Bulk-to-boundary propagators with arbitrary total angular momentum J in soft-wall AdS/QCD

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We derive the equations of motion for the bulk-to-boundary propagators of the anti-de Sitter (AdS) boson and fermion fields with arbitrary total angular momentum J in a soft-wall AdS/QCD model and solve it analytically. It provides the opportunity to study transition form factors induced by these bulk-to-boundary propagators, both for on-shell and off-shell hadrons. This is a continuation of our study of hadron form factors induced by the bulk-to-boundary propagator with total angular momentum $J = 1$ (e.g., electromagnetic form factors of mesons, nucleons, and nucleon resonances).

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I. INTRODUCTION

The soft-wall anti-de Sitter/quantum chromodynamics (AdS/QCD) model proposed in Ref. [1] plays an important role for the description and understanding of hadron structure: mass spectra, form factors, parton distributions, QCD scattering processes (like Drell-Yan, deep-inelastic scattering), etc. The pioneer contribution to the investigation of QCD scattering processes based on gauge/string duality was made in Ref. [2], and the success of the soft-wall model is based on the fact that it provides analytical calculations of hadronic properties. The formalism of the soft-wall AdS/QCD model is based on phenomenological actions formulated in terms of boson and fermion AdS fields, propagating in five-dimensional AdS space. One should stress that the Hamiltonian approach is also widely used, especially in connection with the light-cone formalism, e.g., in the model of Ref. [3]. Four of the five dimensions of the AdS space correspond to the Minkowski subspace and the fifth (holographic) dimension z corresponds to a scale. Conformal and chiral symmetry in the underlying actions are broken by introducing the dilaton field, quadratically dependent on the variable z in the exponential prefactor of the action or in the phenomenological potential. In the case of the Hamiltonian approach [3] the conformal symmetry of the Hamiltonian remains. Based on this action one can solve two problems: (i) The bound-state problem, i.e. derive equations of motion (EOM) for the bulk profiles $\phi(z)$ (the parts of the AdS fields explicitly dependent on the holographic variable z). These profiles obey Schrödinger type equations of motion, which are solved analytically [1]. The solutions of these equations correspond to the hadronic mass spectrum due to the duality of bulk profiles and hadronic wave function. (ii) The scattering problem, i.e. one can derive EOM for the bulk-to-boundary propagators $V(-q^2, z)$ describing the momentum dependence of the AdS field traveling from the AdS interior to its boundary (Minkowski space). In particular, the bulk-to-boundary propagators depend on two variables: holographic coordinate z and q, which is Fourier conjugate to Minkowski coordinate x. Therefore, the main components produced by the AdS/QCD soft-wall action needed for the study of hadron structure are the bulk profiles $\phi(z)$, dual to hadronic wave functions describing the hadrons on the mass shell, and the bulk-to-boundary $V(-q^2, z)$, dual to the off-shell external gauge fields or external hadrons. In particular, hadronic form factors, which are the main focus of the present paper, are the integrals over z of the product of bulk-to-boundary propagators and two bulk profiles. One should stress that up to now, the study of hadronic form factors has been focused on the quantities induced by the bulk-to-boundary propagator with the total angular momentum $J = 1$, dual to electromagnetic field. In particular, following the idea proposed in Ref. [4] one can derive the massless bulk-to-boundary propagator for the field with total angular momentum $J = 1$ and reach its application for the calculation of form factors of mesons and baryons in Euclidean spacetime, which has been done very successfully in Refs. [3,5–16]. For application of the bulk-to-boundary propagators with $J = 0$ and $J = 1$ in Minkowski space see, e.g., Ref. [17]. In fact, the soft-wall AdS/QCD not only provides the correct power scaling description of form factors and helicity amplitudes of all hadrons at large Q^2 in Euclidean spacetime [18]; it is also able to give good agreement with the data at low and intermediate Q^2. Note that up to now only the massless AdS bulk-to-boundary propagator, dual to massless gauge vector fields, has been considered in the context of soft-wall AdS/QCD.

The main objective of the present paper is to extend the soft-wall model formalism for the study of bulk-to-boundary propagators with arbitrary J in the Euclidean spacetime. First, we consider massless bulk-to-boundry propagators, which are relevant for the description of gravitons or light hadrons for which one can apply the massless limit. Second, we extend our results for the case of massive bulk-to-boundary propagators dual to massive gauge bosons and massive hadrons. As a result, we derive analytical expressions of the form factors describing (i) the coupling of off-shell massless gauge bosons (photon, graviton) or massless scalar/pseudoscalar fields of new physics (NP): axions, axionlike particles (ALPs), etc. with two on-shell hadrons; (ii) the coupling of off-shell massive gauge bosons (weak $W^\pm$ and Z^0 bosons) or Higgs H, with two on-shell hadrons; (iii) the coupling of off-shell massless hadrons with two on-shell hadrons; and (iv) the coupling of off-shell massive hadrons with two on-shell hadrons. In all cases the off-shell behavior of gauge fields and hadrons is encoded in the Q^2 behavior of the corresponding bulk-to-boundary propagator. This provides an opportunity to study the off-shell behavior of hadronic form factors, i.e. direct coupling of three particles, when one particle is off-shell and the other two are on-shell. It provides useful insight to lattice QCD and effective field theories, such as chiral perturbation theory (ChPT), heavy hadron ChPT, where direct couplings of hadrons are calculated from first principles (lattice QCD) or provide input parameters for phenomenological Lagrangians.

The paper is organized as follows. In Sec. II, we discuss the derivation of bulk-to-boundary propagators dual to off-shell gauge fields and hadrons. First, we consider the case of boson propagators and then we extend our formalism to the case of fermions. For massive gauge fields and hadrons we propose an extension of the bulk-to-boundary propagators to a massive case. Finally, in Sec. III we present our conclusion.

II. FORMALISM

A. Boson bulk-to-boundary propagator

We start by specifying the AdS_5 metric

$$\begin{aligned} ds^2 &= g_{MN} dx^M dx^N = \eta_{ab} e^{2A(z)} dx^a dx^b \\ &= e^{2A(z)} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2), \\ \eta_{\mu\nu} &= \mathrm{diag}(1, -1, \ldots, -1), \end{aligned} \tag{1}$$

where M and $N = 0, 1, \ldots, 4$ are the base manifold indices, $a = (\mu, z)$ and $b = (\nu, z)$ are the local Lorentz (tangent) indices, and g_{MN} and η_{ab} are curved and flat metric tensors, which are related by the vielbein $\epsilon^a_M(z) = e^{A(z)} \delta^a_M$ as $g_{MN} = \epsilon^a_M \epsilon^b_N \eta_{ab}$. Here z is the holographic coordinate. We restrict our discussion to a conformal-invariant metric with $A(z) = \log(R/z)$, where R is the AdS radius.

The action of the soft-wall AdS/QCD model describing totally symmetric traceless bosonic fields $V_{M_1 \cdots M_J}(x, z)$ with arbitrary integer J was derived in Ref. [1]. In particular, this action has a simplified form in the axial gauge $V_{z\cdots}(x, z) = 0$:

$$S_J = \frac{(-)^J}{2} \int d^4x dz e^{-B_J(z)} \partial_M V_{\mu_1 \cdots \mu_J}(x, z) \partial^M V^{\mu_1 \cdots \mu_J}(x, z), \tag{2}$$

where $\partial_M \otimes \partial^M = \partial_\mu \otimes \partial^\mu - \partial_z \otimes \partial_z$, $B_J(z) = \varphi(z) - (2J - 1)A(z)$, $\varphi(z) = \kappa^2 z^2$ is the dilaton field, and $\kappa \sim 500$ MeV [3,19,20] is the dilaton scale parameter.

The massless bulk-to-boundary propagator $V_J(q, z)$ of the $V_{\mu_1 \cdots \mu_J}(x, z)$ field is given by the Fourier transformation,

$$V_{\mu_1 \cdots \mu_J}(x, z) = \int \frac{d^4q}{(2\pi)^4} e^{-iqx} V_{\mu_1 \cdots \mu_J}(q) V_J(-q^2, z), \tag{3}$$

where q is the Fourier conjugates to x. Next, from Eq. (2) we derive the equation of motion for the propagator $V_J(-q^2, z)$:

$$\partial_z \Big(e^{-B_J(z)} \partial_z V_J(-q^2, z) \Big) + e^{-B_J(z)} q^2 V_J(-q^2, z) = 0, \tag{4}$$

which has an analytical solution in terms of gamma $\Gamma(a)$ and Trikomi $U(a, b, z)$ functions,

$$V_J(Q^2, z) = (\kappa^2 z^2)^J \frac{\Gamma(a_J + 1)}{\Gamma(J)} U(a_J + 1, J + 1, \kappa^2 z^2), \tag{5}$$

where $a_J = a + J - 1$, $a = Q^2/(4\kappa^2)$, $Q^2 = -q^2$ is the Euclidean momentum squared, and an integral representation for the Trikomi function reads

$$U(a,b,c) = \frac{1}{\Gamma(a)} \int_0^\infty dt e^{-ct} t^{a-1} (1+t)^{b-a-1} = \frac{1}{\Gamma(a)} \int_0^1 \frac{dx x^{a-1}}{(1-x)^b} e^{-\frac{cx}{1-x}}. \quad (6)$$

Therefore, an integral representation for the propagator $V_J(Q^2,z)$ is given by

$$V_J(Q^2,z) = \frac{(\kappa^2 z^2)^J}{\Gamma(J)} \int_0^1 \frac{dx x^{a_J}}{(1-x)^{J+1}} e^{-\frac{\kappa^2 z^2 x}{1-x}}. \quad (7)$$

By changing the integration variable $x = y/(y+\kappa^2 z^2)$ one can derive another representation for the $V_J(Q^2,z)$

$$V_J(Q^2,z) = \frac{1}{\Gamma(J)} \int_0^\infty dy y^{J-1} e^{-y} \left(\frac{y}{y+\kappa^2 z^2}\right)^a. \quad (8)$$

Additional useful integral representation for the $V_J(Q^2,z)$, derived from Eq. (7) by partial integration, reads

$$V_J(Q^2,z) = \frac{1}{B(a,J)} \int_0^1 dx x^{a-1} (1-x)^{J-1} e^{-\frac{\kappa^2 z^2 x}{1-x}}, \quad (9)$$

$B(x,y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$ is the beta function.

Now let us consider the properties of the derived bulk-to-boundary propagators. A nice feature of the derived bulk-to-boundary propagator $V_J(Q^2,z)$ is the following: while it was derived for the boson propagators with higher $J \geq 2$, it is also valid for the limit $J = 1$. In particular, in the limit $J = 1$ (the case of the vector bulk-to-boundary propagator), $V_1(Q^2,z)$ reduces to the well-known result [5,8]

$$V_1(Q^2,z) = \kappa^2 z^2 \Gamma(a+1) U(a+1,2,\kappa^2 z^2) = \kappa^2 z^2 \int_0^1 \frac{dx x^a}{(1-x)^2} e^{-\frac{\kappa^2 z^2 x}{1-x}} = \int_0^\infty dy e^{-y} \left(\frac{y}{y+\kappa^2 z^2}\right)^a. \quad (10)$$

The vector bulk-to-boundary propagator obeys the important conditions [5,8]: (i) charge conservation $V_1(0,z) = 1$ at $Q^2 = 0$, (ii) local limit $V_1(Q^2,0) = 1$ at $z = 0$, (iii) confinement $V_1(Q^2,z) \to 0$ at $z \to \infty$, and (iv) it produces power scaling of hadronic form factors $F(Q^2) \sim 1/Q^{2(\tau-1)}$ at large Q^2 [18], where τ is the leading twist of the hadron, which is also the number of its constituent partons.

From the integral representation (8), it immediately follows that all properties (i)–(iv) relevant for the vector propagator are also valid for the propagators with higher $J \geq 2$. In particular, from Eq. (8) it follows that the normalization conditions $V_J(Q^2,0) = V_J(0,z) = 1$ are independent of J. Obviously, the propagator $V_J(Q^2,z)$ has no proper limit $J = 0$. In particular, while at $Q^2 = 0$ and $z = 0$ the scalar propagator $V_0(Q^2,z)$ has the required normalizations $V_0(Q^2,0) = V_0(0,z) = 1$; it vanishes at finite values of Q^2 and z. Therefore, we had to propose an action for the scalar AdS field, which produces a scalar bulk-to-boundary propagator consistent with the following requirements: (i) normalization condition $V_0(Q^2,0) = V_0(0,z) = 1$, (ii) $V_0(Q^2,z)$ is finite at $Q^2 \neq 0$ and $z \neq 0$, (iii) confinement $V_0(Q^2,z) \to 0$ at $z \to \infty$, and (iv) correct power scaling of hadronic form factors $F(Q^2) \sim 1/Q^{2(\tau-1)}$ at large Q^2 [18]. One of such actions, which obeys the above requirements, reads

$$S_0 = \frac{1}{2} \int d^4x dz e^{-B(z)} \partial_M S(x,z) \partial^M S(x,z), \quad (11)$$

where $B(z) = \varphi(z) - A(z)$.

Next, from the action (11) we derive the following equation of motion for the scalar propagator $V_0(-q^2,z)$:

$$\partial_z \Big(e^{-B(z)} \partial_z V_0(-q^2,z)\Big) + e^{-B(z)} q^2 V_0(-q^2,z) = 0, \quad (12)$$

which has the solution $V_0(Q^2,z)$, coinciding with the vector bulk-to-boundary propagator $V_0(Q^2,z) \equiv V_1(Q^2,z)$.

Note that at $Q^2 \to \infty$ the bulk-to-boundary propagator $V_J(Q^2,z)$ for $J \geq 1$ behaves as

$$V_J(Q^2,z) \to \frac{e^{\kappa^2 z^2}}{\Gamma(J)} \left(\frac{Q^2 z^2}{4}\right)^J \int_0^\infty \frac{dt}{t^{J+1}} \exp\left(-t - \frac{Q^2 z^2}{4t}\right) = \frac{2e^{\kappa^2 z^2}}{\Gamma(J)} \left(\frac{Qz}{2}\right)^J K_J(Qz), \quad (13)$$

where $Q = \sqrt{Q^2}$, and

$$K_n(x) = \frac{x^n}{2^{n+1}} \int_0^\infty \frac{dt}{t^{n+1}} \exp\left(-t - \frac{x^2}{4t}\right) \quad (14)$$

is the modified Bessel function of the second kind for arbitrary n [15]. As before, the $Q^2 \to \infty$ asymptotics coincides for $J = 0$ and $J = 1$. It was shown in Ref. [15] that in the case $J = 1$ and in the limit $\kappa \to 0$, the vector bulk-to-boundary propagator $V_1(Q^2,z)$ in the soft-wall AdS/QCD model,

$$V_1(Q^2,z) = Qz K_1(Qz), \quad (15)$$

coincides with the one obtained in the hard-wall AdS/QCD model [4]. Therefore, we make the prediction that the $Q^2 \to \infty$ asymptotics of the bulk-to-boundary propagator in the hard-wall model for arbitrary $J \geq 1$ coincides with the one in the soft-wall model for $\kappa \to 0$:

$$V_J(Q^2, z) = \frac{2}{\Gamma(J)} \left(\frac{Qz}{2}\right)^J K_J(Qz). \tag{16}$$

We should stress that the massless boson bulk-to-boundary propagator $V_J(Q^2, z)$ is mostly relevant for the description of the propagation of massless gauge bosons (photon with $J = 1$ and graviton with $J = 2$) and massless scalar/pseudoscalar of NP (axion, ALPs) with $J = 0$. In the case of massive gauge fields, such as the weak $W^\pm$ and Z^0 bosons or the Higgs H, one should include their masses, which appear after spontaneous breaking of gauge symmetry. We propose to include the finite mass for the bulk-to-boundary propagator by shifting the square of the momentum as $-q^2 = Q^2 \to -q^2 + M^2 = Q^2 + M^2$, where M is the mass of the gauge field or Higgs, taken from data in Ref. [21]:

$$\begin{aligned} M_{W^\pm} &= 80.377 \pm 0.012 \text{ GeV}, \\ M_{Z^0} &= 91.1876 \pm 0.0021 \text{ GeV}, \\ M_H &= 125.25 \pm 0.17 \text{ GeV}. \end{aligned} \tag{17}$$

For example, the massive bulk-to-boundary propagator of the weak bosons and Higgs reads

$$V(Q^2 + M^2, z) = \int_0^\infty dy e^{-y} \left(\frac{y}{y + \kappa^2 z^2}\right)^{a(M^2)}, \tag{18}$$

where $a(M^2) = a + M^2/(4\kappa^2) = (Q^2 + M^2)/(4\kappa^2)$ and $M = M_{W^\pm}, M_{Z^0}, M_H$. One can see that our extension to massive bulk-to-boundary propagators is consistent. It is clear that for $M^2 \gg Q^2$ one can neglect the Q^2 dependence of the propagator $V(Q^2 + M^2, z)$, i.e. in this limit $V(Q^2 + M^2, z) \to V(M^2, z)$ in consistency with the Standard Model (SM). Also, the massless limit $M \to 0$ is straightforward leading to the massless propagator. Here we consider as example the particles of the SM, but this discussion is true for any other massless/massive gauge fields or other structureless particles (axion, ALPs, etc.).

Next we clarify how to use the bulk-to-boundary propagator with arbitrary J for the description of the propagation of composite particles—hadrons. The difference of hadrons from structureless particles is that hadrons in the soft-wall AdS/QCD are described by hadronic wave functions. The latter are dual to the profiles of the AdS fields depending on the holographic variable z. In particular, the meson wave function describing the hadron, which is made by the Fock state with leading twist τ, reads [3,8,19]

$$\phi_{M_\tau}(z) = \sqrt{\frac{2}{\Gamma(\tau - 1)}} \kappa^{\tau-1} z^{\tau-3/2} e^{-\kappa^2 z^2/2}. \tag{19}$$

We should stress that the inclusion of subleading Fock states in the context of the soft-wall model has been considered in Refs. [9,10,13,14,16]. In the present paper we restrict the discussion to the leading Fock state contribution to the structure of the specific hadron. Therefore, the massive bulk-to-boundary propagator with total angular momentum J, dual to massive meson with the same J and made by the leading Fock state with twist τ, must be constructed as a product of the massive bulk-to-boundary propagator

$$V_J(Q^2 + M_M^2, z) = \int_0^\infty dy y^{J-1} e^{-y} \left(\frac{y}{y + \kappa^2 z^2}\right)^{a(M_M^2)} \tag{20}$$

where M_M is the mass of a meson and meson wave function $\phi_{M_\tau}(z)$ (19). For convenience, we denote the bulk-to-boundary propagator dual to massive meson with arbitrary J as

$$\phi_{M_{J,\tau}}(Q^2 + M_M^2, z) = V_J(Q^2 + M_M^2, z) \phi_{M_\tau}(z). \tag{21}$$

One can see that the massive bulk-to-boundary propagator, dual to a hadron (21), obeys important requirements: (i) mass-shell limit $q^2 = -Q^2 = M_M^2$ and (ii) massless limit $M_M^2 = 0$. In particular, in the limit (i): $V_J(Q^2 + M_M^2, z) \to V_J(0, z) = 1$ and therefore

$$\phi_{M_{J,\tau}}(Q^2 + M_M^2, z) \to \phi_{M_{J,\tau}}(0, z) = \phi_{M_\tau}(z). \tag{22}$$

In the limit (ii): $V_J(Q^2 + M_M^2, z) \to V_J(Q^2, z)$ and therefore

$$\phi_{M_{J,\tau}}(Q^2 + M_M^2, z) \to \phi_{M_{J,\tau}}(Q^2, z) = V_J(Q^2, z) \phi_{M_\tau}(z). \tag{23}$$

We summarize the results of this subsection. We derived the set of massless and massive bulk-to-boundary propagators with arbitrary J, dual to massless and massive fields of SM (photon, weak bosons, Higgs) and of NP (axion, ALPs, etc.) and of hadrons. These quantities describe off-shell behavior of SM or NP particles and of hadrons and could be used for the calculation of transition form factors involving off-shell and on-shell states. As we stressed before, it could be useful to provide insight to lattice QCD and effective field theories, where direct couplings of hadrons are calculated from first principles or provide input parameters for phenomenological Lagrangians. Later, we will extend our ideas to the sector of fermion bulk-to-boundary propagators.

In Figs. 1–3 we present two- and three-dimensional plots illustrating the properties of boson bulk-to-boundary propagators, dual to the gauge bosons with $J = 1$ and $J = 2$, and hadrons (mesons) with arbitrary integer J. In particular, in Fig. 1 (left panel) we show results for the bulk-to-boundary propagator $V_1(Q^2, z)$ as a function of Q^2 and z, dual to the massless gauge field with $J = 1$ (like photon). In Fig. 1 (right panel) we show results for the ratio $R_{21}(Q^2, z) =$

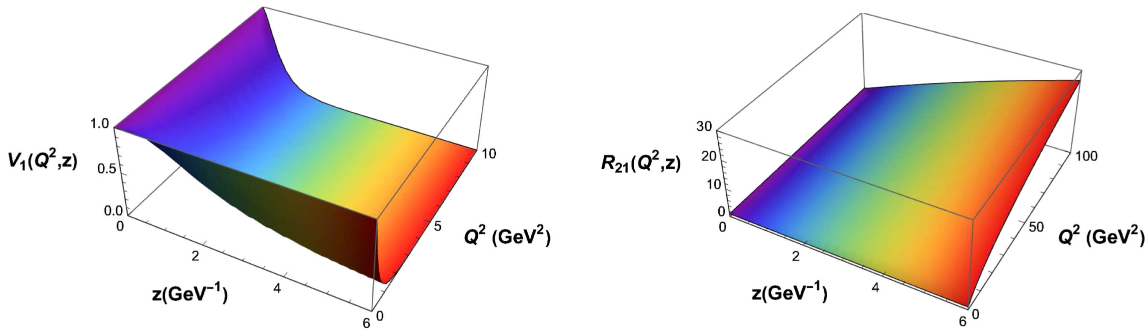

FIG. 1. 3D plots of the massless boson bulk-to-boundary propagators as functions of Q^2 and z, which are dual to massless gauge fields with $J = 1$ and $J = 2$: $V_1(Q^2, z)$ (left panel), ratio $R_{21}(Q^2, z) = V_2(Q^2, z)/V_1(Q^2, z)$ (right panel).

$V_2(Q^2, z)/V_1(Q^2, z)$ of two propagators with $J = 2$ and $J = 1$, dual to the massless gauge fields with $J = 2$ (graviton) and $J = 1$ (photon). One can see that the plot for $V_1(Q^2, z)$ decreases when z and Q^2 increase. The ratio $R_{21}(Q^2, z)$ increases when z and Q^2 increase. One should stress that for both plots the $Q^2 \to \infty$ asymptotics is fully consistent with our analytical prediction in Eq. (13). In particular, at large $Q^2 \to \infty$ the $R_{21}(Q^2, z)$ behaves as

$$R_{21}(Q^2, z) = \frac{Qz}{2}\frac{K_2(Qz)}{K_1(Qz)} = \frac{Q^2z^2}{4}\frac{\int_0^\infty \frac{dt}{t^3}\exp\left(-t - \frac{Q^2z^2}{4t}\right)}{\int_0^\infty \frac{dt}{t^2}\exp\left(-t - \frac{Q^2z^2}{4t}\right)}. \tag{24}$$

In Fig. 2 we show results for the massive bulk-to-boundary propagators $\phi_{M_{J,\tau}}(Q^2 + M_M^2, z)$ as functions of Q^2 and z dual to massive mesons at fixed value of leading twist $\tau = 2$ for specific mesons characterized by total angular momentum J and mass M_M: (a) pion with $J = 0$ and $M_\pi = 0.13957$ GeV, (b) ρ meson with $J = 1$ and $M_\rho = 0.7665$ GeV, (c) a_2 meson with $J = 2$ and $M_{a_2} = 1.3186$ GeV, and (d) ω_3 meson with $J = 3$ and $M_{\omega_3} = 1.67$ GeV [21].

In Fig. 3 we present results for the massive bulk-to-boundary propagator $\phi_{M_{J,\tau}}(Q^2 + M_M^2, z)$ as a function of Q^2 and J at fixed values of $z = 1$ and $z = 2$ GeV^{-1}, for leading twist $\tau = 2$. Figure 3 shows that increasing z leads to a more suppressed behavior of the $\phi_{M_{J,\tau}}(Q^2 + M_M^2, z)$, as expected.

Next we check that the boson bulk-to-boundary propagator with arbitrary J, dual to gauge bosons or mesons, produces the correct power scaling of hadronic form factors $F(Q^2) \sim 1/Q^{2(\tau-1)}$ at large Euclidean values of $Q^2 = -q^2$.

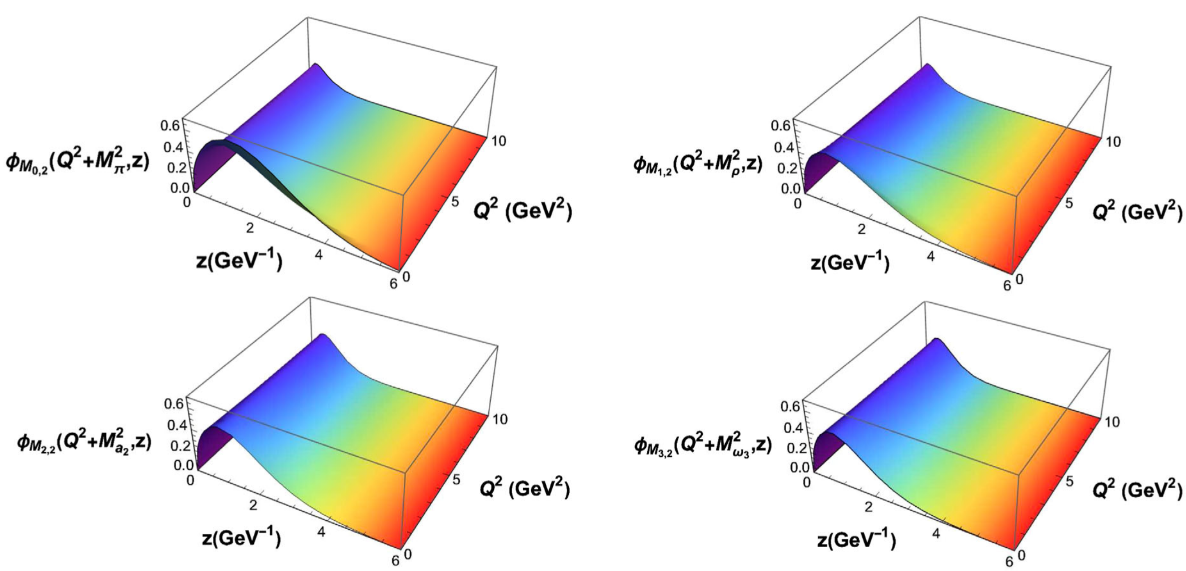

FIG. 2. 3D plots of the massive bulk-to-boundary propagators $\phi_{M_{J,\tau}}(Q^2 + M_M^2, z)$ as functions of Q^2 and z, dual to massive mesons at fixed value of the leading twist $\tau = 2$ for specific mesons: (a) pion with $J = 0$ and $M_\pi = 0.13957$ GeV (left-upper panel), (b) ρ meson with $J = 1$ and $M_\rho = 0.7665$ GeV (right-upper panel), (c) a_2 meson with $J = 2$ and $M_{a_2} = 1.3186$ GeV (left-bottom panel), (d) ω_3 meson with $J = 3$ and $M_{\omega_3} = 1.67$ GeV (right-bottom panel).

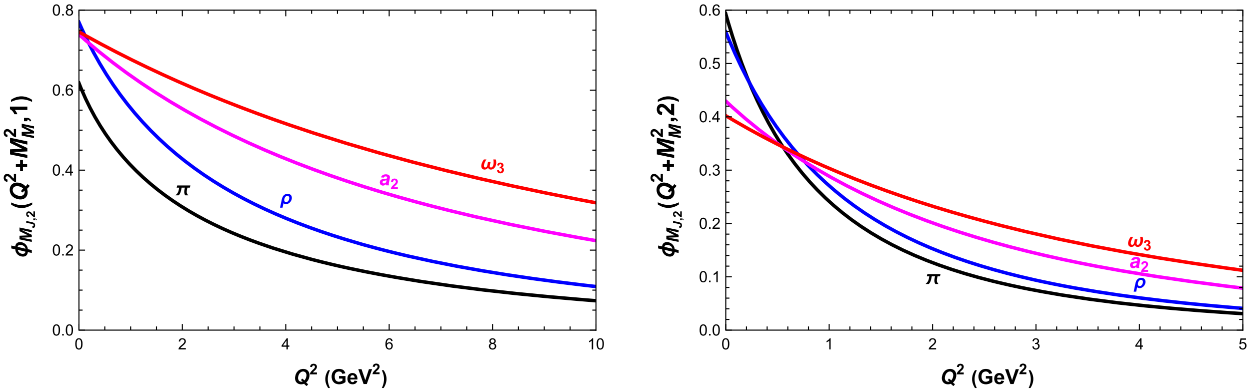

FIG. 3. 2D plots of the massive bulk-to-boundary propagators $\phi_{M_{J,\tau}}(Q^2+M_M^2,z)$ as functions of Q^2 and $J=0$, 1, 2, 3, dual to massive mesons at fixed values of the leading twist $\tau=2$ and holographic coordinate: $z=1$ GeV^{-1} (left panel), $z=2$ GeV^{-1} (right panel).

In particular, we derive the master formulas for the transition meson and baryon form factors $F_{V_J M_{\tau_1} M_{\tau_2}}(Q^2)$ and $F_{V_J B_{\tau_1} B_{\tau_2}}(Q^2)$, which are produced by the integral over the holographic coordinate z of the product of the bulk-to-boundary propagator $V_J(Q^2+M^2,z)$ (off-shell SM or NP bosons with quantum number J) or $\phi_{M_{J,\tau}}(Q^2+M_M^2,z)$ (off-shell meson with quantum numbers J and τ) and two hadron wave functions with arbitrary leading twists τ_1 and τ_2: meson $\phi_{M_{\tau_1}}(z)$ and $\phi_{M_{\tau_2}}(z)$, or baryon $\phi_{B_{\tau_1}}(z)$ and $\phi_{B_{\tau_2}}(z)$ wave functions, respectively [3,5–8].

First, we consider the case of a meson transition form factor induced by off-shell SM or NP bosons, which in the soft-wall AdS/QCD model is given by [3,5,8]

$$F_{V_J M_{\tau_1} M_{\tau_2}}(Q^2) = g_{V_J M_{\tau_1} M_{\tau_2}} \int_0^\infty dz V_J(Q^2+M^2,z)\phi_{M_{\tau_1}}(z)\phi_{M_{\tau_2}}(z),$$
$$= g_{V_J M_{\tau_1} M_{\tau_2}} \frac{\Gamma\Big(J+\frac{\tau_1+\tau_2}{2}-1\Big)}{\Gamma(J)\sqrt{\Gamma(\tau_1-1)\Gamma(\tau_2-1)}} B\bigg(a(M^2)+J,\frac{\tau_1+\tau_2}{2}-1\bigg), \tag{25}$$

where $g_{V_J M_{\tau_1} M_{\tau_2}}$ is the normalization constant, fixed by gauge invariance, from data or from phenomenological approaches. Equation (25) can be also used for the description of the coupling of an off-shell meson with total angular momentum J with two mass-shell mesons with leading twists τ_1 and τ_2. In the soft-wall AdS/QCD model, the form factor (25) for the case $J=1$ was calculated for the first time in Ref. [15].

It can be seen that all the Q^2 dependence of the form factor $F_{V_J M_{\tau_1} M_{\tau_2}}(Q^2)$ is encoded in the beta function. Therefore, the large Q^2 behavior of $F_{V_J M_{\tau_1} M_{\tau_2}}(Q^2)$ is defined by the corresponding behavior of the beta function. At large $Q^2 \gg M^2$ the form factor $F_{V_J M_{\tau_1} M_{\tau_2}}(Q^2)$ has the scaling independent on J,

$$F_{V_J M_{\tau_1} M_{\tau_2}}(Q^2) \sim \frac{1}{Q^{2(\frac{\tau_1+\tau_2}{2}-1)}}. \tag{26}$$

The J dependence only remains in the coupling constant $g_{V_J M_{\tau_1} M_{\tau_2}}$ and the factor

$$\frac{\Gamma(J+\frac{\tau_1+\tau_2}{2}-1)}{\Gamma(J)\sqrt{\Gamma(\tau_1-1)\Gamma(\tau_2-2)}}. \tag{27}$$

In the special case $\tau_1=\tau_2=\tau$ we reproduce the result dictated by quark counting rules [18]

$$F_{V_J M_\tau M_\tau}(Q^2) \sim \frac{1}{Q^{2(\tau-1)}}. \tag{28}$$

Next, by analogy with the meson case, we derive a baryon transition form factor induced by off-shell SM or NP bosons and two on-shell baryons. In the case of two on-shell baryons one should take into account that the baryon AdS spinors are decomposed into two solutions: right-handed $\phi^{(r)}_{B_\tau}(z)$ and left-handed $\phi^{(\ell)}_{B_\tau}(z)$ chiral eigenstates [6–8,10,12]. Here, the leading twist of baryon field τ is related to the angular orbital momentum L as $\tau=3+L$, e.g., for $L=0$ baryons (e.g., nucleons with $J^P=\frac{1}{2}^+$ and Δ isobars with $J^P=\frac{3}{2}^+$) the leading twist equals to $\tau=3$. For fixed internal spin S the total angular momentum J runs from $|L-S|$ to $|L+S|$. Therefore, by changing the value of

L we can generate the solutions for baryonic wave functions with any required value of total angular momentum-parity J^P. In the soft-wall AdS/QCD approach the baryon wave functions with specific leading twist have definite relations with the corresponding meson wave functions. In particular, the baryon wave function $\phi^{(r)}_{B_\tau}(z)$ coincides with the meson wave function $\phi_{M_\tau}(z)$, while $\phi^{(l)}_{B_\tau}(z)$ is related to $\phi^{(r)}_{B_\tau}(z)$ as $\phi^{(l)}_{B_\tau}(z) = \phi^{(r)}_{B_{\tau+1}}(z)$. In particular [8],

$$\phi^{(r)}_{B_\tau}(z) \equiv \phi_{M_\tau}(z) = \sqrt{\frac{2}{\Gamma(\tau-1)}}\kappa^{\tau-1} z^{\tau-3/2} e^{-\kappa^2 z^2/2},$$

$$\phi^{(l)}_{B_\tau}(z) \equiv \phi^{(r)}_{B_\tau+1}(z) = \sqrt{\frac{2}{\Gamma(\tau)}}\kappa^{\tau} z^{\tau-1/2} e^{-\kappa^2 z^2/2}. \tag{29}$$

The form factors describing the coupling of an external boson field with total angular momentum J and two baryons with leading twists τ_1 and τ_2 and specific handedness (r) or (l) are calculated [6,8] by analogy with the case of meson form factors (25). Then we get the following expression using this analogy, with the resulting formula looking very similar:

$$F_{V_J B^{(i_1)}_{\tau_1} B^{(i_2)}_{\tau_2}}(Q^2) = g_{V_J B^{(i_1)}_{\tau_1} B^{(i_2)}_{\tau_2}} \int_0^\infty dz V_J(Q^2+M^2,z) \times \phi^{(i_1)}_{B_{\tau_1}}(z)\phi^{(i_2)}_{B_{\tau_2}}(z), \tag{30}$$

where $g_{V_J B^{(i_1)}_{\tau_1} B^{(i_2)}_{\tau_2}}$ and $i_1, i_2 = l,\ r$ are the normalization constants, which are introduced by analogy with the case of the meson form factors.

By analogy with the meson case we get for $F_{V_J B^{(i_1)}_{\tau_1} B^{(i_2)}_{\tau_2}}(Q^2)$:

$$\begin{aligned}
F_{V_J B^{(r)}_{\tau_1} B^{(r)}_{\tau_2}}(Q^2) &= g_{V_J B^{(r)}_{\tau_1} B^{(r)}_{\tau_2}} \frac{\Gamma(J+\frac{\tau_1+\tau_2}{2}-1)}{\Gamma(J)\sqrt{\Gamma(\tau_1-1)\Gamma(\tau_2-1)}} B\left(a(M^2)+J, \frac{\tau_1+\tau_2}{2}-1\right),\\
F_{V_J B^{(r)}_{\tau_1} B^{(l)}_{\tau_2}}(Q^2) &= g_{V_J B^{(r)}_{\tau_1} B^{(l)}_{\tau_2}} \frac{\Gamma(J+\frac{\tau_1+\tau_2-1}{2})}{\Gamma(J)\sqrt{\Gamma(\tau_1-1)\Gamma(\tau_2)}} B\left(a(M^2)+J, \frac{\tau_1+\tau_2-1}{2}\right),\\
F_{V_J B^{(l)}_{\tau_1} B^{(r)}_{\tau_2}}(Q^2) &= g_{V_J B^{(l)}_{\tau_1} B^{(r)}_{\tau_2}} \frac{\Gamma(J+\frac{\tau_1+\tau_2-1}{2})}{\Gamma(J)\sqrt{\Gamma(\tau_1)\Gamma(\tau_2-1)}} B\left(a(M^2)+J, \frac{\tau_1+\tau_2-1}{2}\right),\\
F_{V_J B^{(l)}_{\tau_1} B^{(l)}_{\tau_2}}(Q^2) &= g_{V_J B^{(l)}_{\tau_1} B^{(l)}_{\tau_2}} \frac{\Gamma(J+\frac{\tau_1+\tau_2}{2})}{\Gamma(J)\sqrt{\Gamma(\tau_1)\Gamma(\tau_2)}} B\left(a(M^2)+J, \frac{\tau_1+\tau_2}{2}\right).
\end{aligned} \tag{31}$$

At large Q^2, the form factors $F_{V_J B^{(i_1)}_{\tau_1} B^{(i_2)}_{\tau_2}}(Q^2)$ scale as

$$\begin{aligned}
F_{V_J B^{(r)}_{\tau_1} B^{(r)}_{\tau_2}}(Q^2) &\sim \frac{1}{Q^{2(\frac{\tau_1+\tau_2}{2}-1)}},\\
F_{V_J B^{(r)}_{\tau_1} B^{(l)}_{\tau_2}}(Q^2) \sim F_{V_J B^{(l)}_{\tau_1} B^{(r)}_{\tau_2}}(Q^2) &\sim \frac{1}{Q^{2(\frac{\tau_1+\tau_2-1}{2})}},\\
F_{V_J B^{(l)}_{\tau_1} B^{(l)}_{\tau_2}}(Q^2) &\sim \frac{1}{Q^{2(\frac{\tau_1+\tau_2}{2})}}.
\end{aligned} \tag{32}$$

Note that the left-handed baryon wave function produces an extra $1/\sqrt{Q^2}$ falloff. In the limiting case $\tau_1 = \tau_2 = \tau$ we reproduce the result dictated by the quark counting rules [18] for the $F_{V_J B^{(r)}_{\tau_1} B^{(r)}_{\tau_2}}(Q^2)$ form factor

$$F_{V_J B^{(r)}_{\tau} B^{(r)}_{\tau}}(Q^2) \sim \frac{1}{Q^{2(\tau-1)}}. \tag{33}$$

The other three form factors have extra $1/\sqrt{Q^2}$ and $1/Q^2$ falloff, respectively,

$$\begin{aligned}
F_{V_J B^{(l)}_{\tau} B^{(r)}_{\tau}}(Q^2) \sim F_{V_J B^{(r)}_{\tau} B^{(l)}_{\tau}}(Q^2) &\sim \frac{1}{Q^{2(\tau-1/2)}},\\
F_{V_J B^{(l)}_{\tau} B^{(l)}_{\tau}}(Q^2) &\sim \frac{1}{Q^{2\tau}}.
\end{aligned} \tag{34}$$

Next we derive analytical expressions for the form factors describing the direct coupling of three hadrons induced by an off-shell meson with quantum numbers J and τ and two on-shell hadrons (two mesons or two baryons) with twists τ_1 and τ_2. As we pointed out before, the off-shell meson is described by the bulk-to-boundary propagator $\phi_{M_{J,\tau}}(Q^2+M_M^2, z)$, while on-shell hadrons by the corresponding hadronic wave functions with leading twists τ_1 and τ_2 defined before in Eqs. (19) and (29).

The coupling of off-shell meson $J \geq 1$ with two on-shell mesons reads

$$F_{M_{J,\tau}M_{\tau_1}M_{\tau_2}}(Q^2) = g_{M_{J,\tau}M_{\tau_1}M_{\tau_2}} \frac{\Gamma\left(\frac{\tau+\tau_1+\tau_2}{2}+J-2\right)}{\Gamma(J)\sqrt{\Gamma(\tau-1)\Gamma(\tau_1-1)\Gamma(\tau_2-1)}} \int_0^1 dx x^{a(M_M^2)+J-1}(1-x)^{\frac{\tau+\tau_1+\tau_2}{2}-3}\left(\frac{2}{3-x}\right)^{\frac{\tau+\tau_1+\tau_2}{2}+J-3}, \quad (35)$$

where $g_{M_{J,\tau}M_{\tau_1}M_{\tau_2}}$ is the normalization constant. For $J=0$ we get

$$F_{M_{0,\tau}M_{\tau_1}M_{\tau_2}}(Q^2) = g_{M_{0,\tau}M_{\tau_1}M_{\tau_2}} \frac{\Gamma\left(\frac{\tau+\tau_1+\tau_2}{2}-1\right)}{\sqrt{\Gamma(\tau-1)\Gamma(\tau_1-1)\Gamma(\tau_2-1)}} \int_0^1 dx x^{a(M_M^2)}(1-x)^{\frac{\tau+\tau_1+\tau_2}{2}-3}\left(\frac{2}{3-x}\right)^{\frac{\tau+\tau_1+\tau_2}{2}+J-2}. \quad (36)$$

At $Q^2 \to \infty$ the form factor (35) is independent on the total angular momentum J of off-shell meson and scales as

$$F_{M_{J,\tau}M_{\tau_1}M_{\tau_2}}(Q^2) \sim \frac{1}{(Q^2)^{\frac{\tau+\tau_1+\tau_2}{2}-2}}. \quad (37)$$

The couplings of off-shell mesons with two on-shell baryons are calculated by analogy with the case of the three-meson coupling discussed above. We get the following relations between three-meson and meson-two-baryon form factors:

$$F_{M_{J,\tau}B^{(r)}_{\tau_1}B^{(r)}_{\tau_2}}(Q^2) = \frac{g_{M_{J,\tau}B^{(r)}_{\tau_1}B^{(r)}_{\tau_2}}}{g_{M_{J,\tau}M_{\tau_1}M_{\tau_2}}} F_{M_{J,\tau}M_{\tau_1}M_{\tau_2}}(Q^2), \quad (38)$$

$$F_{M_{J,\tau}B^{(r)}_{\tau_1}B^{(l)}_{\tau_2}}(Q^2) = \frac{g_{M_{J,\tau}B^{(r)}_{\tau_1}B^{(l)}_{\tau_2}}}{g_{M_{J,\tau}M_{\tau_1}M_{\tau_2+1}}} F_{M_{J,\tau}M_{\tau_1}M_{\tau_2+1}}(Q^2), \quad (39)$$

$$F_{M_{J,\tau}B^{(l)}_{\tau_1}B^{(r)}_{\tau_2}}(Q^2) = \frac{g_{M_{J,\tau}B^{(l)}_{\tau_1}B^{(r)}_{\tau_2}}}{g_{M_{J,\tau}M_{\tau_1+1}M_{\tau_2}}} F_{M_{J,\tau}M_{\tau_1+1}M_{\tau_2}}(Q^2), \quad (40)$$

$$F_{M_{J,\tau}B^{(l)}_{\tau_1}B^{(l)}_{\tau_2}}(Q^2) = \frac{g_{M_{J,\tau}B^{(l)}_{\tau_1}B^{(l)}_{\tau_2}}}{g_{M_{J,\tau}M_{\tau_1+1}M_{\tau_2+1}}} F_{M_{J,\tau}M_{\tau_1+1}M_{\tau_2+1}}(Q^2). \quad (41)$$

B. Fermion bulk-to-boundary propagator

Next we derive the fermion bulk-to-boundary propagator, e.g. the corresponding off-shell baryons. As in the bosons case, first we derive the massless propagator and then extend it to the finite mass case by analogy with bosons. The soft-wall AdS/QCD action has been derived in Ref. [8] for fermions with higher $J \geq 5/2$:

$$\begin{aligned} S_J = \int d^dxdz\sqrt{g}e^{-\varphi(z)}\Big[&\frac{i}{2}\bar{\Psi}^{N_1\cdots N_{J-1/2}}(x,z)\epsilon_a^M\Gamma^a\mathcal{D}_M\Psi_{N_1\cdots N_{J-1/2}}(x,z) \\ &-\frac{i}{2}(\mathcal{D}_M\Psi^{N_1\cdots N_{J-1/2}}(x,z))^\dagger\Gamma^0\epsilon_a^M\Gamma^a\Psi_{N_1\cdots N_{J-1/2}}(x,z) - \bar{\Psi}^{N_1\cdots N_{J-1/2}}(x,z)\Big(\mu+V_F(z)\Big)\Psi_{N_1\cdots N_{J-1/2}}(x,z)\Big], \end{aligned} \quad (42)$$

where $\Psi_{N_1\cdots N_{J-1/2}}$ is the spin-tensor field, $g=|\det g_{MN}| = e^{10A(z)}$, $\mu=(L+3/2)/R$ is the bulk fermion mass, $V_F(z)=\varphi(z)/R$ is the dilaton field-dependent effective potential, and $\mathcal{D}_M$ is the covariant derivative acting on the spin-tensor field defined as

$$\begin{aligned} \mathcal{D}_M\Psi_{N_1\cdots N_{J-1/2}} = \partial_M\Psi_{N_1\cdots N_{J-1/2}} - \Gamma^K_{MN_1}\Psi_{KN_2\cdots N_{J-1/2}} - \cdots \\ - \Gamma^K_{MN_{J-1/2}}\Psi_{N_1\cdots N_{J-3/2}K} \\ - \frac{1}{8}\omega_M^{ab}[\Gamma_a,\Gamma_b]\Psi_{N_1\cdots N_{J-1/2}}. \end{aligned} \quad (43)$$

Here ω_M^{ab} and Γ^K_{MN} are the spin and affine connections, which are defined and related as

$$\omega_M^{ab} = A'(z)(\delta_z^a\delta_M^b - \delta_z^b\delta_M^a) = \epsilon_K^a\Big(\partial_M\epsilon^{Kb} + \epsilon^{Nb}\Gamma^K_{MN}\Big). \quad (44)$$

$\Gamma^a=(\gamma^\mu,-i\gamma^5)$ and $\Gamma^0=\gamma^0$ are the Dirac matrices.

Next, decomposing the fermion field in left- and right-chirality components

$$\begin{aligned} \Psi_{\mu_1\cdots\mu_{J-1/2}}(x,z) &= \Psi^{(l)}_{\mu_1\cdots\mu_{J-1/2}}(x,z) + \Psi^{(r)}_{\mu_1\cdots\mu_{J-1/2}}(x,z), \\ \Psi^{(l/r)} &= \frac{1\mp\gamma^5}{2}\Psi, \qquad \gamma^5\Psi^{(l/r)} = \mp\Psi^{(r/l)} \end{aligned} \quad (45)$$

and performing the Fourier transformation for the $\Psi^{(l)}(x,z)$ and $\Psi^{(r)}(x,z)$ fields in terms of left- and right-handed bulk-to-boundary propagators $F_L^{(l)}(-q^2,z)$ and $F_L^{(r)}(-q^2,z)$ with orbital momentum L (lower index) and left (l) and right (r) chirality (superscript indices)

$$\Psi^{(l/r)}_{\mu_1\cdots\mu_{J-1/2}}(x,z) = \int \frac{d^4q}{(2\pi)^4} e^{-iqx} \Psi^{(l/r)}_{\mu_1\cdots\mu_{J-1/2}}(q) F^{(l/r)}_L(-q^2,z), \tag{46}$$

we derive the equations of motion for the massless fermion bulk-to-boundary propagators $F^{(r)}_L(-q^2,z)$ and $F^{(l)}_L(-q^2,z)$:

$$\partial_z\Big(e^{-B^{(r)}(z)}\partial_z F^{(r)}_L(-q^2,z)\Big) + e^{-B^{(r)}(z)} q^2 F^{(r)}_L(-q^2,z) = 0,$$
$$\partial_z\Big(e^{-B^{(l)}(z)}\partial_z F^{(l)}_L(-q^2,z)\Big) + e^{-B^{(l)}(z)} q^2 F^{(l)}_L(-q^2,z) = 0. \tag{47}$$

Here $B^{(r)}(z) = \phi(z) - (2L+1)A(z)$ and $B^{(l)}(z) = \phi(z) - (2L+3)A(z)$. The equations of motion and their solutions for the fermion propagators are similar to those for the boson propagators (4). We also get the same equations and solutions for the fermion propagators with lower values of $J = \frac{1}{2}$ and $\frac{3}{2}$, whose actions were already discussed in Ref. [8].

We establish the following relations between the solutions for massless boson and fermion bulk-to-boundary propagators:

$$F^{(r)}_L(Q^2,z) = F^{(l)}_{L-1}(Q^2,z) = V_{L+1}(Q^2,z)$$
$$= \frac{1}{\Gamma(L+1)} \int_0^\infty dy y^L e^{-y} \left(\frac{y}{y+\kappa^2 z^2}\right)^a. \tag{48}$$

As in the case of boson propagators, the fermion ones are also properly normalized:

$$F^{(r)}_L(0,z) = F^{(l)}_L(0,z) = F^{(r)}_L(Q^2,0) = F^{(l)}_L(Q^2,0) = 1, \tag{49}$$

and they also vanish at $z\to\infty$. By analogy with the boson case we include the finite mass M in the fermion bulk-to-boundary propagator, via the extension $Q^2 \to Q^2 + M^2$.

As application of the fermion bulk-to-boundary propagators, we consider only the case of their duals—off-shell baryons with quantum numbers of total angular momentum J and mass M_B. In particular, we calculate the form factors describing the coupling of an off-shell baryon with a pair of on-shell meson and baryon. By analogy with the mesons case we define the bulk-to-boundary propagator dual to massive baryon with artbitrary J as the product of the fermion bulk-to-boundary propagator and baryon wave function with specific handedness $i = l, r$:

$$\phi^{(i)}_{B_{L,\tau}}(Q^2+M_B^2,z) = F^{(i)}_L(Q^2+M_B^2,z)\phi^{(i)}_{B_\tau}(z). \tag{50}$$

In this case we have four possibilities, corresponding to the two possible handedness of the fermion bulk-to-boundary propagator and the baryon: (i) right-handed off-shell baryon couples with right-handed on-shell baryon, (ii) right-handed off-shell baryon couples with left-handed on-shell baryon, (iii) left-handed off-shell baryon couples with right-handed on-shell baryon, and (iv) left-handed off-shell baryon couples with left-handed on-shell baryon. For these four possibilities one can produce four types of form factors:

$$F_{B^{(i_1)}_{L,\tau}M_{\tau_1}B^{(i_2)}_{\tau_2}}(Q^2) = g_{B^{(i_1)}_{L,\tau}M_{\tau_1}B^{(i_2)}_{\tau_2}} \int_0^\infty dz \phi^{(i)}_{B_{L,\tau}}(Q^2+M_B^2,z)$$
$$\times \phi_{M_{\tau_1}}(z)\phi^{(i_2)}_{B_{\tau_2}}(z), \tag{51}$$

where $g_{B^{(i_1)}_{L,\tau}M_{\tau_1}B^{(i_2)}_{\tau_2}}$ are the normalization constants.

Baryon form factors $F_{B^{(i_1)}_{L,\tau}M_{\tau_1}B^{(i_2)}_{\tau_2}}(Q^2)$ are related to meson form factors $F_{M_{J,\tau}M_{\tau_1}M_{\tau_2}}(Q^2)$ (35) as

$$F_{B^{(r)}_{L,\tau}M_{\tau_1}B^{(r)}_{\tau_2}}(Q^2) = \frac{g_{B^{(r)}_{L,\tau}M_{\tau_1}B^{(r)}_{\tau_2}}}{g_{M_{L+1,\tau}M_{\tau_1}M_{\tau_2}}} F_{M_{L+1,\tau}M_{\tau_1}M_{\tau_2}}(Q^2), \tag{52}$$

$$F_{B^{(r)}_{L,\tau}M_{\tau_1}B^{(l)}_{\tau_2}}(Q^2) = \frac{g_{B^{(r)}_{L,\tau}M_{\tau_1}B^{(l)}_{\tau_2}}}{g_{M_{L+1,\tau}M_{\tau_1}M_{\tau_2+1}}} F_{M_{L+1,\tau}M_{\tau_1}M_{\tau_2+1}}(Q^2), \tag{53}$$

$$F_{B^{(l)}_{L,\tau}M_{\tau_1}B^{(r)}_{\tau_2}}(Q^2) = \frac{g_{B^{(l)}_{L,\tau}M_{\tau_1}B^{(r)}_{\tau_2}}}{g_{M_{L+2,\tau+1}M_{\tau_1}M_{\tau_2}}} F_{M_{L+2,\tau+1}M_{\tau_1}M_{\tau_2}}(Q^2), \tag{54}$$

$$F_{B^{(l)}_{L,\tau}M_{\tau_1}B^{(l)}_{\tau_2}}(Q^2) = \frac{g_{B^{(l)}_{L,\tau}M_{\tau_1}B^{(l)}_{\tau_2}}}{g_{M_{L+2,\tau+1}M_{\tau_1}M_{\tau_2+1}}} F_{M_{L+2,\tau+1}M_{\tau_1}M_{\tau_2+1}}(Q^2). \tag{55}$$

At large Q^2 these form factors scale as

$$F_{B^{(r)}_{L,\tau}M_{\tau_1}B^{(r)}_{\tau_2}}(Q^2) \sim \frac{1}{Q^{2(\frac{\tau+\tau_1+\tau_2}{2}-1)}},$$
$$F_{B^{(l)}_{L,\tau}M_{\tau_1}B^{(r)}_{\tau_2}}(Q^2) \sim F_{B^{(r)}_{L,\tau}M_{\tau_1}B^{(l)}_{\tau_2}}(Q^2) \sim \frac{1}{Q^{2(\frac{\tau+\tau_1+\tau_2-1}{2})}},$$
$$F_{B^{(l)}_{L,\tau}M_{\tau_1}B^{(l)}_{\tau_2}}(Q^2) \sim \frac{1}{Q^{2(\frac{\tau+\tau_1+\tau_2}{2})}}. \tag{56}$$

For example, for the coupling with leading twist-2 meson and leading twist-3 baryon, one gets

$$F_{B^{(r)}_{L,3}M_2B^{(r)}_3}(Q^2) \sim \frac{1}{Q^6},$$
$$F_{B^{(l)}_{L,3}M_2B^{(r)}_3}(Q^2) \sim F_{B^{(r)}_{L,3}M_2B^{(l)}_3}(Q^2) \sim \frac{1}{Q^7},$$
$$F_{B^{(l)}_{L,3}M_2B^{(l)}_3}(Q^2) \sim \frac{1}{Q^8}. \tag{57}$$

As was expected, the left-handed baryon wave function produces an extra falloff $1/\sqrt{Q^2}$ in comparison with the right-handed one.

III. CONCLUSION

We proposed an extension of the soft-wall AdS/QCD model for the calculation of boson and fermion bulk-to-boundary propagators with arbitrary total angular momentum J. Starting from AdS/QCD actions for boson and fermion fields with arbitrary J, we derived EOMs for the massless boson and fermion bulk-to-boundary propagators. Next we include finite masses of the bulk-to-boundary propagators by shifting the square of the momentum as $-q^2 = Q^2 \to -q^2 + M^2 = Q^2 + M^2$, where M is the mass of the SM or NP fields or hadrons. Bulk-to-boundary propagators obey known and required properties of charge conservation, local limit, and confinement.

The bulk-to-boundary propagators are dual to off-shell SM (NP) fields or off-shell hadrons. This allows one to calculate form factors describing the coupling of two on-shell hadrons (mesons or baryons) with an off-shell SM (NP) field or hadron. The produced form factors are consistent, at large Q^2, with the constituent counting rules [18]. In the case of the bulk-to-boundary propagators, dual to SM (NP) fields, the application of our formalism is relevant for the values of $J = 0, 1, 2$. For the case of bulk-to-boundary propagators dual to off-shell hadrons, we are not limited by upper values of J, because hadrons (both mesons and baryons) with higher J have been searched experimentally and predicted or studied in theoretical approaches [21]. According to the Particle Data Group [21], mesons up to $J = 6$ and baryons up to $J = 15/2$ are known.

We derived the set of analytical formulas describing hadronic form factors with one off-shell and two on-shell particles. This lead to a unique opportunity to study the off-shell behavior of hadronic form factors. Therefore it provides useful insight to lattice QCD and effective field theories, where direct couplings of hadrons are calculated from the first principles or provide input parameters for phenomenological Lagrangians. Our formalism can be straightforwardly extended for study of hadronic form factors with two and three off-shell particles.

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