, q^2)}{18\sqrt{2} m^3_{D'_{s1}} m_{D_s}} (q^2 - 4m_\ell^2) \times \left\{ g_1^S [m^2_{D'_{s1}} - m^2_{D_s} - q^2] + \frac{4 g_2^S}{\Lambda} q^2 m_{D_s} \right\}. \tag{A6}$$

2. $D_{s1} \to D_s \ell^+ \ell^-$

$$A_c(q^2) = g(q^2)[\tau_{3/2}(q^2)]^2 \frac{\lambda^2(m_{D_{s1}}^2, m_{D_s}^2, q^2)}{432 m_{D_{s1}}^5 m_{D_s}^3} q^2 \times [4(m_{D_{s1}} + m_{D_s})^2 + q^2 + 4m_\ell^2], \tag{A7}$$

$$B_c(q^2) = -g(q^2)[\tau_{3/2}(q^2)]^2 \frac{\lambda^2(m_{D_{s1}}^2, m_{D_s}^2, q^2)}{432 m_{D_{s1}}^5 m_{D_s}^3} \times (q^2 - 4m_\ell^2)[4(m_{D_{s1}} + m_{D_s})^2 - q^2], \tag{A8}$$

$$A_s(q^2) = g(q^2) f^2(q^2) \frac{h_T^2}{\Lambda^4} \frac{\lambda^2(m_{D_{s1}}^2, m_{D_s}^2, q^2)}{864 m_{D_{s1}}^5 m_{D_s}} q^2 \times [4q^4 + (q^2 + 4m_\ell^2)(m_{D_{s1}} + m_{D_s})^2], \tag{A9}$$

$$B_s(q^2) = g(q^2) f^2(q^2) \frac{h_T^2}{\Lambda^4} \frac{\lambda^2(m_{D_{s1}}^2, m_{D_s}^2, q^2)}{864 m_{D_{s1}}^5 m_{D_s}} q^2 \times (q^2 - 4m_\ell^2)[(m_{D_{s1}} + m_{D_s})^2 - 4q^2], \tag{A10}$$

$$A_{\rm int}(q^2) = -g(q^2) f(q^2) \tau_{3/2}(q^2) \frac{h_T}{\Lambda^2} \frac{\lambda^2(m_{D_{s1}}^2, m_{D_s}^2, q^2)}{216\sqrt{2} m_{D_{s1}}^5 m_{D_s}^2} q^2 \times (3q^2 - 4m_\ell^2)(m_{D_{s1}} + m_{D_s}), \tag{A11}$$

$$B_{\rm int}(q^2) = g(q^2) f(q^2) \tau_{3/2}(q^2) \frac{h_T}{\Lambda^2} \frac{5\lambda^2(m_{D_{s1}}^2, m_{D_s}^2, q^2)}{216\sqrt{2} m_{D_{s1}}^5 m_{D_s}^2} q^2 \times (q^2 - 4m_\ell^2)(m_{D_{s1}} + m_{D_s}). \tag{A12}$$

3. $D_{s2}^* \to D_s \ell^+ \ell^-$

$$A_c(q^2) = g(q^2)[\tau_{3/2}(q^2)]^2 \frac{\lambda^2(m_{D_{s2}^*}^2, m_{D_s}^2, q^2)}{240 m_{D_{s2}^*}^5 m_{D_s}^3} q^2 (q^2 + 4m_\ell^2), \tag{A13}$$

$$B_c(q^2) = g(q^2)[\tau_{3/2}(q^2)]^2 \frac{\lambda^2(m_{D_{s2}^*}^2, m_{D_s}^2, q^2)}{240 m_{D_{s2}^*}^5 m_{D_s}^3} q^2 (q^2 - 4m_\ell^2), \tag{A14}$$

$$A_s(q^2) = g(q^2) f^2(q^2) \frac{h_T^2}{\Lambda^4} \frac{\lambda^2(m_{D_{s2}^*}^2, m_{D_s}^2, q^2)}{480 m_{D_{s2}^*}^5 m_{D_s}} q^2 \times (q^2 + 4m_\ell^2)(m_{D_{s2}^*} + m_{D_s})^2, \tag{A15}$$

$$B_s(q^2) = g(q^2) f^2(q^2) \frac{h_T^2}{\Lambda^4} \frac{\lambda^2(m_{D_{s2}^*}^2, m_{D_s}^2, q^2)}{480 m_{D_{s2}^*}^5 m_{D_s}} q^2 \times (q^2 - 4m_\ell^2)(m_{D_{s2}^*} + m_{D_s})^2, \tag{A16}$$

$$A_{\rm int}(q^2) = g(q^2) f(q^2) \tau_{3/2}(q^2) \frac{h_T}{\Lambda^2} \frac{\lambda^2(m_{D_{s2}^*}^2, m_{D_s}^2, q^2)}{120\sqrt{2} m_{D_{s2}^*}^5 m_{D_s}^2} q^2 \times (q^2 + 4m_\ell^2)(m_{D_{s2}^*} + m_{D_s}), \tag{A17}$$

$$B_{\rm int}(q^2) = g(q^2) f(q^2) \tau_{3/2}(q^2) \frac{h_T}{\Lambda^2} \frac{\lambda^2(m_{D_{s2}^*}^2, m_{D_s}^2, q^2)}{120\sqrt{2} m_{D_{s2}^*}^5 m_{D_s}^2} q^2 \times (q^2 - 4m_\ell^2)(m_{D_{s2}^*} + m_{D_s}). \tag{A18}$$

4. $D_{s0}^* \to D_s^* \ell^+ \ell^-$

The process is kinematically allowed only for $\ell = e$. The functions in (4.3) are given by

$$A_c(q^2) = g(q^2)[\tau_{1/2}(q^2)]^2 \frac{[(m_{D_{s0}^*} - m_{D_s^*})^2 - q^2]^2}{6 m_{D_{s0}^*}^3 m_{D_s^*}} q^2 \times [(m_{D_{s0}^*} + m_{D_s^*})^2 + q^2 + 4m_\ell^2], \tag{A19}$$

$$B_c(q^2) = -g(q^2)[\tau_{1/2}(q^2)]^2 \frac{[(m_{D_{s0}^*} - m_{D_s^*})^2 - q^2]^2}{6 m_{D_{s0}'}^3 m_{D_s^*}} \times (q^2 - 4m_\ell^2)[(m_{D_{s0}^*} + m_{D_s^*})^2 - q^2], \tag{A20}$$

$$\begin{aligned} A_s(q^2) = g(q^2) f^2(q^2) \frac{[(m_{D_{s0}^*} + m_{D_s^*})^2 - q^2]^2}{48 m_{D_{s0}^*}^3 m_{D_s^*}} q^2 & \times \Big\{ (g_1^S)^2 [(m_{D_{s0}^*} - m_{D_s^*})^2 + q^2 + 4m_\ell^2] \\ & + \frac{8 g_1^S g_2^S}{\Lambda} (m_{D_{s0}^*} - m_{D_s^*})(q^2 + 2m_\ell^2) \\ & + \frac{4(g_2^S)^2}{\Lambda^2} [q^4 + q^2 (m_{D_{s0}^*} - m_{D_s^*})^2 \\ & + 4m_\ell^2 (m_{D_{s0}^*} - m_{D_s^*})^2] \Big\}, \end{aligned} \tag{A21}$$

$$B_s(q^2) = -g(q^2) f^2(q^2) \frac{[(m_{D_{s0}^*} + m_{D_s^*})^2 - q^2]^2}{48 m_{D_{s0}^*}^3 m_{D_s^*}} \times [(m_{D_{s0}^*} - m_{D_s^*})^2 - q^2](q^2 - 4m_\ell^2) \times \left\{ (g_1^S)^2 - \frac{4(g_2^S)^2}{\Lambda^2} q^2 \right\}, \tag{A22}$$

$$A_{\rm int}(q^2) = g(q^2) f(q^2) \tau_{1/2}(q^2) \frac{\lambda(m_{D_{s0}^*}^2, m_{D_s^*}^2, q^2)}{6\sqrt{2} m_{D_{s0}^*}^3 m_{D_s^*}} q^2 \times \left\{ g_1^S [m_{D_{s0}^*}^2 - m_{D_s^*}^2 - q^2 - 4m_\ell^2] - \frac{4 g_2^S}{\Lambda} [2 m_{D_{s0}^*} m_\ell^2 - m_{D_s^*}(q^2 + 2m_\ell^2)] \right\}, \tag{A23}$$

$$B_{\rm int}(q^2) = -g(q^2)f(q^2)\tau_{1/2}(q^2)\frac{\lambda(m^2_{D^*_{s0}}, m^2_{D^*_s}, q^2)}{6\sqrt{2}m^3_{D^*_{s0}}m_{D^*_s}}(q^2-4m^2_\ell) \times\left\{g_1^S[m^2_{D^*_{s0}} - m^2_{D^*_s} + q^2] + \frac{4g_2^S}{\Lambda}m_{D^*_{s0}}q^2\right\}. \quad \text{(A24)}$$

5. $D'_{s1} \to D_s^*\ell^+\ell^-$

$$A_c(q^2) = g(q^2)[\tau_{1/2}(q^2)]^2\frac{[(m_{D'_{s1}} - m_{D^*_s})^2 - q^2]^2}{9m^3_{D'_{s1}}m_{D^*_s}}q^2 \times[(m_{D'_{s1}} + m_{D^*_s})^2 + q^2 + 4m^2_\ell], \quad \text{(A25)}$$

$$B_c(q^2) = -g(q^2)[\tau_{1/2}(q^2)]^2\frac{[(m_{D'_{s1}} - m_{D^*_s})^2 - q^2]^2}{9m^3_{D'_{s1}}m_{D^*_s}} \times(q^2 - 4m^2_\ell)[(m_{D'_{s1}} + m_{D^*_s})^2 - q^2], \quad \text{(A26)}$$

$$A_s(q^2) = g(q^2)f^2(q^2)\frac{[(m_{D'_{s1}} + m_{D^*_s})^2 - q^2]^2}{72m^3_{D'_{s1}}m_{D^*_s}}q^2 \times\left\{(g_1^S)^2[(m_{D'_{s1}} - m_{D^*_s})^2 + q^2 + 4m^2_\ell] + \frac{8g_1^Sg_2^S}{\Lambda}(m_{D'_{s1}} - m_{D^*_s})(q^2 + 2m^2_\ell) + \frac{4(g_2^S)^2}{\Lambda^2}[q^4 + q^2(m_{D'_{s1}} - m_{D^*_s})^2 + 4m^2_\ell(m_{D'_{s1}} - m_{D^*_s})^2]\right\}, \quad \text{(A27)}$$

$$B_s(q^2) = -g(q^2)f^2(q^2)\frac{[(m_{D'_{s1}} + m_{D^*_s})^2 - q^2]^2}{72m^3_{D'_{s1}}m_{D^*_s}} \times[(m_{D'_{s1}} - m_{D^*_s})^2 - q^2](q^2 - 4m^2_\ell) \times\left\{(g_1^S)^2 - \frac{4(g_2^S)^2}{\Lambda^2}q^2\right\}, \quad \text{(A28)}$$

$$A_{\rm int}(q^2) = g(q^2)f(q^2)\tau_{1/2}(q^2)\frac{\lambda(m^2_{D'_{s1}}, m^2_{D^*_s}, q^2)}{9\sqrt{2}m^3_{D'_{s1}}m_{D^*_s}}q^2 \times(m_{D'_{s1}} + m_{D^*_s})\left\{g_1^S(m_{D'_{s1}} - m_{D^*_s}) + \frac{2g_2^S}{\Lambda}q^2\right\}, \quad \text{(A29)}$$

$$B_{\rm int}(q^2) = -g(q^2)f(q^2)\tau_{1/2}(q^2)\frac{\lambda(m^2_{D'_{s1}}, m^2_{D^*_s}, q^2)}{9\sqrt{2}m^3_{D'_{s1}}m_{D^*_s}} \times(q^2 - 4m^2_\ell)(m_{D'_{s1}} + m_{D^*_s}) \times\left\{g_1^S(m_{D'_{s1}} - m_{D^*_s}) + \frac{2g_2^S}{\Lambda}q^2\right\}. \quad \text{(A30)}$$

6. $D_{s1} \to D_s^*\ell^+\ell^-$

$$A_c(q^2) = g(q^2)[\tau_{3/2}(q^2)]^2\frac{\lambda^2(m^2_{D_{s1}}, m^2_{D^*_s}, q^2)}{432m^5_{D_{s1}}m^3_{D^*_s}}q^2 \times[2(m_{D_{s1}} + m_{D^*_s})^2 + 5(q^2 + 4m^2_\ell)], \quad \text{(A31)}$$

$$B_c(q^2) = -g(q^2)[\tau_{3/2}(q^2)]^2\frac{\lambda^2(m^2_{D_{s1}}, m^2_{D^*_s}, q^2)}{432m^5_{D_{s1}}m^3_{D^*_s}} \times(q^2 - 4m^2_\ell)[2(m_{D_{s1}} + m_{D^*_s})^2 - 5q^2], \quad \text{(A32)}$$

$$A_s(q^2) = g(q^2)f^2(q^2)\frac{h_T^2}{\Lambda^4}\frac{\lambda^2(m^2_{D_{s1}}, m^2_{D^*_s}, q^2)}{864m^5_{D_{s1}}m_{D^*_s}}q^2 \times[2q^4 + 5(q^2 + 4m^2_\ell)(m_{D_{s1}} + m_{D^*_s})^2], \quad \text{(A33)}$$

$$B_s(q^2) = g(q^2)f^2(q^2)\frac{h_T^2}{\Lambda^4}\frac{\lambda^2(m^2_{D_{s1}}, m^2_{D^*_s}, q^2)}{864m^5_{D_{s1}}m_{D^*_s}}q^2 \times(q^2 - 4m^2_\ell)[5(m_{D_{s1}} + m_{D^*_s})^2 - 2q^2], \quad \text{(A34)}$$

$$A_{\rm int}(q^2) = g(q^2)f(q^2)\tau_{3/2}(q^2)\frac{h_T}{\Lambda^2}\frac{\lambda^2(m^2_{D_{s1}}, m^2_{D^*_s}, q^2)}{216\sqrt{2}m^5_{D_{s1}}m^2_{D^*_s}}q^2 \times(q^2 + 12m^2_\ell)(m_{D_{s1}} + m_{D^*_s}), \quad \text{(A35)}$$

$$B_{\rm int}(q^2) = g(q^2)f(q^2)\tau_{3/2}(q^2)\frac{h_T}{\Lambda^2}\frac{5\lambda^2(m^2_{D_{s1}}, m^2_{D^*_s}, q^2)}{216\sqrt{2}m^5_{D_{s1}}m^2_{D^*_s}}q^2 \times(q^2 - 4m^2_\ell)(m_{D_{s1}} + m_{D^*_s}). \quad \text{(A36)}$$

7. $D^*_{s2} \to D_s^*\ell^+\ell^-$

$$A_c(q^2) = g(q^2)[\tau_{3/2}(q^2)]^2\frac{\lambda^2(m^2_{D^*_{s2}}, m^2_{D^*_s}, q^2)}{720m^5_{D^*_{s2}}m^3_{D^*_s}}q^2 \times[10(m_{D^*_{s2}} + m_{D_s})^2 + 7(q^2 + 4m^2_\ell)], \quad \text{(A37)}$$

$$B_c(q^2) = -g(q^2)[\tau_{3/2}(q^2)]^2\frac{\lambda^2(m^2_{D^*_{s2}}, m^2_{D^*_s}, q^2)}{720m^5_{D^*_{s2}}m^3_{D^*_s}} \times(q^2 - 4m^2_\ell)[10(m_{D^*_{s2}} + m_{D^*_s})^2 - 7q^2], \quad \text{(A38)}$$

$$A_s(q^2) = g(q^2)[f(q^2)]^2 \frac{h_T^2}{\Lambda^4} \frac{\lambda^2(m^2_{D^*_{s2}}, m^2_{D^*_s}, q^2)}{1440 m^5_{D^*_{s2}} m_{D^*_s}} q^2 \times [10q^4 + 7(q^2 + 4m^2_\ell)(m_{D^*_{s2}} + m_{D^*_s})^2], \quad (A39)$$

$$B_s(q^2) = g(q^2)[f(q^2)]^2 \frac{h_T^2}{\Lambda^4} \frac{\lambda^2(m^2_{D^*_{s2}}, m^2_{D^*_s}, q^2)}{1440 m^5_{D^*_{s2}} m_{D^*_s}} q^2 \times (q^2 - 4m^2_\ell)[7(m_{D^*_{s2}} + m_{D_s})^2 - 10q^2], \quad (A40)$$

$$A_{\rm int}(q^2) = -g(q^2) f(q^2) \tau_{3/2}(q^2) \frac{h_T}{\Lambda^2} \frac{\lambda^2(m^2_{D^*_{s2}}, m^2_{D^*_s}, q^2)}{360\sqrt{2} m^5_{D^*_{s2}} m^2_{D^*_s}} q^2 \times (17q^2 + 28m^2_\ell)(m_{D^*_{s2}} + m_{D^*_s}), \quad (A41)$$

$$B_{\rm int}(q^2) = g(q^2) f(q^2) \tau_{3/2}(q^2) \frac{h_T}{\Lambda^2} \frac{\lambda^2(m^2_{D^*_{s2}}, m^2_{D^*_s}, q^2)}{120\sqrt{2} m^5_{D^*_{s2}} m^2_{D^*_s}} q^2 \times (q^2 - 4m^2_\ell)(m_{D^*_{s2}} + m_{D^*_s}). \quad (A42)$$

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