Charmless semileptonic baryonic $B_{u,d,s}$ decays

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We study $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' l\bar{\nu}$ and $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' \nu\bar{\nu}$ decays with all low lying octet ($\mathcal{B}$) and decuplet ($\mathcal{D}$) baryons using a topological amplitude approach. In tree-induced $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' l\bar{\nu}$ decay modes, we need two tree amplitudes and one annihilation amplitude in $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu}$ decays, one tree amplitude in $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}} l\bar{\nu}$ decays, one tree amplitude in $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}} l\bar{\nu}$ decays and one tree amplitude and one annihilation amplitude in $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}' l\bar{\nu}$ decays. In loop induced $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' \nu\bar{\nu}$ decay modes, similar numbers of penguin-box and penguin-box-annihilation amplitudes are needed. As the numbers of independent topological amplitudes are highly limited, there are plenty of relations on these semileptonic baryonic B_q decay amplitudes. Furthermore, the loop topological amplitudes and tree topological amplitudes have simple relations, as their ratios are fixed by known Cabibbo-Kobayashi-Maskawa (CKM) factors and loop functions. It is observed that the $B^- \to p\bar{p}\mu^-\bar{\nu}$ differential rate exhibits threshold enhancement, which is expected to hold in all other semileptonic baryonic modes. The threshold enhancement effectively squeezes the phase space toward the threshold region and leads to very large SU(3) breaking effects in the decay rates. They are estimated using the measured $B^- \to p\bar{p}\mu^-\bar{\nu}$ differential rate and model calculations. From the model calculations, we find that branching ratios of nonannihilation $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' l\bar{\nu}$ modes are of the orders of 10^{-9}–10^{-6}, while branching ratios of nonpenguin-box-annihilation $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' \nu\bar{\nu}$ modes are of the orders of 10^{-12}–10^{-8}. Modes with relatively unsuppressed rates and good detectability are identified. These modes can be searched experimentally in near future and the rate estimations can be improved when more modes are discovered. Ratios of rates of some loop induced $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' \nu\bar{\nu}$ decays and tree induced $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' l\bar{\nu}$ decays are predicted and can be checked experimentally. They can be tests of the SM. Some implications on $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' l^+ l^-$ decays are also discussed.

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I. INTRODUCTION

Recently, there have been some experimental activities on $\bar{B} \to \mathbf{B}\bar{\mathbf{B}}' l\bar{\nu}$ and $\bar{B} \to \mathbf{B}\bar{\mathbf{B}}' \nu\bar{\nu}$ decays, where $\mathbf{B}\bar{\mathbf{B}}'$ are baryon antibaryon pairs [1–5]. The present experimental results are summarized in Table I. In particular, the branching ratio of $B^- \to p\bar{p}\mu^-\bar{\nu}_\mu$ decay is measured to be $(5.27^{+0.23}_{-0.24} \pm 0.21 \pm 0.15) \times 10^{-6}$ by LHCb [3] and $\mathrm{Br}(B^- \to p\bar{p}l\bar{\nu}) = (5.8^{+2.6}_{-2.3}) \times 10^{-6}$ by Belle [2] (see also [4]), while only upper limit of $\mathrm{Br}(B^- \to \Lambda\bar{p}\nu\bar{\nu}) < 3.0 \times 10^{-5}$ was reported by *BABAR* [5].

Theoretically, the branching ratios of $\bar{B} \to \mathbf{B}\bar{\mathbf{B}}' l\bar{\nu}$ decays were estimated and predicted to be of the order of 10^{-6} to 10^{-4} [6,7]. Some recent studies are devoted to understand the rate of the $B^- \to p\bar{p}l\bar{\nu}$ decay [8,9] as the measured rate is roughly 20 times smaller than a previous theoretical prediction [7], while the shape of the predicted differential rate using QCD counting rules agrees well with data, which exhibits threshold enhancement [3,7]. In this work we will employ the approach of Refs. [10–13], which was used to study two-body baryonic B decays, $\bar{B} \to \mathbf{B}\bar{\mathbf{B}}'$, making use of the well established topological amplitude formalism [14–24]. The decay amplitudes of $\bar{B} \to \mathbf{B}\bar{\mathbf{B}}' l\bar{\nu}$ and $\bar{B} \to \mathbf{B}\bar{\mathbf{B}}' \nu\bar{\nu}$ decays with all low lying octet ($\mathcal{B}$) and decuplet ($\mathcal{D}$) baryons will be decomposed into combinations of several topological amplitudes. As the numbers of topological amplitudes are highly limited, there are many relations of decay amplitudes.

It is well known that a decay rate strongly depends on the masses of the final state particles when the decay is just above the threshold. The rates may vary in orders of magnitudes even if the amplitudes are of similar sizes. One normally does not expect such behavior in B_q decays when large phase spaces are available. From the

TABLE I. Experimental results of $\mathcal{B}^- \to \mathbf{B}\bar{\mathbf{B}}' l\bar{\nu}$ and $\mathbf{B}\bar{\mathbf{B}}'\nu\bar{\nu}$ branching ratios. The upper limits are at 90% confidence level.

Mode	Branching ratio	References
$B^- \to p\bar{p}e^-\bar{\nu}_e$	$(5.8 \pm 3.7 \pm 3.6) \times 10^{-4}$ $(< 5.2 \times 10^{-3})$	CLEO [1]
	$(8.2^{+3.7}_{-3.2} \pm 0.6) \times 10^{-6}$	Belle [2]
	$(8.2^{+4.0}_{-3.3}) \times 10^{-6}$	PDG [4]
$B^- \to p\bar{p}\mu^-\bar{\nu}_\mu$	$(3.1^{+3.1}_{-2.4} \pm 0.7) \times 10^{-6}$	Belle [2]
	$(5.27^{+0.23}_{-0.24} \pm 0.21 \pm 0.15) \times 10^{-6}$	LHCb [3]
	$(5.32 \pm 0.34) \times 10^{-6}$	PDG [4]
$B^- \to p\bar{p}l\bar{\nu}(l = e, \mu)$	$(5.8^{+2.6}_{-2.3}) \times 10^{-6}$	Belle [2], PDG [4]
$B^- \to \Lambda\bar{p}\nu\bar{\nu}$	$(0.4 \pm 1.1 \pm 0.6) \times 10^{-5}$ $(< 3.0 \times 10^{-5})$	*BABAR* [5]

experimental differential rate $d\mathrm{Br}/dm_{p\bar{p}}$ of $B^- \to p\bar{p}\mu^-\bar{\nu}$ decay from LHCb [3] as shown in Fig. 1, one can easily see that the spectrum exhibits prominent threshold enhancement, which is a comment feature in three or more body baryonic B_q decays [6–9,25–29]. Threshold enhancement is expected to hold in all other semileptonic baryonic modes considered in this work as well. The threshold enhancement effectively squeezes the phase space to the threshold region, see Fig. 1, and thus mimics the decay just above threshold situation. Consequently, it amplifies the effects of SU(3) breaking in final state baryon masses and can lead to very large SU(3) breaking effects in the decay rates. The SU(3) breakings in the decay rates from threshold enhancements will be estimated using the measured $B^- \to p\bar{p}\mu^-\bar{\nu}$ differential rate and model calculations with available theoretical inputs from Refs. [8,9], which can reproduce the measured $B^- \to p\bar{p}\mu^-\bar{\nu}$ differential rate.

We will try to identify modes with relatively unsuppressed rates and good detectability. The estimation on

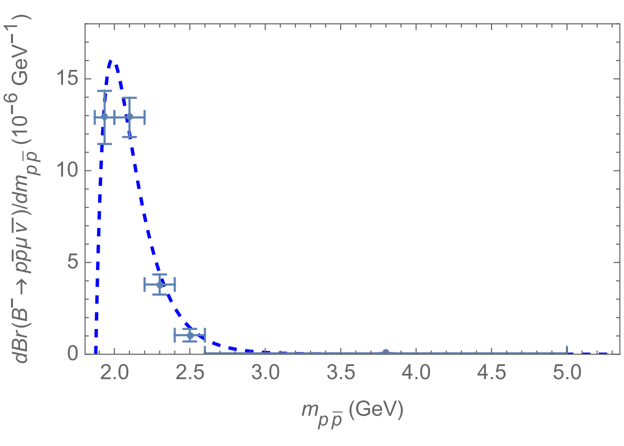


FIG. 1. The experimental differential rate $d\mathrm{Br}/dm_{p\bar{p}}$ of $B^- \to p\bar{p}\mu^-\bar{\nu}$ decay from LHCb [3] exhibits threshold enhancement. The threshold enhancement effectively squeezes the phase space toward the threshold region.

rates can be improved when more modes are discovered. Recently hints of new physics effects in rare B decays are accumulating, see, for example, [30–32]. Given the present situation and the fact that $\bar{B} \to \mathbf{B}\bar{\mathbf{B}}' l\bar{\nu}$ decays are tree induced decay modes, while $\bar{B} \to \mathbf{B}\bar{\mathbf{B}}'\nu\bar{\nu}$ decays are loop induced decay modes, it will be interesting and useful to identify $\bar{B} \to \mathbf{B}\bar{\mathbf{B}}'\nu\bar{\nu}$ and $\bar{B} \to \mathbf{B}\bar{\mathbf{B}}' l\bar{\nu}$ decay modes which have good detectability. Their rate ratios, especially, those insensitive to the modeling of SU(3) breaking from threshold enhancement, can be tests of the Standard Model (SM).

The layout of this paper is as following. We give the formalism for decomposing amplitudes in terms of topological amplitudes and modeling of the topological amplitudes in Sec. II. In Sec. III, results on decay amplitudes in term of topological amplitudes, relations of decay amplitudes and decay rates are provided. Conclusion and discussions are given in Sec. IV, where some comments on $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' l^+ l^-$ decays will also be given. Appendix A concerning the transition matrix elements in the asymptotic limit and Appendix B with some useful formulas for calculating 4-body decay rates are added at the end of the paper.

II. FORMALISM

A. Topological amplitudes

The decay amplitudes of $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' l\bar{\nu}$ and $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'\nu\bar{\nu}$ decays are given by [7,33]

$$\begin{aligned} A(\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' l\bar{\nu}) &= \frac{G_F}{\sqrt{2}} V_{ub} \langle \mathbf{B}\bar{\mathbf{B}}' | \bar{u}_L \gamma_\mu b_L | \bar{B}_q \rangle \bar{l}_L \gamma^\mu \nu_L, \\ A(\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'\nu\bar{\nu}) &= \frac{G_F}{\sqrt{2}} \frac{\alpha_{\mathrm{em}}}{2\pi \sin^2\theta_W} V_{ts}^* V_{tb} D(m_t^2/m_W^2) \\ &\quad \times \langle \mathbf{B}\bar{\mathbf{B}}' | \bar{s}_L \gamma_\mu b_L | \bar{B}_q \rangle \bar{\nu}_L \gamma^\mu \nu_L, \end{aligned} \tag{1}$$

where V_{ub}, V_{ts} and V_{tb} are Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, $D(x)$, $D_0(x)$, and $D_1(x)$ are loop functions with [34]

$$D(x) = D_0(x) + \frac{\alpha_s}{4\pi} D_1(x),$$
$$D_0(x) = \frac{x}{8}\left(-\frac{2+x}{1-x} + \frac{3x-6}{(1-x)^2}\ln x\right),$$
$$D_1(x) = -\frac{23x+5x^2-4x^3}{3(1-x)^2} + \frac{x-11x^2+x^3+x^4}{(1-x)^3}\ln x + \frac{8x+4x^2+x^3-x^4}{2(1-x)^3}\ln^2 x - \frac{4x-x^3}{(1-x)^2} Li_2(1-x) + 8x\frac{dD_0(x)}{dx}\ln\frac{\mu^2}{m_W^2}. \quad (2)$$

Note that the $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' l\bar{\nu}$ decay is governed by the matrix element, $\langle \mathbf{B}\bar{\mathbf{B}}'|\bar{u}_L\gamma_\mu b_L|\bar{B}_q\rangle$, while the $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'\nu\bar{\nu}$ decay is governed by the matrix element, $\langle \mathbf{B}\bar{\mathbf{B}}'|\bar{s}_L\gamma_\mu b_L|\bar{B}_q\rangle$. These two matrix elements are difficult to calculate as they involve baryon pairs $\mathbf{B}\bar{\mathbf{B}}'$ in the final state. Nevertheless they are related by interchanging u and s and, hence, can be related by SU(3) transformations.

It is known that topological amplitude approach is related to SU(3) approach [14,16,18]. We follow the approach similar to the one employed in the study of $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'$ decays [10–13] to decompose $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' l\bar{\nu}$ and $\mathbf{B}\bar{\mathbf{B}}'\nu\bar{\nu}$ decay amplitudes into topological amplitudes.

From Eq. (1), we see that the Hamiltonian governing $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' l\bar{\nu}$ decays has the following flavor structure,

$$(\bar{u}b) = H_T^i(\bar{q}_i b), \quad (3)$$

with

$$H_T^1 = 1, \quad \text{otherwise} \quad H_T^i = 0, \quad (4)$$

where we take $q_{1,2,3} = u,\ d,\ s$ as usual. Similarly, the Hamiltonian governing $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'\nu\bar{\nu}$ decays has the following flavor structure,

$$(\bar{s}b) = H_{PB}^k(\bar{q}_k b), \quad (5)$$

with

$$H_{PB}^3 = 1, \quad \text{otherwise} \quad H_{PB}^k = 0. \quad (6)$$

These H_T and H_{PB} will be used as spurion fields in the following constructions of effective Hamiltonian, $H_{\rm eff}$.

We shall start with $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}' l\bar{\nu}$ decays with $\mathcal{D}$ the low-lying decuplet baryon. The flavor flow diagram for a $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' l\bar{\nu}$ decay is given in Fig. 2. Note that in the case of a $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}' l\bar{\nu}$ decay, the $q_i q_j q_l$ and $\bar{q}^l\bar{q}^j\bar{q}^m$ flavors as shown in Fig. 2 correspond to the following fields,

$$q_i q_j q_l \to \bar{\mathcal{D}}_{ijl}, \qquad \bar{q}^l\bar{q}^j\bar{q}^m \to \mathcal{D}^{jlm}, \quad (7)$$

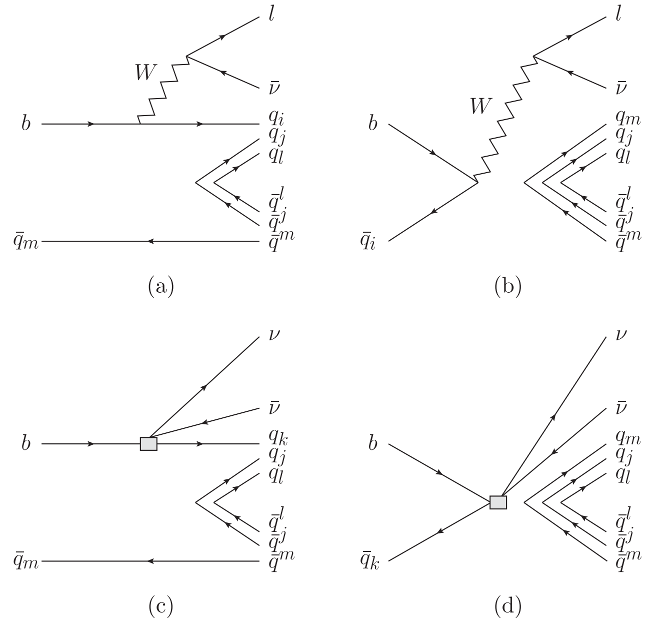


FIG. 2. Topological diagrams of (a) T (tree) and (b) A (annihilation) amplitudes in $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' l\bar{\nu}$ decays and (c) PB (penguin and box) and (d) PBA (penguin-box annihilation) amplitudes in $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'\nu\bar{\nu}$ decays. These are flavor flow diagrams, where gluon lines are not shown.

in the Hamiltonian, respectively, where $\mathcal{D}^{jlm}$ denotes the familiar decuplet field, and, explicitly, we have $\mathcal{D}^{111} = \Delta^{++}$, $\mathcal{D}^{112} = \Delta^+/\sqrt{3}$, $\mathcal{D}^{122} = \Delta^0/\sqrt{3}$, $\mathcal{D}^{222} = \Delta^-$, $\mathcal{D}^{113} = \Sigma^{*-}/\sqrt{3}$, $\mathcal{D}^{123} = \Sigma^{*0}/\sqrt{6}$, $\mathcal{D}^{223} = \Sigma^{*-}/\sqrt{3}$, $\mathcal{D}^{133} = \Xi^{*0}/\sqrt{3}$, $\mathcal{D}^{233} = \Xi^{*-}/\sqrt{3}$, and $\mathcal{D}^{333} = \Omega^-$ (see, for example [35]). By using the above correspondent rule, we obtain the following effective Hamiltonian for $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}' l\bar{\nu}$ decays,

$$H_{\rm eff}(\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}l\bar{\nu}) = 6T_{\mathcal{D}\bar{\mathcal{D}}}\bar{B}_m H_T^i \bar{\mathcal{D}}_{ijl}\mathcal{D}^{ljm} + A_{\mathcal{D}\bar{\mathcal{D}}}\bar{B}_i H_T^i \bar{\mathcal{D}}_{mjl}\mathcal{D}^{ljm}, \quad (8)$$

with $\bar{B}_m = (B^-, \bar{B}^0, \bar{B}_s^0)$. Without lost of generality, the prefactors are assigned for latter purpose.

For the $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}l\bar{\nu}$ decays, we note that the antioctet final state is produced by the $\mathcal{B}_k^j$ field with [35]

$$\mathcal{B} = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix}, \quad (9)$$

where $\mathcal{B}_k^j$ has the following flavor structure $q^j q^a q^b \epsilon_{abk} - \frac{1}{3}\delta_k^j q^c q^a q^b$ [35]. To match the flavor of $\bar{q}^l\bar{q}^j\bar{q}^m$ in the final state as shown in Fig. 2, we use

$$\bar q^l \bar q^j \bar q^m \to \epsilon^{ljb}\mathcal{B}^m_b, \qquad \epsilon^{lbm}\mathcal{B}^j_b, \qquad \epsilon^{bjm}\mathcal{B}^l_b, \tag{10}$$

which are, however, not totally independent, as it can be easily shown that they are subjected to the following relation,

$$\epsilon^{ljb}\mathcal{B}^m_b + \epsilon^{lbm}\mathcal{B}^j_b + \epsilon^{bjm}\mathcal{B}^l_b = 0. \tag{11}$$

Hence we only need two of the terms in the right-hand-side of Eq. (10), and, without loss of generality, the first two terms are chosen. The effective Hamiltonian of the $\bar B \to \mathcal{D}\bar{\mathcal{B}} l\bar\nu$ decays can be obtained by replacing $\mathcal{D}^{ljm}$ in Eq. (8) by $(\mathcal{B}_1)^{ljm} \equiv \epsilon^{ljb}\mathcal{B}^m_b$ and $(\mathcal{B}_2)^{ljm} \equiv \epsilon^{bjm}\mathcal{B}^l_b$, and, consequently, we have

$$\begin{aligned} H_{\rm eff}(\bar B_q \to \mathcal{D}\bar{\mathcal{B}} l\bar\nu) &= \sqrt{6}T_{1\mathcal{D}\bar{\mathcal{B}}}\bar B_m H^i_T \bar{\mathcal{D}}_{ijl}\epsilon^{ljb}\mathcal{B}^m_b + \sqrt{6}T_{2\mathcal{D}\bar{\mathcal{B}}}\bar B_m H^i_T \bar{\mathcal{D}}_{ijl}\epsilon^{bjm}\mathcal{B}^l_b \\ &\quad + \sqrt{6}A_{1\mathcal{D}\bar{\mathcal{B}}}\bar B_i H^i_T \bar{\mathcal{D}}_{mjl}\epsilon^{ljb}\mathcal{B}^m_b + \sqrt{6}A_{2\mathcal{D}\bar{\mathcal{B}}}\bar B_i H^i_T \bar{\mathcal{D}}_{mjl}\epsilon^{bjm}\mathcal{B}^l_b, \end{aligned} \tag{12}$$

where some pre-factors are introduced without lost of generality. Note that the $T_{1\mathcal{D}\bar{\mathcal{B}}}$, $A_{1\mathcal{D}\bar{\mathcal{B}}}$, and $A_{2\mathcal{D}\bar{\mathcal{B}}}$ terms are vanishing and we only have

$$H_{\rm eff}(\bar B_q \to \mathcal{D}\bar{\mathcal{B}} l\bar\nu) = \sqrt{6}T_{\mathcal{D}\bar{\mathcal{B}}}\bar B_m H^i_T \bar{\mathcal{D}}_{ijl}\epsilon^{bjm}\mathcal{B}^l_b, \tag{13}$$

with $T_{2\mathcal{D}\bar{\mathcal{B}}}$ relabeled to $T_{\mathcal{D}\bar{\mathcal{B}}}$.

Similarly for $\bar B \to \mathcal{B}\bar{\mathcal{D}} l\bar\nu$ decays, the $q_i q_k q_l$ flavor in the final state corresponds to $\epsilon_{ika}\bar{\mathcal{B}}^a_l$, $\epsilon_{ial}\bar{\mathcal{B}}^a_k$ and $\epsilon_{akl}\bar{\mathcal{B}}^a_i$, while the last one is redundant, since it can be expressed by the formers using the following relation, $\epsilon_{ika}\bar{\mathcal{B}}^a_l + \epsilon_{ial}\bar{\mathcal{B}}^a_k + \epsilon_{akl}\bar{\mathcal{B}}^a_i = 0$. Hence we replace the $\bar{\mathcal{D}}_{ijl}$ in Eq. (8) by $(\bar{\mathcal{B}}_1)_{ijl} \equiv \epsilon_{ija}\bar{\mathcal{B}}^a_l$ and $(\bar{\mathcal{B}}_2)_{ijl} \equiv \epsilon_{ajl}\bar{\mathcal{B}}^a_i$ and obtain

$$\begin{aligned} H_{\rm eff}(\bar B_q \to \mathcal{B}\bar{\mathcal{D}} l\bar\nu) &= -\sqrt{6}T_{1\mathcal{B}\bar{\mathcal{D}}}\bar B_m H^i_T \epsilon_{ija}\bar{\mathcal{B}}^a_l \mathcal{D}^{ljm} - \sqrt{6}T_{2\mathcal{B}\bar{\mathcal{D}}}\bar B_m H^i \epsilon_{ajl}\bar{\mathcal{B}}^a_i \mathcal{D}^{ljm} \\ &\quad - \sqrt{6}A_{1\mathcal{B}\bar{\mathcal{D}}}\bar B_i H^i_T \epsilon_{mja}\bar{\mathcal{B}}^a_l \mathcal{D}^{ljm} - \sqrt{6}A_{2\mathcal{B}\bar{\mathcal{D}}}\bar B_i H^i \epsilon_{ajl}\bar{\mathcal{B}}^a_m \mathcal{D}^{ljm} \\ &= -\sqrt{6}T_{\mathcal{B}\bar{\mathcal{D}}}\bar B_m H^i_T \epsilon_{ija}\bar{\mathcal{B}}^a_l \mathcal{D}^{ljm} \end{aligned} \tag{14}$$

where the $T_{2\mathcal{B}\bar{\mathcal{D}}}$, $A_{1\mathcal{B}\bar{\mathcal{D}}}$, and $A_{2\mathcal{B}\bar{\mathcal{D}}}$ terms in the equation are vanishing as $\epsilon_{ajl}\mathcal{D}^{ljm} = \epsilon_{ajl}\mathcal{D}^{ljm} = 0$, and $T_{1\mathcal{B}\bar{\mathcal{D}}}$ is relabeled to $T_{\mathcal{B}\bar{\mathcal{D}}}$ in the last step.

To obtain the effective Hamiltonian of $\bar B_q \to \mathcal{B}\bar{\mathcal{B}} l\bar\nu$ decays, we first replace $\bar{\mathcal{D}}_{ijl}$ and $\mathcal{D}^{ljm}$ in Eq. (8) by $(\bar{\mathcal{B}}_1)_{ijl} \equiv \epsilon_{ija}\bar{\mathcal{B}}^a_l$, $(\bar{\mathcal{B}}_2)_{ijl} \equiv \epsilon_{akl}\bar{\mathcal{B}}^a_i$, and $(\mathcal{B}_1)^{lim} \equiv \epsilon^{ljb}\mathcal{B}^m_b$, $(\mathcal{B}_2)^{lim} \equiv \epsilon^{bjm}\mathcal{B}^l_b$, respectively, and obtain

$$\begin{aligned} H_{\rm eff}(\bar B_q \to \mathcal{B}\bar{\mathcal{B}} l\bar\nu) &= -T_{11\mathcal{B}\bar{\mathcal{B}}}\bar B_m H^i_T (\bar{\mathcal{B}}_1)_{ijl}(\mathcal{B}_1)^{ljm} - T_{12\mathcal{B}\bar{\mathcal{B}}}\bar B_m H^i_T (\bar{\mathcal{B}}_1)_{ijl}(\mathcal{B}_2)^{ljm} \\ &\quad - T_{21\mathcal{B}\bar{\mathcal{B}}}\bar B_m H^i_T (\bar{\mathcal{B}}_2)_{ijl}(\mathcal{B}_1)^{ljm} - T_{22\mathcal{B}\bar{\mathcal{B}}}\bar B_m H^i_T (\bar{\mathcal{B}}_2)_{ijl}(\mathcal{B}_2)^{ljm} \\ &\quad - A_{11\mathcal{B}\bar{\mathcal{B}}}\bar B_i H^i_T (\bar{\mathcal{B}}_1)_{mjl}(\mathcal{B}_1)^{ljm} - A_{12\mathcal{B}\bar{\mathcal{B}}}\bar B_i H^i_T (\bar{\mathcal{B}}_1)_{mjl}(\mathcal{B}_2)^{ljm} \\ &\quad - A_{21\mathcal{B}\bar{\mathcal{B}}}\bar B_i H^i_T (\bar{\mathcal{B}}_2)_{mjl}(\mathcal{B}_1)^{ljm} - A_{22\mathcal{B}\bar{\mathcal{B}}}\bar B_i H^i_T (\bar{\mathcal{B}}_2)_{mjl}(\mathcal{B}_2)^{ljm}. \end{aligned} \tag{15}$$

Using the following identity

$$\begin{aligned} -2(\bar{\mathcal{B}}_1)_{ijl}(\mathcal{B}_1)^{ljm} &= (\bar{\mathcal{B}}_2)_{ijl}(\mathcal{B}_1)^{ljm} = -2(\bar{\mathcal{B}}_2)_{ijl}(\mathcal{B}_2)^{ljm}, \\ -2(\bar{\mathcal{B}}_1)_{mjl}(\mathcal{B}_1)^{ljm} &= (\bar{\mathcal{B}}_1)_{mjl}(\mathcal{B}_2)^{ljm} = (\bar{\mathcal{B}}_2)_{mjl}(\mathcal{B}_1)^{ljm} = -2(\bar{\mathcal{B}}_2)_{mjl}(\mathcal{B}_2)^{ljm}, \end{aligned} \tag{16}$$

the above Hamiltonian can be expressed as

$$\begin{aligned} H_{\rm eff}(\bar B_q \to \mathcal{B}\bar{\mathcal{B}} l\bar\nu) &= (-T_{11\mathcal{B}\bar{\mathcal{B}}} + 2T_{21\mathcal{B}\bar{\mathcal{B}}} - T_{22\mathcal{B}\bar{\mathcal{B}}})\bar B_m H^i_T (\bar{\mathcal{B}}_1)_{ijl}(\mathcal{B}_1)^{ljm} - T_{12\mathcal{B}\bar{\mathcal{B}}}\bar B_m H^i_T (\bar{\mathcal{B}}_1)_{ijl}(\mathcal{B}_2)^{ljm} \\ &\quad + (A_{11\mathcal{B}\bar{\mathcal{B}}} - 2A_{12\mathcal{B}\bar{\mathcal{B}}} - 2A_{21\mathcal{B}\bar{\mathcal{B}}} + A_{22\mathcal{B}\bar{\mathcal{B}}})\bar B_i H^i_T (\bar{\mathcal{B}}_1)_{mjl}(\mathcal{B}_1)^{ljm} \\ &= -T_{1\mathcal{B}\bar{\mathcal{B}}}\bar B_m H^i_T \epsilon_{ija}\bar{\mathcal{B}}^a_l \epsilon^{bjm}\mathcal{B}^l_b + T_{2\mathcal{B}\bar{\mathcal{B}}}\bar B_m H^i_T \epsilon_{ija}\bar{\mathcal{B}}^a_l \epsilon^{ljb}\mathcal{B}^m_b + A_{\mathcal{B}\bar{\mathcal{B}}}\bar B_i H^i_T \epsilon_{mja}\bar{\mathcal{B}}^a_l \epsilon^{ljb}\mathcal{B}^m_b, \end{aligned} \tag{17}$$

where the topological amplitudes are redefined as following

$$
\begin{aligned}
T_{1\mathcal{B}\bar{\mathcal{B}}} &\equiv T_{12\mathcal{B}\bar{\mathcal{B}}},\\
T_{2\mathcal{B}\bar{\mathcal{B}}} &\equiv -T_{11\mathcal{B}\bar{\mathcal{B}}} + 2T_{21\mathcal{B}\bar{\mathcal{B}}} - T_{22\mathcal{B}\bar{\mathcal{B}}},\\
A_{\mathcal{B}\bar{\mathcal{B}}} &\equiv A_{11\mathcal{B}\bar{\mathcal{B}}} - 2A_{12\mathcal{B}\bar{\mathcal{B}}} - 2A_{21\mathcal{B}\bar{\mathcal{B}}} + A_{22\mathcal{B}\bar{\mathcal{B}}}.
\end{aligned} \tag{18}
$$

With this all effective Hamiltonians of $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' l\bar{\nu}$ decays with low-lying octet and decuplet baryons are obtained.

The effective Hamiltonian of the $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' \nu\bar{\nu}$ decays can be obtained similarly. We simply give the results in the following equation,

$$
\begin{aligned}
H_{\rm eff}(\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}\nu\bar{\nu}) &= 6PB_{\mathcal{D}\bar{\mathcal{D}}}\bar{B}_m H^k_{PB}\bar{\mathcal{D}}_{kjl}\mathcal{D}^{ljm} + PBA_{\mathcal{D}\bar{\mathcal{D}}}\bar{B}_k H^k_{PB}\bar{\mathcal{D}}_{mjl}\mathcal{D}^{ljm},\\
H_{\rm eff}(\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}\nu\bar{\nu}) &= \sqrt{6}PB_{\mathcal{D}\bar{\mathcal{B}}}\bar{B}_m H^k_{PB}\bar{\mathcal{D}}_{kjl}\epsilon^{bjm}\mathcal{B}^l_b,\\
H_{\rm eff}(\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}\nu\bar{\nu}) &= -\sqrt{6}PB_{\mathcal{B}\bar{\mathcal{D}}}\bar{B}_m H^k_{PB}\epsilon_{kja}\bar{\mathcal{B}}^a_l\mathcal{D}^{ljm},\\
H_{\rm eff}(\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}\nu\bar{\nu}) &= -PB_{1\mathcal{B}\bar{\mathcal{B}}}\bar{B}_m H^k_{PB}\epsilon_{kja}\bar{\mathcal{B}}^a_l\epsilon^{bjm}\mathcal{B}^l_b + PB_{2\mathcal{B}\bar{\mathcal{B}}}\bar{B}_m H^k_{PB}\epsilon_{kja}\bar{\mathcal{B}}^a_l\epsilon^{ljb}\mathcal{B}^m_b + PBA_{\mathcal{B}\bar{\mathcal{B}}}\bar{B}_k H^k_{PB}\epsilon_{mja}\bar{\mathcal{B}}^a_l\epsilon^{ljb}\mathcal{B}^m_b.
\end{aligned} \tag{19}
$$

In summary the effective Hamiltonians of $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' l\bar{\nu}$ and $\mathbf{B}\bar{\mathbf{B}}'\nu\bar{\nu}$ decays for low-lying octet and decuplet baryons are obtained and are shown in Eqs. (8), (13), (14), (17), and (19). The decay amplitudes can be obtained readily by using these effective Hamiltonians. The results of decay amplitudes in terms of these topological amplitudes and relations on the amplitudes will be given explicitly in the next section.

Before we end this section it is important to note that, as shown in Eq. (1), the topological amplitudes PB and T and the topological amplitudes PBA and A should be related in the following manner,

$$
\begin{aligned}
\zeta \equiv \frac{PB_{i\mathcal{B}\bar{\mathcal{B}}}}{T_{i\mathcal{B}\bar{\mathcal{B}}}} = \frac{PBA_{\mathcal{B}\bar{\mathcal{B}}}}{A_{\mathcal{B}\bar{\mathcal{B}}}} = \frac{PB_{\mathcal{B}\bar{\mathcal{D}}}}{T_{\mathcal{B}\bar{\mathcal{D}}}} = \frac{PB_{\mathcal{D}\bar{\mathcal{B}}}}{T_{\mathcal{D}\bar{\mathcal{B}}}} = \frac{PB_{\mathcal{D}\bar{\mathcal{D}}}}{T_{\mathcal{D}\bar{\mathcal{D}}}} = \frac{PBA_{\mathcal{D}\bar{\mathcal{D}}}}{A_{\mathcal{D}\bar{\mathcal{D}}}}\\
= \frac{\alpha_{\rm em}}{2\pi \sin^2\theta_W}\frac{V^*_{ts}V_{tb}}{V_{ub}} D(m_t^2/m_W^2),
\end{aligned} \tag{20}
$$

where numerically we use $|V_{ub}| = 0.0036$ and have $\zeta = -0.037e^{i\phi_3}$, with $\phi_3 = (65.5^{+1.3}_{-1.1})^\circ$ one of the unitary angle in the CKM matrix [36].

B. Modeling the topological amplitudes

In addition to the above decompositions of amplitudes in terms of topological amplitudes, it will be useful to have some numerical results on rates. We will use the available theoretical inputs from Refs. [8,9] in our modeling of the topological amplitudes and we denote them as model 1 and model 2, respectively. They are used as illustration and can be improved when more data are available.

In general the topological amplitudes $T_{1\mathcal{B}\bar{\mathcal{B}}}$, $T_{2\mathcal{B}\bar{\mathcal{B}}}$, and $A_{\mathcal{B}\bar{\mathcal{B}}}$ in $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu}$ decays can be expressed as

$$
\begin{aligned}
T_{i\mathcal{B}\bar{\mathcal{B}}} &= i\frac{G_F}{\sqrt{2}}V_{ub}\bar{l}_L\gamma^\mu\nu_L\bar{u}(p_{\mathcal{B}})\{[g_1^{(i)}\gamma_\mu + ig_2^{(i)}\sigma_{\mu\nu}q^\nu + g_3^{(i)}q_\mu + g_4^{(i)}(p_{\mathcal{B}} + p_{\bar{\mathcal{B}}'})_\mu + g_5^{(i)}(p_{\mathcal{B}} - p_{\bar{\mathcal{B}}'})_\mu]\gamma_5\\
&\quad - [f_1^{(i)}\gamma_\mu + if_2^{(i)}\sigma_{\mu\nu}q^\nu + f_3^{(i)}q_\mu + f_4^{(i)}(p_{\mathcal{B}} + p_{\bar{\mathcal{B}}'})_\mu + f_5^{(i)}(p_{\mathcal{B}} - p_{\bar{\mathcal{B}}'})_\mu]\}v_R(p_{\bar{\mathcal{B}}'}),\\
A_{\mathcal{B}\bar{\mathcal{B}}} &= i\frac{G_F}{\sqrt{2}}V_{ub}\bar{l}_L\gamma^\mu\nu_L\bar{u}(p_{\mathcal{B}})\{[g_1^{(a)}\gamma_\mu + ig_2^{(a)}\sigma_{\mu\nu}q^\nu + g_3^{(a)}q_\mu + g_4^{(a)}(p_{\mathcal{B}} + p_{\bar{\mathcal{B}}'})_\mu + g_5^{(a)}(p_{\mathcal{B}} - p_{\bar{\mathcal{B}}'})_\mu]\gamma_5\\
&\quad - [f_1^{(a)}\gamma_\mu + if_2^{(a)}\sigma_{\mu\nu}q^\nu + f_3^{(a)}q_\mu + f_4^{(a)}(p_{\mathcal{B}} + p_{\bar{\mathcal{B}}'})_\mu + f_5^{(a)}(p_{\mathcal{B}} - p_{\bar{\mathcal{B}}'})_\mu]\}v_R(p_{\bar{\mathcal{B}}'}),
\end{aligned} \tag{21}
$$

with $q \equiv p_{B_q} - p_{\mathbf{B}} - p_{\bar{\mathbf{B}}'}$, $i = 1, 2$, $j = 1, \ldots, 5$, and $f_j^{(i)}$, $g_j^{(i)}$, $f_j^{(a)}$, and $g_j^{(a)}$ denoting form factors. Similarly the topological amplitudes of $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}} l\bar{\nu}$ and $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}} l\bar{\nu}$ decays can be expressed as

$$
\begin{aligned}
T_{\mathcal{B}\bar{\mathcal{D}}} &= i\frac{G_F}{\sqrt{2}}V_{ub}\bar{l}_L\gamma_\mu\nu_L\bar{u}(p_{\mathcal{D}}, \lambda_{\mathcal{B}})\{[g'_1 p_{\mathcal{B}\nu}\gamma_\mu + ig'_2\sigma_{\mu\rho}p_{\mathcal{B}\nu}q^\rho + g'_3 p_{\mathcal{B}\nu}q_\mu + g'_4 p_{\mathcal{B}\nu}p_{\mathcal{B}\mu} + g'_5 g_{\nu\mu} + g'_6 q_\nu\gamma_\mu\\
&\quad + ig'_7\sigma_{\mu\rho}q_\nu q^\rho + g'_8 q_\nu q_\mu + g'_9 q_\nu p_{\mathcal{B}\mu}]\gamma_5 - [f'_1 p_{\mathcal{B}\nu}\gamma_\mu + if'_2\sigma_{\mu\rho}p_{\mathcal{B}\nu}q^\rho + f'_3 p_{\mathcal{B}\nu}q_\mu\\
&\quad + f'_4 p_{\mathcal{B}\nu}p_{\mathcal{B}\mu} + f'_5 g_{\nu\mu} + f'_6 q_\nu\gamma_\mu + if'_7\sigma_{\mu\rho}q_\nu q^\rho + f'_8 q_\nu q_\mu + f'_9 q_\nu p_{\mathcal{B}\mu}]\}v^\nu(p_{\bar{\mathcal{D}}}, \lambda_{\bar{\mathcal{D}}}),
\end{aligned} \tag{22}
$$

and

$$T_{\mathcal{D}\bar{\mathcal{B}}} = i\frac{G_F}{\sqrt{2}}V_{ub}\bar{l}_L\gamma_\mu\nu_L\bar{u}^\nu(p_\mathcal{D},\lambda_\mathcal{D})\{[g''_1 p_{\bar{\mathcal{B}}\nu}\gamma_\mu + ig''_2\sigma_{\mu\rho}p_{\bar{\mathcal{B}}\nu}q^\rho + g''_3 p_{\bar{\mathcal{B}}\nu}q_\mu + g''_4 p_{\bar{\mathcal{B}}\nu}p_{\bar{\mathcal{B}}\mu} + g''_5 g_{\nu\mu} + g''_6 q_\nu\gamma_\mu + ig''_7\sigma_{\mu\rho}q_\nu q^\rho + g''_8 q_\nu q_\mu + g''_9 q_\nu p_{\bar{\mathcal{B}}\mu}]\gamma_5 - [f''_1 p_{\bar{\mathcal{B}}\nu}\gamma_\mu + if''_2\sigma_{\mu\rho}p_{\bar{\mathcal{B}}\nu}q^\rho + f''_3 p_{\bar{\mathcal{B}}\nu}q_\mu + f''_4 p_{\bar{\mathcal{B}}\nu}p_{\bar{\mathcal{B}}\mu} + f''_5 g_{\nu\mu} + f''_6 q_\nu\gamma_\mu + if''_7\sigma_{\mu\rho}q_\nu q^\rho + f''_8 q_\nu q_\mu + f''_9 q_\nu p_{\bar{\mathcal{B}}\mu}]\}v(p_{\bar{\mathcal{B}}},\lambda_{\bar{\mathcal{B}}}), \quad (23)$$

where u^μ, v^μ are the Rarita-Schwinger vector spinors. Finally the tree topological amplitude for $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}' l\bar{\nu}$ decay is given by

$$T_{\mathcal{D}\bar{\mathcal{D}}} = i\frac{G_F}{\sqrt{2}}V_{ub}\bar{l}_L\gamma_\mu\nu_L\bar{u}_\nu(p_\mathcal{D},\lambda_\mathcal{D})\{[g'''_1\gamma_\mu + ig'''_2\sigma_{\mu\rho}q^\rho + g'''_3 q_\mu + g'''_4(p_\mathcal{D}+p_{\bar{\mathcal{D}}'})_\mu + g'''_5(p_\mathcal{D}-p_{\bar{\mathcal{D}}'})_\mu]\gamma_5 - [f'''_1\gamma_\mu + if'''_2\sigma_{\mu\nu}q^\nu + f'''_3 q_\mu + f'''_4(p_\mathcal{D}+p_{\bar{\mathcal{D}}'})_\mu + f'''_5(p_\mathcal{D}-p_{\bar{\mathcal{D}}'})_\mu]\}v^\nu(p_{\bar{\mathcal{D}}},\lambda_{\bar{\mathcal{D}}}) + \ldots, \quad (24)$$

where terms such as $\bar{u}_\nu p^\nu_{\bar{\mathcal{D}}}\{\ldots\}p_{\mathcal{D}\sigma}u^\sigma$, $\bar{u}_\nu q^\nu\{\ldots\}p_{\mathcal{D}\sigma}u^\sigma$, $\bar{u}_\nu p^\nu_{\bar{\mathcal{D}}}\{\ldots\}q_\sigma u^\sigma$, $\bar{u}_\nu q^\nu\{\ldots\}q_\sigma u^\sigma$ are not shown explicitly in the above equation. The annihilation amplitude $A_{\mathcal{D}\bar{\mathcal{D}}}$ can be expressed similarly. Topological amplitudes for loop induced $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'\nu\bar{\nu}$ decays can be obtained using the above equations and Eq. (20).

The topological amplitudes for $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu}$ decays are given in Eq. (21). For illustration we follow Refs. [8,9] to use

$$g_j^{(i)} = \frac{G_j^{(i)}}{t^3}, \qquad f_j^{(i)} = \frac{F_{f_j}^{(i)}}{t^3}, \qquad g_j^{(a)} = f_j^{(a)} = 0, \quad (25)$$

where $G_j^{(i)}$ and $F_j^{(i)}$ are some constants to be specified later, $t \equiv m^2_{\mathbf{B}\bar{\mathbf{B}}'}$ and the last equation corresponds to the $A_{\mathcal{B}\bar{\mathcal{B}}} = PBA_{\mathcal{B}\bar{\mathcal{B}}} = 0$ case. The values of the constants $G_j^{(i)}$ and $F_j^{(i)}$ are extracted from Refs. [8,9] but slightly modified to match the asymptotic relations in Appendix A, where it is known that there are asymptotic relations [37] in the matrix elements of octet and decuplet baryons in the large momentum transfer region, and to match the $B^- \to p\bar{p}l\bar{\nu}$ data. In fact we find that the corresponding $F_{3,4,5}^{(i)}$ used in Ref. [8] do not satisfy the correct asymptotic relations, which can however be satisfied by adding a minus sign to their $F_{3,4,5}^{(i)}$. Nevertheless as we shall see that the modification do not significantly affect the $B^- \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu}$ rates.

The values of $G_j^{(i)}$ and $F_j^{(i)}$ are shown in Table II. Explicitly we use $G_1^{(i)} = \eta_1 m_B(e_\parallel^{(i)}C_{LL} - e_{\overline{\parallel}}^{(i)}C_{RR})/3$, $F_1^{(i)} = \eta_1 m_B(e_\parallel^{(i)}C_{LL} + e_{\overline{\parallel}}^{(i)}C_{RR})/3$, $G_{2,3,4,5}^{(i)} = -F_{2,3,4,5}^{(i)} = -\eta_1 \times 2e_F^{(i)}C_{LR}/3$ with $(C_{LL}, C_{RR}, C_{LR}) = (17.78, -11.67, 6.41)$ GeV^4 [8] for model 1, and $G_1^{(i)} = \eta_2 m_B(e_\parallel^{(i)}D_\parallel - e_{\overline{\parallel}}^{(i)}D_{\overline{\parallel}})/3$, $F_1^{(i)} = \eta_2 m_B(e_\parallel^{(i)}D_\parallel + e_{\overline{\parallel}}^{(i)}D_{\overline{\parallel}})/3$, $G_{2,3,4,5}^{(i)} = -F_{2,3,4,5}^{(i)} = -\eta_2 \times 2e_F^{(i)}D_{2,3,4,5}/3$ with $(D_\parallel, D_{\overline{\parallel}}) = (11.2, 323.3)$ GeV^5 and $D_{2,3,4,5} = (47.7, 442.2, -38.7, -80.7)$ GeV^4 [9] for model 2, where the factors $\eta_1 = 0.93$ and $\eta_2 = 0.75$ are introduced to match the central value of the $B^- \to p\bar{p}l\bar{\nu}$ data and $e^{(i)}_{\parallel,\overline{\parallel},F}$ are given in Eq. (A8). Note that the sign of D_5 is flipped from the one from Ref. [9], to match the definitions of form factors $f_5^{(i)}$ and $g_5^{(i)}$ in Eq. (21).

The topological amplitudes of $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}} l\bar{\nu}$ and $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}} l\bar{\nu}$ decays are given in Eqs. (22) and (23). For simplicity, we only concentrate on the contributions from $g'_{1,2,3,4,5}$, $f'_{1,2,3,4,5}$, $g''_{1,2,3,4,5}$, and $f''_{1,2,3,4,5}$, by assuming that their contributions are dominant. This working assumption can be checked or relaxed when data of $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}} l\bar{\nu}$ and $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}} l\bar{\nu}$ decays become available.

It is known that in the asymptotic limit form factors of octet-octet and octet decuplet are related [37]. As shown in Appendix A in the asymptotic limit $T_{\mathcal{B}\bar{\mathcal{D}}}$, $T_{\mathcal{B}\bar{\mathcal{D}}}$ and $T_{i\mathcal{B}\bar{\mathcal{B}}}$ are related and have similar structure. These impose constrains on the form factors. For simplicity we assume that these form factors have similar forms as the form factors in Eq. (25). Using Eqs. (25), (A12), and (A8), we have

$$g'_{1,2,3,4} = \frac{m_{\bar{\mathcal{D}}}G'_{1,2,3,4}}{t^4}, \qquad g'_5 = \frac{m_{\bar{\mathcal{D}}}G'_5}{t^3}, \qquad f'_{1,2,3,4} = \frac{m_{\bar{\mathcal{D}}}F'_{1,2,3,4}}{t^4}, \qquad f'_5 = \frac{m_{\bar{\mathcal{D}}}F'_5}{t^3},$$
$$g''_{1,2,3,4} = \frac{m_\mathcal{D}G''_{1,2,3,4}}{t^4}, \qquad g''_5 = \frac{m_\mathcal{D}G''_5}{t^3}, \qquad f''_{1,2,3,4} = \frac{m_\mathcal{D}F_{1,2,3,4}}{t^4}, \qquad f''_5 = \frac{m_\mathcal{D}F''_5}{t^3}, \quad (26)$$

with

TABLE II. Values of $G_j^{(i)}$ and $F_j^{(i)}$ for model 1 and model 2. They are extracted from Refs [8,9], respectively, but slightly modified to match the asymptotic relations in Appendix A and to match the $B^- \to p\bar{p}l\bar{\nu}$ data.

Model	$G_1^{(1)}$(GeV^5)	$G_2^{(1)}$(GeV^4)	$G_3^{(1)}$(GeV^4)	$G_4^{(1)}$(GeV^4)	$G_5^{(1)}$(GeV^4)
Model 1	67.02	−1.98	−1.98	−1.98	−1.98
Model 2	−163.00	−11.90	−110.31	9.65	20.13

Model	$G_1^{(2)}$(GeV^5)	$G_2^{(2)}$(GeV^4)	$G_3^{(2)}$(GeV^4)	$G_4^{(2)}$(GeV^4)	$G_5^{(2)}$(GeV^4)
Model 1	96.90	9.89	9.89	9.89	9.89
Model 2	94.07	59.50	551.56	−48.27	−100.66

Model	$F_1^{(1)}$(GeV^5)	$F_2^{(1)}$(GeV^4)	$F_3^{(1)}$(GeV^4)	$F_4^{(1)}$(GeV^4)	$F_5^{(1)}$(GeV^4)
Model 1	−9.06	1.98	1.98	1.98	1.98
Model 2	168.59	11.90	110.31	−9.65	−20.13

Model	$F_1^{(2)}$(GeV^5)	$F_2^{(2)}$(GeV^4)	$F_3^{(2)}$(GeV^4)	$F_4^{(2)}$(GeV^4)	$F_5^{(2)}$(GeV^4)
Model 1	134.94	−9.89	−9.89	−9.89	−9.89
Model 2	−71.72	−59.50	−551.56	48.27	100.66

$$G'_{1,2,3} = -\sqrt{6}G^{(i)}_{1,2,3}, \qquad G'_4 = -\sqrt{6}(G_4^{(i)} + G_5^{(i)}), \qquad G'_5 = \sqrt{\frac{3}{2}}(G_4^{(i)} - G_5^{(i)}),$$
$$F'_{1,2,3} = -\sqrt{6}F^{(i)}_{1,2,3}, \qquad F'_4 = -\sqrt{6}(F_4^{(i)} + F_5^{(i)}), \qquad F'_5 = \sqrt{\frac{3}{2}}(F_4^{(i)} - F_5^{(i)}), \tag{27}$$

but with $(e_\parallel^{(i)}, e_{\overline{\parallel}}^{(i)}, e_F^{(i)})$ in $G_j^{(i)}, F_j^{(i)}$ replaced by $(e'_\parallel, e'_{\overline{\parallel}}, e'_F)$, and

$$G''_{1,2,3} = -\sqrt{6}G^{(i)}_{1,2,3}, \qquad G''_4 = \sqrt{6}(G_5^{(i)} - G_4^{(i)}), \qquad G''_5 = \sqrt{\frac{3}{2}}(G_4^{(i)} + G_5^{(i)}),$$
$$F''_{1,2,3} = -\sqrt{6}F^{(i)}_{1,2,3}, \qquad F''_4 = \sqrt{6}(F_5^{(i)} - F_4^{(i)}), \qquad F''_5 = \sqrt{\frac{3}{2}}(F_4^{(i)} + F_5^{(i)}), \tag{28}$$

but with $(e_\parallel^{(i)}, e_{\overline{\parallel}}^{(i)}, e_F^{(i)})$ in $G_j^{(i)}, F_j^{(i)}$ replaced by $(e''_\parallel, e''_{\overline{\parallel}}, e''_F)$. Note that the above constants are related in the asymptotic limit and, consequently, inputs from model 1 and 2 have been used in the above relations. The values of these constants in model 1 and 2 are given in Table III.

In the model calculations of $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}' l\nu$ and $\mathcal{D}\bar{\mathcal{D}}'\nu\bar{\nu}$ decay rates, we use Eq. (24) for the tree topological amplitude, where we neglect terms, such as $\bar{u}_\nu p^\nu_{\bar{\mathcal{D}}}\{\ldots\}p_{\mathcal{D}\sigma}u^\sigma$, $\bar{u}_\nu q^\nu\{\ldots\}p_{\mathcal{D}\sigma}u^\sigma$, $\bar{u}_\nu p^\nu_{\bar{\mathcal{D}}}\{\ldots\}q_\sigma u^\sigma$, $\bar{u}_\nu q^\nu\{\ldots\}q_\sigma u^\sigma$, for simplicity. This working assumption can be checked or modified once data is available. As in $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu}$ decays, we neglect the contribution from the annihilation topological amplitude, $A_{\mathcal{D}\bar{\mathcal{D}}}$. Using Eqs. (25), (A8), and (A13), the form factors are given by

$$g'''_j = m_{\mathcal{D}} m_{\bar{\mathcal{D}}} \frac{G'''_j}{t^4}, \qquad f'''_j = m_{\mathcal{D}} m_{\bar{\mathcal{D}}} \frac{F'''_j}{t^4}, \tag{29}$$

with

$$G'''_j = -3G_j^{(i)}, \qquad F'''_j = -3F_j^{(i)}, \tag{30}$$

but with $(e_\parallel^{(i)}, e_{\overline{\parallel}}^{(i)}, e_F^{(i)})$ in $G_j^{(i)}, F_j^{(i)}$ replaced by $(e'''_\parallel, e'''_{\overline{\parallel}}, e'''_F)$. Note that in the asymptotic limit the above form factors are related to those in $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu}$ decays via Eq. (A8), and, consequently, inputs from model 1 and 2 have been used. The values of these constants in model 1 and 2 are given in Table IV.

III. RESULTS ON AMPLITUDES

A. Decay amplitudes in terms of topological amplitudes

Using the above Hamiltonian the decompositions of amplitudes for $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu}, \mathcal{B}\bar{\mathcal{D}} l\bar{\nu}, \mathcal{D}\bar{\mathcal{B}} l\bar{\nu}, \mathcal{D}\bar{\mathcal{D}}' l\bar{\nu}$, and $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}'\nu\bar{\nu}, \mathcal{B}\bar{\mathcal{D}}\nu\bar{\nu}, \mathcal{D}\bar{\mathcal{B}}\nu\bar{\nu}, \mathcal{D}\bar{\mathcal{D}}'\nu\bar{\nu}$ decays are shown in Tables V–VIII. These tables are some of the main results of this work.

TABLE III. Values of G'_i, F'_i, G''_i, F''_i for model 1 and model 2.

Model	$G'_1(\mathrm{GeV}^5)$	$G'_2(\mathrm{GeV}^4)$	$G'_3(\mathrm{GeV}^4)$	$G'_4(\mathrm{GeV}^4)$	$G'_5(\mathrm{GeV}^4)$
Model 1	−24.39	4.85	4.85	9.69	0
Model 2	−209.90	29.15	270.21	−72.96	−12.83
Model	$F'_1(\mathrm{GeV}^5)$	$F'_2(\mathrm{GeV}^4)$	$F'_3(\mathrm{GeV}^4)$	$F'_4(\mathrm{GeV}^4)$	$F'_5(\mathrm{GeV}^4)$
Model 1	−117.58	−4.85	−4.85	−9.69	0
Model 2	196.21	−29.15	−270.21	72.96	12.83
Model	$G''_1(\mathrm{GeV}^5)$	$G''_2(\mathrm{GeV}^4)$	$G''_3(\mathrm{GeV}^4)$	$G''_4(\mathrm{GeV}^4)$	$G''_5(\mathrm{GeV}^4)$
Model 1	−24.39	4.85	4.85	0	−4.85
Model 2	−209.90	29.15	270.21	25.66	36.48
Model	$F''_1(\mathrm{GeV}^5)$	$F''_2(\mathrm{GeV}^4)$	$F''_3(\mathrm{GeV}^4)$	$F''_4(\mathrm{GeV}^4)$	$F''_5(\mathrm{GeV}^4)$
Model 1	−117.58	−4.85	−4.85	0	4.85
Model 2	196.21	−29.15	−270.21	−25.66	−36.48

TABLE IV. Values of G'''_i, F'''_i for model 1 and model 2.

Model	$G'''_1(\mathrm{GeV}^5)$	$G'''_2(\mathrm{GeV}^4)$	$G'''_3(\mathrm{GeV}^4)$	$G'''_4(\mathrm{GeV}^4)$	$G'''_5(\mathrm{GeV}^4)$
Model 1	−115.47	5.94	5.94	5.94	5.94
Model 2	115.96	35.70	330.94	−28.96	−60.39
Model	$F'''_1(\mathrm{GeV}^5)$	$F'''_2(\mathrm{GeV}^4)$	$F'''_3(\mathrm{GeV}^4)$	$F'''_4(\mathrm{GeV}^4)$	$F'''_5(\mathrm{GeV}^4)$
Model 1	−58.41	−5.94	−5.94	−5.94	−5.94
Model 2	−132.73	−35.70	−330.94	28.96	60.39

As shown in Table V we have three topological amplitudes, $T_{2\mathcal{B}\bar{\mathcal{B}}}$, $T_{1\mathcal{B}\bar{\mathcal{B}}}$, and $A_{\mathcal{B}\bar{\mathcal{B}}}$, in $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu}$ decays, and three topological amplitudes, $PB_{2\mathcal{B}\bar{\mathcal{B}}}$, $PB_{1\mathcal{B}\bar{\mathcal{B}}}$, and $PBA_{\mathcal{B}\bar{\mathcal{B}}}$, in $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' \nu\bar{\nu}$ decays. As shown in Table VI we need one topological amplitude, $T_{\mathcal{B}\bar{\mathcal{D}}}$, in $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}} l\bar{\nu}$ decays, and one topological amplitude, $PB_{\mathcal{B}\bar{\mathcal{D}}}$, in $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}} \nu\bar{\nu}$ decays. Similarly, as shown in Table VII we have one topological amplitude, $T_{\mathcal{D}\bar{\mathcal{B}}}$, in $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}} l\bar{\nu}$ decays, and one topological amplitude, $PB_{\mathcal{D}\bar{\mathcal{B}}}$, in $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}} \nu\bar{\nu}$ decays. Finally as shown in Table VI we have two topological amplitude, $T_{\mathcal{D}\bar{\mathcal{D}}}$ and $A_{\mathcal{D}\bar{\mathcal{D}}}$, in $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}' l\bar{\nu}$ decays, and two topological amplitudes, $PB_{\mathcal{D}\bar{\mathcal{D}}}$ and $PBA_{\mathcal{D}\bar{\mathcal{D}}}$, in $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}' \nu\bar{\nu}$ decays.

As the numbers of independent topological amplitudes are highly limited comparing to the numbers of the decay modes, there are plenty of relations on $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' l\bar{\nu}$ and $\mathbf{B}\bar{\mathbf{B}}' \nu\bar{\nu}$ decay amplitudes. These relations will be given in the following discussion.

B. Relations of decay amplitudes

As noted previously since the number of topological amplitudes are quite limited, relations of decay amplitudes are expected. The following relations are obtained by using the decomposition of amplitudes shown in Tables V, VI, VII, and VIII.

In $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu}$ decays, we have the following relations on amplitudes,

$$A(\bar{B}^0 \to p\bar{n} l\bar{\nu}) = A(\bar{B}^0_s \to \Sigma^+\overline{\Xi^0} l\bar{\nu}) = \sqrt{2} A(\bar{B}^0_s \to \Sigma^0\overline{\Xi^-} l\bar{\nu}), \tag{31}$$

$$A(B^- \to n\bar{n} l\bar{\nu}) = A(B^- \to \Xi^0\overline{\Xi^0} l\bar{\nu}), \tag{32}$$

$$A(\bar{B}^0 \to \Xi^0\overline{\Xi^-} l\bar{\nu}) = \sqrt{2} A(\bar{B}^0_s \to p\overline{\Sigma^0} l\bar{\nu}) = A(\bar{B}^0_s \to n\overline{\Sigma^-} l\bar{\nu}), \tag{33}$$

$$A(B^- \to p\bar{p} l\bar{\nu}) = A(B^- \to \Sigma^+\overline{\Sigma^+} l\bar{\nu}), \tag{34}$$

$$A(\bar{B}^0 \to \Sigma^+\overline{\Sigma^0} l\bar{\nu}) = -A(\bar{B}^0 \to \Sigma^0\overline{\Sigma^-} l\bar{\nu}) = \sqrt{3} A(\bar{B}^0_s \to p\bar{\Lambda} l\bar{\nu}), \tag{35}$$

TABLE V. Topological amplitudes for $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu}$ and $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' \nu\bar{\nu}$ decays.

Mode	$A(\bar{B}_{q'} \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu})$	Mode	$A(\bar{B}_{q'} \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu})$
$B^- \to p\bar{p}l\bar{\nu}$	$T_{1\mathcal{B}\bar{\mathcal{B}}} + T_{2\mathcal{B}\bar{\mathcal{B}}} + A_{\mathcal{B}\bar{\mathcal{B}}}$	$B^- \to n\bar{n}l\bar{\nu}$	$T_{1\mathcal{B}\bar{\mathcal{B}}} + A_{\mathcal{B}\bar{\mathcal{B}}}$
$B^- \to \Sigma^+\overline{\Sigma^+}l\bar{\nu}$	$T_{1\mathcal{B}\bar{\mathcal{B}}} + T_{2\mathcal{B}\bar{\mathcal{B}}} + A_{\mathcal{B}\bar{\mathcal{B}}}$	$B^- \to \Sigma^0\overline{\Sigma^0}l\bar{\nu}$	$\frac{1}{2}(T_{1\mathcal{B}\bar{\mathcal{B}}} + T_{2\mathcal{B}\bar{\mathcal{B}}}) + A_{\mathcal{B}\bar{\mathcal{B}}}$
$B^- \to \Sigma^-\overline{\Sigma^-}l\bar{\nu}$	$A_{\mathcal{B}\bar{\mathcal{B}}}$	$B^- \to \Xi^-\overline{\Xi^-}l\bar{\nu}$	$A_{\mathcal{B}\bar{\mathcal{B}}}$
$B^- \to \Sigma^0\bar{\Lambda}l\bar{\nu}$	$-\frac{1}{2\sqrt{3}}(T_{1\mathcal{B}\bar{\mathcal{B}}} - T_{2\mathcal{B}\bar{\mathcal{B}}})$	$B^- \to \Xi^0\overline{\Xi^0}l\bar{\nu}$	$T_{1\mathcal{B}\bar{\mathcal{B}}} + A_{\mathcal{B}\bar{\mathcal{B}}}$
$B^- \to \Lambda\overline{\Sigma^0}l\bar{\nu}$	$-\frac{1}{2\sqrt{3}}(T_{1\mathcal{B}\bar{\mathcal{B}}} - T_{2\mathcal{B}\bar{\mathcal{B}}})$	$B^- \to \Lambda\bar{\Lambda}l\bar{\nu}$	$\frac{1}{6}(5T_{1\mathcal{B}\bar{\mathcal{B}}} + T_{2\mathcal{B}\bar{\mathcal{B}}}) + A_{\mathcal{B}\bar{\mathcal{B}}}$
$\bar{B}^0 \to p\bar{n}l\bar{\nu}$	$T_{2\mathcal{B}\bar{\mathcal{B}}}$	$\bar{B}^0 \to \Sigma^+\overline{\Sigma^0}l\bar{\nu}$	$-\frac{1}{\sqrt{2}}(T_{1\mathcal{B}\bar{\mathcal{B}}} + T_{2\mathcal{B}\bar{\mathcal{B}}})$
$\bar{B}^0 \to \Sigma^+\bar{\Lambda}l\bar{\nu}$	$-\frac{1}{\sqrt{6}}(T_{1\mathcal{B}\bar{\mathcal{B}}} - T_{2\mathcal{B}\bar{\mathcal{B}}})$	$\bar{B}^0 \to \Sigma^0\overline{\Sigma^-}l\bar{\nu}$	$\frac{1}{\sqrt{2}}(T_{1\mathcal{B}\bar{\mathcal{B}}} + T_{2\mathcal{B}\bar{\mathcal{B}}})$
$\bar{B}^0 \to \Lambda\overline{\Sigma^-}l\bar{\nu}$	$-\frac{1}{\sqrt{6}}(T_{1\mathcal{B}\bar{\mathcal{B}}} - T_{2\mathcal{B}\bar{\mathcal{B}}})$	$\bar{B}^0 \to \Xi^0\overline{\Xi^-}l\bar{\nu}$	$-T_{1\mathcal{B}\bar{\mathcal{B}}}$
$\bar{B}^0_s \to p\overline{\Sigma^0}l\bar{\nu}$	$-\frac{1}{\sqrt{2}}T_{1\mathcal{B}\bar{\mathcal{B}}}$	$\bar{B}^0_s \to p\bar{\Lambda}l\bar{\nu}$	$-\frac{1}{\sqrt{6}}(T_{1\mathcal{B}\bar{\mathcal{B}}} + 2T_{2\mathcal{B}\bar{\mathcal{B}}})$
$\bar{B}^0_s \to n\overline{\Sigma^-}l\bar{\nu}$	$-T_{1\mathcal{B}\bar{\mathcal{B}}}$	$\bar{B}^0_s \to \Sigma^+\overline{\Xi^0}l\bar{\nu}$	$T_{2\mathcal{B}\bar{\mathcal{B}}}$
$\bar{B}^0_s \to \Sigma^0\overline{\Xi^-}l\bar{\nu}$	$\frac{1}{\sqrt{2}}T_{2\mathcal{B}\bar{\mathcal{B}}}$	$\bar{B}^0_s \to \Lambda\overline{\Xi^-}l\bar{\nu}$	$\frac{1}{\sqrt{6}}(2T_{1\mathcal{B}\bar{\mathcal{B}}} + T_{2\mathcal{B}\bar{\mathcal{B}}})$

Mode	$A(\bar{B}_{q'} \to \mathcal{B}\bar{\mathcal{B}}' \nu\bar{\nu})$	Mode	$A(\bar{B}_{q'} \to \mathcal{B}\bar{\mathcal{B}}' \nu\bar{\nu})$
$B^- \to \Sigma^0\bar{p}\nu\bar{\nu}$	$-\frac{1}{\sqrt{2}}PB_{1\mathcal{B}\bar{\mathcal{B}}}$	$B^- \to \Sigma^-\bar{n}\nu\bar{\nu}$	$-PB_{1\mathcal{B}\bar{\mathcal{B}}}$
$B^- \to \Xi^0\overline{\Sigma^+}\nu\bar{\nu}$	$PB_{2\mathcal{B}\bar{\mathcal{B}}}$	$B^- \to \Xi^-\overline{\Sigma^0}\nu\bar{\nu}$	$\frac{1}{\sqrt{2}}PB_{2\mathcal{B}\bar{\mathcal{B}}}$
$B^- \to \Xi^-\bar{\Lambda}\nu\bar{\nu}$	$\frac{1}{\sqrt{6}}(2PB_{1\mathcal{B}\bar{\mathcal{B}}} + PB_{2\mathcal{B}\bar{\mathcal{B}}})$	$B^- \to \Lambda\bar{p}\nu\bar{\nu}$	$-\frac{1}{\sqrt{6}}(PB_{1\mathcal{B}\bar{\mathcal{B}}} + 2PB_{2\mathcal{B}\bar{\mathcal{B}}})$
$\bar{B}^0 \to \Sigma^+\bar{p}\nu\bar{\nu}$	$-PB_{1\mathcal{B}\bar{\mathcal{B}}}$	$\bar{B}^0 \to \Sigma^0\bar{n}\nu\bar{\nu}$	$\frac{1}{\sqrt{2}}PB_{1\mathcal{B}\bar{\mathcal{B}}}$
$\bar{B}^0 \to \Xi^0\overline{\Sigma^0}\nu\bar{\nu}$	$-\frac{1}{\sqrt{2}}PB_{2\mathcal{B}\bar{\mathcal{B}}}$	$\bar{B}^0 \to \Xi^0\bar{\Lambda}\nu\bar{\nu}$	$\frac{1}{\sqrt{6}}(2PB_{1\mathcal{B}\bar{\mathcal{B}}} + PB_{2\mathcal{B}\bar{\mathcal{B}}})$
$\bar{B}^0 \to \Xi^-\overline{\Sigma^-}\nu\bar{\nu}$	$PB_{2\mathcal{B}\bar{\mathcal{B}}}$	$\bar{B}^0 \to \Lambda\bar{n}\nu\bar{\nu}$	$-\frac{1}{\sqrt{6}}(PB_{1\mathcal{B}\bar{\mathcal{B}}} + 2PB_{2\mathcal{B}\bar{\mathcal{B}}})$
$\bar{B}^0_s \to p\bar{p}\nu\bar{\nu}$	$PBA_{\mathcal{B}\bar{\mathcal{B}}}$	$\bar{B}^0_s \to n\bar{n}\nu\bar{\nu}$	$PBA_{\mathcal{B}\bar{\mathcal{B}}}$
$\bar{B}^0_s \to \Sigma^+\overline{\Sigma^+}\nu\bar{\nu}$	$PB_{1\mathcal{B}\bar{\mathcal{B}}} + PBA_{\mathcal{B}\bar{\mathcal{B}}}$	$\bar{B}^0_s \to \Sigma^0\overline{\Sigma^0}\nu\bar{\nu}$	$PB_{1\mathcal{B}\bar{\mathcal{B}}} + PBA_{\mathcal{B}\bar{\mathcal{B}}}$
$\bar{B}^0_s \to \Sigma^-\overline{\Sigma^-}\nu\bar{\nu}$	$PB_{1\mathcal{B}\bar{\mathcal{B}}} + PBA_{\mathcal{B}\bar{\mathcal{B}}}$	$\bar{B}^0_s \to \Xi^0\overline{\Xi^0}\nu\bar{\nu}$	$PB_{1\mathcal{B}\bar{\mathcal{B}}} + PB_{2\mathcal{B}\bar{\mathcal{B}}} + PBA_{\mathcal{B}\bar{\mathcal{B}}}$
$\bar{B}^0_s \to \Xi^-\overline{\Xi^-}\nu\bar{\nu}$	$PB_{1\mathcal{B}\bar{\mathcal{B}}} + PB_{2\mathcal{B}\bar{\mathcal{B}}} + PBA_{\mathcal{B}\bar{\mathcal{B}}}$	$\bar{B}^0_s \to \Lambda\bar{\Lambda}\nu\bar{\nu}$	$\frac{1}{3}(PB_{1\mathcal{B}\bar{\mathcal{B}}} + 2PB_{2\mathcal{B}\bar{\mathcal{B}}}) + PBA_{\mathcal{B}\bar{\mathcal{B}}}$

$$\begin{aligned}\sqrt{2}A(B^- \to \Sigma^0\bar{\Lambda}l\bar{\nu}) &= \sqrt{2}A(B^- \to \Lambda\overline{\Sigma^0}l\bar{\nu}) = A(\bar{B}^0 \to \Sigma^+\bar{\Lambda}l\bar{\nu}) \\ &= A(\bar{B}^0 \to \Lambda\overline{\Sigma^-}l\bar{\nu}),\end{aligned} \tag{36}$$

$$A(B^- \to \Sigma^-\overline{\Sigma^-}l\bar{\nu}) = A(B^- \to \Xi^-\overline{\Xi^-}l\bar{\nu}), \tag{37}$$

and

$$\begin{aligned}A(B^- \to p\bar{p}l\bar{\nu}) &= A(\bar{B}^0 \to p\bar{n}l\bar{\nu}) + A(B^- \to n\bar{n}l\bar{\nu}) \\ &= 2A(B^- \to \Sigma^0\overline{\Sigma^0}l\bar{\nu}) + A(B^- \to \Sigma^-\overline{\Sigma^-}l\bar{\nu}), \\ 2\sqrt{3}A(B^- \to \Sigma^0\bar{\Lambda}l\bar{\nu}) &= A(\bar{B}^0 \to p\bar{n}l\bar{\nu}) + A(\bar{B}^0 \to \Xi^0\overline{\Xi^-}l\bar{\nu}), \\ \sqrt{6}A(B^- \to \Lambda\bar{\Lambda}l\bar{\nu}) &= A(\bar{B}^0 \to \Sigma^+\bar{\Lambda}l\bar{\nu}) + \sqrt{6}A(B^- \to n\bar{n}l\bar{\nu}), \\ -\sqrt{6}A(\bar{B}^0_s \to p\bar{\Lambda}l\bar{\nu}) &= 2A(\bar{B}^0_s \to \Sigma^+\overline{\Xi^0}l\bar{\nu}) - A(\bar{B}^0_s \to n\overline{\Sigma^-}l\bar{\nu}), \\ \sqrt{6}A(\bar{B}^0_s \to \Lambda\overline{\Xi^-}l\bar{\nu}) &= A(\bar{B}^0_s \to \Sigma^+\overline{\Xi^0}l\bar{\nu}) - 2A(\bar{B}^0_s \to n\overline{\Sigma^-}l\bar{\nu}).\end{aligned} \tag{38}$$

Similarly, for $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' \nu\bar{\nu}$ decays, we have

$$\begin{aligned}A(B^- \to \Xi^0\overline{\Sigma^+}\nu\bar{\nu}) &= \sqrt{2}A(B^- \to \Xi^-\overline{\Sigma^0}\nu\bar{\nu}) = -\sqrt{2}A(\bar{B}^0 \to \Xi^0\overline{\Sigma^0}\nu\bar{\nu}) \\ &= A(\bar{B}^0 \to \Xi^-\overline{\Sigma^-}\nu\bar{\nu}),\end{aligned} \tag{39}$$

TABLE VI. Topological amplitudes for $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}l\bar{\nu}$ and $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}\nu\bar{\nu}$ decays.

Mode	$A(\bar{B}_{q'} \to \mathcal{B}\bar{\mathcal{D}}l\bar{\nu})$	Mode	$A(\bar{B}_{q'} \to \mathcal{B}\bar{\mathcal{D}}l\bar{\nu})$
$B^- \to p\overline{\Delta^+}l\bar{\nu}$	$-\sqrt{2}T_{\mathcal{B}\bar{\mathcal{D}}}$	$B^- \to n\overline{\Delta^0}l\bar{\nu}$	$-\sqrt{2}T_{\mathcal{B}\bar{\mathcal{D}}}$
$B^- \to \Sigma^+\overline{\Sigma^{*+}}l\bar{\nu}$	$\sqrt{2}T_{\mathcal{B}\bar{\mathcal{D}}}$	$B^- \to \Sigma^0\overline{\Sigma^{*0}}l\bar{\nu}$	$-\frac{1}{\sqrt{2}}T_{\mathcal{B}\bar{\mathcal{D}}}$
$B^- \to \Xi^0\overline{\Xi^{*0}}l\bar{\nu}$	$\sqrt{2}T_{\mathcal{B}\bar{\mathcal{D}}}$	$B^- \to \Lambda\overline{\Sigma^{*0}}l\bar{\nu}$	$\sqrt{\frac{3}{2}}T_{\mathcal{B}\bar{\mathcal{D}}}$
$\bar{B}^0 \to p\overline{\Delta^0}l\bar{\nu}$	$-\sqrt{2}T_{\mathcal{B}\bar{\mathcal{D}}}$	$\bar{B}^0 \to n\overline{\Delta^-}l\bar{\nu}$	$-\sqrt{6}T_{\mathcal{B}\bar{\mathcal{D}}}$
$\bar{B}^0 \to \Sigma^+\overline{\Sigma^{*0}}l\bar{\nu}$	$T_{\mathcal{B}\bar{\mathcal{D}}}$	$\bar{B}^0 \to \Sigma^0\overline{\Sigma^{*-}}l\bar{\nu}$	$-T_{\mathcal{B}\bar{\mathcal{D}}}$
$\bar{B}^0 \to \Xi^0\overline{\Xi^{*-}}l\bar{\nu}$	$\sqrt{2}T_{\mathcal{B}\bar{\mathcal{D}}}$	$\bar{B}^0 \to \Lambda\overline{\Sigma^{*-}}l\bar{\nu}$	$\sqrt{3}T_{\mathcal{B}\bar{\mathcal{D}}}$
$\bar{B}^0_s \to p\overline{\Sigma^{*0}}l\bar{\nu}$	$-T_{\mathcal{B}\bar{\mathcal{D}}}$	$\bar{B}^0_s \to n\overline{\Sigma^{*-}}l\bar{\nu}$	$-\sqrt{2}T_{\mathcal{B}\bar{\mathcal{D}}}$
$\bar{B}^0_s \to \Sigma^+\overline{\Xi^{*0}}l\bar{\nu}$	$\sqrt{2}T_{\mathcal{B}\bar{\mathcal{D}}}$	$\bar{B}^0_s \to \Sigma^0\overline{\Xi^{*-}}l\bar{\nu}$	$-T_{\mathcal{B}\bar{\mathcal{D}}}$
$\bar{B}^0_s \to \Xi^0\overline{\Omega^-}l\bar{\nu}$	$\sqrt{6}T_{\mathcal{B}\bar{\mathcal{D}}}$	$\bar{B}^0_s \to \Lambda\overline{\Xi^{*-}}l\bar{\nu}$	$\sqrt{3}T_{\mathcal{B}\bar{\mathcal{D}}}$

Mode	$A(\bar{B}_{q'} \to \mathcal{B}\bar{\mathcal{D}}\nu\bar{\nu})$	Mode	$A(\bar{B}_{q'} \to \mathcal{B}\bar{\mathcal{D}}\nu\bar{\nu})$
$B^- \to \Sigma^+\overline{\Delta^{++}}\nu\bar{\nu}$	$-\sqrt{6}PB_{\mathcal{B}\bar{\mathcal{D}}}$	$B^- \to \Sigma^0\overline{\Delta^+}\nu\bar{\nu}$	$2PB_{\mathcal{B}\bar{\mathcal{D}}}$
$B^- \to \Sigma^-\overline{\Delta^0}\nu\bar{\nu}$	$\sqrt{2}PB_{\mathcal{B}\bar{\mathcal{D}}}$	$B^- \to \Xi^0\overline{\Sigma^{*+}}\nu\bar{\nu}$	$-\sqrt{2}PB_{\mathcal{B}\bar{\mathcal{D}}}$
$B^- \to \Xi^-\overline{\Sigma^{*0}}\nu\bar{\nu}$	$PB_{\mathcal{B}\bar{\mathcal{D}}}$		
$\bar{B}^0 \to \Sigma^+\overline{\Delta^+}\nu\bar{\nu}$	$-\sqrt{2}PB_{\mathcal{B}\bar{\mathcal{D}}}$	$\bar{B}^0 \to \Sigma^0\overline{\Delta^0}\nu\bar{\nu}$	$2PB_{\mathcal{B}\bar{\mathcal{D}}}$
$\bar{B}^0 \to \Sigma^-\overline{\Delta^-}\nu\bar{\nu}$	$\sqrt{6}PB_{\mathcal{B}\bar{\mathcal{D}}}$	$\bar{B}^0 \to \Xi^0\overline{\Sigma^{*0}}\nu\bar{\nu}$	$-PB_{\mathcal{B}\bar{\mathcal{D}}}$
$\bar{B}^0 \to \Xi^-\overline{\Sigma^{*-}}\nu\bar{\nu}$	$\sqrt{2}PB_{\mathcal{B}\bar{\mathcal{D}}}$		
$\bar{B}^0_s \to \Sigma^+\overline{\Sigma^{*+}}\nu\bar{\nu}$	$-\sqrt{2}PB_{\mathcal{B}\bar{\mathcal{D}}}$	$\bar{B}^0_s \to \Sigma^0\overline{\Sigma^{*0}}\nu\bar{\nu}$	$\sqrt{2}PB_{\mathcal{B}\bar{\mathcal{D}}}$
$\bar{B}^0_s \to \Sigma^-\overline{\Sigma^{*-}}\nu\bar{\nu}$	$\sqrt{2}PB_{\mathcal{B}\bar{\mathcal{D}}}$	$\bar{B}^0_s \to \Xi^0\overline{\Xi^{*0}}\nu\bar{\nu}$	$-\sqrt{2}PB_{\mathcal{B}\bar{\mathcal{D}}}$
$\bar{B}^0_s \to \Xi^-\overline{\Xi^{*-}}\nu\bar{\nu}$	$\sqrt{2}PB_{\mathcal{B}\bar{\mathcal{D}}}$		

TABLE VII. Topological amplitudes for $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}l\bar{\nu}$ and $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}\nu\bar{\nu}$ decays.

Mode	$A(\bar{B}_{q'} \to \mathcal{D}\bar{\mathcal{B}}l\bar{\nu})$	Mode	$A(\bar{B}_{q'} \to \mathcal{D}\bar{\mathcal{B}}l\bar{\nu})$
$B^- \to \Delta^+\bar{p}l\bar{\nu}$	$-\sqrt{2}T_{\mathcal{D}\bar{\mathcal{B}}}$	$B^- \to \Delta^0\bar{n}l\bar{\nu}$	$-\sqrt{2}T_{\mathcal{D}\bar{\mathcal{B}}}$
$B^- \to \Sigma^{*+}\overline{\Sigma^+}l\bar{\nu}$	$\sqrt{2}T_{\mathcal{D}\bar{\mathcal{B}}}$	$B^- \to \Sigma^{*0}\overline{\Sigma^0}l\bar{\nu}$	$-\frac{1}{\sqrt{2}}T_{\mathcal{D}\bar{\mathcal{B}}}$
$B^- \to \Xi^{*0}\overline{\Xi^0}l\bar{\nu}$	$\sqrt{2}T_{\mathcal{D}\bar{\mathcal{B}}}$	$B^- \to \Sigma^{*0}\bar{\Lambda}l\bar{\nu}$	$\sqrt{\frac{3}{2}}T_{\mathcal{D}\bar{\mathcal{B}}}$
$\bar{B}^0 \to \Delta^{++}\bar{p}l\bar{\nu}$	$\sqrt{6}T_{\mathcal{D}\bar{\mathcal{B}}}$	$\bar{B}^0 \to \Delta^+\bar{n}l\bar{\nu}$	$\sqrt{2}T_{\mathcal{D}\bar{\mathcal{B}}}$
$\bar{B}^0 \to \Sigma^{*+}\overline{\Sigma^0}l\bar{\nu}$	$-T_{\mathcal{D}\bar{\mathcal{B}}}$	$\bar{B}^0 \to \Sigma^{*0}\overline{\Sigma^-}l\bar{\nu}$	$-T_{\mathcal{D}\bar{\mathcal{B}}}$
$\bar{B}^0 \to \Xi^{*0}\overline{\Xi^-}l\bar{\nu}$	$-\sqrt{2}T_{\mathcal{D}\bar{\mathcal{B}}}$	$\bar{B}^0 \to \Sigma^{*+}\bar{\Lambda}l\bar{\nu}$	$-\sqrt{3}T_{\mathcal{D}\bar{\mathcal{B}}}$
$\bar{B}^0_s \to \Delta^{++}\overline{\Sigma^+}l\bar{\nu}$	$-\sqrt{6}T_{\mathcal{D}\bar{\mathcal{B}}}$	$\bar{B}^0_s \to \Delta^+\overline{\Sigma^0}l\bar{\nu}$	$2T_{\mathcal{D}\bar{\mathcal{B}}}$
$\bar{B}^0_s \to \Delta^0\overline{\Sigma^-}l\bar{\nu}$	$\sqrt{2}T_{\mathcal{D}\bar{\mathcal{B}}}$	$\bar{B}^0_s \to \Sigma^{*+}\overline{\Xi^0}l\bar{\nu}$	$-\sqrt{2}T_{\mathcal{D}\bar{\mathcal{B}}}$
$\bar{B}^0_s \to \Sigma^{*0}\overline{\Xi^-}l\bar{\nu}$	$T_{\mathcal{D}\bar{\mathcal{B}}}$		

Mode	$A(\bar{B}_{q'} \to \mathcal{D}\bar{\mathcal{B}}\nu\bar{\nu})$	Mode	$A(\bar{B}_{q'} \to \mathcal{D}\bar{\mathcal{B}}\nu\bar{\nu})$
$B^- \to \Sigma^{*0}\bar{p}\nu\bar{\nu}$	$-PB_{\mathcal{D}\bar{\mathcal{B}}}$	$B^- \to \Sigma^{*-}\bar{n}\nu\bar{\nu}$	$-\sqrt{2}PB_{\mathcal{D}\bar{\mathcal{B}}}$
$B^- \to \Xi^{*0}\overline{\Sigma^+}\nu\bar{\nu}$	$\sqrt{2}PB_{\mathcal{D}\bar{\mathcal{B}}}$	$B^- \to \Xi^{*-}\overline{\Sigma^0}\nu\bar{\nu}$	$-PB_{\mathcal{D}\bar{\mathcal{B}}}$
$B^- \to \Omega^-\overline{\Xi^0}\nu\bar{\nu}$	$\sqrt{6}PB_{\mathcal{D}\bar{\mathcal{B}}}$	$B^- \to \Xi^{*-}\bar{\Lambda}\nu\bar{\nu}$	$\sqrt{3}PB_{\mathcal{D}\bar{\mathcal{B}}}$
$\bar{B}^0 \to \Sigma^{*+}\bar{p}\nu\bar{\nu}$	$\sqrt{2}PB_{\mathcal{D}\bar{\mathcal{B}}}$	$\bar{B}^0 \to \Sigma^{*0}\bar{n}\nu\bar{\nu}$	$PB_{\mathcal{D}\bar{\mathcal{B}}}$
$\bar{B}^0 \to \Xi^{*0}\overline{\Sigma^0}\nu\bar{\nu}$	$-PB_{\mathcal{D}\bar{\mathcal{B}}}$	$\bar{B}^0 \to \Xi^{*-}\overline{\Sigma^-}\nu\bar{\nu}$	$-\sqrt{2}PB_{\mathcal{D}\bar{\mathcal{B}}}$
$\bar{B}^0 \to \Omega^-\overline{\Xi^-}\nu\bar{\nu}$	$-\sqrt{6}PB_{\mathcal{D}\bar{\mathcal{B}}}$	$\bar{B}^0 \to \Xi^{*0}\bar{\Lambda}\nu\bar{\nu}$	$-\sqrt{3}PB_{\mathcal{D}\bar{\mathcal{B}}}$
$\bar{B}^0_s \to \Sigma^{*+}\overline{\Sigma^+}\nu\bar{\nu}$	$-\sqrt{2}PB_{\mathcal{D}\bar{\mathcal{B}}}$	$\bar{B}^0_s \to \Sigma^{*0}\overline{\Sigma^0}\nu\bar{\nu}$	$\sqrt{2}PB_{\mathcal{D}\bar{\mathcal{B}}}$
$\bar{B}^0_s \to \Sigma^{*-}\overline{\Sigma^-}\nu\bar{\nu}$	$\sqrt{2}PB_{\mathcal{D}\bar{\mathcal{B}}}$	$\bar{B}^0_s \to \Xi^{*0}\overline{\Xi^0}\nu\bar{\nu}$	$-\sqrt{2}PB_{\mathcal{D}\bar{\mathcal{B}}}$
$\bar{B}^0_s \to \Xi^{*-}\overline{\Xi^-}\nu\bar{\nu}$	$\sqrt{2}PB_{\mathcal{D}\bar{\mathcal{B}}}$		

$$-\sqrt{2}A(B^- \to \Sigma^0\bar{p}\nu\bar{\nu}) = -A(B^- \to \Sigma^-\bar{n}\nu\bar{\nu}) = -A(\bar{B}^0 \to \Sigma^+\bar{p}\nu\bar{\nu}) = \sqrt{2}A(\bar{B}^0 \to \Sigma^0\bar{n}\nu\bar{\nu}), \tag{40}$$

$$A(\bar{B}^0_s \to \Sigma^+\overline{\Sigma^+}\nu\bar{\nu}) = A(\bar{B}^0_s \to \Sigma^0\overline{\Sigma^0}\nu\bar{\nu}) = A(\bar{B}^0_s \to \Sigma^-\overline{\Sigma^-}\nu\bar{\nu}), \tag{41}$$

$$A(\bar{B}^0_s \to \Xi^0\overline{\Xi^0}\nu\bar{\nu}) = A(\bar{B}^0_s \to \Xi^-\overline{\Xi^-}\nu\bar{\nu}), \tag{42}$$

$$A(B^- \to \Lambda\bar{p}\nu\bar{\nu}) = A(\bar{B}^0 \to \Lambda\bar{n}\nu\bar{\nu}), \tag{43}$$

$$A(\bar{B}^0_s \to p\bar{p}\nu\bar{\nu}) = A(\bar{B}^0_s \to n\bar{n}\nu\bar{\nu}), \tag{44}$$

and

$$\begin{aligned}
\sqrt{6}A(\bar{B}^0 \to \Xi^0\bar{\Lambda}\nu\bar{\nu}) &= A(\bar{B}^0 \to \Xi^-\overline{\Sigma^-}\nu\bar{\nu}) - A(\bar{B}^0 \to \Sigma^+\bar{p}\nu\bar{\nu}),\\
\sqrt{6}A(B^- \to \Xi^-\bar{\Lambda}\nu\bar{\nu}) &= A(B^- \to \Xi^0\overline{\Sigma^+}\nu\bar{\nu}) - 2A(B^- \to \Sigma^-\bar{n}\nu\bar{\nu}),\\
-\sqrt{6}A(B^- \to \Lambda\bar{p}\nu\bar{\nu}) &= 2A(B^- \to \Xi^0\overline{\Sigma^+}\nu\bar{\nu}) - A(B^- \to \Sigma^-\bar{n}\nu\bar{\nu}),\\
\sqrt{3}A(\bar{B}^0_s \to \Lambda\bar{\Lambda}\nu\bar{\nu}) &= -\sqrt{2}A(B^- \to \Lambda\bar{p}\nu\bar{\nu}) + \sqrt{3}A(\bar{B}^0_s \to p\bar{p}\nu\bar{\nu}).
\end{aligned} \tag{45}$$

For $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}l\bar{\nu}$ decays, there is only one topological amplitude, namely $T_{\mathcal{B}\bar{\mathcal{D}}}$. Therefore, all decay amplitudes are related,

$$\begin{aligned}
-A(B^- \to p\overline{\Delta^+}l\bar{\nu}) &= -A(B^- \to n\overline{\Delta^0}l\bar{\nu}) = A(B^- \to \Sigma^+\overline{\Sigma^{*+}}l\bar{\nu}) = 2A(B^- \to \Sigma^0\overline{\Sigma^{*0}}l\bar{\nu})\\
&= A(B^- \to \Xi^0\overline{\Xi^{*0}}l\bar{\nu}) = \frac{2}{\sqrt{3}}A(B^- \to \Lambda\overline{\Sigma^{*0}}l\bar{\nu}) = -A(\bar{B}^0 \to p\overline{\Delta^0}l\bar{\nu})
\end{aligned}$$

TABLE VIII. Topological amplitudes for $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}' l\bar{\nu}$ and $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}' \nu\bar{\nu}$ decays.

Mode	$A(\bar{B}_{q'} \to \mathcal{D}\bar{\mathcal{D}}' l\bar{\nu})$	Mode	$A(\bar{B}_{q'} \to \mathcal{D}\bar{\mathcal{D}}' l\bar{\nu})$
$B^- \to \Delta^{++}\overline{\Delta^{++}} l\bar{\nu}$	$6T_{\mathcal{D}\bar{\mathcal{D}}} + A_{\mathcal{D}\bar{\mathcal{D}}}$	$B^- \to \Delta^{+}\overline{\Delta^{+}} l\bar{\nu}$	$4T_{\mathcal{D}\bar{\mathcal{D}}}$
$B^- \to \Delta^{0}\overline{\Delta^{0}} l\bar{\nu}$	$2T_{\mathcal{D}\bar{\mathcal{D}}} + A_{\mathcal{D}\bar{\mathcal{D}}}$	$B^- \to \Delta^{-}\overline{\Delta^{-}} l\bar{\nu}$	$A_{\mathcal{D}\bar{\mathcal{D}}}$
$B^- \to \Sigma^{*+}\overline{\Sigma^{*+}} l\bar{\nu}$	$4T_{\mathcal{D}\bar{\mathcal{D}}} + A_{\mathcal{D}\bar{\mathcal{D}}}$	$B^- \to \Sigma^{*0}\overline{\Sigma^{*0}} l\bar{\nu}$	$2T_{\mathcal{D}\bar{\mathcal{D}}} + A_{\mathcal{D}\bar{\mathcal{D}}}$
$B^- \to \Sigma^{*-}\overline{\Sigma^{*-}} l\bar{\nu}$	$A_{\mathcal{D}\bar{\mathcal{D}}}$	$B^- \to \Xi^{*0}\overline{\Xi^{*0}} l\bar{\nu}$	$2T_{\mathcal{D}\bar{\mathcal{D}}} + A_{\mathcal{D}\bar{\mathcal{D}}}$
$B^- \to \Xi^{*-}\overline{\Xi^{*-}} l\bar{\nu}$	$A_{\mathcal{D}\bar{\mathcal{D}}}$	$B^- \to \Omega^{-}\overline{\Omega^{-}} l\bar{\nu}$	$A_{\mathcal{D}\bar{\mathcal{D}}}$
$\bar{B}^0 \to \Delta^{++}\overline{\Delta^{+}} l\bar{\nu}$	$2\sqrt{3}T_{\mathcal{D}\bar{\mathcal{D}}}$	$\bar{B}^0 \to \Delta^{+}\overline{\Delta^{0}} l\bar{\nu}$	$4T_{\mathcal{D}\bar{\mathcal{D}}}$
$\bar{B}^0 \to \Delta^{0}\overline{\Delta^{-}} l\bar{\nu}$	$2\sqrt{3}T_{\mathcal{D}\bar{\mathcal{D}}}$	$\bar{B}^0 \to \Sigma^{*+}\overline{\Sigma^{*0}} l\bar{\nu}$	$2\sqrt{2}T_{\mathcal{D}\bar{\mathcal{D}}}$
$\bar{B}^0 \to \Sigma^{*0}\overline{\Sigma^{*-}} l\bar{\nu}$	$2\sqrt{2}T_{\mathcal{D}\bar{\mathcal{D}}}$	$\bar{B}^0 \to \Xi^{*0}\overline{\Xi^{*-}} l\bar{\nu}$	$2T_{\mathcal{D}\bar{\mathcal{D}}}$
$\bar{B}^0_s \to \Delta^{++}\overline{\Sigma^{*+}} l\bar{\nu}$	$2\sqrt{3}T_{\mathcal{D}\bar{\mathcal{D}}}$	$\bar{B}^0_s \to \Delta^{+}\overline{\Sigma^{*0}} l\bar{\nu}$	$2\sqrt{2}T_{\mathcal{D}\bar{\mathcal{D}}}$
$\bar{B}^0_s \to \Delta^{0}\overline{\Sigma^{*-}} l\bar{\nu}$	$2T_{\mathcal{D}\bar{\mathcal{D}}}$	$\bar{B}^0_s \to \Sigma^{*+}\overline{\Xi^{*0}} l\bar{\nu}$	$4T_{\mathcal{D}\bar{\mathcal{D}}}$
$\bar{B}^0_s \to \Sigma^{*0}\overline{\Xi^{*-}} l\bar{\nu}$	$2\sqrt{2}T_{\mathcal{D}\bar{\mathcal{D}}}$	$\bar{B}^0_s \to \Xi^{*0}\overline{\Omega^{-}} l\bar{\nu}$	$2\sqrt{3}T_{\mathcal{D}\bar{\mathcal{D}}}$

Mode	$A(\bar{B}_{q'} \to \mathcal{D}\bar{\mathcal{D}}' \nu\bar{\nu})$	Mode	$A(\bar{B}_{q'} \to \mathcal{D}\bar{\mathcal{D}}' \nu\bar{\nu})$
$B^- \to \Sigma^{*+}\overline{\Delta^{++}} \nu\bar{\nu}$	$2\sqrt{3}PB_{\mathcal{D}\bar{\mathcal{D}}}$	$B^- \to \Sigma^{*0}\overline{\Delta^{+}} \nu\bar{\nu}$	$2\sqrt{2}PB_{\mathcal{D}\bar{\mathcal{D}}}$
$B^- \to \Sigma^{*-}\overline{\Delta^{0}} \nu\bar{\nu}$	$2PB_{\mathcal{D}\bar{\mathcal{D}}}$	$B^- \to \Xi^{*0}\overline{\Sigma^{*+}} \nu\bar{\nu}$	$4PB_{\mathcal{D}\bar{\mathcal{D}}}$
$B^- \to \Xi^{*-}\overline{\Sigma^{*0}} \nu\bar{\nu}$	$2\sqrt{2}PB_{\mathcal{D}\bar{\mathcal{D}}}$	$B^- \to \Omega^{-}\overline{\Xi^{*0}} \nu\bar{\nu}$	$2\sqrt{3}PB_{\mathcal{D}\bar{\mathcal{D}}}$
$\bar{B}^0 \to \Sigma^{*+}\overline{\Delta^{+}} \nu\bar{\nu}$	$2PB_{\mathcal{D}\bar{\mathcal{D}}}$	$\bar{B}^0 \to \Sigma^{*0}\overline{\Delta^{0}} \nu\bar{\nu}$	$2\sqrt{2}PB_{\mathcal{D}\bar{\mathcal{D}}}$
$\bar{B}^0 \to \Sigma^{*-}\overline{\Delta^{-}} \nu\bar{\nu}$	$2\sqrt{3}PB_{\mathcal{D}\bar{\mathcal{D}}}$	$\bar{B}^0 \to \Xi^{*0}\overline{\Sigma^{*0}} \nu\bar{\nu}$	$2\sqrt{2}PB_{\mathcal{D}\bar{\mathcal{D}}}$
$\bar{B}^0 \to \Xi^{*-}\overline{\Sigma^{*-}} \nu\bar{\nu}$	$4PB_{\mathcal{D}\bar{\mathcal{D}}}$	$\bar{B}^0 \to \Omega^{-}\overline{\Xi^{*-}} \nu\bar{\nu}$	$2\sqrt{3}PB_{\mathcal{D}\bar{\mathcal{D}}}$
$\bar{B}^0_s \to \Delta^{++}\overline{\Delta^{++}} \nu\bar{\nu}$	$PBA_{\mathcal{D}\bar{\mathcal{D}}}$	$\bar{B}^0_s \to \Delta^{+}\overline{\Delta^{+}} \nu\bar{\nu}$	$PBA_{\mathcal{D}\bar{\mathcal{D}}}$
$\bar{B}^0_s \to \Delta^{0}\overline{\Delta^{0}} \nu\bar{\nu}$	$PBA_{\mathcal{D}\bar{\mathcal{D}}}$	$\bar{B}^0_s \to \Delta^{-}\overline{\Delta^{-}} \nu\bar{\nu}$	$PBA_{\mathcal{D}\bar{\mathcal{D}}}$
$\bar{B}^0_s \to \Sigma^{*+}\overline{\Sigma^{*+}} \nu\bar{\nu}$	$2PB_{\mathcal{D}\bar{\mathcal{D}}} + PBA_{\mathcal{D}\bar{\mathcal{D}}}$	$\bar{B}^0_s \to \Sigma^{*0}\overline{\Sigma^{*0}} \nu\bar{\nu}$	$2PB_{\mathcal{D}\bar{\mathcal{D}}} + PBA_{\mathcal{D}\bar{\mathcal{D}}}$
$\bar{B}^0_s \to \Sigma^{*-}\overline{\Sigma^{*-}} \nu\bar{\nu}$	$2PB_{\mathcal{D}\bar{\mathcal{D}}} + PBA_{\mathcal{D}\bar{\mathcal{D}}}$	$\bar{B}^0_s \to \Xi^{*0}\overline{\Xi^{*0}} \nu\bar{\nu}$	$4PB_{\mathcal{D}\bar{\mathcal{D}}} + PBA_{\mathcal{D}\bar{\mathcal{D}}}$
$\bar{B}^0_s \to \Xi^{*-}\overline{\Xi^{*-}} \nu\bar{\nu}$	$4PB_{\mathcal{D}\bar{\mathcal{D}}} + PBA_{\mathcal{D}\bar{\mathcal{D}}}$	$\bar{B}^0_s \to \Omega^{-}\overline{\Omega^{-}} \nu\bar{\nu}$	$6PB_{\mathcal{D}\bar{\mathcal{D}}} + PBA_{\mathcal{D}\bar{\mathcal{D}}}$

$$
\begin{aligned}
&= -\frac{1}{\sqrt{3}} A(\bar{B}^0 \to n\overline{\Delta^{-}} l\bar{\nu}) = \sqrt{2} A(\bar{B}^0 \to \Sigma^{+}\overline{\Sigma^{*0}} l\bar{\nu}) \\
&= -\sqrt{2} A(\bar{B}^0 \to \Sigma^{0}\overline{\Sigma^{*-}} l\bar{\nu}) = A(\bar{B}^0 \to \Xi^{0}\overline{\Xi^{*-}} l\bar{\nu}) = \sqrt{\frac{2}{3}} A(\bar{B}^0 \to \Lambda\overline{\Sigma^{*-}} l\bar{\nu}) \\
&= -\sqrt{2} A(\bar{B}^0_s \to p\overline{\Sigma^{*0}} l\bar{\nu}) = -A(\bar{B}^0_s \to n\overline{\Sigma^{*-}} l\bar{\nu}) = A(\bar{B}^0_s \to \Sigma^{+}\overline{\Xi^{*0}} l\bar{\nu}) \\
&= -\sqrt{2} A(\bar{B}^0_s \to \Sigma^{0}\overline{\Xi^{*-}} l\bar{\nu}) = \frac{1}{\sqrt{3}} A(\bar{B}^0_s \to \Xi^{0}\overline{\Omega^{-}} l\bar{\nu}) \\
&= \sqrt{\frac{2}{3}} A(\bar{B}^0_s \to \Lambda\overline{\Xi^{*-}} l\bar{\nu}).
\end{aligned}
\tag{46}
$$

Similarly, for $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}\nu\bar{\nu}$ decays, there is only one topological amplitude, namely $PB_{\mathcal{B}\bar{\mathcal{D}}}$. Hence, all decay amplitudes are related. Explicitly, we have the following relations,

$$
\begin{aligned}
-\frac{1}{\sqrt{6}} A(B^- \to \Sigma^{+}\overline{\Delta^{++}} \nu\bar{\nu}) &= \frac{1}{2} A(B^- \to \Sigma^{0}\overline{\Delta^{+}} \nu\bar{\nu}) = \frac{1}{\sqrt{2}} A(B^- \to \Sigma^{-}\overline{\Delta^{0}} \nu\bar{\nu}) \\
&= -\frac{1}{\sqrt{2}} A(B^- \to \Xi^{0}\overline{\Sigma^{*+}} \nu\bar{\nu}) = A(B^- \to \Xi^{-}\overline{\Sigma^{*0}} \nu\bar{\nu}) \\
&= -\frac{1}{\sqrt{2}} A(\bar{B}^0 \to \Sigma^{+}\overline{\Delta^{+}} \nu\bar{\nu}) = \frac{1}{2} A(\bar{B}^0 \to \Sigma^{0}\overline{\Delta^{0}} \nu\bar{\nu})
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{\sqrt{6}} A(\bar{B}^0 \to \Sigma^- \overline{\Delta^-} \nu\bar{\nu}) = -A(\bar{B}^0 \to \Xi^0 \overline{\Sigma^{*0}} \nu\bar{\nu}) \\
&= \frac{1}{\sqrt{2}} A(\bar{B}^0 \to \Xi^- \overline{\Sigma^{*-}} \nu\bar{\nu}) = -\frac{1}{\sqrt{2}} A(\bar{B}^0_s \to \Sigma^+ \overline{\Sigma^{*+}} \nu\bar{\nu}) \\
&= \frac{1}{\sqrt{2}} A(\bar{B}^0_s \to \Sigma^0 \overline{\Sigma^{*0}} \nu\bar{\nu}) = \frac{1}{\sqrt{2}} A(\bar{B}^0_s \to \Sigma^- \overline{\Sigma^{*-}} \nu\bar{\nu}) \\
&= -\frac{1}{\sqrt{2}} A(\bar{B}^0_s \to \Xi^0 \overline{\Xi^{*0}} \nu\bar{\nu}) = \frac{1}{\sqrt{2}} A(\bar{B}^0_s \to \Xi^- \overline{\Xi^{*-}} \nu\bar{\nu}). \qquad (47)
\end{aligned}
$$

For $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}l\bar{\nu}$ decays, there is only one topological amplitude ($T_{\mathcal{D}\bar{\mathcal{B}}}$), while for $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}\nu\bar{\nu}$ decays, there is also only one topological amplitude ($PB_{\mathcal{D}\bar{\mathcal{B}}}$). Hence, the decay amplitudes are highly related and we have the following relations for $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}l\bar{\nu}$ decays,

$$
\begin{aligned}
-\frac{1}{\sqrt{2}} A(B^- \to \Delta^+ \bar{p} l\bar{\nu}) &= -\frac{1}{\sqrt{2}} A(B^- \to \Delta^0 \bar{n} l\bar{\nu}) = \frac{1}{\sqrt{2}} A(B^- \to \Sigma^{*+} \overline{\Sigma^+} l\bar{\nu}) \\
&= -\sqrt{2} A(B^- \to \Sigma^{*0} \overline{\Sigma^0} l\bar{\nu}) = \frac{1}{\sqrt{2}} A(B^- \to \Xi^{*0} \overline{\Xi^0} l\bar{\nu}) \\
&= \sqrt{\frac{2}{3}} A(B^- \to \Sigma^{*0} \bar{\Lambda} l\bar{\nu}) = \frac{1}{\sqrt{6}} A(\bar{B}^0 \to \Delta^{++} \bar{p} l\bar{\nu}) \\
&= \frac{1}{\sqrt{2}} A(\bar{B}^0 \to \Delta^+ \bar{n} l\bar{\nu}) = -A(\bar{B}^0 \to \Sigma^{*+} \overline{\Sigma^0} l\bar{\nu}) \\
&= -A(\bar{B}^0 \to \Sigma^{*0} \overline{\Sigma^-} l\bar{\nu}) = -\frac{1}{\sqrt{2}} A(\bar{B}^0 \to \Xi^{*0} \overline{\Xi^-} l\bar{\nu}) \\
&= \frac{1}{\sqrt{3}} A(\bar{B}^0 \to \Sigma^{*+} \bar{\Lambda} l\bar{\nu}) = -\frac{1}{\sqrt{6}} A(\bar{B}^0_s \to \Delta^{++} \overline{\Sigma^+} l\bar{\nu}) \\
&= \frac{1}{2} A(\bar{B}^0_s \to \Delta^+ \overline{\Sigma^0} l\bar{\nu}) = \frac{1}{\sqrt{2}} A(\bar{B}^0_s \to \Delta^0 \overline{\Sigma^-} l\bar{\nu}) \\
&= -\frac{1}{\sqrt{2}} A(\bar{B}^0_s \to \Sigma^{*+} \overline{\Xi^0} l\bar{\nu}) = A(\bar{B}^0_s \to \Sigma^{*0} \overline{\Xi^-} l\bar{\nu}), \qquad (48)
\end{aligned}
$$

and

$$
\begin{aligned}
-A(B^- \to \Sigma^{*0} \bar{p} \nu\bar{\nu}) &= -\frac{1}{\sqrt{2}} A(B^- \to \Sigma^{*-} \bar{n} \nu\bar{\nu}) = \frac{1}{\sqrt{2}} A(B^- \to \Xi^{*0} \overline{\Sigma^+} \nu\bar{\nu}) \\
&= -A(B^- \to \Xi^{*-} \overline{\Sigma^0} \nu\bar{\nu}) = \frac{1}{\sqrt{6}} A(B^- \to \Omega^- \overline{\Xi^0} \nu\bar{\nu}) \\
&= \frac{1}{\sqrt{3}} A(B^- \to \Xi^{*-} \bar{\Lambda} \nu\bar{\nu}) = \frac{1}{\sqrt{2}} A(\bar{B}^0 \to \Sigma^{*+} \bar{p} \nu\bar{\nu}) \\
&= A(\bar{B}^0 \to \Sigma^{*0} \bar{n} \nu\bar{\nu}) = -A(\bar{B}^0 \to \Xi^{*0} \overline{\Sigma^0} \nu\bar{\nu}) \\
&= -\frac{1}{\sqrt{2}} A(\bar{B}^0 \to \Xi^{*-} \overline{\Sigma^-} \nu\bar{\nu}) = -\frac{1}{\sqrt{6}} A(\bar{B}^0 \to \Omega^- \overline{\Xi^-} \nu\bar{\nu}) \\
&= -\frac{1}{\sqrt{3}} A(\bar{B}^0 \to \Xi^{*0} \bar{\Lambda} \nu\bar{\nu}) = -\frac{1}{\sqrt{2}} A(\bar{B}^0_s \to \Sigma^{*+} \overline{\Sigma^+} \nu\bar{\nu}) \\
&= \frac{1}{\sqrt{2}} A(\bar{B}^0_s \to \Sigma^{*0} \overline{\Sigma^0} \nu\bar{\nu}) = \frac{1}{\sqrt{2}} A(\bar{B}^0_s \to \Sigma^{*-} \overline{\Sigma^-} \nu\bar{\nu}) \\
&= -\frac{1}{\sqrt{2}} A(\bar{B}^0_s \to \Xi^{*0} \overline{\Xi^0} \nu\bar{\nu}) = \frac{1}{\sqrt{2}} A(\bar{B}^0_s \to \Xi^{*-} \overline{\Xi^-} \nu\bar{\nu}), \qquad (49)
\end{aligned}
$$

for $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}\nu\bar{\nu}$ decays.

For $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}' l\bar{\nu}$ decays, we have two topological amplitudes, namely $T_{\mathcal{D}\bar{\mathcal{D}}}$ and $A_{\mathcal{D}\bar{\mathcal{D}}}$. The decay amplitudes are related as follows,

$$
\begin{aligned}
\sqrt{3}A(\bar{B}^0 \to \Delta^{++}\overline{\Delta^{+}}l\bar{\nu}) &= \frac{1}{2}A(\bar{B}^0 \to \Delta^{+}\overline{\Delta^{0}}l\bar{\nu}) = \sqrt{3}A(\bar{B}^0 \to \Delta^{0}\overline{\Delta^{-}}l\bar{\nu}) \\
&= \frac{1}{\sqrt{2}}A(\bar{B}^0 \to \Sigma^{*+}\overline{\Sigma^{*0}}l\bar{\nu}) = \frac{1}{\sqrt{2}}A(\bar{B}^0 \to \Sigma^{*0}\overline{\Sigma^{*-}}l\bar{\nu}) \\
&= A(\bar{B}^0 \to \Xi^{*0}\overline{\Xi^{*-}}l\bar{\nu}) = \frac{1}{\sqrt{3}}A(\bar{B}^0_s \to \Delta^{++}\overline{\Sigma^{*+}}l\bar{\nu}) \\
&= \frac{1}{\sqrt{2}}A(\bar{B}^0_s \to \Delta^{+}\overline{\Sigma^{*0}}l\bar{\nu}) = A(\bar{B}^0_s \to \Delta^{0}\overline{\Sigma^{*-}}l\bar{\nu}) \\
&= \frac{1}{2}A(\bar{B}^0_s \to \Sigma^{*+}\overline{\Xi^{0}}l\bar{\nu}) = \frac{1}{\sqrt{2}}A(\bar{B}^0_s \to \Sigma^{*0}\overline{\Xi^{*-}}l\bar{\nu}) \\
&= \frac{1}{\sqrt{3}}A(\bar{B}^0_s \to \Xi^{*0}\overline{\Omega^{-}}l\bar{\nu}) = \frac{1}{2}A(B^- \to \Delta^{+}\overline{\Delta^{+}}l\bar{\nu}),
\end{aligned}
\tag{50}
$$

$$
A(B^- \to \Delta^{0}\overline{\Delta^{0}}l\bar{\nu}) = A(B^- \to \Sigma^{*0}\overline{\Sigma^{*0}}l\bar{\nu}) = A(B^- \to \Xi^{*0}\overline{\Xi^{*0}}l\bar{\nu}),
\tag{51}
$$

$$
\begin{aligned}
A(B^- \to \Delta^{-}\overline{\Delta^{-}}l\bar{\nu}) &= A(B^- \to \Sigma^{*-}\overline{\Sigma^{*-}}l\bar{\nu}) = A(B^- \to \Xi^{*-}\overline{\Xi^{*-}}l\bar{\nu}) \\
&= A(B^- \to \Omega^{-}\overline{\Omega^{-}}l\bar{\nu}),
\end{aligned}
\tag{52}
$$

and

$$
\begin{aligned}
A(B^- \to \Delta^{++}\overline{\Delta^{++}}l\bar{\nu}) &= A(B^- \to \Delta^{0}\overline{\Delta^{0}}l\bar{\nu}) + A(B^- \to \Delta^{-}\overline{\Delta^{-}}l\bar{\nu}), \\
A(B^- \to \Sigma^{*+}\overline{\Sigma^{*+}}l\bar{\nu}) &= A(B^- \to \Delta^{+}\overline{\Delta^{+}}l\bar{\nu}) + A(B^- \to \Sigma^{*-}\overline{\Sigma^{*-}}l\bar{\nu}).
\end{aligned}
\tag{53}
$$

Finally, for $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}' \nu\bar{\nu}$ decays, we have two topological amplitudes, namely $PB_{\mathcal{D}\bar{\mathcal{D}}}$ and $PBA_{\mathcal{D}\bar{\mathcal{D}}}$, giving the following relations on the amplitudes,

$$
\begin{aligned}
\frac{1}{\sqrt{3}}A(B^- \to \Sigma^{*+}\overline{\Delta^{++}}\nu\bar{\nu}) &= \frac{1}{\sqrt{2}}A(B^- \to \Sigma^{*0}\overline{\Delta^{+}}\nu\bar{\nu}) = A(B^- \to \Sigma^{*-}\overline{\Delta^{0}}\nu\bar{\nu}) \\
&= \frac{1}{2}A(B^- \to \Xi^{*0}\overline{\Sigma^{*+}}\nu\bar{\nu}) = \frac{1}{\sqrt{2}}A(B^- \to \Xi^{*-}\overline{\Sigma^{*0}}\nu\bar{\nu}) \\
&= \frac{1}{\sqrt{3}}A(B^- \to \Omega^{-}\overline{\Xi^{*0}}\nu\bar{\nu}) = A(\bar{B}^0 \to \Sigma^{*+}\overline{\Delta^{+}}\nu\bar{\nu}) \\
&= \frac{1}{\sqrt{2}}A(\bar{B}^0 \to \Sigma^{*0}\overline{\Delta^{0}}\nu\bar{\nu}) = \frac{1}{\sqrt{3}}A(\bar{B}^0 \to \Sigma^{*-}\overline{\Delta^{-}}\nu\bar{\nu}) \\
&= \frac{1}{\sqrt{2}}A(\bar{B}^0 \to \Xi^{*0}\overline{\Sigma^{*0}}\nu\bar{\nu}) = \frac{1}{2}A(\bar{B}^0 \to \Xi^{*-}\overline{\Sigma^{*-}}\nu\bar{\nu}) \\
&= \frac{1}{\sqrt{3}}A(\bar{B}^0 \to \Omega^{-}\overline{\Xi^{*-}}\nu\bar{\nu}),
\end{aligned}
\tag{54}
$$

$$
\begin{aligned}
A(\bar{B}^0_s \to \Sigma^{*+}\overline{\Sigma^{*+}}\nu\bar{\nu}) &= A(\bar{B}^0_s \to \Sigma^{*0}\overline{\Sigma^{*0}}\nu\bar{\nu}) = A(\bar{B}^0_s \to \Sigma^{*-}\overline{\Sigma^{*-}}\nu\bar{\nu}), \\
A(\bar{B}^0_s \to \Xi^{*0}\overline{\Xi^{*0}}\nu\bar{\nu}) &= A(\bar{B}^0_s \to \Xi^{*-}\overline{\Xi^{*-}}\nu\bar{\nu}),
\end{aligned}
\tag{55}
$$

$$
\begin{aligned}
A(\bar{B}^0_s \to \Sigma^{*+}\overline{\Sigma^{*+}}\nu\bar{\nu}) &= A(B^- \to \Sigma^{*-}\overline{\Delta^{0}}\nu\bar{\nu}) + A(\bar{B}^0_s \to \Delta^{0}\overline{\Delta^{0}}\nu\bar{\nu}), \\
A(\bar{B}^0_s \to \Xi^{*0}\overline{\Xi^{*0}}\nu\bar{\nu}) &= A(\bar{B}^0 \to \Xi^{*-}\overline{\Sigma^{*-}}\nu\bar{\nu}) + A(\bar{B}^0_s \to \Delta^{-}\overline{\Delta^{-}}\nu\bar{\nu}), \\
A(\bar{B}^0_s \to \Omega^{-}\overline{\Omega^{-}}\nu\bar{\nu}) &= A(\bar{B}^0 \to \Xi^{*-}\overline{\Sigma^{*-}}\nu\bar{\nu}) + A(\bar{B}^0_s \to \Sigma^{*+}\overline{\Sigma^{*+}}\nu\bar{\nu}),
\end{aligned}
\tag{56}
$$

$$A(\bar{B}_s^0 \to \Delta^{++}\overline{\Delta^{++}}\nu\bar{\nu}) = A(\bar{B}_s^0 \to \Delta^{+}\overline{\Delta^{+}}\nu\bar{\nu}) = A(\bar{B}_s^0 \to \Delta^{0}\overline{\Delta^{0}}\nu\bar{\nu}) = A(\bar{B}_s^0 \to \Delta^{-}\overline{\Delta^{-}}\nu\bar{\nu}). \quad (57)$$

The above relations on amplitudes impose relations on rates. For example, we may have three decay modes, where their rates and amplitudes are related as following

$$\Gamma_1 = \sum_i |A_1(i)|^2, \qquad \Gamma_2 = \sum_i |A_2(i)|^2, \qquad \Gamma_3 = \sum_i |A_1(i) + A_2(i)|^2, \quad (58)$$

with i representing the allowed momentum and helicities of final state particles, summing over i indicating integrating over phase space and summing over final state helicities. Note that the following discussion only applies to the SU(3) symmetric case, i.e., we are considering the relation on rates in the SU(3) symmetric limit. Using the triangle inequality in the complex plane, we obtain

$$|A_1(i)|^2 + |A_2(i)|^2 - 2|A_1(i)||A_2(i)| \le |A_1(i) + A_2(i)|^2 \le |A_1(i)|^2 + |A_2(i)|^2 + 2|A_1(i)||A_2(i)|. \quad (59)$$

Sum over i in the above equation and make use of the following inequality,

$$0 \le \sum_i |A_1(i)||A_2(i)| \le \sqrt{\sum_i |A_1(i)|^2}\sqrt{\sum_j |A_2(j)|^2}, \quad (60)$$

we finally obtain the triangle inequality on rates in the SU(3) symmetric limit,

$$(\Gamma_1^{1/2} - \Gamma_2^{1/2})^2 \le \Gamma_3 \le (\Gamma_1^{1/2} + \Gamma_2^{1/2})^2. \quad (61)$$

IV. RESULTS ON RATES

Before we start the discussion on rates it will be useful to recall the detectability of the final state baryons. In Table IX, we identify some octet and decuplet baryons that can decay to all charged final states with unsuppressed branching ratios. Note that modes with antineutron are also detectable, while Δ^+, $\Sigma^{+,0}$, Ξ^0, Σ^{*0}, and Ξ^{*-} can be detected by detecting a π^0 or γ. For example, Δ^+ mainly decays to $p\pi^0$ and $n\pi^+$, while Σ^0 decays to $\Lambda\gamma$. We should pay close attention to the modes that involve these baryons and have large decay rates in the $\bar{B}_q$ decays.

A. $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu}$ and $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}'\nu\bar{\nu}$ decay rates

In this part, we will first give a generic discussion on $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu}$ and $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}'\nu\bar{\nu}$ decays, and the results will be compared to model calculations, where masses of hadrons and lifetimes are taken from Ref. [4].

For $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu}$ decays, the decay amplitudes can be decomposed in terms of three independent topological amplitudes, namely $T_{2\mathcal{B}\bar{\mathcal{B}}}$, $T_{1\mathcal{B}\bar{\mathcal{B}}}$, and $A_{\mathcal{B}\bar{\mathcal{B}}}$, as shown in Table V. As the amplitudes of $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu}$ decays have different combinations of these topological amplitudes, the corresponding branching ratios are denoted with different parameters. Specifically, we use a for the rate with $A = T_{1\mathcal{B}\bar{\mathcal{B}}} + A_{\mathcal{B}\bar{\mathcal{B}}}$, b for the rate with $A = T_{2\mathcal{B}\bar{\mathcal{B}}}$, c for the rate with $A = \frac{1}{2}(T_{1\mathcal{B}\bar{\mathcal{B}}} + T_{2\mathcal{B}\bar{\mathcal{B}}}) + A_{\mathcal{B}\bar{\mathcal{B}}}$, d for the rate with $A = (T_{1\mathcal{B}\bar{\mathcal{B}}} - T_{2\mathcal{B}\bar{\mathcal{B}}})/2$, e for the rate with $A = A_{\mathcal{B}\bar{\mathcal{B}}}$, f for the rate with $A = \frac{1}{6}(5T_{1\mathcal{B}\bar{\mathcal{B}}} + T_{2\mathcal{B}\bar{\mathcal{B}}}) + A_{\mathcal{B}\bar{\mathcal{B}}}$, g for the rate with $A = (2T_{1\mathcal{B}\bar{\mathcal{B}}} + T_{2\mathcal{B}\bar{\mathcal{B}}})/3$, and h for the rate with $A = \frac{1}{3}(T_{1\mathcal{B}\bar{\mathcal{B}}} + 2T_{2\mathcal{B}\bar{\mathcal{B}}}) + A_{\mathcal{B}\bar{\mathcal{B}}}$. In addition, we add tildes for rates with similar amplitudes but without the $A_{\mathcal{B}\bar{\mathcal{B}}}$ terms. For example, $\tilde{a}$ corresponds to the rate with $A = T_{1\mathcal{B}\bar{\mathcal{B}}}$. The same set of alphabets is also used in $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}'\nu\bar{\nu}$ decays as $PB_{i\mathcal{B}\bar{\mathcal{B}}}$ and $PBA_{\mathcal{B}\bar{\mathcal{B}}}$ are proportional to $T_{i\mathcal{B}\bar{\mathcal{B}}}$ and $A_{\mathcal{B}\bar{\mathcal{B}}}$ with a common proportional constant ζ as shown in Eq. (20). Note that the above parameters correspond to the rates in the SU(3) symmetric limit.

Experimentally not only data of the branching ratio of $B^- \to p\bar{p}l\bar{\nu}$ decay is obtained, information of differential rate is also available. The experimental differential rate $d\mathrm{Br}/dm_{p\bar{p}}$ of $B^- \to p\bar{p}\mu^-\bar{\nu}$ decay from LHCb [3] is shown in Fig. 1. The differential rate in Fig. 1 can be well fitted with

$$\frac{d\mathrm{Br}}{dm_{\mathbf{B}\bar{\mathbf{B}}'}} = \frac{N}{(m^2_{\mathbf{B}\bar{\mathbf{B}}'})^\gamma}(m_{\mathbf{B}\bar{\mathbf{B}}'} - m_{\mathbf{B}} - m_{\bar{\mathbf{B}}'}), \quad (62)$$

TABLE IX. Octet and decuplet baryons decaying to all charged final states with unsuppressed branching ratios [4].

Octet/Decuplet	Baryons	All charged final states
Octet, $\mathcal{B}$	p, Λ, Ξ^-	$\Lambda \to p\pi^-$, $\Xi^- \to \Lambda\pi^- \to p\pi^-\pi^-$
Decuplet, $\mathcal{D}$	$\Delta^{++,0}$, $\Sigma^{*\pm}$, Ξ^{*0}, Ω^-	$\Delta^{++,0} \to p\pi^\pm$, $\Sigma^{*\pm} \to \Lambda\pi^\pm \to p\pi^-\pi^\pm$, $\Xi^{*0} \to \Xi^-\pi^+ \to \Lambda\pi^-\pi^+ \to p\pi^-\pi^-\pi^+$, $\Omega^- \to \Lambda K^- \to p\pi^- K^-$

where γ and N are constants. In particular, $\gamma = 9$ is used in Fig. 1 for the plotted blue dashed line. (see also Fig. 3). As noted in Introduction the threshold enhancement is sensitive to the position of the threshold and hence the SU(3) breaking from baryon masses are amplified producing very large SU(3) breaking effects on the integrated decay rates.

In this work we use Eq. (62) to estimate the SU(3) breaking effect from threshold enhancement. Take $B^- \to p\bar{p}l\bar{\nu}$ and $B^- \to \Sigma^+\overline{\Sigma^+}l\bar{\nu}$ decays as examples. As shown in Table V their amplitudes are both equal to $A = T_{1\mathcal{B}\bar{\mathcal{B}}} + T_{2\mathcal{B}\bar{\mathcal{B}}} + A_{\mathcal{B}\bar{\mathcal{B}}}$. Consequently, without SU(3) breaking, their rates should be identical. However, we expect large SU(3) breaking from the threshold enhancement as the masses of p and Σ^+ are different. Using Eq. (62) the ratio of their branching ratios is given by

$$\frac{\mathrm{Br}(B^- \to \Sigma^+\overline{\Sigma^+}l\bar{\nu})}{\mathrm{Br}(B^- \to p\bar{p}l\bar{\nu})} = \frac{\int_{2m_{\Sigma^+}}^{m_B} dm_{\mathcal{B}\bar{\mathcal{B}}'} \frac{N'}{(m^2_{\mathcal{B}\bar{\mathcal{B}}'})^9}(m_{\mathcal{B}\bar{\mathcal{B}}'} - 2m_{\Sigma^+})}{\int_{2m_p}^{m_B} dm_{\mathcal{B}\bar{\mathcal{B}}'} \frac{N}{(m^2_{\mathcal{B}\bar{\mathcal{B}}'})^9}(m_{\mathcal{B}\bar{\mathcal{B}}'} - 2m_p)} = 0.022\frac{N'}{N} = 0.022\sigma, \tag{63}$$

where we define $N'/N \equiv \sigma$. We see that the SU(3) breaking from the threshold enhancement is very large. The decay rates differ by orders of magnitudes. On the other hand, although $N'/N = \sigma$ may contain additional SU(3) breaking from mass differences, it represents a milder SU(3) breaking effect, since the SU(3) breaking from threshold enhancement is already extracted out, the value of σ is expected to be of order one. Consequently, using $\mathrm{Br}(B^- \to p\bar{p}\mu\bar{\nu}) = (5.32 \pm 0.34) \times 10^{-6}$ [4], we expect $\mathrm{Br}(B^- \to \Sigma^+\overline{\Sigma^+}l\bar{\nu}) = (5.32 \times 0.022\sigma) \times 10^{-6}$ with σ an order one parameter. As we shall see later the above estimation agrees well with some recent theoretical calculations [8,9].

With these considerations, the branching ratios of $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}'l\bar{\nu}$ and $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}'\nu\bar{\nu}$ decays are parametrized and are shown in Table X. SU(3) breaking effects from

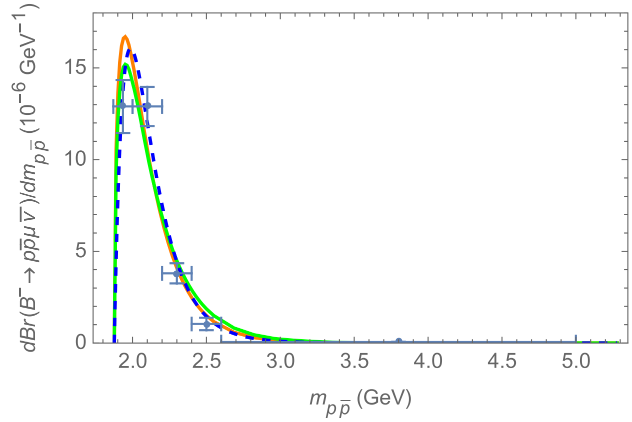


FIG. 3. The experimental differential rate $d\mathrm{Br}/dm_{p\bar{p}}$ of $B^- \to p\bar{p}\mu^-\bar{\nu}$ decay from LHCb [3] can be well fitted with $d\mathrm{Br}/dm_{p\bar{p}} = N(1/m^2_{p\bar{p}})^9(m_{p\bar{p}} - m_p - m_{\bar{p}})$ with blue dashed line. Orange and green solid lines correspond to the differential rates from model 1 and model 2 with inputs basically from refs [8,9], respectively. See text for details.

B_q meson widths and threshold enhancement are included. The order one parameters $\alpha, \beta, \eta, \tilde{\eta}, \tilde{\tilde{\eta}}, \kappa, \tilde{\kappa}, \sigma, \tilde{\sigma}, \tilde{\tilde{\sigma}}, \xi, \tilde{\xi}, \tilde{\tilde{\xi}}$ and $\bar{\alpha}, \bar{\beta}, \bar{\kappa}, \bar{\tilde{\kappa}}$ denote milder SU(3) breaking, where different parameters are used when the baryon masses are different, tilde are used when the combinations of topological amplitudes are different, bars are use when the masses of baryon and antibaryon are switched. From the above example, we expect these parameters to be of order one. We also expect them to be of similar size, and those with bar or tilde be close to those without bar or tilde. We will come back to these later.

There are many parameters in Table X. They are not totally independent, since we only have three independent topological amplitudes. Using the triangle inequality, Eq. (61), the amplitude decomposition in $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'l\bar{\nu}$ decays and the decay rates as shown in Tables V and X, we obtain the following inequalities,

$$\left(\frac{\sqrt{5.32} - \sqrt{e}}{2}\right)^2 \lesssim c \lesssim \left(\frac{\sqrt{5.32} + \sqrt{e}}{2}\right)^2, \qquad \left(\frac{\sqrt{5.32} - \sqrt{e}}{2}\right)^2 \lesssim \tilde{c} \lesssim \left(\frac{\sqrt{5.32} + \sqrt{e}}{2}\right)^2,$$
$$(\sqrt{a} - \sqrt{e})^2 \lesssim \tilde{a} \lesssim (\sqrt{a} + \sqrt{e})^2, \qquad (\sqrt{h} - \sqrt{e})^2 \lesssim \tilde{h} \lesssim (\sqrt{h} + \sqrt{e})^2, \tag{64}$$

$$(\sqrt{5.32} - \sqrt{a})^2 \lesssim b \lesssim (\sqrt{5.32} + \sqrt{a})^2, \qquad (\sqrt{\tilde{c}} - \sqrt{\tilde{a}})^2 \lesssim d \lesssim (\sqrt{\tilde{c}} + \sqrt{\tilde{a}})^2,$$
$$\left(\frac{\sqrt{c} - 2\sqrt{a}}{3}\right)^2 \lesssim f \lesssim \left(\frac{\sqrt{c} + 2\sqrt{a}}{3}\right)^2, \qquad \left(\frac{2\sqrt{\tilde{c}} - \sqrt{\tilde{a}}}{3}\right)^2 \lesssim g \lesssim \left(\frac{2\sqrt{\tilde{c}} + \sqrt{\tilde{a}}}{3}\right)^2,$$
$$\left(\frac{4\sqrt{c} - \sqrt{a}}{3}\right)^2 \lesssim h \lesssim \left(\frac{4\sqrt{c} + \sqrt{a}}{3}\right)^2, \qquad \left(\frac{4\sqrt{\tilde{c}} - \sqrt{\tilde{a}}}{3}\right)^2 \lesssim \tilde{h} \lesssim \left(\frac{4\sqrt{\tilde{c}} + \sqrt{\tilde{a}}}{3}\right)^2, \tag{65}$$

and

TABLE X. Branching ratios of $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu}$ and $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' \nu\bar{\nu}$ decays. The $B^- \to p\bar{p}l\bar{\nu}$ rate is from experimental data [2,4]. Most of the parameters are expected to be of order 1. In particular, we expect $c \simeq \tilde{c} \simeq \sqrt{5.32}/2$, $a \simeq \tilde{a}$, $h \simeq \tilde{h}$ and $e \ll 5.32$, satisfying Eqs. (67)–(69). The last factors are from the SU(3) breaking from threshold enhancement, and we expect $\alpha, \beta, \eta, \tilde{\eta}, \tilde{\tilde{\eta}}, \kappa, \tilde{\kappa}, \sigma, \tilde{\sigma}, \tilde{\tilde{\sigma}}, \xi, \tilde{\xi}, \tilde{\tilde{\xi}}$ and $\bar{\alpha}, \bar{\beta}, \bar{\kappa}, \bar{\bar{\kappa}}$ being of order unity. See text for details.

Mode	$\mathrm{Br}(\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu})(10^{-6})$	Mode	$\mathrm{Br}(\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu})(10^{-6})$
$B^- \to p\bar{p}l\bar{\nu}$	5.32 ± 0.34 [4]	$B^- \to n\bar{n}l\bar{\nu}$	$a \times (0.978)$
$B^- \to \Sigma^+\overline{\Sigma^+}l\bar{\nu}$	$5.32 \times (0.0225\sigma)$	$B^- \to \Sigma^0\overline{\Sigma^0}l\bar{\nu}$	$c \times (0.0215\sigma)$
$B^- \to \Sigma^-\overline{\Sigma^-}l\bar{\nu}$	$e \times (0.0202\tilde{\tilde{\sigma}})$	$B^- \to \Xi^-\overline{\Xi^-}l\bar{\nu}$	$e \times (0.00416\tilde{\tilde{\xi}})$
$B^- \to \Sigma^0\bar{\Lambda}l\bar{\nu}$	$\frac{d}{3} \times (0.0364\eta)$	$B^- \to \Xi^0\overline{\Xi^0}l\bar{\nu}$	$a \times (0.00452\tilde{\xi})$
$B^- \to \Lambda\overline{\Sigma^0}l\bar{\nu}$	$\frac{d}{3} \times (0.0364\eta)$	$B^- \to \Lambda\bar{\Lambda}l\bar{\nu}$	$f \times (0.0626\tilde{\eta})$
$\bar{B}^0 \to p\bar{n}l\bar{\nu}$	$0.93b \times (0.989)$	$\bar{B}^0 \to \Sigma^+\overline{\Sigma^0}l\bar{\nu}$	$1.85\tilde{c} \times (0.0220\sigma)$
$\bar{B}^0 \to \Sigma^+\bar{\Lambda}l\bar{\nu}$	$0.62d \times (0.0372\eta)$	$\bar{B}^0 \to \Sigma^0\overline{\Sigma^-}l\bar{\nu}$	$1.85\tilde{c} \times (0.0208\sigma)$
$\bar{B}^0 \to \Lambda\overline{\Sigma^-}l\bar{\nu}$	$0.62d \times (0.0352\eta)$	$\bar{B}^0 \to \Xi^0\overline{\Xi^-}l\bar{\nu}$	$0.9\tilde{a} \times (0.00434\tilde{\xi})$
$\bar{B}_s^0 \to p\overline{\Sigma^0}l\bar{\nu}$	$0.47\tilde{a} \times (0.131\beta)$	$\bar{B}_s^0 \to p\bar{\Lambda}l\bar{\nu}$	$1.40\tilde{h} \times (0.236\alpha)$
$\bar{B}_s^0 \to n\overline{\Sigma^-}l\bar{\nu}$	$0.93\tilde{a} \times (0.125\beta)$	$\bar{B}_s^0 \to \Sigma^+\overline{\Xi^0}l\bar{\nu}$	$0.93b \times (0.00988\kappa)$
$\bar{B}_s^0 \to \Sigma^0\overline{\Xi^-}l\bar{\nu}$	$0.47b \times (0.00927\kappa)$	$\bar{B}_s^0 \to \Lambda\overline{\Xi^-}l\bar{\nu}$	$1.40g \times (0.0152\tilde{\kappa})$

Mode	$\sum_\nu \mathrm{Br}(\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' \nu\bar{\nu})(10^{-8})$	Mode	$\sum_\nu \mathrm{Br}(\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' \nu\bar{\nu})(10^{-8})$
$B^- \to \Sigma^0\bar{p}\nu\bar{\nu}$	$0.20\tilde{a} \times (0.131\bar{\beta})$	$B^- \to \Sigma^-\bar{n}\nu\bar{\nu}$	$0.40\tilde{a} \times (0.125\bar{\beta})$
$B^- \to \Xi^0\overline{\Sigma^+}\nu\bar{\nu}$	$0.40b \times (0.00988\bar{\kappa})$	$B^- \to \Xi^-\overline{\Sigma^0}\nu\bar{\nu}$	$0.20b \times (0.00927\bar{\kappa})$
$B^- \to \Xi^-\bar{\Lambda}\nu\bar{\nu}$	$0.60g \times (0.0152\bar{\bar{\kappa}})$	$B^- \to \Lambda\bar{p}\nu\bar{\nu}$	$0.60\tilde{h} \times (0.236\bar{\alpha})$
$\bar{B}^0 \to \Sigma^+\bar{p}\nu\bar{\nu}$	$0.37\tilde{a} \times (0.134\bar{\beta})$	$\bar{B}^0 \to \Sigma^0\bar{n}\nu\bar{\nu}$	$0.19\tilde{a} \times (0.130\bar{\beta})$
$\bar{B}^0 \to \Xi^0\overline{\Sigma^0}\nu\bar{\nu}$	$0.19b \times (0.00968\bar{\kappa})$	$\bar{B}^0 \to \Xi^0\bar{\Lambda}\nu\bar{\nu}$	$0.56g \times (0.0159\bar{\bar{\kappa}})$
$\bar{B}^0 \to \Xi^-\overline{\Sigma^-}\nu\bar{\nu}$	$0.37b \times (0.00899\bar{\kappa})$	$\bar{B}^0 \to \Lambda\bar{n}\nu\bar{\nu}$	$0.56\tilde{h} \times (0.233\bar{\alpha})$
$\bar{B}_s^0 \to p\bar{p}\nu\bar{\nu}$	$0.37e$	$\bar{B}_s^0 \to n\bar{n}\nu\bar{\nu}$	$0.37e \times (0.978)$
$\bar{B}_s^0 \to \Sigma^+\overline{\Sigma^+}\nu\bar{\nu}$	$0.37a \times (0.0225\tilde{\sigma})$	$\bar{B}_s^0 \to \Sigma^0\overline{\Sigma^0}\nu\bar{\nu}$	$0.37a \times (0.0215\tilde{\sigma})$
$\bar{B}_s^0 \to \Sigma^-\overline{\Sigma^-}\nu\bar{\nu}$	$0.37a \times (0.0202\tilde{\sigma})$	$\bar{B}_s^0 \to \Xi^0\overline{\Xi^0}\nu\bar{\nu}$	$1.98 \times (0.00452\xi)$
$\bar{B}_s^0 \to \Xi^-\overline{\Xi^-}\nu\bar{\nu}$	$1.98 \times (0.00416\xi)$	$\bar{B}_s^0 \to \Lambda\bar{\Lambda}\nu\bar{\nu}$	$0.37h \times (0.0626\tilde{\tilde{\eta}})$

$$
(\sqrt{5.32} - \sqrt{b})^2 \lesssim a \lesssim (\sqrt{5.32} + \sqrt{b})^2, \qquad (\sqrt{\tilde{c}} - \sqrt{b})^2 \lesssim d \lesssim (\sqrt{\tilde{c}} + \sqrt{b})^2,
$$
$$
\left(\frac{4\sqrt{\tilde{c}} - \sqrt{b}}{3}\right)^2 \lesssim g \lesssim \left(\frac{4\sqrt{\tilde{c}} + \sqrt{b}}{3}\right)^2, \qquad \left(\frac{2\sqrt{\tilde{c}} - \sqrt{b}}{3}\right)^2 \lesssim \tilde{h} \lesssim \left(\frac{2\sqrt{\tilde{c}} + \sqrt{b}}{3}\right)^2. \tag{66}
$$

Although the above inequalities can constrain the sizes of these parameters, it will be useful to reduce the number of the parameters. Note that the rates proportional to e are governed by annihilation $A_{\mathcal{B}\bar{\mathcal{B}}}$ or penguin-box-annihilation $PBA_{\mathcal{B}\bar{\mathcal{B}}}$ diagrams. It is known that these contributions a re usually much suppressed than tree and penguin contributions. For example, in two-body baryonic B_q decays, $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}'$ decays, the tree dominated mode $B^- \to p\bar{p}$ and penguin dominated mode $B^- \to \Lambda\bar{p}$ was observed with branching ratios at 10^{-8} and 10^{-6} levels, respectively [38–40], while $\bar{B}_s \to p\bar{p}$ decay, which is an exchange and penguin-annihilation mode is not yet observed with the upper limit pushed down to 10^{-9} level [40]. It is therefore reasonable to consider the case where the annihilation $A_{\mathcal{B}\bar{\mathcal{B}}}$ and penguin-box-annihilation $PBA_{\mathcal{B}\bar{\mathcal{B}}}$ contributions are highly suppressed, i.e., $e \ll \mathcal{O}(1)$. Nevertheless this assumption can be checked by searching pure annihilation (penguin-box-annuhilation) modes, $B^- \to \Sigma^-\overline{\Sigma^-}l\bar{\nu}$, $B^- \to \Xi^-\overline{\Xi^-}l\bar{\nu}$, $\bar{B}_s^0 \to p\bar{p}\nu\bar{\nu}$, and $\bar{B}_s^0 \to n\bar{n}\nu\bar{\nu}$ decays, as their rates are proportional to e. In particular, as the $\bar{B}_s^0 \to p\bar{p}\nu\bar{\nu}$ mode has good detectability, see Table IX, it is a good place to verify the above assumption.

Applying the above assumption to the relations Eq. (68), we obtain,

$$
e \ll 5.32, \qquad c \simeq \tilde{c} \simeq \frac{5.32}{4}, \qquad \tilde{a} \simeq a, \qquad \tilde{h} \simeq h, \tag{67}
$$

$$
\begin{aligned}
&(\sqrt{5.32}-\sqrt{a})^2 \lesssim b \lesssim (\sqrt{5.32}+\sqrt{a})^2, \qquad (0.5\sqrt{5.32}-\sqrt{a})^2 \lesssim d \lesssim (0.5\sqrt{5.32}+\sqrt{a})^2,\\
&\left(\frac{\sqrt{5.32}-4\sqrt{a}}{6}\right)^2 \lesssim f \lesssim \left(\frac{\sqrt{5.32}+4\sqrt{a}}{6}\right)^2, \qquad \left(\frac{\sqrt{5.32}-\sqrt{a}}{3}\right)^2 \lesssim g \lesssim \left(\frac{\sqrt{5.32}+\sqrt{a}}{3}\right)^2,\\
&\left(\frac{2\sqrt{5.32}-\sqrt{a}}{3}\right)^2 \lesssim h, \qquad \tilde{h} \lesssim \left(\frac{2\sqrt{5.32}+\sqrt{a}}{3}\right)^2,
\end{aligned} \tag{68}
$$

and

$$
\begin{aligned}
&(\sqrt{5.32}-\sqrt{b})^2 \lesssim a \lesssim (\sqrt{5.32}+\sqrt{b})^2, \qquad (0.5\sqrt{5.32}-\sqrt{b})^2 \lesssim d \lesssim (0.5\sqrt{5.32}+\sqrt{b})^2,\\
&\left(\frac{5\sqrt{5.32}-4\sqrt{b}}{6}\right)^2 \lesssim f \lesssim \left(\frac{5\sqrt{5.32}+4\sqrt{b}}{6}\right)^2, \qquad \left(\frac{2\sqrt{5.32}-\sqrt{b}}{3}\right)^2 \lesssim g \lesssim \left(\frac{2\sqrt{5.32}+\sqrt{b}}{3}\right)^2,\\
&\left(\frac{\sqrt{5.32}-\sqrt{b}}{3}\right)^2 \lesssim h, \qquad \tilde{h} \lesssim \left(\frac{\sqrt{5.32}+\sqrt{b}}{3}\right)^2.
\end{aligned} \tag{69}
$$

These are the inequalities we shall employ in this work.

The parameters a, b, c and so on in Table X need to satisfy the above triangular inequalities, Eqs. (67)–(69). At this moment we do not have enough data to verify them. Nevertheless, it will be useful to make use of model calculations in Sec. II B for illustration.

Branching ratios of $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu}$ and $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' \nu\bar{\nu}$ decays in model 1 and model 2 can be obtained by using $T_{i\mathcal{B}\bar{\mathcal{B}}}$ and $A_{\mathcal{B}\bar{\mathcal{B}}}$, as shown in Eq. (21), with inputs as shown in Table II, and formulas of decay rates collected in Appendix B. The results are shown in Table XI. They can be compared to the results given in Refs. [8,9], where $\mathrm{Br}(B^- \to p\bar{p}l\bar{\nu}) = (5.21 \pm 0.34) \times 10^{-6}$ [8], $(5.3 \pm 0.2) \times 10^{-6}$ [9], $\mathrm{Br}(B^- \to n\bar{n}l\bar{\nu}) = (0.68 \pm 0.10) \times 10^{-6}$, $\mathrm{Br}(B^- \to \Sigma^+\overline{\Sigma^+}l\bar{\nu}) = (0.24 \pm 0.02) \times 10^{-6}$, $\mathrm{Br}(B^- \to \Sigma^0\overline{\Sigma^0}l\bar{\nu}) = (0.06 \pm 0.01) \times 10^{-6}$, $\mathrm{Br}(B^- \to \Sigma^0\bar{\Lambda}l\bar{\nu}) = (0.014 \pm 0.004) \times 10^{-6}$, $\mathrm{Br}(B^- \to \Lambda\overline{\Sigma^0}l\bar{\nu}) = (0.014 \pm 0.004) \times 10^{-6}$, $\mathrm{Br}(B^- \to \Xi^0\overline{\Xi^0}l\bar{\nu}) = (0.008 \pm 0.001) \times 10^{-6}$, $\mathrm{Br}(B^- \to \Lambda\bar{\Lambda}l\bar{\nu}) = (0.08 \pm 0.01) \times 10^{-6}$ [8], $\mathrm{Br}(\bar{B}^0_s \to p\bar{\Lambda}l\bar{\nu}) = (2.1 \pm 0.6) \times 10^{-6}$, $\sum_\nu \mathrm{Br}(B^- \to \Lambda\bar{p}\nu\bar{\nu}) = (3.5 \pm 1.0) \times 10^{-8}$ and $\sum_\nu \mathrm{Br}(\bar{B}_s \to \Lambda\bar{\Lambda}\nu\bar{\nu}) = (0.8 \pm 0.2) \times 10^{-8}$ [9] are reported. We find that results in model 1 agree with or close to those in Ref. [8], while the results on $\sum_\nu \mathrm{Br}(B^- \to \Lambda\bar{p}\nu\bar{\nu})$ and $\sum_\nu \mathrm{Br}(\bar{B}_s \to \Lambda\bar{\Lambda}\nu\bar{\nu})$ in model 2 differs to those in Ref. [9] by factors of 7. Results on all other modes in Table XI are new.

Model 1 and model 2 have similar results on some modes, but very different results on some other modes. For example, their rates in the $B^- \to p\bar{p}l\bar{\nu}$ decay are identical by construction, $B^- \to \Sigma^+\overline{\Sigma^+}l\bar{\nu}$ rates as well as $B^- \to \Sigma^0\overline{\Sigma^0}l\bar{\nu}$ rates are similar, but the $B^- \to \Sigma^0\bar{\Lambda}l\bar{\nu}$ rate in model 2 is larger than the one in model 1 by one order of magnitude, the $\bar{B}^0 \to p\bar{n}l\bar{\nu}$ rate in model 2 is larger than the one in model 1 by a factor of 3, and the $B^- \to n\bar{n}l\bar{\nu}$ rate in model 2 is larger than the one in model 1 by a factor of 19. Note that the amplitudes of $B^- \to p\bar{p}l\bar{\nu}$, $B^- \to \Sigma^+\overline{\Sigma^+}l\bar{\nu}$, and $B^- \to \Sigma^0\overline{\Sigma^0}l\bar{\nu}$ are to $T_{1\mathcal{B}\bar{\mathcal{B}}} + T_{2\mathcal{B}\bar{\mathcal{B}}}$, $B^- \to \Sigma^0\bar{\Lambda}l\bar{\nu}$, $B^- \to n\bar{n}l\bar{\nu}$ and $\bar{B}^0 \to p\bar{n}l\bar{\nu}$ are proportional to $T_{1\mathcal{B}\bar{\mathcal{B}}} - T_{2\mathcal{B}\bar{\mathcal{B}}}$, $T_{1\mathcal{B}\bar{\mathcal{B}}}$, and $T_{2\mathcal{B}\bar{\mathcal{B}}}$, respectively. These results imply that model 1 (2) has constructive (destructive) interference of $T_{1\mathcal{B}\bar{\mathcal{B}}}$ and $T_{2\mathcal{B}\bar{\mathcal{B}}}$ in $B^- \to p\bar{p}l\bar{\nu}$ decay, but destructive (constructive) interference of $T_{1\mathcal{B}\bar{\mathcal{B}}}$ and $-T_{2\mathcal{B}\bar{\mathcal{B}}}$ in $B^- \to \Sigma^0\bar{\Lambda}l\bar{\nu}$ decay, and $|T_{1,2\mathcal{B}\bar{\mathcal{B}}}|$ in model 2 are larger than those in model 1. These two models are complementary. It is therefore useful to consider both of them.

In Fig. 3 the differential rates $d\mathrm{Br}/dm_{p\bar{p}}$ of $B^- \to p\bar{p}\mu^-\bar{\nu}$ from model 1 and model 2 are shown and are compared to data. The differential rates from model 1 and 2 agree with data and are similar to each other.

The expectations of the orders of magnitudes of α, β and so on to be of order one and the triangular inequalities, Eqs. (67)–(69), on a, b and so on can be checked by comparing Table X with the results in model 1 and model 2 as shown in Table XI. The findings are shown in Table XII. The values of the parameters $\beta, \bar{\beta}, \sigma, \tilde{\sigma}, \kappa, \bar{\kappa}, \xi, \tilde{\xi}$ are indeed of order one and are in the range of 1.47–3.19, and their values in model 1 and model 2 are similar with differences at most 13%, even though these two models have very different interference patterns. The ratios of $\frac{\bar{\alpha}}{\alpha}, \frac{\bar{\beta}}{\beta}, \frac{\bar{\kappa}}{\kappa}, \frac{\bar{\bar{\kappa}}}{\bar{\kappa}}$ are close to one and are in the range of 0.86–0.93 and again their values in model 1 and model 2 are similar. The bounds on $\alpha, \bar{\alpha}, \eta, \tilde{\eta}, \tilde{\tilde{\eta}}, \bar{\kappa}, \bar{\bar{\kappa}}$ in model 1 are more restrictive than those in model 2, but they all allow these parameters to be of order one. We do not see any violation of the triangular inequalities, Eqs. (67)–(69). The values of a and b in model 1 and 2 confirm that $T_{1\mathcal{B}\bar{\mathcal{B}}}$ and $T_{2\mathcal{B}\bar{\mathcal{B}}}$ are constructive in model 1, but destructive in model 2, and $|T_{1,2\mathcal{B}\bar{\mathcal{B}}}|$ in model 2 are larger than those in model 1. These two models are indeed different but they give similar results on these parameters. Furthermore, our expectations on these parameters are basically verified in these two models.

TABLE XI. Branching ratios of $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu}$ and $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' \nu\bar{\nu}$ decays in model 1 and model 2. Br_1 and Br_2 denote results in model 1 and model 2, respectively.

Mode	$\mathrm{Br}_1(10^{-6})$	$\mathrm{Br}_2(10^{-6})$	Mode	$\mathrm{Br}_1(10^{-6})$	$\mathrm{Br}_2(10^{-6})$
$B^- \to p\bar{p}l\bar{\nu}$	5.32	5.32	$B^- \to n\bar{n}l\bar{\nu}$	0.41	7.81
$B^- \to \Sigma^+\overline{\Sigma^+}l\bar{\nu}$	0.26	0.26	$B^- \to \Sigma^0\overline{\Sigma^0}l\bar{\nu}$	0.064	0.061
$B^- \to \Sigma^-\overline{\Sigma^-}l\bar{\nu}$	$\simeq 0$	$\simeq 0$	$B^- \to \Xi^-\overline{\Xi^-}l\bar{\nu}$	$\simeq 0$	$\simeq 0$
$B^- \to \Sigma^0\bar{\Lambda}l\bar{\nu}$	0.019	0.18	$B^- \to \Xi^0\overline{\Xi^0}l\bar{\nu}$	0.0044	0.094
$B^- \to \Lambda\overline{\Sigma^0}l\bar{\nu}$	0.019	0.18	$B^- \to \Lambda\bar{\Lambda}l\bar{\nu}$	0.062	0.47
$\bar{B}^0 \to p\bar{n}l\bar{\nu}$	3.40	9.40	$\bar{B}^0 \to \Sigma^+\overline{\Sigma^0}l\bar{\nu}$	0.12	0.12
$\bar{B}^0 \to \Sigma^+\bar{\Lambda}l\bar{\nu}$	0.036	0.34	$\bar{B}^0 \to \Sigma^0\overline{\Sigma^-}l\bar{\nu}$	0.12	0.11
$\bar{B}^0 \to \Lambda\overline{\Sigma^-}l\bar{\nu}$	0.034	0.33	$\bar{B}^0 \to \Xi^0\overline{\Xi^-}l\bar{\nu}$	0.0040	0.084
$\bar{B}^0_s \to p\overline{\Sigma^0}l\bar{\nu}$	0.044	0.87	$\bar{B}^0_s \to p\bar{\Lambda}l\bar{\nu}$	1.01	1.28
$\bar{B}^0_s \to n\overline{\Sigma^-}l\bar{\nu}$	0.086	1.68	$\bar{B}^0_s \to \Sigma^+\overline{\Xi^0}l\bar{\nu}$	0.10	0.24
$\bar{B}^0_s \to \Sigma^0\overline{\Xi^-}l\bar{\nu}$	0.048	0.11	$\bar{B}^0_s \to \Lambda\overline{\Xi^-}l\bar{\nu}$	0.048	0.10
Mode	$\sum_\nu \mathrm{Br}_1(10^{-8})$	$\sum_\nu \mathrm{Br}_2(10^{-8})$	Mode	$\sum_\nu \mathrm{Br}_1(10^{-8})$	$\sum_\nu \mathrm{Br}_2(10^{-8})$
$B^- \to \Sigma^0\bar{p}\nu\bar{\nu}$	0.017	0.32	$B^- \to \Sigma^-\bar{n}\nu\bar{\nu}$	0.031	0.62
$B^- \to \Xi^0\overline{\Sigma^+}\nu\bar{\nu}$	0.038	0.089	$B^- \to \Xi^-\overline{\Sigma^0}\nu\bar{\nu}$	0.018	0.042
$B^- \to \Xi^-\bar{\Lambda}\nu\bar{\nu}$	0.018	0.041	$B^- \to \Lambda\bar{p}\nu\bar{\nu}$	0.39	0.52
$\bar{B}^0 \to \Sigma^+\bar{p}\nu\bar{\nu}$	0.031	0.61	$\bar{B}^0 \to \Sigma^0\bar{n}\nu\bar{\nu}$	0.015	0.30
$\bar{B}^0 \to \Xi^0\overline{\Sigma^0}\nu\bar{\nu}$	0.017	0.041	$\bar{B}^0 \to \Xi^0\bar{\Lambda}\nu\bar{\nu}$	0.017	0.039
$\bar{B}^0 \to \Xi^-\overline{\Sigma^-}\nu\bar{\nu}$	0.032	0.076	$\bar{B}^0 \to \Lambda\bar{n}\nu\bar{\nu}$	0.36	0.48
$\bar{B}^0_s \to p\bar{p}\nu\bar{\nu}$	$\simeq 0$	$\simeq 0$	$\bar{B}^0_s \to n\bar{n}\nu\bar{\nu}$	$\simeq 0$	$\simeq 0$
$\bar{B}^0_s \to \Sigma^+\overline{\Sigma^+}\nu\bar{\nu}$	0.0081	0.17	$\bar{B}^0_s \to \Sigma^0\overline{\Sigma^0}\nu\bar{\nu}$	0.0078	0.16
$\bar{B}^0_s \to \Sigma^-\overline{\Sigma^-}\nu\bar{\nu}$	0.0074	0.15	$\bar{B}^0_s \to \Xi^0\overline{\Xi^0}\nu\bar{\nu}$	0.028	0.028
$\bar{B}^0_s \to \Xi^-\overline{\Xi^-}\nu\bar{\nu}$	0.026	0.026	$\bar{B}^0_s \to \Lambda\bar{\Lambda}\nu\bar{\nu}$	0.097	0.12

From Table XI we find that the $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu}$ branching ratios are of the orders 10^{-8}–10^{-6} for nonannihilation modes, while the branching ratios of $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' \nu\bar{\nu}$ decays are of the orders of 10^{-11}–10^{-8} for nonpenguin-box-annihilation modes. From Tables IX and XI, we see that the following modes have good detectability and relatively unsuppressed rates, they are $B^- \to p\bar{p}l\bar{\nu}$, $\bar{B}^0 \to p\bar{n}l\bar{\nu}$, $\bar{B}^0_s \to p\bar{\Lambda}l\bar{\nu}$, $B^- \to \Lambda\bar{p}\nu\bar{\nu}$, $\bar{B}^0 \to \Lambda\bar{n}\nu\bar{\nu}$, and $\bar{B}^0_s \to \Lambda\bar{\Lambda}\nu\bar{\nu}$ decays. It is reasonable that the $B^- \to p\bar{p}l\bar{\nu}$ decay is the first detected mode as it has a large rate with very good detectability. In fact its rate is the largest one in model 1, but is the third largest one in model 2, where $\bar{B}^0 \to p\bar{n}l\bar{\nu}$ and $B^- \to n\bar{n}l\bar{\nu}$ decays have larger rates but poorer detectability. It will be useful to search for these modes to differentiate these two models and to understand the interference patterns of $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu}$ decay amplitudes.

From Tables X, we obtain

$$\frac{\sum_\nu \mathrm{Br}(B^- \to \Lambda\bar{p}\nu\bar{\nu})}{\mathrm{Br}(\bar{B}^0_s \to p\bar{\Lambda}l\bar{\nu})} = 4.29\frac{\bar{\alpha}}{\alpha}\times\left(\frac{0.0036}{|V_{ub}|}\right)^2 \times 10^{-3},$$
$$\frac{\sum_\nu \mathrm{Br}(\bar{B}^0 \to \Lambda\bar{n}\nu\bar{\nu})}{\mathrm{Br}(\bar{B}^0_s \to p\bar{\Lambda}l\bar{\nu})} = 3.94\frac{\bar{\alpha}}{\alpha}\times\left(\frac{0.0036}{|V_{ub}|}\right)^2 \times 10^{-3}. \quad (70)$$

The ratio $\bar{\alpha}/\alpha$ is expected to be close to one. In model 1 and 2, we have $\bar{\alpha}/\alpha = 0.90$ and 0.95, respectively, as shown in Table XII, which are indeed close to one. Hence the ratios are not sensitive to the SU(3) breakings from threshold enhancement as they are mostly canceled out. Furthermore, the ratios do not rely on the assumption of neglecting annihilation A and penguin-box-annihilation PBA contributions, as these modes are free from these contributions, see Table V. As the $\bar{B}^0_s \to p\bar{\Lambda}l\bar{\nu}$ decay are tree level decay modes, while the $B^- \to \Lambda\bar{p}\nu\bar{\nu}$ and $\bar{B}^0 \to \Lambda\bar{n}\nu\bar{\nu}$ decays are governed by penguin and box diagrams, the above ratios can be tests of SM.

B. $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}l\bar{\nu}$, $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}\nu\bar{\nu}$, $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}l\bar{\nu}$, and $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}\nu\bar{\nu}$ decay rates

We now consider the rates of $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}l\bar{\nu}$ and $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}\nu\bar{\nu}$ decays and the rates of $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}l\bar{\nu}$ and $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}\nu\bar{\nu}$ decays. As shown in Table VI, in $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}l\nu$ decays, there is only one topological amplitude, namely $T_{\mathcal{B}\bar{\mathcal{D}}}$, while in $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}\nu\bar{\nu}$ decays, there is also only one topological amplitude, namely $PB_{\mathcal{B}\bar{\mathcal{D}}}$, but $PB_{\mathcal{B}\bar{\mathcal{D}}}$ and $T_{\mathcal{B}\bar{\mathcal{D}}}$ are related by ζ as shown in Eq. (20). Similar features also hold in

TABLE XII. Values of various parameters in model 1 and model 2. The bounds of the parameters $d\eta, \ldots h\tilde{\tilde{\eta}}$ are obtained using triangular inequalities, Eqs. (68) and (69), while the bounds of the parameters $\alpha, \bar{\alpha}, \eta, \tilde{\eta}, \tilde{\tilde{\eta}}, \tilde{\kappa}, \bar{\tilde{\kappa}}$ are obtained using the values and bounds of the parameters $d\eta, \ldots h\tilde{\tilde{\eta}}$.

Parameters	Values (Model 1)	Bounds (Model 1)	Values (Model 2)	Bounds (Model 2)
a	0.42	0.15–17.90	7.99	0.80–30.34
b	3.70	2.75–8.74	10.25	0.27–26.34
$d\eta$	1.56	$(0.59–3.25)\eta$	14.89	$(4.20–15.83)\eta$
$f\tilde{\eta}$	0.99	$(0.41–0.67)\tilde{\eta}$	7.44	$(2.25–5.14)\tilde{\eta}'$
$g\tilde{\kappa}$	2.24	$(0.80–0.97)\tilde{\kappa}$	4.81	$(0.22–2.93)\tilde{\kappa}$
$g\bar{\tilde{\kappa}}$	1.96	$(0.80–0.97)\bar{\tilde{\kappa}}$	4.45	$(0.22–2.93)\bar{\tilde{\kappa}}$
$\tilde{h}\alpha$	3.08	$(1.75–1.99)\alpha$	3.88	$(0.35–3.37)\alpha$
$\tilde{h}\,\bar{\alpha}$	2.77	$(1.75–1.99)\bar{\alpha}$	3.67	$(0.35–3.37)\bar{\alpha}$
$h\tilde{\tilde{\eta}}$	4.17	$(1.75–1.99)\tilde{\tilde{\eta}}$	5.26	$(0.35–3.37)\tilde{\tilde{\eta}}$
α	⋯	1.55–1.76	⋯	1.15–10.93
$\bar{\alpha}$	⋯	1.39–1.59	⋯	1.09–10.34
β	1.72	⋯	1.79	⋯
$\bar{\beta}$	1.47	⋯	1.54	⋯
η	⋯	0.48–2.62	⋯	0.94–3.55
$\tilde{\eta}$	⋯	1.49–2.43	–	1.45–3.31
$\tilde{\tilde{\eta}}$	⋯	2.10–2.39	⋯	1.56–14.83
σ	2.21	⋯	2.13	⋯
$\tilde{\sigma}$	2.30	⋯	2.49	⋯
κ	2.96	⋯	2.55	⋯
$\bar{\kappa}$	2.59	⋯	2.21	⋯
$\tilde{\kappa}$	⋯	2.31–2.79	⋯	1.64–21.73
$\bar{\tilde{\kappa}}$	⋯	2.02–2.45	⋯	1.52–20.11
ξ	3.19	⋯	3.13	⋯
$\tilde{\xi}$	2.32	⋯	2.61	⋯
$\left(\frac{\bar{\alpha}}{\alpha}, \frac{\bar{\beta}}{\beta}, \frac{\bar{\kappa}}{\kappa}, \frac{\bar{\tilde{\kappa}}}{\tilde{\kappa}}\right)$	(0.90, 0.85, 0.87, 0.88)	⋯	(0.95, 0.86, 0.87, 0.93)	⋯

$\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}l\bar{\nu}$ and $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}\nu\bar{\nu}$ decays by using Table VII and Eq. (20), but with $T_{\mathcal{D}\bar{\mathcal{B}}}$ and $PB_{\mathcal{D}\bar{\mathcal{B}}}$. The decay rates of $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}l\nu, \mathcal{B}\bar{\mathcal{D}}\nu\bar{\nu}$ decays and $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}'l\bar{\nu}, \mathcal{D}\bar{\mathcal{B}}\nu\bar{\nu}$ decays are parametrized in term of a' and a'', respectively, where the rates correspond to $A = T_{\mathcal{B}\bar{\mathcal{D}}}$ and $T_{\mathcal{D}\bar{\mathcal{B}}}$ are denoted as a' and a'', respectively. Note that the above parameters correspond to the rates in the SU(3) symmetric limit.

As in $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}'l\bar{\nu}$ and $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}'\nu\bar{\nu}$ decays, we expect to see threshold enhancement in the differential rates of these modes. Likewise the SU(3) breaking on $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}l\bar{\nu}$ and $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}l\bar{\nu}$ rates from the threshold enhancement can be estimated as in the $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}'l\bar{\nu}$ case, once the corresponding differential rates of some modes are known. However, at the moment no such information is available yet. We should make use of some model calculations to obtain information on the differential rates of these modes. As we shall see the SU(3) breaking from the threshold enhancement can be estimated using Eq. (62) and similar procedure in the discussion around Eq. (63) but with $\gamma = 7$. In Tables XIII and XIV the decay rates of $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}l\nu, \mathcal{B}\bar{\mathcal{D}}\nu\bar{\nu}$ decays and $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}l\bar{\nu}, \mathcal{D}\bar{\mathcal{B}}\nu\bar{\nu}$ decays are shown. The parameters $\beta^{\prime,\prime\prime}$, $\kappa^{\prime,\prime\prime}$, $\sigma^{\prime,\prime\prime}$ and so on are used to denote milder SU(3) breaking effects and are expected to be of order one. The relative sizes of $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}l\bar{\nu}(\nu\bar{\nu})$ and $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}l\bar{\nu}(\nu\bar{\nu})$ decay rates can be readily read from Tables XIII and XIV.

At this moment we do not have enough data to verify Tables XIII and XIV. As in the case of $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}'l\bar{\nu}$ and $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}'\nu\bar{\nu}$ decays we will make use of model 1 and model 2 for illustration. Using inputs from Sec. II B and the formulas given Appendix B, the branching ratios of $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}l\bar{\nu}$ and $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}\nu\bar{\nu}$ in model 1 and 2 are obtained and are shown in Table XV, while the branching ratios of $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}l\bar{\nu}$ and $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}\nu\bar{\nu}$ in model 1 and 2 are obtained and are shown in Table XVI. These results are new.

From Tables XV and XVI, we see that the branching ratios of $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}l\bar{\nu}$ and $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}l\bar{\nu}$ decays are in the ranges of 10^{-9}–10^{-8} and 10^{-9}–10^{-7} in model 1 and 2, respectively, while $\sum_\nu \mathrm{Br}(\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}l\nu\nu)$ and $\sum_\nu \mathrm{Br}(\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}l\nu\nu)$ are in the ranges of 10^{-12}–10^{-10} and 10^{-11}–10^{-10} in model 1 and 2, respectively. The rates in model 2 are greater than those in model 1 by a factor of 4–7. This

TABLE XIII. Branching ratios of $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}l\bar{\nu}$ and $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}\nu\bar{\nu}$ decays. The last factors are the SU(3) breaking from threshold enhancement estimated using Eq. (62) with $\gamma = 7$, and parameters $\beta', \kappa', \sigma', \xi', \omega'$ are expected to be of order 1.

Mode	$\mathrm{Br}(\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}l\bar{\nu})(10^{-8})$	Mode	$\mathrm{Br}(\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}l\bar{\nu})(10^{-8})$
$B^- \to p\overline{\Delta^+}l\bar{\nu}$	$2a'$	$B^- \to n\overline{\Delta^0}l\bar{\nu}$	$2a' \times (0.993)$
$B^- \to \Sigma^+\overline{\Sigma^{*+}}l\bar{\nu}$	$2a' \times (0.130\sigma')$	$B^- \to \Sigma^0\overline{\Sigma^{*0}}l\bar{\nu}$	$\frac{1}{2}a' \times (0.128\sigma')$
$B^- \to \Xi^0\overline{\Xi^{*0}}l\bar{\nu}$	$2a' \times (0.0384\xi')$	$B^- \to \Lambda\overline{\Sigma^{*0}}l\bar{\nu}$	$\frac{3}{2}a' \times (0.184\sigma')$
$\bar{B}^0 \to p\overline{\Delta^0}l\bar{\nu}$	$1.85a'$	$\bar{B}^0 \to n\overline{\Delta^-}l\bar{\nu}$	$5.56a' \times (0.993)$
$\bar{B}^0 \to \Sigma^+\overline{\Sigma^{*0}}l\bar{\nu}$	$0.93a' \times (0.130\sigma')$	$\bar{B}^0 \to \Sigma^0\overline{\Sigma^{*-}}l\bar{\nu}$	$0.93a' \times (0.125\sigma')$
$\bar{B}^0 \to \Xi^0\overline{\Xi^{*-}}l\bar{\nu}$	$1.85a' \times (0.0379\xi')$	$\bar{B}^0 \to \Lambda\overline{\Sigma^{*-}}l\bar{\nu}$	$2.78a' \times (0.181\sigma')$
$\bar{B}^0_s \to p\overline{\Sigma^{*0}}l\bar{\nu}$	$0.93a' \times (0.444\beta')$	$\bar{B}^0_s \to n\overline{\Sigma^{*-}}l\bar{\nu}$	$1.86a' \times (0.434\beta')$
$\bar{B}^0_s \to \Sigma^+\overline{\Xi^{*0}}l\bar{\nu}$	$1.86a' \times (0.0662\kappa')$	$\bar{B}^0_s \to \Sigma^0\overline{\Xi^{*-}}l\bar{\nu}$	$1.86a' \times (0.0643\kappa')$
$\bar{B}^0_s \to \Xi^0\overline{\Omega^-}l\bar{\nu}$	$5.59a' \times (0.0215\omega')$	$\bar{B}^0_s \to \Lambda\overline{\Xi^{*-}}l\bar{\nu}$	$2.79a' \times (0.0906\kappa')$

Mode	$\sum_\nu \mathrm{Br}(\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}\nu\bar{\nu})(10^{-10})$	Mode	$\sum_\nu \mathrm{Br}(\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}\nu\bar{\nu})(10^{-10})$
$B^- \to \Sigma^+\overline{\Delta^{++}}\nu\bar{\nu}$	$2.40a' \times (0.269\beta')$	$B^- \to \Sigma^0\overline{\Delta^+}\nu\bar{\nu}$	$1.60a' \times (0.264\beta')$
$B^- \to \Sigma^-\overline{\Delta^0}\nu\bar{\nu}$	$0.80a' \times (0.258\beta')$	$B^- \to \Xi^0\overline{\Sigma^{*+}}\nu\bar{\nu}$	$0.80a' \times (0.0734\kappa')$
$B^- \to \Xi^-\overline{\Sigma^{*0}}\nu\bar{\nu}$	$0.40a' \times (0.0709\kappa')$		
$\bar{B}^0 \to \Sigma^+\overline{\Delta^+}\nu\bar{\nu}$	$0.74a' \times (0.269\beta')$	$\bar{B}^0 \to \Sigma^0\overline{\Delta^0}\nu\bar{\nu}$	$1.48a' \times (0.264\beta')$
$\bar{B}^0 \to \Sigma^-\overline{\Delta^-}\nu\bar{\nu}$	$2.22a' \times (0.258\beta')$	$\bar{B}^0 \to \Xi^0\overline{\Sigma^{*0}}\nu\bar{\nu}$	$0.37a' \times (0.0731\kappa')$
$\bar{B}^0 \to \Xi^-\overline{\Sigma^{*-}}\nu\bar{\nu}$	$0.74a' \times (0.0698\kappa')$		
$\bar{B}^0_s \to \Sigma^+\overline{\Sigma^{*+}}\nu\bar{\nu}$	$0.75a' \times (0.130\sigma')$	$\bar{B}^0_s \to \Sigma^0\overline{\Sigma^{*0}}\nu\bar{\nu}$	$0.75a' \times (0.128\sigma')$
$\bar{B}^0_s \to \Sigma^-\overline{\Sigma^{*-}}\nu\bar{\nu}$	$0.75a' \times (0.123\sigma')$	$\bar{B}^0_s \to \Xi^0\overline{\Xi^{*0}}\nu\bar{\nu}$	$0.75a' \times (0.0384\xi')$
$\bar{B}^0_s \to \Xi^-\overline{\Xi^{*-}}\nu\bar{\nu}$	$0.75a' \times (0.0368\xi')$		

mostly corresponds to the fact that $|T_{\mathcal{B}\bar{\mathcal{D}}}|$ and $|T_{\mathcal{D}\bar{\mathcal{B}}}|$ in model 2 are greater than those in model 1, as reflected through the sizes of a' and a'' as shown in Table XVII. This is not surprising as $|T_{1\mathcal{B}\bar{\mathcal{B}}}|$ and $|T_{2\mathcal{B}\bar{\mathcal{B}}}|$ in $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}'l\bar{\nu}$ decays in model 2 are greater than those in model 1, as reflected in the sizes of a and b as shown in Table XII. From Table XVII, we see that the parameters $\beta'^{,\prime\prime}, \kappa'^{,\prime\prime}, \sigma'^{,\prime\prime}, \xi'^{,\prime\prime}$ and ω' denoting milder SU(3) breaking are indeed of order one and are similar in model 1 and 2 in most cases.

From Tables XIII, XV and IX, we find that $\bar{B}^0 \to p\overline{\Delta^0}l\bar{\nu}$ and $\bar{B}^0 \to \Sigma^0\overline{\Delta^0}\nu\bar{\nu}$ have relatively unsuppressed rates and good detectability. In particular, we have the follow rate ratio of the loop induced mode and tree induced modes,

$$\frac{\sum_\nu \mathrm{Br}(\bar{B}^0 \to \Sigma^0\overline{\Delta^0}\nu\bar{\nu})}{\mathrm{Br}(\bar{B}^0 \to p\overline{\Delta^0}l\bar{\nu})} = 2.11\beta' \times \left(\frac{0.0036}{|V_{ub}|}\right)^2 \times 10^{-3}, \quad (71)$$

where β' is of order one. In fact as shown in Table XVII, we have $\beta' = 1.01$ and 1.03 in model 1 and 2, respectively. The ratio in Eq. (71) can be a test of SM.

From Tables XIV, XVI and IX, we find that $\bar{B}^0 \to \Delta^{++}\bar{p}l\bar{\nu}$, $\bar{B}^0 \to \Sigma^{*+}\bar{\Lambda}l\bar{\nu}$, $B^- \to \Delta^+\bar{p}l\bar{\nu}$, $B^- \to \Delta^0\bar{n}l\bar{\nu}$, $\bar{B}^0 \to \Sigma^{*+}\bar{p}\nu\bar{\nu}$ and $B^- \to \Sigma^{*-}\bar{n}\nu\bar{\nu}$ decays have good detectability and relatively unsuppressed rates. The rate ratios of these loop induced modes and tree induced modes can be sensible tests of SM. For example, we have the following rate ratio,

$$\frac{\sum_\nu \mathrm{Br}(\bar{B}^0 \to \Sigma^{*+}\bar{p}\nu\bar{\nu})}{\mathrm{Br}(\bar{B}^0 \to \Delta^{++}\bar{p}l\bar{\nu})} = 4.55\kappa'' \times \left(\frac{0.0036}{|V_{ub}|}\right)^2 \times 10^{-4}, \quad (72)$$

where κ'' is of order one. In fact, as shown in Table XVII, we have $\kappa'' = 1.86$ and 2.24 in model 1 and 2, respectively.

The differential rates $d\mathrm{Br}/dm_{p\overline{\Delta^0}}$ of $\bar{B}^0 \to p\overline{\Delta^0}l^-\bar{\nu}$ decay and $d\mathrm{Br}/dm_{\Delta^{++}\bar{p}}$ of $\bar{B}^0 \to \Delta^{++}\bar{p}l^-\bar{\nu}$ decay from model 1 and model 2 are plotted in Fig. 4. They can be compared to the dashed lines plotted using Eq. (62) with $\gamma = 7$. They clearly exhibit threshold enhancement as expected.

C. $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}'l\bar{\nu}$ and $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}'\nu\bar{\nu}$ decay rates

As shown in Table VIII, the $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}'l\nu$ decay amplitudes are governed by tree $T_{\mathcal{D}\bar{\mathcal{D}}}$ and annihilation $A_{\mathcal{D}\bar{\mathcal{D}}}$ amplitudes, while the $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}'\nu\bar{\nu}$ decay amplitudes are governed by penguin-box PB and penguin-box-annihilation PBA amplitudes. The penguin-box and tree amplitudes are

TABLE XIV. Branching ratios of $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}l\bar{\nu}$ and $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}\nu\bar{\nu}$ decays. The last factors are from the SU(3) breaking from threshold enhancement estimated using Eq. (62) with $\gamma = 7$, and parameters $\beta'', \kappa'', \sigma'', \xi'', \omega''$ are expected to be of order one.

Mode	$\mathrm{Br}(\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}l\bar{\nu})(10^{-8})$	Mode	$\mathrm{Br}(\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}l\bar{\nu})(10^{-8})$
$B^- \to \Delta^+\bar{p}l\bar{\nu}$	$2a''$	$B^- \to \Delta^0\bar{n}l\bar{\nu}$	$2a'' \times (0.991)$
$B^- \to \Sigma^{*+}\overline{\Sigma^+}l\bar{\nu}$	$2a'' \times (0.0660\sigma'')$	$B^- \to \Sigma^{*0}\overline{\Sigma^0}l\bar{\nu}$	$\frac{1}{2}a'' \times (0.0643\sigma'')$
$B^- \to \Xi^{*0}\overline{\Xi^0}l\bar{\nu}$	$2a'' \times (0.0130\xi'')$	$B^- \to \Sigma^{*0}\bar{\Lambda}l\bar{\nu}$	$\frac{3}{2}a'' \times (0.104\sigma'')$
$\bar{B}^0 \to \Delta^{++}\bar{p}l\bar{\nu}$	$5.56a''$	$\bar{B}^0 \to \Delta^+\bar{n}l\bar{\nu}$	$1.85a'' \times (0.991)$
$\bar{B}^0 \to \Sigma^{*+}\overline{\Sigma^0}l\bar{\nu}$	$0.93a'' \times (0.0646\sigma'')$	$\bar{B}^0 \to \Sigma^{*0}\overline{\Sigma^-}l\bar{\nu}$	$0.93a'' \times (0.0624\sigma'')$
$\bar{B}^0 \to \Xi^{*0}\overline{\Xi^-}l\bar{\nu}$	$1.85a'' \times (0.0125\xi'')$	$\bar{B}^0 \to \Sigma^{*+}\bar{\Lambda}l\bar{\nu}$	$2.78a'' \times (0.105\sigma'')$
$\bar{B}^0_s \to \Delta^{++}\overline{\Sigma^+}l\bar{\nu}$	$5.59a'' \times (0.173\beta'')$	$\bar{B}^0_s \to \Delta^+\overline{\Sigma^0}l\bar{\nu}$	$3.73a'' \times (0.170\beta'')$
$\bar{B}^0_s \to \Delta^0\overline{\Sigma^-}l\bar{\nu}$	$1.86a'' \times (0.164\beta'')$	$\bar{B}^0_s \to \Sigma^{*+}\overline{\Xi^0}l\bar{\nu}$	$1.86a'' \times (0.0308\kappa'')$
$\bar{B}^0_s \to \Sigma^{*0}\overline{\Xi^-}l\bar{\nu}$	$0.93a'' \times (0.0294\kappa'')$		

Mode	$\sum_\nu \mathrm{Br}(\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}\nu\bar{\nu})(10^{-10})$	Mode	$\sum_\nu \mathrm{Br}(\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}\nu\bar{\nu})(10^{-10})$
$B^- \to \Sigma^{*0}\bar{p}\nu\bar{\nu}$	$0.40a'' \times (0.339\beta'')$	$B^- \to \Sigma^{*-}\bar{n}\nu\bar{\nu}$	$0.80a'' \times (0.328\beta'')$
$B^- \to \Xi^{*0}\overline{\Sigma^+}\nu\bar{\nu}$	$0.80a'' \times (0.0270\kappa'')$	$B^- \to \Xi^{*-}\overline{\Sigma^0}\nu\bar{\nu}$	$0.40a'' \times (0.0260\kappa'')$
$B^- \to \Omega^-\overline{\Xi^0}\nu\bar{\nu}$	$2.40a'' \times (0.00602\omega'')$	$B^- \to \Xi^{*-}\bar{\Lambda}\nu\bar{\nu}$	$1.20a'' \times (0.0408\kappa'')$
$\bar{B}^0 \to \Sigma^{*+}\bar{p}\nu\bar{\nu}$	$0.74a'' \times (0.341\beta'')$	$\bar{B}^0 \to \Sigma^{*0}\bar{n}\nu\bar{\nu}$	$0.37a'' \times (0.336\beta'')$
$\bar{B}^0 \to \Xi^{*0}\overline{\Sigma^0}\nu\bar{\nu}$	$0.37a'' \times (0.0263\kappa'')$	$\bar{B}^0 \to \Xi^{*-}\overline{\Sigma^-}\nu\bar{\nu}$	$0.74a'' \times (0.0251\kappa'')$
$\bar{B}^0 \to \Omega^-\overline{\Xi^-}\nu\bar{\nu}$	$2.24a'' \times (0.00580\omega'')$	$\bar{B}^0 \to \Xi^{*0}\bar{\Lambda}\nu\bar{\nu}$	$1.11a'' \times (0.0416\kappa'')$
$\bar{B}^0_s \to \Sigma^{*+}\overline{\Sigma^+}\nu\bar{\nu}$	$0.75a'' \times (0.0660\sigma'')$	$\bar{B}^0_s \to \Sigma^{*0}\overline{\Sigma^0}\nu\bar{\nu}$	$0.75a'' \times (0.0643\sigma'')$
$\bar{B}^0_s \to \Sigma^{*-}\overline{\Sigma^-}\nu\bar{\nu}$	$0.75a'' \times (0.0611\sigma'')$	$\bar{B}^0_s \to \Xi^{*0}\overline{\Xi^0}\nu\bar{\nu}$	$0.75a'' \times (0.0130\xi'')$
$\bar{B}^0_s \to \Xi^{*-}\overline{\Xi^-}\nu\bar{\nu}$	$0.75a'' \times (0.0123\xi'')$		

related by a proportional constant ζ, while the penguin-box-annihilation and annihilation amplitudes are related by the same constant, see Eq. (20). The $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}'l\nu$ decay rates can be parametrized by 5 parameters, namely, a''', b''', c''', d''', and e''', where the first four are contributed from tree and annihilation amplitudes, with the following amplitudes $T_{\mathcal{D}\bar{\mathcal{D}}}$, $T_{\mathcal{D}\bar{\mathcal{D}}} + A_{\mathcal{D}\bar{\mathcal{D}}}/6, T_{\mathcal{D}\bar{\mathcal{D}}} + A_{\mathcal{D}\bar{\mathcal{D}}}/4, T_{\mathcal{D}\bar{\mathcal{D}}} + A_{\mathcal{D}\bar{\mathcal{D}}}/2$, respectively, while the last one is only from the annihilation amplitude, $A_{\mathcal{D}\bar{\mathcal{D}}}$. The same set of parameters can be used in $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}'\nu\bar{\nu}$ decay rates as the topological amplitudes are proportional to those in $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}'l\nu$ decays by the common factor, ζ. Note that the above parameters correspond to the rates in the SU(3) symmetric limit.

Using the triangle inequality Eq. (61), we obtain the following relations,

$$\begin{aligned}(\sqrt{a'''} - \sqrt{e'''}/6)^2 &\lesssim b''' \lesssim (\sqrt{a'''} + \sqrt{e'''}/6)^2,\\ (\sqrt{a'''} - \sqrt{e'''}/4)^2 &\lesssim c''' \lesssim (\sqrt{a'''} + \sqrt{e'''}/4)^2,\\ (\sqrt{a'''} - \sqrt{e'''}/2)^2 &\lesssim d''' \lesssim (\sqrt{a'''} + \sqrt{e'''}/2)^2. \end{aligned} \quad (73)$$

As in the case of $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}'l\bar{\nu}$ decays, it is expected that the contributions from annihilation amplitudes to be much suppressed than others. Consequently, we should have $e''' \ll a'''$ and the above inequalities lead to the following relations,

$$a''' \simeq b''' \simeq c''' \simeq d''' \gg e'''. \quad (74)$$

As in other $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'l\bar{\nu}(\nu\bar{\nu})$ decays, threshold enhancements in the differential rates of $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}'l\bar{\nu}$ and $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}'\nu\bar{\nu}$ decays are anticipated. They will lead to large SU(3) breaking effect on $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}'l\bar{\nu}$ and $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}'\nu\bar{\nu}$ decay rates. The SU(3) breaking on rates from the threshold enhancement can be estimated as in the $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}'l\bar{\nu}$ case, once the differential rate of a $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}'l\bar{\nu}$ decay mode is measured. Apparently, no such information is available at this moment. We should make use of some model calculations to obtain information on the differential rates of these modes for illustration. As we shall see, in the model calculations the SU(3) breaking can be estimated using Eq. (62) with $\gamma = 10$ employing a similar procedure in the discussion around Eq. (63), and the related parameters $\beta''', \sigma''', \kappa''', \xi''', \upsilon''', \omega'''$ denoting milder SU(3) breaking are expected to be of order 1.

With these considerations the rates of $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}'l\nu$ and $\mathcal{D}\bar{\mathcal{D}}'\nu\bar{\nu}$ decays are shown in Table XVIII. With parameters $a''' \simeq b''' \simeq c''' \simeq d''' \gg e'''$ and SU(3) breaking parameters $\beta''', \sigma''', \kappa''', \xi''', \upsilon''', \omega'''$ of order 1, the relative sizes of

TABLE XV. Branching ratios of $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}l\bar{\nu}$ and $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}\nu\bar{\nu}$ decays in model 1 and model 2. Br_1 and Br_2 denote results in model 1 and model 2, respectively.

Mode	$\mathrm{Br}_1(10^{-8})$	$\mathrm{Br}_2(10^{-8})$	Mode	$\mathrm{Br}_1(10^{-8})$	$\mathrm{Br}_2(10^{-8})$
$B^- \to p\overline{\Delta^+}l\bar{\nu}$	2.30	14.48	$B^- \to n\overline{\Delta^0}l\bar{\nu}$	2.28	14.38
$B^- \to \Sigma^+\overline{\Sigma^{*+}}l\bar{\nu}$	0.22	1.48	$B^- \to \Sigma^0\overline{\Sigma^{*0}}l\bar{\nu}$	0.054	0.36
$B^- \to \Xi^0\overline{\Xi^{*0}}l\bar{\nu}$	0.046	0.32	$B^- \to \Lambda\overline{\Sigma^{*0}}l\bar{\nu}$	0.25	1.65
$\bar{B}^0 \to p\overline{\Delta^0}l\bar{\nu}$	2.13	13.44	$\bar{B}^0 \to n\overline{\Delta^-}l\bar{\nu}$	6.35	40.01
$\bar{B}^0 \to \Sigma^+\overline{\Sigma^{*0}}l\bar{\nu}$	0.10	0.68	$\bar{B}^0 \to \Sigma^0\overline{\Sigma^{*-}}l\bar{\nu}$	0.098	0.66
$\bar{B}^0 \to \Xi^0\overline{\Xi^{*-}}l\bar{\nu}$	0.042	0.29	$\bar{B}^0 \to \Lambda\overline{\Sigma^{*-}}l\bar{\nu}$	0.45	2.30
$\bar{B}^0_s \to p\overline{\Sigma^{*0}}l\bar{\nu}$	0.48	3.08	$\bar{B}^0_s \to n\overline{\Sigma^{*-}}l\bar{\nu}$	0.94	5.99
$\bar{B}^0_s \to \Sigma^+\overline{\Xi^{*0}}l\bar{\nu}$	0.10	0.70	$\bar{B}^0_s \to \Sigma^0\overline{\Xi^{*-}}l\bar{\nu}$	0.049	0.34
$\bar{B}^0_s \to \Xi^0\overline{\Omega^-}l\bar{\nu}$	0.069	0.49	$\bar{B}^0_s \to \Lambda\overline{\Xi^{*-}}l\bar{\nu}$	0.22	1.51

Mode	$\sum_\nu \mathrm{Br}_1(10^{-10})$	$\sum_\nu \mathrm{Br}_2(10^{-10})$	Mode	$\sum_\nu \mathrm{Br}_1(10^{-10})$	$\sum_\nu \mathrm{Br}_2(10^{-10})$
$B^- \to \Sigma^+\overline{\Delta^{++}}\nu\bar{\nu}$	0.64	4.27	$B^- \to \Sigma^0\overline{\Delta^+}\nu\bar{\nu}$	0.42	2.79
$B^- \to \Sigma^-\overline{\Delta^0}\nu\bar{\nu}$	0.20	1.36	$B^- \to \Xi^0\overline{\Sigma^{*+}}\nu\bar{\nu}$	0.043	0.30
$B^- \to \Xi^-\overline{\Sigma^{*0}}\nu\bar{\nu}$	0.021	0.14			
$\bar{B}^0 \to \Sigma^+\overline{\Delta^+}\nu\bar{\nu}$	0.20	1.32	$\bar{B}^0 \to \Sigma^0\overline{\Delta^0}\nu\bar{\nu}$	0.39	2.59
$\bar{B}^0 \to \Sigma^-\overline{\Delta^-}\nu\bar{\nu}$	0.57	3.78	$\bar{B}^0 \to \Xi^0\overline{\Sigma^{*0}}\nu\bar{\nu}$	0.020	0.14
$\bar{B}^0 \to \Xi^-\overline{\Sigma^{*-}}\nu\bar{\nu}$	0.038	0.26			
$\bar{B}^0_s \to \Sigma^+\overline{\Sigma^{*+}}\nu\bar{\nu}$	0.097	0.65	$\bar{B}^0_s \to \Sigma^0\overline{\Sigma^{*0}}\nu\bar{\nu}$	0.095	0.64
$\bar{B}^0_s \to \Sigma^-\overline{\Sigma^{*-}}\nu\bar{\nu}$	0.090	0.61	$\bar{B}^0_s \to \Xi^0\overline{\Xi^{*0}}\nu\bar{\nu}$	0.021	0.14
$\bar{B}^0_s \to \Xi^-\overline{\Xi^{*-}}\nu\bar{\nu}$	0.019	0.14			

TABLE XVI. Same as Table XV but for branching ratios of $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}l\bar{\nu}$ and $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}\nu\bar{\nu}$ decays.

Mode	$\mathrm{Br}_1(10^{-8})$	$\mathrm{Br}_2(10^{-8})$	Mode	$\mathrm{Br}_1(10^{-8})$	$\mathrm{Br}_2(10^{-8})$
$B^- \to \Delta^+\bar{p}l\bar{\nu}$	2.70	12.38	$B^- \to \Delta^0\bar{n}l\bar{\nu}$	2.68	12.28
$B^- \to \Sigma^{*+}\overline{\Sigma^+}l\bar{\nu}$	0.26	1.45	$B^- \to \Sigma^{*0}\overline{\Sigma^0}l\bar{\nu}$	0.065	0.36
$B^- \to \Xi^{*0}\overline{\Xi^0}l\bar{\nu}$	0.056	0.37	$B^- \to \Sigma^{*0}\bar{\Lambda}l\bar{\nu}$	0.30	1.63
$\bar{B}^0 \to \Delta^{++}\bar{p}l\bar{\nu}$	7.51	34.45	$\bar{B}^0 \to \Delta^+\bar{n}l\bar{\nu}$	2.48	11.40
$\bar{B}^0 \to \Sigma^{*+}\overline{\Sigma^0}l\bar{\nu}$	0.12	0.66	$\bar{B}^0 \to \Sigma^{*0}\overline{\Sigma^-}l\bar{\nu}$	0.12	0.64
$\bar{B}^0 \to \Xi^{*0}\overline{\Xi^-}l\bar{\nu}$	0.050	0.33	$\bar{B}^0 \to \Sigma^{*+}\bar{\Lambda}l\bar{\nu}$	0.56	3.04
$\bar{B}^0_s \to \Delta^{++}\overline{\Sigma^+}l\bar{\nu}$	2.05	9.47	$\bar{B}^0_s \to \Delta^+\overline{\Sigma^0}l\bar{\nu}$	1.34	6.20
$\bar{B}^0_s \to \Delta^0\overline{\Sigma^-}l\bar{\nu}$	0.65	3.02	$\bar{B}^0_s \to \Sigma^{*+}\overline{\Xi^0}l\bar{\nu}$	0.14	0.80
$\bar{B}^0_s \to \Sigma^{*0}\overline{\Xi^-}l\bar{\nu}$	0.069	0.38			

Mode	$\sum_\nu \mathrm{Br}_1(10^{-10})$	$\sum_\nu \mathrm{Br}_2(10^{-10})$	Mode	$\sum_\nu \mathrm{Br}_1(10^{-10})$	$\sum_\nu \mathrm{Br}_2(10^{-10})$
$B^- \to \Sigma^{*0}\bar{p}\nu\bar{\nu}$	0.21	1.16	$B^- \to \Sigma^{*-}\bar{n}\nu\bar{\nu}$	0.42	2.26
$B^- \to \Xi^{*0}\overline{\Sigma^+}\nu\bar{\nu}$	0.045	0.29	$B^- \to \Xi^{*-}\overline{\Sigma^0}\nu\bar{\nu}$	0.022	0.14
$B^- \to \Omega^-\overline{\Xi^0}\nu\bar{\nu}$	0.030	0.23	$B^- \to \Xi^{*-}\bar{\Lambda}\nu\bar{\nu}$	0.099	0.64
$\bar{B}^0 \to \Sigma^{*+}\bar{p}\nu\bar{\nu}$	0.40	2.16	$\bar{B}^0 \to \Sigma^{*0}\bar{n}\nu\bar{\nu}$	0.20	1.07
$\bar{B}^0 \to \Xi^{*0}\overline{\Sigma^0}\nu\bar{\nu}$	0.020	0.13	$\bar{B}^0 \to \Xi^{*-}\overline{\Sigma^-}\nu\bar{\nu}$	0.039	0.25
$\bar{B}^0 \to \Omega^-\overline{\Xi^-}\nu\bar{\nu}$	0.027	0.21	$\bar{B}^0 \to \Xi^{*0}\bar{\Lambda}\nu\bar{\nu}$	0.094	0.60
$\bar{B}^0_s \to \Sigma^{*+}\overline{\Sigma^+}\nu\bar{\nu}$	0.12	0.63	$\bar{B}^0_s \to \Sigma^{*0}\overline{\Sigma^0}\nu\bar{\nu}$	0.11	0.62
$\bar{B}^0_s \to \Sigma^{*-}\overline{\Sigma^-}\nu\bar{\nu}$	0.11	0.59	$\bar{B}^0_s \to \Xi^{*0}\overline{\Xi^0}\nu\bar{\nu}$	0.025	0.16
$\bar{B}^0_s \to \Xi^{*-}\overline{\Xi^-}\nu\bar{\nu}$	0.024	0.16			

TABLE XVII. Values of various parameters in model 1 and model 2.

Parameters	Values (Model 1)	Values (Model 2)
a'	1.15	7.24
β'	1.01	1.03
κ'	0.72	0.78
σ'	0.74	0.79
ξ'	0.52	0.57
ω'	0.50	0.56

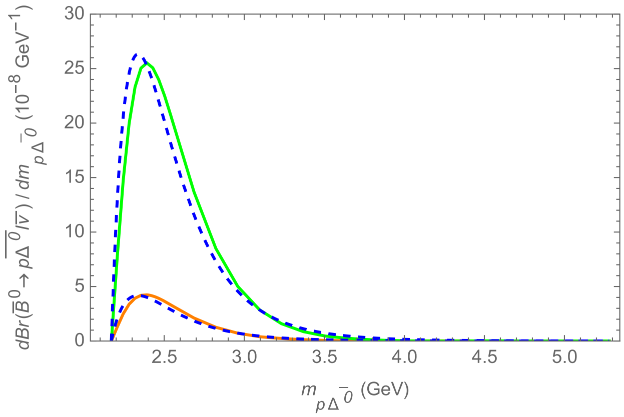
Parameters	Values (Model 1)	Values (Model 2)
a''	1.35	6.19
β''	1.57	1.58
κ''	1.86	2.24
σ''	1.49	1.78
ξ''	1.58	2.29
ω''	1.53	2.59

$\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}' l\nu$ and $\mathcal{D}\bar{\mathcal{D}}'\nu\bar{\nu}$ decay rates can be readily read from the table. From Tables XVIII and IX we note that $B^- \to \Delta^{++}\overline{\Delta^{++}} l\bar{\nu}$ decay mode has the largest rate and good detectability. It is also among the least suppressed modes by SU(3) breaking effect from the threshold enhancement even if $\gamma = 10$ in Eq. (62) is not borne out. For $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}'\nu\bar{\nu}$ modes, $B^- \to \Sigma^{*+}\overline{\Delta^{++}}\nu\bar{\nu}$ decay has relatively unsuppressed rate and good detectability. The above assumption of neglecting annihilation contributions can be checked by searching the $B^- \to \Sigma^{*-}\overline{\Sigma^{*-}} l\bar{\nu}$ decay mode, which is a pure annihilation mode but with final states of good detectability.

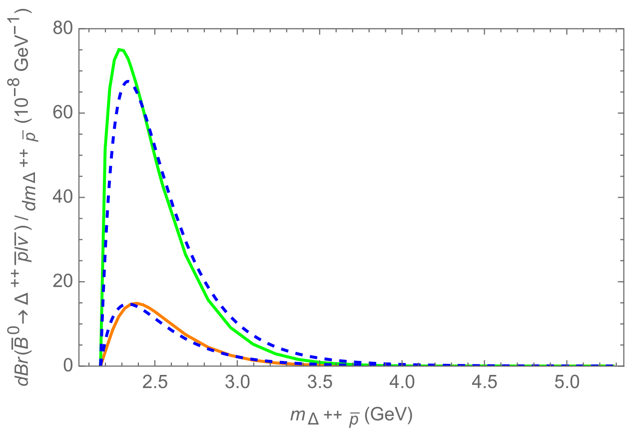
Using inputs from Sec. II B and the formulas given Appendix B, the branching ratios of $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}' l\bar{\nu}$ and $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}'\nu\bar{\nu}$ in model 1 and 2 are obtained and are shown in Table XIX. These results are new.

From Table XIX, we see that the branching ratios of $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}' l\bar{\nu}$ decays of nonannihilation modes are in the ranges of 10^{-9}–10^{-7} in model 1 and 2, while $\sum_\nu \mathrm{Br}(\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}' l\nu\nu)$ are in the ranges of 10^{-12}–10^{-10} and 10^{-11}–10^{-10} in model 1 and 2, respectively. The rates in model 2 are greater than those in model 1 by a factor of 3. This corresponds to the fact that $|T_{\mathcal{D}\bar{\mathcal{D}}}|$ in model 2 is greater than one the in model 1 as reflected through the sizes of a''' in these two models as shown in Table XX. This is not surprising as we noted previously, the sizes of topological amplitudes $|T_{1\mathcal{B}\bar{\mathcal{B}}}|$, $|T_{2\mathcal{B}\bar{\mathcal{B}}}|$ $|T_{\mathcal{B}\bar{\mathcal{D}}}|$ and $|T_{\mathcal{D}\bar{\mathcal{B}}}|$ in model 2 are greater than those in model 1.

The differential rates $d\mathrm{Br}/dm_{\Delta^{++}\overline{\Delta^{++}}}$ of $B^- \to \Delta^{++}\overline{\Delta^{++}} l^-\bar{\nu}$ decay from model 1 and model 2 are shown in Fig. 5, they can be compared to the one plotted using Eq. (62) with $\gamma = 10$. They clearly exhibit threshold enhancement as in other $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' l\bar{\nu}$ decays.



(a)



(b)

FIG. 4. (a) Differential rate $d\mathrm{Br}/dm_{p\overline{\Delta^0}}$ of $\bar{B}^0 \to p\overline{\Delta^0} l^-\bar{\nu}$ decay from model 1 (orange solid line) and model 2 (green solid line). The dashed lines are $d\mathrm{Br}/dm_{p\overline{\Delta^0}}$ using Eq. (62) with $\gamma = 7$. (b) Same as (a) but for the differential rate $d\mathrm{Br}/dm_{\Delta^{++}\bar{p}}$ of $\bar{B}^0 \to \Delta^{++}\bar{p} l^-\bar{\nu}$ decay.

From Table XX, we see that the parameters $\beta''', \kappa''', \sigma''', \xi''', \upsilon''', \omega', \bar{\beta}''', \bar{\kappa}'''$ and $\bar{\omega}'''$ denoting milder SU(3) breaking are indeed of order one and are similar in model 1 and 2. Furthermore, $\bar{\beta}'''/\beta'''$, $\bar{\kappa}'''/\kappa'''$ and $\bar{\omega}'''/\omega'''$ are close to one as expected.

By taking into account the sensitivity of detection, see Table IX, and decay rates, see Tables XVIII and XIX, we find that the following decay modes have relatively unsuppressed rates and good detectability, they are $B^- \to \Delta^{++}\overline{\Delta^{++}} l\bar{\nu}$, $B^- \to \Delta^0\overline{\Delta^0} l\bar{\nu}$, $B^- \to \Sigma^{*+}\overline{\Sigma^{*+}} l\bar{\nu}$, $\bar{B}^0_s \to \Delta^{++}\overline{\Sigma^{*+}} l\bar{\nu}$, $\bar{B}^0_s \to \Delta^0\overline{\Sigma^{*-}} l\bar{\nu}$, $\bar{B}^0_s \to \Sigma^{*+}\overline{\Xi^{*0}} l\bar{\nu}$, $B^- \to \Sigma^{*+}\overline{\Delta^{++}}\nu\bar{\nu}$, $B^- \to \Sigma^{*-}\overline{\Delta^0}\nu\bar{\nu}$, $B^- \to \Xi^{*0}\overline{\Sigma^{*+}}\nu\bar{\nu}$, $\bar{B}^0_s \to \Sigma^{*+}\overline{\Sigma^{*+}}\nu\bar{\nu}$, $\bar{B}^0_s \to \Sigma^{*-}\overline{\Sigma^{*-}}\nu\bar{\nu}$, $\bar{B}^0_s \to \Xi^{*0}\overline{\Xi^{*0}}\nu\bar{\nu}$ decays.

Ratios of rates from loop induced modes and tree induced modes are sensible test of the SM. From Table XVIII, we obtain

TABLE XVIII. Branching ratios of $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}' l\bar{\nu}$ and $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}' \nu\bar{\nu}$ decays. Parameters a''', b''', c''', d''' are expected to be of similar sizes, while e''' is expected to be much suppressed. The last factors are SU(3) breaking from threshold enhancement, estimated uisng Eq. (62) with $\gamma = 10$, and parameters $\beta''', \sigma''', \kappa''', \xi''', \upsilon''', \omega'''$ are expected to be of order 1.

Mode	$\text{Br}(\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}' l\bar{\nu})(10^{-8})$	Mode	$\text{Br}(\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}' l\bar{\nu})(10^{-8})$
$B^- \to \Delta^{++}\overline{\Delta^{++}} l\bar{\nu}$	$36b'''$	$B^- \to \Delta^{+}\overline{\Delta^{+}} l\bar{\nu}$	$16a'''$
$B^- \to \Delta^{0}\overline{\Delta^{0}} l\bar{\nu}$	$4d'''$	$B^- \to \Delta^{-}\overline{\Delta^{-}} l\bar{\nu}$	e'''
$B^- \to \Sigma^{*+}\overline{\Sigma^{*+}} l\bar{\nu}$	$16c''' \times (0.125\sigma''')$	$B^- \to \Sigma^{*0}\overline{\Sigma^{*0}} l\bar{\nu}$	$4d''' \times (0.124\sigma''')$
$B^- \to \Sigma^{*-}\overline{\Sigma^{*-}} l\bar{\nu}$	$e''' \times (0.118\sigma''')$	$B^- \to \Xi^{*0}\overline{\Xi^{*0}} l\bar{\nu}$	$4d''' \times (0.0198\xi''')$
$B^- \to \Xi^{*-}\overline{\Xi^{*-}} l\bar{\nu}$	$e''' \times (0.0191\xi''')$	$B^- \to \Omega^{-}\overline{\Omega^{-}} l\bar{\nu}$	$e''' \times (0.00407\upsilon''')$
$\bar{B}^0 \to \Delta^{++}\overline{\Delta^{+}} l\bar{\nu}$	$11.1a'''$	$\bar{B}^0 \to \Delta^{+}\overline{\Delta^{0}} l\bar{\nu}$	$14.8a'''$
$\bar{B}^0 \to \Delta^{0}\overline{\Delta^{-}} l\bar{\nu}$	$11.1a'''$	$\bar{B}^0 \to \Sigma^{*+}\overline{\Sigma^{*0}} l\bar{\nu}$	$7.4a''' \times (0.124\sigma''')$
$\bar{B}^0 \to \Sigma^{*0}\overline{\Sigma^{*-}} l\bar{\nu}$	$7.4a''' \times (0.121\sigma''')$	$\bar{B}^0 \to \Xi^{*0}\overline{\Xi^{*-}} l\bar{\nu}$	$3.7a''' \times (0.0195\xi''')$
$\bar{B}^0_s \to \Delta^{++}\overline{\Sigma^{*+}} l\bar{\nu}$	$11.2a''' \times (0.343\beta''')$	$\bar{B}^0_s \to \Delta^{+}\overline{\Sigma^{*0}} l\bar{\nu}$	$7.5a''' \times (0.341\beta''')$
$\bar{B}^0_s \to \Delta^{0}\overline{\Sigma^{*-}} l\bar{\nu}$	$3.7a''' \times (0.333\beta''')$	$\bar{B}^0_s \to \Sigma^{*+}\overline{\Xi^{*0}} l\bar{\nu}$	$14.9a''' \times (0.0486\kappa''')$
$\bar{B}^0_s \to \Sigma^{*0}\overline{\Xi^{*-}} l\bar{\nu}$	$7.5a''' \times (0.0474\kappa''')$	$\bar{B}^0_s \to \Xi^{*0}\overline{\Omega^{-}} l\bar{\nu}$	$11.2a''' \times (0.00883\omega''')$

Mode	$\sum_\nu \text{Br}(\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}' \nu\bar{\nu})(10^{-10})$	Mode	$\sum_\nu \text{Br}(\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}' \nu\bar{\nu})(10^{-10})$
$B^- \to \Sigma^{*+}\overline{\Delta^{++}} \nu\bar{\nu}$	$4.80a''' \times (0.343\bar{\beta}''')$	$B^- \to \Sigma^{*0}\overline{\Delta^{+}} \nu\bar{\nu}$	$3.20a''' \times (0.341\bar{\beta}''')$
$B^- \to \Sigma^{*-}\overline{\Delta^{0}} \nu\bar{\nu}$	$1.60a''' \times (0.333\bar{\beta}''')$	$B^- \to \Xi^{*0}\overline{\Sigma^{*+}} \nu\bar{\nu}$	$6.40a''' \times (0.0486\bar{\kappa}''')$
$B^- \to \Xi^{*-}\overline{\Sigma^{*0}} \nu\bar{\nu}$	$3.20a''' \times (0.0474\bar{\kappa}''')$	$B^- \to \Omega^{-}\overline{\Xi^{*0}} \nu\bar{\nu}$	$4.80a''' \times (0.00883\bar{\omega}''')$
$\bar{B}^0 \to \Sigma^{*+}\overline{\Delta^{+}} \nu\bar{\nu}$	$1.48a''' \times (0.343\bar{\beta}''')$	$\bar{B}^0 \to \Sigma^{*0}\overline{\Delta^{0}} \nu\bar{\nu}$	$2.97a''' \times (0.341\bar{\beta}''')$
$\bar{B}^0 \to \Sigma^{*-}\overline{\Delta^{-}} \nu\bar{\nu}$	$4.45a''' \times (0.333\bar{\beta}''')$	$\bar{B}^0 \to \Xi^{*0}\overline{\Sigma^{*0}} \nu\bar{\nu}$	$2.97a''' \times (0.0484\bar{\kappa}''')$
$\bar{B}^0 \to \Xi^{*-}\overline{\Sigma^{*-}} \nu\bar{\nu}$	$5.93a''' \times (0.0464\bar{\kappa}''')$	$\bar{B}^0 \to \Omega^{-}\overline{\Xi^{*-}} \nu\bar{\nu}$	$4.45a''' \times (0.00867\bar{\omega}''')$
$\bar{B}^0_s \to \Delta^{++}\overline{\Delta^{++}} \nu\bar{\nu}$	$0.37e'''$	$\bar{B}^0_s \to \Delta^{+}\overline{\Delta^{+}} \nu\bar{\nu}$	$0.37e'''$
$\bar{B}^0_s \to \Delta^{0}\overline{\Delta^{0}} \nu\bar{\nu}$	$0.37e'''$	$\bar{B}^0_s \to \Delta^{-}\overline{\Delta^{-}} \nu\bar{\nu}$	$0.37e'''$
$\bar{B}^0_s \to \Sigma^{*+}\overline{\Sigma^{*+}} \nu\bar{\nu}$	$1.49d''' \times (0.125\sigma''')$	$\bar{B}^0_s \to \Sigma^{*0}\overline{\Sigma^{*0}} \nu\bar{\nu}$	$1.49d''' \times (0.124\sigma''')$
$\bar{B}^0_s \to \Sigma^{*-}\overline{\Sigma^{*-}} \nu\bar{\nu}$	$1.49d''' \times (0.118\sigma''')$	$\bar{B}^0_s \to \Xi^{*0}\overline{\Xi^{*0}} \nu\bar{\nu}$	$5.96c''' \times (0.0198\xi''')$
$\bar{B}^0_s \to \Xi^{*-}\overline{\Xi^{*-}} \nu\bar{\nu}$	$5.96c'''' \times (0.0191\xi''')$	$\bar{B}^0_s \to \Omega^{-}\overline{\Omega^{-}} \nu\bar{\nu}$	$13.42b'''' \times (0.00407\upsilon''')$

$$\frac{\sum_\nu \text{Br}(\bar{B}^0_s \to \Sigma^{*+}\overline{\Sigma^{*+}}\nu\bar{\nu})}{\text{Br}(B^- \to \Delta^0\overline{\Delta^0}l\bar{\nu})} = 4.66\sigma''' \times \left(\frac{0.0036}{|V_{ub}|}\right)^2 \times 10^{-4},$$
$$\frac{\sum_\nu \text{Br}(\bar{B}^0_s \to \Sigma^{*-}\overline{\Sigma^{*-}}\nu\bar{\nu})}{\text{Br}(B^- \to \Delta^0\overline{\Delta^0}l\bar{\nu})} = 4.40\sigma''' \times \left(\frac{0.0036}{|V_{ub}|}\right)^2 \times 10^{-4}, \tag{75}$$

and

$$\frac{\sum_\nu \text{Br}(B^- \to \Sigma^{*+}\overline{\Delta^{++}}\nu\bar{\nu})}{\text{Br}(\bar{B}^0_s \to \Delta^{++}\overline{\Sigma^{*+}}l\bar{\nu})} = 4.29\frac{\bar{\beta}'''}{\beta'''} \times \left(\frac{0.0036}{|V_{ub}|}\right)^2 \times 10^{-3},$$
$$\frac{\sum_\nu \text{Br}(B^- \to \Sigma^{*-}\overline{\Delta^{0}}\nu\bar{\nu})}{\text{Br}(\bar{B}^0_s \to \Delta^{0}\overline{\Sigma^{*-}}l\bar{\nu})} = 4.29\frac{\bar{\beta}'''}{\beta'''} \times \left(\frac{0.0036}{|V_{ub}|}\right)^2 \times 10^{-3},$$
$$\frac{\sum_\nu \text{Br}(B^- \to \Xi^{*0}\overline{\Sigma^{*+}}\nu\bar{\nu})}{\text{Br}(\bar{B}^0_s \to \Sigma^{*+}\overline{\Xi^{*0}}l\bar{\nu})} = 4.29\frac{\bar{\kappa}'''}{\kappa'''} \times \left(\frac{0.0036}{|V_{ub}|}\right)^2 \times 10^{-3}, \tag{76}$$

where σ''' is expected to be of order one, while $\bar{\beta}'''/\beta'''$ and $\bar{\kappa}'''/\kappa'''$ are expected to be close to one. In fact as shown in Table XX, we have $\sigma''' = 1.40(1.43)$, $\bar{\beta}'''/\beta''' = 0.83(0.85)$ and $\bar{\kappa}'''/\kappa''' = 0.81(0.83)$ in model 1 (2), which are indeed agree with the above expectations. Note that the ratios in Eqs. (75) and (76) do not involve the small e''' assumption, and the ratios in Eq. (76) are less sensitive to the SU(3) breaking from threshold enhancement. These ratios can be checked experimentally.

V. DISCUSSIONS AND CONCLUSION

We study the decay amplitudes and rates of $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' l\bar{\nu}$ and $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}' \nu\bar{\nu}$ decays with all low lying octet ($\mathcal{B}$) and decuplet ($\mathcal{D}$) baryons using a topological amplitude approach. The decay amplitudes are decomposed into combinations of topological amplitudes. In $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}' l\bar{\nu}$ decays we need three topological amplitudes, namely two tree amplitudes, $T_{2\mathcal{B}\bar{\mathcal{B}}}$, $T_{1\mathcal{B}\bar{\mathcal{B}}}$ and one annihilation

TABLE XIX. Branching ratios of $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}' l\bar{\nu}$ and $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}'\nu\bar{\nu}$ decays in model 1 and 2. Br_1 and Br_2 denote results in model 1 and model 2, respectively. Those with vanishing rates are pure annihilation or penguin-box annihilation modes.

Mode	$\mathrm{Br}_1(10^{-8})$	$\mathrm{Br}_2(10^{-8})$	Mode	$\mathrm{Br}_1(10^{-8})$	$\mathrm{Br}_2(10^{-8})$
$B^- \to \Delta^{++}\overline{\Delta^{++}}l\bar{\nu}$	16.92	48.26	$B^- \to \Delta^{+}\overline{\Delta^{+}}l\bar{\nu}$	7.52	21.45
$B^- \to \Delta^{0}\overline{\Delta^{0}}l\bar{\nu}$	1.88	5.36	$B^- \to \Delta^{-}\overline{\Delta^{-}}l\bar{\nu}$	0	0
$B^- \to \Sigma^{*+}\overline{\Sigma^{*+}}l\bar{\nu}$	1.32	3.83	$B^- \to \Sigma^{*0}\overline{\Sigma^{*0}}l\bar{\nu}$	0.33	0.95
$B^- \to \Sigma^{*-}\overline{\Sigma^{*-}}l\bar{\nu}$	0	0	$B^- \to \Xi^{*0}\overline{\Xi^{*0}}l\bar{\nu}$	0.060	0.18
$B^- \to \Xi^{*-}\overline{\Xi^{*-}}l\bar{\nu}$	0	0	$B^- \to \Omega^{-}\overline{\Omega^{-}}l\bar{\nu}$	0	0
$\bar{B}^0 \to \Delta^{++}\overline{\Delta^{+}}l\bar{\nu}$	5.23	14.93	$\bar{B}^0 \to \Delta^{+}\overline{\Delta^{0}}l\bar{\nu}$	6.98	19.90
$\bar{B}^0 \to \Delta^{0}\overline{\Delta^{-}}l\bar{\nu}$	5.23	14.93	$\bar{B}^0 \to \Sigma^{*+}\overline{\Sigma^{*0}}l\bar{\nu}$	0.61	1.77
$\bar{B}^0 \to \Sigma^{*0}\overline{\Sigma^{*-}}l\bar{\nu}$	0.60	1.72	$\bar{B}^0 \to \Xi^{*0}\overline{\Xi^{*-}}l\bar{\nu}$	0.055	0.16
$\bar{B}^0_s \to \Delta^{++}\overline{\Sigma^{*+}}l\bar{\nu}$	2.59	7.45	$\bar{B}^0_s \to \Delta^{+}\overline{\Sigma^{*0}}l\bar{\nu}$	1.72	4.94
$\bar{B}^0_s \to \Delta^{0}\overline{\Sigma^{*-}}l\bar{\nu}$	0.84	2.42	$\bar{B}^0_s \to \Sigma^{*+}\overline{\Xi^{*0}}l\bar{\nu}$	0.64	1.85
$\bar{B}^0_s \to \Sigma^{*0}\overline{\Xi^{*-}}l\bar{\nu}$	0.31	0.91	$\bar{B}^0_s \to \Xi^{*0}\overline{\Omega^{-}}l\bar{\nu}$	0.093	0.27

Mode	$\sum_\nu \mathrm{Br}_1(10^{-10})$	$\sum_\nu \mathrm{Br}_2(10^{-10})$	Mode	$\sum_\nu \mathrm{Br}_1(10^{-10})$	$\sum_\nu \mathrm{Br}_2(10^{-10})$
$B^- \to \Sigma^{*+}\overline{\Delta^{++}}\nu\bar{\nu}$	0.92	2.70	$B^- \to \Sigma^{*0}\overline{\Delta^{+}}\nu\bar{\nu}$	0.61	1.79
$B^- \to \Sigma^{*-}\overline{\Delta^{0}}\nu\bar{\nu}$	0.30	0.88	$B^- \to \Xi^{*0}\overline{\Sigma^{*+}}\nu\bar{\nu}$	0.22	0.66
$B^- \to \Xi^{*-}\overline{\Sigma^{*0}}\nu\bar{\nu}$	0.11	0.32	$B^- \to \Omega^{-}\overline{\Xi^{*0}}\nu\bar{\nu}$	0.031	0.095
$\bar{B}^0 \to \Sigma^{*+}\overline{\Delta^{+}}\nu\bar{\nu}$	0.28	0.83	$\bar{B}^0 \to \Sigma^{*0}\overline{\Delta^{0}}\nu\bar{\nu}$	0.57	1.66
$\bar{B}^0 \to \Sigma^{*-}\overline{\Delta^{-}}\nu\bar{\nu}$	0.83	2.44	$\bar{B}^0 \to \Xi^{*0}\overline{\Sigma^{*0}}\nu\bar{\nu}$	0.10	0.30
$\bar{B}^0 \to \Xi^{*-}\overline{\Sigma^{*-}}\nu\bar{\nu}$	0.20	0.59	$\bar{B}^0 \to \Omega^{-}\overline{\Xi^{*-}}\nu\bar{\nu}$	0.029	0.086
$\bar{B}^0_s \to \Delta^{++}\overline{\Delta^{++}}\nu\bar{\nu}$	0	0	$\bar{B}^0_s \to \Delta^{+}\overline{\Delta^{+}}\nu\bar{\nu}$	0	0
$\bar{B}^0_s \to \Delta^{0}\overline{\Delta^{0}}\nu\bar{\nu}$	0	0	$\bar{B}^0_s \to \Delta^{-}\overline{\Delta^{-}}\nu\bar{\nu}$	0	0
$\bar{B}^0_s \to \Sigma^{*+}\overline{\Sigma^{*+}}\nu\bar{\nu}$	0.15	0.43	$\bar{B}^0_s \to \Sigma^{*0}\overline{\Sigma^{*0}}\nu\bar{\nu}$	0.14	0.43
$\bar{B}^0_s \to \Sigma^{*-}\overline{\Sigma^{*-}}\nu\bar{\nu}$	0.14	0.41	$\bar{B}^0_s \to \Xi^{*0}\overline{\Xi^{*0}}\nu\bar{\nu}$	0.11	0.33
$\bar{B}^0_s \to \Xi^{*-}\overline{\Xi^{*-}}\nu\bar{\nu}$	0.11	0.32	$\bar{B}^0_s \to \Omega^{-}\overline{\Omega^{-}}\nu\bar{\nu}$	0.049	0.15

TABLE XX. Values of various parameters in model 1 and model 2. Other parameters can be obtained by using this table and Eq. (74).

Parameters	Values (Model 1)	Values (Model 2)
a'''	0.47	1.34
e'''	0	0
β'''	1.43	1.45
κ'''	1.87	1.91
σ'''	1.40	1.43
ξ'''	1.62	1.65
υ'''	1.93	2.03
ω'''	2.01	2.06
$\bar{\beta}'''$	1.19	1.22
$\bar{\kappa}'''$	1.51	1.58
$\bar{\omega}'''$	1.58	1.66
$\bar{\beta}'''/\beta'''$	0.83	0.85
$\bar{\kappa}'''/\kappa'''$	0.81	0.83
$\bar{\omega}'''/\omega'''$	0.79	0.81

amplitude, $A_{\mathcal{B}\bar{\mathcal{B}}}$. In $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}l\bar{\nu}$ decays only one tree amplitude, $T_{\mathcal{B}\bar{\mathcal{D}}}$, is needed. Likewise in $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}l\bar{\nu}$ decays, we only need one tree amplitude, $T_{\mathcal{D}\bar{\mathcal{B}}}$. Lastly in $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}'l\bar{\nu}$ decays, two topological amplitudes, namely a tree amplitude, $T_{\mathcal{D}\bar{\mathcal{D}}}$, and an annihilation amplitude, $A_{\mathcal{D}\bar{\mathcal{D}}}$, are needed. In loop induced decay modes, we have three topological amplitudes, namely two penguin-box amplitudes $PB_{2\mathcal{B}\bar{\mathcal{B}}}$, $PB_{1\mathcal{B}\bar{\mathcal{B}}}$ and one penguin-box-annihilation amplitude, $PBA_{\mathcal{B}\bar{\mathcal{B}}}$, in $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}'\nu\bar{\nu}$ decays, one topological amplitude, namely a penguin-box amplitude, $PB_{\mathcal{B}\bar{\mathcal{D}}}$, in $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}\nu\bar{\nu}$ decays, one topological amplitude, namely a penguin-box amplitude, $PB_{\mathcal{D}\bar{\mathcal{B}}}$, in $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}\nu\bar{\nu}$ decays, two topological amplitudes, namely a penguin-box amplitude, $PB_{\mathcal{D}\bar{\mathcal{D}}}$, and a penguin-box-annihilation amplitude, $PBA_{\mathcal{D}\bar{\mathcal{D}}}$, in $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}'\nu\bar{\nu}$ decays. As the numbers of independent topological amplitudes are highly limited, there are plenty of relations on these $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'l\bar{\nu}$ and $\mathbf{B}\bar{\mathbf{B}}'\nu\bar{\nu}$ decay amplitudes. Furthermore, the loop topological amplitudes and tree topological amplitudes have simple relations, as

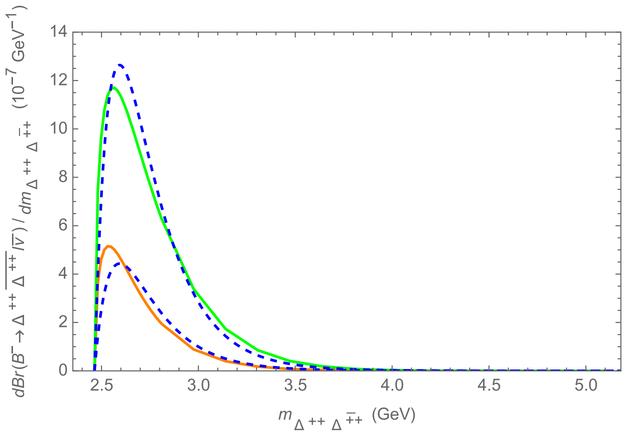


FIG. 5. Differential rate $d\mathrm{Br}/dm_{\Delta^{++}\overline{\Delta^{++}}}$ of $B^-\to\Delta^{++}\overline{\Delta^{++}}l^-\bar\nu$ decay from model 1 (orange solid line) and model 2 (green solid line). The dashed lines are $d\mathrm{Br}/dm_{\Delta^{++}\overline{\Delta^{++}}}$ using Eq. (62) with $\gamma=10$.

their ratios are determined by the CKM factors and loop functions.

It is known that the $B^-\to p\bar p\mu^-\bar\nu$ differential rate exhibits threshold enhancement, which is expected to hold in all other $\bar B_q\to\mathbf{B}\bar{\mathbf{B}}'l\bar\nu(\nu\bar\nu)$ decay modes. These B_q decays have large phase space and one normally does expect the SU(3) breaking in baryon masses to have large SU(3) breaking effects on the B_q decay rates. However, the threshold enhancement effectively squeezes the phase space to the threshold region and thus mimics the decay just above threshold situation. It amplifies the effects of SU(3) breaking in final state baryon masses, consequently, the decay rates may differ by orders of magnitudes even if their amplitudes are of similar sizes. In this work, the $B^-\to p\bar p\mu^-\bar\nu$ differential rate and model calculations with available theoretical inputs from Ref. [8,9], which can reproduce the observed differential rate, are used to estimate the SU(3) breaking from threshold enhancement. We find that the differential rates $d\mathrm{Br}/dm_{\mathbf{B}\bar{\mathbf{B}}'}$ of $\bar B_q\to\mathbf{B}\bar{\mathbf{B}}'l\bar\nu(\nu\bar\nu)$ decays can be parametrized as

$$\frac{d\mathrm{Br}}{dm_{\mathbf{B}\bar{\mathbf{B}}'}}=\frac{N}{(m^2_{\mathbf{B}\bar{\mathbf{B}}'})^\gamma}(m_{\mathbf{B}\bar{\mathbf{B}}'}-m_{\mathbf{B}}-m_{\bar{\mathbf{B}}'}),\qquad(77)$$

with γ and N some constants, and we obtain $\gamma=9,7,7,10$ for $\bar B_q\to\mathcal{B}\bar{\mathcal{B}}'l\bar\nu(\nu\bar\nu)$, $\bar B_q\to\mathcal{B}\bar{\mathcal{D}}l\bar\nu(\nu\bar\nu)$, $\bar B_q\to\mathcal{D}\bar{\mathcal{B}}l\bar\nu(\nu\bar\nu)$ and $\bar B_q\to\mathcal{D}\bar{\mathcal{D}}'l\bar\nu(\nu\bar\nu)$ decays, respectively. SU(3) breaking from threshold enhancement can be estimated using the above equation. The estimations on SU(3) breaking from threshold enhancement are supported by model calculations and can be improved once differential rates of other modes are measured.

Note that as shown in Figs. 1 and 3, although $\gamma=9$ agrees with $d\mathrm{Br}/dm_{p\bar p}$ of $B^-\to p\bar p\mu^-\bar\nu$ decay from LHCb [3] and the theoretical calculations using inputs from Refs. [8,9], which made use of QCD counting rules, there are only four data points with non-negligible uncertainties in the plots. Therefore the reliability of the value of γ remains to be checked when more data is available. In $\bar B_q\to\mathcal{B}\bar{\mathcal{D}}l\bar\nu(\nu\bar\nu)$, $\bar B_q\to\mathcal{D}\bar{\mathcal{B}}l\bar\nu(\nu\bar\nu)$ and $\bar B_q\to\mathcal{D}\bar{\mathcal{D}}'l\bar\nu(\nu\bar\nu)$ decays, the values of γ are determined by comparing to numerical results of the theoretical calculations. It should be noted that there are some assumptions employed in the theoretical calculations. Therefore these γ should be taken as illustrations for the moment. The estimations on SU(3) breaking from threshold enhancement can be improved once differential rates of these modes are measured. In particular, in $\bar B_q\to\mathcal{B}\bar{\mathcal{D}}l\bar\nu(\mathcal{D}\bar{\mathcal{B}}l\bar\nu)$ decays, as there is only one topological amplitude, namely $T_{\mathcal{B}\bar{\mathcal{D}}}$ $(T_{\mathcal{D}\bar{\mathcal{B}}})$, the rates of all other modes [including $\bar B_q\to\mathcal{B}\bar{\mathcal{D}}\nu\bar\nu(\mathcal{D}\bar{\mathcal{B}}\nu\bar\nu)$ modes] can be estimated without resorting to model calculations, once the total rate and the differential rate of a single mode is measured. The same situation also applies to $\bar B_q\to\mathcal{D}\bar{\mathcal{D}}'l\bar\nu(\nu\bar\nu)$ decays as long as the decuplet and antidecuplet baryons are not related by charge conjugation, as there is only one topological amplitude, namely $T_{\mathcal{D}\bar{\mathcal{D}}}$ $(PB_{\mathcal{D}\bar{\mathcal{D}}})$ in these decays. It is therefore interesting to see the experimental results in these modes.

In the model calculations, we find that the $\bar B_q\to\mathcal{B}\bar{\mathcal{B}}'l\bar\nu$ branching ratios are of the orders 10^{-8}–10^{-6} for non-annihilation modes, while the branching ratios of $\bar B_q\to\mathcal{B}\bar{\mathcal{B}}'\nu\bar\nu$ decays are of the orders of 10^{-11}–10^{-8} for non-penguin-box-annihilation modes. The branching ratios of $\bar B_q\to\mathcal{B}\bar{\mathcal{D}}l\bar\nu$ and $\bar B_q\to\mathcal{D}\bar{\mathcal{B}}l\bar\nu$ decays are in the ranges of 10^{-9}–10^{-7} while $\sum_\nu\mathrm{Br}(\bar B_q\to\mathcal{B}\bar{\mathcal{D}}l\nu\nu)$ and $\sum_\nu\mathrm{Br}(\bar B_q\to\mathcal{D}\bar{\mathcal{B}}l\nu\nu)$ are in the ranges of 10^{-12}–10^{-10}. The branching ratios of $\bar B_q\to\mathcal{D}\bar{\mathcal{D}}'l\bar\nu$ decays of nonannihilation modes are in the ranges of 10^{-9}–10^{-7}, while $\sum_\nu\mathrm{Br}(\bar B_q\to\mathcal{D}\bar{\mathcal{D}}'l\nu\nu)$ are in the ranges of 10^{-12}–10^{-10}.

Modes with relatively unsuppressed rates and good detectability are identified as following. In $\bar B_q\to\mathcal{B}\bar{\mathcal{B}}'l\bar\nu$ and $\bar B_q\to\mathcal{B}\bar{\mathcal{B}}'\nu\bar\nu$ decays, we have $B^-\to p\bar pl\bar\nu$, $\bar B^0\to p\bar nl\bar\nu$, $\bar B^0_s\to p\bar\Lambda l\bar\nu$, $B^-\to\Lambda\bar p\nu\bar\nu$, $\bar B^0\to\Lambda\bar n\nu\bar\nu$, and $\bar B^0_s\to\Lambda\bar\Lambda\nu\bar\nu$ decays. In $\bar B_q\to\mathcal{B}\bar{\mathcal{D}}l\bar\nu$ and $\bar B_q\to\mathcal{B}\bar{\mathcal{D}}\nu\bar\nu$ decays, $\bar B^0\to p\overline{\Delta^0}l\bar\nu$ and $\bar B^0\to\Sigma^0\overline{\Delta^0}\nu\bar\nu$ have unsuppressed rates and good detectability. While in $\bar B_q\to\mathcal{D}\bar{\mathcal{B}}l\bar\nu$ and $\bar B_q\to\mathcal{D}\bar{\mathcal{B}}\nu\bar\nu$ decays, $\bar B^0\to\Delta^{++}\bar pl\bar\nu$, $\bar B^0\to\Sigma^{*+}\bar\Lambda l\bar\nu$, $B^-\to\Delta^+\bar pl\bar\nu$, $B^-\to\Delta^0\bar nl\bar\nu$, $\bar B^0\to\Sigma^{*+}\bar p\nu\bar\nu$ and $B^-\to\Sigma^{*-}\bar n\nu\bar\nu$ decay modes are identified. Finally in $\bar B_q\to\mathcal{D}\bar{\mathcal{D}}'l\bar\nu$ and $\bar B_q\to\mathcal{D}\bar{\mathcal{D}}'\nu\bar\nu$ decays, we find that the following decay modes have unsuppressed rates and good detectability, they are $B^-\to\Delta^{++}\overline{\Delta^{++}}l\bar\nu$, $B^-\to\Delta^0\overline{\Delta^0}l\bar\nu$, $B^-\to\Sigma^{*+}\overline{\Sigma^{*+}}l\bar\nu$, $\bar B^0_s\to\Delta^{++}\overline{\Sigma^{*+}}l\bar\nu$, $\bar B^0_s\to\Delta^0\overline{\Sigma^{*-}}l\bar\nu$, $\bar B^0_s\to\Sigma^{*+}\overline{\Xi^{*0}}l\bar\nu$, $B^-\to\Sigma^{*+}\overline{\Delta^{++}}\nu\bar\nu$, $B^-\to\Sigma^{*-}\overline{\Delta^0}\nu\bar\nu$, $B^-\to\Xi^{*0}\overline{\Sigma^{*+}}\nu\bar\nu$, $\bar B^0_s\to\Sigma^{*+}\overline{\Sigma^{*+}}\nu\bar\nu$, $\bar B^0_s\to\Sigma^{*-}\overline{\Sigma^{*-}}\nu\bar\nu$, $\bar B^0_s\to\Xi^{*0}\overline{\Xi^{*0}}\nu\bar\nu$ decays. These modes can be searched experimentally in near future.

Ratios of rates of some loop induced $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'\nu\bar{\nu}$ decays and tree induced $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'l\bar{\nu}$ decays are predicted and can be checked experimentally. They can be tests of the SM. In particular, we predict

$$\frac{\sum_\nu \mathrm{Br}(B^- \to \Lambda\bar{p}\nu\bar{\nu})}{\mathrm{Br}(\bar{B}^0_s \to p\bar{\Lambda}l\bar{\nu})} = 4.29\frac{\bar{\alpha}}{\alpha}\times\left(\frac{0.0036}{|V_{ub}|}\right)^2\times 10^{-3},$$
$$\frac{\sum_\nu \mathrm{Br}(\bar{B}^0 \to \Lambda\bar{n}\nu\bar{\nu})}{\mathrm{Br}(\bar{B}^0_s \to p\bar{\Lambda}l\bar{\nu})} = 3.94\frac{\bar{\alpha}}{\alpha}\times\left(\frac{0.0036}{|V_{ub}|}\right)^2\times 10^{-3}, \quad (78)$$

for $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}'\nu\bar{\nu}, \mathcal{B}\bar{\mathcal{B}}'l\bar{\nu}$ decays,

$$\frac{\sum_\nu \mathrm{Br}(\bar{B}^0 \to \Sigma^0\overline{\Delta^0}\nu\bar{\nu})}{\mathrm{Br}(\bar{B}^0 \to p\overline{\Delta^0}l\bar{\nu})} = 2.11\beta'\times\left(\frac{0.0036}{|V_{ub}|}\right)^2\times 10^{-3}, \quad (79)$$

for $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}\nu\bar{\nu}, \mathcal{B}\bar{\mathcal{D}}l\bar{\nu}$ decays,

$$\frac{\sum_\nu \mathrm{Br}(\bar{B}^0 \to \Sigma^{*+}\bar{p}\nu\bar{\nu})}{\mathrm{Br}(\bar{B}^0 \to \Delta^{++}\bar{p}l\bar{\nu})} = 4.55\kappa''\times\left(\frac{0.0036}{|V_{ub}|}\right)^2\times 10^{-4}, \quad (80)$$

for $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}\nu\bar{\nu}, \mathcal{D}\bar{\mathcal{B}}l\bar{\nu}$ decays, and

$$\frac{\sum_\nu \mathrm{Br}(\bar{B}^0_s \to \Sigma^{*+}\overline{\Sigma^{*+}}\nu\bar{\nu})}{\mathrm{Br}(B^- \to \Delta^0\overline{\Delta^0}l\bar{\nu})} = 4.66\sigma'''\times\left(\frac{0.0036}{|V_{ub}|}\right)^2\times 10^{-4},$$
$$\frac{\sum_\nu \mathrm{Br}(\bar{B}^0_s \to \Sigma^{*-}\overline{\Sigma^{*-}}\nu\bar{\nu})}{\mathrm{Br}(B^- \to \Delta^0\overline{\Delta^0}l\bar{\nu})} = 4.40\sigma'''\times\left(\frac{0.0036}{|V_{ub}|}\right)^2\times 10^{-4}, \quad (81)$$

and

$$\frac{\sum_\nu \mathrm{Br}(B^- \to \Sigma^{*+}\overline{\Delta^{++}}\nu\bar{\nu})}{\mathrm{Br}(\bar{B}^0_s \to \Delta^{++}\overline{\Sigma^{*+}}l\bar{\nu})} = 4.29\frac{\bar{\beta}'''}{\beta'''}\times\left(\frac{0.0036}{|V_{ub}|}\right)^2\times 10^{-3},$$
$$\frac{\sum_\nu \mathrm{Br}(B^- \to \Sigma^{*-}\overline{\Delta^0}\nu\bar{\nu})}{\mathrm{Br}(\bar{B}^0_s \to \Delta^0\overline{\Sigma^{*-}}l\bar{\nu})} = 4.29\frac{\bar{\beta}'''}{\beta'''}\times\left(\frac{0.0036}{|V_{ub}|}\right)^2\times 10^{-3},$$
$$\frac{\sum_\nu \mathrm{Br}(B^- \to \Xi^{*0}\overline{\Sigma^{*+}}\nu\bar{\nu})}{\mathrm{Br}(\bar{B}^0_s \to \Sigma^{*+}\overline{\Xi^{*0}}l\bar{\nu})} = 4.29\frac{\bar{\kappa}'''}{\kappa'''}\times\left(\frac{0.0036}{|V_{ub}|}\right)^2\times 10^{-3}, \quad (82)$$

for $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}'\nu\bar{\nu}, \mathcal{D}\bar{\mathcal{D}}'l\bar{\nu}$ decays. The parameters β', κ'', and σ''' are expected to be of order one, while the ratios $\bar{\alpha}/\alpha$, $\bar{\beta}'''/\beta'''$ and $\bar{\kappa}'''/\kappa'''$ are expected to be close to one. These expectations are supported by model calculations. Note that the ratios in Eqs. (78) and (82) are insensitive to the SU(3) breaking from threshold enhancement, while those in Eqs. (79)–(81) do depend on the estimations of SU(3) breaking from threshold enhancement, which, however, can be checked and improved when more modes are discovered. The ratios which are insensitive to the modeling of SU(3) breaking from threshold enhancement can be tests of the SM.

The approach developed in this work can be applied to some other related modes. In particular, $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'l^+l^-$ decays can be studied using a similar method. Given the fact that the final states have good detectability, these are interesting modes to be studied [41]. Further investigation is needed as the governing operators in $H_{\rm eff}$, see for example [42], are more complicated, where they have structures beyond the simple $V-A$ form considered in this work [see Eq. (1)], and the (differential) decay rates can be rather different. Nevertheless as long as the SU(3) flavor structure of the amplitudes is concerned, the decompositions of $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'l^+l^-$ decay amplitudes are identical to those in $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'\nu\bar{\nu}$ amplitudes presented in Tables V, VI, VII and VIII. Relations of $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'\nu\bar{\nu}$ amplitudes as shown in Sec. III B are applicable to $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'l^+l^-$ amplitudes. Consequently, although the absolute sizes of decay rates and the shapes of differential rates need further investigation, the relative decay rates of $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'l^+l^-$ decays can be estimated using the results given in this work, especially for modes with simple topological structures, such as $\bar{B}_q \to \mathcal{B}\bar{\mathcal{D}}'\nu\bar{\nu}$, $\bar{B}_q \to \mathcal{D}\bar{\mathcal{B}}'\nu\bar{\nu}$ decays and some $\bar{B}_q \to \mathcal{D}\bar{\mathcal{D}}'\nu\bar{\nu}$ decay modes. By naïvely using the results on $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'\nu\bar{\nu}$ decay rates in Tables XIII, XIV and XVIII, it is expected that the following $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'l^+l^-$ modes should have relatively unsuppressed rates and good detectability. These modes are $\bar{B}^0 \to \Sigma^0\overline{\Delta^0}l^+l^-$, $\bar{B}^0 \to \Sigma^{*+}\bar{p}l^+l^-$, $B^- \to \Sigma^{*+}\overline{\Delta^{++}}l^+l^-$, $B^- \to \Sigma^{*-}\overline{\Delta^0}l^+l^-$, and $B^- \to \Xi^{*0}\overline{\Sigma^{*+}}l^+l^-$ decays. In addition to the above modes, although having much complicated topological structure, the $\bar{B}_q \to \mathcal{B}\bar{\mathcal{B}}'l^+l^-$ decay modes should also be searched, especially the $B^- \to \Lambda\bar{p}l^+l^-$ decay, which has good detectability. It will be interesting to search for them.

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APPENDIX A: $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'$ MATRIX ELEMENTS IN THE ASYMPTOTIC LIMIT

We discuss $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'$ transition matrix elements in the asymptotic limit in this appendix. We follow Ref. [37] to obtain the asymptotic limit of these matrix elements. The wave function of a octet or decuplet baryon with helicity $\lambda = -1/2$ can be expressed as

$$|\mathbf{B};\downarrow\rangle \sim \frac{1}{\sqrt{3}}(|\mathbf{B};\downarrow\uparrow\downarrow\rangle + |\mathbf{B};\downarrow\downarrow\uparrow\rangle + |\mathbf{B};\uparrow\downarrow\downarrow\rangle), \quad (A1)$$

which are composed of 13-, 12- and 23-symmetric terms, respectively. For octet baryons, we have

$$
\begin{aligned}
|p;\downarrow\uparrow\downarrow\rangle &= \left[\frac{d(1)u(3)+u(1)d(3)}{\sqrt{6}}u(2)-\sqrt{\frac{2}{3}}u(1)d(2)u(3)\right]|\downarrow\uparrow\downarrow\rangle,\\
|n;\downarrow\uparrow\downarrow\rangle &= (-|p;\downarrow\uparrow\downarrow\rangle \quad \text{with} \quad u\leftrightarrow d),\\
|\Sigma^+;\downarrow\uparrow\downarrow\rangle &= (-|p;\downarrow\uparrow\downarrow\rangle \quad \text{with} \quad d\to s),\\
|\Sigma^0;\downarrow\uparrow\downarrow\rangle &= \left[-\frac{u(1)d(3)+d(1)u(3)}{\sqrt{3}}s(2)+\frac{u(2)d(3)+d(2)u(3)}{2\sqrt{3}}s(1)+\frac{u(1)d(2)+d(1)u(2)}{2\sqrt{3}}s(3)\right]|\downarrow\uparrow\downarrow\rangle,\\
|\Sigma^-;\downarrow\uparrow\downarrow\rangle &= (|p;\downarrow\uparrow\downarrow\rangle \quad \text{with} \quad u,d\to d,s),\\
|\Lambda;\downarrow\uparrow\downarrow\rangle &= \left[\frac{d(2)u(3)-u(2)d(3)}{2}s(1)+\frac{u(1)d(2)-d(1)u(2)}{2}s(3)\right]|\downarrow\uparrow\downarrow\rangle,\\
|\Xi^0;\downarrow\uparrow\downarrow\rangle &= (|p;\downarrow\uparrow\downarrow\rangle \quad \text{with} \quad u,d\to s,u),\\
|\Xi^-;\downarrow\uparrow\downarrow\rangle &= (-|p;\downarrow\uparrow\downarrow\rangle \quad \text{with} \quad u\to s),
\end{aligned}
\tag{A2}
$$

and for decuplet baryons, we have

$$
\begin{aligned}
&|\Delta^{++};\downarrow\uparrow\downarrow\rangle = u(1)u(2)u(3)|\downarrow\uparrow\downarrow\rangle, \qquad |\Delta^-;\downarrow\uparrow\downarrow\rangle = d(1)d(2)d(3)|\downarrow\uparrow\downarrow\rangle,\\
&|\Delta^+;\downarrow\uparrow\downarrow\rangle = \frac{1}{\sqrt{3}}[u(1)u(2)d(3)+u(1)d(2)u(3)+d(1)u(2)u(3)]|\downarrow\uparrow\downarrow\rangle,\\
&|\Delta^0;\downarrow\uparrow\downarrow\rangle = (|\Delta^+;\downarrow\uparrow\downarrow\rangle \quad \text{with} \quad u\leftrightarrow d), \qquad |\Sigma^{*+};\downarrow\uparrow\downarrow\rangle = (|\Delta^+;\downarrow\uparrow\downarrow\rangle \quad \text{with} \quad d\leftrightarrow s),\\
&|\Sigma^{*0};\downarrow\uparrow\downarrow\rangle = \frac{1}{\sqrt{6}}[u(1)d(2)s(3)+\text{permutation}]|\downarrow\uparrow\downarrow\rangle,\\
&|\Omega^-;\downarrow\uparrow\downarrow\rangle = (|\Delta^{++};\downarrow\uparrow\downarrow\rangle \quad \text{with} \quad u\to s),
\end{aligned}
\tag{A3}
$$

for the $|\mathbf{B};\downarrow\uparrow\downarrow\rangle$ parts. while the 12- and 23-symmetric parts can be easily obtained by suitable permutation.

The transition matrix element can be expressed as

$$
\begin{aligned}
\langle \mathbf{B}\bar{\mathbf{B}}'|\bar{q}_L\gamma_\mu b_L|\bar{B}_{q'}\rangle = {}& i\bar{u}_L(p_\mathbf{B})\gamma_\mu v_L(p_{\bar{\mathbf{B}}'})\mathcal{G}_L\\
&+ i\bar{u}_R(p_\mathbf{B})\gamma_\mu v_R(p_{\bar{\mathbf{B}}'})\mathcal{G}_R\\
&+ i\bar{u}_L(p_\mathbf{B})\mathbf{F}_\mu v_R(p_{\bar{\mathbf{B}}'}),
\end{aligned}
\tag{A4}
$$

where $\mathbf{F}_\mu$ can be expressed as

$$
\mathbf{F}_\mu = a_F\sigma_{\mu\nu}q^\nu + b_F q_\mu + c_F(p_\mathbf{B}+p_{\bar{\mathbf{B}}'})_\mu + d_F(p_\mathbf{B}-p_{\bar{\mathbf{B}}'})_\mu,
\tag{A5}
$$

with $q\equiv p_{B_q}-p_\mathbf{B}-p_{\bar{\mathbf{B}}'}$ and form factors a_F, b_F, c_F, and d_F. These $\mathcal{G}_L$, $\mathcal{G}_R$, and $\mathbf{F}_\mu$ depends on the decaying meson and the final state baryon pair.

We use spacelike case for illustration. Using the approach similar to those in [10–13,25,37] the above form factors $\mathcal{G}_L$, $\mathcal{G}_R$, and $\mathbf{F}_\mu$ can be expressed in terms of three universal form factors, $\mathcal{G}_\parallel$, $\mathcal{G}_{\overline{\parallel}}$, and $\mathcal{F}_\mu$ as following,

$$
\mathcal{G}_L = e_\parallel\mathcal{G}_\parallel, \qquad \mathcal{G}_R = e_{\overline{\parallel}}\mathcal{G}_{\overline{\parallel}}, \qquad \mathbf{F}_\mu = e_F\mathcal{F}_\mu,
\tag{A6}
$$

where the coefficients $e_\parallel$, $e_{\overline{\parallel}}$, and e_F are given by

$$
\begin{aligned}
e_\parallel &= (\langle\mathbf{B};\downarrow\uparrow\downarrow|O[q'_L(1)\to q_L(1)]|\mathbf{B}';\downarrow\uparrow\downarrow\rangle\\
&\quad + \langle\mathbf{B};\downarrow\uparrow\downarrow|O[q'_L(3)\to q_L(3)]|\mathbf{B}';\downarrow\uparrow\downarrow\rangle),\\
e_{\overline{\parallel}} &= \langle\mathbf{B};\uparrow\downarrow\uparrow|O[q'_L(2)\to q_L(2)]|\mathbf{B}';\uparrow\downarrow\uparrow\rangle,\\
e_F &= (\langle\mathbf{B};\downarrow\downarrow\uparrow|O[q'_R(1)\to q_L(1)]|\mathbf{B}';\uparrow\downarrow\uparrow\rangle\\
&\quad + \langle\mathbf{B};\uparrow\downarrow\downarrow|O[q'_R(3)\to q_L(3)]|\mathbf{B}';\uparrow\downarrow\uparrow\rangle).
\end{aligned}
\tag{A7}
$$

Note that q' is the anti-quark in $\bar{B}_{q'}$ meson and q is the quark in the $\bar{q}_L\gamma_\mu b_L$ current. Applying $Q[q'_L(1)\to q_L(1)]$ to $|\mathbf{B}';\downarrow\uparrow\downarrow\rangle$ changes the parallel spin $q'(1)|\downarrow\rangle$ part of $|\mathbf{B}';\downarrow\uparrow\downarrow\rangle$ to a parallel spin $q(1)|\downarrow\rangle$ part, where the flavor is changed from q' to q, and likewise for the operation of $O[q'_L(3)\to q_L(3)]$ on $|\mathbf{B}';\downarrow\uparrow\downarrow\rangle$. As the operation involves only the parallel spin component, the coefficient is called $e_\parallel$ and the correspondent form factor is $\mathcal{G}_\parallel$. Likewise $e_{\overline{\parallel}}$ involves only the anti-parallel spin component, while e_F involves operations that flip the spin of the quark in addition to changing the flavor from q' to q. Note that annihilation diagram is not included in the above analysis, as the flavor flow structure is different, see Fig. 2, where, as far as the flavor structure is concern, $\bar{B}_{q'}$ is annihilated by the current and the baryon pair is created from vacuum. The coefficients $e_\parallel$, $e_{\overline{\parallel}}$ and e_F for all relevant transitions

TABLE XXI. The coefficients $(e_{\parallel}, e_{\overline{\parallel}}, e_F)$ for various $\langle \mathbf{B}\bar{\mathbf{B}}'|\bar{q}_L\gamma_\mu b_L|\bar{B}_{q'}\rangle$ matrix elements.

$\langle \mathbf{B}\bar{\mathbf{B}}'\|\bar{q}_L\gamma_\mu b_L\|\bar{B}_{q'}\rangle$	$(e_{\parallel}, e_{\overline{\parallel}}, e_F)$	$\langle \mathbf{B}\bar{\mathbf{B}}'\|\bar{q}_L\gamma_\mu b_L\|\bar{B}_{q'}\rangle$	$(e_{\parallel}, e_{\overline{\parallel}}, e_F)$
$\langle p\bar{p}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	$(5,1,-2)$	$\langle n\bar{n}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	$(1,2,\frac{1}{2})$
$\langle \Sigma^+\overline{\Sigma^+}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	$(5,1,-2)$	$\langle \Sigma^0\overline{\Sigma^0}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	$(\frac{5}{2},\frac{1}{2},-1)$
$\langle \Sigma^0\bar{\Lambda}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	$(\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2})$	$\langle \Xi^0\overline{\Xi^0}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	$(1,2,\frac{1}{2})$
$\langle \Lambda\overline{\Sigma^0}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	$(\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2})$	$\langle \Lambda\bar{\Lambda}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	$(\frac{3}{2},\frac{3}{2},0)$
$\langle p\bar{n}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(4,-1,-\frac{5}{2})$	$\langle \Sigma^+\overline{\Sigma^0}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(-\frac{5}{\sqrt{2}},-\frac{1}{\sqrt{2}},\sqrt{2})$
$\langle \Sigma^+\bar{\Lambda}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(\sqrt{\frac{3}{2}},-\sqrt{\frac{3}{2}},-\sqrt{\frac{3}{2}})$	$\langle \Sigma^0\overline{\Sigma^-}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(\frac{5}{\sqrt{2}},\frac{1}{\sqrt{2}},-\sqrt{2})$
$\langle \Lambda\overline{\Sigma^-}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(\sqrt{\frac{3}{2}},-\sqrt{\frac{3}{2}},-\sqrt{\frac{3}{2}})$	$\langle \Xi^0\overline{\Xi^-}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(-1,-2,-\frac{1}{2})$
$\langle p\overline{\Sigma^0}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(-\frac{1}{\sqrt{2}},-\sqrt{2},-\frac{1}{2\sqrt{2}})$	$\langle p\bar{\Lambda}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(-3\sqrt{\frac{3}{2}},0,\frac{3}{2}\sqrt{\frac{3}{2}})$
$\langle n\overline{\Sigma^-}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(-1,-2,-\frac{1}{2})$	$\langle \Sigma^+\overline{\Xi^0}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(4,-1,-\frac{5}{2})$
$\langle \Sigma^0\overline{\Xi^-}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(2\sqrt{2},-\frac{1}{\sqrt{2}},-\frac{5}{2\sqrt{2}})$	$\langle \Lambda\overline{\Xi^-}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(\sqrt{6},\sqrt{\frac{3}{2}},-\frac{1}{2}\sqrt{\frac{3}{2}})$
$\langle \Sigma^0\bar{p}\|\bar{s}_L\gamma_\mu b_L\|B^-\rangle$	$(-\frac{1}{\sqrt{2}},-\sqrt{2},-\frac{1}{2\sqrt{2}})$	$\langle \Sigma^-\bar{n}\|\bar{s}_L\gamma_\mu b_L\|B^-\rangle$	$(-1,-2,-\frac{1}{2})$
$\langle \Xi^0\overline{\Sigma^+}\|\bar{s}_L\gamma_\mu b_L\|B^-\rangle$	$(4,-1,-\frac{5}{2})$	$\langle \Xi^-\overline{\Sigma^0}\|\bar{s}_L\gamma_\mu b_L\|B^-\rangle$	$(2\sqrt{2},-\frac{1}{\sqrt{2}},-\frac{5}{2\sqrt{2}})$
$\langle \Xi^-\bar{\Lambda}\|\bar{s}_L\gamma_\mu b_L\|B^-\rangle$	$(\sqrt{6},\sqrt{\frac{3}{2}},-\frac{1}{2}\sqrt{\frac{3}{2}})$	$\langle \Lambda\bar{p}\|\bar{s}_L\gamma_\mu b_L\|B^-\rangle$	$(-3\sqrt{\frac{3}{2}},0,\frac{3}{2}\sqrt{\frac{3}{2}})$
$\langle \Sigma^+\bar{p}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(-1,-2,-\frac{1}{2})$	$\langle \Sigma^0\bar{n}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(\frac{1}{\sqrt{2}},\sqrt{2},\frac{1}{2\sqrt{2}})$
$\langle \Xi^0\overline{\Sigma^0}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(-2\sqrt{2},\frac{1}{\sqrt{2}},\frac{5}{2\sqrt{2}})$	$\langle \Xi^0\bar{\Lambda}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(\sqrt{6},\sqrt{\frac{3}{2}},-\frac{1}{2}\sqrt{\frac{3}{2}})$
$\langle \Xi^-\overline{\Sigma^-}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(4,-1,-\frac{5}{2})$	$\langle \Lambda\bar{n}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(-3\sqrt{\frac{3}{2}},0,\frac{3}{2}\sqrt{\frac{3}{2}})$
$\langle \Sigma^+\overline{\Sigma^+}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(1,2,\frac{1}{2})$	$\langle \Sigma^0\overline{\Sigma^0}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(1,2,\frac{1}{2})$
$\langle \Sigma^-\overline{\Sigma^-}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(1,2,\frac{1}{2})$	$\langle \Xi^0\overline{\Xi^0}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(5,1,-2)$
$\langle \Xi^-\overline{\Xi^-}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(5,1,-2)$	$\langle \Lambda\bar{\Lambda}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(3,0,-\frac{3}{2})$

considered in this work are obtained accordingly and are shown in Tables XXI, XXII, XXIII, XXIV.

By comparing the Tables XXI, XXII, XXIII, XXIV and Tables V, VI, VII, VIII, we found the following correspondences of topological amplitudes and $(e_{\parallel}, e_{\overline{\parallel}}, e_F)$,

$$\begin{aligned} T_{1\mathcal{B}\bar{\mathcal{B}}}&:\ (e^{(1)}_{\parallel}, e^{(1)}_{\overline{\parallel}}, e^{(1)}_F) = \left(1, 2, \frac{1}{2}\right), \\ T_{2\mathcal{B}\bar{\mathcal{B}}}&:\ (e^{(2)}_{\parallel}, e^{(2)}_{\overline{\parallel}}, e^{(2)}_F) = \left(4, -1, -\frac{5}{2}\right), \\ T_{\mathcal{B}\bar{\mathcal{D}}}&:\ (e'_{\parallel}, e'_{\overline{\parallel}}, e'_F) = \left(1, -1, \frac{1}{2}\right), \\ T_{\mathcal{D}\bar{\mathcal{B}}}&:\ (e''_{\parallel}, e''_{\overline{\parallel}}, e''_F) = \left(1, -1, \frac{1}{2}\right), \\ T_{\mathcal{D}\bar{\mathcal{D}}}&:\ (e'''_{\parallel}, e'''_{\overline{\parallel}}, e'''_F) = \left(1, \frac{1}{2}, \frac{1}{2}\right), \end{aligned} \tag{A8}$$

and similar relations for $P_{i\mathcal{B}\bar{\mathcal{B}},\mathcal{B}\bar{\mathcal{D}},\mathcal{D}\bar{\mathcal{B}},\mathcal{D}\bar{\mathcal{D}}}$.

In general, the topological amplitudes, $T_{i\mathcal{B}\bar{\mathcal{B}}}$, $T_{\mathcal{B}\bar{\mathcal{D}}}$ $T_{\mathcal{D}\bar{\mathcal{B}}}$, and $T_{\mathcal{D}\bar{\mathcal{D}}}$, are given in Eqs. (21)–(24). It is useful to show that $T_{\mathcal{B}\bar{\mathcal{D}}}$, $T_{\mathcal{D}\bar{\mathcal{B}}}$, and $T_{\mathcal{D}\bar{\mathcal{D}}}$ have the structure of $T_{i\mathcal{B}\bar{\mathcal{B}}}$ in the asymptotic limit. Note that the Rarita-Schwinger vector spinor u_μ can be expressed in terms of Dirac spinors and polarization vectors as following [43]

$$\begin{aligned} u_\mu\left(\vec{p}, \pm\frac{3}{2}\right) &= \epsilon_\mu(\vec{p}, \pm 1) u\left(\vec{p}, \pm\frac{1}{2}\right), \\ u_\mu\left(\vec{p}, \pm\frac{1}{2}\right) &= \frac{1}{\sqrt{3}}\epsilon_\mu(\vec{p}, \pm 1) u\left(\vec{p}, \mp\frac{1}{2}\right) \\ &\quad + \sqrt{\frac{2}{3}}\epsilon_\mu(\vec{p}, 0) u\left(\vec{p}, \pm\frac{1}{2}\right), \end{aligned} \tag{A9}$$

where $\epsilon_\mu(\vec{p}, \lambda)$ is the polarization vector,

$$\epsilon_\mu(\vec{p}, 0) = \left(\frac{|\vec{p}|}{m}, \frac{E}{m}\hat{n}\right), \quad \epsilon_\mu(\vec{p}, \pm 1) = (0, \vec{\epsilon}(\vec{0}, \pm 1)). \tag{A10}$$

with $\hat{n} \equiv \vec{p}/|\vec{p}|$ and $\vec{\epsilon}(\vec{0}, \pm 1)\cdot\hat{n} = 0$, and, for example, in the case of $\vec{p} = (0, 0, p)$, we have $\hat{n} = \hat{z}$ and $\vec{\epsilon}(\vec{0}, \pm 1) = \mp(1, \pm i, 0)/\sqrt{2}$. Spinors $v^\mu(\vec{p}, \lambda)$ have similar relations.

TABLE XXII. The coefficients $(e_\parallel, e_{\overline{\parallel}}, e_F)$ for various $\langle \mathbf{B}\bar{\mathbf{B}}'|\bar{q}_L\gamma_\mu b_L|\bar{B}_{q'}\rangle$ matrix elements.

$\langle \mathbf{B}\bar{\mathbf{B}}'\|\bar{q}_L\gamma_\mu b_L\|\bar{B}_{q'}\rangle$	$(e_\parallel, e_{\overline{\parallel}}, e_F)$	$\langle \mathbf{B}\bar{\mathbf{B}}'\|\bar{q}_L\gamma_\mu b_L\|\bar{B}_{q'}\rangle$	$(e_\parallel, e_{\overline{\parallel}}, e_F)$
$\langle p\overline{\Delta^+}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	$(-\sqrt{2},\sqrt{2},-\frac{1}{\sqrt{2}})$	$\langle n\overline{\Delta^0}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	$(-\sqrt{2},\sqrt{2},-\frac{1}{\sqrt{2}})$
$\langle \Sigma^+\overline{\Sigma^{*+}}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	$(\sqrt{2},-\sqrt{2},\frac{1}{\sqrt{2}})$	$\langle \Sigma^0\overline{\Sigma^{*0}}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	$(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},-\frac{1}{2\sqrt{2}})$
$\langle \Xi^0\overline{\Xi^{*0}}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	$(\sqrt{2},-\sqrt{2},\frac{1}{\sqrt{2}})$	$\langle \Lambda\overline{\Sigma^{*0}}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	$(\sqrt{\frac{3}{2}},-\sqrt{\frac{3}{2}},\frac{1}{2}\sqrt{\frac{3}{2}})$
$\langle p\overline{\Delta^0}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(-\sqrt{2},\sqrt{2},-\frac{1}{\sqrt{2}})$	$\langle n\overline{\Delta^-}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(-\sqrt{6},\sqrt{6},-\sqrt{\frac{3}{2}})$
$\langle \Sigma^+\overline{\Sigma^{*0}}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(1,-1,\frac{1}{2})$	$\langle \Sigma^0\overline{\Sigma^{*-}}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(-1,1,-\frac{1}{2})$
$\langle \Xi^0\overline{\Xi^{*-}}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(\sqrt{2},-\sqrt{2},\frac{1}{\sqrt{2}})$	$\langle \Lambda\overline{\Sigma^{*-}}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(\sqrt{3},-\sqrt{3},\frac{\sqrt{3}}{2})$
$\langle p\overline{\Sigma^{*0}}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(-1,1,-\frac{1}{2})$	$\langle n\overline{\Sigma^{*-}}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(-\sqrt{2},\sqrt{2},-\frac{1}{\sqrt{2}})$
$\langle \Sigma^+\overline{\Xi^{*0}}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(\sqrt{2},-\sqrt{2},\frac{1}{\sqrt{2}})$	$\langle \Sigma^0\overline{\Xi^{*-}}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(-1,1,-\frac{1}{2})$
$\langle \Xi^0\overline{\Omega^-}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(\sqrt{6},-\sqrt{6},\sqrt{\frac{3}{2}})$	$\langle \Lambda\overline{\Xi^{*-}}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(\sqrt{3},-\sqrt{3},\frac{\sqrt{3}}{2})$
$\langle \Sigma^+\overline{\Delta^{++}}\|\bar{s}_L\gamma_\mu b_L\|B^-\rangle$	$(-\sqrt{6},\sqrt{6},-\sqrt{\frac{3}{2}})$	$\langle \Sigma^0\overline{\Delta^+}\|\bar{s}_L\gamma_\mu b_L\|B^-\rangle$	$(2,-2,1)$
$\langle \Sigma^-\overline{\Delta^0}\|\bar{s}_L\gamma_\mu b_L\|B^-\rangle$	$(\sqrt{2},-\sqrt{2},\frac{1}{\sqrt{2}})$	$\langle \Xi^0\overline{\Sigma^{*+}}\|\bar{s}_L\gamma_\mu b_L\|B^-\rangle$	$(-\sqrt{2},\sqrt{2},-\frac{1}{\sqrt{2}})$
$\langle \Xi^-\overline{\Sigma^{*0}}\|\bar{s}_L\gamma_\mu b_L\|B^-\rangle$	$(1,-1,\frac{1}{2})$		
$\langle \Sigma^+\overline{\Delta^+}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(-\sqrt{2},\sqrt{2},-\frac{1}{\sqrt{2}})$	$\langle \Sigma^0\overline{\Delta^0}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(2,-2,1)$
$\langle \Sigma^-\overline{\Delta^-}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(\sqrt{6},-\sqrt{6},\sqrt{\frac{3}{2}})$	$\langle \Xi^0\overline{\Sigma^{*0}}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(-1,1,-\frac{1}{2})$
$\langle \Xi^-\overline{\Sigma^{*-}}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(\sqrt{2},-\sqrt{2},\frac{1}{\sqrt{2}})$		
$\langle \Sigma^+\overline{\Sigma^{*+}}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(-\sqrt{2},\sqrt{2},-\frac{1}{\sqrt{2}})$	$\langle \Sigma^0\overline{\Sigma^{*0}}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(\sqrt{2},-\sqrt{2},\frac{1}{\sqrt{2}})$
$\langle \Sigma^-\overline{\Sigma^{*-}}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(\sqrt{2},-\sqrt{2},\frac{1}{\sqrt{2}})$	$\langle \Xi^0\overline{\Xi^{*0}}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(-\sqrt{2},\sqrt{2},-\frac{1}{\sqrt{2}})$
$\langle \Xi^-\overline{\Xi^{*-}}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(\sqrt{2},-\sqrt{2},\frac{1}{\sqrt{2}})$		

When $|\vec{p}| \gg m$, $\epsilon_\mu(\vec{p},0)$ dominates over $\epsilon_\mu(\vec{p},\pm1)$ and, consequently, $u^\mu(\vec{p},\pm1/2)$ and $v^\mu(\vec{p},\pm1/2)$ dominate over $u^\mu(\vec{p},\pm3/2)$ and $v^\mu(\vec{p},\pm3/2)$, respectively, and they can be approximated as

$$u_\mu\left(\vec{p},\pm\frac{1}{2}\right) \simeq \sqrt{\frac{2}{3}}\frac{p_\mu}{m}u\left(\vec{p},\pm\frac{1}{2}\right), \qquad v_\mu\left(\vec{p},\pm\frac{1}{2}\right) \simeq \sqrt{\frac{2}{3}}\frac{p_\mu}{m}v\left(\vec{p},\pm\frac{1}{2}\right). \tag{A11}$$

Using the above relations and Eqs. (22)–(24), in the large momentum limit, we should have

$$\begin{aligned} T_{\mathcal{B}\bar{\mathcal{D}}} \simeq{}& i\frac{G_F}{\sqrt{2}}V_{ub}\bar{l}_L\gamma_\mu\nu_L\delta_{|\lambda_{\bar{\mathcal{D}}}|,1/2} \\ &\times \bar{u}(p_{\mathcal{D}},\lambda_{\mathcal{B}})\sqrt{\frac{2}{3}}\frac{1}{m_{\bar{\mathcal{D}}}}\bigg\{\Big[(g'_1 p_{\mathcal{B}}\cdot p_{\bar{\mathcal{D}}} + g'_6 q\cdot p_{\bar{\mathcal{D}}})\gamma_\mu + i(g'_2 p_{\mathcal{B}}\cdot p_{\bar{\mathcal{D}}} + g'_7 q\cdot p_{\bar{\mathcal{D}}})\sigma_{\mu\rho}q^\rho \\ &\quad + (g'_3 p_{\mathcal{B}}\cdot p_{\bar{\mathcal{D}}} + g'_8 q\cdot p_{\bar{\mathcal{D}}})q_\mu + (g'_4 p_{\mathcal{B}}\cdot p_{\bar{\mathcal{D}}} + g'_9 q\cdot p_{\bar{\mathcal{D}}})p_{\mathcal{B}\mu} + g'_5 p_{\bar{\mathcal{D}}\mu}\Big]\gamma_5 \\ &\quad - \Big[(f'_1 p_{\mathcal{B}}\cdot p_{\bar{\mathcal{D}}} + f'_6 q\cdot p_{\bar{\mathcal{D}}})\gamma_\mu + i(f'_2 p_{\mathcal{B}}\cdot p_{\bar{\mathcal{D}}} + f'_7 q\cdot p_{\bar{\mathcal{D}}})\sigma_{\mu\rho}q^\rho \\ &\quad + (f'_3 p_{\mathcal{B}}\cdot p_{\bar{\mathcal{D}}} + f'_8 q\cdot p_{\bar{\mathcal{D}}})q_\mu + (f'_4 p_{\mathcal{B}}\cdot p_{\bar{\mathcal{D}}} + f'_9 q\cdot p_{\bar{\mathcal{D}}})p_{\mathcal{B}\mu} + f'_5 p_{\bar{\mathcal{D}}\mu}\Big]\bigg\}v(p_{\bar{\mathcal{D}}},\lambda_{\bar{\mathcal{D}}}), \end{aligned}$$

TABLE XXIII. The coefficients $(e_\parallel, e_{\overline{\parallel}}, e_F)$ for various $\langle \mathbf{B}\bar{\mathbf{B}}'|\bar{q}_L\gamma_\mu b_L|\bar{B}_{q'}\rangle$ matrix elements.

$\langle \mathbf{B}\bar{\mathbf{B}}'\|\bar{q}_L\gamma_\mu b_L\|\bar{B}_{q'}\rangle$	$(e_\parallel, e_{\overline{\parallel}}, e_F)$	$\langle \mathbf{B}\bar{\mathbf{B}}'\|\bar{q}_L\gamma_\mu b_L\|\bar{B}_{q'}\rangle$	$(e_\parallel, e_{\overline{\parallel}}, e_F)$
$\langle \Delta^+\bar{p}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	$(-\sqrt{2}, \sqrt{2}, -\frac{1}{\sqrt{2}})$	$\langle \Delta^0\bar{n}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	$(-\sqrt{2}, \sqrt{2}, -\frac{1}{\sqrt{2}})$
$\langle \Sigma^{*+}\overline{\Sigma^+}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	$(\sqrt{2}, -\sqrt{2}, \frac{1}{\sqrt{2}})$	$\langle \Sigma^{*0}\overline{\Sigma^0}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	$(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{1}{2\sqrt{2}})$
$\langle \Xi^{*0}\overline{\Xi^0}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	$(\sqrt{2}, -\sqrt{2}, \frac{1}{\sqrt{2}})$	$\langle \Sigma^{*0}\bar{\Lambda}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	$(\sqrt{\frac{3}{2}}, -\sqrt{\frac{3}{2}}, \frac{1}{2}\sqrt{\frac{3}{2}})$
$\langle \Delta^{++}\bar{p}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(\sqrt{6}, -\sqrt{6}, \sqrt{\frac{3}{2}})$	$\langle \Delta^+\bar{n}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(\sqrt{2}, -\sqrt{2}, \frac{1}{\sqrt{2}})$
$\langle \Sigma^{*+}\overline{\Sigma^0}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(-1, 1, -\frac{1}{2})$	$\langle \Sigma^{*0}\overline{\Sigma^-}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(-1, 1, -\frac{1}{2})$
$\langle \Xi^{*0}\overline{\Xi^-}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(-\sqrt{2}, \sqrt{2}, -\frac{1}{\sqrt{2}})$	$\langle \Sigma^{*+}\bar{\Lambda}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(-\sqrt{3}, \sqrt{3}, -\frac{\sqrt{3}}{2})$
$\langle \Delta^{++}\overline{\Sigma^+}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(-\sqrt{6}, \sqrt{6}, -\sqrt{\frac{3}{2}})$	$\langle \Delta^+\overline{\Sigma^0}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(2, -2, 1)$
$\langle \Delta^0\overline{\Sigma^-}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(\sqrt{2}, -\sqrt{2}, \frac{1}{\sqrt{2}})$	$\langle \Sigma^{*+}\overline{\Xi^0}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(-\sqrt{2}, \sqrt{2}, -\frac{1}{\sqrt{2}})$
$\langle \Sigma^{*0}\overline{\Xi^-}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(1, -1, \frac{1}{2})$		
$\langle \Sigma^{*0}\bar{p}\|\bar{s}_L\gamma_\mu b_L\|B^-\rangle$	$(-1, 1, -\frac{1}{2})$	$\langle \Sigma^{*-}\bar{n}\|\bar{s}_L\gamma_\mu b_L\|B^-\rangle$	$(-\sqrt{2}, \sqrt{2}, -\frac{1}{\sqrt{2}})$
$\langle \Xi^{*0}\overline{\Sigma^+}\|\bar{s}_L\gamma_\mu b_L\|B^-\rangle$	$(\sqrt{2}, -\sqrt{2}, \frac{1}{\sqrt{2}})$	$\langle \Xi^{*-}\overline{\Sigma^0}\|\bar{s}_L\gamma_\mu b_L\|B^-\rangle$	$(-1, 1, -\frac{1}{2})$
$\langle \Omega^-\overline{\Xi^0}\|\bar{s}_L\gamma_\mu b_L\|B^-\rangle$	$(\sqrt{6}, -\sqrt{6}, \sqrt{\frac{3}{2}})$	$\langle \Xi^{*-}\bar{\Lambda}\|\bar{s}_L\gamma_\mu b_L\|B^-\rangle$	$(\sqrt{3}, -\sqrt{3}, \frac{\sqrt{3}}{2})$
$\langle \Sigma^{*+}\bar{p}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(\sqrt{2}, -\sqrt{2}, \frac{1}{\sqrt{2}})$	$\langle \Sigma^{*0}\bar{n}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(1, -1, \frac{1}{2})$
$\langle \Xi^{*0}\overline{\Sigma^0}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(-1, 1, -\frac{1}{2})$	$\langle \Xi^{*-}\overline{\Sigma^-}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(-\sqrt{2}, \sqrt{2}, -\frac{1}{\sqrt{2}})$
$\langle \Omega^-\overline{\Xi^-}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(-\sqrt{6}, \sqrt{6}, -\sqrt{\frac{3}{2}})$	$\langle \Xi^{*0}\bar{\Lambda}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(-\sqrt{3}, \sqrt{3}, -\frac{\sqrt{3}}{2})$
$\langle \Sigma^{*+}\overline{\Sigma^+}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(-\sqrt{2}, \sqrt{2}, -\frac{1}{\sqrt{2}})$	$\langle \Sigma^{*0}\overline{\Sigma^0}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(\sqrt{2}, -\sqrt{2}, \frac{1}{\sqrt{2}})$
$\langle \Sigma^{*-}\overline{\Sigma^-}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(\sqrt{2}, -\sqrt{2}, \frac{1}{\sqrt{2}})$	$\langle \Xi^{*0}\overline{\Xi^0}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(-\sqrt{2}, \sqrt{2}, -\frac{1}{\sqrt{2}})$
$\langle \Xi^{*-}\overline{\Xi^-}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(\sqrt{2}, -\sqrt{2}, \frac{1}{\sqrt{2}})$		

$$
\begin{aligned}
T_{\mathcal{D}\bar{\mathcal{B}}} \simeq\, & i\frac{G_F}{\sqrt{2}}V_{ub}\bar{l}_L\gamma_\mu\nu_L\delta_{|\lambda_\mathcal{D}|,1/2} \\
& \times \bar{u}(p_\mathcal{D},\lambda_\mathcal{D})\sqrt{\frac{2}{3}}\frac{1}{m_\mathcal{D}}\bigg\{\Big[(\bar{g}''_1 p_{\bar{\mathcal{B}}}\cdot p_\mathcal{D} + \bar{g}''_6 q\cdot p_\mathcal{D})\gamma_\mu + i(\bar{g}''_2 p_{\bar{\mathcal{B}}}\cdot p_\mathcal{D} + \bar{g}''_7 q\cdot p_\mathcal{D})\sigma_{\mu\rho}q^\rho \\
& + (g''_3 p_{\bar{\mathcal{B}}}\cdot p_\mathcal{D} + g''_8 q\cdot p_\mathcal{D})q_\mu + g''_5 p_{\mathcal{D}\mu} + (g''_4 p_{\bar{\mathcal{B}}}\cdot p_\mathcal{D} + g''_9 q\cdot p_\mathcal{D})p_{\bar{\mathcal{B}}\mu}\Big]\gamma_5 \\
& - \Big[(f''_1 p_{\bar{\mathcal{B}}}\cdot p_\mathcal{D} + f''_6 q\cdot p_\mathcal{D})\gamma_\mu + i(f''_2 p_{\bar{\mathcal{B}}}\cdot p_\mathcal{D} + f''_7 q\cdot p_\mathcal{D})\sigma_{\mu\rho}q^\rho \\
& + (f''_3 p_{\bar{\mathcal{B}}}\cdot p_\mathcal{D} + f''_8 q\cdot p_\mathcal{D})q_\mu + f''_5 p_{\mathcal{D}\mu} + (f''_4 p_{\bar{\mathcal{B}}}\cdot p_\mathcal{D} + f''_9 q\cdot p_\mathcal{D})p_{\bar{\mathcal{B}}\mu}\Big]\bigg\}v(p_{\bar{\mathcal{B}}},\lambda_{\bar{\mathcal{B}}}), \qquad \text{(A12)}
\end{aligned}
$$

and

$$
\begin{aligned}
T_{\mathcal{D}\bar{\mathcal{D}}} \simeq\, & i\frac{G_F}{\sqrt{2}}V_{ub}\bar{l}_L\gamma_\mu\nu_L\delta_{|\lambda_\mathcal{D}|,1/2}\delta_{|\lambda_{\bar{\mathcal{D}}}|,1/2} \\
& \times \bar{u}(p_\mathcal{D},\lambda_\mathcal{D})\frac{2}{3}\frac{p_\mathcal{D}\cdot p_{\bar{\mathcal{D}}}}{m_\mathcal{D} m_{\bar{\mathcal{D}}}}\bigg\{\Big[g'''_1\gamma_\mu + ig'''_2\sigma_{\mu\rho}q^\rho + g'''_3 q_\mu + g'''_4(p_\mathcal{D} + p_{\bar{\mathcal{D}}'})_\mu \\
& + g'''_5(p_\mathcal{D} - p_{\bar{\mathcal{D}}'})_\mu\Big]\gamma_5 - \Big[f'''_1\gamma_\mu + if'''_2\sigma_{\mu\nu}q^\nu + f'''_3 q_\mu + f'''_4(p_\mathcal{D} + p_{\bar{\mathcal{D}}'})_\mu \\
& + f'''_5(p_\mathcal{D} - p_{\bar{\mathcal{D}}'})_\mu\Big]\bigg\}v(p_{\bar{\mathcal{D}}},\lambda_{\bar{\mathcal{D}}}). \qquad \text{(A13)}
\end{aligned}
$$

TABLE XXIV. The coefficients $(e_\parallel, e_{\overline{\parallel}}, e_F)$ for various $\langle \mathbf{B}\bar{\mathbf{B}}'|\bar{q}_L\gamma_\mu b_L|\bar{B}_{q'}\rangle$ matrix elements.

$\langle \mathbf{B}\bar{\mathbf{B}}'\|\bar{q}_L\gamma_\mu b_L\|\bar{B}_{q'}\rangle$	$(e_\parallel, e_{\overline{\parallel}}, e_F)$	$\langle \mathbf{B}\bar{\mathbf{B}}'\|\bar{q}_L\gamma_\mu b_L\|\bar{B}_{q'}\rangle$	$(e_\parallel, e_{\overline{\parallel}}, e_F)$
$\langle \Delta^{++}\overline{\Delta^{++}}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	(6, 3, 3)	$\langle \Delta^{+}\overline{\Delta^{+}}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	(4, 2, 2)
$\langle \Delta^{*0}\overline{\Delta^{0}}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	(2, 1, 1)	$\langle \Sigma^{*+}\overline{\Sigma^{*+}}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	(4, 2, 2)
$\langle \Sigma^{*0}\overline{\Sigma^{*0}}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	(2, 1, 1)	$\langle \Xi^{*0}\overline{\Xi^{*0}}\|\bar{u}_L\gamma_\mu b_L\|B^-\rangle$	(2, 1, 1)
$\langle \Delta^{++}\overline{\Delta^{+}}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(2\sqrt{3}, \sqrt{3}, \sqrt{3})$	$\langle \Delta^{+}\overline{\Delta^{0}}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	(4, 2, 2)
$\langle \Delta^{0}\overline{\Delta^{-}}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(2\sqrt{3}, \sqrt{3}, \sqrt{3})$	$\langle \Sigma^{*+}\overline{\Sigma^{*0}}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(2\sqrt{2}, \sqrt{2}, \sqrt{2})$
$\langle \Sigma^{*0}\overline{\Sigma^{*-}}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(2\sqrt{2}, \sqrt{2}, \sqrt{2})$	$\langle \Xi^{*0}\overline{\Xi^{*-}}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	(2, 1, 1)
$\langle \Delta^{++}\overline{\Sigma^{*+}}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(2\sqrt{3}, \sqrt{3}, \sqrt{3})$	$\langle \Delta^{+}\overline{\Sigma^{*0}}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(2\sqrt{2}, \sqrt{2}, \sqrt{2})$
$\langle \Delta^{0}\overline{\Sigma^{*-}}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	(2, 1, 1)	$\langle \Sigma^{*+}\overline{\Xi^{0}}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	(4, 2, 2)
$\langle \Sigma^{*0}\overline{\Xi^{*-}}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(2\sqrt{2}, \sqrt{2}, \sqrt{2})$	$\langle \Xi^{*0}\overline{\Omega^{-}}\|\bar{u}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	$(2\sqrt{3}, \sqrt{3}, \sqrt{3})$
$\langle \Sigma^{*+}\overline{\Delta^{++}}\|\bar{s}_L\gamma_\mu b_L\|B^-\rangle$	$(2\sqrt{3}, \sqrt{3}, \sqrt{3})$	$\langle \Sigma^{*0}\overline{\Delta^{+}}\|\bar{s}_L\gamma_\mu b_L\|B^-\rangle$	$(2\sqrt{2}, \sqrt{2}, \sqrt{2})$
$\langle \Sigma^{*-}\overline{\Delta^{0}}\|\bar{s}_L\gamma_\mu b_L\|B^-\rangle$	(2, 1, 1)	$\langle \Xi^{*0}\overline{\Sigma^{*+}}\|\bar{s}_L\gamma_\mu b_L\|B^-\rangle$	(4, 2, 2)
$\langle \Xi^{*-}\overline{\Sigma^{*0}}\|\bar{s}_L\gamma_\mu b_L\|B^-\rangle$	$(2\sqrt{2}, \sqrt{2}, \sqrt{2})$	$\langle \Omega^{-}\overline{\Xi^{*0}}\|\bar{s}_L\gamma_\mu b_L\|B^-\rangle$	$(2\sqrt{3}, \sqrt{3}, \sqrt{3})$
$\langle \Sigma^{*+}\overline{\Delta^{+}}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	(2, 1, 1)	$\langle \Sigma^{*0}\overline{\Delta^{0}}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(2\sqrt{2}, \sqrt{2}, \sqrt{2})$
$\langle \Sigma^{*-}\overline{\Delta^{-}}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(2\sqrt{3}, \sqrt{3}, \sqrt{3})$	$\langle \Xi^{*0}\overline{\Sigma^{*0}}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(2\sqrt{2}, \sqrt{2}, \sqrt{2})$
$\langle \Xi^{*-}\overline{\Sigma^{*-}}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	(4, 2, 2)	$\langle \Omega^{-}\overline{\Xi^{*-}}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0\rangle$	$(2\sqrt{3}, \sqrt{3}, \sqrt{3})$
$\langle \Sigma^{*+}\overline{\Sigma^{*+}}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	(2, 1, 1)	$\langle \Sigma^{*0}\overline{\Sigma^{*0}}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	(2, 1, 1)
$\langle \Sigma^{*-}\overline{\Sigma^{*-}}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	(2, 1, 1)	$\langle \Xi^{*0}\overline{\Xi^{*0}}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	(4, 2, 2)
$\langle \Xi^{*-}\overline{\Xi^{*-}}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	(4, 2, 2)	$\langle \Omega^{-}\overline{\Omega^{-}}\|\bar{s}_L\gamma_\mu b_L\|\bar{B}^0_s\rangle$	(6, 3, 3)

Comparing the above equations and Eq. (21), we see that $T_{\mathcal{B}\bar{\mathcal{D}}}$, $T_{\mathcal{D}\bar{\mathcal{B}}}$, and $T_{\mathcal{D}\bar{\mathcal{D}}}$ indeed have the structure of $T_{i\mathcal{B}\bar{\mathcal{B}}}$ in the asymptotic limit. Their asymptotic form can be obtained by using Eqs. (A4), (A8), and the above equations.

APPENDIX B: FORMULAS OF DECAY RATES FOR $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'l\bar{\nu}$ AND $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'\nu\bar{\nu}$ DECAYS

The $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'l\bar{\nu}$ and $\bar{B}_q \to \mathbf{B}\bar{\mathbf{B}}'\nu\bar{\nu}$ decays involve 4-body decays. The decay rate of a 4-body decay is given by [7,33,44]

$$d\Gamma = \frac{|M|^2}{4(4\pi)^6 m^3_{B_q}}\beta X ds\, dt\, d\cos\theta_{\mathbf{B}}\, d\cos\theta_{\mathbf{L}}\, d\phi, \qquad \text{(B1)}$$

where s is the invariant mass squared of the lepton pair, t is the invariant mass squared of the baryon pair, $\theta_{\mathbf{B}(\mathbf{L})}$ is the angle of the baryon $\mathbf{B}$ (the lepton l [or ν]) in the baryon pair (lepton pair) rest frame with respect to the opposite direction of the lepton pair (baryon pair) total 3-momentum direction, ϕ is the angle between the baryon and the lepton planes, and

$$X = \left(\frac{(m^2_{B_q} - s - t)^2}{4} - st\right)^{1/2},$$
$$\beta = \frac{1}{t}[t^2 - 2t(m^2_{\mathbf{B}} + m^2_{\bar{\mathbf{B}}}) + (m^2_{\mathbf{B}} - m^2_{\bar{\mathbf{B}}})^2]^{1/2}. \qquad \text{(B2)}$$

The ranges of s, t, $\theta_{\mathbf{B}}$, $\theta_{\mathbf{L}}$ and ϕ are

$$0 \le s \le (m_{B_q} - t^{1/2})^2, \qquad (m_{\mathbf{B}} + m_{\bar{\mathbf{B}}})^2 \le t \le m^2_{B_q},$$
$$0 \le \theta_{\mathbf{B},\mathbf{L}} \le \pi, \qquad 0 \le \phi \le 2\pi, \qquad \text{(B3)}$$

where the masses of leptons are neglected.

The amplitude squared $|M|^2$ can be obtained by using $T_{i\mathcal{B}\bar{\mathcal{B}}}$, $T_{\mathcal{B}\bar{\mathcal{D}}}$, $T_{\mathcal{D}\bar{\mathcal{B}}}$, and $T_{\mathcal{D}\bar{\mathcal{D}}}$ as shown in Eqs. (21)–(24) with the help of FeynCalc [45–47]. The scalar products of momenta and the contracted Levi-Civita symbol need to be expressed in terms of s, t, $\theta_{\mathbf{B}}$, $\theta_{\mathbf{L}}$ and ϕ before the integration $\int d\Gamma$ can be carried out. The expressions have been worked out in Ref. [44]. Defining

$$P \equiv p_{\mathbf{B}} + p_{\bar{\mathbf{B}}}, \qquad Q \equiv p_{\mathbf{B}} - p_{\bar{\mathbf{B}}},$$
$$L \equiv p_{l(\nu)} + p_{\bar{\nu}}, \qquad N \equiv p_{l(\nu)} - p_{\bar{\nu}}, \qquad \text{(B4)}$$

one has [44]

$$P^2 = t, \qquad P\cdot Q = m^2_{\mathbf{B}} - m^2_{\bar{\mathbf{B}}}, \qquad Q^2 = 2(m^2_{\mathbf{B}} + m^2_{\bar{\mathbf{B}}}) - t,$$
$$L^2 = s, \qquad L\cdot N = 0, \qquad N^2 = -s, \qquad \text{(B5)}$$

$$L\cdot P=\frac{1}{2}(m_{B_q}^2-t-s),\qquad L\cdot Q=\beta X\cos\theta_{\mathbf{B}}+\frac{m_{\mathbf{B}}^2-m_{\bar{\mathbf{B}}}^2}{t}L\cdot P,\qquad N\cdot P=X\cos\theta_{\mathbf{L}},$$
$$N\cdot Q=\frac{m_{\mathbf{B}}^2-m_{\bar{\mathbf{B}}}^2}{t}X\cos\theta_{\mathbf{L}}+\beta(L\cdot P)\cos\theta_{\mathcal{B}}\cos\theta_{\mathbf{L}}-\sqrt{st}\beta\sin\theta_{\mathbf{B}}\sin\theta_{\mathbf{L}}\cos\phi, \tag{B6}$$

$$p_B\cdot P=\frac{1}{2}(m_{B_q}^2-s+t),\qquad p_B\cdot Q=\frac{(m_{\mathbf{B}}^2-m_{\bar{\mathbf{B}}}^2)(m_{B_q}^2-s+t)}{2t}+\beta X\cos\theta_{\mathbf{B}},$$
$$p_B\cdot L=\frac{1}{2}(m_{B_q}^2+s-t),\qquad p_B\cdot N=X\cos\theta_{\mathbf{L}}, \tag{B7}$$

and

$$\epsilon_{\mu\nu\rho\sigma}N^\mu P^\nu p_B^\rho Q^\sigma=\sqrt{st}\beta X\sin\theta_{\mathbf{B}}\sin\theta_{\mathbf{L}}\sin\phi, \tag{B8}$$

with $\epsilon_{0123}=-1$.

In $\bar{B}_q\to\mathcal{B}\bar{\mathcal{D}}l\bar{\nu}(\nu\bar{\nu})$, $\bar{B}_q\to\mathcal{D}\bar{\mathcal{B}}l\bar{\nu}(\nu\bar{\nu})$ and $\bar{B}_q\to\mathcal{D}\bar{\mathcal{D}}'l\bar{\nu}(\nu\bar{\nu})$ decays, the calculation of $|M|^2$ involve polarization sums of Rarita-Schwinger vector spinors. The following formulas for polarization sums [see, for example, Eq. (4.31) of Ref. [43]] are needed,

$$\sum_{\lambda=-3/2}^{3/2}u_\mu(p,\lambda)\bar{u}_\nu(p,\lambda)=-(\not{p}+m)\left(G_{\mu\nu}-\frac{1}{3}G_{\mu\sigma}G_{\nu\lambda}\gamma^\sigma\gamma^\lambda\right),$$
$$\sum_{\lambda=-3/2}^{3/2}v_\mu(p,\lambda)\bar{v}_\nu(p,\lambda)=-(\not{p}-m)\left(G_{\mu\nu}-\frac{1}{3}G_{\mu\sigma}G_{\nu\lambda}\gamma^\sigma\gamma^\lambda\right), \tag{B9}$$

where $G_{\mu\nu}$ is defined as

$$G_{\mu\nu}\equiv g_{\mu\nu}-\frac{p_\mu p_\nu}{m^2}. \tag{B10}$$

Note that in the above formulas the signs of m differ from those in Ref. [43]. It is useful to check that in the large momentum limit, we have

$$\sum_{\lambda=-3/2}^{3/2}u_\mu(p,\lambda)\bar{u}_\nu(p,\lambda)\simeq(\not{p}+m)\frac{2}{3m}p_\mu p_\nu=\frac{2}{3m^2}p_\mu p_\nu\sum_{\lambda=-1/2}^{1/2}u(p,\lambda)\bar{u}(p,\lambda),$$
$$\sum_{\lambda=-3/2}^{3/2}v_\mu(p,\lambda)\bar{v}_\nu(p,\lambda)\simeq(\not{p}-m)\frac{2}{3m^2}p_\mu p_\nu,=\frac{2}{3m^2}p_\mu p_\nu\sum_{\lambda=-1/2}^{1/2}v(p,\lambda)\bar{v}(p,\lambda), \tag{B11}$$

which agree with Eq. (A11). Note that in our calculations involving Rarita-Schwinger vector spinors, only the exact polarization formulas in Eq. (B9) are used.

With the above formulas $\bar{B}_q\to\mathbf{B}\bar{\mathbf{B}}'l\bar{\nu}$ and $\bar{B}_q\to\mathbf{B}\bar{\mathbf{B}}'\nu\bar{\nu}$ decay rates can be readily obtained once the topological amplitudes are given.

[1] N. E. Adam *et al.* (CLEO Collaboration), Search for $B \to \bar{p} e \bar{\nu}_e X$ decay using a partial reconstruction method, Phys. Rev. D **68**, 012004 (2003).
[2] K. J. Tien *et al.* (Belle Collaboration), Evidence for semileptonic $B^- \to p\bar{p} l^- \bar{\nu}_l$ decays, Phys. Rev. D **89**, 011101 (2014).
[3] R. Aaij *et al.* (LHCb Collaboration), Observation of the semileptonic decay $B^+ \to p\bar{p}\mu^+\nu_\mu$, J. High Energy Phys. 03 (2020) 146.
[4] R. L. Workman *et al.* (Particle Data Group), Review of particle physics, Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).
[5] J. P. Lees *et al.* (*BABAR* Collaboration), Search for $B^- \to \Lambda\bar{p}\nu\bar{\nu}$ with the *BABAR* experiment, Phys. Rev. D **100**, 111101 (2019).
[6] W. S. Hou and A. Soni, Pathways to Rare Baryonic B Decays, Phys. Rev. Lett. **86**, 4247 (2001).
[7] C. Q. Geng and Y. K. Hsiao, Semileptonic $B^- \to p\bar{p}\ell^-\bar{\nu}_\ell$ decays, Phys. Lett. B **704**, 495 (2011).
[8] C. Q. Geng, C. W. Liu, and T. H. Tsai, Revisiting semileptonic $B^- \to p\bar{p}\ell^-\bar{\nu}_\ell$ decays, Phys. Rev. D **104**, 113002 (2021).
[9] Y. K. Hsiao, Semileptonic baryonic B decays, Eur. Phys. J. C **83**, 300 (2023).
[10] C.-K. Chua, Charmless two body baryonic B decays, Phys. Rev. D **68**, 074001 (2003).
[11] C. K. Chua, Charmless two-body baryonic $B_{u,d,s}$ decays revisited, Phys. Rev. D **89**, 056003 (2014).
[12] C. K. Chua, Rates and CP asymmetries of charmless two-body baryonic $B_{u,d,s}$ decays, Phys. Rev. D **95**, 096004 (2017).
[13] C. K. Chua, Two-body baryonic $B_{u,d,s}$ and B_c to charmless final state decays, Phys. Rev. D **106**, 036015 (2022).
[14] D. Zeppenfeld, SU(3) relations For B meson decays, Z. Phys. C **8**, 77 (1981).
[15] L. L. Chau and H. Y. Cheng, Analysis of exclusive two-body decays of charm mesons using the quark diagram scheme, Phys. Rev. D **36**, 137 (1987).
[16] M. J. Savage and M. B. Wise, SU(3) predictions for non-leptonic B meson decays, Phys. Rev. D **39**, 3346 (1989); **40**, 3127(E) (1989).
[17] L. L. Chau, H. Y. Cheng, W. K. Sze, H. Yao, and B. Tseng, Charmless nonleptonic rare decays of B mesons, Phys. Rev. D **43**, 2176 (1991); **58**, 019902(E) (1998).
[18] M. Gronau, O. F. Hernandez, D. London, and J. L. Rosner, Decays of B mesons to two light pseudoscalars, Phys. Rev. D **50**, 4529 (1994).
[19] M. Gronau, O. F. Hernandez, D. London, and J. L. Rosner, Electroweak penguins and two-body B decays, Phys. Rev. D **52**, 6374 (1995).
[20] C. W. Chiang, M. Gronau, J. L. Rosner, and D. A. Suprun, Charmless $B \to PP$ decays using flavor SU(3) symmetry, Phys. Rev. D **70**, 034020 (2004).
[21] H. Y. Cheng, C. W. Chiang, and A. L. Kuo, Updating $B \to PP, VP$ decays in the framework of flavor symmetry, Phys. Rev. D **91**, 014011 (2015).
[22] X. G. He and W. Wang, Flavor SU(3) topological diagram and irreducible representation amplitudes for heavy meson charmless hadronic decays: Mismatch and equivalence, Chin. Phys. C **42**, 103108 (2018).
[23] X. G. He, Y. J. Shi, and W. Wang, Unification of flavor SU(3) analyses of heavy hadron weak decays, Eur. Phys. J. C **80**, 359 (2020).
[24] D. Wang, C. P. Jia, and F. S. Yu, A self-consistent framework of topological amplitude and its $SU(N)$ decomposition, J. High Energy Phys. 09 (2020) 126.
[25] C. K. Chua, W. S. Hou, and S. Y. Tsai, Charmless three-body baryonic B decays, Phys. Rev. D **66**, 054004 (2002).
[26] C. K. Chua, W. S. Hou, and S. Y. Tsai, Understanding $B \to D^* N\bar{N}$ and its implications, Phys. Rev. D **65**, 034003 (2002).
[27] H. Y. Cheng and K. C. Yang, Charmless exclusive baryonic B decays, Phys. Rev. D **66**, 014020 (2002).
[28] C. K. Chua and W. S. Hou, Three body baryonic $\bar{B} \to \Lambda\bar{p}\pi$ decays and such, Eur. Phys. J. C **29**, 27 (2003).
[29] X. Huang, Y. K. Hsiao, J. Wang, and L. Sun, Baryonic B meson decays, Adv. High Energy Phys. **2022**, 4343824 (2022).
[30] W. Altmannshofer and P. Stangl, New physics in rare B decays after Moriond 2021, Eur. Phys. J. C **81**, 952 (2021).
[31] C. Cornella, D. A. Faroughy, J. Fuentes-Martin, G. Isidori, and M. Neubert, Reading the footprints of the B-meson flavor anomalies, J. High Energy Phys. 08 (2021) 050.
[32] L. S. Geng, B. Grinstein, S. Jäger, S. Y. Li, J. Martin Camalich, and R. X. Shi, Implications of new evidence for lepton-universality violation in $b \to \ell + \ell-$ decays, Phys. Rev. D **104**, 035029 (2021).
[33] C. Q. Geng and Y. K. Hsiao, Rare $B^- \to \Lambda\bar{p}\nu\bar{\nu}$ decay, Phys. Rev. D **85**, 094019 (2012).
[34] T. Inami and C. S. Lim, Effects of superheavy quarks and leptons in low-energy weak processes $K_L \to \mu\bar{\mu}$, $K^+ \to \pi^+\nu\bar{\nu}$ and $K^0 \leftrightarrow \bar{K}^0$, Prog. Theor. Phys. **65**, 297 (1981); **65**, 1772(E) (1981); G. Belanger and C. Q. Geng, New range of mixing parameters and rare K decays, Phys. Rev. D **43**, 140 (1991).
[35] T. D. Lee, Particle physics and introduction to field theory, Contemp. Concepts Phys. **1**, 1 (1981); H. Georgi, *Weak Interactions and Modern Particle Theory* (Benjamin/Cummings, Menlo Park, USA, 1984).
[36] J. Charles *et al.* (CKMfitter Group), CP violation and the CKM matrix: Assessing the impact of the asymmetric B factories, Eur. Phys. J. C **41**, 1 (2005).
[37] S. J. Brodsky, G. P. Lepage, and S. A. Zaidi, Weak and electromagnetic form-factors of baryons at large momentum transfer, Phys. Rev. D **23**, 1152 (1981).
[38] R. Aaij *et al.* (LHCb Collaboration), Evidence for the two-body charmless baryonic decay $B^+ \to p\bar{\Lambda}$, J. High Energy Phys. 04 (2017) 162.
[39] R. Aaij *et al.* (LHCb Collaboration), First Observation of the Rare Purely Baryonic Decay $B^0 \to p\bar{p}$, Phys. Rev. Lett. **119**, 232001 (2017).
[40] LHCb Collaboration, Search for the rare hadronic decay $B_s^0 \to p\bar{p}$, Phys. Rev. D **108**, 012007 (2023).

[41] The author would like to thank the referee for pointing this out.

[42] W. Altmannshofer, P. Ball, A. Bharucha, A. J. Buras, D. M. Straub, and M. Wick, Symmetries and asymmetries of $B \to K^* \mu^+ \mu^-$ decays in the standard model and beyond, J. High Energy Phys. 01 (2009) 019.

[43] T. Moroi, Effects of the gravitino on the inflationary universe, arXiv:hep-ph/9503210.

[44] H. Y. Cheng, C. Y. Cheung, W. Dimm, G. L. Lin, Y. C. Lin, T. M. Yan, and H. L. Yu, Heavy quark and chiral symmetry predictions for semileptonic decays anti-$B \to D(D*)\pi$ lepton anti-neutrino, Phys. Rev. D **48**, 3204 (1993).

[45] R. Mertig, M. Bohm, and A. Denner, FEYN CALC: Computer algebraic calculation of Feynman amplitudes, Comput. Phys. Commun. **64**, 345 (1991).

[46] V. Shtabovenko, R. Mertig, and F. Orellana, New developments in FeynCalc 9.0, Comput. Phys. Commun. **207**, 432 (2016).

[47] V. Shtabovenko, R. Mertig, and F. Orellana, FeynCalc 9.3: New features and improvements, Comput. Phys. Commun. **256**, 107478 (2020).