Quark exchange effects in single flavored dibaryons

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(Received 9 May 2023; accepted 29 August 2023; published 29 September 2023)

We reveal the quark exchange effects related to both the kinetic energy and various interactions in the single flavored dibaryon bound states with $^1S_0$ in the quark models. The hadron covalent bond can be established by the shared identical quarks due to the quark exchange effect between two colorless baryons. Such hadron covalent bond plays a decisive role in the deuteronlike di-Ω_{ccc} and di-Ω_{bbb} covalent molecule states. The σ-meson exchange is indispensable in the deuteronlike di-Δ^{++} and compact di-Ω states. The hadron covalent bond clearly appears in the di-Δ^{++} state but is hidden in the di-Ω state. The chromomagnetic interaction is always repulsive in the di-Δ^{++}, di-Ω, di-Ω_{ccc}, and di-Ω_{bbb} states. The color-electric interaction is strongly attractive in the di-Ω state but weakly attractive or repulsive in the di-Δ^{++}, di-Ω_{ccc}, and di-Ω_{bbb} states.

DOI: 10.1103/PhysRevD.108.054037

I. INTRODUCTION

The nuclear force is a residual color force among colorless nucleons, much like the van der Waals forces among electric neutral molecules with the exception of their energy scale. Its typical characters are the short-range repulsion and medium-range attraction. It is a fundamental and central subject of nuclear physics and has been intensively studied since Yukawa proposed one pion exchange theory [1]. With the developments of both experiment and computational physics, one can generalize the nuclear force from the nucleon-nucleon (NN) to other dibaryon systems involving strange, charm and bottom flavors [2–14]. The generalization is significantly important for describing the nuclear force, nuclear structure and dense matter relevant to nuclear physics and astrophysics [15–17].

The Fermi-Dirac statistics requires that identical fermions must be antisymmetrized to satisfy the Pauli exclusion principle. In nuclear physics, the identical quark exchange effect between different nucleons plays a critical role in the behaviors of nuclei [18]. For example, most of European Muon Collaboration effect can be attributed to the quark exchange effect between nucleons in three-nucleon systems [19]. In molecular physics, the electrons are shared by nuclei and their delocalization is an important effect contributing to the formation of molecule covalent bond. Similarly, is there the hadron covalent bond due to the shared identical quarks originating from the quark exchange effects? The covalent hadron molecules were proposed, where the light identical quarks are assumed to be shared by the heavy quarks [20,21]. The hydrogen moleculelike T_{bb}^{-} properly manifests such hadron covalent bond [22,23].

In principle, the heavy identical quarks, if any, could also present such quark exchange effects in the heavy hadron molecules as the light identical quarks do. Admittedly, the heavy quark exchange effects are weaker than that of light identical quarks because the exchange effects should be depressed by the large mass of the heavy quarks. The single flavored dibaryons, di-Δ^{++}, di-Ω, di-Ω_{ccc}, and di-Ω_{bbb}, cover from about 2.4 GeV to 30 GeV. Such a wide energy region allows us to comprehensively address various dynamic mechanisms of the low-energy strong interactions and their quark exchange effects. Technically, the single flavored dibaryons possess the same flavor symmetry so that it is convenient to perform a systematical investigation. In the channel with $^1S_0$, the maximum attraction of the dibaryons is expected in comparison to other channels because the Pauli exclusion principle between identical quarks at short distances does not operate in this channel.

In this work, we attempt to systematically inspect the most promising single flavored dibaryon bound states and figure out their binding energy and spatial configuration from the perspective of quark models. More importantly, we prepare to unveil such quark exchange effect and analyze various underlying binding mechanisms in the dibaryon bound states very carefully.

After the introduction, the paper is organized as follows. In Sec. II we describe the quark models for nuclear force. In Sec. III we briefly introduce the trial wave functions for ground state baryons and dibaryons. In Sec. IV we present the numerical results and discussions. In the last section we list a brief summary.

II. QUARK MODELS FOR NUCLEAR FORCE

The strong interactions are widely described by quantum chromodynamics (QCD) in the standard model of particle

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physics. However, the *ab initio* calculation of the hadron spectroscopy and the hadron-hadron interaction directly from QCD is very difficult due to the complicated nonperturbative natures. Therefore, the QCD-inspired constituent quark model is a powerful implement in obtaining physical insight for these complicated strong interacting systems. We apply naive quark model (NQM) and chiral quark model (ChQM) of the Salamanca group to investigate the single flavored dibaryons in this work. Those models were developed based on the reasonable description of the natures of baryons and the NN interactions.

A. Naive quark model

Naive quark model generally includes an effective one-gluon-exchange (OGE) potential $V^{\rm oge}$ directly coming from the OGE diagram in QCD [24] and an artificial quark confinement potential $V^{\rm con}$. The model can provide a very good description of the light baryons [25,26]. In the NN interactions, the model can obtain the short-range repulsive core by the spin-spin part of the inter quark interaction between nucleons and the Pauli exclusion principle enforced by the quark structure of the nucleon [27]. However, the medium-range attraction is absent [28,29]. The model hamiltonian used in this work reads

$$H_n = \sum_{i=1}^{n}\left(m_i + \frac{\mathbf{p}_i^2}{2m_i}\right) - T_{\rm cm} + \sum_{i<j}^{n}(V_{ij}^{\rm oge} + V_{ij}^{\rm con}),$$
$$V_{ij}^{\rm oge} = \frac{\alpha_s}{4}\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j\left(\frac{1}{r_{ij}} - \frac{\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j}{6m_im_jr_0^2(\mu_{ij})r_{ij}}e^{-\frac{r_{ij}}{r_0(\mu_{ij})}}\right),$$
$$V_{ij}^{\rm con} = -a_c\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j r_{ij}^2. \tag{1}$$

m_i and $\mathbf{p}_i$ are the mass and momentum of the quark q_i, respectively. $T_{\rm cm}$ is the center-of-mass kinetic energy. $\boldsymbol{\lambda}_i$ and $\boldsymbol{\sigma}_i$ stand for the SU(3) Gell-Mann matrices and SU(2) Pauli matrices, respectively. r_{ij} is the distance between two quarks q_i and q_j and μ_{ij} is their reduced mass, $r_0(\mu_{ij}) = \frac{\hat{r}_0}{\mu_{ij}}$. The quark-gluon coupling constant α_s adopts an effective scale-dependent form,

$$\alpha_s(\mu_{ij}) = \frac{\alpha_0}{\ln\frac{\mu_{ij}^2}{\Lambda_0^2}}. \tag{2}$$

The model parameters a_c, $\hat{r}_0$, Λ_0, and α_0 can be determined by fitting the ground state baryon spectrum.

B. Chiral quark model

To achieve medium- and long-range behaviors of nuclear force, the hybrid quark model was established by introducing one π-meson exchange and one σ-meson exchange on the baryon level [30]. The effective meson-exchange potential between two nucleons were considered to simulate the effects of the meson cloud surrounding the quark core. In this way, the implementation of chiral symmetry at the quark potential level was needed for the sake of consistency [31]. The constituent quark mass appears because of the spontaneous breaking of the chiral symmetry at some momentum scale. Once a constituent quark mass is generated, such quarks have to interact through Goldstone bosons. σ-meson as well as π-meson exchanges on the quark level were introduced in the NQM, i.e., SU(2) ChQM. The model can well describe the hadron spectra, NN phase shifts and the deuteron [32–34]. Subsequently, the extended model, SU(3) ChQM, was employed to investigate the nucleon-hyperon and hyperon-hyperon interactions [35]. In the light quark sector (u, d, and s), the meson-exchange interactions V_{ij}^{π}, V_{ij}^{K}, V_{ij}^{η}, and V_{ij}^{σ} are included and the relative parameters are taken from Ref. [36]. Note that the vector meson exchange interactions are excluded to avoid the possible double counting of the short-range repulsion in the model study of the baryon-baryon interactions [37]. In the heavy quark sector (b and c), the meson-exchange interaction does not happen because the chiral symmetry is explicitly broken.

III. WAVE FUNCTIONS

The wave function of ground state baryons with isospin I and angular momentum J can be written as the direct products of color part χ_c, isospin-spin part η_{is}, and spatial part ψ,

$$\Phi_{IJ}^{B} = \chi_c \otimes \eta_{is} \otimes \psi. \tag{3}$$

The spin-flavor symmetry $\mathrm{SU_{sf}}(6) \supset \mathrm{SU_s}(2) \otimes \mathrm{SU_f}(3)$ is taken into account in the SU(3) ChQM. The spin-flavor symmetry $\mathrm{SU_{sf}}(4) \supset \mathrm{SU_s}(2) \otimes \mathrm{SU_f}(2)$ is involved in the NQM. The color singlet χ_c is antisymmetrical so that the spatial ψ must be symmetrical for identical quarks in the ground state baryons.

We define a set of Jacobi coordinates $\mathbf{r}_{ij}$, $\mathbf{r}_{ijk}$ and $\mathbf{R}_{\rm cm}$,

$$\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j, \mathbf{r}_{ijk} = \frac{m_i\mathbf{r}_i + m_j\mathbf{r}_j}{m_i + m_j} - \mathbf{r}_k, \tag{4}$$

$\mathbf{R}_{\rm cm}$ stands for the center of mass of baryons. In the center of mass frame, the symmetrical spatial wave functions of baryons composed of three identical particles can be expressed as

$$\psi = \phi(\mathbf{r}_{12})\phi(\mathbf{r}_{123}) + \phi(\mathbf{r}_{13})\phi(\mathbf{r}_{132}) + \phi(\mathbf{r}_{23})\phi(\mathbf{r}_{231}). \tag{5}$$

For baryons with only two identical particles, we just consider their antisymmetry in the simplest way because we pay more attention to the residual interaction between two colorless baryons than the properties of the individual baryon. The spatial wave function can be written as

$$\psi = \phi(\mathbf{r}_{12})\phi(\mathbf{r}_{123}), \tag{6}$$

where the quarks 1 and 2 are identical particles. For baryons with three different particles, the spatial wave function is also taken as Eq. (6), where the quarks 1 and 2 are the two light quarks. In fact, the influence of the simplification on the baryon is not obvious in comparison of the case including the Jacobi coordinate in Eq. (5).

Accurate model calculations are a primary requirement for the exact understanding the properties of dibaryons. The Gaussian expansion method (GEM) has been proven to be rather powerful to solve the few-body problem in nuclear physics [38]. According to the GEM, the relative motion wave functions $\phi(\mathbf{x})$ is expanded as the superpositions of a set of Gaussian functions with different sizes,

$$\phi(\mathbf{x}) = \sum_{n=1}^{n_{\max}} c_n N_{nl} x^l e^{-\nu_n x^2} Y_{lm}(\hat{\mathbf{x}}), \tag{7}$$

where $\mathbf{x}$ represents $\mathbf{r}_{ij}$ and $\mathbf{r}_{ijk}$. More details about the GEM can be found in Ref. [38].

The wave function of the ground state dibaryons with defined isospin-spin can be expressed as

$$\Psi_{IJ}^{\rm Dibaryon} = \sum_{\xi} c_\xi \mathcal{A}\{[\Phi^{\rm B_1}_{I_1J_1C_1}\Phi^{\rm B_2}_{I_2J_2C_2}]_{IJ}F(\boldsymbol{\rho})\}, \tag{8}$$

where $\Phi^{\rm B_1}_{I_1J_1C_1}$ and $\Phi^{\rm B_2}_{I_2J_2C_2}$ are the wave functions of the individual baryon and the subscripts C_i denote their color representations. In principe, the dibaryons should be the mixture of color singlet and hidden color octet. Here, we mainly focus on the quark exchange effect between two colorless baryons similar to the chemical covalent bond. The hidden color effect is left for the future work. $\mathcal{A}$ is antisymmetrization operator acting on the identical quarks belonging to two different baryons. ξ stands for all possible isospin-spin-color combinations $\{I_1, I_2, J_1, J_2, C_1, C_2\}$ that can be coupled into the quantum numbers of the dibaryon. The coefficients c_ξ can be determined by the dynamics of the dibaryon. $F(\boldsymbol{\rho})$ is the relative motion wave function between two baryons and can also be expanded by a set of Gaussian bases.

IV. NUMERICAL RESULTS AND DISCUSSIONS

A. Model parameters and baryon spectra

The u- and d-quark mass $m_{u,d}$ is taken to be one third of that of nucleon. With the MINUIT program [39], other model parameters can be determined by fitting ground state baryon spectrum by accurately solving the three-body Schrödinger equation. The parameters and ground state baryon spectrum are presented in Tables I and II, respectively.

In addition, we calculate the mass root-mean-square (rms) radius of quark core of baryons with their eigenvectors. The mass rms radius was defined as [40,41]

TABLE I. Model parameters. Quark masses and Λ_0 unit in MeV, a_c unit in MeV · fm^{-2}, r_0 unit in MeV · fm and α_0 is dimensionless.

Parameter	$m_{u,d}$	m_s	m_c	m_b	a_c	α_0	Λ_0	r_0
NQM	313	450	1633	4991	118	3.03	67.7	90.8
ChQM	313	500	1614	4982	45.6	3.76	21.9	95.7

$$\langle \mathbf{r}^2 \rangle^{\frac{1}{2}} = \left(\sum_{i=1}^{3} \frac{m_i \langle (\mathbf{r}_i - \mathbf{R}_{\rm cm})^2 \rangle}{m_1 + m_2 + m_3} \right)^{\frac{1}{2}}. \tag{9}$$

We list the numerical results in Table II, which are close to those in Refs. [40,41]. The mass rms radius is not an observable, but it is nevertheless a very interesting quantity, which gives the size of the baryons in the constituent quark models. In general, the mass rms radius of quark core is smaller than physical radius of baryons because the contributions from the meson cloud surrounding the valence quarks are not included in the model calculations.

B. Natures of di-Δ^{++}, di-Ω, di-Ω_{ccc}, and di-Ω_{bbb}

Binding energies. Using the well-defined trial wave function, we can obtain the eigenvalue and eigenvector of the single flavored dibaryons with $^1S_0$ by accurately solving the six-body Schrödinger equation in the quark models. Subsequently, we can arrive at their binding energy $E_b = E_6(\rho) - E_6(\infty)$, where $E_6(\rho)$ denotes the minimum of the dibaryons at the average separation ρ between two baryons and $E_6(\infty)$ is the mass of two isolated baryons in the models. Such a subtraction procedure can greatly reduce the influence of the inaccurate model parameters and hadron spectra on the binding energy, which is properly exhibited in study of the deuteronlike molecular state T_{cc}^+ [22]. To illustrate the formation mechanism of the bound dibaryons, we calculate and decompose the contribution to E_b from each part of the model Hamiltonian. We present the binding energy and various contributions in Table III.

Oka *et al.* found that the di-Δ^{++} state with $^1S_0$ cannot be bound in the similar NQM [42], which is strengthened by the present work. The di-Δ^{++} state can establish a shallow bound dibaryons with a binding energy about 8 MeV in the ChQM. The previous ChQM studies on the state indicated that it is a deep bound state with a binding energy about 10 to 50 MeV [43,44]. Quark delocalization and color screening model, where the σ-meson exchange effect is replaced with a hybrid confinement potential and quark delocalization, also gave similar results [44,45]. In one word, all of the models that provide the intermediate range attraction of nuclear force support the existence of the bound di-Δ^{++} state. Exactly, the di-Δ^{++} state is a resonance rather than a bound state in the quark models because it can decay into the $pp\pi^+\pi^+$ channel.

TABLE II. Mass spectra of baryon ground states unit in MeV and mass rms radius of quark core unit in fm. PDG is the abbreviation of particle data group. The "×" denotes that the state does not exist in the experiment.

		NQM	ChQM	PDG			NQM	ChQM	PDG
Baryon	$I(J^P)$	Mass, Radius	Mass, Radius	Mass	Baryon	$I(J^P)$	Mass, Radius	Mass, Radius	Mass
$\Delta(1232)$	$\frac{3}{2}(\frac{3}{2}^+)$	1234, 0.51	1242, 0.64	1232	$\Omega_c(2770)^0$	$0(\frac{3}{2}^+)$	2768, 0.38	2751, 0.48	2766
$\Sigma^*(1385)$	$1(\frac{3}{2}^+)$	1393, 0.50	1391, 0.61	1385	$\Xi_{cc}^{++}(3622)$	$\frac{1}{2}(\frac{1}{2}^+)$	3635, 0.33	3636, 0.43	3622
$\Xi^*(1530)$	$\frac{1}{2}(\frac{3}{2}^+)$	1537, 0.48	1521, 0.58	1530	$\Lambda_b^0(5620)$	$0(\frac{1}{2}^+)$	5624, 0.23	5607, 0.29	5620
$\Omega(1672)$	$0(\frac{1}{2}^+)$	1668, 0.47	1653, 0.55	1672	$\Sigma_b(5810)$	$1(\frac{1}{2}^+)$	5810, 0.24	5814, 0.31	5808
$N(939)$	$\frac{1}{2}(\frac{1}{2}^+)$	942, 0.47	938, 0.54	939	$\Sigma_b^*(5830)$	$1(\frac{3}{2}^+)$	5838, 0.24	5826, 0.32	5830
$\Sigma(1192)$	$1(\frac{1}{2}^+)$	1178, 0.46	1206, 0.54	1192	$\Xi_b(5792)$	$\frac{1}{2}(\frac{1}{2}^+)$	5790, 0.24	5816, 0.30	5790
$\Xi(1315)$	$\frac{1}{2}(\frac{1}{2}^+)$	1321, 0.45	1336, 0.50	1315	$\Xi_b'(5935)$	$\frac{1}{2}(\frac{1}{2}^+)$	5927, 0.25	5937, 0.32	5935
$\Lambda(1116)$	$0(\frac{1}{2}^+)$	1121, 0.46	1109, 0.51	1116	$\Xi_b(5955)$	$\frac{1}{2}(\frac{3}{2}^+)$	5955, 0.25	5949, 0.32	5955
$\Lambda_c^+(2286)$	$0(\frac{1}{2}^+)$	2288, 0.35	2270, 0.44	2285	$\Omega_b^-(6046)$	$0(\frac{1}{2}^+)$	6052, 0.25	6064, 0.32	6046
$\Sigma_c(2455)$	$1(\frac{1}{2}^+)$	2440, 0.36	2463, 0.47	2455	Ξ_{cc}^{++}	$\frac{1}{2}(\frac{3}{2}^+)$	3718, 0.34	3667, 0.44	×
$\Sigma_c(2520)$	$1(\frac{3}{2}^+)$	2517, 0.37	2493, 0.48	2520	Ξ_{bb}^0	$\frac{1}{2}(\frac{1}{2}^+)$	10244, 0.23	10264, 0.29	×
$\Xi_c(2467)$	$\frac{1}{2}(\frac{1}{2}^+)$	2462, 0.36	2485, 0.44	2466	Ξ_{bb}^0	$\frac{1}{2}(\frac{3}{2}^+)$	10277, 0.23	10277, 0.29	×
$\Xi_c'(2578)$	$\frac{1}{2}(\frac{1}{2}^+)$	2566, 0.36	2591, 0.46	2578	Ω_{ccc}	$0(\frac{3}{2}^+)$	4881, 0.32	4791, 0.39	×
$\Xi_c(2645)$	$\frac{1}{2}(\frac{3}{2}^+)$	2641, 0.37	2622, 0.48	2645	Ω_{bbb}	$0(\frac{3}{2}^+)$	14666, 0.21	14662, 0.25	×
$\Omega_c(2695)^0$	$0(\frac{1}{2}^+)$	2698, 0.36	2721, 0.46	2695					

TABLE III. Binding energy E_b and the contribution of each part in the Hamiltonian to E_b, ΔV^{con}, ΔV^{coul}, ΔV^{cm}, ΔT, ΔV^{σ}, ΔV^{π}, and ΔV^{η} are confinement term, Coulomb term, chromomagnetic term, kinetic energy, σ-, π-, and η-meson exchange term, respectively, unit in MeV. $\langle \mathbf{r}^2\rangle^{\frac{1}{2}}$ is the size of a single baryon, $\langle \boldsymbol{\rho}^2\rangle^{\frac{1}{2}}$ is the distance between two baryons and d is the distance predicted by the Heisenberg uncertainty-relation formula, unit in fm.

Dibaryon	Model	E_b	ΔV^{con}	ΔV^{coul}	ΔV^{cm}	ΔT	ΔV^{σ}	ΔV^{π}	ΔV^{η}	$\langle \mathbf{r}^2\rangle^{\frac{1}{2}}$	$\langle \boldsymbol{\rho}^2\rangle^{\frac{1}{2}}$	d
di-Δ^{++}	NQM	Unbound								0.51	∞	∞
	ChQM	−7.57	−2.99	0.69	12.79	−4.14	−27.42	12.12	1.36	0.64	2.48	2.04
di-Ω	NQM	Unbound								0.47	∞	
	ChQM	−61.66	−18.93	−20.72	33.99	44.35	−116.97	0.00	16.62	0.55	1.03	0.61
di-Ω_{ccc}	NQM	−0.54	0.13	1.91	2.56	−5.14				0.32	3.71	3.84
	ChQM	−1.16	−0.28	1.08	2.44	−4.40				0.39	2.34	2.65
di-Ω_{bbb}	NQM	−1.07	−0.25	−0.25	1.30	−1.87				0.21	1.96	1.57
	ChQM	−1.08	0.05	−0.07	1.18	−2.24				0.25	1.80	1.57

In the NQM, the di-Ω state with $^1S_0$ is unbound because of the absence of the binding mechanism. However, it becomes a deep bound state with a binding energy of about 62 MeV in the ChQM owing to the strongly σ-meson exchange. Other versions of SU(3) ChQM also preferred the deep bound di-Ω state and its binding energy is around 80–120 MeV [43,46,47]. Recently, lattice QCD predicted that the binding energy of the di-Ω state is about $1.6(6)(^{+0.7}_{-0.6})$ MeV with a large volume and nearly physical pion mass [6]. Subsequently, the quark delocalization and color screening model and QCD sum rule also suggested the existence of a loosely molecular di-Ω state [47,48]. Comparatively speaking, the ChQMs provide the strongly attraction for the di-Ω state due to the σ-meson exchange, which may be pushed down by the introduction of the vector meson exchanges. The vector meson exchanges were used to reduce the strongly attraction also induced by the σ-exchange in the doubly heavy state T_{cc}^+ [49].

With regard to the fully heavy quark systems, the NQM and ChQM do not exist any dissimilarities except for their model parameters in this work. The di-Ω_{ccc} and di-Ω_{bbb} states with $^1S_0$ can establish very shallow bound states with a binding energy around 1 MeV. Quark delocalization and color screening model also gave similar results [50]. Therefore, the shallow di-Ω_{ccc} and di-Ω_{bbb} bound states seem to be independent of quark models. The extended one-boson-exchange model including heavy meson exchange prefers to describe the di-Ω_{ccc} and di-Ω_{bbb} as shallow bound states [51]. In the lattice QCD, the di-Ω_{ccc} is a loose bound state [12] while the di-Ω_{bbb} prefers a very deep bound state [13].

Spatial configurations. We can precisely calculate the average distance, $\langle \rho^2 \rangle^{\frac{1}{2}}$ in Table III, between two baryons with the eigenvector. Combining the average distance with the mass rms radius of baryons, we figure out the spatial configuration of the dibaryon bound states. In the di-Δ^{++}, di-Ω_{ccc}, and di-Ω_{bbb} states, the average distances $\langle \rho^2 \rangle^{\frac{1}{2}}$ are obviously larger than the sum of the mass rms radius $\langle \mathbf{r}^2 \rangle^{\frac{1}{2}}$ of the corresponding baryons. They are deuteronlike states because two baryons are very far apart from each other and do not overlap entirely. The di-Ω state is a compact state rather than a loose deuteronlike state because two Ωs are partly overlapped from its $\langle \mathbf{r}^2 \rangle^{\frac{1}{2}}$ and $\langle \rho^2 \rangle^{\frac{1}{2}}$, which is supported by Ref. [46]. If taking into account the contributions from meson cloud surrounding the valence quarks to the size of Ω, two Ωs are strongly overlapped in the di-Ω state.

In general, the average distance between two baryons is related to the binding energy E_b of the dibaryon states. One can therefore roughly estimate the distance between two completely separated baryons by the Heisenberg uncertainty-relation formula [52],

$$d \approx \frac{\hbar c}{\sqrt{2\mu E_b}}, \tag{10}$$

where μ is the reduced mass of two baryons. This formula was proposed to roughly estimate the size of the state $X(3872)$ described as a $D^0\bar{D}^{*0}$ molecule [52]. For the deuteron, one can verify that the formula is effective. For the deuteronlike di-Δ^{++}, di-Ω_{ccc} and di-Ω_{bbb} states, the differences between $\langle \rho^2 \rangle^{\frac{1}{2}}$ and d are obviously smaller than the sizes of the deuteronlike states. For the compact di-Ω state, the difference is 0.42 fm so that it cannot be ignored relative to the size predicted by the formula. The di-Δ resonance $d^*(2380)$ reported by the WASA-at-COSY Collaboration is very similar to the di-Ω state because both of them are deeply bound states [53]. However, the reliable information about the spatial configuration of the state $d^*(2380)$ is unavailable so far [54]. The reliability of this formula is an open question in the estimating the size of compact multiquark states.

C. Quark exchange effects and binding mechanisms

Chromomagnetic and color-electric interactions. Both the chromomagnetic and color-electric interactions depend on the color factor $\langle \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j \rangle$ so that their contributions to the binding energy come from the quark exchange effects between two colorless objects. From Table III, one can see that the chromomagnetic interaction provide some repulsions in all of the bound single flavored dibaryons predicted by our models. The repulsion is in the order of tens MeV in the di-Δ^{++} and di-Ω states but less than 3 MeV in the di-Ω_{ccc} and di-Ω_{bbb} states due to the large mass of heavy quarks. The contributions from the color-electric interaction, i.e., the color Coulomb plus color confinement, are small in the deuteronlike di-Δ^{++}, di-Ω_{ccc}, and di-Ω_{bbb} states. The reason is that the Coulomb interaction is inverse proportional to the distance and the effective interacting range of the confinement potential is around 1 fm. For the same reason, the color-electric interaction provides a stronger attraction in the compact di-Ω state. On the whole, the chromomagnetic and color-electric interactions can just provide a small quantity of attractions even a few of repulsions. In this way, none of bound single flavored dibaryons can be produced completely by means of the chromomagnetic and color-electric interactions, which approves the conclusion about the stability of fully heavy dibaryons in the extended chromomagnetic model [55].

Meson exchange interactions. The meson exchange interactions are independent of colors. Their contributions to the binding energy come from both the direct term (main) and the quark exchange effects. The σ-meson exchange provides a strongly attraction in the both di-Δ^{++} and di-Ω states with $^1S_0$ in the ChQM while the π- and η-meson exchanges are repulsive in the states. The total contribution from the σ-, π-, and η-meson exchange is attractive. Exactly similar to the deuteron, the σ-meson exchange plays a predominant role in the formation of the di-Δ^{++} and di-Ω states with $^1S_0$ in the ChQM. The absence of σ-meson exchange in the NQM directly leads to the disappearance of the di-Δ^{++} and di-Ω bound states. The one boson exchange model based on the nuclear force was extended to predict the existence of di-Ω_{ccc} and di-Ω_{bbb} by introducing charmonium and bottomonia exchange potential [51]. In this work, the di-Ω_{ccc} and di-Ω_{bbb} states can establish bound states independence of any meson exchanges. That is to say, the meson exchanges in heavy quark sector are not indispensable in the formation of the dibaryon bound states, which implies that there may exist some novel binding mechanism.

Hadron covalent bond. Assuming the size of baryons does not change obviously in their interaction, the kinetic energy contribution ΔT to the binding energy is the sum of the relative motion part between two baryons and the exchange kinetic term introduced by exchanging identical quarks. The study on the nucleon-nucleon system indicated that the exchange kinetic term can reduce the total kinetic energy, i.e., the term is negative [56]. Hoodbhoy and Jaffe pointed out that the reduction is equivalent to a softening of the quark momentum distribution [19].

In the dibaryon systems, the identical quark exchange permits a quark in one baryon to roam into the other baryon, which can effectively expand the Hilbert space of the systems. The delocalized identical quarks are shared by

TABLE IV. Dependence of binding energy E_b and various contributions to E_b on the heavy quark mass, unit in MeV.

M_Q	Model	E_b	ΔV^{con}	ΔV^{coul}	ΔV^{cm}	ΔT
1500	NQM	−0.46	0.09	1.81	2.55	−4.90
	ChQM	−1.11	−0.31	1.10	2.50	−4.40
2000	NQM	−0.65	0.21	1.97	2.42	−5.26
	ChQM	−1.25	−0.16	1.06	2.23	−4.39
2500	NQM	−0.71	0.24	1.74	2.14	−4.84
	ChQM	−1.27	0.00	1.10	1.97	−4.34
3000	NQM	−0.75	0.18	1.25	1.89	−4.07
	ChQM	−1.22	0.10	1.08	1.75	−4.16
3500	NQM	−0.81	0.03	0.55	1.70	−3.10
	ChQM	−1.16	0.14	0.93	1.57	−3.80
4000	NQM	−0.93	−0.17	−0.11	1.56	−2.21
	ChQM	−1.11	0.13	0.61	1.41	−3.27
4500	NQM	−1.04	−0.20	−0.22	1.38	−2.00
	ChQM	−1.10	0.09	0.11	1.29	−2.59
5000	NQM	−1.08	−0.26	−0.25	1.29	−1.86
	ChQM	−1.07	0.04	−0.07	1.18	−2.22

the dibaryon so that the hadron covalent bond similar to the molecular one can establish. The most intuitive representation of such hadron covalent bond is the reduction of the total kinetic energy of the system because of the Heisenberg uncertainty relation. In other words, the hadron covalent bond can provide an effective binding mechanism. As can be seen from ΔT in Table III, the effect of the hadron covalent bond in the deuteronlike di-Δ^{++}, di-Ω_{ccc}, and di-Ω_{bbb} states conspicuously emerge because of the small relative motion energy between two remarkably separated baryons. However, the effect in the compact di-Ω state is hidden by the larger relative motion kinetic energy between two overlapped Ωs.

In the di-Δ^{++} and di-Ω states, the main binding mechanism is the σ-meson exchange or its alternative effect while the hadron covalent bond is secondary. In strong contrast, the absolute predominant binding mechanism in the di-Ω_{ccc} and di-Ω_{bbb} states is the hadron covalent bond so that we can call di-Ω_{ccc} and di-Ω_{bbb} bound states the covalent hadron molecules. Note that the large mass of heavy quarks depresses the repulsive chromomagnetic interaction, which is beneficial to establish the covalent hadron molecules.

Dependence of binding mechanisms on the heavy quark mass. In order to clear the dependence of various mechanisms on the heavy quark mass, we calculate the binding energy E_b and various contributions to E_b in the context of the heavy quark mass varying from 1500 MeV to 5000 MeV with a step size of 500 MeV. The numerical results are presented in Table IV. One can find that the chromomagnetic term ΔV^{cm} and kinetic energy term ΔT dominant the properties of singled heavy flavor dibaryon states because they directly depend on the heavy quark mass. Their signs do not change in the range of heavy quark mass. Relatively speaking, the confinement term ΔV^{con} and coulomb term ΔV^{coul} are weak and trivial for the formation of the bound dibaryon states. With the increase of heavy quark mass, the Coulomb term ΔV^{coul} is generally diminished while the confinement term ΔV^{con} first increases and then decreases. The interval span range from m_c to m_b is so large that the signs of each term are opposite in the di-Ω_{ccc} and di-Ω_{bbb} states.

V. SUMMARY

In this work, we systematically investigate the single flavored dibaryons, di-Δ^{++}, di-Ω, di-Ω_{ccc}, and di-Ω_{bbb}, with $^1S_0$ in the quark models. In the calculation, we employ the Gaussian expansion method, a high-precision numerical method. The di-Δ^{++}, di-Ω_{ccc}, and di-Ω_{bbb} states can establish the deuteronlike bound state with a binding energy about several MeV. However, the di-Ω state is a compact deep bound state with a binding energy about 62 MeV.

Similar to chemical molecule covalent bond, the hadron covalent bond between two colorless baryons can be established by the shared identical quarks induced by the identical quark exchange effects. As a novel binding mechanism, it plays a decisive role in the deuteronlike di-Ω_{ccc} and di-Ω_{bbb} states so that we call them covalent molecule states. Like the deuteron, the σ-meson exchange play a dominant role in the light di-Δ^{++} and di-Ω states. The hadron covalent bond clearly appears in the di-Δ^{++} state but is hidden in the di-Ω state by the larger relative motion kinetic energy between two overlapped Ωs. The chromomagnetic interaction is always repulsive in the single flavored dibaryon states. The color-electric interaction is strongly attractive in the di-Ω state but weakly attractive or repulsive in the other dibayon states.

ACKNOWLEDGMENTS

The author thanks Prof. S. L. Zhu for helpful discussions. This research is supported by the Fundamental Research Funds for the Central Universities under Contract No. SWU118111.

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