Quark model with hidden local symmetry and its application to T_{cc}

Bing-Ran He,[1,*] Masayasu Harada,[2,3,4,†] and Bing-Song Zou[5,6,7,‡]

[1]*Department of Physics, Nanjing Normal University, Nanjing 210023, People's Republic of China*
[2]*Department of Physics, Nagoya University, Nagoya, 464-8602, Japan*
[3]*Kobayashi-Maskawa Institute for the Origin of Particles and the Universe, Nagoya University, Nagoya, 464-8602, Japan*
[4]*Advanced Science Research Center, Japan Atomic Energy Agency, Tokai 319-1195, Japan*
[5]*CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China*
[6]*School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China*
[7]*School of Physics, Peking University, Beijing 100871, China*

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We propose a chiral quark model including the ω and ρ meson contributions in addition to the π and σ meson contributions. We show that the masses of the ground state baryons such as the nucleon, Λ_c and Λ_b are dramatically improved in the model with the vector mesons compared with the one without them. The study of the tetraquark T_{cc} is also performed in a coupled channel calculation and the resultant mass is much closer to its experimental value than the result without vector meson contribution. This approach can be applied to the future study of multiquark systems.

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I. INTRODUCTION

Since the proposal of the quark model, lots of people have spent great effort to describe mesons and baryons in a systematic way (see, e.g., Refs. [1–8]).

Existence of the chiral symmetry and its spontaneous symmetry breaking is one of the most important features in the low-energy region for hadrons including light quarks. In Ref. [9], the energy scale of the spontaneous chiral symmetry breaking (SχSB) is shown to be larger than the confinement scale, and a chiral quark model was proposed in which the chiral symmetry is spontaneously broken to generate the quark mass and the pion as the Nambu-Goldstone boson associated with the SχSB. The model includes the pion in addition to the gluon which provides the color force including the confining effect. In the chiral quark model, the pseudoscalar mesons can be exchanged between a quark and an antiquark as well as between two quarks, which is understood as shown in Fig. 1. Since the exchanged mesons are classified by the chiral symmetry, such models are generally called chiral quark models.

*hebingran@njnu.edu.cn
†harada@hken.phys.nagoya-u.ac.jp
‡zoubs@itp.ac.cn

There are several scenarios to construct a chiral quark model. Shimizu [10] studied the quark exchange effects by using pseudoscalar exchange and confining potential (CON). Obukhovsky and Kusainov [11] employed scalar and pseudoscalar exchange, one gluon exchange (OGE) and CON to study nucleon nucleon (NN) scattering and the baryon spectrum. Glozman and Riska [12,13] included pseudoscalar and vector meson exchange and also CON to study baryon spectrum. Dai *et al.* [14] included scalar, pseudoscalar and vector meson exchange together with OGE and CON to study the phase shifts of nucleon nucleon scattering. They found that when a vector meson is included, the phase shift is obviously improved in the $^1S_0$ channel. Vijande *et al.* [15,16] and Valcarce *et al.* [17] included scalar, pseudoscalar, OGE and CON to study the meson and baryon spectrum. They did not include the vector mesons for avoiding the double counting. It was stressed [15,16] that the chiral quark model beautifully reproduces the mass spectra of mesons constructed from light and heavy quarks, and also the nucleon-nucleon scattering phase shift. However, it is known that spectra of baryons are hard to be reproduced since the "σ meson" provides too much strong attractive force between two quarks [17]. Actually, if the best fitted parameters of the chiral quark model shown in Ref. [15] were used, three ground states of baryons would get extremely lower values than experimental results, i.e., $M_N^{(\mathrm{exp})} - M_N^{(\mathrm{theo})} = 262$ MeV, $M_{\Lambda_c}^{(\mathrm{exp})} - M_{\Lambda_c}^{(\mathrm{theo})} = 322$ MeV and $M_{\Lambda_b}^{(\mathrm{exp})} - M_{\Lambda_b}^{(\mathrm{theo})} = 359$ MeV.

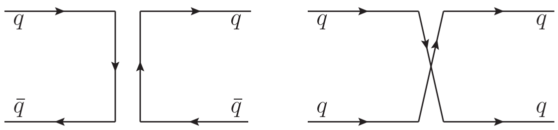

FIG. 1. Meson exchange between $q\bar{q}$ (left) and qq (right).

After the observation of $X(3872)$ in 2003, lots of exotic states are observed experimentally and studied theoretically (see, e.g., Refs. [18–27]). Recently, the discovered T_{cc} state [28,29] provides a new opportunity to check the validity of the chiral quark model. Actually, a problem similar to the one for the above baryons happens in the study of the tetraquark T_{cc}: the diquark picture of T_{cc} gives very deep binding energy and the resultant mass is far below the DD^* threshold energy, $\delta m = M_{T_{cc}} - M_D - M_{D^*} \sim -(130\text{–}185)$ MeV [30–35] to be compared with $-0.36 \pm 0.04^{+0.004}_{-0}$ MeV of the experimental observation [29]. We note that the wave function of the T_{cc} and the above three baryons contain a "good diquark" constructed from two light quarks.

The problem existing in the $I = 0$ and $S = 0$ light diquark channel indicates some important interactions are missing in the chiral quark model. We notice that, in Ref. [9], the chiral symmetry breaking scale is estimated as about $\Lambda_\chi \sim 1.2$ GeV, and that the chiral quark model is constructed as an effective field theory applicable below Λ_χ. The inclusion of vector meson is studied in lots of effective models, e.g., one boson exchange model in nucleon interaction [36–38], the Skyrme model [37,39–44], in addition to the quark models mentioned above [12–14]. Furthermore, in the framework of the hidden local symmetry (HLS) [37,45,46], these vector mesons are strongly related to the spontaneous chiral symmetry breaking.

In this paper, we construct an effective field theory for the chiral quark including the vector mesons in addition to scalar and pseudoscalar mesons using the framework of the HLS, to study mesons, baryons, and also exotic states. We note that, in the HLS, the model possesses the chiral symmetry without including the axial-vector mesons [45,46]. We will show that the ω meson provides the attractive force between a quark and an antiquark, while repulsive force is generated between two quarks due to its G parity. This property is contrasted to the one for the colored force: the colored force gives attractive force in both channels (although strengths are different). Thus, we expect that the inclusion of the ω meson with changing the colored force cures the light diquark problem in the chiral quark model, while the success for the light mesons is kept. We should note that, since the ω meson is not exchanged by mesons including a heavy quark, we only modify the colored force in the middle and long range. Furthermore, since the gluon and vector mesons play different roles in $q\bar{q}$ and qq channels, we expect that the double counting is avoided by adjusting coupling constants of the gluon exchange and vector meson exchange.

II. THE QUARK MODEL WITH HIDDEN LOCAL SYMMETRY

In this paper, we only discuss the u, d, c, b quarks for simplicity. We leave the inclusion of the strange quark for future work [47]. Now, the Hamiltonian of the present model is written as

$$H = \sum_{i=1}\left(m_i + \frac{p_i^2}{2m_i}\right) - T_{\rm CM} + \sum_{j>i=1}(V_{ij}^{\rm CON} + V_{ij}^{\rm OGE} + V_{ij}^{\sigma} + V_{ij}^{\pi} + V_{ij}^{\omega} + V_{ij}^{\rho}), \tag{1}$$

where m_i and p_i are the mass and the momentum of the ith quark, and $T_{\rm CM}$ is the kinetic energy of the center of mass of the system. $V_{ij}^{\rm CON}$, $V_{ij}^{\rm OGE}$, V_{ij}^{σ} and V_{ij}^{π} represent the potential of confinement, one-gluon exchange, σ and π exchange, which are given in Refs. [15,17]. V_{ij}^{ω} and V_{ij}^{ρ} are the potential of ω and ρ exchange. They take the same form except the isospin dependence. It is convenient to define the common part $V_{ij}^{v=\omega,\rho}$ as $V_{ij}^{\omega} = V_{ij}^{v=\omega}$ and $V_{ij}^{\rho} = \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j V_{ij}^{v=\rho}$ for qq ($q = u$, d) potential, where $\boldsymbol{\tau}_i$ is the flavor $SU(2)$ Pauli matrices of the ith quark. The V_{ij}^{v} in the spatial coordinate is given as

$$\begin{aligned} V_{ij}^{v} = \frac{\Lambda_v^2}{\Lambda_v^2 - m_v^2}\Bigg\{&\frac{g_v^2}{4\pi} m_v\left[Y(m_v r) - \left(\frac{\Lambda_v}{m_v}\right)Y(\Lambda_v r)\right] + \frac{m_v^3}{m_i m_j}\left(\frac{g_v(2f_v + g_v)}{16\pi} + \frac{\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j}{6}\frac{(f_v + g_v)^2}{4\pi}\right) \\ &\times\left[Y(m_v r) - \left(\frac{\Lambda_v}{m_v}\right)^3 Y(\Lambda_v r)\right] - \boldsymbol{S}_+\cdot\boldsymbol{L}\frac{g_v(4f_v + 3g_v)}{8\pi}\frac{m_v^3}{m_i m_j}\times\left[G(m_v r) - \left(\frac{\Lambda_v}{m_v}\right)^3 G(\Lambda_v r)\right] \\ &- \boldsymbol{S}_{ij}\frac{(f_v + g_v)^2}{4\pi}\frac{m_v^3}{12 m_i m_j}\times\left[H(m_v r) - \left(\frac{\Lambda_v}{m_v}\right)^3 H(\Lambda_v r)\right]\Bigg\}. \end{aligned} \tag{2}$$

Here m_v, Λ_v, g_v and f_v are the mass, cutoff, electric and magnetic coupling constants of the relevant vector meson, respectively. $\boldsymbol{\sigma}_i$ is the spin $SU(2)$ Pauli matrices of the ith quark, $\boldsymbol{S}_+ = \frac{\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j}{2}$, $\boldsymbol{S}_{ij} = 3(\boldsymbol{\sigma}_i\cdot\hat{r}_{ij})(\boldsymbol{\sigma}_j\cdot\hat{r}_{ij}) - \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j$. $Y(x) = e^{-x}/x$, $H(x) = (1 + 3/x + 3/x^2)Y(x)$ and $G(x) = (1/x + 1/x^2)Y(x)$. The potentials of V_ω and V_ρ for $q\bar{q}$ ($q = u$, d) is

TABLE I. Model parameters. The naming scheme is the same as Ref. [15].

$m_u = m_d$ (MeV)	303.3	m_σ (fm^{-1})	3.42
m_c (MeV)	1696.0	m_π (fm^{-1})	0.7
m_b (MeV)	5039.6	m_ω (fm^{-1})	3.97
a_c (MeV)	364.5	m_ρ (fm^{-1})	3.93
μ_c (fm^{-1})	0.6	$\Lambda_\sigma = \Lambda_\pi$ (fm^{-1})	4.2
Δ (MeV)	100.28	$\Lambda_\omega = \Lambda_\rho$ (fm^{-1})	7.2
a_s	0.731	$g_{ch}^2/(4\pi)$	0.54
α_0	2.742	g_ω	3.841
Λ_0 (fm^{-1})	0.304	g_ρ	0.675
μ_0 (MeV)	241.205	f_ω	−1.416
$\hat{r}_0$ (MeV · fm)	95.621	f_ρ	0.581
$\hat{r}_g$ (MeV · fm)	155.066		

obtained by performing a G-parity transformation of that in the qq case.

We solve the Schrödinger equation of the Hamiltonian (1) by using the Gaussian expansion method [48]. For studying the effects of vector mesons, we keep the model parameters of the π and σ potentials as Ref. [15]. We fit the model parameters of vector meson potentials and color potential to several mesons and baryons. We show best fitted values of model parameters in Table I. Here, we newly introduce eight parameters, i.e., m_ω, m_ρ, Λ_ω, Λ_ρ, g_ω, g_ρ, f_ω, and f_ρ, with other parameters in the same naming scheme as Ref. [15]. The vector masses m_ω and m_ρ are taken from PDG [49]. The cutoffs Λ_ω and Λ_ρ of vector mesons are not sensitive to get the result: here we take a typical value of 7.2 fm^{-1}. The wave functions for the meson and the baryon can be found in [50].

III. RESULTS

We show the meson masses calculated in the present model in Fig. 2, and baryon masses in Fig. 3.

The blue block represents the error calculated as $\mathrm{Err(sys)} = \sqrt{\mathrm{Err(exp)}^2 + \mathrm{Err(the)}^2}$, where Err(exp) is the experimental error taken from PDG [49], while Err(the) represents the model limitation error as we do not include isospin breaking effects, and also ignore mixing effects (e.g. S-D, P-F mixing between meson states). We take Err(the) as about 40 MeV for ground state and 80 MeV for excited state of mesons and baryons. We minimize $\chi^2 = \sum_i (\frac{m_i(\mathrm{the}) - m_i(\mathrm{exp})}{\mathrm{Err_i(sys)}})^2$ of the system, where m_i(the) and m_i(exp) are theoretical and experimental mass of each particle, respectively. The total χ^2 of the present work is about 15.12. The degrees of freedom (d.o.f.) of the present work is d.o.f. $= 51 - (23 - 3) = 31$, which is calculated in the following way: (i) we used 51 particles (42 mesons and 9 baryons) to minimize χ^2, (ii) the total parameter of the present model is 23 but we fixed three parameters m_π, m_ω and m_ρ as experiment values. Thus, $\chi^2/\mathrm{d.o.f.} \simeq 15.12/31 \simeq 0.4877$ indicate the reliability of the present model is about 99.26%. The orange blocks for mesons are taken from Ref. [15], the corresponding baryon spectrum are calculated using the same parameter. The green blocks show the predictions of mass spectra of the present model.

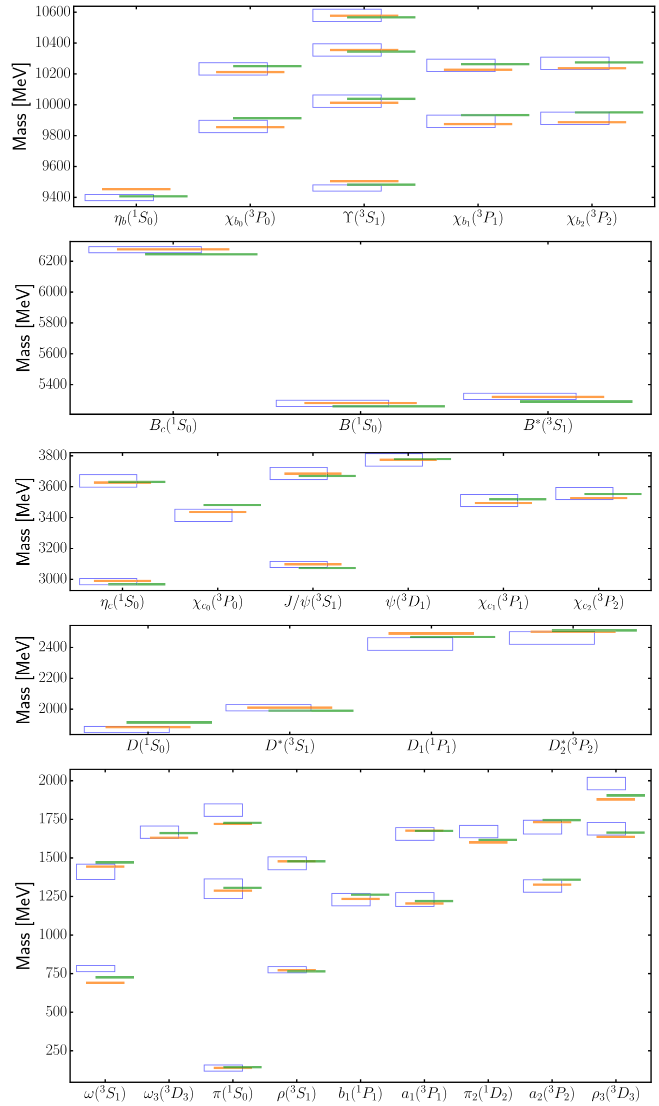

FIG. 2. Mass spectrum in $b\bar{b}$ (first panel), $c\bar{b}$ and $b\bar{q}$ (second panel), $c\bar{c}$ (third panel), $q\bar{c}$ (fourth panel) and $q\bar{q}$ (fifth panel) systems. The green lines show the predictions in the present model including vector mesons, while the orange lines the ones in the model without vector mesons.

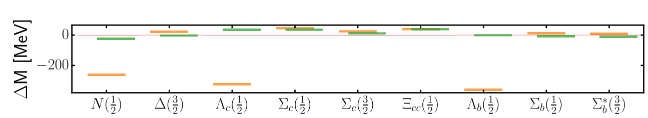

FIG. 3. Mass spectrum of baryon system. The meaning of colors is as in Fig. 2.

We would like to stress that the spectra of ground-state baryons N, Λ_c and Λ_b, which all contain a good diquark, are dramatically improved as one can easily see from Fig. 3, while the present model reproduces the meson spectra as good as the original model in Ref. [15] as shown in Fig. 2. From the values of parameters listed in Table I, this is the consequence of the ω exchange contribution: As we stated above, the ω meson provides the attractive force between a quark and an antiquark and the repulsive force between two quarks.

Here, to understand Table I we clarify the operators included in the potentials with the sign of each contribution, which are summarized in Table II. Here we only discuss $L = 0$ states for simplicity, in such a case only central force remains. The problem of deep binding energy occurs when the good light diquark appears ($I = 0$ and $S = 0$). To solve this problem, we have four possible ways of modification to provide a repulsive force between qq: (i) modify terms proportional to the operator “1” in all I and S channels, (ii) modify terms to $\tau_i\tau_j$ in the $I = 0$ channel, (iii) modify terms to $\sigma_i\sigma_j$ in the $S = 0$ channel, (iv) modify terms to $\sigma_i\sigma_j \cdot \tau_i\tau_j$ in $I = 0$ and $S = 0$ channels. We should note that we kept V_π and V_σ the same as in Ref. [15], since Ref. [15] has already given a good description of meson spectra and also phase shift of baryon-baryon scattering. This implies that $g_\rho + f_\rho$ in V_ρ cannot be too big since it generates the contribution proportional to the $\sigma_i\sigma_j \cdot \tau_i\tau_j$ operator which is already saturated by the contribution from V_π, and thus way (iv) is ruled out. The modification of way (ii) is done by V_ρ (terms proportional to g_ρ) playing a role similar to the one by a_0 meson exchange, which is introduced to explain the ω-ρ masses in Ref. [16]. Since a_0 only gives a small modification of meson spectra, as can be seen from the predicted mass $\omega(^3S_1)$ in Fig. 2, g_ρ should be very small. As a result there remains only two ways, i.e., (i) and (iii), which are achieved only by the inclusion of a ω meson contribution proportional to g_ω and $f_\omega + g_\omega$. Here, let us briefly explain why the ω meson contribution can cure the problem of deep binding of a good light diquark in the $S = 0$ channel. As seen in Table II, the one-gluon exchange potential $V^{\rm OGE}$ contains the $\sigma_i\sigma_j$ operator, which gives the attractive force in both $q\bar{q}$ and qq channels, while the $\sigma_i\sigma_j$ term in V_ω generates an attractive force in the $q\bar{q}$ channel but a repulsive force in the qq channel. For the operator 1, the change of $V^{\rm OGE}$ and V_ω can be compensated by $V^{\rm CON}$, i.e., inclusion of ω together with the weakening of color potential make the attractive force in the qq channel smaller with keeping the force in the $q\bar{q}$ channel intact. As a conclusion, way (i) and (iii) with taking $|g_\rho| \sim |f_\rho| \sim 0$ is the minimal modification to the model in Ref. [15].

We next explain a mechanism why the spectra of mesons including a heavy quark, $c\bar{c}$, $q\bar{c}$ and $b\bar{b}$ systems, are as good as in Refs. [15,17] although the color potential $V^{\rm OGE} + V^{\rm CON}$, which is the only force acting in these systems, is modified. Since the effective range of the heavy quark sector is rather smaller than that of the light sector, we can modify the color potential $V_{ij}^{\rm CON} + V_{ij}^{\rm OGE}$ in the middle and long range parts without changing the short range part to maintain the main features of mesons made of heavy quarks. In the present work we achieved this modification by keeping the forms in Ref. [15] and just changing the values of parameters. In Fig. 4, we show the color potential $(V_{\rm OGE} + V_{\rm CON})r^2$ of Υ (left) and ω (right). This shows that the color potential is changed slightly in both heavy and light quark sectors when the distance is larger than about 0.25 fm. The typical distance between two heavy quarks such as $\Upsilon(1S)$ in the present analysis is about 0.21 fm, so that the potential relevant for mesons made from b and $\bar{b}$ is not changed. Thus, we can see that, from Fig. 2 the spectra of mesons including heavy quarks are in reasonable agreement with experiment. On the other hand, a typical distance between q and $\bar{q}$ in mesons of the ground state, e.g., $\rho(1S)$ is about 0.83 fm, so that the color attractive force is actually weakened. The attractive force mediated by the ω meson between q and $\bar{q}$ compensates the weakening effect of the color force to provide good agreement of the predictions of the present model with experiment as can be seen in Fig. 2. For further improvement of the spectrum, we might need to modify the forms of color potential, which we leave as a future work.

Based on the above success in the meson and baryon sectors, we now perform a coupled channel calculation of T_{cc} and T_{bb} tetraquarks. The T_{cc} and T_{bb} have eight

TABLE II. List of operators included in the potentials for $L = 0$ states and the sign of the contributions. The left (right) side of slash represents qq ($q\bar{q}$), respectively.

	1	$\tau_i\tau_j$	$\sigma_i\sigma_j$	$\sigma_i\sigma_j \cdot \tau_i\tau_j$
σ [15]	−/−			
π [15]				+/−
a_0 [16]		−/+		
OGE [15]	−/−		+/+	
CON [15]	+/+			
ω (this work)	+/−		+/−	
ρ (this work)		+/+		+/+

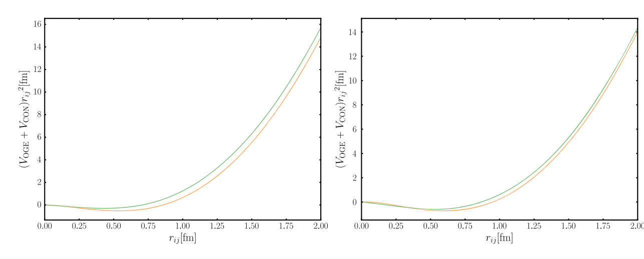

FIG. 4. Potential $(V_{\rm OGE} + V_{\rm CON})r^2$ of Υ (left) and ω (right). Orange and green lines represent Ref. [15] and the present study, respectively.

TABLE III. Root-mean-square distance (fm) between two quarks of T_{cc} and T_{bb} in coupled channel calculation.

	r_{cc}	$r_{\bar{q}c}$	$r_{\bar{q}\bar{q}}$	$r_{\bar{q}b}$	r_{bb}
T_{cc}	1.56	1.24	1.70		
T_{bb}			0.75	0.65	0.37

channels, with six of meson-meson structure and two of diquark-antidiquark structure. The detailed wave functions of T_{cc} and T_{bb} can be found in Refs. [34,35]. In the coupled channel calculation, the nonzero components of T_{cc}/T_{bb} are as follows: $[DD^*]_{1\otimes1}(39\%)/[BB^*]_{1\otimes1}(12\%)$, $[D^*D]_{1\otimes1}(39\%)/[B^*B]_{1\otimes1}(12\%)$, $[D^*D^*]_{1\otimes1}(5\%)/[B^*B^*]_{1\otimes1}(32\%)$ and $[(cc)^*(\bar{q}\bar{q})]_{\bar{3}\otimes3}(17\%)/[(bb)^*(\bar{q}\bar{q})]_{\bar{3}\otimes3}$ (44%). The subscripts $1\otimes1$ and $\bar{3}\otimes3$ denote color singlet-singlet and antitriplet-triplet, respectively. We find that after channel coupling the mass of T_{cc}/T_{bb} is $4.9/88.2$ MeV below the DD^*/BB^* threshold. This result demonstrates a significant improvement in T_{cc} compared to the previous value of 180 MeV, which was obtained in the model without vector mesons. The deep binding of T_{bb} is due to the smaller distance between bb compared to cc, and feels stronger color electric force which is proportional to $1/r$. The root-mean-square (rms) distance between two quarks included in T_{cc} and T_{bb} is given in Table III. Combining the results for the ratios of the components with the rms distances in Table III, we conclude that T_{cc} is a meson-meson molecular state, while T_{bb} is a compact tetraquark state.

IV. SUMMARY

We add the vector meson exchange effects to a chiral quark model to produce a repulsive force between two light quarks. This effect dramatically improves the spectra of the baryons including a good diquark, while meson spectra are in good agreement with experiments. In addition, the predicted mass of T_{cc} is much closer to its experimental value than the result without vector mesons. This approach can be applied to the future study of multiquark systems.

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