Baryogenesis from sphaleron decoupling

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The electroweak sphaleron process breaks the baryon number conservation within the realms of the Standard Model of particle physics. Recently, it was pointed out that its decoupling may provide the out-of-equilibrium condition required for baryogenesis. In this paper, we study such a scenario taking into account the baryon-number wash-out effect of the sphaleron itself to improve the estimate. We clarify the amount of CP violation required for this scenario to explain the observed asymmetry.

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I. INTRODUCTION

Baryogenesis, the origin of baryon asymmetry of the Universe (BAU), is yet an unsolved problem. Sakharov [1] identified three necessary conditions for baryogenesis: (1) baryon-number nonconservation, (2) C and CP violation, and (3) deviation from thermal equilibrium. Various scenarios (e.g., [2–4]; for recent review, see Ref. [5]) satisfying these conditions have been proposed to explain the observed baryon-to-entropy ratio [6,7]

$$\frac{n_B}{s} \simeq 9 \times 10^{-11}, \tag{1}$$

with n_B and s being the baryon number density and entropy density, respectively. All scenarios that reproduce this value known to date are based on physics beyond the Standard Model of particle physics. Electroweak (EW) baryogenesis [2] and leptogenesis [3] are some of the promising examples. EW baryogenesis requires a more complicated Higgs sector, so that CP violation can be enhanced and a first order EW phase transition (EWPT) instead of a smooth crossover [8], as in the Standard Model of particle physics (SM) [9–11] with the Higgs boson mass around 125 GeV [12,13], can appear to create an inequilibrium environment, while leptogenesis requires right-handed Majorana neutrinos whose decay [3] or oscillation [14,15] provides the inequilibrium process.

The sphaleron process [16–18] plays a key role in both electroweak baryogenesis and leptogenesis, because it breaks baryon-number conservation. It changes the gauge field configuration topologically, resulting in the violation of baryon B and lepton L charges [19] due to the chiral anomaly [20,21], while conserving $B - L$. Specifically, an energy barrier exists between topologically different vacua in the space of field configuration [22], and the sphaleron process describes the barrier crossing between different vacua through thermal fluctuations. Each topologically distinct vacuum is characterized by an integer N_{CS}, and the sphaleron process increasing it by unity will generate nine quarks (three baryons), while the one decreasing it by unity will generate nine antiquarks (-3 baryons) [19].

The sphaleron process can occur within the SM. However, it has been believed that CP violation in the SM is insufficient to generate enough baryon number [23–27], and the smooth crossover at EW symmetry breaking [9–11] in the SM with the 125 GeV Higgs [12,13] does not realize the required inequilibrium environment, and hence the EW baryogenesis does not work in the SM. It would be appealing to explain baryon asymmetry within the SM instead of using new physics, and it is important to see if it is truly impossible to solve this problem using known physics.

Recently, Kharzeev *et al.* [28] proposed an interesting scenario, trying to realize baryogenesis within the SM.

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They argued that when the Higgs boson acquires a nonvanishing vacuum expectation value (VEV), the energy barrier acquires a scale, and various sizes of sphaleronlike configurations with different energy contribute to the baryon-number violating transition. Sphalerons start decoupling in sequence according to their energy, providing an out-of-equilibrium condition required for baryogenesis, and this sphaleron decoupling process continues until sphaleron processes freeze out completely. The number of quarks and that of antiquarks generated during those decoupling processes are different due to the CP violation that originated from the Cabbibo-Kobayashi-Maskawa (CKM) matrix, and they claimed that, unlike the common belief [23–27], it is possible to pick up sufficient CP violation from the CKM matrix, and this provides a source of baryon number. Their estimate showed the baryon-to-photon ratio generated from this source is only 1 order of magnitude smaller than the observed value.

In this paper, we study the scenario more quantitatively by formulating a kinetic equation for the BAU. We point out that their estimation did not involve the wash-out effect of baryon number due to the sphalerons that are still in equilibrium. We give the criteria of which sizes of sphaleronlike configurations are in or out of equilibrium to incorporate the wash-out effect in the kinetic equation. We determine the required amount of CP violation for this scenario to explain the observed BAU, which turns out to be 2 to 3 orders of magnitude larger than that evaluated in Ref. [28] for the SM.

The paper is organized as follows. In the next section, we give a brief review of the EW sphaleron and baryogenesis from its decoupling studied in Ref. [28]. We then formulate the kinetic equation for the baryon number by identifying the source term and the wash-out term around the sphaleron decoupling temperature. In Sec. III, we solve it numerically to give an estimate for the resulting BAU. We give our concluding remarks in Sec. IV.

II. FORMULATION OF THE KINETIC EQUATION

A. Baryon number violation in the SM

Let us start with reviewing the baryon-number violation in the SM. The baryon number in a system changes through the chiral anomaly [20,21] when the quantum tunneling process between the topologically different degenerate vacua takes place. At zero temperature, electroweak SU(2) instanton which mediates such a tunneling is found to be exponentially suppressed, $[\exp(-8\pi^2/g^2)]^2 \sim 10^{-173}$ [19]. While this guarantees the stability of the BAU, this process would not be its origin.

At finite temperature, baryon-number violation takes place by a transition over the energy barrier between topologically different vacua through thermal fluctuations. When the temperature becomes lower than the EW scale, the energy barrier gains a scale determined by the Higgs expectation value. The transition proceeds by crossing around the field configuration at the top of the energy barrier, which is called a sphaleron, and this transition process is called a sphaleron process.

The sphaleron is a spherically symmetric static solution of the field equation of the SU(2)-Higgs theory.[1] The ansatz for the sphaleron configuration adopted in Ref. [17] is given as follows. Let us introduce profile functions, $f(\xi)$ and $h(\xi)$, which describe the relevant physical degrees of freedom of the SU(2) gauge field W_i^a and the Higgs doublet field φ as

$$W_i^a \sigma^a \mathrm{d}x^i = -\frac{2\mathrm{i}}{g} f(\xi) \mathrm{d}U^\infty (U^\infty)^{-1},$$
$$\varphi = \frac{v}{\sqrt{2}} h(\xi) U^\infty \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad i = 1, 2, 3, \quad (2)$$

with σ^a, g, and v being the Pauli matrices, the SU(2) coupling constant, and the VEV of the Higgs field, respectively. U^∞ denotes a two-dimensional matrix

$$U^\infty = \frac{1}{r} \begin{pmatrix} x^3 & x^1 + \mathrm{i}x^2 \\ -x^1 + \mathrm{i}x^2 & x^3 \end{pmatrix},$$
$$r \equiv \sqrt{\sum_i x^{i2}}, \qquad \xi \equiv gvr. \quad (3)$$

Here we have adopted the gauge fixing $W_0 = 0$. Reference [17] found that the following ansatz,

$$f(\xi) = \begin{cases} \frac{\xi^2}{\Xi(\Xi+4)} & \xi \leq \Xi \\ 1 - \frac{4}{\Xi+4} \exp\left[\frac{1}{2}(\Xi - \xi)\right] & \xi \geq \Xi, \end{cases} \quad (4)$$

$$h(\xi) = \begin{cases} \frac{\sigma\Omega+1}{\sigma\Omega+2} \frac{\xi}{\Omega} & \xi \leq \Omega \\ 1 - \frac{\Omega}{\sigma\Omega+2} \frac{1}{\xi} \exp\left[\sigma(\Omega - \xi)\right] & \xi \geq \Omega, \end{cases} \quad (5)$$

fit the numerical solutions relatively well, which we will use in the following. Here $\sigma = \sqrt{2\lambda/g^2}$ (λ is the Higgs quartic coupling), and the boundary conditions

$$f(0) = h(0) = 0, \qquad \lim_{r\to\infty} f(r) = \lim_{r\to\infty} h(r) = 1, \quad (6)$$

are imposed. The values of Ξ and Ω are determined by the minimization condition of the sphaleron mass (or energy) [29],

[1] In the SM, hyper U(1) gauge field is also involved through the nonzero weak mixing angle, but its effect has been turned out to give just a perturbative modification [17].

$$M = \frac{4\pi v}{g}\int_0^\infty \mathrm{d}\xi\left[4(f')^2 + \frac{8}{\xi^2}f^2(1-f)^2 + \frac{\xi^2}{2}(h')^2 + h^2(1-f)^2 + \frac{\xi^2}{16}\sigma^2(h^2-1)^2\right], \tag{7}$$

where the prime denotes the derivative with respect to ξ. At finite temperature, we use the coupling constants as well as the Higgs expectation value evaluated at the temperature scale of interest. Around the EW scale, for Higgs boson mass around 125 GeV we extrapolate the results for the three-dimensional effective theory [29] as

$$\frac{\bar{\lambda}_3}{\bar{g}_3^2} \approx 0.22, \qquad \bar{g}_3^2 \approx 0.39, \tag{8}$$

and find numerically $\Xi_0 = 1.467$ and $\Omega_0 = 1.701$. Later we also consider sphaleronlike configurations parametrized by a single parameter a with $\Xi = a\Xi_0$ and $\Omega = a\Omega_0$. Since Ξ and Ω represent the size of the SU(2) gauge field and Higgs field, respectively, a means the size of sphaleronlike configurations.

Analytically, to evaluate the transition rate per unit time and unit volume, or the sphaleron rate, below EW temperature, one calculates the ensemble average of "probability current" at the top of the energy barrier, by taking the path integral around the sphaleron background. The sphaleron rate is given as [18]

$$\Gamma_{\mathrm{sph}} \sim TW\mathrm{Det}'\{3\}\exp[-M(T)/T], \tag{9}$$

where $\mathrm{Det}'\{3\}$ is a three-dimensional determinant obtained by integrating fluctuations around the sphaleron background and W is the volume factor coming from the zero modes associated with the translational invariance. $M(T)$ is the temperature-dependent sphaleron mass calculated using Eq. (7), evaluated with the temperature-dependent Higgs VEV, $v(T)$. At temperatures higher than the EW scale, the energy barrier for the sphaleron no longer exists, but the baryon-number violating transition process is governed by the parameter g^2T. From the dimensional estimate, the sphaleron rate is evaluated as [18,30]

$$\Gamma_{\mathrm{sph}} \sim \alpha_{\mathrm{W}}^5 T^4, \tag{10}$$

where $\alpha_{\mathrm{W}} \equiv g^2/4\pi$. Here the additional factor α_{W} is due to the plasma damping effect [30].

Since it is difficult to go beyond the approximate formulas (9) and (10) calculating the precise values of their coefficients analytically, numerical analysis is often performed but from a different perspective, namely, based on the diffusion equation of the Chern-Simons number, which yields

$$\Gamma_{\mathrm{sph}}(T) \equiv \lim_{V,t\to\infty}\frac{\langle Q(t)^2\rangle_T}{Vt}. \tag{11}$$

Here the bracket represents thermal average, and $Q(t)$ is the SU(2) topological charge,

$$Q(t) \equiv N_{\mathrm{CS}}(t) - N_{\mathrm{CS}}(0) = \frac{1}{32\pi^2}\int_0^t \mathrm{d}t' \int \mathrm{d}^3x\,\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}W^{\mu\nu}W^{\rho\sigma}. \tag{12}$$

This determines the diffusion rate of the Chern-Simons number, including but not limited to the process described by a sphaleron at $T < T_{\mathrm{EW}}$. In this way, the numerical calculation that evaluates Eq. (11) determines the full transition rate.

Using a recent lattice study [31] in light of the Higgs boson with a mass around 125 GeV, one can fit the sphaleron rate as

$$\begin{cases} \Gamma_{\mathrm{lattice,sph}}(T)/T^4 = (8.0 \pm 1.3)\times 10^{-7} & (T > T_{\mathrm{EW}}) \\ \log(\Gamma_{\mathrm{lattice,sph}}(T)/T^4) = (0.83 \pm 0.01)T/\mathrm{GeV} - (147.7 \pm 1.9) & (T < T_{\mathrm{EW}}), \end{cases} \tag{13}$$

with $T_{\mathrm{EW}} = (159.5 \pm 1.5)$ GeV. We can also fit the temperature dependence of the Higgs VEV as

$$\frac{v(T)}{T} \approx 3\sqrt{1 - \frac{T}{T_{\mathrm{EW}}}}. \tag{14}$$

The qualitative feature of these results can be explained by the analytical investigation above; see Eqs. (9) and (10). In other words, the lattice results quantitatively determine the unknown numerical coefficients in these expressions. In the following, we focus on the temperature regime below T_{EW}. We incorporate transitions induced by configurations whose size is larger or smaller than the sphaleron solution introducing a size parameter, a, of the sphaleronlike configuration, motivated by the argument of Ref. [28], and examine the transition rate through the configuration with each size.

B. Sphaleron decoupling and baryogenesis

Reference [28] suggested that the decoupling of sphaleron, which is an out-of-equilibrium process, can generate

BAU. To investigate this possibility, we need to look at the process in depth. The sphaleron process would proceed through not only the exact sphaleron solution [well fit by Eqs. (4) and (5)] but also higher energy field configurations excited by thermal fluctuations. While in the analytic calculation they are taken into account as the prefactor of the exponential term in the transition rate evaluated by integrating over fluctuations around the exact sphaleron solution with the Gaussian approximation, we here take into account the sphaleronlike configuration with different sizes, similar to the treatment of Ref. [28]. That is, we express the sphaleron rate obtained by the lattice calculation [31] as

$$\Gamma_{\mathrm{lattice,sph}}(T) = \int \mathrm{d}a \Gamma_{\mathrm{sph}}(T,a), \quad \text{with}$$
$$\Gamma_{\mathrm{sph}}(T,a) = \frac{\mathrm{e}^{-M(T,a)/T}}{\int \mathrm{d}a' \mathrm{e}^{-M(T,a')/T}} \cdot \Gamma_{\mathrm{lattice,sph}}(T), \tag{15}$$

where $M(T,a)$ is the energy of a sphaleronlike configuration with the size a. In other words, we interpret that the results of the lattice simulation reflect the contributions from the field configuration with different sizes. Figure 1 shows the distribution of the sphaleron rate with respect to the size of the field configuration and temperatures below the EW scale, $T = 157.0$, 152.0, 147.0, 142.0, 137.0 GeV. It is peaked at $a = 1$ corresponding to the true saddle point solution and becomes less than the Hubble scale for some smaller and larger values of a at two points.

As the temperature decreases, the range of the sphaleron size above the horizontal line gets smaller, meaning that too large and too small sphalerons continuously decouple. Reference [28] argued that this process is out of equilibrium

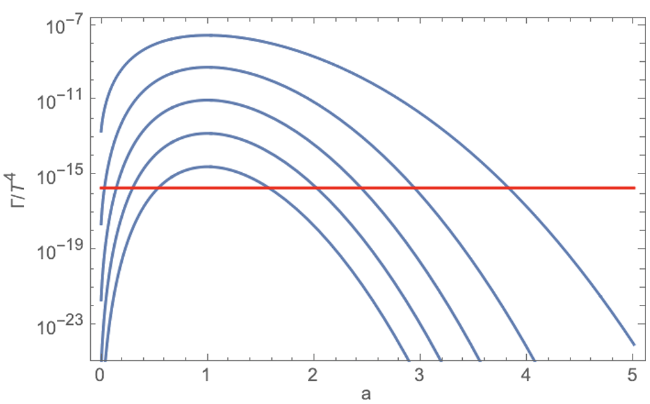

FIG. 1. The sphaleron rate of different sizes and temperatures; see Eq. (15). The temperature is taken as $T = 157.0$, 152.0, 147.0, 142.0, 137.0 GeV from top to bottom, respectively. The red horizontal line represents the Hubble parameter normalized by the temperature, H/T, in the relevant temperature range. The temperature of the onset of the EW symmetry breaking is taken as $T_{\mathrm{EW}} = 160$ GeV and practically the integration range in Eq. (15) is taken as $0.001 \le a \le 5$.

and contributes to baryon number generation, because no upward-going fluctuations occur while decoupling and all configurations go down the energy barrier with these particular sizes of sphalerons. This kind of decoupling occurs below T_{EW} when the energy barrier gains a scale, and ends when all the sphaleron processes freeze. The authors of [28] used the sphaleron rate at decoupling, namely, when it coincides with the Hubble rate, to represent the rate of baryon generation.

They also argued that the difference between the probability of production of one baryon number and that of reduction of one baryon number for sphaleron processes can be as large as

$$A_{\mathrm{CP}} \sim 0.25 \times 10^{-9}. \tag{16}$$

Supposing that this process is active from the onset of the EW symmetry breaking to the sphaleron freezeout, $T_{\mathrm{FO}} \simeq 130$ GeV (extrapolated from lattice simulation [31]), the resulting baryon asymmetry has been estimated as

$$\frac{n_B}{s} = 3A_{\mathrm{CP}} \times H_{\mathrm{EW}} \times (t_{\mathrm{FO}} - t_{\mathrm{EW}})/s_{\mathrm{EW}}$$
$$\sim 10^{-12}\left(\frac{A_{\mathrm{CP}}}{0.25 \times 10^{-9}}\right), \tag{17}$$

where the factor 3 is the absolute value of the baryon number produced from a sphaleron [28]. The subscripts EW and FO represent that the variables are evaluated at the EW symmetry breaking and sphaleron freezeout, respectively. This value is apparently just 1 order of magnitude smaller than the observed BAU, $n_B/s \sim 9 \times 10^{-11}$ [6,7]. The authors of Ref. [28] concluded that this would be consistent with the present Universe within the uncertainty of the estimate of the CP violation A_{CP}.

C. Sphaleron wash out and kinetic equation

Let us now point out that the estimate in Ref. [28] did not include the effect from the sphalerons that are still in equilibrium, which tend to wash out the baryon number. In this subsection, we formulate the kinetic equation for the baryon asymmetry that includes this effect and also the source term due to the sphaleronlike configurations that are about to decouple. In the next section, we will evaluate it numerically.

At high enough temperature, the sphaleron process tends to wash out the $B+L$ (the summation of baryon and lepton number) asymmetry [2]. When we do not take into account the baryon production from the sphaleron decoupling and any other source term, the kinetic equation for the baryon asymmetry around the EW scale is obtained as [32]

$$\frac{\mathrm{d}n_B}{\mathrm{d}t} + 3Hn_B = -\Gamma_B n_B, \qquad \Gamma_B = \frac{39}{4}\frac{\Gamma_{\mathrm{sph}}(T)}{T^3}, \tag{18}$$

where we have assumed that there was no initial $B - L$ asymmetry. Strictly speaking, the coefficient in Γ_B depends on the Higgs VEV and the equilibrium condition for the rapid processes such as the top Yukawa interaction [32,33]. But its variation during the process around $T \lesssim 10^2$ TeV is quite small (a few percent) [33,34], and hence in the following, we take it as a constant, 39/4, which is often used in literature, e.g., [18,35]).

We now rewrite the kinetic equation Eq. (18) for the baryogenesis scenario from the sphaleron decoupling. As discussed in the previous section, we introduce the source term from decoupling sphalerons and modify the wash-out term, too.

To determine the source term precisely, we first recall the sphaleron decoupling condition,

$$\frac{\Gamma_{\mathrm{sph}}(T)}{T^3} \lesssim H(T).$$

Since this condition is determined by the comparison between the Hubble expansion and the rate of the process, a similar condition can be set for each size of sphaleronlike configuration. We assume the decoupling temperature of the sphaleronlike configuration with its size a, $T_*(a)$, by the solution of

$$\frac{\Gamma_{\mathrm{sph}}(T_*(a), a)}{T_*^3(a)} = cH(T_*(a)), \tag{19}$$

where c is a parameter of order of unity. Its precise value may be determined by investigating the decoupling process quantitatively, which is left for future research. We assume that when the rate $\Gamma_{\mathrm{sph}}(T_*(a), a)$ becomes lower than $cT_*^3(a)H(T_*(a))$, the "size a configuration" decouples. In [28], c was set to 4/39 to calculate t_{FO} in (17) (see also Refs. [31,36]). We will use it as a reference value.

With these criteria, one can modify the kinetic equation as follows. For given c and a temperature T, we can draw a horizontal line for $cH(T)/T$ in Fig. 1 (the red line corresponds to the case with $c = 1$). At each temperature, T, we identify the points where the curve of $\Gamma_{\mathrm{sph}}(T, a)$ crosses the horizontal line $cH(T_*)/T_*$, which we call $a = a_l$ and $a = a_u$ with $a_l \leq a_u$. Sphaleronlike configurations with size $a_l \leq a \leq a_u$ contribute to washing out the asymmetry while those with $a \leq a_l$ and $a \geq a_u$ may produce net baryon number. As the temperature drops, the curve will fall below the horizontal line. We define this critical temperature as T_c. Then taking into account the factor for the efficiency of baryon-number production, $3 \times A_{\mathrm{CP}}$, we determine the source term for the baryon-number generation, $P(T)$, as

$$P(T) = \begin{cases} \left(\int_{a_{\min}}^{a_l} \mathrm{d}a\Gamma_{\mathrm{sph}}(T, a) + \int_{a_u}^{a_{\max}} \mathrm{d}a\Gamma_{\mathrm{sph}}(T, a)\right) \cdot 3A_{\mathrm{CP}}, & \text{for } T_c < T \leq T_{\mathrm{EW}}, \\ \Gamma_{\mathrm{lattice,sph}} \cdot 3A_{\mathrm{CP}}, & \text{for } T \leq T_c, \end{cases} \tag{20}$$

and the wash-out rate as

$$\Gamma_B(T) = \frac{39}{4T^3}\Gamma_{\mathrm{washout,sph}}(T) = \begin{cases} \frac{39}{4T^3}\int_{a_l}^{a_u} \mathrm{d}a\Gamma_{\mathrm{sph}}(T, a), & \text{for } T_c < T \leq T_{\mathrm{EW}}, \\ 0, & \text{for } T \leq T_c. \end{cases} \tag{21}$$

For practical purposes, in calculating $\Gamma_{\mathrm{sph}}(T, a)$ and $P(T)$, we choose $a_{\min} = 0.001$ and $a_{\max} = 5$. If we do not find a_l for $P(T)$, we just omit the first term in Eq. (20). The contributions from the omitted part are exponentially small, and hence the results are unaffected. The main contribution of the source term calculated in this way comes from the sphalerons near the horizontal red line as we can see in Fig. 1 where the vertical axis is of logarithmic scale.

By taking them together, the modified kinetic equation for our scenario is given as

$$-HT\frac{\mathrm{d}n_B}{\mathrm{d}T} + 3Hn_B = -\frac{39}{4}\frac{\Gamma_{\mathrm{washout,sph}}(T)}{T^3}n_B + P(T), \tag{22}$$

where we have used the relation $H = 1/2t \propto T^2$ during the radiation dominated era.

III. RESULTING BARYON-TO-ENTROPY RATIO

Now we solve the kinetic equation numerically to determine the resultant baryon asymmetry. For this purpose, first we evaluate the source term and wash-out term. For temperatures in the range $125.0\,\mathrm{GeV} \leq T \leq 157.0\,\mathrm{GeV}$ every 0.1 GeV, we evaluate $\Gamma_{\mathrm{sph}}(T, a)$ for $0.001 < a < 5$ by using Eqs. (4), (5), (7), (13), and (15). Then for a given c, we can determine a_l and a_u as a function of T. We first determine the net decoupling temperature T_* by the condition $\Gamma_{\mathrm{lattice}}(T_*)/T_*^4 = cH(T_*)/T_*$, then determine a_l and a_u by the solution of $\Gamma_{\mathrm{sph}}(T, a_{l,u}(T))/T^4 = cH(T_*)/T_*$, which makes it possible to determine T_c as a solution of $\max[\Gamma(T_c, a)/T_c^4] = cH(T_*)/T_*$. With these $a_l(T)$ and $a_u(T)$ we calculate $P(T)$ and $\Gamma_{\mathrm{washout,sph}}(T)$ by using Eqs. (20) and (21). We find that the wash-out term is well fit as

$$\Gamma_{\mathrm{washout,sph}}(T) = 10^{-17}T^4 \times \begin{cases} \exp(n_1 T + n_2), & \text{for } n_5 \text{ GeV} < T < T_{\mathrm{EW}}, \\ \exp(n_3 T + n_4), & \text{for } T_c < T < n_5 \text{ GeV}, \end{cases} \tag{23}$$

while the source term is well fit as

$$P(T) = 10^{-17}A_{\mathrm{CP}}T^4 \times \begin{cases} \exp(m_1 T + m_2), & \text{for } m_9 \text{ GeV} < T < T_{\mathrm{EW}} \text{ GeV}, \\ \exp(m_3 T + m_4), & \text{for } m_8 \text{ GeV} < T < m_9 \text{ GeV}, \\ \exp[(m_6 T + m_7)/(T - m_5)], & \text{for } T_c < T < m_8 \text{ GeV}, \end{cases} \tag{24}$$

where n_1 to n_4 as well as m_1 to m_9 depend on c. Figures 2 and 3 show $\Gamma_{\mathrm{washout,sph}}(T)$ and $P(T)$ numerically obtained for $c = 0.01$, 0.1, and 1, respectively, which clearly show the fitting functions Eqs. (23) and (24) work very well. Note that we find $T_* = 128.7$, 131.5, 134.3 GeV and $T_c = 128.0$, 130.8, 133.6 GeV, for $c = 0.01$, 0.1, 1, respectively.

Using these fitting results, we solve the kinetic equation (22) for $T < T_{\mathrm{EW}} = 160$ GeV taking $n_B = 0$ as the initial condition. Figure 4 shows the typical evolution of baryon asymmetry for $c = 0.1$. We can see that baryon asymmetry stops growing at some temperature below $T_c = 130.8$ GeV and becomes constant at $T \lesssim 125$ GeV.

The resulting baryon asymmetry at 120 GeV, when it has already become a constant, is plotted as a function of c in Fig. 5. Here we have set $A_{\mathrm{CP}} = 0.25 \times 10^{-9}$ [28] and used $s = \frac{4\pi^2}{90} g_* T^3$ for the process calculated above to convert T^3 to s. The baryon-to-entropy ratio increases as c increases as expected. Considering that the result is proportional to the CP violation parameter A_{CP}, which has not been calculated quantitatively, we give a general expression of our conclusion:

$$\frac{n_B}{s} \sim 3 \times 10^{-14} \left(\frac{c}{4/39} \right)^{0.92} \frac{A_{\mathrm{CP}}}{0.25 \times 10^{-9}}. \tag{25}$$

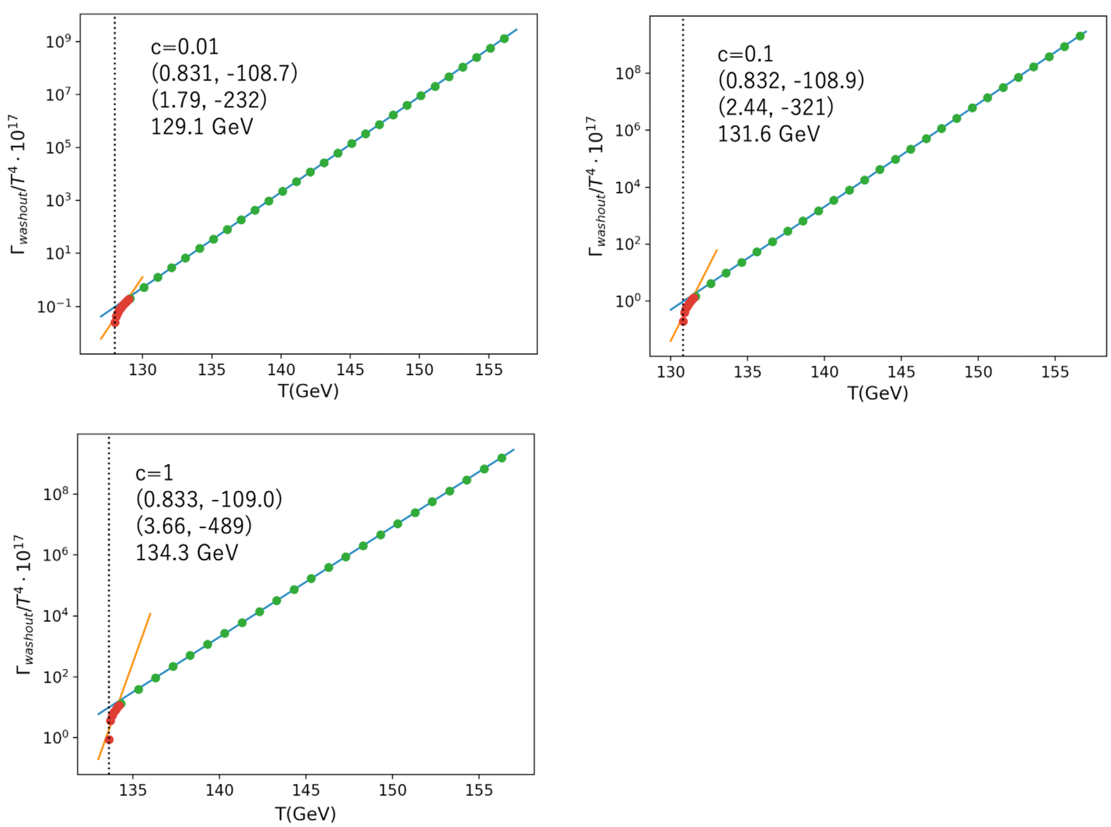

FIG. 2. Numerically evaluated values of the wash-out term, $\Gamma_{\mathrm{washout,sph}}/T^4$, [see Eq. (21)] as well as its fitting curve [Eq. (23)] for $c = 0.01$, 0.1 and 1. The numbers in each figure are the parameters in the fitting curve: from top to bottom, (n_1, n_2), (n_3, n_4), and n_5 GeV. The vertical dotted lines represent $T = T_c$.

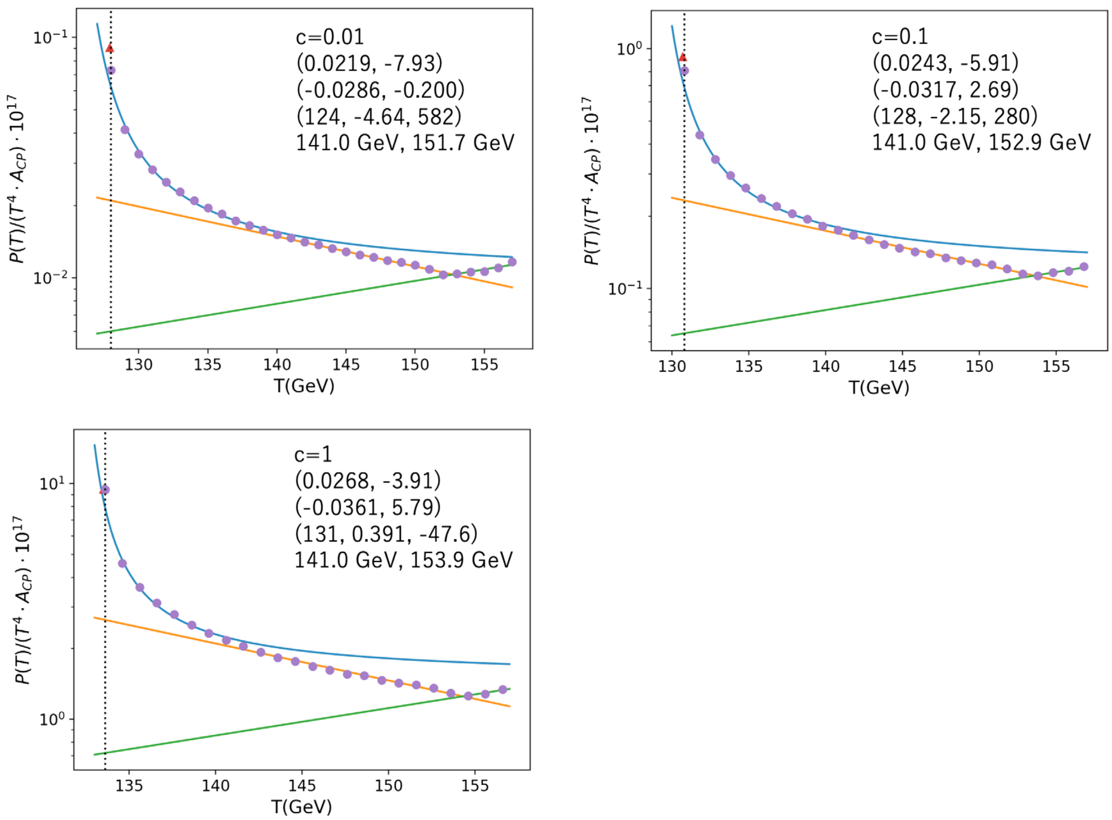

FIG. 3. Numerically evaluated values of the source term, $P/(T^4 \cdot A_{\mathrm{CP}})$ [see Eq. (20)], as well as its fitting curve [Eq. (24)]. The vertical dotted line represents $T = T_c$. The red triangle denotes the value obtained from the lattice result [Eq. (13)] at ($T_c - 0.1$ GeV). The numbers in each figure are the parameters in the fitting curve: from top to bottom, (m_1, m_2), (m_3, m_4), (m_5, m_6, m_7), and m_8 GeV, m_9 GeV.

Comparing with the observed value of baryon-to-entropy ratio, [6,7]

$$\frac{n_B}{s} \simeq 9 \times 10^{-11}, \tag{26}$$

the baryon-to-entropy ratio in our calculation is 2 to 3 orders lower than the observed value. We conclude that, unfortunately, as long as we take the parameter c of order of unity, this scenario is impossible to explain the present BAU, even with an optimistic estimate of the CP violation. On the other hand, we expect that the present result also qualitatively applies to extensions of the SM, as long as the temperature scale of electroweak symmetry breaking and Higgs VEV does not differ dramatically from what we

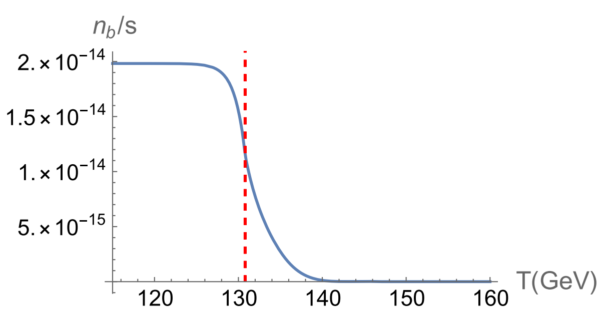

FIG. 4. The time evolution of the baryon-to-entropy ratio in the case $c = 0.1$. The red dashed line represents $T = T_c = 130.8$ GeV.

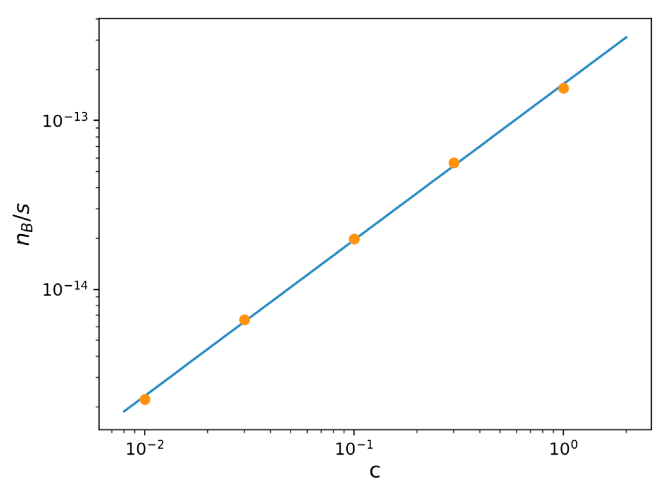

FIG. 5. The net baryon-to-entropy ratio evaluated at $T = 120$ GeV for $c = 0.01$, 0.03, 0.1, 0.3, and 1, as well as its fitting function [Eq. (25)]. Here we have taken $A_{\mathrm{CP}} = 0.25 \times 10^{-9}$.

studied here. Therefore, if some new physics enhances the CP violation up to $A_{\mathrm{CP}} \sim 10^{-6}$, the present BAU might be explained. Compared with the standard EW baryogenesis [37], it is more economical since we do not have to make the EW symmetry breaking a strong first-order PT.

One might be concerned that when the sphaleron size is too small, quantum fluctuations become large and the classical field configuration loses sense. Typical amplitudes of fluctuations of $h(\xi)$ and $f(\xi)$ can be estimated as [38]

$$|\delta h| \simeq \frac{\sigma_h}{\Omega}, \tag{27}$$

$$|\delta f| \simeq \frac{\sigma_f}{\Xi}, \tag{28}$$

where σ_h and σ_f are constants of order unity. This suggests that the notion of the sphaleronlike configuration with $a < 0.1$ might be questionable. To check the effect of possible noncontributions from these parameters, we took $a \in [0.1, 5]$ and repeated the procedures above. We have found that the result does not change, because sphaleronlike configurations with $a < 0.1$ do not contribute much. Therefore, our conclusion is unchanged even if we take into account this concern.

IV. CONCLUSION

Reference [28] proposed a scenario of baryogenesis within the SM, where the decoupling of sphalerons provides the inequilibrium process required by Sakharov's conditions. They focused on the size distribution of sphalerons, and pointed out that sphalerons of different sizes and rates keep decoupling from the moment of EWPT to that of freeze-out of the entire sphaleron process. Sphaleron decoupling described in such a way can provide a source for the generation of baryon number. With an estimate of the difference between probability of the production of quarks and antiquarks in the sphaleron process, or the CP violation, A_{CP}, they gave an estimate of n_B/s, which was only 1 order smaller than the observed BAU. They argued that the BAU can be explained by the SM taking into account the uncertainty in their crude estimation of A_{CP}.

In this paper, we have studied the scenario following the idea of Ref. [28] with constructing a more proper kinetic equation for the BAU. We have taken into account the wash-out effect from the sphalerons that are still in equilibrium, which was not incorporated in Ref. [28]. We have calculated the rate distribution for sphalerons of different sizes at different temperatures, and evaluated the source term and wash-out term in the kinetic equation. Including this effect, we have estimated n_B/s numerically, which turned out to be 2 to 3 orders smaller than the observed BAU, with the amounts of CP violation used in [28]. Thus we conclude the scenario of baryogenesis in the SM proposed in [28] does not work unless A_{CP} is enhanced by a factor of 10 or 10^2 compared with their optimistic value. In other words, in order to make use of the sphaleron decoupling discussed here to realize adequate baryogenesis, we must introduce some new ingredients to increase the amount of CP violation to the level of $A_{\mathrm{CP}} \sim 10^{-6}$.

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