Constraining supermassive primordial black holes with magnetically induced gravitational waves

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Primordial black holes (PBHs) can answer a plethora of cosmic conundrums, including the origin of cosmic magnetic fields. In particular, supermassive PBHs with masses $M_{\rm PBH} > 10^{10} M_\odot$ and furnished with a plasma disk moving around them can generate through the Biermann battery mechanism a seed primordial magnetic field that can later be amplified to provide the magnetic field threading the intergalactic medium. In this article, we derive the gravitational-wave (GW) signal induced by the magnetic anisotropy of such a population of magnetized PBHs. Interestingly enough, by using GW constraints from big bang nucleosynthesis and an effective model for the galactic/turbulent dynamo amplification of the magnetic field, we set conservative upper bound constraints on the abundances of supermassive PBHs at formation time, $\Omega_{\rm PBH,f}$ as a function of their masses, namely, that $\Omega_{\rm PBH,f} \leq 2.5 \times 10^{-10} \left(\frac{M}{10^{10} M_\odot}\right)^{45/22}$. Remarkably, these constraints are comparable and, in some mass ranges, even tighter compared to the constraints on $\Omega_{\rm PBH,f}$ from large-scale structure probes, hence promoting the portal of magnetically induced GWs as a new probe to explore the enigmatic nature of supermassive PBHs.

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I. INTRODUCTION

The origin of the primordial magnetic fields (MFs) threading the intergalactic medium constitutes one of the longstanding issues in cosmology. These cosmic MFs can play a crucial role in the processes of particle acceleration through the intergalactic medium [1] and the propagation of cosmic rays [2] while at the same time significantly affecting the Universe's thermal state between inflation and recombination [3–5].

Among their generation mechanisms, there have been proposed processes related to phase transitions in the early Universe [6,7], primordial scalar [8–10] and vector perturbations [11,12], and astrophysical ones seeding battery-induced MFs [13]. In particular, in the last years there has been a rekindled interest in connecting the origin of primordial MFs with primordial black holes (PBHs) [14–16]. As was recently shown in [16], supermassive PBHs furnished with a disk can generate through the Biermann-battery mechanism the seed for the primordial MFs of 10^{-18} G observed on intergalactic scales [17].

Interestingly enough, PBHs, first introduced in the 1970s, can address many modern cosmological enigmas. In particular, they can naturally account for a fraction or even the totality of dark matter [18,19], explaining as well large-scale structure formation through Poisson fluctuations [20–22] and providing the seeds of the supermassive black holes residing in galactic centers [23–26]. At the same time, they are associated with numerous gravitational-wave (GW) signals related to PBH merger events [27–29], Hawking radiation [30–32], and enhanced cosmological [33–35] adiabatic and isocurvature perturbations [36–39]. For recent reviews, see [40,41].

In this article, we derive the GW signal induced by the magnetic anisotropic stress of a population of magnetized supermassive PBHs. Accounting as well for GW constraints from big bang nucleosynthesis (BBN), we are able to set tight constraints on the abundances of supermassive PBHs, promoting in this way magnetically induced GWs (MIGWs) as a novel portal to shed light on the field of PBH physics.

II. SEED PRIMORDIAL MAGNETIC FIELD À LA BIERMANN

PBH accretion disks were recently proposed as a candidate to generate the seed primordial MFs threading the intergalactic medium [14–16]. In particular, the *ab initio* generation of a seed MF requires the relative motion between negative and positive charges, the conditions for which can be achieved in a highly turbulent medium such as the primordial plasma between BBN and recombination [42–45]. Under such conditions, a Biermann-battery mechanism operates whenever the energy density and temperature gradients are not parallel to each other [46].

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Consequently, one is inevitably met with the following magnetic field induction equation:

$$\partial_t \boldsymbol{B} = \nabla(\boldsymbol{u}\times\boldsymbol{B}) - \frac{ck_B}{e}\frac{\nabla\rho\times\nabla T}{\rho}, \quad (1)$$

where the second term on the right-hand side is the Biermann-battery one.

In order to derive the Biermann-battery-induced MF one should assume an equation of state for the vortex-like moving plasma around the black hole. Doing so, we assume a locally isothermal disk around the PBH [47], where the density and the pressure are related through the following relation:

$$p(R,\phi,z) = \rho(R,\phi,z)c_s^2(R), \quad (2)$$

where (R,ϕ,z) are the cylindrical coordinates. This equation of state can describe quite well a gas that radiates internal energy gained by shocks [48], here produced by the turbulent motion of the primordial plasma expected after BBN and before the recombination era [42–45].

In the end, accounting for the random spatial distribution of PBHs and considering monochromatic PBH mass distributions, after a long but straightforward calculation (for more details see [16]) one can extract the MF power spectrum, which can be recast as[1]

$$P_B(k,t_s) \simeq 4\times 10^{-86} q^2 \ell_R^4 \Omega_{\rm PBH,f}^2 \times \left(\frac{M}{10^{10}M_\odot}\right)^2 \left(\frac{k}{\rm Mpc^{-1}}\right)^3 \ [\mathrm{G^2\,Mpc^3}], \quad (3)$$

where $\ell_R = R_{\rm d}/R_{\rm ISCO}$ is the ratio of the radius of the disk, $R_{\rm d}$, and the radius of the innermost stable circular orbit, $R_{\rm ISCO}$, and $q = H_{\rm d}/R_{\rm ISCO}$ is the ratio of the thickness of the disk $H_{\rm d}$ and $R_{\rm ISCO}$ which is less than 1 since Eq. (5) was extracted within the thin-disk limit where one is usually met with sub-Eddington accretion [50–52]. It is important to notice that the above-mentioned MF power spectrum was extracted at saturation time t_s, namely, at the end of the linear growth phase of the MF [see the Biermann-battery term in Eq. (1)] and for scales larger than the PBH mean separation scale, so as not to enter the nonlinear regime. This imposes a UV-cutoff scale $k_{\rm UV}$ which can be recast straightforwardly as $k_{\rm UV} = 10^{19}M_\odot\Omega_{\rm PBH,f}^{1/3}\ \mathrm{Mpc}^{-1}/M$ [16].

One can also derive the mean MF amplitude, which is defined as

$$\langle|\boldsymbol{B}_k|\rangle \equiv \sqrt{\frac{k^3 P_B(k)}{2\pi^2}}. \quad (4)$$

Accounting for cosmic expansion, i.e., $B\sim a^{-2}$ and plugging Eq. (3) into Eq. (4), one gets the mean MF amplitude on intergalactic scales, i.e., $k\sim 100\ \mathrm{Mpc}^{-1}$, which reads as

$$B\sim 10^{-30} q \left(\frac{\ell_{\rm R}}{10^6}\right)^2 \left(\frac{M_{\rm PBH}}{10^{14}M_\odot}\right)^{5/2}\ (\mathrm{G}). \quad (5)$$

Interestingly enough, by taking typical values of $q\sim$ 0.001–1 and varying the parameter ℓ_R within the range $\ell_R\in[10^2,10^{11}]$ depending on the accretion rate [53], one can produce for PBH masses $M\in[10^{10},10^{16}]M_\odot$[2] a seed primordial MF of the order of 10^{-32}–10^{-29} G, which is actually the minimum seed MF amplitude needed to give rise, through dynamo/turbulent amplification, to the present-day average magnetic field of order 10^{-18} G on intergalactic scales [54].

III. MAGNETIC FIELD ANISOTROPIC STRESS

Let us now extract the magnetic anisotropic stress induced by such an MF power spectrum. In particular, regarding the stress-energy tensor associated with a magnetic field $\boldsymbol{B}$, this can be recast in the following covariant form:

$$T_{ij}^{(B)} \equiv \frac{1}{4\pi}\left[\frac{B^2 g_{ij}}{2} - B_i B_j\right]. \quad (6)$$

From Eq. (6), one can define an associated anisotropic stress as follows:

$$\Pi_{ij}(\boldsymbol{k}) \equiv \left(P_i^l P_j^m - \frac{P_{ij}P^{lm}}{2}\right) T_{lm}(\boldsymbol{k}), \quad (7)$$

where P_{ij} is a projection matrix defined as $P_{ij}\equiv\delta_{ij}-\hat{\boldsymbol{k}}_i\hat{\boldsymbol{k}}_j$ and $\hat{\boldsymbol{k}} = \boldsymbol{k}/k$. From Eq. (7) one can define the equal-time two-point correlator of the magnetic anisotropic stress as

$$\langle\Pi_{ij}(\boldsymbol{k},\eta)\Pi_{ij}(\boldsymbol{q},\eta)\rangle \equiv \Pi_B(k,\eta)\delta(\boldsymbol{k},\boldsymbol{q}), \quad (8)$$

where $\Pi_B(\boldsymbol{k},\eta)$ is the power spectrum of the magnetic anisotropic stress related to the magnetic field power spectrum $P_B(k,\eta)$ as follows [55]:

[1]To extract Eq. (3) we followed the prescription described in the Appendix of [16] considering that the PBH mass is of the order of the mass within the cosmological horizon at the time of PBH formation [49].

[2]We need to point out here that in order to generate the seed primordial MF necessary to give rise to an MF amplitude of the order 10^{-18} G, threading the intergalactic medium, through the Biermann-battery mechanism, one needs to consider PBH masses higher than $10^{10}M_\odot$, as it was shown in [16].

$$\Pi_B(k,\eta) = \int \mathrm{d}^3\boldsymbol{q} P_B(q,\eta) P_B(|\boldsymbol{q}-\boldsymbol{k}|,\eta)(1+\gamma^2)(1+\beta^2), \tag{9}$$

where $\gamma = \hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{q}}$ and $\beta = \hat{\boldsymbol{k}}\cdot\widehat{\boldsymbol{k}-\boldsymbol{q}}$.

Introducing now the auxiliary variables v and u such that $u = |\boldsymbol{q}-\boldsymbol{k}|/k$ and $v = q/k$, after some algebraic manipulations one can recast the above equation in the following form:

$$\Pi_B(k,\eta) = 2\pi k^3 \int_0^{\frac{k_{\mathrm{UV}}}{k}} \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u P_B(kv,\eta) P_B(ku,\eta) uv \times \left[1+\frac{(1+v^2-u^2)^2}{4v^2}\right] \times \left[1+\left(1-\frac{1+v^2-u^2}{2v}\right)^2\right]. \tag{10}$$

Finally, by plugging Eq. (3) into Eq. (9) one gets that

$$\Pi_B(k,\eta) \simeq 10^{-170} q^4 \ell_R^8 \Omega_{\mathrm{PBH,f}}^4 10^{-174}\left(\frac{k}{\mathrm{Mpc}^{-1}}\right)^9 \times \left(\frac{M}{10^{10}M_\odot}\right)^4 f\left(\frac{k_{\mathrm{UV}}}{k}\right) \mathrm{G}^4\,\mathrm{Mpc}^3, \tag{11}$$

where $f(\frac{k_{\mathrm{UV}}}{k})$ is the double integral over u and v which is defined as follows:

$$f\left(\frac{k_{\mathrm{UV}}}{k}\right) \equiv \int_0^{\frac{k_{\mathrm{UV}}}{k}} \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u u^4 v^4 \left[1+\frac{(1+v^2-u^2)^2}{4v^2}\right] \times \left[1+\left(1-\frac{1+v^2-u^2}{2v}\right)^2\right]. \tag{12}$$

After performing the integration one can show that

$$f\left(y \equiv \frac{k_{\mathrm{UV}}}{k}\right) = \frac{32}{45}y^9 - \frac{131y^8}{60} + \frac{293y^7}{98} - \frac{3323y^6}{1680} + \frac{27229y^5}{50400} - \frac{2y^4}{105} + \frac{26y^3}{3465} - \frac{2y^2}{1155} + \frac{8y}{15015} - \frac{19711}{6306300}. \tag{13}$$

In the region away from the UV-cutoff scale k_{UV}, namely, when $k_{\mathrm{UV}}/k \gg 1$, one obtains that $f(y \equiv \frac{k_{\mathrm{UV}}}{k}) \simeq (32/45)(k_{\mathrm{UV}}/k)^9$.

IV. MAGNETICALLY INDUCED GRAVITATIONAL WAVES

Having extracted the power spectrum of the magnetic anisotropic stress, here we study the dynamics of the tensor perturbations $h_{\boldsymbol{k}}$ induced by such an anisotropic stress. In particular, the equation of motion for $h_{\boldsymbol{k}}$ can be recast as [55]

$$h_{\boldsymbol{k}}^{s,\prime\prime} + 2\mathcal{H}h_{\boldsymbol{k}}^{s,\prime} + k^2 h_{\boldsymbol{k}}^s = \frac{8\pi G}{a^2}\sqrt{\Pi_B(k,\eta)}, \tag{14}$$

where $s = (+), (\times)$ stands for the two polarization states of the tensor modes in general relativity.

An analytic solution to Eq. (14) can be obtained by using the Green's function formalism. Namely, one gets that

$$h_{\boldsymbol{k}}^s(\eta) = 8\pi G \int_{\eta_0}^{\eta} \mathrm{d}\bar{\eta}\, g_{\boldsymbol{k}}(\eta,\bar{\eta}) a^2(\bar{\eta}) \frac{\sqrt{\Pi_B(k,\bar{\eta})}}{a^2(\bar{\eta})}, \tag{15}$$

where $g_{\boldsymbol{k}}(\eta,\bar{\eta})$ is the Green's function that is the solution of the homogeneous equation (14) without the source term (see [55] for more details). For the case of a radiation-dominated (RD) era, $w = 1/3$, when PBHs typically form, one gets that

$$k g_{\boldsymbol{k}}^{\mathrm{RD}}(\eta,\bar{\eta}) = \sin(x-\bar{x}). \tag{16}$$

One then can define the GW spectral abundance as $\Omega_{\mathrm{GW}}(\eta,k) \equiv \frac{1}{\rho_{\mathrm{c}}}\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}\ln k}$, where $\rho_{\mathrm{c}} = 3H^2/(8\pi G)$ is the critical energy density, and show that it can be recast as [35,56]

$$\Omega_{\mathrm{GW}}(\eta,k) = \frac{1}{24}\left[\frac{k}{\mathcal{H}(\eta)}\right]^2 \bar{\mathcal{P}}_h(\eta,k), \tag{17}$$

where $\mathcal{H}$ is the conformal Hubble parameter and $\mathcal{P}_h(\eta,k)$ is the tensor power spectrum defined as

$$\mathcal{P}_h(\eta,k) \equiv \frac{k^3|h_{\boldsymbol{k}}|^2}{2\pi^2}. \tag{18}$$

The bar denotes averaging over the subhorizon oscillations of the tensor field, which is done in order to extract the envelope of the GW spectrum at those scales.

Combining Eqs. (11), (15), (16), (18), and (17), one obtains that the GW spectral abundance is

$$\Omega_{\mathrm{GW}}(\eta,k) = 10^{-70} I^2(x)\left(\frac{k}{\mathrm{Mpc}^{-1}}\right) \times \left(\frac{10^{10}M_\odot}{M}\right)^4 \ell_R^8 \Omega_{\mathrm{PBH,f}}^7, \tag{19}$$

where $x = k\eta$ and $I(x)$ is defined as

$$I(x) \equiv \int_{x_{\mathrm{dyn}}}^{x} \mathrm{d}\bar{x} \frac{\sin(x-\bar{x})}{\bar{x}^2}\left[1-\frac{x_{\mathrm{dyn}}^2}{\bar{x}^2}\right]^2, \tag{20}$$

where η_{dyn} stands for the conformal disk dynamical time and $x_{\mathrm{dyn}} = k\eta_{\mathrm{dyn}} \simeq 1$ since the disk very quickly establishes its hydrostatic equilibrium on the vertical axis soon after the PBH formation time, which is standardly considered as the time when the typical size of the collapsing overdensity region $r \sim 1/k$ crosses the cosmological horizon, i.e., when $k = aH$. Thus, one has that $\eta_{\mathrm{dyn}} \simeq \eta_{\mathrm{f}}$ and that $x_{\mathrm{dyn}} = k\eta_{\mathrm{dyn}} \simeq k\eta_{\mathrm{f}} = k/(a_{\mathrm{f}}H_{\mathrm{f}}) = 1$.

V. NONAMPLIFIED MAGNETICALLY INDUCED GRAVITATIONAL WAVES

In what follows, we account for the contribution of the magnetic anisotropic stress during the time interval where the Biermann-battery mechanism operates within its linear growth regime up to $t = t_{\mathrm{s}}$, hence underestimating the GW signal and considering that the tensor modes propagate as free GWs up to our present day soon after the end of the era of the linear growth of $\boldsymbol{B}$ at $t = t_{\mathrm{s}}$.

Thus, for $x = x_{\mathrm{s}} = \sqrt{2}x_{\mathrm{dyn}}$ [16] one can check numerically that $I^2(x_{\mathrm{s}}) \simeq 10^{-5}$. Accounting for the fact that $a^2H^2 \propto a^{-2}$ in the RD era, one gets that

$$\Omega_{\mathrm{GW}}(\eta_{\mathrm{s}}, k) = 10^{-75}\left(\frac{k}{\mathrm{Mpc}^{-1}}\right) \times \left(\frac{10^{10}M_{\odot}}{M}\right)^{4}\ell_R^8\Omega_{\mathrm{PBH,f}}^7. \tag{21}$$

One can then compute the GW spectral abundance $\Omega_{\mathrm{GW}}(\eta, k)$ during our present epoch as follows:

$$\Omega_{\mathrm{GW}}(\eta_0, k) = \frac{\rho_{\mathrm{GW}}(\eta_0, k)}{\rho_{\mathrm{c}}(\eta_0)} = \frac{\rho_{\mathrm{GW}}(\eta_{\mathrm{s}}, k)}{\rho_{\mathrm{c}}(\eta_{\mathrm{s}})}\left(\frac{a_{\mathrm{s}}}{a_0}\right)^4\frac{\rho_{\mathrm{c}}(\eta_{\mathrm{s}})}{\rho_{\mathrm{c}}(\eta_0)} = c_g\Omega_{\mathrm{r}}^{(0)}\Omega_{\mathrm{GW}}(\eta_{\mathrm{s}}, k), \tag{22}$$

where $c_g = \frac{\rho_{\mathrm{r,s}}a_{\mathrm{s}}^4}{\rho_{\mathrm{r,0}}a_0^4} \simeq 0.4$ [57], $\Omega_{\mathrm{r}}^{(0)} \sim 10^{-5}$, and we have also taken into account that $\Omega_{\mathrm{GW}} \sim a^{-4}$. The index 0 refers to the present time. Finally, $\Omega_{\mathrm{GW}}(\eta_0, k)$ will be recast as

$$\Omega_{\mathrm{GW}}(\eta_0, k) \simeq 3\times 10^{-81}\left(\frac{k}{\mathrm{Mpc}^{-1}}\right)\left(\frac{10^{10}M_{\odot}}{M}\right)^4 q^4\ell_R^8\Omega_{\mathrm{PBH,f}}^7. \tag{23}$$

The GW frequency will be given by $f = k/(2\pi a_0)$, where we conventionally take $a_0 = 1$. Thus, since $k \leq k_{\mathrm{UV}}$, one can extract an upper bound constraint on the GW frequency, namely, that

$$f \leq f_{\max} = \frac{k_{\mathrm{UV}}}{2\pi} = 10^5\frac{M_{\odot}}{M}\Omega_{\mathrm{PBH,f}}^{1/3} \leq 3\times 10^{-7}\left(\frac{10^{10}M_{\odot}}{M}\right)^{5/6} \leq 3\times 10^{-7}\ (\mathrm{Hz}), \tag{24}$$

since $M > 10^{10}M_{\odot}$ and $\Omega_{\mathrm{PBH,f}} < 2.6\times 10^{-5}\sqrt{\frac{M}{10^{10}M_{\odot}}}$ so as not to overproduce PBHs at matter-radiation equality. Therefore, the relevant MIGW signal is far away from the frequency bands of the Laser Interferometer Space Antenna (LISA) [58,59], the Einstein Telescope (ET) [60], and the Big Bang Observer (BBO) [61] GW detectors. Potentially, it can be well within the Square Kilometer Array [62], NANOGrav [63], and pulsar timing array [64,65] frequency detection bands. However, for smaller-mass PBHs furnished with a disk, which however will not be able to seed primordial MFs [16], the GW frequency will increase and can be well within the LISA, ET, and BBO frequency detection bands.

Apart from the GW frequency, one should also check the GW amplitude (23). In particular, considering that $k \leq k_{\mathrm{UV}}$, $\ell_R \leq 10^{11}$ and accounting for Eq. (23) and the fact that for $M > 10^{10}M_{\odot}$, $\Omega_{\mathrm{PBH,f}} < 2.6\times 10^{-5}\left(\frac{M}{10^{10}M_{\odot}}\right)^{1/2}$, one gets an upper bound on $\Omega_{\mathrm{GW}}(\eta_0)$ that reads as

$$\Omega_{\mathrm{GW}}(\eta_0) < 7\times 10^{-18}\left(\frac{10^{10}M_{\odot}}{M}\right)^{4/3} \leq 7\times 10^{-18}, \tag{25}$$

which is slightly below the lowest GW sensitivity of current and future GW detectors, being of the order of 10^{-15}. Consequently, these magnetically induced GWs can hardly be detected by current/future GW detectors.

VI. AMPLIFIED MAGNETICALLY INDUCED GRAVITATIONAL WAVES

However, up to now, we have not accounted for the turbulent and galactic dynamo or the magnetorotational instability [66–68] amplification which will play significant roles after matter-radiation equality during the nonlinear growth of the matter perturbations. These amplification mechanisms can significantly enhance the MF amplitude and ultimately the GW signal, potentially putting it within the sensitivity bands of current/future GW detectors. Therefore, to account for these effects we introduce the amplification factor $\alpha(k)$ as the ratio between the amplified MF and the nonamplified MF, i.e.,

$$\alpha(k) = \frac{B^{\mathrm{amplified}}(k)}{B^{\mathrm{nonamplified}}(k)}. \tag{26}$$

To extract the function $\alpha(k)$ over the different scales k at hand, one should run high-cost numerical magnetohydrodynamics simulations to account for the various turbulent/galactic dynamo and instability processes, which is beyond the scope of this work. Thus, in order to make quantitative predictions for the GW signal, we assume an effective power-law toy model for $\alpha(k)$ which can be recast as

$$\alpha(k) = \alpha(k_*)\left(\frac{k}{k_*}\right)^{n_B}, \tag{27}$$

where k_* is a pivot scale and n_B is the amplification spectral index which should be greater than or equal to zero since on small scales one expects to have a greater MF amplification.[3] In what follows, we consider as our pivot scale the characteristic intergalactic scale $k_* = 100\ \mathrm{Mpc}^{-1}$ where we know from observations that $B \sim 10^{-18}$ G [17], and thus $\alpha(k_*)$ will read as

$$\alpha(k_* = 100\ \mathrm{Mpc}^{-1}) = \frac{10^{-18}}{10^{-30}\ \mathrm{G}q(\frac{\ell_R}{10^6})^2(\frac{M_{\mathrm{PBH}}}{10^{14}M_\odot})^{5/2}} = \frac{10^{22}}{q}\left(\frac{10^6}{\ell_R}\right)^2\left(\frac{10^{10}M_\odot}{M}\right)^{5/2}, \quad (28)$$

where we have used Eq. (5) for the nonamplified MF amplitude on intergalactic scales.

This amplification effect will give an extra $a^2(k)$ factor at the level of the MF power spectrum $P_B(k)$ since $P_B(k) \propto B_k^2$. Finally, one is met with a rough overall $a^4(k)$ amplification factor at the level of the GW signal since $\Omega_{\mathrm{GW}} \propto \int\int P_B^2$, as we can see from Eqs. (15), (10), and (17). Thus, multiplying Eq. (23) by $\alpha^4(k)$ gives

$$\Omega_{\mathrm{GW}}(k,\eta_0) \simeq 3\times 10^{55-8n_B}\left(\frac{k}{\mathrm{Mpc}^{-1}}\right)^{4n_B+1} \times \left(\frac{10^{10}M_\odot}{M}\right)^{14}\Omega^7_{\mathrm{PBH,f}}. \quad (29)$$

VII. CONSTRAINTS ON SUPERMASSIVE PRIMORDIAL BLACK HOLES

Now let us see how one can constrain the abundances of such supermassive PBHs using the aforementioned GW portal. Interestingly enough, one can set an upper bound on $\Omega_{\mathrm{PBH,f}}$ by accounting for the contribution of the GWs to the effective number of extra neutrino species ΔN_{eff}. In particular, one can translate the upper bound from *Planck* for the amplitude of GWs today, namely, that $\Omega_{\mathrm{GW}}(\eta_0) \leq 10^{-6}$ [74,75], to an upper bound on $\Omega_{\mathrm{PBH,f}}$.

As we see from Eq. (29), for $k \geq 100\ \mathrm{Mpc}^{-1}$ the GW amplitude increases with n_B. Thus, in order to get a conservative constraint on $\Omega_{\mathrm{PBH,f}}$ we choose the flat case where $n_B = 0$. Finally, by using Eq. (29) for $n_B = 0$ and setting $k = k_{\mathrm{UV}}$, since at $k \sim k_{\mathrm{UV}}$ one gets the maximum GW amplitude (see Fig. 1), we obtain a conservative upper bound constraint on $\Omega_{\mathrm{PBH,f}}$ by just simply requiring that $\Omega_{\mathrm{GW}}(\eta_0) \leq 10^{-6}$, which reads as[4]

$$\Omega_{\mathrm{PBH,f}} \leq 2.5\times 10^{-10}\left(\frac{M}{10^{10}M_\odot}\right)^{45/22}. \quad (30)$$

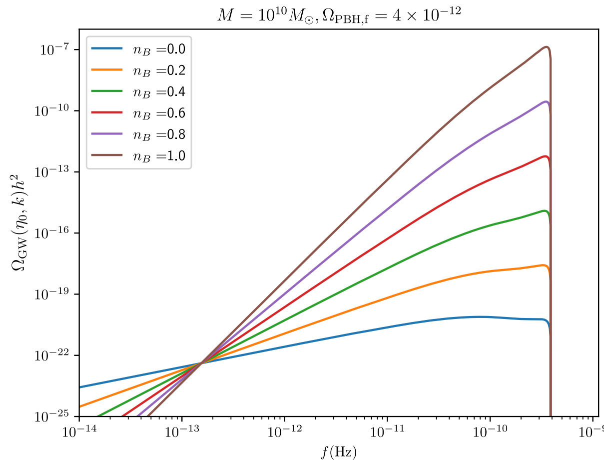

FIG. 1. GW spectrum induced by a population of magnetized PBHs with mass $M = 10^{10}M_\odot$ and initial PBH abundance at formation time $\Omega_{\mathrm{PBH,f}} = 4\times 10^{-12}$ for different values of the amplification spectral index n_B.

Remarkably, this upper bound constraint on $\Omega_{\mathrm{PBH,f}}$ is comparable and in some mass regions tighter compared to constraints on $\Omega_{\mathrm{PBH,f}}$ from large-scale structure probes, which were derived by simply requiring that galaxies should not form too early [76,77]. Interestingly, if we increase the amplification spectral index n_B we get tighter constraints on $\Omega_{\mathrm{PBH,f}}$ up to $10^{15}M_\odot$. See Fig. 2 for more details. Therefore, the portal of GWs induced by magnetized PBHs is inevitably promoted as a new probe to explore the enigmatic nature of supermassive PBHs.

However, it is important to highlight that we did not consider μ and y distortions of the cosmic microwave background affected by PBH formation which put strong constraints on $\Omega_{\mathrm{PBH,f}}$ assuming Gaussian primordial perturbations [78,79]. These strong constraints can in general be evaded by assuming non-Gaussian cosmological perturbations [80–82], and hence we do not consider them in this work.

[3] Concerning the order of magnitude for the amplitude of the MFs in the Universe, there is strong evidence for a pregalactic seed magnetic fields of the order of 10^{-16}–10^{-18} G [17,69] on intergalactic scales, while on galactic scales we observe MFs with present-day amplitudes of up to 10^{-7} G [70–72]. On smaller scales, the MF intensity is strongly influenced by the presence of interstellar gas and the proximity to stars. For instance, in the vicinity of the Earth the interplanetary magnetic field is 10^{-4} G [73].

[4] It is important to highlight here that the constraint (30) on the PBH abundances is valid only for PBH masses higher than $10^{10}M_\odot$. This is because our pivot amplification factor $\alpha(k_*)$ was computed at the intergalactic scale $k_{\mathrm{intg}} = 100\ \mathrm{Mpc}^{-1}$ assuming that our Biermann-battery mechanism can give rise to present-day intergalactic MFs of the order of 10^{-18} G [17,69]. If now we use the proposed Biermann-battery mechanism with smaller-mass PBHs, which are not able to give rise to the present-day intergalactic MFs as it was shown in [16], we will not be able to have an estimate on the pivot amplification factor and thus on the present-day MIGW signal.

VIII. DISCUSSION

The origin of cosmic MFs constitutes one of the longstanding issues in cosmology. Among their generation mechanisms, the portal of magnetized PBHs seeding battery-induced cosmic MFs seems one of the most promising.

In this article, we have considered a population of supermassive PBHs furnished with a locally isothermal disk which can generate through the Biermann-battery mechanism a seed primordial MF on intergalactic scales. In particular, we derived for the first time the GWs induced by the magnetic anisotropic stress of such a population of magnetized PBHs.

Interestingly enough, by accounting for the contribution of the MIGWs to the effective number of extra neutrino species $\Delta N_{\rm eff}$ and adopting an effective model for the galactic/turbulent dynamo amplification of the magnetic field, we set upper bound constraints on the abundances of supermassive PBHs at formation time, $\Omega_{\rm PBH,f}$, as a function of their masses, which reads as

$$\Omega_{\rm PBH,f} \leq 2.5\times 10^{-10}\left(\frac{M}{10^{10}M_\odot}\right)^{45/22}. \quad (31)$$

In particular, as one may see from Fig. 2, these constraints are comparable and in some mass ranges even tighter compared to constraints on $\Omega_{\rm PBH,f}$ derived from clusters of galaxies. One should also account for constraints on supermassive PBHs due to dark matter halo accretion after matter-radiation equality, as discussed in [83] where a mass-independent upper bound constraint on the contribution of PBHs to dark matter, $f_{\rm PBH} \equiv \Omega_{\rm PBH}/\Omega_{\rm DM}$, of the order of 3×10^{-9} was derived. However, the aforementioned accretion constraint is not so robust for PBH masses larger than $10^4 M_\odot$, such as the ones considered here, since one finds super-Eddington accretion in this high mass range.

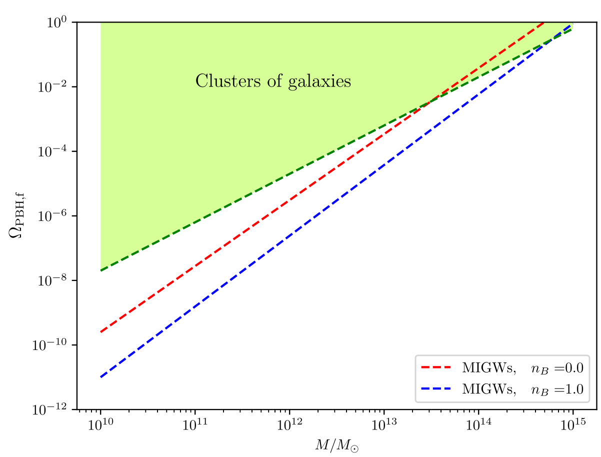

FIG. 2. Constraints on the initial PBH abundance at formation time $\Omega_{\rm PBH,f}$ as a function of the PBH mass M for $n_B = 0$ (dashed red curve) and $n_B = 1$ (dashed blue curve). In the green region we show the constraints on $\Omega_{\rm PBH,f}$ from galaxy clusters [76].

At this point, it is worth mentioning that in the present work we considered thin accretion disks usually exhibiting sub-Eddington accretion [50–52], which in our case operate only during the linear growth phase of the magnetic field that lasts a few dynamical times [16]. Note also that the only place in our analysis where one is met with a dependence on the accretion model is the dimensionless parameter ℓ_R giving us the radial size of the disk, which in general depends on the accretion rate [53]. Interestingly enough, this parameter cancels out in the final expression (29) for the GW signal today since one needs to multiply $\alpha^4 \propto \ell_R^{-4}$ [see Eqs. (27) and (28)] by Eq. (23). In order to extract a potentially more stringent accretion constraint on the PBH abundances in the mass region $M_{\rm PBH} > 10^4 M_\odot$, one needs in principle to run dedicated hydrodynamical simulations in a cosmological setting, going beyond the scope of the current work. Thus, in the absence of numerical simulations for accretion in the very-high-PBH-mass regime [83], it can be claimed that the portal of MIGWs can act as a novel alternative probe to constrain the abundances of supermassive PBHs.

This portal of MIGWs can also be used in order to constrain lower-mass PBHs furnished with Biermann-battery-induced MFs which, however, do not generate the necessary seed primordial MF to give rise to an MF amplitude of the order of 10^{-18} G, threading the intergalactic medium [16]. Nevertheless, there exist other MF-generation mechanisms, like the cosmic battery one [84–86], which is able to give a very high MF amplification on intergalactic scales when operating on lower black hole masses, i.e., $M < 10^{10} M_\odot$.

Furthermore, it is important to emphasize here that within this work we assumed the standard PBH formation scenario where PBHs form out of the collapse of enhanced cosmological perturbations with a mass of the order of that within the cosmological horizon [87] at the time of PBH formation, remaining agnostic on the specific cosmological model giving rise to enhanced cosmological perturbations. In order to access the exact PBH mass distribution, one should choose a particular cosmological model giving rise to PBH formation and account for the critical collapse scaling law for the PBH mass spectrum [88,89] as well for the effect of primordial non-Gaussianities which are necessary to avoid the μ and y distortion constraints. These effects lead in principle to extended PBH mass functions. In this work, we assumed for simplicity a monochromatic PBH mass distribution. This choice can be sufficiently justified only for sharply peaked primordial curvature power spectra which, in the presence of non-Gaussian cosmological perturbations, lead to nearly monochromatic PBH mass distributions [90,91]. However, our work can be easily generalized to also account for extended PBH mass distributions using the formalism developed in [15,16].

We should also point out that since we used a simplified effective model in order to capture the MF amplification due to galactic/turbulent dynamo and various instability processes, we underestimated the GW amplitude and therefore set conservative constraints on $\Omega_{\rm PBH,f}$. Full magnetohydrodynamical numerical simulations are required, however, in order account for the convective term in the MF induction equation and the aforementioned MF amplification effects.

Finally, let us highlight that the formalism developed in this article regarding the derivation of the magnetically induced GWs is quite generic and can be applied to any population of magnetized PBHs [92], e.g., PBHs with magnetic charge [93] or Kerr-Newman PBHs [94], thus promoting the portal of MIGWs to a new GW counterpart associated with PBHs that is potentially detectable by current/future GW detectors.

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