Toward UV model of kinetic mixing and portal matter. IV. Quartification

Thomas G. Rizzo[*]

SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, California 94025, USA

As is well known, trinification, i.e., the extension of the Standard Model (SM) to $[SU(3)]^3 = SU(3)_c \times SU(3)_L \times SU(3)_R$ as occurs in E_6 models, allows for a partial unification of the gauge forces even though quarks and leptons remain in separate multiplets so that no heavy gauge or scalar fields exist which can generate proton decay. The extension of this idea to quartification, by including an additional $SU(3)'$ factor, has also been considered in the literature maintaining the basic attributes of trinification but now allowing, e.g., for a more symmetric treatment of quarks and leptons at the price of new matter fields and gauge interactions. In this paper, we will consider this $SU(3)'$ to be the "dark" gauge group, now containing the familiar $U(1)_D$ subgroup, under which the SM fields are all neutral, which is associated with kinetic mixing (KM) and the existence of a light, $\lesssim 1$ GeV dark photon. This setup naturally predicts the existence of color-singlet portal matter fields, carrying both electromagnetic and $U(1)_D$ dark charges, that are necessary to generate this KM at the one-loop level and whose masses are directly tied with those of the many new gauge bosons that originate from the extended gauge sector. In this paper, after a discussion of the detailed structure of this model, we present a broad survey of the collider phenomenology of the large set of new fields that must necessarily arise from this setup in a simplified version involving only a single generation of fermions. We demonstrate that several new signatures may be anticipated at the LHC as well as at future hadron and lepton colliders if such models are realized in nature.

I. INTRODUCTION AND BACKGROUND DISCUSSION

The Standard Model (SM) of particle physics, though very successful, faces a number of significant obstacles in explaining the world that we see. One of the most challenging and longstanding of these is the nature of dark matter (DM) for which there is no SM candidate: what is it and how does it interact with us, if at all, beyond the obvious gravitational interactions by which we know it to exist? A perhaps reasonable expectation is that DM and SM fields will interact rather weakly through some new, as yet unknown, mediator which itself is not a part of the SM and by whose action the DM obtains the value of the relic density as measured by Planck [1]. It is not unreasonable to ask how the SM might be extended to account for this possibility and how a more unified description which includes these new interactions may be achieved.

Such questions in one form or another have been under discussion for quite some time and the consideration of

various DM candidates now extends back several decades. As is well known, the searches for these "traditional" DM candidates, such as the QCD axion [2–4] and the family of weakly interacting massive particles, i.e., WIMPS [5,6], continue to push downwards into ever lower cross section regimes and wider over larger ranges of possible masses with increasing sensitivities. Unfortunately, these direct detection experiments, as well as those searching for indirect signatures or via the direct production of DM at the 13 TeV LHC [7–11], have all so far produced negative results, thus excluding increasing large regions of the parameter spaces of many specific models. These results and others have, over the last decade or so, led the community to greatly expand upon this set of traditional candidates with many new ideas for the possible nature of DM and its interactions with the SM. During this time interval it has become clear that both the coupling strength of DM to (at least some of) the fields of the SM as well as its possible mass can both span previously unexpected large ranges [12–17] which will require a wide variety of very broad and very deep searches to provide even partially adequate coverage. In a similar fashion, it has been found that many distinct types of interactions between DM and the fields of the SM are possible, many of which are best classified by the use of the effective field theory approach employing "portals." Such portals can produce interaction structures which are either renormalizable (i.e., dimension

≤ 4) or nonrenormalizable (i.e., dimension > 4) depending upon the specific setup. For these models to work, an additional set of new fields also needs to be introduced which acts as mediators linking the SM to the DM and potentially also to an enlarged, nontrivial, dark sector of which the DM candidate is only the lightest stable state— possibly due to, e.g., the existence of some new, at least approximately conserved, quantum number.

Among the various classes of these portals, significant attention has been given in the literature to the renormalizable kinetic mixing/vector portal [18–20] scenario which is based upon the existence of a new dark gauge interaction and which has significant model building flexibility. In such a scenario, one finds that even in the most simple realization, over significant ranges of the model parameters, it is possible for the DM to reach its observed abundance via the usual WIMP-like thermal freeze-out mechanism [21,22]. Unlike in traditional WIMP models, however, this will now occur only for sub-GeV DM masses by employing this previously mentioned new non-SM dark gauge interaction that, because of its weakness, has so far evaded detection by other means. The simplest construction of this kinds has only a few moving pieces assuming only the existence of this new gauge interaction based on the $U(1)_D$ gauge group, with a coupling g_D, under which it it is postulated that the SM fields are all neutral, implying that they do not carry dark charges, i.e., thus $Q_D = 0$. The new $U(1)_D$ gauge boson is then termed as the "dark photon" (DP) [23,24], which we will generally denote as D. In the usual setup, in order to obtain the observed relic density by thermal means, this new $U(1)_D$ is assumed to be spontaneously broken at or below the scale of ∼ few GeV and thus both the DM and DP will have comparable masses. This symmetry breaking is most simply accomplished via the (sub-)GeV scale vev(s) of at least one (if not several) new scalar(s), the dark Higgs(es), similar to the symmetry breaking which occurs in the SM. While the DP couples to DM it does not do so at tree level with SM fields but, within this framework, is generated via renormalizable kinetic mixing (KM) at the one-loop level between the $U(1)_D$ and the SM $U(1)_Y$ gauge fields and whose strength is then described by a small, dimensionless parameter, ϵ. Since the SM fields have $Q_D = 0$ and DM has no SM charges, these loops must arise from a set of new fields, usually being vectorlike (VL) fermions and/or complex scalars, which we call portal matter (PM) [25–39], that must carry *both* SM and $U(1)_D$ dark charges. Transforming back to the familiar canonically normalized gauge fields to remove the KM and after both the SM and $U(1)_D$ gauge symmetries are broken, this now leads to an effective coupling of the DP to SM fields of the form $\simeq e\epsilon Q_{em}$ (up to small correction terms or order $m_D^2/m_Z^2 \ll 1$). Further, for both DM and a DP being sub-GeV in mass, one finds that the magnitude of the parameter ϵ is constrained by experiment to very roughly lie in the $\epsilon \sim 10^{-(3-4)}$ range, a number

which we might have already expected from it originating due to a loop. Importantly, in such a setup, for *p*-wave annihilating DM or for pseudo-Dirac DM with a sufficiently large mass splitting, it is found the rather tight constraints arising from the CMB on DM annihilation into electromagnetically interacting final states can also be avoided [1,40–42] for an overlapping range of model parameters.

Finally, another interesting feature of this class of setups, in the conventionally chosen normalization and in the IR limit, is that the parameter ϵ can be determined in terms of the properties of the PM fields that appear in these vacuum polarizationlike graphs and is given by the sum

$$\epsilon = \frac{g_D e}{24\pi^2} \sum_i (\eta_i N_{c_i} Q_{em,i} Q_{D_i}) \ln \frac{m_i^2}{\mu^2}, \qquad (1)$$

where $e(g_D)$ is the usual QED ($U(1)_D$) gauge coupling and $m_i(Q_{em,i}, Q_{D_i}, N_{c_i})$ are the mass (electric charge, dark charge, number of colors) of the ith PM field. We note that, e.g., $\eta_i = 1(1/2)$ if the PM particle is assumed to be a chiral fermion (complex scalar).[1] We then see that if the condition

$$\sum_i \eta_i N_{c_i} Q_{em,i} Q_{D_i} = 0 \qquad (2)$$

is also satisfied, as might perhaps be expected within a fully unified description, then ϵ also becomes a finite and, in principle, a calculable quantity. Such an observation may already whet our appetite to search for such a more enveloping framework for the KM scenario. However, as we will see in the discussion below, this condition is not automatically satisfied in this only partially unified model without the introduction of some additional (likely scalar) fields beyond this minimal setup.

It seems to be advantageous to go beyond this effective theory to further our understanding of how this (apparently) simple KM mechanism fits together into a single scheme with the SM, something that we, and others, have begun to examine in pathfinder mode employing various bottom-up and top-down approaches in a recent series of papers [25,26,28–35,38,39]. Two common features that one finds from following our general approach are the embedding of the $U(1)_D$ dark Abelian symmetry into some larger, non-Abelian G_{Dark}, e.g., with an $SU(2)_I \times U(1)_{Y_I}$ [26] gauge symmetry [43] being its simplest manifestation and the appearance of at least some of the SM fields in common gauge group representations with the PM fields. In such setups, the PM mass generation is generally the result of the $G_{\text{Dark}} \to U(1)_D$ symmetry breaking and so, with $O(1)$

---

[1]In the model framework below, there are two gauge bosons that can also play this PM role and so will also contribute to the sum above as we will discuss below.

Yukawa couplings, will share a similar overall mass scale with the associated heavy gauge bosons. This was seen explicitly in Ref. [26] whose PM content and the $G_{\text{Dark}} = SU(2)_I \times U(1)_{Y_I}$ gauge group were both inspired by E_6 [44,45]. More recently, we extended this idea of a more general product group $G = G_{\text{SM}} \times G_{\text{Dark}}$ from a top-down perspective [34] and also via an augmentation of G_{SM} to that of the left-right symmetric model (LRM) [46–50] while maintaining $G_{\text{Dark}} = SU(2)_I \times U(1)_{Y_I}$ wherein the PM and RH-neutrino mass scales were shown to be related. Interestingly, one generally finds that attempts to "unify" the SM with the KM portal naturally brings in some of the other portals as well, e.g., the Higgs and neutrino portals will also frequently appear.

Following the cue from these previous studies, in this paper we consider identifying $G_{\text{SM}} = SU(3)_c \times SU(3)_L \times SU(3)_R$, i.e., the trinification [51] subgroup of E_6, while also assuming that $G_{\text{Dark}} = SU(3)'$, thus forming a quartification model. Such a class of partially unified models has been discussed in the literature for both DM as well as in other contexts [52] and in some cases allows for greater flexibility than many conventional unification approaches involving a single gauge group or a product of two groups. While the general setup that we will obtain following this path will share some easily recognizable common characteristics with one or both of the previously examined $G_{\text{Dark}} = SU(2)_I \times U(1)_{Y_I}$ model classes, it will present us with numerous new and interesting features that we will begin to explore in the present work. Unfortunately, due to the necessarily numerous new fields of all varieties that will appear in the current study (even ignoring the fact of there being three generations) it is quite difficult to make many specific phenomenological predictions that can provide more that suggestive tests for this setup beyond a few specific examples without making further assumptions about, e.g., the relative sizes/orderings of the large mass scale vevs. These specific choices will induce different possible paths of symmetry breaking down to the SM $\times$ $U(1)_D$ thus leading to quite different, quantitative predictions for, e.g., the masses and, in some cases, the couplings of the many new color singlet gauge bosons within this setup. However, there remains many aspects that most of these paths will share, at least at the semiquantitative level, and we will concentrate our efforts in exploring those phenomenological tests here. Specifically, we will examine the capabilities of the LHC as well as future hadron and lepton colliders to explore the essential components of this setup: the new gauge bosons and their interplay with the VL fermions and SM fields. As we will see, this scenario shares many of the features that we had earlier encountered in our analyses of one or both of the previously examined E_6 and Pati-Salam motivated models but with different (and sometimes simultaneous) aspects of additional simplicity and complexity. Unfortunately, while a step forward, this type of setup is not yet fully unified in the traditional sense.

The outline of this paper is as follows: Following the present Introduction and Background discussion, in Sec. II we will present a broad outline of the model setup and framework, setting the stage for the analysis in the later sections. Sections III and IV will then individually examine the various fermionic sectors of this setup, i.e., the generation of the Dirac and Majorana fermion masses together with the corresponding mixings between the PM and SM/LRM fermion fields. The KM and gauge symmetry breaking which takes place in several distinct steps at a hierarchy of mass scales and the resulting gauge boson masses and mixings that will be important at the electroweak scale and below will then be discussed separately for the non-Hermitian and Hermitian gauge fields in Secs. V and VI, respectively. A further examination of a sample of some of the (mostly collider oriented) phenomenological implications and tests of this scenario that were not touched upon in any detail earlier as part of the model development will then be presented in Sec. VII. As we will see below, given the numerous moving parts for this model, it is quite difficult to capture all of the interesting new physics potential of this setup within this introductory survey in more than a suggestive manner. Finally, a summary and discussion of our results, possible future avenues of exploration and our subsequent conclusions can then be found in Sec. VIII.

II. OVERVIEW OF MODEL SETUP AND FRAMEWORK

Although the details of the fermion embedding, the interpretation, and low energy phenomenology of the $SU(3)'$ gauge group will differ here from that in much of the literature, the essential features and the basic mechanics of the general quartification model setup will remain unaltered [52]. However, in what follows we will *not*, e.g., assume that an additional Z_4 exchange symmetry among the four $SU(3)$ groups is also present and we will not be directly interested in specific unification issues at very high mass scales. This freedom, e.g., allows for some asymmetric treatment in the necessary symmetry breaking of the various gauge group factors as we will encounter in the discussion below. In terms of the diagonal generators of the $SU(3)_c \times SU(3)_L \times SU(3)' \times SU(3)_R$ gauge group, the electric charge in this version of quartification will be given by the somewhat uncommon but highly symmetric decomposition

$$Q_{em} = T_{3L} - \frac{Y_L}{2} + T_{3R} - \frac{Y_R}{2} + T'_3 - \frac{Y'}{2}, \qquad (3)$$

where $Y_a/2 = T_{8a}/\sqrt{3}$ with $a = (L, R,')$. Similarly, as we will see below, the dark charge, Q_D, associated with the familiar $U(1)_D$ gauge group as described above, is just given by a linear combination of the two diagonal $SU(3)'$ generators,[2] up to an overall sign convention, as

---

[2]The origin of this expression will be explained shortly.

$$Q_D = T_3' - \sqrt{3}T_8'. \tag{4}$$

As noted above, since we will not be concerned here with the potential vast wealth of flavor phenomenology associated with this kind of setup, we will limit ourselves in our discussion below to the consideration of a only single fermion generation so that we can emphasize the corresponding wealth of nonflavor physics. To this end, we will label all the SM fermion fields by their first generation labels. Interestingly, a respectable fraction of the phenomenology that we will encounter in the present model has already been seen in our earlier work on PM [25,26,28–35,38,39] but in other and generally more simplified settings. Although related at the "unification" scale, at accessible collider energies we will treat the three gauge couplings for $3_L3'3_R$, i.e., g_L, g', g_R as generally independent quantities although, for ease and clarity of presentation at some later points, we will assume they have a common value.

In terms of this $SU(3)_c \times SU(3)_L \times SU(3)' \times SU(3)_R$ quartification gauge group, which hereafter we will frequently abbreviate simply as $3_c3_L3'3_R$, helped by the left-right gauge symmetries and the VL nature of the new fields, the 36, two-component, left-handed fermion fields of a single generation are found to transform in an anomaly-free manner with equal numbers of triplets and antitriplets for each of the $SU(3)$ group factors combined with singlets. In addition to the 15 SM fields plus the familiar RH-neutrino, there will also now be eight new VL fermions: one electroweak singlet, VL quark (i.e., 6 degrees of freedom) plus three charged and four neutral VL leptons (i.e., 14 degrees of freedom). This full set of fermions for a single generation is given by the sum of the four representations:

$$[q](3, \bar{3}, 1, 1) + [l](1, 3, \bar{3}, 1)$$
$$+ [l^c](1, 1, 3, \bar{3}) + [q^c](\bar{3}, 1, 1, 3), \tag{5}$$

where we want to especially note that all of the color (anti) triplet fields are seen to be singlets under the new $SU(3)'$ group so that in this framework the PM fields must necessarily be color singlets, e.g., VL leptons and/or complex scalars. The q, q^c fields, being $SU(3)'$ singlets, are the familiar, canonical ones that also appear in E_6-inspired trinification models [51]:

$$[q] = \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix}, \qquad [q^c] = \begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix}, \tag{6}$$

with h being, to those familiar with E_6 phenomenology [44,45], a $Q_{em} = -1/3$, color-triplet, electroweak iso-singlet. However, unlike in other previously considered PM scenarios, here h is *not* a PM field as it (and all the

quarks) is a $Q_D = 0$ singlet under the $SU(3)'$ group and so is simply a "conventional" VL quark as has been sought at the LHC, excluding a mass below ~ 1.2 TeV assuming decays [53] only to the third generation SM fermions.[3] Clearly, the fact that quarks do not carry any $SU(3)'$ charges will greatly impact the production of the various new states we will encounter below at hadron colliders. The l, l^c fields, on the other hand, are here somewhat different due to, amongst other things, the relationship between the diagonal $SU(3)'$ generators and Q_{em} as well as the necessity of keeping SM fields free of any Q_D charge. Below, for these "leptonic" fields, and following somewhat closely the nomenclature of, e.g., the recent work of Ma in Ref. [52] to make contact with the existing literature, we see that

$$[l] = \begin{pmatrix} \nu & E_1^c & N_1 \\ e & N_1^c & E_1 \\ S_1 & E_3^c & S_2 \end{pmatrix}, \quad [l^c] = \begin{pmatrix} \nu^c & e^c & S_1^c \\ E_2 & N_2 & E_3 \\ N_2^c & E_2^c & S_2^c \end{pmatrix}, \tag{7}$$

where the $S_{1,2}(E_3)$ and their conjugates have $|Q_{em}| = 0(1)$ and are 2_L2_R singlets, $(N_1, E_1)[(N_2, E_2)]^T$ with both helicities together forming VL doublets with $Q_{em} = 0, -1$, respectively, under $2_L[2_R]$ and are singlets under $2_R[2_L]$, while $(\nu, e)^T$ + H.c. are just the conventional SM fields with the addition of the RH neutrino. Since the usual $U(1)_D$ gauge group must be a diagonal subgroup of the $SU(3)'$ group, the familiar dark charge Q_D must be a linear combination of the two diagonal generators, $T_{3,8}'$, chosen in a way such that the SM fields ν, e, by construction, have $Q_D = 0$. Normalizing to unit charges, this uniquely fixes, up to a sign choice, the relationship $Q_D = T_3' - \sqrt{3}T_8'$ as given above when these generators act on an $SU(3)'$ (anti) triplet. From this we see that $Q_D(S_1) = 0$ whereas all the other new lepton fields, N_i, E_i, S_2, will all carry $Q_D = -1$ and those which also have $Q_{em} \neq 0$ can be identified as true PM fields. Generally, leptonlike PM will, of course, be somewhat more difficult to produce and observe at a hadron collider as previously noted.

Lastly, although we will not be directly discussing unification issues in the analysis that follows, we note in passing that with the definition of Q_{em} given above, as well as the details of the fermion representation structure, we can calculate the value of $\sin^2 \theta_w$ at the scale at which the product of the gauge symmetries, $G = [SU(3)]^4$, begin to break via the relation

$$\sin^2 \theta_w = \frac{\text{Tr} T_{3L}^2}{\text{Tr} Q_{em}^2} = \frac{1}{4}, \tag{8}$$

---

[3]One possible difference is that in this setup h may be sufficiently massive as to have decays through non-SM mediators or into non-SM final states but these are unlikely to be dominant.

obtaining an interesting result that has been previously discussed in the quartification literature [52] and which is quite suggestive. Similarly, one finds that the analog of this quantity for the dark gauge group, $s_I^2 = \sin^2\theta_I$, as was employed in our earlier analyses of the $SU(2)_I \times U(I)_{Y_I}$, E_6-inspired model [26], satisfies a similar relationship and is given by

$$\sin^2\theta_I = \frac{\mathrm{Tr}\,T_3'^2}{\mathrm{Tr}\,Q_D^2} = \frac{1}{4}. \qquad (9)$$

Given the complexity of the setup that we will examine below, the opportunities for both KM and mass mixing between the DP and the many more massive neutral gauge boson states are rampant. Fortunately, the absence of biadjoint representations somewhat restricts this possibility. Of course, in the IR, this KM is in practice given simply by Eq. (1) above.

III. DIRAC FERMION MASSES

In order to generate the various fermion masses, we recall that we can form $SU(3)$ singlets either as a product of a **3** with a $\bar{\mathbf{3}}$ or by taking antisymmetric products of three **3**'s or $\bar{\mathbf{3}}$'s. This results in a somewhat restrictive, though sufficient, setup for the fermion mass terms and, as we will see, also the need to introduce an additional Higgs scalar representation, not coupling to the fermions, in order to properly generate all of the heavy gauge boson masses. In particular, as $SU(3)_c$ must remain unbroken, the independent set of Higgs fields that can be employed to generate the fermion masses can be at most three in number and must transform as bitriplets under the remaining broken $SU(3)$ groups. Specifically, in order to obtain the Dirac masses for all of these fermions, it is sufficient to introduce the three Higgs fields, H_i, $i = 1 - 3$, which transform in obvious manners with respect to the group $3_c 3_L 3' 3_R$ based on the transformation properties of the fermion representations themselves as described above. A respectable fraction of these scalars will be eaten as they play the role of the Goldstone bosons for the many heavy gauge boson fields that we will encounter below. These $3 \times 9 = 27$ complex scalars are given by

$$H_1 = (1,3,1,\bar{3}), \quad H_2 = (1,3,\bar{3},1), \quad H_3 = (1,1,3,\bar{3}), \qquad (10)$$

so that the following Yukawa couplings (with repeated indices summed over) are allowed:

$$\begin{aligned} \mathcal{L}_y = {}& y_1 q_{ij} q_{jk}^c (H_1)_{ki} + y_2 l_{ij} l_{jk}^c (H_1^\dagger)_{ki} \\ & + \epsilon_{ijk}\epsilon_{\alpha\beta\gamma}\left[y_3 l_{i\alpha} l_{j\beta} (H_2)_{k\gamma} + y_4 l_{i\alpha}^c l_{j\beta}^c (H_3)_{k\gamma} \right] + \mathrm{H.c.} \end{aligned} \qquad (11)$$

In the analysis presented here, for simplicity, we will ignore possible CP violation and assume that all of these Yukawa couplings (as well as the many vevs that we will soon encounter below) are all real. Note that *none* of the scalars appearing in representation H_1 carry a $Q_D \neq 0$ charge. More generally, knowing the relationships of both Q_{em} and Q_D to the relevant gauge group generators, we see that each of the H_i is found to contain five neutral scalars so that, in total, it is possible to contemplate 15 distinct vevs associated with the various required mass scales. In particular, the following vevs for the neutral fields within these Higgs scalars, H_i, can be considered:

$$\sqrt{2}\langle H_1 \rangle = \begin{pmatrix} v_1 & 0 & v_4 \\ 0 & v_2 & 0 \\ v_5 & 0 & v_3 \end{pmatrix}, \qquad (12)$$

which are solely responsible for quark masses and mixings but also contribute to the lepton masses as well, and

$$\sqrt{2}\langle H_2 \rangle = \begin{pmatrix} u_1 & 0 & x_1 \\ 0 & x_2 & 0 \\ u_2 & 0 & x_3 \end{pmatrix}, \quad \sqrt{2}\langle H_3 \rangle = \begin{pmatrix} u_3 & 0 & u_4 \\ 0 & x_4 & 0 \\ x_5 & 0 & x_6 \end{pmatrix}, \qquad (13)$$

which govern the mass generation for the combined leptonic and PM sectors. All of the vevs, v_i, within H_1 correspond to scalars with $Q_D = 0$ but, whereas $v_{1,2,4}$ will lead to the breaking of $SU(2)_L$, and so must be $\lesssim 100$ GeV or so, the vevs $v_{3,5}$ do not and either will simply lead to the breaking $SU(2)_R$ or to the more general breaking of $3_L 3_R \to 2_L 2_R 1_{L+R}$ so we can easily imagine them being at the ~ 10 TeV mass scale or above. Similarly, while the vevs u_{2-4} in $H_{2,3}$ all correspond with $Q_D = 0$ scalars, they do not break $SU(2)_L$ and so are also expected to be large, again $\gtrsim 10$ TeV. On the other hand, the vev u_1, while still having $Q_D = 0$, breaks $SU(2)_L$ and so, like $v_{1,2,4}$ above, is expected to be of order $\lesssim 100$ GeV. The remaining six vevs in $H_{2,3}$, the x_i, all arise from scalars having $|Q_D| = 1$ so will lead to the breaking of $U(1)_D$ and thus are required to lie at the much smaller $\lesssim 1$ GeV scale. Due to the hierarchy of scales of $\sim O(10^2)$ or possibly greater between the three sets of vevs, it will sometimes be convenient to treat their effects iteratively. Although rather obvious, it should be noted that whereas the vevs v_i will lead to a breaking of only $3_L 3_R$, the vevs u_i will instead break either $3_L 3'$ or $3_R 3'$ and so, in particular, link the scales associated with $SU(3)_R / SU(2)_R$ and $SU(3)'$ breaking, similar to those discussed in earlier work [38]. Table I summarizes the quantum number and transformation properties of this large set of vevs that are responsible for, at least partly, both gauge symmetry breaking and fermion mass generation; those appearing in H_1 appear almost universally in the

TABLE I. Higgs scalar vacuum expectation values. The properties of the multiple vevs contained in the three Higgs scalar representations, H_{1-3}, that are discussed in the text and which are responsible for generating both Dirac and Majorana fermion masses and which contribute to gauge symmetry breaking. The horizontal line in the middle of the table separates the $Q_D = 0$ from the $Q_D \neq 0$ vevs and acts as a guide to the eye. Here $Q_D = T'_3 - \sqrt{3} T'_8$. It is to be noted that both of the vevs, $x_{2,4}$, will also violate lepton number by 2 units, i.e., are $\Delta L = 2$, as is also discussed in the text.

vev	T_{3L}	$Y_L/2$	T_{3R}	$Y_R/2$	T'_3	$Y'/2$	Q_D
v_1	1/2	1/6	−1/2	−1/6	0	0	0
v_2	−1/2	1/6	1/2	−1/6	0	0	0
v_3	0	−1/3	0	1/3	0	0	0
v_4	1/2	1/6	0	1/3	0	0	0
v_5	0	−1/3	−1/2	−1/6	0	0	0
u_1	1/2	1/6	0	0	−1/2	−1/6	0
u_2	0	−1/3	0	0	−1/2	−1/6	0
u_3	0	0	−1/2	−1/6	1/2	1/6	0
u_4	0	0	0	1/3	1/2	1/6	0
x_1	1/2	1/6	0	0	0	1/3	−1
x_2	−1/2	1/6	0	0	1/2	−1/6	1
x_3	0	−1/3	0	0	0	1/3	−1
x_4	0	0	1/2	−1/6	−1/2	1/6	1
x_5	0	0	−1/2	−1/6	0	−1/3	−1
x_6	0	0	0	1/3	0	−1/3	−1

quartification literature [52]. Note that among the set of v_i, u_i vevs, both the same and opposite signs for their values of T_{3L}, Y_L (and also T_{3R}, Y_R) appear *but* only like sign values of T'_3, Y' appear; this is dictated by the requirement of Q_D conservation. Once the $U(1)_D$ generator is broken by the small x_i vevs we indeed see both the same as well as opposite signs of T'_3, Y' now appearing.

Interestingly, we note that unless there exists some relationship among the various vevs it is difficult to break only one of the $SU(3)$ group factors without also breaking another unless some care is taken which can have a significant impact on the pattern of how all these symmetries must eventually be broken down to $U(1)_{em}$. Note that neglecting the $Q_D \neq 0$ vevs is an excellent first approximation in obtaining the fermion masses below.

The mass terms generated by the H_1 vevs (all of which have $Q_D = 0$) with a single Yukawa coupling for the quarks are the simplest to consider and take the form

$$\frac{y_1}{\sqrt{2}} \left(d^c d v_1 + u^c u v_2 + h^c h v_3 + h^c d v_4 + d^c h v_5 \right) + \text{H.c.},$$

(14)

where we see that, as might be expected, there is a single term for the u-type quarks whereas the d, h fields undergo a mixing quite similar to what one finds in the E_6-inspired

scenarios, forming a 2×2 mass matrix of the form $D^c M_{-1/3} D$, where $D = (d, h)^T$ and

$$M_{-1/3} = \frac{y_1}{\sqrt{2}} \begin{pmatrix} v_1 & v_5 \\ v_4 & v_3 \end{pmatrix},$$

(15)

that can be diagonalized as usual by a biunitary transformation, which in this very simple case can be expressed via the mixing angles between the two right-handed and two left-handed fields, respectively. To leading order in the small vev ratios, these are given by the expressions

$$\tan 2\phi_R^{-1/3} \simeq \frac{2 v_3 v_5}{v_5^2 - v_3^2}, \qquad \tan 2\phi_L^{-1/3} \simeq \frac{-2(v_1 v_5 + v_4 v_3)}{v_3^2 + v_5^2}.$$

(16)

From these we see that we might expect $\phi_L^{-1/3}$ to be relatively small, $\lesssim 10^{-2}$, due to the appearance of the ratio of the small to large vevs, whereas $\phi_R^{-1/3}$ may be significantly larger since only the large multi-TeV scale vevs appear to leading order in this particular case. The possible relative sizes of $v_{3,5}$ (as well as the other large vevs) will become further clarified when we discuss the gauge symmetry breaking and the corresponding various new gauge boson masses later below.

As is easily imagined, the mass terms for the leptonic sector are significantly more complex as they involve two fermion representations, can arise from all three sets of $Q_D = 0$ Higgs vevs in the $\langle H_i \rangle$ as expressed above, as well as involving multiple mass scales. We note that in the absence of any of the $Q_D \neq 0$ vevs we can uniquely assign lepton number to all of the fields in both l and l^c and also to the corresponding scalars lying in $H_{2,3}$, e.g., it is easy to see that in this approximation all of the E_i share the same lepton number as does the SM charged lepton, here represented as an electron.

We begin by first considering the $Q_{em} = -1$ states which form a 4×4 mass matrix, $\mathcal{E}^c M_{-1} \mathcal{E}$, where $\mathcal{E} = (e, E_1, E_2, E_3)^T$ and

$$\sqrt{2} M_{-1} = \begin{pmatrix} y_2 v_2 & 0 & B & C \\ D & y_3 u_2 & y_2 v_1 & y_2 v_4 \\ 0 & y_2 v_2 & y_4 u_4 & -y_4 u_3 \\ F & -y_3 u_1 & y_2 v_5 & y_2 v_3 \end{pmatrix}.$$

(17)

Similarly, for the $Q_{em} = 0$ fermions, the various mass terms now lead to the 5×5 *Dirac* mass matrix, $\mathcal{N}^c M_0^D \mathcal{N}$, where the neutral fermion basis is chosen to be as $\mathcal{N} = \nu, S_1, N_1, N_2, S_2$ and

$$\sqrt{2}M_0^D = \begin{pmatrix} y_2v_1 & y_2v_5 & 0 & -B & 0 \\ y_2v_4 & y_2v_3 & 0 & -C & 0 \\ -D & -F & -y_3u_2 & y_2v_2 & y_3u_1 \\ 0 & 0 & y_2v_1 & -y_4u_4 & y_2v_5 \\ 0 & 0 & y_2v_4 & y_4u_3 & y_2v_3 \end{pmatrix}. \quad (18)$$

In both of these matrices, the shared entries represented by the upper case roman letters B, C, D, F are somewhat special in that they arise from the rather small $Q_D \neq 0$ vevs, $x_i \lesssim 1$ GeV and play an essential role in the phenomenology of the new states appearing in this setup (i.e., particle decays) but can be essentially neglected when discussing the (Dirac) masses themselves to a very good approximation. Note that the lepton number violating vevs, $x_{2,4}$, do *not* appear here. In particular, we have that

$$B = -y_4x_6, \quad C = y_4x_5, \quad D = -y_3x_3, \quad F = y_3x_1. \quad (19)$$

We will return to the *Majorana* mass terms that are generated by the remaining $Q_D \neq 0$, $\Delta L = 2$ vevs, $x_{2,4}$, in the next section below.

There are many observations that one can make about the overall structure of the matrices M_{-1} and M_0^D with respect to their apparent subcomponents that result from both the locations of null entries as well as the hierarchy of the various vevs which make their appearances within them. In the case of the $Q_{em} = -1$ fermions, we first observe that, unsurprisingly, in the absence of the small $Q_D \neq 0$ vevs, x_i, the electron is "isolated" and does not mix with the remaining VL E_i states since, unlike SM fields, they all have $Q_D = -1$. However, once these small vevs are turned on, $e - E_i$ mixing of a generally chiral nature is induced allowing for the 2-body decays $E_i \rightarrow e + \mathrm{DP}$. As has frequently been discussed and is by now well known [25,26,28–35,38,39], due to the enhanced coupling of the DP's longitudinal component (or the $O(1)$ Yukawa coupling of the equivalent Goldstone boson [54]) arising from the large ratio of the mass of the PM lepton to that of the DP, this is the dominant PM decay mode over essentially all of the entire model parameter space and provides the production/decay signature for these particles at colliders [31], i.e., an oppositely charged, same-flavor charged lepton pair plus MET, assuming the DP decays to DM or is sufficiently long lived. In any case, naively, we would roughly expect the size of this SM-PM mixing to be on the order of the ratio of the relevant vevs/mass scales, i.e., ~ 1 GeV$/ \sim 10$ TeV $\sim 10^{-4} \sim \epsilon$, or so as has been noted in previous works, which is essentially just the inverse of the large mass ratio enhancement discussed above. At the opposite end of the mass spectrum, we see that when we turn off the ~ 100 GeV, $SU(2)_L$ violating vevs, $v_{1,2,4}$ as well as u_1, one finds that E_1 no longer mixes with $E_{2,3}$ while both the left- and right-handed components

of these later two states can still mix significantly depending upon the relative sizes of the remaining large vevs. The masses and mixings among these three states will depend upon the values and mass ordering among the set of large vevs which is clearly correlated with the pattern of gauge symmetry breaking within this setup but in any case the mixing of E_1 with $E_{2,3}$ is expected to be of order ~ 100 GeV$/ \sim 10$ TeV $\sim 10^{-2}$. However, the corresponding mixing between the remaining states $E_{2,3}$ is likely to be $\sim O(1)$ for both helicities unless special hierarchies amongst the various vevs are preferred by the gauge symmetry breaking patterns to be discussed below.

In the case of the Dirac masses for the neutral fermions, we find a somewhat similar pattern outside of some essential differences due to, amongst other things, there now being five fields that mix instead of four and that the SM ν as well as S_1 fields both have $Q_D = 0$ here. We also observe that the $SU(2)_{L,R}$ partners (N_1, E_1) and (N_2, E_2) are (separately) degenerate in the absence of mixing as might be expected. We similarly note that S_2, E_3, which are both $SU(2)_{L,R}$ singlets, are also found to be degenerate in this same limit. In addition, as we will discuss in the next section, Majorana mass terms are likely also present amongst some of these neutral fields, inducing further mixings beyond those discussed below, which arise solely from the lepton-number violating subset of the Q_D-violating vevs, $x_{2,4}$. Here we observe that the most essential difference, as far as the Dirac mass terms are concerned, with respect to the $Q_{em} = -1$ case, is that there exists an additional fermion, S_1, which, together with the SM ν, is also a $Q_D = 0$ fermion. In the limit that the $Q_D \neq 0$ vevs can be ignored, this 5×5 mass matrix breaks down into a 2×2 block, M_{ν, S_1}, for the $Q_D = 0$ fields, ν, S_1, as well as a 3×3 block for the $Q_D = -1$ fields $N_{1,2}$, S_2; recall that $N_{1,2}$ share isodoublets with the previously discussed electrically charged PM fields $E_{1,2}$.

Our first observation is that apart from an overall Yukawa coupling, the submatrix M_{ν, S_1} is identical to $M_{-1/3}$ and so can be diagonalized by the same rotations as described above. This mixing allows for the $2_L 2_R$ isosinglet to have suppressed decays through both SM as well as LRM-like interactions. The 3×3 submatrix is, essentially, apart from some signs, observed to be the transpose/Hermitian conjugate of the 3×3 submatrix appearing in M_{-1} above and, like there, if the $SU(2)_L$-violating vevs are turned off, one sees that the corresponding field N_1 no longer mixes with the remaining states N_2, S_2. This implies that, similar to the case of E_1 above, the mixing between N_1 and both N_2, S_2 will be of order ~ 100 GeV$/ \sim 10$ TeV $\sim 10^{-2}$. We again note that due to the $SU(2)_{L,R}$ symmetries, the pairs of states N_1, E_1 and N_2, E_2 are degenerate prior to both their mixings with the other states as well as loop radiative corrections; this is also observed to be true for the states S_2, E_3 due to the extended $SU(3)_{L,R,'}$ symmetries. This implies that 2-body decays,

such as, e.g., $E_1 \to N_1 W_{\text{SM}}$, even if kinematically allowed, will more than likely be highly suppressed by the very small phase space availability; even less likely are any on-shell, 2-body decays via $W_R^\pm$ since these new gauge states are substantially more massive. The parallel with the $Q_{em} = -1$ states is further strengthened by the presence of the $Q_D \neq 0$ vevs that allow for both $N_{1,2}$ to directly mix in a chiral manner with ν, S_1 while S_2 mixes only indirectly with them and must first mix with $N_{1,2}$ beforehand. As in the $Q_{em} = -1$ case, we might expect this Q_D-violation-induced mixing to be $\sim 1 \text{ GeV}/ \sim 10 \text{ TeV} \sim 10^{-4} \sim \epsilon$, allowing for the presence of the dominant decay modes $(N_{1,2}, S_2) \to (\nu, S_1) + \text{DP}$, while the DP in most cases will likely appear simply as MET. As will be seen below, S_1 can be produced through the production and decay of U_L, the mixing of U_L with the SM W, the mixing of S_1 with the SM ν, or some combination of these mechanisms.

IV. NEUTRAL FERMION MAJORANA MASSES

As noted in the previous section, in the neutral fermion sector, all of the $Q_D = 0$ vevs produce only Dirac mass terms but two of the Q_D-violating vevs, $x_{2,4}$, which correspond to scalar fields of opposite Q_D charge from those sourcing the other x_i, can generate $\Delta L = 2$ Majorana mass terms. Since $H_{2,3}$ are the scalar analogs of l, l^c in that they transform under $3_c 3_L 3'3_R$ in exactly the same manner, we observe that the "central" neutral member of these multiplets are distinctive in this manner as can be seen from Table I. However, these vevs only result in the following rather simple pair of Majorana mass terms linking four of the five neutral fermion fields which can then be used to define the corresponding left- and right-handed Majorana mass matrices, $M_{L,R}$:

$$y_3 \left(\nu S_2 - N_1 S_1 \right) \frac{x_2}{\sqrt{2}} + y_4 \left(\nu^c S_2^c - N_1^c S_1^c \right) \frac{x_4}{\sqrt{2}} \to M_L + M_R,$$

$$(20)$$

which are in some sense, almost conjugates of each other; note that N_2 is not involved in this interaction. These matrices, when combined with M_0^D, will then form a more general 10×10 Majorana mass matrix. With the perhaps naive expectations that $x_2 \sim x_4$ and $y_3 \sim y_4$ we might expect then that $M_L \simeq M_R$. Needless to say, in comparison to most of the entries in the neutral Dirac mass matrix given above, these Majorana mass matrices are quite sparse, having nonzero elements that are all quite small, $\lesssim 1$ GeV, as the relevant vevs are both $U(1)_D$ violating as well as being lepton-number violating. As a result they will have little influence on the heavy neutral lepton states (except for the possibility of making them pseudo-Dirac with extremely small fractional mass splittings in some cases [55] as we encountered in our previous study based on the Pati-Salam setup [38]) and they are not capable of

explaining the observed light neutrino masses without further extensions of this setup as has been noted elsewhere in the quartification literature [52].

V. NON-HERMITIAN GAUGE BOSON MASSES AND MIXINGS

Apart from QCD, the gauge boson sector of the $3_c 3_L 3'3_R$ quartification setup is rather complex with a total of 24 gauge fields of which 6 are Hermitian while the remainder form nine pairs of complex, i.e., non-Hermitian fields including the SM $W_L^\pm$ as well as the corresponding $W_R^\pm$, familiar from the LRM. It should be noted that not all of these non-Hermitian fields carry electric charge, i.e., have $Q_{em} = \pm 1$. In this section, the masses of these non-Hermitian gauge bosons (NHGB) will be discussed and, as we will see, an additional Higgs scalar, Ω, beyond the three discussed above, the H_i, that are responsible for fermion mass generation, will need to be introduced to satisfy our model building assumptions. The corresponding discussion for the six Hermitian fields will be the subject of the next section. Except for rare circumstances and in the absence of any large mixing effects, the couplings of the NHGB to the various fermions will generally be chiral, a notable exception being, e.g., that of the SM[LRM] $W_L[W_R]$ to the $(N_1, E_1)^T[(N_2, E_2)^T]$ doublet which is vectorlike.

Before beginning, the first observation to make is that, apart from $W_L^\pm$, all of these nine NHGB need to be heavy, likely in excess of several TeV, to avoid the many LHC searches [56–58] and so must have their masses generated by combinations of the vevs $v_{3,5}, u_{2–4}$ above as well as possibly by others of a similar magnitude. A second observation is that the three NHGB arising from $SU(3)'$ all carry different values $Q_D \neq 0$ (as well as different values of Q_{em}) and so, in the limit that $U(1)_D$ remains unbroken, they will not mix with the remaining six fields or even with each other. A third observation is, given the definition of the electric charge in terms of the gauge group generators above, that the three pairs of NHGB associated with each of the $SU(3)_a$ gauge will consist of two pairs of fields with $Q_{em} = \pm 1$, which we will generically refer to as $W_a^\pm, U_a^\pm$, while the remaining pair of NHGB is electrically neutral, which we will refer to as $V_a^{0(\dagger)}$. These two observations, when combined, tell us that the full 9×9 mass matrix for the NHGB in the limit of $U(1)_D$ conservation will consist of three blocks: a diagonal 3×3 block for the three $Q_D \neq 0$, $SU(3)'$ fields, a 2×2 block for the $Q_D = 0$, neutral fields $V_{L,R}$, and the remaining 4×4 block for the electrically charged $Q_D = 0$ fields $W_{L,R}, U_{L,R}$, respectively. Lastly, it is useful to be reminded about what the roles are for the three classes of gauge bosons, W_a, U_a, and V_a, in the sense of which fermion fields, that here lie in **3**'s or $\bar{\mathbf{3}}$'s of $SU(3)_a$, are connected to each other by them. If we label the three triplet fermions a $(f_1, f_2, f_3)^T$, then W

connects $f_1 - f_2$ while V connects $f_1 - f_3$ and U connects $f_2 - f_3$. Thus, e.g., W_L connects (ν, e) and (d, u) in the familiar way, while V_L connects (ν, S_1) and (d, h) and U_L connects (e, S_1) and (u, h), respectively. In practice, amongst other things, this implies that W_a and U_a will have opposite values of Q_{em} so that, e.g., W_L mixes with $U_L^\dagger$ and not with U_L. Finally, with this notation, one finds that $Q_D(W', V', U') = (1, -1, -2)$ and, given that $Q_{em}(V') = 0$, we see that this NHGB plays essentially the same role (in the leptonic sector) as did the W_I gauge field in the $G_{\text{Dark}} = SU(2)_I \times U(1)_{Y_I}$ model encountered in several of our earlier works, while the W' acts in a similar manner being its electrically charged partner. The gauge field U', on the other hand, in the absence of any mixing, will only connect pairs of states with nonzero values of Q_D and so will not easily be produced or probed in a simple fashion, especially at a hadron collider while future searches in conventional W' channels at FCC-hh may reach as high as ~40 TeV [56,57,59,60].

It is interesting to note that since the W' and U' gauge bosons carry no-zero values of both Q_D and Q_{em}, they too can act as PM fields, something we had not previously encountered but is an obvious result of the embedding of $U(1)_D$ within $SU(3)'$ and that fact that Q_{em} also depends upon the diagonal $SU(3)'$ generators. Another interesting feature of these NHGB states is also to be observed: while $W_{L,R}$, $U_{L,R}$, and $V_{L,R}$ will clearly carry zero lepton number, $L = 0$, this is *not* generally true for the $SU(3)'$ NHGBs. In fact, one finds that W', U' carry $|L| = 2$ while V' remains an $L = 0$ state. We will return to this issue below when we consider the mixing induced by the small $x_{2,4} \neq 0$ vevs which we have seen above generate $\Delta L = 2$ Majorana mass terms among the new neutral fermions.

With this preparation we will now consider the 4×4 (M_{44}^2), 2×2 (M_{22}^2), and 3×3 (M_{33}^2) blocks of the full 9×9 NHGB mass-squared matrix that are obtained in the approximate limit that we can ignore the $\lesssim 1$ GeV, $Q_D \neq 0$ vevs; we will then return and discuss the perturbing effects that these additional vevs will have. As a first step, we will assume that the only Higgs vevs that are relevant are those that are employed above to generate the various fermion masses and whose properties are already given in Table I. In the $(W_L, U_L^\dagger, W_R, U_R^\dagger)$ basis,[4] we find that

$$M_{44}^2 = \frac{1}{4}\begin{pmatrix} g_L^2(v_1^2 + v_2^2 + v_4^2 + u_1^2) & g_L^2(v_1 v_5 + v_3 v_4 + u_1 u_2) & 2g_L g_R v_1 v_2 & 2g_L g_R v_2 v_4 \\ g_L^2(v_1 v_5 + v_3 v_4 + u_1 u_2) & g_L^2(v_3^2 + v_5^2 + u_2^2) & 2g_L g_R v_2 v_5 & 2g_L g_R v_2 v_3 \\ 2g_L g_R v_1 v_2 & 2g_L g_R v_2 v_5 & g_R^2(v_1^2 + v_2^2 + v_5^2) & g_R^2(v_1 v_4 + v_3 v_5 + u_3 u_4) \\ 2g_L g_R v_2 v_4 & 2g_L g_R v_2 v_3 & g_R^2(v_1 v_4 + v_3 v_5 + u_3 u_4) & g_R^2(v_3^2 + v_4^2 + u_4^2) \end{pmatrix},$$

(21)

whereas, in the (V_L, V_R) basis we obtain

$$M_{22}^2 = \frac{1}{4}\begin{pmatrix} g_L^2 X_L & 2g_L g_R(v_1 v_3 + v_4 v_5) \\ 2g_L g_R(v_1 v_3 + v_4 v_5) & g_R^2 X_R \end{pmatrix},$$

(22)

where $X_L = v_1^2 + v_3^2 + v_4^2 + v_5^2 + u_1^2 + u_2^2$ and $X_R = v_1^2 + v_3^2 + v_4^2 + v_5^2 + u_3^2 + u_4^2$. Note that $V_L - U_L$ have their masses split by electroweak scale vevs only as they form an $SU(2)_L$ doublet. Finally, in the $(W', U'^\dagger, V')$ basis we find

$$M_{33}^2 = \frac{g'^2}{4} X' \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

(23)

where $X' = u_1^2 + u_2^2 + u_3^2 + u_4^2$.

There are many things to observe about these results, beginning with the most obvious ones arising from M_{33}^2: U' is massless and W', V' are degenerate. The masslessness of U', alluded to above, is the result of the requirement of preserving $U(1)_{em}$ as well as $U(1)_D$ down to low mass scales so that the Higgs in the (anti)triplet representations only break $SU(3)'$ down to $SU(2)'$. Clearly U' must be a rather massive field with $M_{U'} \gtrsim$ a few TeV or so and thus we must employ an additional Higgs scalar with vevs beyond those found in the H_i which transforms differently under $SU(3)'$. The simplest possibility is to employ an $SU(3)'$ octet, i.e., Ω, that is a $(1, 1, 8, 1)$ representation under $3_c 3_L 3' 3_R$, which will not influence the fermion masses discussed above, will not contribute to the masses of the Hermitian gauge bosons that we will discuss in the next section, nor to the masses of the other NHGB which are all $SU(3)'$ singlets. Note that this additional scalar representation represents an explicit breaking of the apparent symmetry Z_4 among the gauge fields and fermions dictated by anomaly freedom and the various $[SU(3)]^4$ group factors but, as we have noted above, we have *not* assumed that such a symmetry exists as part of our present discussion. We will return to this issue later below. As is well known, the vevs of an $SU(3)$ octet acting on (anti) triplet representations can be expressed as

---

[4] Recall that, e.g., W_L and $U_L^\dagger$ carry the same electric charge.

$$\langle\Omega\rangle \sim \begin{pmatrix} w_1 & 0 & 0 \\ 0 & w_2 & 0 \\ 0 & 0 & w_3 \end{pmatrix}, \tag{24}$$

subject to the constraint that the sum of these vevs satisfies $\sum_i w_i = 0$ due to the tracelessness of the octet. This results in a shift, ΔM_{33}^2, in M_{33}^2 that is given by

$$\Delta M_{33}^2 = \frac{g'^2}{8} \begin{pmatrix} (w_2 - w_1)^2 & 0 & 0 \\ 0 & (w_3 - w_2)^2 & 0 \\ 0 & 0 & (w_3 - w_1)^2 \end{pmatrix}, \tag{25}$$

thus resolving our problem with the U' mass (under the assumption that the vevs $w_i \gtrsim$ a few TeV or so and that $w_3 \neq w_2$) and also simultaneously removes the $W' - V'$ mass degeneracy, not that this was a problem in any way. From this result it is easy to imagine that the U' may be the lightest of the three $SU(3)'$ NHGB.

Turning now to M_{22}^2, we see both diagonal entries involve the squares of large vevs whereas the off-diagonal terms also involve the weak-scale vevs, $v_{1,4}$, so that this matrix is diagonalized via the small angle

$$\tan 2\phi_{22} = \frac{2\kappa(v_1 v_3 + v_4 v_5)}{X_L - \kappa^2 X_R}, \tag{26}$$

where $\kappa = g_R/g_L \simeq 1$, and which we might thus expect to be of order $\sim 10^{-2}$ or so. Again, with $\kappa \simeq 1$, the masses resulting for both of the eigenstates, $V_{1,2}$, will clearly lie in the range of at least several TeV.

Lastly, M_{44}^2 is seen to be the most complex of these three sets of mass-squared blocks but one immediate observation is that the limit that the electroweak scale vevs are neglected, not only is SM W_L massless but its mixings with the remaining three states, who dominantly obtain their masses from the larger vevs, will all vanish. Looking a bit closer we see that whereas we might expect that the mixing of the W_L with $W_R, U_R^\dagger$ to be of order $\sim(100\,\text{GeV}/10\,\text{TeV})^2 \sim 10^{-4}$, its mixing with $U_L^\dagger$ is potentially much larger, of order $\sim 100\,\text{GeV}/10\,\text{TeV} \sim 10^{-2}$, which *could* lead to significant phenomenological consequences as it, e.g., "depletes" a small amount of the $\bar{u}d$-type coupling into a $\bar{u}h$-type coupling. However, since an *identical* effect occurs in the leptonic sector (since gauge boson mixing has nothing to do with the fermion sector at tree level) in that the $\bar{\nu}e$-type coupling is also diverted into an $\bar{S}_1 e$-type coupling, the effect of this tree-level $W_L - U_L^\dagger$ mixing may be more subtle than it first appears, requiring a consistent treatment since it is a "universal" effect experienced by both the quark and lepton sectors. In the other parts of this matrix, we see the $W_R - U_R^\dagger$ mixing can be $O(1)$ since it involves only the large vevs (*assuming that*

there is also no significant hierarchy amongst these large vevs which may or may not be the case depending upon how the guage symmetries are broken) while the mixing of these two fields with $U_L^\dagger$ is also seen to naively be relatively small, of $O(10^{-2})$. In a manner similar to $W_L - W_R$ mixing, we see that $U_L - U_R$ mixing may also roughly lie at the level of 10^{-2}.

Since $W_R - U_R^\dagger$ mixing is allowed to be significant $\sim O(1)$, they will likely both couple to the $\bar{u}d$ + H.c. initial state though the lighter of the two mass eigenstates, $\tilde{W}$, would have the largest production cross section but would now have two distinct leptonic decay modes. Since SM neutrinos are likely Dirac in the present setup, $\tilde{W}$ will decay into both the $e\nu \to e + \text{MET}$ as well as the $eS_1^c \to eejj$ final states with somewhat comparable branching fractions, allowing for phase space. This implies that a signal may be observable in more than one search mode [56,57].

It should be noted that whereas $W_L - U_L^\dagger$ mixing will result in a small downward shift in the SM W mass, the SM Z mass, as we will see below, is also pushed downward due to a similar mixing. However, we note that the combination of these two effects, at tree level, will most likely not provide any explanation for the apparent upward shift in the W mass, relative to that of the Z, as measured by CDFII [61].

Overall, we see that apart from the (approximate) SM W_L mass eigenstate, all of the NHGB masses will lie at the (at least) several TeV scale, but their specific mass spectrum is quite sensitive to the various potentially large mixings among these states as well as the specific ordering of the values of the multiple high-scale vevs $v_{3,5}, u_{2-4}$ as well as w_{1-3}.

Finally, we now discuss how this picture of the NHGB masses and mixings is altered when we turn on the small Q_D-violating vevs, x_i, appearing in the Higgs fields $H_{2,3}$ above. Clearly, some previously unmixed states with the same value of Q_{em} can now mix but the most important effect is most obviously the specific induced mixing between states of *different* Q_D that were forbidden to mix previously. Mixings can now also occur between states with differing lepton number as the $x_{2,4}$ violate this quantity as was noted earlier. Specifically, this also means that, e.g., there is now a small induced mixing of V' *and* $V'^\dagger$ with both $V_{L,R}$ and $V_{L,R}^\dagger$ (all of which mixings are lepton number conserving). There are also correspondingly small mixings of the W' with the fields $W_{L,R}$, etc., with specific sets of x_i participating, and, furthermore, there is now an induced mixing between the two $SU(3)'$ fields $W' - U'^\dagger$ at second order, $\sim x_i x_j$. We note that there is *no* mixing between the U' and either $W_{L,R}$ or $U_{L,R}$ to leading (i.e., linear) order in x_i as all of the vevs are $|\Delta Q_D| = 1$ and these states differ by two units of Q_D as well as by lepton number. However, still at leading order in the x_i, there is such mixing between the $U_{L,R}$ and the W'. One might

expect that, roughly speaking, the size of these typical mixings to be or order, e.g., $\phi_{W_{L,R}W'} \sim x_{2,4}/(v_{3,5}, u_{2-4}) \sim 10^{-(4-5)}$ or so. Phenomenologically, while these tiny mixings have very little effect on any of the NHGB masses, they will now allow (via the $x_{2,4}$ $\Delta L = 2$ vevs) for new decay paths, e.g., $W' \to W_{L,R} + D$ assuming that the W' is more massive than the W_R. As in the well-known example of PM decay induced by a similar type of mixing, the longitudinal coupling of the DP, i.e., D, (and also that of the SM/LRM $W_{L,R}$) is enhanced by a double ratio of large masses, $M_{W'}^2/(M_{W_{L,R}}M_D) \sim 10^4$ or so, which is likely sufficient to overcome the correspondingly expected tiny mixing angle suppression so that this may become one of (if not solely) the dominant W' decay mode(s); in this example, the corresponding (single) production signature at a collider would then be just $W + \text{MET}$ for which, e.g., LHC searches exist [62] assuming these W' states are kinematically accessible, which is one of the issues that we will return to below. Explicitly, to leading order in these large mass ratios, we obtain, e.g., the following expression this partial width:

$$\Gamma(W' \to W_L + D) \simeq \frac{\alpha_D M_{W'}}{48} \left[\phi_{W_L W'} \frac{M_{W'}^2}{M_{W_L} M_D} \right]^2, \quad (27)$$

where $\alpha_D = g_D^2/4\pi$ with g_D being the DP's $U(1)_D$ gauge coupling, $\sim g'$ as we will find in the next section below, and where we expect the expression in the square bracket to be $O(1)$.

While some of the NHGB may be easily produced by a hadron collider, this will not be true for all of them as a subset will only interact with $Q_D \neq 0$ sector fields and not directly with those of the SM except via mixing which leads us directly to the some of the discussions in later sections. However, the same mixing that allows for the $W' \to W_L + D$ decay process can also be used to singly produce a W' together with a DP at a hadron collider via, e.g., an off-shell transverse W_L exchange in the s-channel. In such a case, one would find that the cross section scales as the square of $\phi_{W_L W'} \frac{M_{W'}}{M_D}$, which is seen to be a factor of $\frac{M_{W_L}}{M_{W'}} \sim 10^{-2}$ smaller in amplitude than that appearing in the square bracket above and which was presumed to be $O(1)$. This would then imply that the cross section for this W' production process is relatively suppressed by a factor of roughly $\sim 10^{-4}$ and so would be "difficult" to observe at best unless the W' were not to bee too massive.

VI. HERMITIAN GAUGE BOSON MASSES AND MIXINGS

In some ways, the mass-squared matrix for the six Hermitian gauge bosons (HGB), i.e., W_{3a}, W_{8a}, $(a = L, R, ')$, is more complex than that encountered

above in the NHGB case although we have several guideposts thanks to the expected hierarchy of symmetry breaking scales. For example, in the limit that we can ignore the ~ 1 GeV, Q_D-violating vevs, this mass-squared matrix will have two zero eigenvalues corresponding to both the DP as well as the usual SM photon and, furthermore, in the limit of vanishing $SU(2)_L$-violating vevs, the SM Z must also be massless. Finding a convenient and useful basis for this matrix is, however, somewhat nontrivial and clearly the above set of HGB, W_{3a}, W_{8a}, is itself not a very useful choice in this regard. Given the various Higgs multiplets introduced above, we have sufficient freedom to generate all of the HGB masses as we will see below.

One obvious observation is that, due to the definition of Q_D and the fact that $U(1)_D$ remains unbroken down to low mass scales, we can decompose the $SU(3)'$ terms corresponding to the diagonal generators appearing in that part of the covariant derivative (dropping Lorentz indices) as the two orthogonal combinations[5]

$$g'(T_3' W_3' + T_8' W_8') = g' \frac{\sqrt{3}}{2} [T_3' + T_8'/\sqrt{3}] W_+' $$
$$+ \frac{g'}{2} [T_3' - \sqrt{3} T_8'] \tilde{D}, \quad (28)$$

with the definitions

$$W_+' = \frac{\sqrt{3}}{2} [W_3' + W_8'/\sqrt{3}], \quad \tilde{D} = \frac{1}{2} [W_3' - \sqrt{3} W_8']. \quad (29)$$

Here we recognize the expression in the square bracket associated with gauge field $\tilde{D}$ as just equal to Q_D (up to a possible sign) such that *if* $\tilde{D}$ were to be identified with the DP, D, with this chosen normalization the usual $U(1)_D$ gauge coupling must then be just $g_D = g'/2 = g' s_I$. In the limit where we can neglect the contributions arising from the $Q_D \neq 0$ vevs, x_i, as a good approximation, we can simply omit the appearance of this DP term in the covariant derivative when constructing the HGB mass-squared matrix which, effectively, now becomes only 5×5 since $\tilde{D}$ neither has a mass or any off-diagonal mixing terms with the other HGB states. When combined with the quantum numbers of the various Higgs fields in Table I, this result motivates us to consider the following useful (but not always physically intuitive) basis for these neutral gauge fields:

$$A_a = g_a(W_{3a} + W_{8a}/\sqrt{3}), \qquad B_a = g_a \frac{2}{\sqrt{3}} W_{8a}, \quad (30)$$

---

[5]This normalization has been chosen so that, as usual, $\text{Tr}[G_i]^2 = 1/2$, where the G_i are the combinations of the $SU(3)'$ generators appearing in the brackets, obtained for either the fundamental **3** or antifundamental **3̄** representation.

with, as usual, $a = L, R, '$. It is important to note that the gauge couplings have been absorbed into these definitions; note also that $A' \sim W'_+$ as defined in the equation above, apart from a normalization factor. Interestingly, in this basis, instead of the $SU(3)'$ part of the covariant derivative decomposition above, we find that, e.g.,

$$g'(T'_3 W'_3 + T'_8 W'_8) = T'_3 A' - \frac{1}{2}[T'_3 - \sqrt{3}T'_8]B', \quad (31)$$

and similarly for $a = L, R$. Note that, due to the possible Q_D sign assignment ambiguity, apart from an absorbed gauge coupling, here B' might *alternatively* be identified as the DP, D, again with $g_D = g'/2$ as before.[6] More generally, we can define the B_a fields as coupling to the generalized charges Q_a with $Q' = -Q_D$. In this basis, neglecting the $Q_D \neq 0$ vevs, x_i, for the moment as we did above, the B' field decouples and remains massless and so the general 6×6 HGB mass squared matrix again reduces to one which is 5×5 by the earlier argument.

In this A_L, A_R, B_L, B_R, A' basis, we can write this truncated (the B' or DP now being omitted) 5×5 HGB mass squared matrix as

$$M^2_{HGB-5} = \frac{1}{4} \begin{pmatrix} v_1^2 + v_2^2 + v_4^2 + u_1^2 & -v_1^2 - v_2^2 & -v_2^2 & v_2^2 + v_4^2 & -u_1^2 \\ -v_1^2 - v_2^2 & v_1^2 + v_2^2 + v_5^2 + u_3^2 & v_2^2 + v_5^2 & -v_2^2 & -u_3^2 \\ -v_2^2 & v_2^2 + v_5^2 & v_2^2 + v_3^2 + v_5^2 + u_2^2 & -v_2^2 - v_3^2 & u_2^2 \\ v_2^2 + v_4^2 & -v_2^2 & -v_2^2 - v_3^2 & v_2^2 + v_3^2 + v_4^2 + u_4^2 & u_4^2 \\ -u_1^2 & -u_3^2 & u_2^2 & u_4^2 & U \end{pmatrix}, \quad (32)$$

where we have defined the quantity $U = \sum_i u_i^2$. It is important to remember when viewing this matrix that the gauge couplings do not appear as they have been absorbed into the definitions of the set of the gauge fields, A_a, B_a. Here we observe that the first row and column consists solely of combinations of the $SU(2)_L$-breaking vevs so that, in the limit where the squares of the electroweak-scale breaking can be neglected and only the mult-TeV scale vevs are relevant, the A_L gauge boson will decouple as "massless." In such a case, this matrix can effectively be further truncated to the much simpler, lower right-hand, 4×4 block with the vevs $v_{1,2,4}, u_1$ all set to zero in this approximation. Explicitly, this matrix is found to be given as

$$M^2_{HGB-4} \simeq \frac{1}{4} \begin{pmatrix} v_5^2 + u_3^2 & v_5^2 & 0 & -u_3^2 \\ v_5^2 & v_3^2 + v_5^2 + u_2^2 & -v_3^2 & u_2^2 \\ 0 & -v_3^2 & v_3^2 + u_4^2 & u_4^2 \\ -u_3^2 & u_2^2 & u_4^2 & u_2^2 + u_3^2 + u_4^2 \end{pmatrix}. \quad (33)$$

In this same limit, we should expect this truncated 4×4 mass-squared matrix to have one null eigenvalue whose associated field, which well call $\mathcal{H}$, which together with A_L, will form the familiar W_3, B_Y ones of the SM. In fact, not surprisingly, it is easily seen that in this truncated basis (and here taking $g_L = g_R = g' = g$ as is frequently done in the quartification literature [52] for clarity of presentation except where noted), that $\mathcal{H} \sim (1, -1, -1, 1)^T$, i.e., the combination of fields $\sim (A_R - B_L - B_R + A')$, forms the null eigenvector corresponding to a massless HGB state.[7]

Interestingly, if we return to the 5×5 matrix above, also including the additional electroweak scale vevs, then in this more general case we would, perhaps unsurprisingly, find that the corresponding null eigenvector is instead $\sim (1, 1, -1, -1, 1)^T$, on its way to being identified with the photon given the definition of Q_{em}. Returning now to the 4×4 case at hand, we note that, since all these vevs are large, we might expect that these four states, $A_R, B_{L,R}$, and A', will in general mix together rather strongly unless the vevs have some sort of associated hierarchy about which we have no *a priori* knowledge; this implies that general expression for all the mass eigenvalues and eigenstates in terms of these vevs would be hardly enlightening.

One possibility is to make a choice of basis based on familiar dynamics which suggests that an examination of these HGB states in a phenomenological basis. Since, from

---

[6]One could instead just redefine $D = -B'$ and ignore this sign issue altogether.

[7]Importantly, we recall that each of the remaining three mass eigenstates/eigenvectors must be orthogonal to this state as well as to each other.

the large vev perspective, three of these HGB are clearly either massless or close to being massless eigenstates to a fairly good approximation, one might consider employing, e.g., $\gamma, D, Z_{SM}, Z_R, \tilde{B}, A'$, where Z_R is the familiar and well-studied Z' of the LRM and $\tilde{B} \sim B_L - B_R \sim g_L W_{8L} - g_R W_{8R}$. On the other hand, it is may be more useful to explore the nature of this matrix under variously motivated assumptions about the vevs themselves, e.g., one might expect that the product $3_L 3_R$ breaks at a very high scale to $2_L 2_R 1_L 1_R \to 2_L 2_R 1_{L+R}$, similar in nature to the LRM, although a piece of both A', B' will also be necessary to form the familiar 1_{B-L} gauge group factor as can be seen from the definition of Q_{em} in terms of the set of diagonal generators. Note that $\tilde{B}$ is just the orthogonal combination to that which appears in the gauge field for $U(1)_{B-L}$, and will be discussed in more detail below. We recall that in the well-known trinification model [51] limit this field is seen to be just the result of the reorganization and/or diagonal breaking of the product $1_L 1_R \to 1_{B-L} 1_{\tilde{B}}$.

The field $\tilde{B}$, in fact, provides an excellent (but simplified) case in point for both studying the influence of vev hierarchies as well as a phenomenological motivation for choosing a particular HGB mass eigenstate basis as we see that the large vev, v_3, preserves, e.g., in the generalized charge notation above, the combination $Q_L/2 + Q_R/2$ but breaks the difference $Q_L/2 - Q_R/2$. To clarify for demonstration purposes, consider for simplicity rewriting only the relevant pieces in the covariant derivative and neglecting the contributions from the other gauge fields (while again dropping Lorentz indices and also restoring the different gauge couplings here for more clarity, i.e., $A_a \to g_a \tilde{A}_a$, etc.) as

$$g_L(T_{3L}W_{3L} + T_{8L}W_{8L}) + (L \to R)$$
$$= g_L T_{3L} \tilde{A}_L + g_R T_{3R} \tilde{A}_{3R} - g_L \frac{Q_L}{2} \tilde{B}_L - g_R \frac{Q_R}{2} \tilde{B}_R, \quad (34)$$

and we now recall that the vev v_3 has $Q_L = -Q_R$ from Table I. For further purposes of demonstration, let us concentrate on the second pair of terms and imagine in this example that we live in a model realization where this vev, v_3, is significantly greater than all of the other large vevs so that we can neglect their effects. Then we see that one combination of the $\tilde{B}_{L,R}$, i.e., $B_1 = c\tilde{B}_L - s\tilde{B}_R$, will get a mass whereas the second, $B_2 = c\tilde{B}_R + s\tilde{B}_L$, where $(s,c) = (g_R, g_L)/\sqrt{g_L^2 + g_R^2}$, remains massless. Rewriting the covariant derivative terms, we see that these two mass eigenstates will couple as (recalling that $\kappa = g_R/g_L \simeq 1$)

$$\frac{g_L}{\sqrt{1+\kappa^2}} \left[\frac{Q_L}{2} - \kappa^2 \frac{Q_R}{2}\right] B_1 + \frac{g_L \kappa}{\sqrt{1+\kappa^2}} \left[\frac{Q_L}{2} + \frac{Q_R}{2}\right] B_2, \quad (35)$$

where, restoring all the constants, we can now identify the previously mentioned field, $\tilde{B} = B_L - B_R \sim B_1$, and the massless B_2 field will subsequently become (part of) the usual LRM/SM $B - L$/hypercharge gauge boson after further symmetry breaking and mixing.

It thus appears attractive to examine the matrix M_{HGB-4}^2 in some more detail when certain vevs, or sets of them, become large relative to the others forcing certain symmetries to be broken while others respected. Of course, we cannot treat any given scenario too seriously but they can provide us with guideposts for the many scenarios that one might imagine. As was noted earlier, given the set of vevs in the Table I above it is impossible to break $SU(3)'$ without also breaking $SU(3)_R$ since the set of vevs u_{2-4} all carry $SU(3)_R$ quantum numbers; the reverse, however, is not true as the vevs $v_{3,5}$ do *not* carry $SU(3)'$ quantum numbers. Let us now turn to a specific example of this approach which presents a reasonable setup, which we will call scenario I, generalizing on the discussion above. We have already seen that a (very) large v_3 will break $3_L 3_R \to 2_L 2_R 1_L 1_R \to 2_L 2_R 1_{L+R}$; we also simultaneously observe that if $u = u_2 = u_4$ is also very large it will act somewhat similarly while simultaneously breaking $SU(3)'$ but will also leave $SU(2)_R$ intact. This being the case, let us consider the limit where $v_3^2, u^2 \gg v_5^2, u_3^2$ [but still neglecting the $SU(2)_L$-breaking vevs] as a working scenario; then one finds (in this approximation), in addition to the massless field described above, two (very) heavy fields: $\tilde{B}$, also known as, Z_1, corresponding to the eigenvector $\sim (0,1,-1,0)^T$ as we might have expected, with $M_{\tilde{Z}_1}^2 = g^2(2v_3^2 + u^2)/4$, and also $Z_2 \sim (0,1,1,2)^T$, with $M_{Z_2}^2 = 3g^2 u^2/4$. Finally, there is also a somewhat lighter field (though being still quite heavy in comparison to the electroweak scale), $Z_3 \sim (3,1,1,-1)^T$, with $M_{Z_3}^2 = g^2(v_5^2 + u_3^2)/3$. Note that none of the Z_i appear very much like either Z_R, or A' due to the rather large amount of mixing that one finds in this scenario I. This is, perhaps, not overly surprising as such large mixings between Z_R and the $SU(2)_I$'s heavy partner of the DP, the A' analog in that analysis there termed Z_I, was also found to occur in earlier work [38].

To demonstrate the efficacy/transparency of taking the $g_L = g_R = g'$ limit in obtaining the results above, we note that if we had *not* assumed the equality of the gauge couplings, then the 4×4 eigenvector for the massless HGB field would instead be $\sim (1, -g_R/g_L, -1, g_R/g')^T$ and the eigenvector for the gauge field $\tilde{B}$ would now be $\sim (0, 1, \gamma, \delta)^T$ where we have defined

$$\gamma = -\frac{g_R}{g_L} + \frac{g_R}{g'}\delta, \qquad \delta = \frac{g'(\lambda - c)}{g_L g'^2 u^2 - g_L g_R^2 v_3^2},$$
$$c = (g_L^2 + g_R^2)v_3^2 + g_L^2 u^2, \qquad d = (g_R^2 + 2g'^2)u^2, \quad (36)$$

and

$$\lambda = \frac{1}{2}(c+d) + \frac{1}{2}[(c-d)^2 + T]^{1/2}, \qquad (37)$$

with $T = (g_L^2 - g_R^2)(g'^2 u^2 - g_R^2 v_3^2)$ and the corresponding mass-squared eigenvalue would instead have been $M_{\tilde{Z}_1}^2 = \lambda/4$, none of which is very transparent or overly useful other than observing that the results are g' independent. We clearly see, however, that as $g_R \to g_L$, i.e., $\kappa \to 1$, one finds that both T, $\delta \to 0$ so that $\lambda \to c$ and $\gamma \to -1$ and we staightforwardly recover the previously obtained simplified result.

Due to their interplay in the phenomenology, it is interesting to consider the corresponding set of masses for the NHGB in this same scenario I under the $g_L = g_R = g'$ simplifying assumption employing the set of expressions above and in which limit mixing between these states can be approximately ignored. Here we see in this approximation that W_R is very likely to be the lightest amongst these states with $M_{W_R}^2 = g^2 v_5^2/4$, while for $U_{L,R}$ one finds $M_{U_{L,R}}^2 = g^2(u^2 + v_3^2 + v_5^2)/4$, $g^2(u^2 + v_3^2)/4$, respectively, and for $V_{L,R}$, $M_{V_{L,R}}^2 = g^2(u^2 + v_3^2 + v_5^2)/4, g^2(u^2 + u_3^2 + v_3^2 + v_5^2)/4$, respectively. We also see that for $O(1)$ Yukawa couplings, the various new fermions will have masses interspersed in this same mass range. Also we note the equality of the U_L and V_L masses due to the (at this point) unbroken $SU(2)_L$ gauge symmetry. Furthermore, before turning on any of the w_i vevs, one finds that $M_{W',V'}^2 = g^2(2u^2 + u_3^2)/4$ and, obviously, the contributions for the w_i will only make these states heavier. Of course, the U' mass is set only by the values of the w_i themselves.

By way of contrast, we now consider a second scenario (II) where we imagine a common scale for the large vevs in H_1, i.e., $v_3 = v_5 = v$, which do not break $SU(3)'$, and a similar common scale for the large vevs in $H_{2,3}$ that do, i.e., $u_{2-4} = u$, but, to be general, with no specific ordering between v and u. In such a case, we find instead the following mass eigenstates, still assuming a common gauge coupling for simplicity: $Z_1 \sim (1, -1, -1, -3)^T$ with a mass $M_{Z_1}^2 = g^2 u^2$, $Z_2 \sim (1, 0, 1, 0)^T$ with mass $M_{Z_2}^2 = g^2(v^2 + u^2)/4$, and $Z_3 \sim (1, 2, -1, 0)^T$ with $M_{Z_3}^2 = g^2(3v^2 + u^2)/4$; note that, necessarily, $M_{Z_3}^2 > M_{Z_2}^2$ while the mass ordering of Z_1 and Z_3 depends on whether u or v is the larger. Again, we see that, in this case, all three of the very heavy states are quite highly mixed with none of them appearing to be much like Z_R, A' or even $\tilde{B}$ and this result is also observed to be insensitive to any further assumptions with respect to possible hierarchies between v^2 and u^2. In the corresponding NHGB sector, ignoring mixing effects, the W_R again appears as the lightest of these new gauge boson states with $M_{W_R}^2 = g^2 v^2/4$, while for

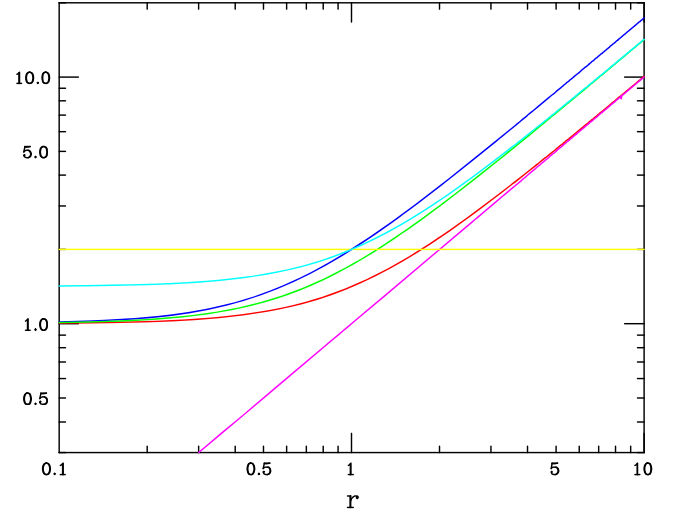
FIG. 1. Sample mass spectrum of the various NHGB and HGB for scenario II in units of $M_0 = gu/2$ as a function of the vev ratio $r = v/u$, as discussed in the text. For values of r slightly above unity, the curves, from top to bottom, correspond to the masses of the Z_3 (blue), V_R (cyan), $U_L = V_L$ (green), Z_1 (yellow), $U_R = Z_2$ (red), and W_R (magenta), respectively.

$U_{L,R}$ one finds $M_{U_{L,R}}^2 = g^2(u^2 + 2v^2)/4, g^2(u^2 + v^2)/4$, respectively, and now for $V_{L,R}$, one obtains instead $M_{V_{L,R}}^2 = g^2(u^2 + 2v^2)/4, g^2(2u^2 + 2v^2)/4$, respectively. We again note the equality of the V_L and U_L masses in the limit that $SU(2)_L$ remains unbroken and that the W_R tends to be the lightest state. Furthermore, before turning on any of the w_i vevs, in this case one finds that $M_{W',V'}^2 = 3g^2 u^2/4$ with the contributions for the w_i further increasing these masses while also providing one for the U'. From these considerations (although only based on our experience with this pair of specific breaking scenarios) it is clear that the HGB and NHGB will always have overlapping yet hierarchal mass spectra. Though quantitatively different from scenario I, scenario II possesses essentially the same qualitative features although, apart from the overall scale, the mass spectrum in this case depends upon a single parameter, the vev ratio $r = v/u$, as can be seen in Fig. 1. Of course, even in this simplified case, we find that mass relations can be somewhat complex due to this single parameter sensitivity especially when $r \sim 1$ although this situation does simplify if either the $r \ll 1$ or $r \ll 1$ limits are realized, resulting in some further degeneracy among these new gauge boson states.

Of course, many other orderings of the large scale vevs are possible and will correspond to differing symmetry breaking patterns.

In either of the scenarios above, and more generally, once the electroweak scale vevs, $v_{1,2,4}, u_1$, are turned on, the two, non-DP, massless states $\mathcal{H}$ and A_L of the 5×5 mass squared matrix will mix to form the familiar SM fields, γ and Z_{SM}, as usual and we see that the amount of mass

mixing between the Z and the other more massive Z_i, which generically live at the few TeV scale and above, will always be suppressed by mixing angles which are or order $\sim M^2_{Z_{SM}}/M^2_{Z_i} \sim 10^{-4}$, consistent with any requirements from the (tree-level) electroweak constraints [63]. The size of the different $Z_{SM} - Z_i$ mixings will also be somewhat sensitive to the relative magnitudes of the set of $v_{1,2,4}, u_1$ vevs. From the structure of the 5×5 mass-squared matrix above we see that these same vevs will also induce very small corrections to the Z_i masses and the mixings among these HGB beyond those which we have already encountered but which we can safely ignore numerically amongst just these heavy states. Also, as noted, once the $SU(2)_L$ gauge symmetry is broken, the degeneracy among some of the NHGB states, e.g., U_L, V_L, will be lifted although this mass splitting will remain relatively small.

Apart from these specific spectrum scenarios, some further intricacies are introduced once the $Q_D \neq 0$ vevs turn on, as they produce the DP mass itself and also induce the usual mass mixings between the DP and all of the other more massive HGB states, Z_{SM} and Z_i, as was seen in the corresponding case of the NHGB discussed earlier. Since these mass mixings are generally of order $M^2_D/M^2_{Z_{SM,i}}$, we see that the dominant one (via the $x_{1,2}$ vevs) is that with Z_{SM} and thus the DP essentially picks up a small Z_{SM}-like coupling to the SM fermions, which is a familiar and frequently occurring feature of many DP models. As usual, due to the small magnitudes of these vevs, these newly induced mass mixings will generally have very little influence on the masses and couplings of the heavier HGB eigenstates themselves as already discussed.

One other effect of these $Q_D \neq 0$ vevs, which has also been observed in our earlier work, is to generate a small mixing between the HGB Z_{SM}, Z_i states (which all have $Q_D = 0$) and the electrically neutral NHGB $|Q_D| = 1$ states via the Hermitian structure $\sim Z_{SM}(V' + V'^\dagger)$, which, e.g., allows them to have new decay paths such as $V' \rightarrow Z_{SM}D$, somewhat analogous to the $W' \rightarrow W_L D$ decay previously discussed. It is to be noted that this particular coupling preserves lepton number as V' has an $L = 0$ assignment. Overall, the lepton-number violating vevs $x_{2,4}$ have far less direct impact in the HGB sector than in the NHGB one since here such vevs can only appear quadratically (instead of linearly in lowest order in the NHGB case) and they are already relatively small, $\lesssim 1$ GeV.

As a last point of discussion before ending this section, we can easily see that there are several reasons why we should already suspect that the current model, as so far described, remains incomplete; one might strongly suspect that this is at least partially due to the lack of a fully unified setup. First, we have already seen the necessity of introducing the additional Higgs scalar field, Ω, beyond those required to generate the corresponding fermion masses, to complete the corresponding mass generation for the NHGB

gauge fields. This explicitly breaks the apparent Z_4 symmetry that this model would otherwise possess. Second, we have not yet discussed the identity of the DM field itself which, at the very least, must carry $Q_D \neq 0$ while also being an $3_c 3_L 3_R$ singlet with $Q_{em} = 0$, properties not possessed by any of the fermion fields or among the many scalars in any of the H_i introduced above. Indeed, as mentioned previously, the CMB constraints on annihilating DM are most easily satisfied when the DM is a p-wave annihilating complex scalar, ϕ, which does *not* get a vev so as to maintain its stability. Since all of the diagonal members of the Ω Higgs field are seen to obtain vevs to complete the NHGB mass generation process as just discussed, only the off-diagonal field, Ω_{13}, could potentially be a DM candidate (albeit with significant fine-tuning), with the alternative being to introduce an additional, vev-less, scalar representation with all of the necessary properties. Finally, since the gauge and fermion fields in this setup are completely fixed and the Higgs scalar fields necessary to break the gauge symmetries and generate the relevant fermions masses have all been introduced, one might ask whether or not the KM parameter ϵ, as was defined above in terms of the field content of the model, is finite or not, i.e., is Eq. (2) satisfied automatically? Certainly this is *not* the case given the field content of the single fermion generation, $n_g = 1$, toy model described above as can be easily seen. While q, q^c, and H_1 make no contribution here as they all have $Q_D = 0$, both l and l^c, being color-singlet chiral fermions, will each contribute a factor of 3. Correspondingly, the fields $H_{2,3}$, being complex scalars, will each make an additional contribution of $3/2$ to this sum, assuming that the relevant fields are physical and do not become Goldstone bosons. The scalar Higgs field, Ω, on the other hand, also yields a further donation of $3/2$ to the sum (again assuming that all of the contributing scalar fields remain physical after SSB) and so the only remaining possible addition to the total, as has been mentioned above, will then arise from the U' and W' NHGB as they carry both Q_D and $Q_{em} \neq 0$. To determine their contributions, we follow the work in Ref. [64] which determines that the value of η_i for a massive gauge boson with $g = 2$ at the tree level, as is the case in renormalizable theories, i.e., $\eta_i = -10$. Hence, these two gauge bosons will then yield a total contribution to the sum of -30. However, we must subtract from this the contribution that arises from the relevant eaten Goldstone bosons which we have previously included as part of this sum, an amount equivalent to that obtained from Ω. In any case, it is clear from this analysis that the total sum appearing in Eq. (2) is *not* identically zero (and is, in fact, found to be equal to -21) implying that ϵ is not finite in this setup without the presence of some (to avoid issues with any gauge anomalies) large number of additional scalar fields. For the general case of an arbitrary number of fermion generations, assuming the same scalar content as

above, we find that this sum is now given by $6n_g + 3 - 30$ so that for the realistic case of $n_g = 3$, we still do not obtain a null result although we are appreciably closer to our goal. Here too, a (now somewhat smaller) number of additional scalar fields in the correct representations that are $3_c3_L3_R$ singlets could render ϵ finite.

VII. PHENOMENOLOGICAL DISCUSSION

The complexities of the current setup are many, mostly due to the signifciant parameter freedom controlling the mass spectrum thus preventing the details of its phenomenology from being easily explorable in all generality. However, as we have seen, specific scenarios with a fixed set of assumptions are clearly much more amenable to such analyses. If anything, this setup is awash in new particles at least some of which may be produced at the LHC and/or FCC-hh as well as at future multi-TeV lepton colliders. It predicts eight new NHGB and four new HGB plus a host of vectorlike fermions, i.e., one new quark, three new charged leptons, four new neutral leptons in addition to a right-handed neutrino for each generation as well over a dozen new neutral and charged Higgs scalars. Here we limit our discussion to only a small subset of the new gauge bosons and fermions that we have encountered and which have somewhat complex interactions with the conventional SM fields due to their unusual quantum number assignments. While much of this phenomenology will certainly depend upon the details of both the mass hierarchy and the mixings among the various new states, in some cases a few model-independent conclusions can be drawn. Given the complexity of the model, in many (if not most) cases, it is also impossible to separate out the consequences of existence of these new gauge bosons from those of the new fermions. This is especially true in the limit that the mixings effects induced by the $Q_D \neq 0$ vevs in the various production processes are neglected since many of these new fields connect SM fields to the new ones and their relative locations in the mass spectrum are extremely flexible. There is not much new to say in the present context regarding the "conventional" $SU(2)_R$ LRM gauge bosons, W_R, Z_R, where these particles are close to mass eigenstates except for possible purposes of comparisons,as they has already been the subject of many analyses [56,57,65] which, at least semiquantatively, will still apply to the present situation. Similarly, there is not much new to say about the new isosinglet vectorlike quark, h, as it too is rather conventional in that it is by now a well-studied $Q_D = 0$ state [53], with the only exceptions being when it participates in the production processes for *other* new heavy states or it becomes sufficiently heavy to have on-shell, 2-body decays into these new states, e.g., $h \to uW_R^-$. These however, will in many cases simply mimic some of the more familiar SM modes such as $h \to uW_{SM}$. The production of some of the other new states will also mimic familiar production modes that have been well studied.

For example, as was noted above, the $eS_1^{(c)}$ final state can be produced either via $\nu - S_1$ and/or $W - U^\dagger$-type mixings from initial state quarks, i.e., $\bar{u}d + \text{H.c.}$, similarly to a conventional heavy neutrino [56,57]. Likewise, these same mixings will allow for familiar decays such as, e.g., $S_1 \to eW, W \to jj$. The production of and signatures for the new PM leptons, E_i, N_i, are also already familiar from earlier work, being dominantly pair produced at a hadron/lepton collider via the SM/LRM electroweak gauge boson interactions and decaying back into the corresponding SM field plus a DP as was noted above. As noted, these, like h, can also act as "intermediaries" or important components of other more interesting interactions.

It is to be noted, given the discussion of the HGB in the previous section, that it is not always obvious what is the best basis for making phenomenological predictions involving these states due to the rather large amount of mixing involved among the heavier ones (i.e., those outside of γ, D, and Z_{SM}) and one is forced to consider the various specific symmetry breaking scenarios such as I and II above for overall guidance if one wants to make precise predictions. More globally we simply just refer to these three heavy HGB states collectively as Z_i (as above) and they will share many common features at the semiquantitative level outside of certain corners of parameter space. The reason for this is that, fortunately, as we have seen above, the Z_i are all sufficiently well mixed implying, e.g., that they all will have some reasonable couplings to the SM quarks, via the $T_{3L,3R}$ and $Y_{L,R}$ generators. This implies that they may be made resonantly at a hadron collider and decay into the familiar dilepton final state as is traditionally employed for LHC Z' searches provided that they are kinematically accessible. For example, in the case of scenario I, we see that Z_1 is fairly typical in that its (making the standard assumption that only SM final states are kinematically allowed) width to mass ratio is $\simeq 0.017$ and its leptonic branching fraction is $\simeq 0.083$ when $\kappa = 1$, values which are not unusual for a new Z' [45,66–68].

Not surprisingly, the current 13 TeV LHC constraints from ATLAS [69] on the, e.g., scenario I Z_i's are not too dissimilar from those on the SSM standard reference model, Z'_{SSM}, of $\simeq 5.1$ TeV. Following this analysis we obtain a lower bound of roughly $\simeq 4.33$ (4.28, 4.72) TeV on the masses of $Z_1(Z_2, Z_3)$, respectively, assuming only decays to SM fields in the final state and making the $g_L = g_R = g'$ assumption as employed in the previous section as can be seen in Fig. 2. Note that the reaches for the Z_i are somewhat suppressed relative to the usual SSM guidepost at least partially due either the absence or suppression of their couplings to up-type quarks. In all cases, however, these reaches may improve by up to $\sim 20\%$–25% at the HL-LHC given the significantly greater integrated luminosity and the slightly higher value of $\sqrt{s}$. For the 100 TeV FCC-hh, employing the same integrated luminosity (30 ab^{-1}) and search criteria as in Refs. [60,70], we find the corresponding
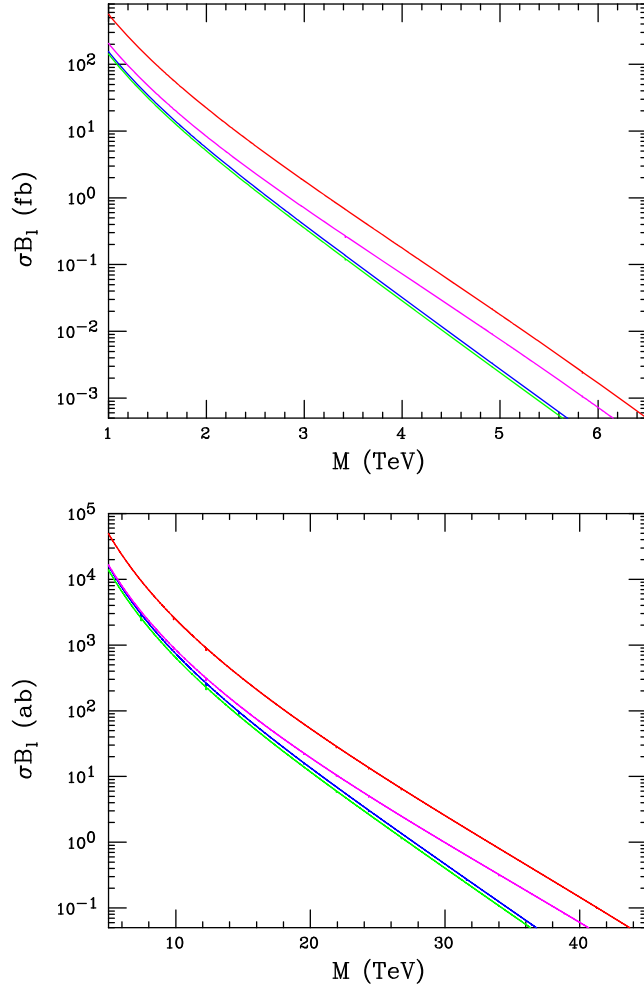
FIG. 2.   Top: production cross section times leptonic branching fraction for the three Z_i NHGB in scenario I in comparison to Z_{SSM} at the 13 TeV LHC as a function of their mass under the assumptions as discussed in the text. From top to bottom the curves correspond to Z_{SSM}, Z_3, Z_1, and Z_2, respectively. Bottom: same as the top panel but now for the 100 TeV FCC-hh.

reaches of roughly $\simeq 35.2$ (34.8, 38.9) TeV for the $Z_{1,2,3}$, which are comparable to that of the SSM Z' result of 42 TeV. Of course, if the Z_i have additional important non-SM decay modes these search reaches will be somewhat degraded by the lower leptonic branching fraction but this possibility also opens up new paths for discovery involving signatures produced by some subset of the other new fields that needed to be introduced in this setup. The HGB in scenario II will semiquantitatively behave quite similarly.

While these new heavy HGB can be treated reasonably symmetrically, the situation with the NHGB will be somewhat different as generally the mixing amongst most of these states is rather small. The NHGB will clearly fall into two main categories: those that are part of the $SU(3)'$ gauge group and those that are not due to the possible couplings of the SM fields to others that have nonzero values of Q_D (in the absence of the $Q_D \neq 0$ vevs). As will be noted,

we will omit discussions of the production and signatures of all such states that which are purely of the canonical heavy-W-like dilepton pair type.

Beginning with these NHGB, the states $U_{L,R}$ and $V_{L,R}$ are of immediate interest as, in the absence of mixing with the $SU(3)'$ NHGB, these are the only new ones (apart from W_R) that will couple directly to the SM quarks and so are more readily made at hadron colliders such as the LHC and FCC-hh. There are two conventional mechanisms by which such states may be produced in the absence of significant mixing with the other NHGB states (e.g., W_R in the case of U_R): associated production together with the heavy h vectorlike quark or via pair production. Note that in the absence of large mixing these NHGB cannot be produced singly (on their own) as a resonance as can, e.g., the SM W or the W_R in the LRM, since they couple to $\bar{u}h$ and $\bar{d}h$ (and *not* to $\bar{u}d$), respectively. Unlike in previous studies, where the analog of h carried $Q_D \neq 0$, here $h - d$-mixing induced processes such as, e.g., $V_{L,R,'}D$ production cannot occur in this setup. In the case of associated production, which is the least model dependent since it (roughly speaking) only depends upon the particle masses, $U_{L,R}$ ($V_{L,R}$) can be made via gluon-quark fusion in the initial state, i.e., $gu(d) \rightarrow hU_{L,R}(V_{L,R}) + $H.c. and, as might be expected, is only limited by the available collider phase space. Figures 3 and 4 show the production cross sections for these processes at the 13 TeV LHC and 100 TeV FCC-hh, respectively, for different values of m_h, as functions of the $U_{L,R}$ and $V_{L,R}$ masses assuming for simplicity that $\kappa = g_R/g_L = 1$. The analogous process, $gd \rightarrow hD$, does not occur in the present setup since here h has $Q_D = 0$.

The corresponding signatures for these production processes will depend, to some extent, upon the ordering of the h mass (and whose decays are well known) relative to the $U_{L,R}, V_{L,R}$ NHBG masses as well as on the masses of leptonlike states S_i, E_i, and N_i into which the U's and V's might also pair-wise decay to (approximately) conserve Q_D. For $U_{L(R)}$, the simplest final state will likely be that of $eS_1^{(c)}$ with a clean, very high p_T charged lepton, while for the case of $V_{L(R)}$, with $S_1^{(c)} \rightarrow \nu^{(c)}$, MET will likely be the most important part of the signature depending, of course, on how the $S_1^{(c)}$ itself decays.

The second process, $U_{L,R}, V_{L,R}$ NHGB pair production, is the result of s-channel exchange (by at least some) of the set of HGBs as well as the t-channel (u-channel) exchange of the h quark, required to maintain unitarity, and is much more highly dependent upon the details of the model than is associated production. Of course in the original weak basis, the only contributing s-channel exchanges for $U_{L(R)}, V_{L(R)}$ pair production are the corresponding $W_{3L(R)}, W_{8L(R)}$, but in the mass eigenstate basis the situation is much more complex due to the rather nontrivial mixing among the Z_i as described above. For example, for the case of $U_{L(R)}U_{L(R)}^\dagger$ production, the Z_{SM},
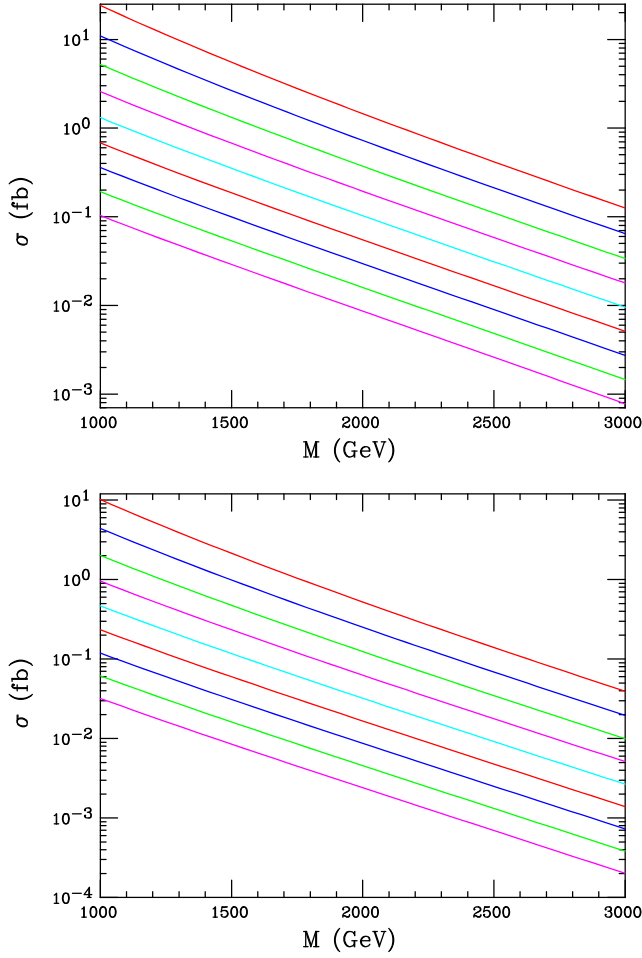
FIG. 3. Top: $gu \to U_{L,R}h + $ H.c. associated production cross section as a function of the $U_{L,R}$ mass assuming a SM gauge coupling, $g_R = g_L$ and, from top to bottom, that $m_h = 1, 1.25, \ldots 3$ TeV at the 13 TeV LHC. Bottom: same as the top panel but now for the $gd \to V_{L,R}h + $ H.c. associated production cross section as a function of the $V_{L,R}$ mass.

FIG. 4. Same as the previous figure but now, from top to bottom, for $m_h = 5, 7.5, 10, \ldots 25$ TeV at the 100 TeV FCC-hh.

the three Z_i, as well as the photon will contribute to this process in the s-channel. Specifically, in addition to the masses of the h and U/V which appear in the expression for the associated production cross section, these additional s-channel exchange contributions will also be sensitive to the masses of, e.g., the Z_i as well as the mixing angles among the weak eigenstates making any model-independent predictions for these cross sections impossible. However, the masses and couplings of the various states may be such that one (or more) of the Z_i, which is kinematically accessible, so can be resonantly produced at a hadron collider, may decay into on-shell pairs of U's and/or V's and so will provide the dominant mechanism for accessing these states, with significant cross sections. In both scenarios I and II, introduced in the previous section, it was found that the masses of the NHGB and HGB are generally comparable and interspersed with each other. For example, in scenario I, both $Z_{1,2}$ are sufficiently massive to
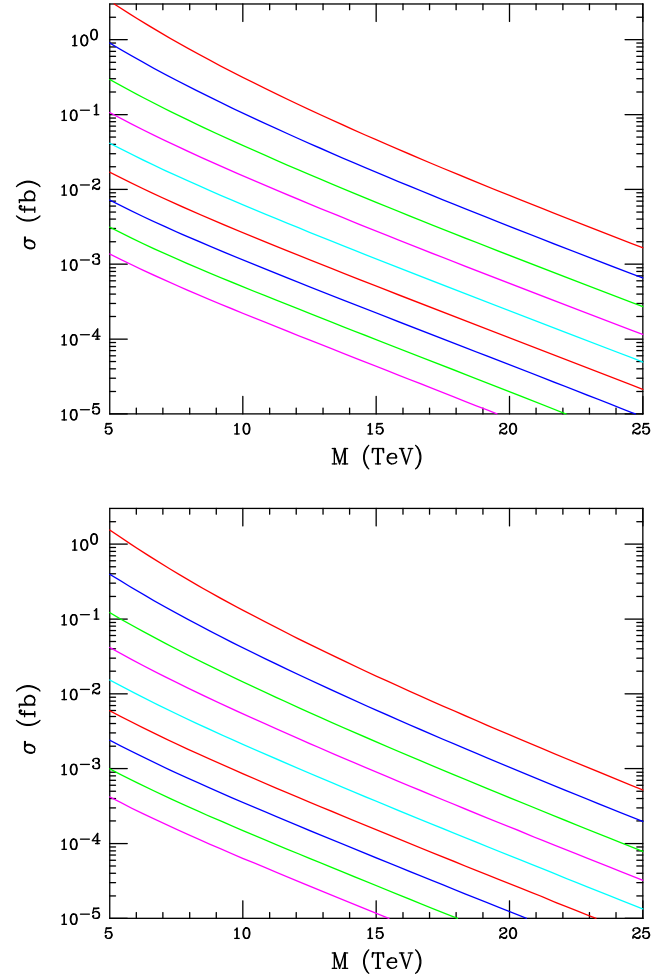
decay into $W_R^+ W_R^-$, with Z_2 also being allowed to on-shell decay to $V_{L,R} V_{L,R}^\dagger$; other modes may be possible depending upon the specific relative magnitudes amongst the various vevs. Scenario II shows a similar pattern although differing in the specifics.

Away from any resonances, it is more difficult to ascertain pair production rates in general due to the many contributing amplitudes. However, as a simple toy example, we again consider the case of $V_{L,R} V_{L,R}^\dagger$ pair production far above the SM Z peak but below the Z_i resonance regions and assuming that only one of the Z_i's is the dominant contributor (Z') to the cross section. The h exchange amplitude in the t-channel is then "fixed" by the specific assumed value of the h mass. This is somewhat similar to what occurs in the previously examined E_6-inspired model. Figure 5 shows some examples of this scenario for various values of $M_{Z'}/M_V$ for both the $\sqrt{s} = 13$ TeV LHC and the $\sqrt{s} = 100$ TeV FCC-hh where we see this cross section is not very sensitive to the mass of the h since the Z' contribution is resonant. As the Z' mass increases relative to that of the $V_{L,R}$, the cross section is
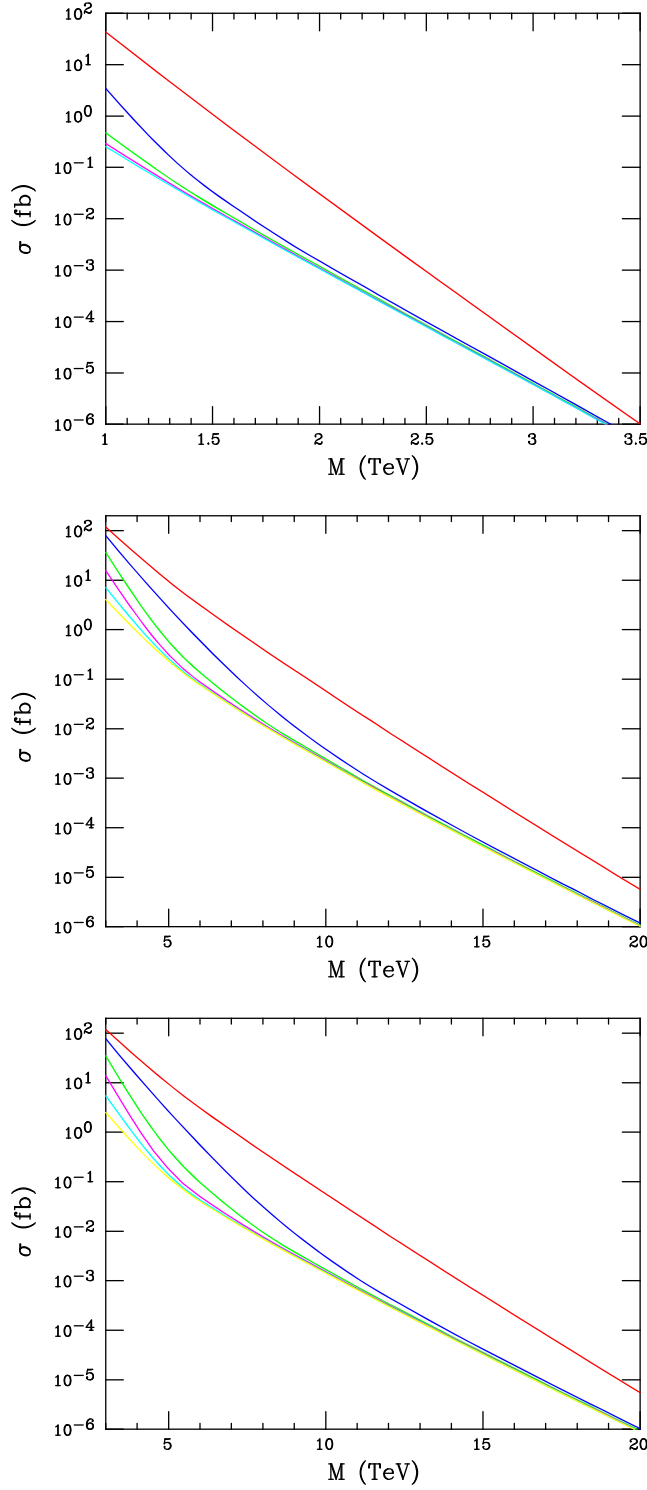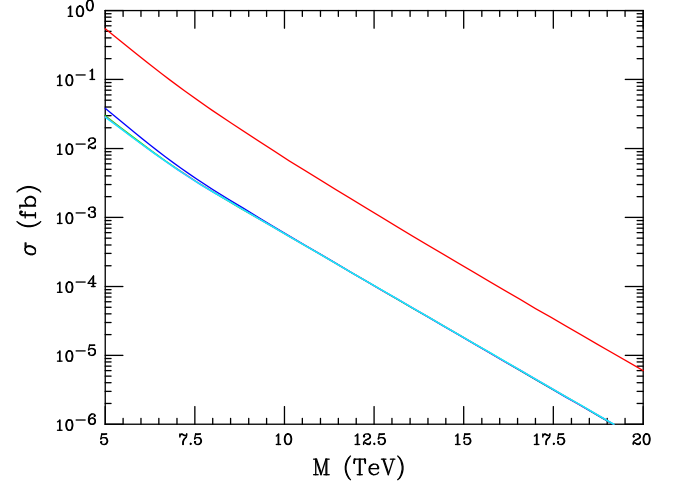
FIG. 6. Same as in the previous figure and assuming that $m_h = 7$ TeV at the $\sqrt{s} = 100$ TeV FCC-hh but now for $M/M_{Z'} = 0.5, 1, 2, 3, 4$, respectively, from top to bottom on the left-hand side of the panel.

FIG. 5. $V_{L,R} V_{L,R}^\dagger$ pair production cross section as a function of the mass of $V_{L,R}$, M, at (top) the $\sqrt{s} = 13$ TeV LHC with $m_h = 2$ TeV assuming $M_{Z'}/M = 3, 5, 7, \ldots$ from top to bottom. Middle (bottom): same as the previous panel but now for the $\sqrt{s} = 100$ TeV FCC-hh with $m_h = 3(7)$ TeV and assuming the same gauge boson mass ratios as above. $g_R = g_L$ is also assumed in all panels.

found to decrease until $M_{Z'}/M_V \sim 7$ is reached at which point this resonance effect essentially saturates. Even though the Z' is resonant, its mass has become sufficiently large that it ends up making only a rather small contribution to the overall production cross section. If the Z' mass is such that it cannot (or can barely) resonantly contribute to $V_{L,R} V_{L,R}^\dagger$ pair production, i.e., below $2M_V$, we instead obtain results as typically shown in Fig. 6 for the particular case of the FCC-hh. Here we see that while there is some enhancement exactly on/near resonance, once $2M_V$ exceeds $M_{Z'}$ this contribution essentially becomes $M_{Z'}$ independent as might be expected. In more realistic scenarios, the situation will be somewhat more complex than presented above for numerous reasons, although we might expect that these toy examples have captured at least some of the possibilities.

For the case of the $SU(3)'$ NHGB, in the limit that they remain unmixed with the other NHGB fields, the situation is somewhat different as the SM quarks do not carry any of the corresponding $SU(3)'$ quantum numbers so that these gauge bosons are not easily produced at a hadron collider, other than by pair production. At lepton colliders, however, other processes may are possible, e.g., $e^+ e^- \to V' + D$, with D being the DP, via t-/u-channel exchange of one or more of the $Q_D \neq 0$, E_i PM fields. Recall from our discussion above that the effective $e E_i D$ couplings for longitudinal DPs, λ_i, are both suppressed by the $e E_i$-mixing angles, $\phi_{e E_i} \sim 10^{-4}$, while simultaneously being enhanced by factors of the large mass ratios $M_{E_i}/M_D \sim 10^4$ so that we expect $\lambda_i = g \phi_{e E_i} M_{E_i}/M_D \sim O(1)$. This, together with the IR behavior of the amplitude, leads us to anticipate sizable cross sections for this process right up to the
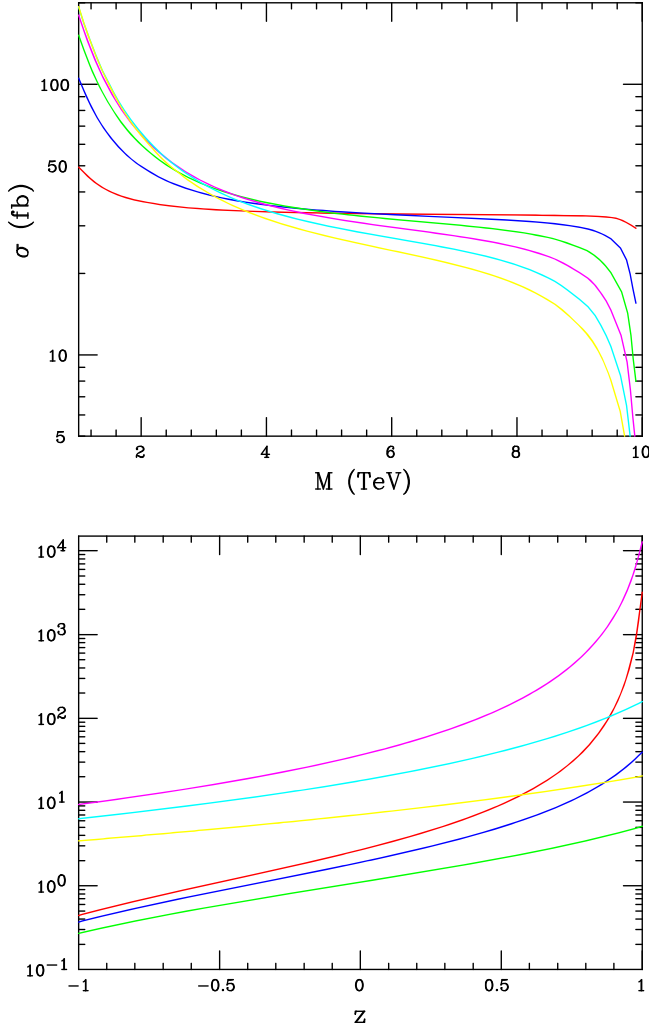
FIG. 7. Top: production cross section for the $V'D +$ H.c. final state at a $\sqrt{s} = 10$ TeV lepton collider as a function of the V' mass M assuming PM lepton masses from 0.5 to 6.5 TeV from top to bottom on the right-hand side of the panel in steps of 1 TeV. Bottom: unnormalized angular distributions, where $z = \cos\theta$, for the same process and $\sqrt{s}$ as in the top panel assuming that, from top to bottom on the left side of the panel, $(M_{V'}, M_E) = (8,1), (8,3), (8,5), (4,1), (4,3)$, and $(4, 5)$ TeV, respectively, where M_E is the PM lepton mass. In the top panel, the parameter λ, as described in the text, is assumed to be equal to unity.

kinematic production threshold, i.e., $M_{V'} \lesssim \sqrt{s}$. Figure 7 shows a sample set of production cross section results for an assumed $\sqrt{s} = 10$ TeV lepton collider under the further assumption that a single PM lepton, E, exchange is dominant showing that (i) the result is not overly sensitive to the specific PM E mass and (ii) as might be expected, the cross sections is highly peaked along the beam direction. Once produced, the V' will likely dominantly decay in a not very boosted fashion back into the eE final state, assuming it is kinematically allowed, i.e., that $M_{V'} > M_E$, producing a final state with wide-angle, opposite sign leptons.

This process can still happen in a 3-body mode when E is the more massive, $V' \to eE^* \to e^+e^-D$, but in either case the final state will still appear as two opposite sign leptons plus missing energy/momentum.

As noted the $V' + D$ production process cannot occur at a hadron collider via h exchange in the t-/u-channel but may occur via, e.g., $Z_{\mathrm{SM},i}$ mixing with V' induced by the $Q_D \neq 0$ vevs as was mentioned earlier. However, since the relevant mixing angle is likely to be of order $\sim M_D^2/M_{V'}^2$, i.e., quadratic in the small mass ratio unlike in the case of fermions or the NHGB, one finds that the rate for this process is numerically suppressed and so is not very useful in probing this setup. We note, however, that if relevant Z_i is more massive than V' then it is possible for this suppression to be at least partially offset by a resonance enhancement provided the Z_i is within the kinematically accessible mass range of the collider as was noted earlier.

VIII. DISCUSSION AND CONCLUSIONS

As is well known, the kinetic mixing of the SM photon and the dark photon allows for the possibility that thermal dark matter, lying in the sub-GeV mass range, can reproduce the observed relic abundance of dark matter for a reasonable range of model parameters while still satisfying all other known constraints. However, the generation of such KM relies on the existence of a new set of particles, here termed portal matter, that will transform nontrivially under both the SM as well as the DP's $U(1)_D$ gauge groups. Given the electroweak constraints, such states will most likely consist of heavy vectorlike fermions and/or new scalars, some of which must acquire vevs to break $U(1)_D$ as well as any larger gauge structure into which it may be embedded. If such particles do indeed exist and generate this KM portal, what are their properties, how can they be discovered, and how can we explore their detailed natures? How will they plus the dark photon fit into a more UV-complete theoretical framework with the fields of the SM, and what other additional structure is necessary to achieve this? In a recent series of papers, we have begun to explore these issues following both bottom-up and top-down approaches to model building by employing the guidance provided by some basic frameworks that will naturally contain at least some of the necessary ingredients to construct successful scenarios of this kind.

In the present paper, we have continued to explore these possibilities, motivated by our earlier analyses of both the E_6 and Pati-Salam inspired setups. In the past, we have considered UV structures in the form of product groups, i.e., $G = G_{\mathrm{SM}} \times G_{\mathrm{Dark}}$, where G_{SM} was identified as either the conventional SM, $3_c 2_L 1_Y$, in the case of E_6 or, effectively, the LRM, $3_c 2_L 2_R 1_{B-L}$, in the corresponding Pati-Salam setup. For either of these possibilities it was assumed that $G_{\mathrm{Dark}} = SU(2)_I \times U(1)_{Y_I}$, again inspired by E_6. In both of theses cases, there was at most only a partial symmetry directly linking the full SM and dark sectors, but

these two setups were found to share some general (and obvious) necessary common features: the natural occurrence of both new VL fermions and/or charged scalars that can play the role of PM as well the extended gauge group sector that partly connect SM fields to the PM ones at the multi-TeV mass scale. In this paper, we have considered a scenario based on anomaly-free quartification, a quark-lepton symmetric setup previously considered in the literature in other contexts, where we now identify $G_{SM} = 3_c 3_L 3_R$, the familiar trinification group arising from E_6, and where also $G_{Dark} = SU(3)'$ so that now $G = [SU(3)]^4$, which clearly displays an obvious symmetry between the visible and dark sectors that was absent from our previously examined scenarios. Although not a true *unification* in the very traditional sense, since quarks and leptons remain in different representations and the value of ϵ remains uncalculable with the minimal field content, this brings us much closer to goal of realizing a full UV theory than either of our earlier attempts. In this setup, $U(1)_D$ is simply a diagonal subgroup of $SU(3)'$ with Q_D being a linear combination of the two diagonal generators, $T'_{3,8}$, that is uniquely determined by the requirement that SM fields will all have $Q_D = 0$. However, a general, assumption-free, phenomenological analysis of this type of setup is made somewhat challenging due to the significant, but necessary, augmentation of the gauge, fermion, and Higgs scalar (containing multiple vevs at various scales) sectors beyond those of the SM or LRM which is viewed as being realized at large mass scales, $\gtrsim 10$ TeV. In particular, we are confronted by a set of eight new, non-Hermitian plus three new, Hermitian gauge bosons (in addition to the DP) and, per generation and ignoring color degrees of freedom, eight additional VL fermions as well as the RH neutrino. Of these, one is an "ordinary," $Q_D = 0$, color triplet, weak isosinglet quark, h, another a $Q_D = 0$, isosinglet neutral lepton, S_1, while the remainder are both charged and neutral leptons, all of which carry dark charges, E_i, N_i, S_2. Thus, unusually, the PM in this model necessarily consists solely of only color-singlet fields [39], which for both scalar and fermion result in relatively low discovery reaches for these new states at hadron colliders. While both h and S_1 are fairly conventional VL fermion states and will decay to SM fields via the usual SM (or LRM) gauge and Higgs bosons, e.g., $h \to dZ$, the PM fermion fields, as we had found in earlier work, will instead dominantly decay to a corresponding analog SM field plus the DP, e.g., $E_i \to eD$. Uniquely in the present setup, two of the NHGB, W', U', can *also* act as PM since they carry nonzero values of both Q_D and Q_{em}; this is correlated with the fact that $U(1)_D$ is an Abelian subgroup of $SU(3)'$ and that Q_{em} also partially depends upon the same two $SU(3)'$ diagonal generators. Since the setup is only partially unified, ϵ is not finite and calculable if only the minimal particle content is present but can be made so for the case of three fermion generations with the addition of several complex scalar fields which can be chosen to be $3_c 3_L 3_R$ singlets.

As we have seen, this scenario leads to numerous testable predictions, at least at the semiquantitative level, which are the result of the extended gauge structure, the existence of PM fields/new VL fermions, as well as our other model-building requirements, e.g., that the DP's $U(1)_D$ gauge symmetry remains unbroken until the ∼GeV mass scale is reached. As we have neglected flavor physics issues in this discussion for simplicity, the primary tests of the current setup will necessarily directly involve these new gauge bosons and fermions and be provided by high energy hadron and/or lepton colliders. Since the set of three new heavy HGB, Z_i, all couple to various linear combinations of the four group generators $T_{3R,8L,8R,3'}$, they will necessarily couple to at least some of the SM quarks and leptons and so, apart from the details discussed above, will generally behave somewhat similarly to the many Z' gauge bosons already encountered in the literature. However, if they are sufficiently massive, they may also decay to pairs of both new VL fermions, including PM, as well as some of the other gauge bosons, which may be fortuitous as some of these other states can be difficult to produce at colliders by other means with large cross sections. This is especially true for the $SU(3)'$ NHGB, W', U', and V', as these states do not directly couple to the SM quarks in the proton (except via mixing with the other NHGB states) and so can only be produced in association with PM or in pairs unlike, e.g., the W_R in the LRM. Unfortunately, the mass spectrum and, in particular, the mass ordering of these new gauge bosons (and the VL fermions as well) is highly model dependent and can have a significant impact on detailed phenomenological tests of this setup. For example, in the two sample scenarios, I and II, analyzed above it was observed that the W_R was the lightest among the set of new gauge boson states, although this result can easily be seen not to be true in all generality but only reflects the particular choices we made with respect to the assumed ordering of spectrum of the five multi-TeV vevs. However, even with this handicap some quite general model signatures were obtainable and analyzed in the previous sections and which will be further discussed elsewhere.

The KM portal remains a very attractive approach to linking the SM and dark sectors and providing a window for sub-GeV DM; it is hoped significant experimental evidence for this idea will soon be obtained. Further exploration of these types of PM scenarios is clearly necessary.

[1] N. Aghanim *et al.* (Planck Collaboration), Astron. Astrophys. **641,** A6 (2020); **652,** C4(E) (2021).

[2] M. Kawasaki and K. Nakayama, Annu. Rev. Nucl. Part. Sci. **63,** 69 (2013).

[3] P. W. Graham, I. G. Irastorza, S. K. Lamoreaux, A. Lindner, and K. A. van Bibber, Annu. Rev. Nucl. Part. Sci. **65,** 485 (2015).

[4] I. G. Irastorza and J. Redondo, Prog. Part. Nucl. Phys. **102,** 89 (2018).

[5] G. Arcadi, M. Dutra, P. Ghosh, M. Lindner, Y. Mambrini, M. Pierre, S. Profumo, and F. S. Queiroz, Eur. Phys. J. C **78,** 203 (2018).

[6] L. Roszkowski, E. M. Sessolo, and S. Trojanowski, Rep. Prog. Phys. **81,** 066201 (2018).

[7] K. Pachal, Dark Matter Searches at ATLAS and CMS, in *Talk Given at the Eighth Edition of the Large Hadron Collider Physics Conference* (SISSA, Trieste, 2020).

[8] E. Aprile *et al.* (XENON Collaboration), Phys. Rev. Lett. **121,** 111302 (2018).

[9] A. Albert *et al.* (Fermi-LAT and DES Collaborations), Astrophys. J. **834,** 110 (2017).

[10] C. Amole *et al.* (PICO Collaboration), Phys. Rev. D **100,** 022001 (2019).

[11] J. Aalbers *et al.* (LZ Collaboration), Phys. Rev. Lett. **131,** 041002 (2023).

[12] J. Alexander *et al.*, arXiv:1608.08632.

[13] M. Battaglieri *et al.*, arXiv:1707.04591.

[14] G. Bertone and T. M. P. Tait, Nature (London) **562,** 51 (2018).

[15] J. Cooley, T. Lin, W. H. Lippincott, T. R. Slatyer, T. T. Yu, D. S. Akerib, T. Aramaki, D. Baxter, T. Bringmann, R. Bunker *et al.*, arXiv:2209.07426.

[16] A. Boveia, T. Y. Chen, C. Doglioni, A. Drlica-Wagner, S. Gori, W. H. Lippincott, M. E. Monzani, C. Prescod-Weinstein, B. Shakya, T. R. Slatyer *et al.*, arXiv:2210.01770.

[17] P. Schuster, N. Toro, and K. Zhou, Phys. Rev. D **105,** 035036 (2022).

[18] B. Holdom, Phys. Lett. **166B,** 196 (1986); **178,** 65 (1986); K. R. Dienes, C. F. Kolda, and J. March-Russell, Nucl. Phys. **B492,** 104 (1997); F. Del Aguila, Acta Phys. Pol. B **25,** 1317 (1994); K. S. Babu, C. F. Kolda, and J. March-Russell, Phys. Rev. D **54,** 4635 (1996); T. G. Rizzo, Phys. Rev. D **59,** 015020 (1998).

[19] There has been a huge amount of work on this subject; see, for example, D. Feldman, B. Kors, and P. Nath, Phys. Rev. D **75,** 023503 (2007); D. Feldman, Z. Liu, and P. Nath, Phys. Rev. D **75,** 115001 (2007); M. Pospelov, A. Ritz, and M. B. Voloshin, Phys. Lett. B **662,** 53 (2008); M. Pospelov, Phys. Rev. D **80,** 095002 (2009); H. Davoudiasl, H. S. Lee, and W. J. Marciano, Phys. Rev. Lett. **109,** 031802 (2012); Phys. Rev. D **85,** 115019 (2012); R. Essig *et al.*, arXiv:1311.0029; E. Izaguirre, G. Krnjaic, P. Schuster, and N. Toro, Phys. Rev. Lett. **115,** 251301 (2015); M. Khlopov, Int. J. Mod. Phys. A **28,** 1330042 (2013); For a general overview and introduction to this framework, see D. Curtin, R. Essig, S. Gori, and J. Shelton, J. High Energy Phys. 02 (2015) 157.

[20] T. Gherghetta, J. Kersten, K. Olive, and M. Pospelov, Phys. Rev. D **100,** 095001 (2019).

[21] G. Steigman, Phys. Rev. D **91,** 083538 (2015).

[22] K. Saikawa and S. Shirai, J. Cosmol. Astropart. Phys. 08 (2020) 011.

[23] M. Fabbrichesi, E. Gabrielli, and G. Lanfranchi, *The Physics of the Dark Photon* (Springer, Cham, 2021).

[24] M. Graham, C. Hearty, and M. Williams, Annu. Rev. Nucl. Part. Sci. **71,** 37 (2021).

[25] T. G. Rizzo, Phys. Rev. D **99,** 115024 (2019).

[26] T. D. Rueter and T. G. Rizzo, Phys. Rev. D **101,** 015014 (2020).

[27] J. H. Kim, S. D. Lane, H. S. Lee, I. M. Lewis, and M. Sullivan, Phys. Rev. D **101,** 035041 (2020).

[28] T. D. Rueter and T. G. Rizzo, arXiv:2011.03529.

[29] G. N. Wojcik and T. G. Rizzo, Phys. Rev. D **105,** 015032 (2022).

[30] T. G. Rizzo, J. High Energy Phys. 11 (2021) 035.

[31] T. G. Rizzo, arXiv:2202.02222.

[32] G. N. Wojcik, Phys. Rev. D **108,** 035006 (2023).

[33] T. G. Rizzo, Phys. Rev. D **106,** 035024 (2022).

[34] T. G. Rizzo, Phys. Rev. D **106,** 095024 (2022).

[35] G. N. Wojcik, L. L. Everett, S. T. Eu, and R. Ximenes, Phys. Lett. B **841,** 137931 (2023).

[36] A. Carvunis, N. McGinnis, and D. E. Morrissey, J. High Energy Phys. 01 (2023) 014.

[37] S. Verma, S. Biswas, A. Chatterjee, and J. Ganguly, Phys. Rev. D **107,** 115024 (2023).

[38] T. G. Rizzo, Phys. Rev. D **107,** 095014 (2023).

[39] G. N. Wojcik, L. L. Everett, S. T. Eu, and R. Ximenes, arXiv:2303.12983.

[40] T. R. Slatyer, Phys. Rev. D **93,** 023527 (2016).

[41] H. Liu, T. R. Slatyer, and J. Zavala, Phys. Rev. D **94,** 063507 (2016).

[42] R. K. Leane, T. R. Slatyer, J. F. Beacom, and K. C. Ng, Phys. Rev. D **98,** 023016 (2018).

[43] For related work on the possibilities of KM and DM physics employing this same gauge group, see M. Bauer and P. Foldenauer, Phys. Rev. Lett. **129,** 171801 (2022).

[44] See, for example, F. Gursey, P. Ramond, and P. Sikivie, Phys. Lett. **60B,** 177 (1976); Y. Achiman and B. Stech, Phys. Lett. **77B,** 389 (1978); Q. Shafi, Phys. Lett. **79B,** 301 (1978).

[45] J. L. Hewett and T. G. Rizzo, Phys. Rep. **183,** 193 (1989).

[46] J. C. Pati and A. Salam, Phys. Rev. D **10,** 275 (1974); **11,** 703(E) (1975).

[47] R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11,** 566 (1975).

[48] R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11,** 2558 (1975).

[49] G. Senjanovic and R. N. Mohapatra, Phys. Rev. D **12,** 1502 (1975).

[50] R. N. Mohapatra, *Unification and Supersymmetry: The Frontiers of Quark-Lepton Physics* (Springer, New York, NY, 1986).

[51] The literature on this possibility is quite extensive; see, for example, K. S. Babu, X. G. He, and S. Pakvasa, Phys. Rev. D **33,** 763 (1986); J. Sayre, S. Wiesenfeldt, and S. Willenbrock, Phys. Rev. D **73,** 035013 (2006); S. Willenbrock, Phys. Lett. B **561,** 130 (2003); J. E. Kim, Phys. Lett. B **591,** 119 (2004); P. V. Dong, D. T. Huong, F. S. Queiroz, J. W. F. Valle, and C. A. Vaquera-Araujo, J. High Energy Phys. 04 (2018) 143; G. M. Pelaggi, A. Strumia, and S. Vignali, J. High Energy Phys. 08 (2015)

130; J. Hetzel and B. Stech, Phys. Rev. D **91**, 055026 (2015); K. S. Babu, S. Jana, and A. Thapa, J. High Energy Phys. 02 (2022) 051; K. S. Babu, B. Bajc, V. Susič, O. Rodríguez, R. H. Benavides, W. A. Ponce, and E. Rojas, Phys. Rev. D **95**, 014009 (2017); S. J. Lonsdale and R. R. Volkas, Phys. Rev. D **90**, 083501 (2014); **91**, 129906(E) (2015).

[52] See, for example, E. Ma, Phys. Lett. B **820**, 136561 (2021); C. Kownacki, E. Ma, N. Pollard, O. Popov, and M. Zakeri, Phys. Lett. B **777**, 121 (2018); C. Kownacki, E. Ma, N. Pollard, O. Popov, and M. Zakeri, Nucl. Phys. **B928**, 520 (2018); J. B. Dent, T. W. Kephart, H. Päs, and T. J. Weiler, arXiv:2009.04443; D. A. Eby, P. H. Frampton, X. G. He, and T. W. Kephart, Phys. Rev. D **84**, 037302 (2011); K. L. McDonald, Int. J. Mod. Phys. A **22**, 5023 (2007); A. Demaria and K. L. McDonald, Phys. Rev. D **75**, 056006 (2007); A. Demaria, C. I. Low, and R. R. Volkas, Phys. Rev. D **74**, 033005 (2006); E. Ma, Mod. Phys. Lett. A **20**, 1953 (2005); A. Demaria, C. I. Low, and R. R. Volkas, Phys. Rev. D **72**, 075007 (2005); **73**, 079902(E) (2006); S. L. Chen and E. Ma, Mod. Phys. Lett. A **19**, 1267 (2004); K. S. Babu, E. Ma, and S. Willenbrock, Phys. Rev. D **69**, 051301 (2004).

[53] J. M. Alves, G. C. Branco, A. L. Cherchiglia, J. T. Penedo, P. M. F. Pereira, C. C. Nishi, M. N. Rebelo, and J. I. Silva-Marcos, arXiv:2304.10561.

[54] M. S. Chanowitz and M. K. Gaillard, Nucl. Phys. **B261**, 379 (1985); B. W. Lee, C. Quigg, and H. B. Thacker, Phys. Rev. D **16**, 1519 (1977); J. M. Cornwall, D. N. Levin, and G. Tiktopoulos, Phys. Rev. D **10**, 1145 (1974); **11**, 972(E) (1975); G. J. Gounaris, R. Kogerler, and H. Neufeld, Phys. Rev. D **34**, 3257 (1986).

[55] Such heavy neutral lepton states have been discussed in a number of different contexts; see for example, A. Das, P. S. Bhupal Dev, and N. Okada, Phys. Lett. B **735**, 364 (2014); A. de Gouvea, W. C. Huang, and J. Jenkins, Phys. Rev. D **80**, 073007 (2009); G. Anamiati, M. Hirsch, and E. Nardi, J. High Energy Phys. 10 (2016) 010; P. Hernández, J. Jones-Pérez, and O. Suarez-Navarro, Eur. Phys. J. C **79**, 220 (2019); D. Chang and O. C. W. Kong, Phys. Lett. B **477**, 416 (2000); S. Bahrami, M. Frank, D. K. Ghosh, N. Ghosh, and I. Saha, Phys. Rev. D **95**, 095024 (2017).

[56] See, for example, A. M. Sirunyan et al. (CMS Collaboration), Phys. Lett. B **820**, 136535 (2021); G. Aad et al. (ATLAS Collaboration), J. High Energy Phys. 03 (2020) 145; A. Tumasyan et al. (CMS Collaboration), J. High Energy Phys. 04 (2022) 047; G. Aad et al. (ATLAS Collaboration), Phys. Rev. D **100**, 052013 (2019); A. Tumasyan et al. (CMS Collaboration), J. High Energy Phys. 07 (2022) 067; A. M. Sirunyan et al. (CMS Collaboration), J. High Energy Phys. 05 (2020) 033; ATLAS Collaboration, Report No. ATLAS-CONF-2021-043.

[57] ATLAS Collaboration, Report No. ATLAS-CONF-2022-028.

[58] A. Tumasyan et al. (CMS Collaboration), arXiv:2307.06959.

[59] T. G. Rizzo, Phys. Rev. D **89**, 095022 (2014).

[60] M. Nemevšek and F. Nesti, arXiv:2306.12104.

[61] T. Aaltonen et al. (CDF Collaboration), Science **376**, 170 (2022).

[62] M. Aaboud et al. (ATLAS Collaboration), J. High Energy Phys. 10 (2018) 180.

[63] R. L. Workman et al. (Particle Data Group), Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).

[64] V. S. Vanyashin and M. V. Terentev, Zh. Eksp. Teor. Fiz. **48**, 565 (1965); Sov. Phys. JETP **21**, 375 (1965).

[65] M. Nemevšek and F. Nesti, arXiv:2306.12104.

[66] P. Langacker, Rev. Mod. Phys. **81**, 1199 (2009).

[67] A. Leike, Phys. Rep. **317**, 143 (1999).

[68] T. G. Rizzo, arXiv:hep-ph/0610104.

[69] G. Aad et al. (ATLAS Collaboration), Phys. Lett. B **796**, 68 (2019).

[70] C. Helsens, D. Jamin, M. L. Mangano, T. G. Rizzo, and M. Selvaggi, Eur. Phys. J. C **79**, 569 (2019).