Impact of the chiral asymmetry and a magnetic field on the passage of an energetic test parton in a QCD medium

Ritesh Ghosh,[1,*] Mohammad Yousuf Jamal,[2,†] and Manu Kurian[3,‡]

[1]*School of Physical Sciences, National Institute of Science Education and Research, An OCC of Homi Bhabha National Institute, Jatni, Khurda 752050, India*
[2]*School of Physical Sciences, Indian Institute of Technology Goa, Ponda 403401, Goa, India*
[3]*RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA*

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We study the dependence of soft contribution to the energy loss of a test parton moving with a high velocity on the chiral imbalance and magnetic field in the QCD medium. A semiclassical approach is adopted to estimate the parton energy loss that takes into account the back-reaction on the parton due to the polarization effects of the QCD medium while traversing through the medium. We find that the motion of the parton is sensitive to the chiral asymmetry in the medium. Further, we investigate the effect of magnetic field-induced anisotropy on the energy transfer between the moving parton and the medium. Our results show that the energy loss of the parton is strongly influenced by the strength of the magnetic field as well as the relative orientation of the motion of the parton and the direction of the magnetic field in the medium.

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I. INTRODUCTION

The heavy-ion collision (HIC) experiments at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) provide a unique opportunity to produce and investigate the properties of an extremely hot and dense state of matter: the quark-gluon plasma (QGP), which is believed to resemble the state of the universe shortly after the big bang [1–4]. The success of relativistic viscous hydrodynamics in describing the evolution of the QGP medium is considered one of the theoretical breakthroughs in this area of research [5,6]. The dissipative processes and the associated transport parameters rely on the understanding of the nonequilibrium physics of the QGP. Much research has been devoted to the extraction of the transport coefficients by employing the hydrodynamics approach and data fitting methods to the observables associated with the final state particles [7,8]. The data-driven methods are currently getting wider attention in this field as the physics of the QGP is entering a high-precision science era [9–11].

The class of *hard probes* such as jets, heavy quarks, *etc* offers another possible way to characterize the properties of the QGP. These energetic patrons penetrate and travel through the QGP and lose their energy due to several interactions in the medium. Therefore, proper knowledge of the energy loss of a fast-moving parton is essential for the quantitative description of the jet quenching phenomenon. The theoretical modeling of the energy loss mechanism of the energetic partons and its dependence on various observables of collision experiments play a significant role in elucidating the underlying physics. Several efforts have been made to understand the transport properties of the QGP through the investigation of jets while considering both the collisional and radiative processes of the energetic parton [12–18]. The heavy quarks are another promising candidates that can probe the evolution history of the QGP as they are created in the very initial stages of the HIC [19–25]. The motion of the energetic partons and their energy loss depend on the properties of the medium. There have been some attempts to explore the parton interaction with the nonequilibrium plasma or unstable medium [26–32].

The chiral anomaly and parity-violating effects have recently attracted substantial theoretical and experimental interest in the study of the QGP and strong interaction. The interaction of the chiral fermions/quarks with the nontrivial gluonic field produces asymmetry between left- and right-handed chiral fermions that yield chiral imbalance [33]. Furthermore, the recent observations [34,35] and the associated studies [36,37] of the enhanced directed flow of heavy flavor mesons revealed the existence of a strong magnetic field (for reviews on the strong magnetic field in

[*]riteshghosh1994@gmail.com
[†]mohammad@iitgoa.ac.in
[‡]mkurian@bnl.gov

collision experiments, see Refs. [38–41]) in the initial stages of the HICs. This generated magnetic field breaks the rotational symmetry of the medium, and induces anisotropy in the produced medium that, in turn, affects the thermodynamic and transport properties of the QCD medium as the charged particle motion is influenced by the field [42–53]. References [54–56] showed that the charm quark momentum diffusion critically depends on the preferred direction and strength of the magnetic field. The magnetic field, together with the chiral imbalance, leads to novel phenomena such as chiral magnetic effect (CME) and is one of the active areas of ongoing studies in contemporary physics [41,57–60]. The dependence of collisional energy loss on the helicity of the fermion traversing the plasma has been recently studied in Ref. [61]. Both the magnetic field and chiral imbalance are expected to influence the passage of partons through the medium, as the momentum broadening and energy loss of energetic partons depend on the properties of the medium. It is an exciting task to study the impact of chiral asymmetry and magnetic field on the energy loss pattern of the moving parton in the medium, and this sets the motivation for the present study.

A large fraction of the total energy loss of the equilibrated plasma is carried by particles with momentum in order $\sim T$ where T is the temperature of the medium (hard modes). In addition, there are gauge fields (soft modes) in the plasma with momentum $\sim gT$ in which g is the coupling constant. The soft modes can be considered as classical fields as they are highly occupied (Ref. [62]). While traversing through the medium, the test parton interacts with hard and soft modes of the QGP. It is important to emphasize that the energetic test parton initial momentum is considered much larger than T. The hard contribution of parton energy loss that originates from the elastic collisions with the plasma constituents (with momentum $\sim T$) and inelastic collision (radiative processes) have been well studied in several studies [12,63]. On the other hand, the soft contribution of the parton energy loss has received much less attention in comparison to contributions from the hard modes. This is due to the fact that soft modes carry a small fraction of total plasma energy. But the interaction frequency of classical fields with test parton is non-negligible due to their high occupation number in the plasma. Physically, the soft part of energy loss corresponds to the interaction of the parton with soft collective excitations of the medium. The authors of Refs. [27,62] have shown that the soft contribution to energy loss plays an important role in the total energy loss of the test parton in the medium and in the phenomenology of jet quenching.

In the current work, for the first time, we analyze the impact of chiral imbalance, strength, and orientation of the magnetic field on the soft contribution of the parton energy loss. A semiclassical approach is utilized to set up the formalism for energy loss experienced by an energetic parton that incorporates its interactions with the chromodynamic fields in the QCD medium. The motion of the test parton in plasma is described with Wong's equations by treating it as a classical particle with $SU(N_c)$ color charge. The change in the color field configuration due to the passage of the parton is embedded through the linearized Yang-Mills equations. We consider three different choices of plasma, namely (i) isotropic, (ii) chiral asymmetric, and (iii) magnetized QCD medium. While traversing the medium, the backreaction exerted on the parton medium is taken into account by analyzing the polarization effects of the medium. We observe that the chiral asymmetry affects the parton energy loss mechanism. Further, we show that the presence of the magnetic field suppresses the energy loss. Another crucial finding is the magnetic field-induced anisotropy in the parton energy loss, which indicates that the relative orientation of the parton's motion and the magnetic field's direction strongly influences the energy loss.

We organize the manuscript as follows. In Sec. II, we present the formalism for the energy loss of an energetic parton moving in three different choices of QCD medium. Section III is devoted to the results and discussions. Finally, we summarize the analysis with an outlook in Sec. IV.

Notations and conventions: In the present analysis, we consider $c = k_B = \hbar = 1$, $g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$, $N_c = 3$, and $N_f = 2$. The quantity q_f denotes the particle's electric charge with flavor f. A four-vector is defined as $X^\mu = (x^0, |\mathbf{x}|)$ with the component of the three-vector is described with the Latin indices x^i where $i = (1, 2, 3)$.

II. FORMALISM

A highly energetic test parton, while passing through the QGP medium, loses energy due to its interactions with the color fields. The energy loss experienced by the parton can be measured through the work of the retarding forces acting on the parton in the medium from the induced chromoelectric field that generated due to its motion. The dynamics of the test parton in chromodynamic fields can be described with Wong's equations [64] that in the Lorentz covariant form given as,

$$\frac{dX^\mu(\tau)}{d\tau} = V^\mu(\tau), \tag{1}$$

$$\frac{dP^\mu(\tau)}{d\tau} = g_s q^a(\tau) F_a^{\mu\nu}(X(\tau)) V_\nu(\tau), \tag{2}$$

$$\frac{dq^a(\tau)}{d\tau} = -g_s f^{abc} V_\mu(\tau) A_b^\mu(X(\tau)) q_c(\tau), \tag{3}$$

with τ, $X^\mu(\tau)$, $V^\mu(\tau)$, and $P^\mu(\tau)$ as the proper time, position, velocity, and momentum of the test parton. Here, g_s denotes the strong running coupling constant; q_a is the parton color charge; $F^{\mu\nu}$ represents chromodynamic field tensor; A^μ is the gauge potential; f^{abc} describes the structure constant of $SU(N_c)$ group; and a defines the color index with $a = 1, 2, \ldots, N_c^2 - 1$. We follow two assumptions to solve Wong's equations; first, we choose the gauge condition $V_\mu A_a^\mu(X) = 0$, which indicates the gauge potential vanishes on the particle's trajectory [27,65]. Second, we consider that the energy of the moving parton is comparatively much larger than its energy loss in the medium [65,66]. Next, considering $\mu = 0$ component in Eq. (2) and replacing proper time with time $t = \gamma\tau$, we get,

$$\frac{dE}{dt} = g_s q^a \mathbf{v} \cdot \mathbf{E}_a(t, \mathbf{x} = \mathbf{v}t), \tag{4}$$

where $\mathbf{E}_a(t, \mathbf{x} = \mathbf{v}t)$ is the chromo-electric field induced due to the motion of the parton with the energy, E and velocity, $\mathbf{v}$. The energy loss can also be described in terms of the color current $\mathbf{j}_a$ generated by the energetic moving parton as,

$$\frac{dE}{dt} = \int d^3\mathbf{x} \mathbf{E}_a(t, \mathbf{x}) \cdot \mathbf{j}_a(t, \mathbf{x}), \tag{5}$$

with $\mathbf{j}_a(t, \mathbf{x}) = g_s q^a \mathbf{v} \delta^{(3)}(\mathbf{x} - \mathbf{v}t)$. The form of $\mathbf{E}_a$ generated due to the test parton motion in the medium can be obtained by solving the linearized Yang-Mills equation. Employing the conventional way, i.e., Fourier transforming the linearized differential equation to algebraic forms, the induced chromo-electric can be obtained as [67,68],

$$\mathrm{E}_a^i(K) = i\omega \Delta^{ij}(K) j_a^j(K), \tag{6}$$

where $K^\mu = (\omega, \mathbf{k})$. The gluon propagator $\Delta^{ij}(K)$ and the external current $j_a^j(K)$ take the following forms [69],

$$\Delta^{ij} = [(|\mathbf{k}|^2 - \omega^2)\delta^{ij} - k^i k^j + \Pi^{ij}(K)]^{-1}, \tag{7}$$

$$j_a^j(K) = \frac{i g_s q^a v^j}{\omega - \mathbf{k} \cdot \mathbf{v} + i0^+}. \tag{8}$$

Here, $\Pi^{ij}(K)$ is the gluon self-energy tensor that captures the medium effects. Using Eqs. (7) and (8) in Eq. (6), one can obtain the induced field, and that, in turn, gives the change of parton energy in the Fourier space. Next, to get back to the coordinate space, we perform the inverse Fourier transformation, which after averaging over color indices and completing ω-integration using the residue theorem (as the integrand has a pole at $\omega = \mathbf{k} \cdot \mathbf{v}$) can be written as,

$$\left\langle \frac{dE}{dx} \right\rangle = i \frac{1}{|\mathbf{v}|} g_s^2 C_F v^i v^j \int \frac{d^3\mathbf{k}}{(2\pi)^3} \omega \Delta^{ij}, \tag{9}$$

where C_F is the Casimir invariant of $SU(N_c)$ and $\omega = \mathbf{k} \cdot \mathbf{v}$. Notably, the change of energy of the parton depends on its velocity and the gluon propagator. The latter has a strong dependence on the choice of the medium. Depending on the medium properties and initial conditions, the energy change can be negative or positive. If parton loses energy while interacting with the medium, $-\langle \frac{dE}{dx} \rangle$ should be positive; otherwise, if in some special cases, the test parton gains energy, $-\langle \frac{dE}{dx} \rangle$ should be negative [28,66]. Next, we shall discuss the gluon propagator (that depends of the gluon self-energy or the dielectric permittivity of the medium) and the energy loss mechanism for three different systems.

A. Moving parton in isotropic medium

For an isotropic medium, the gluon self-energy tensor can be decomposed in terms of longitudinal projection operator $B^{\mathrm{ij}} = \frac{k^i k^j}{|\mathbf{k}|^2}$ and transverse projection operator $A^{\mathrm{ij}} = \delta^{\mathrm{ij}} - \frac{k^i k^j}{|\mathbf{k}|^2}$ as,

$$\Pi^{ij}(K) = A^{ij} \Pi_T(K) + B^{ij} \Pi_L(K), \tag{10}$$

where Π_L and Π_T are the longitudinal and transverse form factors, respectively. The $\Pi^{ij}(K)$ can be related to the induced current in the medium via linear response theory. The current induced in the medium can be quantified in terms of the deviation of the medium particle distribution functions that can be obtained by solving the Boltzmann-Vlasov transport equation. Hence, the form factors can be obtained by solving the transport equation and employing linear response theory and can be expressed as [70–72],

$$\Pi_T(K) = m_D^2 \frac{\omega^2}{2|\mathbf{k}|^2} \left[1 - \frac{K^2}{2\omega|\mathbf{k}|} \ln\left(\frac{\omega + |\mathbf{k}|}{\omega - |\mathbf{k}|} \right) \right], \tag{11}$$

$$\Pi_L(K) = -m_D^2 \frac{\omega^2}{k^2} \left[1 - \frac{\omega}{2k} \ln\left(\frac{\omega + k}{\omega - k} \right) \right], \tag{12}$$

where $K^2 = \omega^2 - |\mathbf{k}|^2$ and $m_D^2 = (N_f + 2N_c)\frac{g_s^2 T^2}{6}$ is the Debye screening mass. Employing Eq. (10) in Eq. (7), the gluon propagator for an isotropic medium can be written as,

$$\Delta^{ij} = \frac{1}{C_T} A^{ij} + \frac{1}{C_L} B^{ij}, \tag{13}$$

with $C_T = -\omega^2 + |\mathbf{k}|^2 + \Pi_T$ and $C_L = -\omega^2 + \Pi_L$. By substituting Eq. (13) in Eq. (9), the loss of the moving parton in an isotropic medium can be obtained as,

$$-\left\langle \frac{dE}{dx} \right\rangle = \frac{\alpha_s C_F}{2\pi^2 |\mathbf{v}|} \int d^3\mathbf{k} \frac{\omega}{|\mathbf{k}|^2} \{ \omega^2 \mathrm{Im}(-\omega^2 + \Pi_L)^{-1} + (|\mathbf{k}|^2 |\mathbf{v}|^2 - \omega^2) \mathrm{Im}(-\omega^2 + k^2 + \Pi_T)^{-1} \}_{\omega = \mathbf{k} \cdot \mathbf{v}}. \tag{14}$$

The medium effects are incorporated through the form factors of the gluon self-energy and coupling constant.

We can rewrite Eq. (14) (as estimated in other parallel studies [67,73,74]) in terms of the longitudinal $\epsilon_L(K)$ and transverse $\epsilon_T(K)$ components of the dielectric permittivity $\epsilon^{ij}(K)$ of the medium as,

$$-\left\langle\frac{dE}{dx}\right\rangle = \frac{\alpha_s C_F}{2\pi^2|\mathbf{v}|}\int d^3\mathbf{k}\,\frac{\omega}{|\mathbf{k}|^2}\{\mathrm{Im}(\epsilon_L(K))^{-1} + (|\mathbf{k}|^2|\mathbf{v}|^2-\omega^2)\mathrm{Im}(\omega^2\epsilon_T(K)-|\mathbf{k}|^2)^{-1}\}_{\omega=\mathbf{k}\cdot\mathbf{v}}, \tag{15}$$

where the relation between gluon self-energy and dielectric permittivity is taken as,

$$\epsilon^{ij}(K) = \delta^{ij} - \frac{1}{\omega^2}\Pi^{\mathrm{ij}}(K). \tag{16}$$

It is important to emphasize that the Eqs. (14) and (15) describe the energy loss due to the polarization effects of the medium, i.e., the change in parton energy due to its interaction with collective excitations of the medium. For the numerical calculation, we took the upper limit of the integration $k_{\max}\sim E$, i.e., the initial energy of the parton. For the longitudinal boost–invariant expansion (noninteracting QGP), the temperature evolution takes the following form [75]

$$T(\tau) = T_0\left(\frac{\tau_0}{\tau}\right)^{1/3}, \tag{17}$$

where τ is proper time with $T_0 \equiv T(\tau_0) = 600$ MeV for $\tau_0 = 0.25$ fm for the LHC energy. The effects of medium expansion on the energy loss can enter through the temperature profile of the medium. The detailed description is presented in Ref. [24] for charm quarks. It is important to note that the medium expansion can further give rise to nonequilibrium effects on the parton motion. The nonequilibrium effects to parton energy in the evolving medium are discussed in the Appendix in detail.

B. Moving parton in chiral imbalance medium

The asymmetry between right-handed and left-handed fermions can be quantified in terms of the chiral chemical potential, $\mu_5 \equiv \mu_R - \mu_L$. The chiral plasma with a finite μ_5 can be described within the kinetic theory with Berry curvature terms [76–79]. In the small gauge field A^μ, using the linear response theory, we can express $\Pi^{\mu\nu}$ in terms of induced current density in the chiral medium, which can be obtained from the linearized transport equation with Berry curvature correction. The parity-violating terms are entering through the Berry curvature terms, and Π^{ij} with finite μ_5 can be described as [79],

$$\Pi^{ij}(K) = \Pi^{ij}_+(K) + \Pi^{ij}_-(K), \tag{18}$$

where $\Pi^{ij}_+(K)$ and $\Pi^{ij}_-(K)$ are parity even and parity odd parts of self-energy, respectively, and can be defined as,

$$\Pi^{ij}_+(K) = \frac{m_D^2\omega}{4\pi}\int d\mathbf{u}\,\frac{u^i u^j}{U\cdot K + i\epsilon}, \tag{19}$$

$$\Pi^{ij}_-(K) = \frac{\mu_5 g_s^2}{4\pi^2}\,i\epsilon^{ijk}k^k\left(1-\frac{\omega^2}{|\mathbf{k}|^2}\right)\left[1-\frac{\omega}{2|\mathbf{k}|}\ln\left(\frac{\omega+|\mathbf{k}|}{\omega-|\mathbf{k}|}\right)\right]. \tag{20}$$

Here, $U^\mu = (1,\mathbf{u})$ with $\mathbf{u} = \frac{\mathbf{k}}{|\mathbf{k}|}$, the indices i, j, k represent the spatial components, and the Debye mass in the chiral medium takes the form as,

$$m_D^2 = (N_f + 2N_c)\frac{g_s^2 T^2}{6} + N_f\frac{g_s^2}{2\pi^2}(\mu_R^2+\mu_L^2). \tag{21}$$

Due to the parity-violating term, an additional projector operator is required for the tensorial decomposition of Π^{ij} in the chiral medium as compared to the isotropic case. At finite μ_5, gluon propagator can be written as [79],

$$\Delta^{ij} = \frac{C_T}{C_T^2 - C_A^2}A^{ij} + \frac{1}{C_L}B^{ij} - \frac{C_A}{C_T^2 - C_A^2}C^{ij}, \tag{22}$$

where $C^{ij} = i\epsilon^{ijk}\frac{k^k}{|\mathbf{k}|}$ is the antisymmetric tensor and C_A that denotes the parity violating component of the self-energy takes the form as,

$$C_A \equiv \Pi_A = \mu_5\frac{g_s^2|\mathbf{k}|}{4\pi^2}\left(1-\frac{\omega^2}{|\mathbf{k}|^2}\right)\left[1-\frac{\omega}{2|\mathbf{k}|}\ln\left(\frac{\omega+|\mathbf{k}|}{\omega-|\mathbf{k}|}\right)\right]. \tag{23}$$

Notably, C_A is proportional to chiral chemical potential and in the limit $\mu_5 = 0$, Eq. (22) reduce back to the result of isotropic case as described in Eq. (13). Energy loss of the parton can be influenced by the chiral asymmetry in the medium and can be quantified in terms Δ^{ij}. Employing Eq. (22) in Eq. (9), the energy loss of the parton in a chiral imbalance plasma can be written as,

$$\left\langle\frac{dE}{dx}\right\rangle = \frac{ig_s^2 C_F}{|\mathbf{v}|}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{\omega}{|\mathbf{k}|^2}\left\{\omega^2\frac{1}{C_L} + (|\mathbf{k}|^2|\mathbf{v}|^2-\omega^2)\times\frac{C_T}{C_T^2-C_A^2} - i|\mathbf{k}|\mathbf{v}\cdot(\mathbf{v}\times\mathbf{k})\frac{C_A}{C_T^2-C_A^2}\right\}_{\omega=\mathbf{k}\cdot\mathbf{v}}. \tag{24}$$

By substituting the forms of C_L, C_T, and C_A in Eq. (24) we obtain,

$$-\left\langle\frac{dE}{dx}\right\rangle = \frac{\alpha_s C_F}{2\pi^2|\mathbf{v}|}\int d^3\mathbf{k}\,\frac{\omega}{|\mathbf{k}|^2}\left\{\omega^2 \mathrm{Im}(-\omega^2+\Pi_L)^{-1} + (|\mathbf{k}|^2|\mathbf{v}|^2-\omega^2)\mathrm{Im}\left(\frac{-\omega^2+|\mathbf{k}|^2+\Pi_T}{(-\omega^2+|\mathbf{k}|^2+\Pi_T)^2-\Pi_A^2}\right)\right\}_{\omega=\mathbf{k}\cdot\mathbf{v}} \quad (25)$$

The energy loss term that originates from contraction of the antisymmetric projection operator C^{ij} given in Eq. (22) vanishes, however, the chiral effects still enter through the Debye mass and parity violating scalar component of the self-energy Π_A. In the case of vanishing chiral chemical potential, Eq. (25) reduces back to the form of test parton energy loss expression for an isotropic case as defined in Eq. (14).

C. Moving partons in magnetized QCD medium

The magnetic field is a source of anisotropy in the system, which will be reflected in the medium's QCD thermodynamics and transport processes. The motion of the test parton will be affected by the strength **B** and direction of the magnetic field **n** in the medium. In the magnetized medium, the tensor basis for a symmetric second-rank tensor will be different from that of an isotropic medium due to the inclusion of the additional magnetic field vector. Using vectors k^i and n^i with $n^2 = 1$, we can construct the spacial part of the gluon self-energy. In the analysis, we consider various cases with different directions of the test parton velocity **v** to analyze the impact of the relative orientation of **v** and **n** on the parton energy loss. In the presence of a magnetic field, Π^{ij} can be decomposed in terms of basis tensors which depend on k^i and n^i as,

$$\Pi^{ij} = aN^{ij} + bB^{ij} + cR^{ij} + dQ^{ij}, \quad (26)$$

where the projection operators can be defined as follows,

$$N^{ij} = -\frac{\hat{k}^i\tilde{n}^j + \hat{k}^j\tilde{n}^i}{\sqrt{\tilde{n}^2}}, \quad (27)$$

$$B^{ij} = \frac{k^i k^j}{|\mathbf{k}|^2}, \quad (28)$$

$$R^{ij} = -\delta^{ij} + \frac{k^i k^j}{|\mathbf{k}|^2} - \frac{\tilde{n}^i\tilde{n}^j}{\tilde{n}^2}, \quad (29)$$

$$Q^{ij} = \frac{\tilde{n}^i\tilde{n}^j}{\tilde{n}^2}. \quad (30)$$

In the presence of a magnetic field, defining $\tilde{n}^i = A^{ij}n^j$ vector to describe the tensor basis is convenient. Here, a, b, c, and d are Lorentz-invariant form factors and can be obtained from the following relations,

$$a = \frac{1}{2}N^{ij}\Pi^{ij}, \quad (31)$$

$$b = B^{ij}\Pi^{ij}, \quad (32)$$

$$c = R^{ij}\Pi^{ij}, \quad (33)$$

$$d = Q^{ij}\Pi^{ij}. \quad (34)$$

One can get the similar forms of the tensor structures in presence of the momentum anisotropy as well with n^i representing the anisotropic direction [80,81]. The form factors can be calculated from the one-loop gluon self-energy. The fermion loop is shown in Fig. 1, which is affected by the magnetic field. We are considering the strong magnetic field limit with the energy hierarchy $T < \sqrt{|q_f B|}$. The form factors containing the gluon loop, ghost loop, and magnetic field modified quark loop contributions are given as [82],

$$a = \sum_f \frac{g_s^2 q_f B}{4\pi^2} e^{-k_\perp^2/2q_f B}\frac{\omega k_z}{\omega^2-k_z^2}\sqrt{\frac{k_\perp^2 K^4}{\omega^2|\mathbf{k}|^4}}, \quad (35)$$

$$b = \frac{N_c g_s^2 T^2}{3}\left(\frac{K^4}{\omega^2|\mathbf{k}|^2}\right)[1-\mathcal{T}_K(\omega,|\mathbf{k}|)] - \sum_f \frac{g_s^2 q_f B}{4\pi^2}\left(\frac{K^4}{\omega^2|\mathbf{k}|^2}\right)e^{-k_\perp^2/2q_f B}\frac{k_z^2}{\omega^2-k_z^2}, \quad (36)$$

$$c = \frac{N_c g_s^2 T^2}{3}\frac{1}{2}\left[\frac{\omega^2}{|\mathbf{k}|^2} - \frac{K^2}{|\mathbf{k}|^2}\mathcal{T}_K(\omega,|\mathbf{k}|)\right], \quad (37)$$

$$d = \frac{N_c g_s^2 T^2}{3}\frac{1}{2}\left[\frac{\omega^2}{|\mathbf{k}|^2} - \frac{K^2}{|\mathbf{k}|^2}\mathcal{T}_K(\omega,|\mathbf{k}|)\right] + \sum_f \frac{g_s^2 q_f B}{4\pi^2}\left(\frac{K^4}{\omega^2|\mathbf{k}|^2}\right)e^{-k_\perp^2/2q_f B}\frac{k_z^2}{\omega^2-k_z^2}, \quad (38)$$

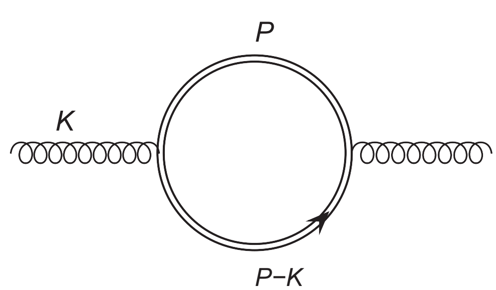

FIG. 1. One loop gluon self-energy. The double line denotes the modified quark propagator in the presence of a strong magnetic field.

where the angular integral reads as $\mathcal{T}_K(\omega,|\mathbf{k}|)=\int\frac{d\Omega}{4\pi}\frac{\omega}{\omega-\mathbf{k}\cdot\hat{p}}$ with $\hat{p}$ is unit vector along $\mathbf{p}$, i.e., $\hat{p}=\mathbf{p}/|\mathbf{p}|$ and $k_\perp=\sqrt{k_x^2+k_y^2}$. The general structure of the gluon effective propagator in the magnetized medium can be written as,

$$\Delta^{ij}=C_1B^{ij}+C_2R^{ij}+C_3Q^{ij}+C_4N^{ij}, \tag{39}$$

where $C_{1,..4}$ are related to the form factors as,

$$\begin{aligned}C_1&=\frac{K^2-d}{(K^2-b)(K^2-d)-a^2},\\ C_2&=\frac{1}{K^2-c},\\ C_3&=\frac{K^2-b}{(K^2-b)(K^2-b)-a^2},\\ C_4&=\frac{a}{(K^2-b)(K^2-d)-a^2}.\end{aligned} \tag{40}$$

Employing Eq. (39) in Eq. (9), we can estimate the parton energy change in the magnetized medium as,

$$\begin{aligned}\left\langle\frac{dE}{dx}\right\rangle=\frac{ig_sC_F}{|\mathbf{v}|}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\omega\Bigg\{&C_1\frac{\omega^2}{|\mathbf{k}|^2}-C_2\Bigg(|\mathbf{v}|^2-\frac{\omega^2}{|\mathbf{k}|^2}\\ &+\frac{1}{\tilde{n}^2}(\mathbf{v}\cdot\mathbf{n})^2+2\frac{k_z\omega}{\tilde{n}^2|\mathbf{k}|^2}(\mathbf{v}\cdot\mathbf{n})+\frac{k_z^2\omega^2}{|\mathbf{k}|^4\tilde{n}^2}\Bigg)\\ &+\frac{C_3}{\tilde{n}^2}\left((\mathbf{v}\cdot\mathbf{n})^2+2\frac{k_z\omega}{|\mathbf{k}|^2}(\mathbf{v}\cdot\mathbf{n})+\frac{k_z^2\omega^2}{|\mathbf{k}|^4}\right)\\ &-\frac{2C_4}{\sqrt{\tilde{n}^2}}\frac{\omega}{|\mathbf{k}|}\left((\mathbf{v}\cdot\mathbf{n})+\frac{k_z\omega}{|\mathbf{k}|^2}\right)\Bigg\}.\end{aligned} \tag{41}$$

Here, we define $(\mathbf{k}\cdot\mathbf{n})=k_z$ as the magnetic field is fixed along the $\hat{z}$-direction in the present analysis. The term $(\mathbf{v}\cdot\mathbf{n})$ describes the relative orientation of the parton with respect to the direction of the magnetic field and has a direct influence on the energy loss. This can give rise to anisotropy in the energy loss of the parton in the magnetized medium. In addition to that, the magnetic field effects are entering through the form factors as described in Eqs. (35)–(38). For the perpendicular case, where the parton is moving transverse to the direction of the magnetic field, i.e., $(\mathbf{v}\cdot\mathbf{n})=0$, Eq. (41) can be further simplified as,

$$\begin{aligned}\left\langle\frac{dE}{dx}\right\rangle=\frac{ig_sC_F}{|\mathbf{v}|}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\omega\Bigg\{&C_1\frac{\omega^2}{|\mathbf{k}|^2}-C_2\left(|\mathbf{v}|^2-\frac{\omega^2}{|\mathbf{k}|^2}\right)\\ &+(C_3-C_2)\frac{k_z^2\omega^2}{|\mathbf{k}|^4\tilde{n}^2}-\frac{2C_4}{\sqrt{\tilde{n}^2}}\frac{k_z\omega^2}{|\mathbf{k}|^3}\Bigg\}.\end{aligned} \tag{42}$$

Next, we shall discuss the results obtained from various sections.

III. RESULTS

Our primary findings are the energy loss of a test parton in the presence of chiral imbalance and a strong magnetic field in the QGP medium. The impact of chiral asymmetry and medium temperature over the energy loss of the fast-moving parton within the massless limit is depicted in Fig. 2 (left panel). In this figure, the imbalance between right-handed and left-handed fermions is measured using a chiral chemical potential $\mu_5=\mu_R-\mu_L$. For the quantitative estimation, we consider $\mu_L^2+\mu_R^2=\frac{1}{2}(\mu_5^2+\mu_V^2)$ with $\mu_V=\mu_R+\mu_L$. The induced current due to the passage of the parton depends on the chirality of the medium and is reflected in the energy loss mechanism as described in Eq. (25). The passage of the energetic parton is influenced by the chiral effects of the medium that enter through the medium's screening mass and parity-violating component of the self-energy Π_A as defined in Eqs. (21) and (23), respectively. Similar to the case of an isotropic medium, the parton energy loss in the chiral imbalance medium increases with an increase in the temperature of the medium. The ratio of energy loss of a massless parton

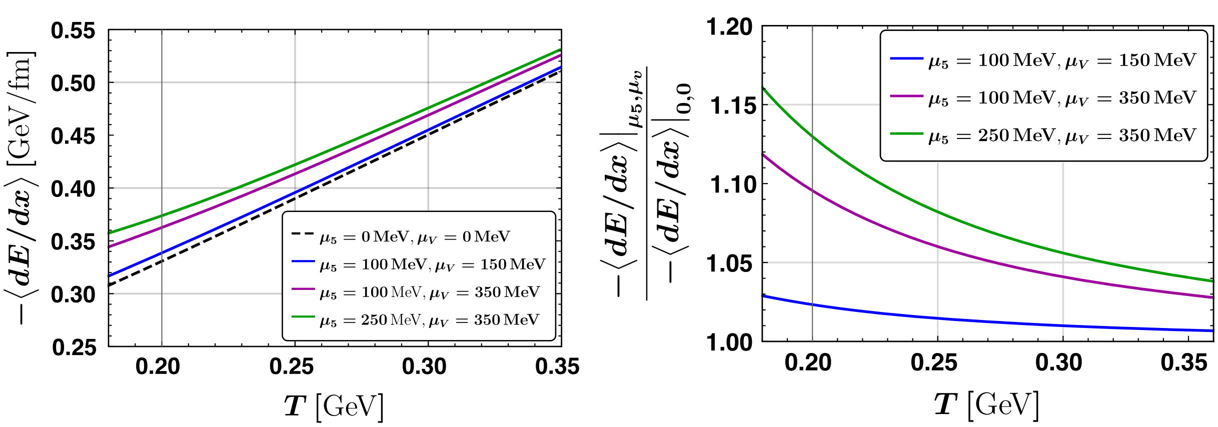

FIG. 2. Temperature dependence of the energy loss of a fast-moving parton in a chiral imbalance medium (left panel). The ratio of energy loss in a chiral medium to that of an isotropic medium is plotted as a function of temperature (right panel).

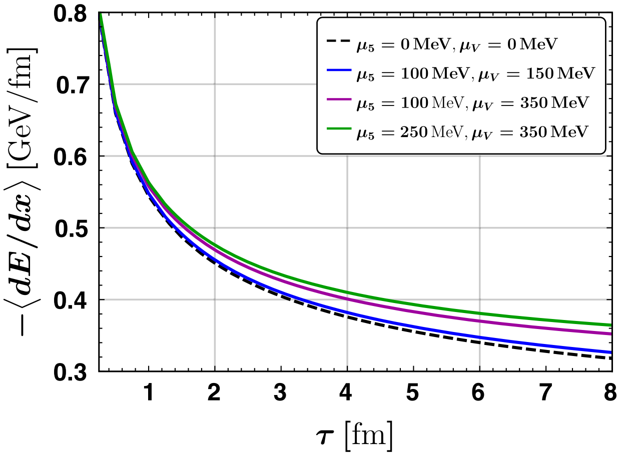

FIG. 3. Time evolution of energy loss of a massless fast-moving parton in a chiral imbalance medium.

in the chiral imbalance medium to the isotropic medium is shown in Fig. 2 (right panel). We observe an increment in the energy loss in the presence of the chiral asymmetry compared to the isotropic QGP medium with vanishing chemical potential, which has a strong dependence on the values of μ_R and μ_L. The proper time evolution of energy change of the parton moving in the medium with longitudinal boost–invariant expansion is plotted in Fig. 3. It is observed that the chiral effects of the medium on the parton energy loss is more visible in the later stages of the evolution.

The energy loss of a parton in a left- and right-hand dominated medium at a fixed temperature is plotted in Fig. 4. The region above and below the diagonal line $\mu_R = \mu_L$ represent the left- and right-hand fermions dominated regions, respectively. Notably, the energy loss is symmetric at a fixed temperature in both regions. This is because the parton energy loss in chiral medium remains as a parity

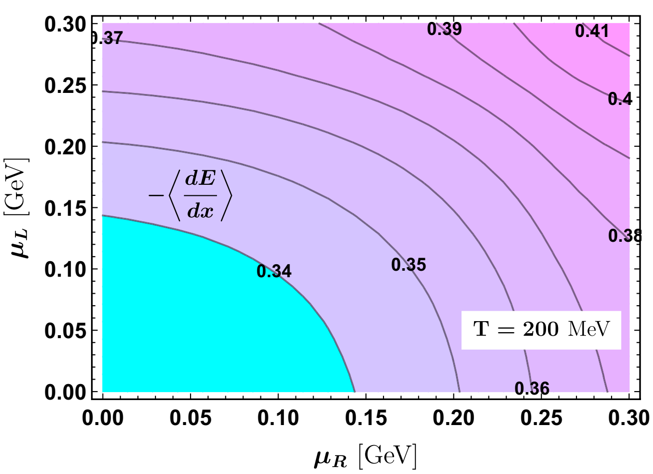

FIG. 4. Energy loss of a massless energetic parton in the chiral medium at $T = 200$ MeV. Curved lines denote constant energy loss contours and $\langle \frac{dE}{dx} \rangle$ is described with the unit [GeV/fm].

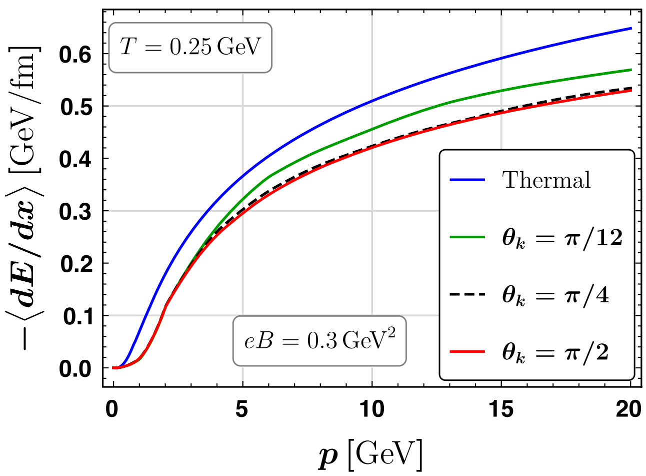

FIG. 5. Momentum dependence of the parton energy loss in a magnetized QCD medium for various choices of angle between parton velocity and magnetic field in the medium (θ_k). The mass of fast moving parton is chosen as $m = 1.25$ GeV.

conserving term as $-\langle \frac{dE}{dx} \rangle$ depends on μ_5^2 as described in Eq. (25). It is seen that the parton energy loss increases with the chemical potential. The curved lines that represent constant energy loss contours indicate that $-\langle \frac{dE}{dx} \rangle$ in a chiral medium is not only depending on the value of μ_R and μ_L but also the difference between them.

In Fig. 5, we illustrated the momentum behavior of anisotropy generated due to the strong magnetic field to the energy loss mechanism. It is more relevant to the case of heavy quarks than the massless test parton, as they expect to witness a strong magnetic field in the initial stages of the collision. Here, we target the charm quark energy loss case with $m = 1.25$ GeV, $T = 250$ MeV, and $eB = 0.3\ \mathrm{GeV}^2$ to ensure the strong field approximation. It is crucial to underline that the heavy quark does not undergo Landau-level dynamics owing to its large mass and initial momentum in the medium. However, the QCD medium properties and hence, the polarization effects will change with the inclusion of the magnetic field. They can indirectly affect charm quark energy loss in the magnetized medium. We observe that the energy loss of the charm quark traveling through a magnetized medium is sensitive to the magnetic field in the plasma. Notably, the energy loss strongly depends on the relative direction of the charm quark's velocity and magnetic field in the medium. The impact is more pronounced at the high momentum regime. It is seen that the energy loss is more when the motion is transverse to the direction of the magnetic field.

The dependence of the strength of the magnetic field and temperature on the energy loss of a massless parton is shown in Fig. 6. We fix the relative orientation of parton motion and magnetic field by choosing $\theta_k = \pi/2$. For an energetic parton, the magnetic field suppresses the energy loss compared to that in the thermal medium due to the dimensionally reduced Landau level motion of the charged

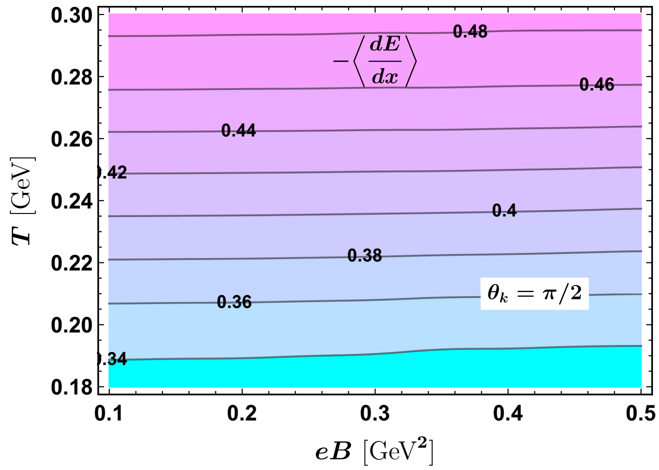

FIG. 6. Energy loss (in [GeV/fm]) of an energetic massless parton as a function of temperature and magnetic field.

fermions in the plasma. The impact of the magnetic field strength is more pronounced in the low-temperature regimes. As the temperature rises, the suppression of parton energy loss that arises from the Landau kinematics is not sensitive to the strength of the magnetic field. The observation holds true for other choices of θ_k as well.

IV. SUMMARY AND OUTLOOK

In conclusion, we have studied the energy loss of a fast-moving parton for three scenarios within the QGP medium, viz., the isotropic, chiral asymmetric, and strongly magnetized QGP medium. To do so, we employed Wong's equations combined with linearized Yang-Mills equations that describe the backreaction exerted on the energetic parton by the medium while traversing through it. In the isotropic case, it is found that the energy loss increases with the temperature of the medium. It further increases with the momentum of the test parton. Similar trends are found for the chiral asymmetric medium as well. Moreover, the energy loss turned out to be sensitive to the chiral chemical potential, especially at the low-temperature regimes. We further investigated the heavy parton energy loss in a magnetized QCD medium. It is noticed that the magnetic field induces anisotropy in the medium that, in turn, suppresses the energy loss. Furthermore, the energy loss is found to have a strong dependence on the relative orientation of the parton's velocity and the magnetic field. It is seen parton loses more energy when it travels transversely to the direction of the magnetic field compared to other directions.

The current focus is on the parton energy loss due to the polarization effects of the medium. The current formalism (by treating test parton as a classical particle with a color charge and its dynamics within Wong's equations) holds true only for the soft interactions. For the phenomenological studies, this analysis should be combined with the contribution from that of the hard modes. The estimation of scattering amplitude $2 \to 2$ elastic process with medium particles (momentum $\sim T$) and especially the soft gluon emission $2 \to 3$ of a moving particle in the presence of an arbitrary magnetic field and chiral chemical potential requires further attention and is beyond the scope of this study. Another interesting direction is to set up the formalism by considering the fluctuations in a magnetized QCD medium and exploring the possibility of energy gain of the parton. These are interesting aspects to explore shortly.

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APPENDIX: EFFECT OF ANISOTROPIC MOMENTUM DISTRIBUTIONS ON PARTON ENERGY LOSS

The medium evolution can give rise to nonequilibrium effects on the parton energy loss. The effect of anisotropic momentum distributions of plasma constituents in the nonequilibrium medium on the parton energy loss can be quantified as follows: The nonequilibrium distribution function can be defined as

$$f^{(\mathrm{aniso})} = f_0 + \delta f, \tag{A1}$$

where f_0 is the equilibrium isotropic part and δf is the nonequilibrium correction part which can be described as follows [81],

$$\delta f = -\frac{\xi}{2E_k T}(\mathbf{k}\cdot\mathbf{a})^2 (f_0)^2 \exp\left(\frac{E_k}{T}\right). \tag{A2}$$

Here, ξ is the anisotropic parameter that determines the deviation of the momentum distribution from equilibrium and $\mathbf{a}$ is the unit vector that represents the direction of momentum anisotropy in the medium. The spacelike components of the self-energy tensor can be described as,

$$\Pi^{ij} = -g^2 \int \frac{d^3\mathbf{p}}{(2\pi)^3} v^i \partial^l f^{(\mathrm{aniso})}(\mathbf{p}) \left(\delta^{jl} + \frac{v^j k^l}{K\cdot V + i\epsilon}\right), \tag{A3}$$

with $V^\mu = (1, \frac{\mathbf{k}}{\omega})$. We can further decompose the self-energy into four structure functions in the anisotropic medium as,

$$\Pi^{ij} = \alpha A^{ij} + \beta B^{ij} + \gamma Y^{ij} + \delta Z^{ij}, \tag{A4}$$

where A^{ij} and B^{ij} are the conventional transverse and longitudinal projection operators. Here, $Y^{ij} = \frac{\tilde{a}^i \tilde{a}^j}{\tilde{a}^2}$ and $Z^{ij} = k^i \tilde{a}^j + k^j \tilde{a}^i$ with $\tilde{a}^i = A^{ij} a_j$. The detailed calculation of the structure functions α, β, γ, δ are presented in Ref. [81]. We can obtain the effective propagator from Eq. (A4). Following the similar formalism employed in our manuscript, we can obtain the energy loss of the parton in the anisotropic medium and quantify the nonequilibrium correction to the energy loss as [27],

$$\left\langle \frac{dE}{dx} \right\rangle = -i \frac{1}{|\mathbf{v}|} g_s^2 C_F v^i v^j \int \frac{d^3\mathbf{k}}{(2\pi)^3} \omega [\Delta_1(\omega)(A^{ij} - Y^{ij}) + \Delta_2(\omega)((\omega^2 - |\mathbf{k}|^2 - \alpha - \gamma)B^{ij} + (\omega^2 - \beta)Y^{ij} + \delta Z^{ij})], \tag{A5}$$

where $\Delta_1 = \omega^2 - |\mathbf{k}|^2 - \alpha$ and $\Delta_2 = (\omega^2 - \beta)(\omega^2 - |\mathbf{k}|^2 - \alpha - \gamma) - |\mathbf{k}|^2 \delta^2 \tilde{a}^2$. It is important to focus that in the limit $\xi \to 0$, γ and δ vanishes and Eq. (A4) reduce back to that of isotropic case. The nonequilibrium corrections will not be significant in the near-equilibrium region ($\xi \ll 1$). In a recent study [24], some of us have shown that the nonequilibrium corrections are negligible in the momentum evolution of charm quark in a $1 + 3 - \mathrm{D}$ expanding QGP medium. However, these nonequilibrium corrections are important to maintain the theoretical consistency of the analysis in an evolving medium. Fluctuations on chromodynamic fields in the preequilibrium phase can also affect the energy loss pattern especially in the very early phase of evolution [28].

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