

Gluon generalized parton distributions and angular momentum in a light-cone spectator model

Chentao Tan and Zhun Lu ^{*}

School of Physics, Southeast University, Nanjing 211189, China

 (Received 26 January 2023; accepted 7 September 2023; published 29 September 2023)

We study the leading twist gluon generalized parton distributions and the gluon angular momentum inside the proton within a light-cone spectator model. Using the light-cone wave functions derived from the model, we provide the expressions of these distributions at the particular kinematical point $\xi = 0$ in the overlap representation. The numerical results of the H^g , E^g , \tilde{H}^g , H_T^g , and E_T^g as functions of x at different Δ_T are presented. Particularly, H^g , \tilde{H}^g at nonzero Δ_T are different from their forward counterparts, the unpolarized distribution f_1^g and the helicity distribution g_1^g , respectively. We also obtain the total angular momentum of the gluon contributed to the proton spin $J^g = 0.19$, which is consistent with the recent lattice calculation after the uncertainties are considered. The kinetic orbital angular momentum is also calculated and is negative in our model.

DOI: [10.1103/PhysRevD.108.054038](https://doi.org/10.1103/PhysRevD.108.054038)

I. INTRODUCTION

The study of the hadronic structure in terms of the quark and gluon degrees of freedom is one of the important tasks in hadronic physics. Although the parton distribution functions (PDFs), which encode the distributions of the longitudinal momentum and polarization carried by quarks and gluons in a fast moving hadron, are most widely used in the investigation of the hadronic structure, a more comprehensive picture can be obtained from the general parton distributions (GPDs) [1–3]. These objects are experimentally accessible through the hard exclusive reactions, such as deep virtual Compton scattering and deep virtual meson production [4–9].

One of the necessities of studying GPDs is their relationships with mass decomposition [10,11] and spin decomposition of hadrons. In particular, Ji [12] derived a gauge-invariant decomposition of the nucleon spin in terms of the quark spin, quark orbital angular momentum (OAM), and the gluon angular momentum. Furthermore, the angular momentum sum rule can relate the moments of GPDs to the corresponding form factors defined through the expectation value of certain operators, which gives the spin and (orbit) angular momentum of partons evaluated at $t = 0$. The quark OAM and spin-orbit correlations in the nucleon and the pion meson have been calculated by different

models [13–16]. In addition, Fourier transforming GPDs with respect to the transverse momentum transfer Δ_T yields the impact-parameter-dependent distributions $f(x, b_T^2)$ [17–20], which encode how partons are distributed in the transverse plane along with the longitudinal momentum fraction of partons inside the hadron.

The quark GPDs in the nucleon and the meson have been widely studied from theoretical aspect [21–27]. However, the knowledge of the gluon GPDs [2,28] is rather limited. Nevertheless, the twist-2 gluon GPDs have been calculated in a quark-target model [29] that is different from the realistic situation of the target state. In Ref. [29], the authors also studied the relations between the gluon GPDs and transverse-momentum-dependent parton distributions (TMDs). For H^g and E^g , the Balitsky-Fadin-Kuraev-Lipatov resummation technique [30,31] is established and widely used in phenomenology, and this means that the study on how they evolve mainly focuses on the small- x region [32,33]. Similarly, the gluon OAM can also be calculated using the sum rule of the corresponding GPDs [34,35], and the methods of extracting it from experimental observables have been proposed [36–43].

In this work, we study the leading-twist gluon GPDs and the kinetic gluon OAM from an intuitive model concerning the gluon structure of the proton, which can be considered as a useful complement to the phenomenological analysis on the experimental data and other model calculations. The approach of the study follows the one applied in Ref. [44], in which a spectator model was applied to generate the gluon degree of freedom from the proton target to calculate the gluon Sivers function. In the approach, the proton is regarded as a two-particle system composed by an active

^{*}zhunlu@seu.edu.cn

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

gluon and a spectator particle that contains three valence quarks. A similar model has also been applied to calculate the T-even gluon TMDs [45] in which the spectral function and more complicated form factors are considered. Within the spectator model, we can obtain the expression of the gluon GPDs using the overlap representation [46,47] in terms of the light-cone wave functions of the proton Fock state. Based on this, we choose the Brodsky-Huang-Lepage prescription [48] for the coupling of the nucleon-gluon spectator vertex to calculate the analytical forms of these distributions. We calculate the five T-even gluon GPDs H^g , E^g , \tilde{H}^g , H_T^g , and E_T^g at $\xi = 0$. The GPDs then can be used to study the total angular momentum of the gluon contributed to the proton spin via Ji's sum rule. As a by-product, we also investigate the x dependence of the gluon kinetic OAM.

The rest of the paper is organized as follows. In Sec. II, the definition of the leading-twist gluon GPDs is provided via the light-cone correlation function. In Sec. III, we present the analytic expressions of the gluon T-even GPDs in the overlap representation within light-cone spectator model. In Sec. IV, we provide the results of the GPDs as functions of x at different Δ_T . In Sec. V, some conclusions are given.

II. DEFINITION OF THE GLUON GPDs

In this section, we present the definitions of the gluon GPDs of the proton. Unless specified otherwise, we will follow the conventions used in Ref. [2]. Figure 1 describes the kinematics for the gluon GPDs. The momenta of the incoming and outgoing proton are given by

$$p = P - \frac{1}{2}\Delta, \quad p' = P + \frac{1}{2}\Delta, \quad (1)$$

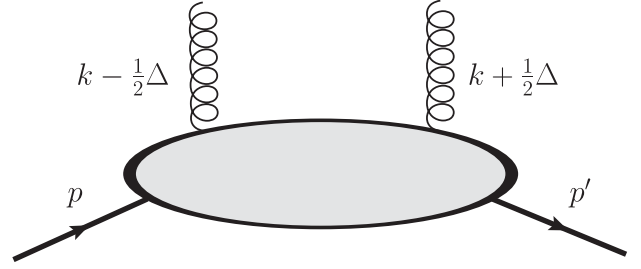


FIG. 1. Kinematics for gluon GPDs.

where $p^2 = p'^2 = M^2$, with M as the proton mass. In a physical process, $P = (p + p')/2$ is the average momentum of the initial and final proton, and $\Delta = p' - p$ is the momentum transfer to the proton. The GPDs depend on three variables,

$$x = \frac{k^+}{P^+}, \quad \xi = -\frac{\Delta^+}{2P^+}, \quad t = \Delta^2, \quad (2)$$

where k and x are the average momentum and the average plus-momentum fraction carried by the active gluon, respectively; skewness ξ is the fraction of the transferred momentum, and t is the square of the momentum transfer. In the light-cone coordinate, a general four-vector is defined as

$$a^\pm = \frac{1}{\sqrt{2}}(a^0 \pm a^3), \quad \mathbf{a}_T = (a_1, a_2). \quad (3)$$

Similar to the definitions of PDFs, the GPDs are also defined through the relevant correlation functions. For the leading-twist gluon GPDs, the light-cone correlator reads [2]

$$F^{g[ij]}(x, \Delta; \lambda, \lambda') = \frac{1}{xP^+} \int \frac{dz^-}{2\pi} e^{ik \cdot z} \langle p'; \lambda' | F_a^{+j} \left(-\frac{1}{2}z \right) \mathcal{W}_{ab} \left(-\frac{1}{2}z; \frac{1}{2}z \right) F_b^{+i} \left(\frac{1}{2}z \right) | p; \lambda \rangle \Big|_{z^+=0^+, z_T=\mathbf{0}_T}, \quad (4)$$

where λ and λ' denote the helicity of the initial and final proton, respectively. The gluon field strength tensor $F_a^{\mu\nu}(x)$ has the standard form

$$F_a^{\mu\nu}(x) = \partial^\mu A_a^\nu(x) - \partial^\nu A_a^\mu(x) + g f_{abc} A_b^\mu(x) A_c^\nu(x), \quad (5)$$

with f_{abc} being the structure constants of the $SU_c(3)$ group. To ensure the color gauge invariance of the correlator (4), the Wilson line running along the path

$$\mathcal{W}_{ab} \left(-\frac{1}{2}z; \frac{1}{2}z \right) \Big|_{z^+=0^+, z_T=\mathbf{0}_T} = \left[0^+, -\frac{1}{2}z^-, \mathbf{0}_T; 0^+, \frac{1}{2}z^-, \mathbf{0}_T \right]_{ab} = \mathcal{P} \exp \left[-g \int_{-\frac{1}{2}z^-}^{\frac{1}{2}z^-} dy^- f_{abc} A_c^+(0^+, y^-, \mathbf{0}_T) \right] \quad (6)$$

is included, which couples to the gluon field strength tensor through the coupling constant g . \mathcal{P} denotes all possible ordered paths followed by the gluon field A . The following two tensors

$$\delta_T^{ij} = -g^{ij}, \quad \epsilon_T^{ij} = \epsilon^{-+ij} \quad (7)$$

are applied to define the chiral-even gluon GPDs, while the symmetric operator \hat{S}

$$\hat{S}O^{ij} = \frac{1}{2}(O^{ij} + O^{ji} - \delta_T^{ij}O^{mm}), \quad (8)$$

is used to define the chiral-odd gluon GPDs, with O^{ij} as a general tensor. Then the twist-2 gluon GPDs can be obtained from the correlator by projecting onto the tensors in Eqs. (7) and (8) as follows:

$$F^g(x, \Delta; \lambda, \lambda') = \delta_T^{ij}F^{g[ij]}(x, \Delta; \lambda, \lambda') = \frac{1}{2P^+}\bar{u}(p', \lambda')\left(\gamma^+H^g(x, \xi, t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}E^g(x, \xi, t)\right)u(p, \lambda), \quad (9)$$

$$\tilde{F}^g(x, \Delta; \lambda, \lambda') = i\epsilon_T^{ij}F^{g[ij]}(x, \Delta; \lambda, \lambda') = \frac{1}{2P^+}\bar{u}(p', \lambda')\left(\gamma^+\gamma_5\tilde{H}^g(x, \xi, t) + \frac{\Delta^+\gamma_5}{2M}\tilde{E}^g(x, \xi, t)\right)u(p, \lambda), \quad (10)$$

$$\begin{aligned} F_T^{g,ij}(x, \Delta; \lambda, \lambda') &= -\hat{S}F^{g[ij]}(x, \Delta; \lambda, \lambda') \\ &= \frac{\hat{S}}{2P^+}\frac{P^+\Delta_T^i - \Delta^+P_T^i}{2MP^+}\bar{u}(p', \lambda')\left(i\sigma^{+j}H_T^g(x, \xi, t) + \frac{\gamma^+\Delta_T^j - \Delta^+\gamma_T^j}{2M}E_T^g(x, \xi, t)\right. \\ &\quad \left.+ \frac{P^+\Delta_T^j - \Delta^+P_T^j}{M^2}\tilde{H}_T^g(x, \xi, t) + \frac{\gamma^+P_T^j - P^+\gamma_T^j}{M}\tilde{E}_T^g(x, \xi, t)\right)u(p, \lambda). \end{aligned} \quad (11)$$

Note that Eqs. (9) and (10) define the four chiral-even GPDs H^g , E^g , \tilde{H}^g , \tilde{E}^g , while Eq. (11) expresses the four chiral-odd GPDs H_T^g , E_T^g , \tilde{H}_T^g , \tilde{E}_T^g , respectively. Besides, E^g , H_T^g , and \tilde{E}_T^g are the helicity-flipped GPDs.

If we choose a particular kinematical point $\xi = 0$, which implies the plus-momentum transfer $\Delta^+ = 0$, the right-hand side of Eqs. (9)–(11) will be simplified considerably. With the help of the spin vector S of the proton and the

transverse component Δ_T of the momentum transfer, we find [29]

$$F^g(x, \Delta_T; S) = H^g(x, 0, -\Delta_T^2) - \frac{i\epsilon_T^{ij}\Delta_T^i S_T^j}{2M}E^g(x, 0, -\Delta_T^2), \quad (12)$$

$$\tilde{F}^g(x, \Delta_T; S) = \lambda\tilde{H}^g(x, 0, -\Delta_T^2), \quad (13)$$

$$\begin{aligned} F_T^{g,ij}(x, \Delta_T; S) &= \frac{\hat{S}\Delta_T^i\Delta_T^j}{4M^2}\left(E_T^g(x, 0, -\Delta_T^2) + 2\tilde{H}_T^g(x, 0, -\Delta_T^2)\right) + \frac{i\hat{S}\Delta_T^i\epsilon_T^{jk}S_T^k}{2M}\left(H_T^g(x, 0, -\Delta_T^2) + \frac{\Delta_T^2}{4M^2}\tilde{H}_T^g(x, 0, -\Delta_T^2)\right) \\ &\quad - \frac{i\hat{S}\Delta_T^i\epsilon_T^{jk}(2\Delta_T^k\Delta_T \cdot S_T - S_T^k\Delta_T^2)}{8M^3}\tilde{H}_T^g(x, 0, -\Delta_T^2). \end{aligned} \quad (14)$$

Because of the choice $\xi = 0$, the GPDs \tilde{E}^g and \tilde{E}_T^g will not show up in the above expressions.

III. GLUON GPDs IN THE OVERLAP REPRESENTATION WITHIN SPECTATOR MODEL

The gluon GPDs have been first studied by models in Refs. [29,49], in which the authors applied the quark-target model inspired by perturbative QCD, i.e., the gluon is produced from the radiation off the parent quark. In the case where the target is a proton, the minimum Fock state for the proton that contains a gluon is $|qqqg\rangle$. As the four-body system is rather complicated, here we resort to a more phenomenological approach to assume that the three quarks can be grouped into a spectator particle [44,45]. Thus, in this model, in which the degree of freedom of a gluon is

present, the proton can be viewed as a composite system formed by an active gluon and a spectator particle X ,

$$|p; S\rangle \rightarrow |g_{s_g}X_{s_X}(uud)\rangle, \quad (15)$$

where s_g and s_X denote the spins of the gluon and the spectator particle, respectively. In principle, the spin quantum number of the spectator can be $s_X = 1/2$ or $3/2$. According to the angular momentum conservation, for a $s_X = \frac{1}{2}$ spectator, the orbital angular momentum of the gluon may be $L = 0$ or $L = 1$. While for a $s_X = \frac{3}{2}$ spectator, the orbital angular momentum of the gluon has to be at least $L = 2$. As the contribution from high OAM components is assumed to be much smaller than that from the low OAM components, in this calculation we only consider the spin-1/2 component and ignore the contribution from a $s_X = \frac{3}{2}$ spectator, following Refs. [44,45].

Then the Fock-state expansion of the proton with $J_z = +1/2$ has the following form:

$$|\Psi_{\text{two particle}}^{\uparrow}(p^+, \mathbf{p}_T = \mathbf{0}_T)\rangle = \int \frac{d^2\mathbf{k}_T dx}{16\pi^3 \sqrt{x(1-x)}} \left[\psi_{+1+\frac{1}{2}}^{\uparrow}(x, \mathbf{k}_T) \left| +1, +\frac{1}{2}; xp^+, \mathbf{k}_T \right\rangle + \psi_{+1-\frac{1}{2}}^{\uparrow}(x, \mathbf{k}_T) \left| +1, -\frac{1}{2}; xp^+, \mathbf{k}_T \right\rangle \right. \\ \left. + \psi_{-1+\frac{1}{2}}^{\uparrow}(x, \mathbf{k}_T) \left| -1, +\frac{1}{2}; xp^+, \mathbf{k}_T \right\rangle + \psi_{-1-\frac{1}{2}}^{\uparrow}(x, \mathbf{k}_T) \left| -1, -\frac{1}{2}; xp^+, \mathbf{k}_T \right\rangle \right], \quad (16)$$

where $\psi_{s_g^z s_X^z}^{\uparrow(\downarrow)}(x, \mathbf{k}_T)$ are the wave functions corresponding to the two-particle states $|s_g^z, s_X^z; xp^+, \mathbf{k}_T\rangle$, with s_g^z and s_X^z being the z components of the spins of the gluon and spectator, respectively. Here $\uparrow(\downarrow)$ denotes that the z component J_z of the proton spin S equals $1/2(-1/2)$. Thus, the light-cone wave functions of the Fock-state component of the proton (16) can be expressed as

$$\begin{aligned} \psi_{+1+\frac{1}{2}}^{\uparrow}(x, \mathbf{k}_T) &= -\sqrt{2} \frac{-k_T^1 + ik_T^2}{x(1-x)} \phi, \\ \psi_{+1-\frac{1}{2}}^{\uparrow}(x, \mathbf{k}_T) &= -\sqrt{2} \left(M - \frac{M_X}{1-x} \right) \phi, \\ \psi_{-1+\frac{1}{2}}^{\uparrow}(x, \mathbf{k}_T) &= -\sqrt{2} \frac{k_T^1 + ik_T^2}{x} \phi, \\ \psi_{-1-\frac{1}{2}}^{\uparrow}(x, \mathbf{k}_T) &= 0, \end{aligned} \quad (17)$$

which is similar to the light-cone Fock-state wave functions of the physical electron given in Ref. [50]. Here, M_X is the spectator mass, and ϕ denotes the wave function in the momentum space,

$$\phi(x, \mathbf{k}_T) = \frac{\lambda \sqrt{x(1-x)}}{x(1-x)M^2 - (1-x)\mathbf{k}_T^2 - x(\mathbf{k}_T^2 + M_X^2)}, \quad (18)$$

where M_g is the gluon mass for which we fix $M_g = 0$, and λ denotes the coupling of the nucleon-gluon spectator vertex. To simulate the nonperturbative physics of the vertex, we choose the Brodsky-Huang-Lepage prescription for the coupling λ [48],

$$\lambda \rightarrow N_\lambda \exp\left(-\frac{\mathcal{M}^2}{2\beta_1^2}\right), \quad (19)$$

where N_λ is a strength parameter of the vertex, β_1 is a cutting-off parameter, and \mathcal{M} is the invariant mass of the two-particle system,

$$\mathcal{M}^2 = \frac{\mathbf{k}_T^2}{x} + \frac{\mathbf{k}_T^2 + M_X^2}{1-x}. \quad (20)$$

Similarity, the Fock-state expansion of the proton with $J_z = -1/2$ is given by

$$|\Psi_{\text{two particle}}^{\downarrow}(p^+, \mathbf{p}_T = \mathbf{0}_T)\rangle = \int \frac{d^2\mathbf{k}_T dx}{16\pi^3 \sqrt{x(1-x)}} \left[\psi_{+1+\frac{1}{2}}^{\downarrow}(x, \mathbf{k}_T) \left| +1, +\frac{1}{2}; xp^+, \mathbf{k}_T \right\rangle + \psi_{+1-\frac{1}{2}}^{\downarrow}(x, \mathbf{k}_T) \left| +1, -\frac{1}{2}; xp^+, \mathbf{k}_T \right\rangle \right. \\ \left. + \psi_{-1+\frac{1}{2}}^{\downarrow}(x, \mathbf{k}_T) \left| -1, +\frac{1}{2}; xp^+, \mathbf{k}_T \right\rangle + \psi_{-1-\frac{1}{2}}^{\downarrow}(x, \mathbf{k}_T) \left| -1, -\frac{1}{2}; xp^+, \mathbf{k}_T \right\rangle \right], \quad (21)$$

where

$$\begin{aligned} \psi_{+1+\frac{1}{2}}^{\downarrow}(x, \mathbf{k}_T) &= 0, \\ \psi_{+1-\frac{1}{2}}^{\downarrow}(x, \mathbf{k}_T) &= -\sqrt{2} \frac{-k_T^1 + ik_T^2}{x} \phi, \\ \psi_{-1+\frac{1}{2}}^{\downarrow}(x, \mathbf{k}_T) &= -\sqrt{2} \left(M - \frac{M_X}{1-x} \right) \phi, \\ \psi_{-1-\frac{1}{2}}^{\downarrow}(x, \mathbf{k}_T) &= -\sqrt{2} \frac{k_T^1 + ik_T^2}{x(1-x)} \phi. \end{aligned} \quad (22)$$

It is worth noting that Eqs. (17) and (22) are obtained using light-cone perturbation theory introducing theory uncertainties.

Similar to the analytical results of the gluon GPDs in the quark-target model [29], we can write the GPDs in the overlap representation using the light-cone wave functions as

$$H^g(x, 0, -\Delta_T^2) = C_F \sum_{s_g^z s_X^z} \int \frac{d^2\mathbf{k}_T}{32\pi^3} \psi_{s_g^z s_X^z}^{\uparrow\star}(x^{\text{out}}, \mathbf{k}_T^{\text{out}}) \psi_{s_g^z s_X^z}^{\uparrow}(x^{\text{in}}, \mathbf{k}_T^{\text{in}}) + \psi_{s_g^z s_X^z}^{\downarrow\star}(x^{\text{out}}, \mathbf{k}_T^{\text{out}}) \psi_{s_g^z s_X^z}^{\downarrow}(x^{\text{in}}, \mathbf{k}_T^{\text{in}}), \quad (23)$$

$$\frac{\Delta_-}{2M} E^g(x, 0, -\Delta_T^2) = C_F \int \frac{d^2 \mathbf{k}_T}{16\pi^3} \psi_{+1-\frac{1}{2}}^{\uparrow\star}(x^{\text{out}}, \mathbf{k}_T^{\text{out}}) \psi_{+1-\frac{1}{2}}^\downarrow(x^{\text{in}}, \mathbf{k}_T^{\text{in}}) + \psi_{-1+\frac{1}{2}}^{\uparrow\star}(x^{\text{out}}, \mathbf{k}_T^{\text{out}}) \psi_{-1+\frac{1}{2}}^\downarrow(x^{\text{in}}, \mathbf{k}_T^{\text{in}}), \quad (24)$$

$$\begin{aligned} \tilde{H}^g(x, 0, -\Delta_T^2) &= C_F \int \frac{d^2 \mathbf{k}_T}{32\pi^3} \psi_{+1\pm\frac{1}{2}}^{\uparrow\star}(x^{\text{out}}, \mathbf{k}_T^{\text{out}}) \psi_{+1\pm\frac{1}{2}}^\uparrow(x^{\text{in}}, \mathbf{k}_T^{\text{in}}) - \psi_{-1\pm\frac{1}{2}}^{\uparrow\star}(x^{\text{out}}, \mathbf{k}_T^{\text{out}}) \psi_{-1\pm\frac{1}{2}}^\uparrow(x^{\text{in}}, \mathbf{k}_T^{\text{in}}) \\ &\quad + \psi_{-1\pm\frac{1}{2}}^{\downarrow\star}(x^{\text{out}}, \mathbf{k}_T^{\text{out}}) \psi_{-1\pm\frac{1}{2}}^\downarrow(x^{\text{in}}, \mathbf{k}_T^{\text{in}}) - \psi_{+1\pm\frac{1}{2}}^{\downarrow\star}(x^{\text{out}}, \mathbf{k}_T^{\text{out}}) \psi_{+1\pm\frac{1}{2}}^\downarrow(x^{\text{in}}, \mathbf{k}_T^{\text{in}}), \end{aligned} \quad (25)$$

$$-\frac{k_-}{M} H_T^g(x, 0, -\Delta_T^2) = C_F \int \frac{d^2 \mathbf{k}_T}{16\pi^3} \psi_{+1-\frac{1}{2}}^{\uparrow\star}(x^{\text{out}}, \mathbf{k}_T^{\text{out}}) \psi_{+1-\frac{1}{2}}^\downarrow(x^{\text{in}}, \mathbf{k}_T^{\text{in}}) - \psi_{-1+\frac{1}{2}}^{\uparrow\star}(x^{\text{out}}, \mathbf{k}_T^{\text{out}}) \psi_{-1+\frac{1}{2}}^\downarrow(x^{\text{in}}, \mathbf{k}_T^{\text{in}}), \quad (26)$$

$$-\frac{\Delta_T^1 \Delta_T^2}{2M^2} E_T^g(x, 0, -\Delta_T^2) = C_F i \int \frac{d^2 \mathbf{k}_T}{16\pi^3} \psi_{+1+\frac{1}{2}}^{\uparrow\star}(x^{\text{out}}, \mathbf{k}_T^{\text{out}}) \psi_{-1+\frac{1}{2}}^\uparrow(x^{\text{in}}, \mathbf{k}_T^{\text{in}}) - \psi_{-1+\frac{1}{2}}^{\uparrow\star}(x^{\text{out}}, \mathbf{k}_T^{\text{out}}) \psi_{+1+\frac{1}{2}}^\uparrow(x^{\text{in}}, \mathbf{k}_T^{\text{in}}), \quad (27)$$

$$\tilde{H}_T^g(x, 0, -\Delta_T^2) = 0, \quad (28)$$

and those of the final-state wave functions are given by

where $C_F = (N_c^2 - 1)/(2N_c)$, with $N_c = 3$ being the number of the color degree of freedom. We find that $\tilde{H}_T^g(x, 0, -\Delta_T^2)$ in Eq. (28) vanishes, which is same as the case in the quark-target model. In addition, Eqs. (23)–(28) are tree-level results; hence, in principle, higher-order corrections can be carried out. Here the arguments of the initial-state wave functions are given by

$$\begin{aligned} \mathbf{k}_T^{\text{in}} &= \mathbf{k}_T - (1 - x^{\text{in}}) \frac{\Delta_T}{2}, \\ x^{\text{in}} &= \frac{x + \xi}{1 + \xi}, \end{aligned} \quad (29)$$

$$\begin{aligned} \mathbf{k}_T^{\text{out}} &= \mathbf{k}_T + (1 - x^{\text{out}}) \frac{\Delta_T}{2}, \\ x^{\text{out}} &= \frac{x - \xi}{1 - \xi}. \end{aligned} \quad (30)$$

Note that Eqs. (23)–(28) hold at $\xi = 0$, so Eqs. (29) and (30) can be simplified further.

By substituting the light-cone wave functions (17) and (22) into the overlap representation (23)–(27) of GPDs, we obtain the following analytical results:

$$\begin{aligned} H^g(x, 0, -\Delta_T^2) &= \frac{N_\lambda^2}{24\pi^3 x} \int d^2 \mathbf{k}_T \exp\left(-\frac{4\mathbf{k}_T^2 + 4xM_X^2 + (1-x)^2 \Delta_T^2}{4x(1-x)\beta_1^2}\right) \\ &\quad \times \frac{4x^2(M_X - M(1-x))^2 + (1 + (1-x)^2)(4\mathbf{k}_T^2 - (1-x)^2 \Delta_T^2)}{D^g(x, \Delta_T, \mathbf{k}_T)}, \end{aligned} \quad (31)$$

$$E^g(x, 0, -\Delta_T^2) = -\frac{N_\lambda^2}{3\pi^3} \int d^2 \mathbf{k}_T \exp\left(-\frac{4\mathbf{k}_T^2 + 4xM_X^2 + (1-x)^2 \Delta_T^2}{4x(1-x)\beta_1^2}\right) \frac{M(M_X - M(1-x))(1-x)^2}{D^g(x, \Delta_T, \mathbf{k}_T)}, \quad (32)$$

$$\begin{aligned} \tilde{H}^g(x, 0, -\Delta_T^2) &= \frac{N_\lambda^2}{24\pi^3} \int d^2 \mathbf{k}_T \exp\left(-\frac{4\mathbf{k}_T^2 + 4xM_X^2 + (1-x)^2 \Delta_T^2}{4x(1-x)\beta_1^2}\right) \\ &\quad \times \frac{4x(M_X - M(1-x))^2 + (1 + (1-x))(4\mathbf{k}_T^2 - (1-x)^2 \Delta_T^2)}{D^g(x, \Delta_T, \mathbf{k}_T)}, \end{aligned} \quad (33)$$

$$H_T^g(x, 0, -\Delta_T^2) = -\frac{N_\lambda^2}{3\pi^3} \int d^2 \mathbf{k}_T \exp\left(-\frac{4\mathbf{k}_T^2 + 4xM_X^2 + (1-x)^2 \Delta_T^2}{4x(1-x)\beta_1^2}\right) \frac{M(M_X - M(1-x))(1-x)}{D^g(x, \Delta_T, \mathbf{k}_T)}, \quad (34)$$

$$E_T^g(x, 0, -\Delta_T^2) = -\frac{N_\lambda^2(1-x)}{3\pi^3 x} \int d^2 \mathbf{k}_T \exp\left(-\frac{4\mathbf{k}_T^2 + 4xM_X^2 + (1-x)^2 \Delta_T^2}{4x(1-x)\beta_1^2}\right) \frac{\frac{M^2}{\Delta_T^1 \Delta_T^2} (4\mathbf{k}_T^1 k_T^2 - (1-x)^2 \Delta_T^1 \Delta_T^2)}{D^g(x, \Delta_T, \mathbf{k}_T)}, \quad (35)$$

where

$$D^g(x, \Delta_T, \mathbf{k}_T) = \left[\left(\mathbf{k}_T - \frac{1}{2}(1-x)\Delta_T \right)^2 + xM_X^2 - x(1-x)M^2 \right] \left[\left(\mathbf{k}_T + \frac{1}{2}(1-x)\Delta_T \right)^2 + xM_X^2 - x(1-x)M^2 \right]. \quad (36)$$

We also check that H^g and \tilde{H}^g reduce to the unpolarized PDF $f_1^g(x)$ and helicity PDF $g_1^g(x)$ as $\Delta_T \rightarrow 0$. Finally, comparing the gluon GPDs in Eqs. (31)–(35) with those in the quark-target model [29], we find that the former will reduce to the latter after the following substitutions:

$$\begin{aligned} \exp\left(-\frac{4\mathbf{k}_T^2 + 4xM_X^2 + (1-x)^2\Delta_T^2}{4x(1-x)\beta_1^2}\right) &\rightarrow 1, \\ N_\lambda &\rightarrow g, \\ M_X, M &\rightarrow m. \end{aligned}$$

IV. NUMERICAL RESULTS

In order to present the numerical results of the gluon GPDs, we need to specify the values of parameters M , M_X , β_1 , and N_λ in our model. We choose [44]

$$\begin{aligned} N_\lambda &= 5.026, & M_X &= 0.943 \text{ GeV}, \\ \beta_1 &= 2.092 \text{ GeV}, & M &= 0.938 \text{ GeV}, \end{aligned} \quad (37)$$

which were obtained from a fit of the model result to the leading-order set of the Glück-Reya-Vogt (GRV98) [51] gluon PDF. In this fit, we choose the lowest allowed scale of GRV98 parametrization as the model scale, which is $\mu_0^2 = 0.8 \text{ GeV}^2$. After integrating out \mathbf{k}_T and substituting the corresponding parameter values in Eqs. (31)–(35), we obtain the numerical results of GPDs.

In Fig. 2, we plot the GPDs $H^g(x, 0, -\Delta_T^2)$, $E^g(x, 0, -\Delta_T^2)$, $\tilde{H}^g(x, 0, -\Delta_T^2)$, $H_T^g(x, 0, -\Delta_T^2)$, and $E_T^g(x, 0, -\Delta_T^2)$ as functions of x at $\Delta_T = 0.5, 1, \text{ and } 1.5 \text{ GeV}$, respectively. It is shown that $H^g(x, 0, -\Delta_T^2)$ has two features that are different from those of the unpolarized gluon PDF $f_1^g(x)$. First, in the small- x region $H^g(x, 0, -\Delta_T^2)$ is negative, while $f_1^g(x)$ is positive in the whole region $0 < x < 1$. That is because there is an additional term $-(1-x)^2\Delta_T^2$ in the numerator of the expression (31). Second, $H^g(x, 0, -\Delta_T^2)$ vanishes as x approaches 0 when $\Delta_T \neq 0$. This is different from $f_1^g(x)$, which is nonzero in the small- x region. $H^g(x, 0, -\Delta_T^2)$ turns out to be positive in the large- x region; i.e., there is a node in the x dependence of $H^g(x, 0, -\Delta_T^2)$. Concerning the Δ_T dependence, the size of $H^g(x, 0, -\Delta_T^2)$ decreases with increasing Δ_T , and the node position moves toward higher x .

The GPDs $E^g(x, 0, -\Delta_T^2)$ and $H_T^g(x, 0, -\Delta_T^2)$ share similar shape since $E^g = xH_T^g$ in our model. That is, they are both negative in the entire x region, and the peak of the curve moves to the higher- x region with increasing Δ_T . The

GPD $\tilde{H}^g(x, 0, -\Delta_T^2)$ is negative in the smaller- x region and is positive in the larger- x region. Again there is a node in the x dependence of $\tilde{H}^g(x, 0, -\Delta_T^2)$, which is similar to the case of $H^g(x, 0, -\Delta_T^2)$. Finally, the chiral-odd GPD $E_T^g(x, 0, -\Delta_T^2)$ is positive in the entire x region. It has substantial magnitude in the small- x region, while it is largely suppressed in the region $x > 0.6$.

In order to study the impact of the initial scale applied in the fit of $f_1^g(x)$, we also perform the fit at two different scales: $\mu_0^2 = 1.2$ and $\mu_0^2 = 1.6 \text{ GeV}^2$. We then calculate the x dependences of $H(x, 0, -\Delta_T^2)$ calculated from the three fits and plot them in Fig. 3 for comparison. We find that the sizes of GPDs monotonically decrease with increasing scale. Thus, the difference of the initial scale can change the size of $H(x, 0, -\Delta_T^2)$, while it will not alter the shape of $H(x, 0, -\Delta_T^2)$.

The GPDs provide unique opportunity to explore the spin structure of the nucleon. According to Ji's sum rule [12], the following moment gives rise to gluon contribution to the nucleon spin:

$$J^g = \frac{1}{2} \int dx \{x[H^g(x, 0, 0) + E^g(x, 0, 0)]\}. \quad (38)$$

We apply our model results for $H^g(x, 0, 0)$ and $E^g(x, 0, 0)$ to perform the calculation, yielding

$$J^g = 0.190. \quad (39)$$

This result is consistent with the recent lattice result $J^g = 0.187$ calculated by the ETM Collaboration [52], and this lattice result is given in the $\overline{\text{MS}}$ scheme at the scale of 2 GeV. However, we note that the two results are obtained at different scales and may not be compared directly.

Using the GPDs H^g , E^g , and \tilde{H}^g , we also calculate the gluon OAM inside the nucleon from the expression [12]

$$L_z^g = \frac{1}{2} \int dx \{x[H^g(x, 0, 0) + E^g(x, 0, 0)] - \tilde{H}^g(x, 0, 0)\} \quad (40)$$

$$\equiv \int dx L_z^g(x). \quad (41)$$

This definition corresponds to the kinetic OAM of the gluon [53–55], and $L_z^g(x)$ denotes the x dependence of the unintegrated OAM. Our numerical result shows that $L_z^g = -0.123$, which means that the gluon kinetic OAM

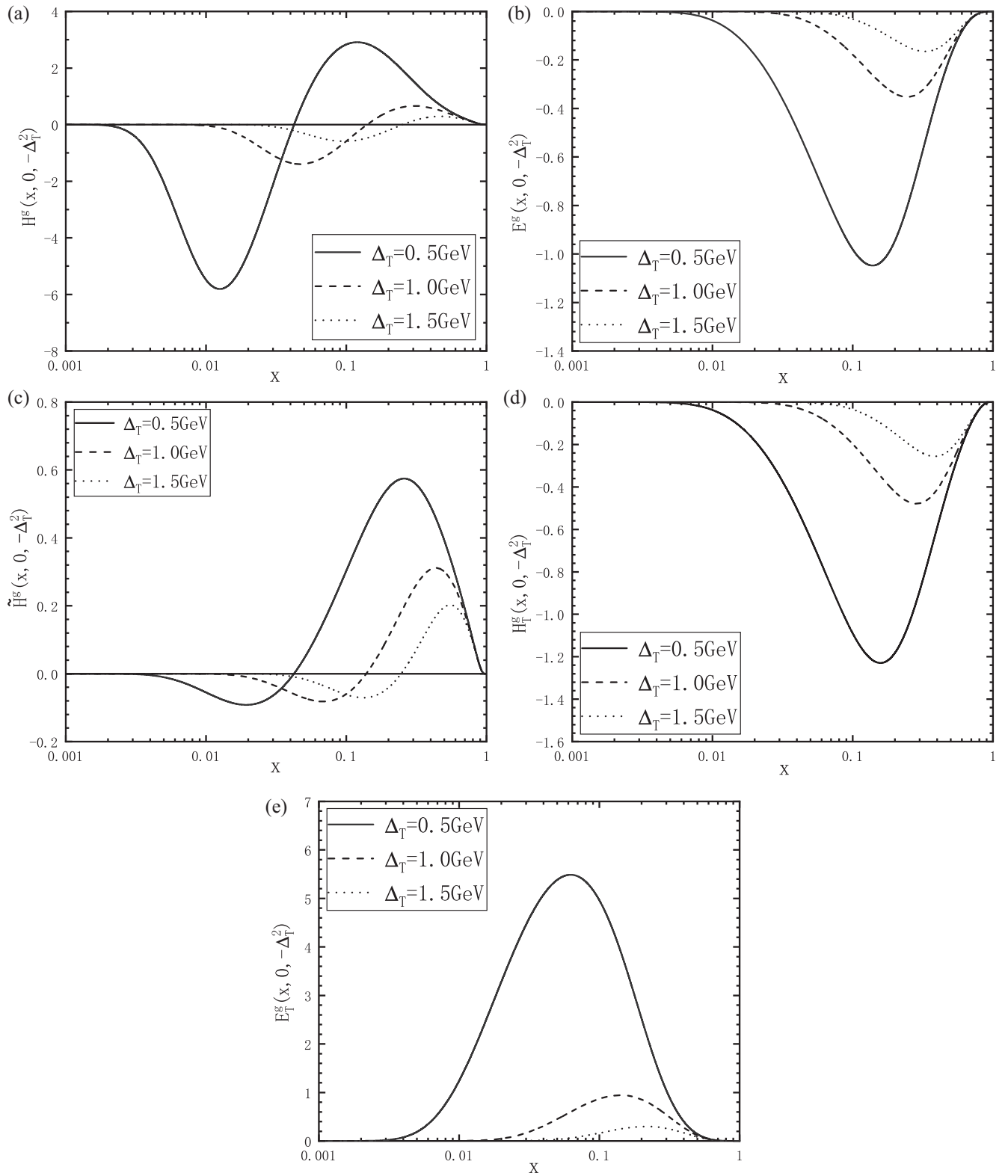


FIG. 2. The dependence of the gluon GPDs $H^g(x, 0, -\Delta_T^2)$, $E^g(x, 0, -\Delta_T^2)$, $\tilde{H}^g(x, 0, -\Delta_T^2)$, $H_T^g(x, 0, -\Delta_T^2)$, and $E_T^g(x, 0, -\Delta_T^2)$ on x at $\xi = 0$ when $\Delta_T = 0.5, 1.0, 1.5$ GeV, respectively.

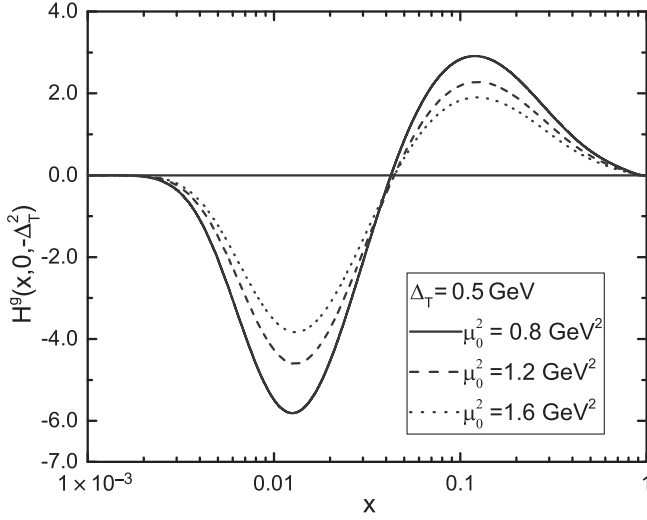


FIG. 3. The dependence of the gluon GPDs $H^g(x, 0, -\Delta_T^2)$ on the initial scale μ_0^2 used in the fit. The solid line, dashed line, and the dotted line correspond to the results with the initial scale $\mu_0^2 = 0.8, 1.2,$ and 1.6 GeV^2 , respectively.

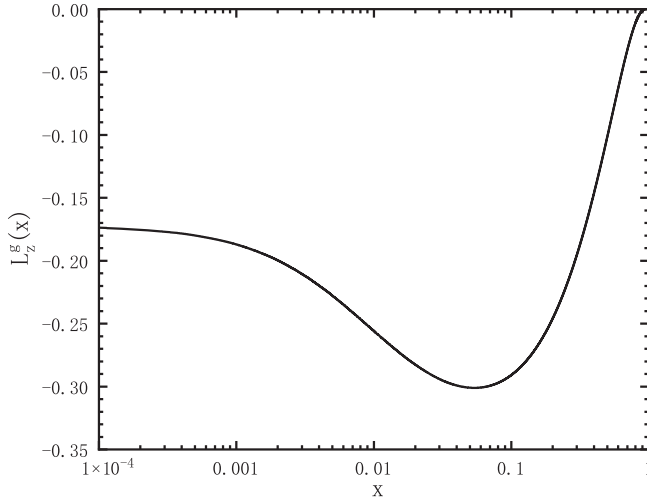


FIG. 4. The dependence of the kinetic gluon OAM $L_z^g(x)$ on x in the proton.

is negative. In order to present the contribution of the gluon OAM at different x , we plot the unintegrated OAM $L_z^g(x)$ as a function of x in Fig. 4. We find that $L_z^g(x)$ is negative in the entire $0 < x < 1$ region. It is also interesting to point out that the contribution in the very small- x region is not zero. The “distribution” peaks at around $x = 0.05$, and it

decreases rapidly when x approaches 1. Another observation is that in our model there is substantial cancellation between $xH^g(x, 0, 0)$ and $\tilde{H}^g(x, 0, 0)$ because these two GPDs are both positive. Thus, in our result the sign of $L_z^g(x)$ is almost determined by $(1/2)xE^g(x, 0, 0)$.

V. CONCLUSION

In this work, we studied the leading-twist gluon GPDs of the proton as well as the kinetic gluon OAM by employing a light-cone spectator model. In the study, we took a special kinematic point $\xi = 0$. Thus, among the eight leading-twist GPDs, H^g , E^g , \tilde{H}^g , H_T^g , E_T^g , and \tilde{H}_T^g survive in this limit. These GPDs can be expressed as the overlap of the proton wave functions for the in and out states within the light-cone formalism. In a first approximation, we treated the proton as a composite system formed by an active gluon and a spectator particle X to get the proton wave functions. We chose the Brodsky-Huang-Lepage prescription for the coupling of the proton-gluon spectator vertex to simulate the nonperturbative physics.

Based on these model assumptions, we obtained the analytic results of the GPDs H^g , E^g , \tilde{H}^g , H_T^g , and E_T^g . The GPD \tilde{H}_T^g is found to be zero in this model, similar to the case of the quark GPDs. We found that, in the intermediate- x and large- x regions, H^g and \tilde{H}^g are positive, while in the small- x region they are negative and vanish as x approaches 0 and $\Delta_T \neq 0$. This is different from the PDFs $f_1^g(x)$ and $g_1^g(x)$, which are positive in the whole region $0 < x < 1$ and are nonzero in the small- x region. Nodes were found in the x dependence of GPDs H^g and $\tilde{H}^g(x, 0, -\Delta_T^2)$. It was shown that the GPDs $E^g(x, 0, -\Delta_T^2)$ and $H_T^g(x, 0, -\Delta_T^2)$ share similar shape since $E^g = xH_T^g$ in our model, i.e., they are both negative in the entire x region. The chiral-odd GPD $E_T^g(x, 0, -\Delta_T^2)$ is positive in the entire x region and has substantial magnitude in the small- x region. Using Ji’s sum rule, we also calculated the total angular momentum of the gluon and obtained $J^g = 0.190$, which agrees with recent lattice results within uncertainty. The kinetic OAM of the gluon in the same model is found to be $L_z^g = -0.123$. Our study may provide useful theoretical constraints on the gluon GPDs and angular momentum. Further experimental measurements are needed to verify these predictions.

ACKNOWLEDGMENTS

This work is partially supported by the National Natural Science Foundation of China under Grant No. 12150013.

- [1] D. Müller, D. Robaschik, B. Geyer, F.M. Dittes, and J. Hořejši, *Fortschr. Phys.* **42**, 101 (1994).
- [2] M. Diehl, *Phys. Rep.* **388**, 41 (2003).
- [3] A. V. Belitsky and A. V. Radyushkin, *Phys. Rep.* **418**, 1 (2005).
- [4] X. D. Ji, *Phys. Rev. D* **55**, 7114 (1997).
- [5] A. V. Radyushkin, *Phys. Rev. D* **56**, 5524 (1997).
- [6] X. D. Ji and J. Osborne, *Phys. Rev. D* **58**, 094018 (1998).
- [7] X. D. Ji, *J. Phys. G* **24**, 1181 (1998).
- [8] J. Blumlein, B. Geyer, and D. Robaschik, *Nucl. Phys.* **B560**, 283 (1999).
- [9] K. Goeke, M. V. Polyakov, and M. Vanderhaeghen, *Prog. Part. Nucl. Phys.* **47**, 401 (2001).
- [10] C. Lorcé, H. Moutarde, and A. P. Trawiński, *Eur. Phys. J. C* **79**, 89 (2019).
- [11] Y. Hatta, A. Rajan, and K. Tanaka, *J. High Energy Phys.* **12** (2018) 008.
- [12] X. D. Ji, *Phys. Rev. D* **53**, 6653 (1996).
- [13] C. Lorcé, *Phys. Lett. B* **735**, 344 (2014).
- [14] M. Engelhardt, J. Green, N. Hasan, T. Izubuchi, C. Kallidonis, S. Krieg, S. Liuti, S. Meinel, J. Negele, A. Pochinsky *et al.*, *Proc. Sci. LATTICE2021* (2022) 413 [arXiv:2112.13464].
- [15] C. Lorcé, *Int. J. Mod. Phys. Conf. Ser.* **37**, 1560036 (2015).
- [16] C. Tan and Z. Lu, *Phys. Rev. D* **105**, 034004 (2022).
- [17] M. Burkardt, *Phys. Rev. D* **62**, 071503 (2000); **66**, 119903 (E) (2002).
- [18] M. Burkardt, *Int. J. Mod. Phys. A* **18**, 173 (2003).
- [19] S. Bondarenko, E. Levin, and J. Nyiri, *Eur. Phys. J. C* **25**, 277 (2002).
- [20] C. Riedl, *Acta Phys. Pol.* **53**, 1 (2022).
- [21] B. Pasquini, M. Pincetti, and S. Boffi, *Phys. Rev. D* **72**, 094029 (2005).
- [22] B. Pasquini and S. Boffi, *Phys. Rev. D* **73**, 094001 (2006).
- [23] S. Meissner, A. Metz, and M. Schlegel, *J. High Energy Phys.* **08** (2009) 056.
- [24] S. Meissner, A. Metz, M. Schlegel, and K. Goeke, *Rev. Sci. Instrum.* **80**, 114901 (2009).
- [25] T. Frederico, E. Pace, B. Pasquini, and G. Salme, *Phys. Rev. D* **80**, 054021 (2009).
- [26] M. Burkardt and B. Pasquini, *Eur. Phys. J. A* **52**, 161 (2016).
- [27] B. Pasquini, S. Rodini, and A. Bacchetta, *Phys. Rev. D* **100**, 054039 (2019).
- [28] M. V. Polyakov, *Phys. Rev. D* **73**, 094001 (2006).
- [29] S. Meissner, A. Metz, and K. Goeke, *Phys. Rev. D* **76**, 034002 (2007).
- [30] E. A. Kuraev, L. N. Lipatov, and V. S. Fadin, *Sov. Phys. JETP* **45**, 199 (1977).
- [31] I. I. Balitsky and L. N. Lipatov, *Sov. J. Nucl. Phys.* **28**, 822 (1978).
- [32] F. Gelis, E. Iancu, J. Jalilian-Marian, and R. Venugopalan, *Annu. Rev. Nucl. Part. Sci.* **60**, 463 (2010).
- [33] Y. Hatta and J. Zhou, *Phys. Rev. Lett.* **129**, 252002 (2022).
- [34] P. Kroll, *Mod. Phys. Lett. A* **35**, 2050093 (2020).
- [35] Y. Hatta and S. Yoshida, *J. High Energy Phys.* **10** (2012) 080.
- [36] S. V. Goloskokov and P. Kroll, *Eur. Phys. J. C* **59**, 809 (2009).
- [37] X. Ji, F. Yuan, and Y. Zhao, *Phys. Rev. Lett.* **118**, 192004 (2017).
- [38] Y. Hatta, Y. Nakagawa, F. Yuan, Y. Zhao, and B. Xiao, *Phys. Rev. D* **95**, 114032 (2017).
- [39] B. Pire and L. Szymanowski, *Phys. Rev. D* **96**, 114008 (2017).
- [40] S. Bhattacharya, A. Metz, and J. Zhou, *Phys. Lett. B* **771**, 396 (2017); **810**, 135866(E) (2020).
- [41] S. Bhattacharya, A. Metz, V. K. Ojha, J. Y. Tsai, and J. Zhou, *Phys. Lett. B* **833**, 137383 (2022).
- [42] B. Pire, L. Szymanowski, and J. Wagner, *Phys. Rev. D* **104**, 094002 (2021).
- [43] S. Bhattacharya, R. Boussarie, and Y. Hatta, *Phys. Rev. Lett.* **128**, 182002 (2022).
- [44] Z. Lu and B. Q. Ma, *Phys. Rev. D* **94**, 094022 (2016).
- [45] A. Bacchetta, F. G. Celiberto, M. Radici, and P. Taels, *Eur. Phys. J. C* **80**, 733 (2020).
- [46] M. Diehl, T. Feldmann, R. Jakob, and P. Kroll, *Nucl. Phys.* **B596**, 33 (2001); **605**, 647(E) (2001).
- [47] S. J. Brodsky, M. Diehl, and D. S. Hwang, *Nucl. Phys.* **B596**, 99 (2001).
- [48] S. J. Brodsky, T. Huang, and G. P. Lepage, *Springer Tracts Mod. Phys.* **100**, 81 (1982).
- [49] K. Goeke, S. Meissner, A. Metz, and M. Schlegel, *Phys. Lett. B* **637**, 241 (2006).
- [50] S. J. Brodsky, D. S. Hwang, B. Q. Ma, and I. Schmidt, *Nucl. Phys.* **B593**, 311 (2001).
- [51] M. Glück, E. Reya, and A. Vogt, *Eur. Phys. J. C* **5**, 461 (1998).
- [52] C. Alexandrou, S. Bacchio, M. Constantinou, J. Finkenrath, K. Hadjiyiannakou, K. Jansen, G. Koutsou, H. Panagopoulos, and G. Spanoudes, *Phys. Rev. D* **101**, 094513 (2020).
- [53] X. S. Chen, X. F. Lu, W. M. Sun, F. Wang, and T. Goldman, *Phys. Rev. Lett.* **100**, 232002 (2008).
- [54] M. Wakamatsu, *Phys. Rev. D* **81**, 114010 (2010).
- [55] E. Leader and C. Lorcé, *Phys. Rep.* **541**, 163 (2014).