Energy transfer between gravitational waves and quantum matter

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We study the interaction between gravitational waves and quantum matter such as Bose-Einstein condensates, superfluid helium, or ultracold solids, explicitly taking into account the changes of the trapping potential induced by the gravitational wave. As a possible observable, we consider the change of energy due to the gravitational wave, for which we derive rigorous bounds in terms of kinetic energy and particle number. Finally, we discuss implications for possible experimental tests.

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I. INTRODUCTION

Gravitational waves had been predicted shortly after the publication of Einstein's field equations of general relativity [1,2], but their experimental detection was thought to be impossible for a long time in view of the smallness of the expected signals. It took roughly half a century until Weber constructed a detector for gravitational waves based on resonant mass antennas known as Weber bars [3–5]. Although Weber's initial results and claims of having detected a signal could not be reproduced by other groups, his endeavors should still be considered pioneering experiments, paving the way for later developments.

A whole century after their prediction, gravitational waves have been detected at LIGO [6,7], marking a major breakthrough and the beginning of a new era in modern physics. Note that one should distinguish two major detection schemes for gravitational waves: At interferometers such as LIGO, one measures the changes of the arm lengths and the resulting interference patters *during* the passage of the gravitational wave. In contrast, the resonant excitation of a Weber bar can be measured *after* the gravitational wave passed by.

In the following, we theoretically investigate detection schemes of the second type, see also [8–10]. Instead of Weber bars, we consider more general resonant mass antennas represented by quantum matter such as Bose-Einstein condensates, superfluid helium, or ultracold solids. To some extent, these studies are motivated by recent and partly controversial discussions regarding the use of Bose-Einstein condensates as gravitational wave detectors, see, e.g., [11–20]. More generally, the weakness of the interaction with gravitational waves and the resulting smallness of the signal motivates a quantum description. The aforementioned examples for quantum matter may offer certain advantages, e.g., regarding temperature, purity, or experimental control, see also [21–28].

As a typical observable, we consider the change in energy induced by the gravitational wave. Note that matter-wave interferometers, which have also been proposed as gravitational wave detectors [29–35], are typically based on detection schemes of the first type—and are thus not considered here.

II. GRAVITATIONAL WAVES

For simplicity, we consider linearly polarized gravitational waves propagating in a fixed direction. Other waves can be written as linear combinations of such solutions. In a suitable coordinate system, the metric reads (using natural units $\hbar = c = \varepsilon_0 = \mu_0 = 1$)

$$ds^2 = dt^2 - [1+h]dx^2 - [1-h]dy^2 - dz^2, \qquad (1)$$

where the function $h(t-z)$ describes the gravitational wave. However, as its wavelength is much larger than the characteristic length scales in the laboratory while its period is shorter than the duration of the experiment, we use the approximation $h(t-z) \approx h(t)$ in what follows. Furthermore, since h is extremely small, $h = \mathcal{O}(10^{-22})$, we neglect quadratic terms $\mathcal{O}(h^2)$ in the following (as usual in the linearized theory of gravitational waves). As a consequence, the metric determinant can be approximated by unity $\sqrt{-g} = 1 + \mathcal{O}(h^2)$.

A. Massive particles

Before investigating the implications of the metric (1) for the quantum Hamiltonian in Sec. III, let us briefly discuss the impact on classical point particles and electromagnetic waves, which will also be relevant for changes of the trapping potential.

Since the Christoffel symbols corresponding to Newton's gravitational acceleration vanish $\Gamma^i_{00} = 0$, massive particles at rest with respect to the coordinates (1), i.e., at constant positions x, y, and z, are solutions to the geodesic equations. As a result, the heavy mirrors used in LIGO, for example, do not change their positions x, y, and z during the passage of the gravitational wave. However, their physical distance (1) changes, which can be measured by light rays, for instance.

For moving particles, on the other hand, the gravitational wave does generate an effective force. Considering a nonrelativistic motion in the x, y plane for simplicity, the Christoffel symbols Γ^i_{0i} are given by $\pm\dot{h}/2$ and correspond to the acceleration $\dot{u}^x = -\dot{h}u^x$ and $\dot{u}^y = \dot{h}u^y$ in terms of the four-velocity u^μ. The Christoffel symbols Γ^0_{ij} then yield the change of energy $\dot{u}^t = \dot{h}(u_y^2 - u_x^2)/2$.

B. Electromagnetic waves

Next, let us consider electromagnetic waves propagating in the background metric (1) which are described by the Maxwell equations $\nabla_\mu F^{\mu\nu} = 0$ with the electromagnetic field-strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and the vector potential A_μ. Their dispersion relation can already be read off the metric (1)

$$\Omega^2 = -g^{ij}K_iK_j = [1-h]K_x^2 + [1+h]K_y^2 + K_z^2. \quad (2)$$

Since the changes of the amplitudes A_i induced by the gravitational wave depend on their polarization, let us first consider the cases of fixed polarizations along the coordinate axes for simplicity.

First, the identity $\nabla_\mu F^{\mu\nu} = \partial_\mu(\sqrt{-g}F^{\mu\nu})/\sqrt{-g}$ leads to the wave equation for the polarization $A_z(t,x,y)$,

$$(\partial_t^2 - \partial_x[1-h]\partial_x - \partial_y[1+h]\partial_y)A_z = 0. \quad (3)$$

After a spatial Fourier transformation, this reduces to the differential equation $\ddot{A}_z + \Omega^2 A_z = 0$ of a parametric harmonic oscillator with the time-dependent frequency $\Omega(t)$ given by Eq. (2) for $K_z = 0$. Since the frequency Ω of the electromagnetic waves (e.g., optical lasers) is much larger than that of the gravitational waves $\omega \ll \Omega$, we may employ the WKB approximation and deduce a scaling of the amplitude A_z with $1/\sqrt{\Omega}$. One way to obtain this result is to consider the conserved Wronskian which reads $W = A_z^*\dot{A}_z - \dot{A}_z^*A_z$ and thus simplifies to $W \approx -2i\Omega|A_z^2|$.

Second, let us consider the fixed polarization $A_x(t,y,z)$, for which we find the wave equation

$$(\partial_t[1-h]\partial_t - \partial_y^2 - \partial_z[1-h]\partial_z)A_x = 0. \quad (4)$$

In this case, the conserved Wronskian contains an additional metric factor $W = [1-h](A_x^*\dot{A}_x - \dot{A}_x^*A_x)$ and thus the amplitude A_x scales with $1/\sqrt{[1-h]\Omega}$.

Obviously, the third case $A_y(t,x,z)$ is completely analogous to the second after replacing $1-h$ by $1+h$. The behavior of general polarizations A_i can be inferred from the wave equation in temporal gauge $A_0 = 0$,

$$\partial_t(g^{ij}\partial_t A_j) = K_2^{ij}A_j, \quad (5)$$

where the matrix K_2^{ij} contains bilinear forms of the wave numbers K_i as well as metric factors $1 \pm h$. In this case, the conserved Wronskian reads $W = A_i^* g^{ij}\dot{A}_j - \dot{A}_i^* g^{ij} A_j$ which can again be used to infer the scaling of the amplitude A_i. Note, however, that the transversality condition $K_i g^{ij}\dot{A}_j = 0$ implies small changes of the polarization direction induced by the gravitational wave—unless K_i or A_j are oriented along the eigenvectors of g_{ij}, i.e., the coordinate axes.

In summary, both the frequency Ω as well as the amplitudes A_i of the electromagnetic waves acquire small corrections of the form $1 + \zeta h$ due to the gravitational wave, where the various values of ζ depend on the propagation and polarization directions of the electromagnetic waves.

III. MATTER HAMILTONIAN

In flat space-time, i.e., without the gravitational wave, we assume that the matter can be described by the standard nonrelativistic many-body Hamiltonian

$$\hat{H}_0 = \int d^3r \left[\frac{1}{2m}(\nabla\hat{\Psi}^\dagger)\cdot(\nabla\hat{\Psi}) + V_0(\boldsymbol{r})\hat{\Psi}^\dagger\hat{\Psi}\right] + \frac{1}{2}\int d^3r d^3r' \hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}^\dagger(\boldsymbol{r}')W(\boldsymbol{r},\boldsymbol{r}')\hat{\Psi}(\boldsymbol{r}')\hat{\Psi}(\boldsymbol{r}), \quad (6)$$

with bosonic or fermionic field operators $\hat{\Psi}$ and $\hat{\Psi}^\dagger$, the static trapping potential V_0 and the interaction W.

In order to describe the response to a gravitational wave, we first have to determine the corresponding changes in the Hamiltonian. As already shown in [14,36], for example, the kinetic term is modified quite intuitively by inserting the metric g^{ij} into the scalar product between the field gradients, i.e., $(\nabla\hat{\Psi}^\dagger)\cdot(\nabla\hat{\Psi})$ is replaced by $-(\partial_i\hat{\Psi}^\dagger)g^{ij}(\partial_j\hat{\Psi})$. The change of the trapping potential V will be discussed below. Assuming that the interaction W is isotropic and short ranged, we neglect its modification due to the gravitational wave.

A. Energy transfer

Now we are in the position to study how the energy of the matter changes due to its interaction with the gravitational wave. To this end, we employ the Heisenberg picture where

$$\frac{\mathrm{d}\hat{H}}{\mathrm{d}t} = \left(\frac{\partial \hat{H}}{\partial t}\right)_{\mathrm{expl}} = \frac{\partial \hat{H}}{\partial h}\dot{h}$$
$$= \dot{h}\int \mathrm{d}^3 r\left[\frac{\partial V}{\partial h}\hat{\Psi}^\dagger\hat{\Psi} + \frac{(\partial_y\hat{\Psi}^\dagger)(\partial_y\hat{\Psi}) - (\partial_x\hat{\Psi}^\dagger)(\partial_x\hat{\Psi})}{2m}\right]. \tag{7}$$

Taking expectation values yields the change of the total energy $E = \langle\hat{H}\rangle$. The very general expression (7) already allows us to infer important consequences. In analogy to time-dependent perturbation theory, we may replace the expectation values (such as $\langle\hat{\Psi}^\dagger\hat{\Psi}\rangle$) in the above integrand to lowest order in h by their undisturbed expressions (such as $\langle\hat{\Psi}^\dagger\hat{\Psi}\rangle_0$) in flat space-time because there is already a factor of $\dot{h}$ in front of the integral,

$$\dot{E} = \dot{h}\int \mathrm{d}^3 r\left\langle\frac{\partial V}{\partial h}\hat{\Psi}^\dagger\hat{\Psi} + \frac{(\partial_y\hat{\Psi}^\dagger)(\partial_y\hat{\Psi}) - (\partial_x\hat{\Psi}^\dagger)(\partial_x\hat{\Psi})}{2m}\right\rangle_0 + \mathcal{O}(h^2). \tag{8}$$

As a result, if this undisturbed (i.e., initial) state is a stationary state with respect to the $\hat{H}_0$-dynamics—such as the ground state or a thermal equilibrium state—the above expectation value would be independent of time. In this case, the time integration of Eq. (8) becomes trivial and thus there is no energy shift to linear order in h. In order to obtain such a first-order energy shift, one should prepare a nonstationary state (e.g., vibrating or oscillating) such that the expectation values oscillate—ideally in resonance with $\dot{h}$ to maximize the energy transfer.

As another consequence of the general expression (7), we may estimate the maximum amount of energy which can be transferred. To this end, we exploit the non-negativity of the operators $\hat{\Psi}^\dagger\hat{\Psi}$ and $(\partial_i\hat{\Psi}^\dagger)(\partial_i\hat{\Psi})$ which allows us to derive the rigorous upper bound

$$\dot{E} \le |\dot{h}|_{\mathrm{max}}\left(\left|\frac{\partial V}{\partial h}\right|_{\mathrm{max}}\langle\hat{N}\rangle + \langle\hat{E}_{\mathrm{kin}}\rangle_{\mathrm{max}}\right), \tag{9}$$

in terms of the total particle number $\langle\hat{N}\rangle$ and the kinetic energy $\langle\hat{E}_{\mathrm{kin}}\rangle$ of the matter. Note that the former is conserved, i.e., $\langle\hat{N}\rangle$ is constant, while the latter $\langle\hat{E}_{\mathrm{kin}}\rangle$ may vary with time due to an exchange between kinetic, potential and interaction energy.

B. Electromagnetic analogy

It might be illuminating to compare the energy transfer by gravitational waves discussed above to the well-known case of electromagnetic waves. Again assuming that our quantum system is much smaller than the wavelength of the electromagnetic field (dipole approximation), we may effectively describe it by a purely time-dependent vector potential $\boldsymbol{A}(t)$. Then the Hamiltonian (6) becomes

$$\hat{H}_0 = \frac{1}{2m}\int \mathrm{d}^3 r\Big([\boldsymbol{\nabla} + iq\boldsymbol{A}]\hat{\Psi}^\dagger\Big)\cdot\Big([\boldsymbol{\nabla} - iq\boldsymbol{A}]\hat{\Psi}\Big) + \int \mathrm{d}^3 r V_0(\boldsymbol{r})\hat{\Psi}^\dagger\hat{\Psi} + \frac{1}{2}\int \mathrm{d}^3 r \mathrm{d}^3 r' \hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}^\dagger(\boldsymbol{r}')W(\boldsymbol{r},\boldsymbol{r}')\hat{\Psi}(\boldsymbol{r}')\hat{\Psi}(\boldsymbol{r}). \tag{10}$$

If we assume that the electromagnetic field does neither affect the potential $V_0(\boldsymbol{r})$ nor the interaction $W(\boldsymbol{r},\boldsymbol{r}')$, the analog of Eq. (7) reads

$$\dot{E} = \dot{\boldsymbol{A}}\cdot\int \mathrm{d}^3 r\left[iq\frac{\hat{\Psi}^\dagger\boldsymbol{\nabla}\hat{\Psi} - (\boldsymbol{\nabla}\hat{\Psi}^\dagger)\hat{\Psi}}{2m} + q^2\frac{\boldsymbol{A}\hat{\Psi}^\dagger\hat{\Psi}}{m}\right]. \tag{11}$$

For a purely time-dependent vector potential $\boldsymbol{A}(t)$, the second term $\propto q^2$ yields the total particle number $\hat{N}$. Since $\hat{N}$ is conserved, this term does not generate a net energy shift. The same line of reasoning would apply to the term $\partial V/\partial h$ in Eq. (7) if $\partial V/\partial h$ was purely time dependent. Still, it is advantageous to keep this second term $\propto q^2$ in order to retain gauge invariance.

Altogether, we find that Eq. (7) is analogous to the well-known Poynting theorem in electrodynamics as the integrand of Eq. (11) represents the current density $\boldsymbol{j}$. Thus, $\dot{E}$ can be bound in analogy to Eq. (9) by electric field $|\dot{\boldsymbol{A}}|_{\mathrm{max}}$, current density $|\boldsymbol{j}|_{\mathrm{max}}$, and volume.

IV. TOY MODEL

In order to understand the above result (7) by means of a simple toy model, let us consider two classical and non-relativistic point particles of mass m on circular orbits around their joint center of mass

$$\boldsymbol{r}_\pm(t) = \pm R\begin{pmatrix}\cos(\omega_{\mathrm{rot}}t)\\ \sin(\omega_{\mathrm{rot}}t)\\ 0\end{pmatrix}. \tag{12}$$

Besides the force holding the masses on their circular orbits, the gravitational wave induces a small additional acceleration, as given by the geodesic equations already discussed in Sec. II A, i.e., $\dot{u}^x = -\dot{h}u^x$ and $\dot{u}^y = \dot{h}u^y$ as well as $\dot{u}^t = \dot{h}(u_y^2 - u_x^2)/2$. The resulting change in energy is thus given by $\dot{E} = \dot{h}(E_{\mathrm{kin}}^y - E_{\mathrm{kin}}^x)$, in analogy to Eq. (8). If the frequency ω of the gravitational wave equals twice the rotational frequency ω_{rot}, we obtain a resonant transfer of energy, see Appendix B and [37,38] as well as [39] and references therein.

It might be illuminating to insert some numbers and to estimate the resulting orders of magnitude. Assuming a gravitational wave with a frequency ω in the kHz regime and an amplitude of $h = \mathcal{O}(10^{-22})$, we may estimate the energy ΔE transferred after an interaction time T of

100 cycles, i.e., $\omega T = \mathcal{O}(10^2)$. Then, demanding that this energy shift $\Delta E = \mathcal{O}(h\omega T E_{\mathrm{kin}})$ corresponds to one excitation quantum $\hbar\omega$ in the kHz regime, we would need an initial kinetic energy of order 10^8 eV or 10^{-11} J.

Even though it would be easy to prepare such an initial kinetic energy for mesoscopic or macroscopic matter distributions, actually detecting an energy shift of one excitation quantum $\hbar\omega$ on top of this huge background is certainly extremely challenging. As a way around this obstacle, one could consider the change in vibrational energy E_{vib} instead of rotational energy E_{rot}. The acceleration induced by the gravitational wave has also components in the radial direction, which lead to a change in vibrational energy of order

$$\Delta E_{\mathrm{vib}} = \mathcal{O}\Big(h\omega T\sqrt{E_{\mathrm{vib}}E_{\mathrm{rot}}}\Big), \tag{13}$$

if an initial vibration of the barbell is present, i.e., $\dot{R} \neq 0$. In order to obtain resonant energy transfer, the vibrational frequency ω_{vib} should match $|\omega \pm 2\omega_{\mathrm{rot}}|$. In the following, we assume that all three frequencies are in the kHz regime. Then, if the initial quantum state of the vibrational mode corresponds to a few excitation quanta (say, ten $\hbar\omega_{\mathrm{vib}}$), an energy shift of one excitation quantum $\hbar\omega_{\mathrm{vib}}$ would require a rotational energy $E_{\mathrm{rot}} = 10^8$ J.

Of course, this value is now much larger than in the previous case (10^8 eV or 10^{-11} J), but it is not completely out of reach. For example, a barbell with $m = \mathcal{O}(100\ \mathrm{kg})$ and $R = \mathcal{O}(\mathrm{m})$, rotating with $\omega_{\mathrm{rot}} = \mathcal{O}(\mathrm{kHz})$, would have such a rotational energy $E_{\mathrm{rot}} = \mathcal{O}(10^8\ \mathrm{J})$. Obviously, controlling the vibrational modes to the desired accuracy would still be very challenging and probably requires a barbell levitating or suspended in ultrahigh vacuum etc. On the other hand, the impressive experimental progress regarding controlling and cooling down vibrational modes of macroscopic objects (see, e.g., [40,41]) gives rise to the hope that such an experiment may not be totally out of reach.

As an alternative, one could envision two concentric and corotating barbells at a right angle and consider the scissorslike motion instead of the vibrational mode. In doing so, one can find basically the same energy transfer as given by Eq. (13).

V. BOSE-EINSTEIN CONDENSATES

A. Trapping potential

After this simple toy model, let us apply our results to Bose-Einstein condensates. To this end, we first have to determine how the trapping potential V changes. As already mentioned, this will depend on its explicit physical realization in general. As an extreme case, if the shape of V is only determined by the positions of effectively force-free masses at rest (such as the mirrors in LIGO), it would not change at all during the passage of a gravitational wave.

However, for more realistic scenarios, one would expect V to vary. As a concrete example, let us consider optical traps which are often used to confine Bose-Einstein condensates. They may consist of a superposition of standing laser beams in various directions. As discussed in Sec. II B, these electromagnetic waves respond to gravitational waves via modification factors of the form $1+\zeta h$ in front of their frequencies and amplitudes where the ζ values are typically of order unity and depend on polarization and propagation direction.

The atoms in the Bose-Einstein condensate are then polarized by the electromagnetic waves where their polarizability scales with $1/(\Omega^2 - \Omega_{\mathrm{res}}^2)$ in terms of the frequencies Ω of the electromagnetic wave and the relevant atomic resonance Ω_{res} (blue or red detuned atoms). Assuming that the Ω_{res} do not change (see Appendix A), these polarizabilities get also modified by the gravitational wave via the change in Ω. Actually, if Ω is close to the resonance frequency Ω_{res}, the response to gravitational waves is enhanced, but going too close to resonance can be problematic.

In addition to these effects already occurring for free electromagnetic waves, one should also include their sources and boundary conditions (i.e., mirrors) which may induce further factors of $1+\zeta h$. Since these various factors of $1+\zeta h$ stem from different effects, their values of ζ will typically be different and hence they will not cancel each other in general.

In order to accommodate all these different factors of $1+\zeta h$, we employ the standard harmonic approximation $V_0(\boldsymbol{r}) = \boldsymbol{r}\cdot\boldsymbol{M}_0\cdot\boldsymbol{r}$ for the trapping potential V_0 near its minimum (which we set to $\boldsymbol{r}=0$) with some matrix $\boldsymbol{M}_0$. Then, in view of the above considerations, the most general form for the modifications due to the gravitational wave can be cast into the form

$$V(t,\boldsymbol{r}) = \boldsymbol{r}\cdot(\boldsymbol{M}_0 + h\boldsymbol{M}_1)\cdot\boldsymbol{r} + h\boldsymbol{F}_1\cdot\boldsymbol{r} + hV_1. \tag{14}$$

The perturbations $\boldsymbol{M}_1$, $\boldsymbol{F}_1$, and V_1 account for all the factors $1+\zeta h$ mentioned above and thus depend on the amplitudes, polarizations, frequencies and propagation directions of the various laser beams as well as the atomic resonances (and the mirrors etc.).

In addition to the modification $\boldsymbol{M}_1$ of the shape of the potential, which one would naturally expect from a gravitational wave, one can also have a shift in position $\boldsymbol{F}_1$ (in asymmetric scenarios) and in energy V_1. Since the total particle number $\hat{N}$ commutes with the Hamiltonian, the term V_1 has no effect (unless we observe interference between two Bose-Einstein condensates with different V_1).

B. Excitations

In order to study the excitations in the Bose-Einstein condensate induced by the gravitational wave, we employ the standard mean-field approximation $\hat{\Psi} \to \psi_{\mathrm{c}} + \delta\psi$, where ψ_{c} denotes the undisturbed wave function of the condensate (i.e., in the absence of the gravitational wave) while $\delta\psi$ are the perturbations. Linearizing in $\delta\psi$ then yields the Bogoliubov–de Gennes equations which now acquire a source term due to the gravitational wave,

$$\left(i\partial_t + \frac{\boldsymbol{\nabla}^2}{2m} - V_0 - 2g|\psi_{\mathrm{c}}|^2\right)\delta\psi - g\psi_{\mathrm{c}}^2\delta\psi^* = h\left(\frac{\partial_y^2 - \partial_x^2}{2m} + \frac{\partial V}{\partial h}\right)\psi_{\mathrm{c}}. \tag{15}$$

Assuming rotational symmetry for the undisturbed condensate (i.e., for ψ_{c} and V_0), we find that the direct interaction $\propto (\partial_y^2 - \partial_x^2)$ in the first term of the second line in Eq. (15) generates quadrupolar excitations $\delta\psi$, as expected from a gravitational wave. However, the indirect interaction via changes in the trapping potential may also generate other (e.g., dipolar) excitations $\delta\psi$, provided that such contributions (e.g., $\boldsymbol{F}_1$) occur in Eq. (14).

In order to make the connection to fluid dynamics more apparent, we use the Madelung split $\psi_{\mathrm{c}} = \sqrt{\rho}e^{iS}$ in terms of condensate density ρ and phase S where the perturbation $\delta\psi$ is then represented by $\delta\rho$ and δS. In this form, Eq. (15) splits into two real equations,

$$(\partial_t + \boldsymbol{\nabla}\cdot\boldsymbol{v})\delta\rho + \boldsymbol{\nabla}\cdot\left(\frac{\rho}{m}\boldsymbol{\nabla}\delta S\right) = h[\partial_y(\rho v_y) - \partial_x(\rho v_x)] \tag{16}$$

and (reinserting $\hbar$ for the discussion below)

$$(\partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla})\delta S + g\delta\rho + \frac{\hbar^2}{4m}\frac{\delta\rho\boldsymbol{\nabla}^2\sqrt{\rho} - \rho\boldsymbol{\nabla}^2(\delta\rho/\sqrt{\rho})}{\rho^{3/2}} = h\left[\frac{m}{2}(v_y^2 - v_x^2) - \frac{\partial V}{\partial h} + \frac{\hbar^2}{2m}\frac{\partial_x^2\sqrt{\rho} - \partial_y^2\sqrt{\rho}}{\sqrt{\rho}}\right], \tag{17}$$

where $\boldsymbol{v} = \boldsymbol{\nabla}S/m$ is the condensate velocity.

For length scales much larger than the healing length, we may neglect the "quantum-pressure" terms $\propto \hbar^2$ in Eq. (17) such that the two first-order equations above can be combined into one second-order equation,

$$(\partial_t + \boldsymbol{\nabla}\cdot\boldsymbol{v})(\partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla})\delta S - \boldsymbol{\nabla}\cdot\left(\frac{g\rho}{m}\boldsymbol{\nabla}\delta S\right) = \left[\frac{m}{2}(\partial_t + \boldsymbol{\nabla}\cdot\boldsymbol{v})(v_y^2 - v_x^2) - (\partial_t + \boldsymbol{\nabla}\cdot\boldsymbol{v})\frac{\partial V}{\partial h}\right]h + gh[\partial_x(\rho v_x) - \partial_y(\rho v_y)]. \tag{18}$$

As an extremely simple example, we may consider homogeneous condensates at rest for which the above equation simplifies to $(\partial_t^2 - c_{\mathrm{s}}^2\boldsymbol{\nabla}^2)\delta S = \dot{h}\partial V/\partial h$ with the speed of sound $c_{\mathrm{s}}^2 = g\rho/m$. In this case, the generated fluctuations $\delta S = \mathcal{O}(h)$ can be obtained via the well-known retarded Green function of the d'Alembertian. Note, however, that these first-order fluctuations $\delta S = \mathcal{O}(h)$ do not generate a first-order energy shift because the background state is stationary, as explained in Sec. III A.

C. Estimate of energy transfer

Finally, let us exemplify the rigorous bound (9) for a general nonstationary state of a Bose-Einstein condensate. As in Sec. IV, the first factor $\dot{h}$ can be estimated by the typical frequencies $\omega = \mathcal{O}(\mathrm{kHz})$ and amplitudes $h = \mathcal{O}(10^{-22})$ of gravitational waves.

In order to estimate the derivative $\partial V/\partial h$, we may start from the harmonic approximation (14). For optical traps, the order of magnitude of the potential strength is set by the recoil energy $E_{\mathrm{R}} = k^2/(2m)$ which is typically in the μK regime. Here $k = 2\pi/\lambda$ is the momentum of the photons forming the optical trap and m the mass of the trapped atoms (e.g., rubidium). In the absence of further large numbers, one would expect that $\boldsymbol{M}_0$ and $\boldsymbol{M}_1$ in Eq. (14) scale with $\mathcal{O}(E_{\mathrm{R}}/\lambda^2)$ while $\boldsymbol{F}_1 = \mathcal{O}(E_{\mathrm{R}}/\lambda)$ and $V_1 = \mathcal{O}(E_{\mathrm{R}})$. Of course, extending the harmonic approximation (14) to large distances $\boldsymbol{r}$, the derivative $\partial V/\partial h$ would grow formally without any bound. This artifact can be avoided by limiting the maximum distance $\boldsymbol{r}$ to the size of the condensate or the region of applicability of the harmonic approximation (14). Both are set by the optical wavelength $\lambda = \mathcal{O}$ (μm) such that we arrive at $\partial V/\partial h = \mathcal{O}(E_{\mathrm{R}})$.

The remaining term in Eq. (9) is the kinetic energy $\langle\hat{E}_{\mathrm{kin}}\rangle_{\mathrm{max}}$ of the condensate. Obviously, the maximum kinetic energy per atom should not exceed the total potential depth of order $\mathcal{O}(E_{\mathrm{R}})$ in order to stay trapped. Thus, we have $\langle\hat{E}_{\mathrm{kin}}\rangle_{\mathrm{max}} \leq \mathcal{O}(NE_{\mathrm{R}})$, where $N = \langle\hat{N}\rangle$ is the total number of atoms in the condensate.

Altogether we arrive at the following order-of-magnitude estimate for the energy shift:

$$\Delta E \leq \mathcal{O}(h\omega T N E_{\mathrm{R}}), \tag{19}$$

where T is the interaction time (e.g., the duration of the gravitational wave). Inserting a typical amplitude $h = \mathcal{O}(10^{-22})$, a number of cycles $\omega T = \mathcal{O}(10^2)$, a rather large atom number $N = \mathcal{O}(10^9)$, and a characteristic potential strength $E_{\mathrm{R}} = \mathcal{O}$ (μK), we find an energy shift ΔE in the atto-Kelvin regime—which is probably too small to be measurable. Note that this is not the energy shift per particle, but the energy shift for the whole condensate.

Turning this argument (19) around, an energy shift of $\Delta E = \mathcal{O}(10\ \mathrm{nK})$ corresponding to the energy $\hbar\omega$ of a single kHz phonon would require a characteristic potential strength (and energy per atom) of order 10 Kelvin, which is also beyond current experimental capabilities.

VI. CONCLUSIONS

We study the interaction between gravitational waves and quantum matter and find two major coupling mechanisms. First, the gravitational wave encoded in the metric g^{ij} directly affects the kinetic term $(\nabla\hat{\Psi}^\dagger)\cdot(\nabla\hat{\Psi})$ which is replaced by $-(\partial_i\hat{\Psi}^\dagger)g^{ij}(\partial_j\hat{\Psi})$. Second, the gravitational wave may indirectly couple to matter by modifying its trapping potential V (see Appendix A).

As a possible observable, we consider the energy transfer ΔE between the gravitational wave and matter. For stationary initial states, we find that this energy transfer ΔE vanishes to first order in the amplitude h of the gravitational wave. For arbitrary initial states, we derive a general rigorous bound for the energy transfer ΔE in terms of particle number and initial kinetic energy.

As a first example, we discuss a simple toy model in the form of a rotating barbell. For quite moderate rotational energies E_{rot}, the energy transfer ΔE_{rot} can exceed one excitation quantum $\hbar\omega$, but actually measuring this small change on top of a huge background E_{rot} is very challenging. As a possible remedy, one might consider the change of the vibrational energy ΔE_{vib} instead. Demanding that this change E_{vib} exceeds one excitation quantum $\hbar\omega$ then requires a rotational energy E_{rot} which is much larger (assuming a reasonably small initial vibrational energy E_{vib}), but not necessarily out of reach.

As a second example, we apply our findings to Bose-Einstein condensates, where we discuss the gravitationally induced modifications of the trapping potential V for the explicit example of optical traps. Assuming rotational symmetry of the undisturbed condensate, we find that the direct interaction mechanism involving the kinetic term generates quadrupolar excitations (as expected) while the indirect coupling via the potential V may also induce other (e.g., dipolar) excitations—depending on the specific realization of the trap.

Quite generally, inserting typical orders of magnitude of Bose-Einstein condensates into the rigorous bound for the energy transfer ΔE, we find that it is probably far too small to be detectable with present-day technology—at least in the absence of further large numbers which may enhance the signal.

VII. OUTLOOK

As we may infer from the rigorous bound, one way to increase the possible energy transfer ΔE could be to consider other forms of matter such as superfluid helium or ultracold solids containing more particles and thus admitting higher kinetic energies. For example, one could envisage levitating helium droplets or barbells which display quadrupolar vibrations or rotations in resonance with the gravitational wave. In this case, it might be easier to achieve an energy transfer ΔE corresponding to one or more excitation quanta $\hbar\omega$. Of course, detecting such a small change of energy experimentally is another challenge.

Let us provide a rough estimate for the associated orders of magnitude in the case of superfluid helium [42,43]. Inserting a surface tension of $\sigma \approx 3.7\times10^{-4}$ N/m and a density of $\rho \approx 125$ kg/m^3, we find that vibrational modes with frequencies ω_{vib} in the kHz range translate to length scales $(\omega_{\mathrm{vib}}^2\rho/\sigma)^{-1/3}$ in the millimeter regime. Lower frequencies ω_{vib} correspondingly translate to longer length scales (e.g., droplet size). As another way of increasing the length scale, one could envision a helium droplet around a solid core which attracts the surrounding helium film and thus generates an additional restoring force (similar to gravity waves in water). In this way, it should be possible to reach kinetic energies in the $\mathcal{O}(10^{-11}$ J) range required for transferring one excitation quantum $\hbar\omega$ from the gravitational wave to the helium droplet, see Sec. IV. However, as also discussed in Sec. IV, because measuring this single excitation quantum $\hbar\omega$ on top of the huge background is challenging, one could try to have a larger energy of $\mathcal{O}(10^8$ J) in one mode (e.g., rotation) in order to obtain the transfer of one excitation quantum $\hbar\omega$ into another mode (e.g., vibration). Inserting the speed of (first) sound of $c_s \approx 240$ m/s, this other mode could be a sound mode of helium in a container with a size in the meter range. Inserting the above density of $\rho \approx 125$ kg/m^3, such a container could hold enough helium to reach the required rotational energy of $\mathcal{O}(10^8$ J), see Sec. IV. One possible realization could then be a rotating barbell with two large helium containers at its two ends. Nevertheless, even though the orders of magnitude match quite nicely, it is clear that such an experiment would still be very challenging.

Going a bit further, let us discuss these scenarios in some more detail. If the levitating helium droplets or barbells display quadrupolar vibrations or rotations in resonance with the gravitational wave, the sign of their energy shift ΔE depends on the relative phase between the gravitational wave and the quadrupolar vibration or rotation. If they are in phase, the energy increases $\Delta E > 0$ but if they are out of phase (by a phase shift of $\Delta\varphi = \pi$), the energy decreases $\Delta E < 0$. In analogy to photons as quanta of electromagnetic waves, we may use the picture of gravitons as quanta of gravitational waves. Then, the first case corresponds to the absorption of gravitons, while the second scenario describes the stimulated emission of gravitons. Such a stimulated emission scenario may be our best chance to actually emit gravitons in a controlled earth-bound experiment—but it would still be a challenging experiment. However, it would mark the important step from merely observing a natural phenomenon to actually manipulating it. Of course, detecting the gravitons emitted in this way would then be yet another challenge.

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APPENDIX A: ATOMIC EIGENSTATES

For the sake of completeness and as another illustration for the impact of gravitational waves, let us investigate the induced modifications of the atomic eigenstates. For simplicity, let us start with the nonrelativistic hydrogen atom as described by the undisturbed Hamiltonian,

$$\hat{H}_0 = \frac{\hat{\boldsymbol{p}}^2}{2m} + V(\hat{r}), \tag{A1}$$

where $V(\hat{r}) = -q^2/(4\pi\hat{r})$ denotes the Coulomb potential. Then, in complete analogy to the Hamiltonian (6), the impact of the gravitational wave can be encoded in the perturbation Hamiltonian [44,45],

$$\hat{H}_1 = h\left[\frac{\hat{p}_y^2 - \hat{p}_x^2}{2m} + q^2\frac{\hat{x}^2 - \hat{y}^2}{8\pi\hat{r}^3}\right] = \hat{H}_1^{\text{kin}} + \hat{H}_1^{\text{pot}}. \tag{A2}$$

The deformation of the Coulomb potential can be derived via replacing the flat space-time Laplace operator $\boldsymbol{\nabla}^2$ in the Poisson equation for $V(\boldsymbol{r})$ by the Laplace-Beltrami operator $-\partial_i g^{ij}\partial_j$, see also [47].

Since h is slowly varying in comparison to the atomic frequencies, we may estimate the lowest-order variations of the eigenstates via stationary perturbation theory. The first-order shift of the eigenenergies is determined by the expectation values of the perturbation Hamiltonian (A2) in the undisturbed eigenstates. Obviously, the expectation value in the 1 s ground state vanishes in view of rotational invariance $\langle 1\text{s}|\hat{H}_1|1\text{s}\rangle = 0$. More generally, matrix elements $\langle n,\ell,m|\hat{H}_1|n',\ell',m'\rangle$ can only yield nonvanishing contributions if $m' = m \pm 2$. Thus, one might expect an energy shift for p_x orbitals, for example, see also [46]. Indeed, the expectation value of $\hat{H}_1^{\text{kin}}$ corresponds to the difference between the average kinetic energies in the x and the y direction and yields a nonzero result,

$$\langle 2\text{p}_x|\hat{H}_1^{\text{kin}}|2\text{p}_x\rangle = -h\frac{q^2}{80\pi a_{\text{B}}}, \tag{A3}$$

where a_{B} is the Bohr radius. Apart from the small prefactor h, this energy shift is in the eV regime and thus one might expect it to be measurable. However, one should not forget the second contribution $\hat{H}_1^{\text{pot}}$. Calculating its expectation value $\langle 2\text{p}_x|\hat{H}_1^{\text{pot}}|2\text{p}_x\rangle$, one finds that it precisely cancels the above contribution (A3) leading to a vanishing energy shift $\langle 2\text{p}_x|\hat{H}_1|2\text{p}_x\rangle = 0$ to lowest order, consistent with the results of [48–51].

This cancellation is perhaps not too surprising because a constant h can be interpreted as a trivial change of coordinates $x \to [1 + h/2]x$ and $y \to [1 - h/2]y$, which should not affect any physical quantities such as energies. However, such a change of coordinates is consistent with modifications of the wave functions and thus nondiagonal matrix elements can be nonvanishing, such as

$$\langle 1\text{s}|\hat{H}_1|3\text{d}_{x^2-y^2}\rangle = h\frac{q^2}{128\pi a_{\text{B}}}, \tag{A4}$$

see also [52,53]. As a consequence, transition matrix elements could also change (in those coordinates).

In view of the above argument based on coordinate independence (i.e., general covariance), the cancellation of the energy shifts to lowest order in h should remain valid in the general case. As an example, let us briefly discuss the Dirac equation. To lowest order in h, the metric (1) can be incorporated by a modification of the Dirac matrices $\gamma^x \to [1 - h/2]\gamma^x$ and $\gamma^y \to [1 + h/2]\gamma^y$ while γ^z and γ^t remain unchanged. Again using that h is slowly varying, we may neglect the Fock-Ivanenko (spin connection) coefficients because they scale with the derivative $\dot{h} = \mathcal{O}(\omega h)$ and are thus suppressed for small ω. As a consequence, the Dirac perturbation Hamiltonian has a structure very similar to the Schrödinger case (A2). The potential part $\hat{H}_1^{\text{pot}}$ stemming from the deformation of the Coulomb potential is basically the same, while the kinetic part $\hat{H}_1^{\text{kin}}$ reads $h[\alpha^y i\partial_y - \alpha^x i\partial_x]/2$, where the $\alpha^i = \gamma^0\gamma^i$ are the velocity matrices in the Dirac representation—in analogy to the Schrödinger case (A2).

APPENDIX B: ROTATING FRAME

For studying rotating matter distributions such as the barbell, it is often useful to transform into the rotating frame. Assuming that potential V_0 and interaction W are isotropic, the Hamiltonian (6) in the rotating frame reads

$$\begin{aligned}\hat{H}_0^{\text{rot}} = \int d^3r\bigg[&\frac{1}{2m}\Big(\boldsymbol{\nabla}\hat{\Psi}^\dagger - im[\boldsymbol{\omega}_{\text{rot}}\times\boldsymbol{r}]\hat{\Psi}^\dagger\Big)\\ &\cdot\Big(\boldsymbol{\nabla}\hat{\Psi} + im[\boldsymbol{\omega}_{\text{rot}}\times\boldsymbol{r}]\hat{\Psi}\Big) + V_0(\boldsymbol{r})\hat{\Psi}^\dagger\hat{\Psi}\\ &-\frac{m}{2}(\boldsymbol{\omega}_{\text{rot}}\times\boldsymbol{r})^2\hat{\Psi}^\dagger\hat{\Psi}\bigg]\\ &+\frac{1}{2}\int d^3r d^3r'\hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}^\dagger(\boldsymbol{r}')W(\boldsymbol{r},\boldsymbol{r}')\hat{\Psi}(\boldsymbol{r}')\hat{\Psi}(\boldsymbol{r}).\end{aligned} \tag{B1}$$

While the interaction term in the last line does not change (due to the assumed isotropy), the first three lines now contain the kinetic energy in the rotating frame $E_{\rm kin}^{\rm rot}$ plus the effective potential $V_{\rm eff}(\boldsymbol{r}) = V_0(\boldsymbol{r}) - m(\boldsymbol{\omega}_{\rm rot} \times \boldsymbol{r})^2/2$. In order to ensure stability, we assume that V_0 is stronger than the centrifugal potential $m(\boldsymbol{\omega}_{\rm rot} \times \boldsymbol{r})^2/2$.

Quite importantly, the effective Hamiltonian (B1) is independent of time. This allows us to prepare an initial state which is stationary or even static in the rotating frame. Furthermore, the well-known analogy to charged particles in a magnetic field described by the effective vector potential $\boldsymbol{A}_{\rm eff} \propto \boldsymbol{\omega}_{\rm rot} \times \boldsymbol{r}$ enables us to transfer many of the concepts to the rotating case. For example, the conserved current contains an additional term from $\boldsymbol{A}_{\rm eff}$:

$$\hat{\boldsymbol{j}}_{\rm rot} = \frac{1}{2mi}\left[\hat{\Psi}^\dagger \boldsymbol{\nabla}\hat{\Psi} - {\rm H.c.}\right] + (\boldsymbol{\omega}_{\rm rot} \times \boldsymbol{r})\hat{\Psi}^\dagger\hat{\Psi}. \tag{B2}$$

Now let us study the impact of the gravitational wave (1). Assuming a rotation around the z axis for simplicity with an angle of $\varphi(t) = \omega_{\rm rot} t$, the induced interaction Hamiltonian becomes

$$\hat{H}_{\rm int} = h(t)\int {\rm d}^3 r\left[\cos(2\omega_{\rm rot}t)\frac{(\partial_y\hat{\Psi}^\dagger)(\partial_y\hat{\Psi}) - (\partial_x\hat{\Psi}^\dagger)(\partial_x\hat{\Psi})}{2m} - \sin(2\omega_{\rm rot}t)\frac{(\partial_x\hat{\Psi}^\dagger)(\partial_y\hat{\Psi}) + (\partial_y\hat{\Psi}^\dagger)(\partial_x\hat{\Psi})}{2m}\right], \tag{B3}$$

where we have again omitted the changes of V_0 and W induced by the gravitational wave.

Now let us estimate the energy transfer in analogy to Sec. III A. As an important difference to that section, the terms such as $(\partial_y\hat{\Psi}^\dagger)(\partial_y\hat{\Psi})$ can no longer be directly bound by the kinetic energy $E_{\rm kin}^{\rm rot}$ which now contains more contributions and is given in the first two lines of Eq. (B1). In order to place a bound on these additional terms we assume that the initial (unperturbed) state is static in the rotating frame which implies $\langle\hat{\boldsymbol{j}}_{\rm rot}\rangle_0 = 0$. Using this assumption and the Cauchy-Schwarz inequality, we finally arrive at

$$\dot{E} \le |\dot{h}|_{\rm max}\left(2E_{\rm kin}^{\rm rot} + m\int {\rm d}^3 r\,(\boldsymbol{\omega}_{\rm rot} \times \boldsymbol{r})^2\langle\hat{\Psi}^\dagger\hat{\Psi}\rangle_0\right) + \mathcal{O}(h^2). \tag{B4}$$

Quite intuitively, apart from the kinetic energy within the rotating frame, we also obtain a contribution from the rotation itself—which can be bound by the total particle number and the maximum spatial extent of the matter distribution (e.g., barbell).

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