

Cold quark matter in a quasiparticle model: Thermodynamic consistency and stellar properties

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The strong coupling in the effective quark mass was usually taken as a constant in a quasiparticle model while it is, in fact, running with an energy scale. With a running coupling, however, the thermodynamic inconsistency problem appears in the conventional treatment. We show that the renormalization subtraction point should be taken as a function of the summation of the biquadratic chemical potentials if the quark's current masses vanish, in order to ensure full thermodynamic consistency. Taking the simplest form, we study the properties of up-down (ud) quark matter, and confirm that the revised quasiparticle model fulfills the quantitative criteria for thermodynamic consistency. Moreover, we find that the maximum mass of an ud quark star can be larger than 2 times the solar mass, reaching up to $2.31M_{\odot}$, for reasonable model parameters. However, to further satisfy the upper limit of tidal deformability $\tilde{\Lambda}_{1.4} \leq 580$ observed in the event GW170817, the maximum mass of an ud quark star can only be as large as $2.08M_{\odot}$, namely $M_{\max} \lesssim 2.08M_{\odot}$. In other words, our results indicate that the measured tidal deformability for event GW170817 places an upper bound on the maximum mass of ud quark stars, but does not rule out the possibility of the existence of quark stars composed of ud quark matter, with a mass of about 2 times the solar mass.

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I. INTRODUCTION

The accepted theory for a precise description of the strong interaction between quarks and gluons is quantum chromodynamics (QCD) [1], in which the running coupling $\alpha_s(\Lambda^2)$ acts as one of the basic parameters. The relevant theoretical and experimental research shows that QCD has two special features: color confinement and asymptotic freedom [2–4]. The former has been proposed because a single quark cannot be observed from experiments even at the present time, but the quantum number of the so-called color is indispensable in order to explain the confusion of the requisite antisymmetry wave function of hadrons. The latter reveals that the interaction between quarks gets weaker with increasing energy scales. These two extraordinary features of QCD happen at two extreme energy

scales: the asymptotic freedom is related to the high-energy region while the color confinement comes up in the infrared domain. Owing to the difficulties in the nonperturbative regime and the consistent implementation of finite density in lattice simulations, some QCD-inspired phenomenological models have been constructed to study the properties of strongly interacting quark matter, e.g., the MIT bag model [5–8], Nambu-Jona-Lasinio (NJL) model [9–14], perturbative QCD model [15–17], density-dependent quark mass model [18–22], etc.

Employing a two-flavor NJL model including the vector interactions, Yuan *et al.* [23] found that there is a possibility for both ud and strange (uds) quark matter being absolutely stable, depending on the interplay of the confinement with quark vector interaction and the exchange interaction channels. As stated by Buballa in Ref. [24], however, the NJL model does not support the idea that uds quark matter is more stable than the ud quark matter. In particular, by using an effective theory with Yukawa coupling to quarks Holdom *et al.* [25] demonstrated that ud quark matter generally has lower bulk energy per baryon than normal nuclei and uds quark matter. Moreover, within a confining quark matter model, Cao *et al.* [26] demonstrated

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that the reported GW190814's secondary component with a mass greater than 2 times the solar mass could be an ud quark star. After the proposal that ud quark matter is more stable than uds quark matter, the study of the properties of strongly interacting ud quark matter and its astrophysical implications has recently received increased interest in the literature, e.g., Refs. [27–33].

The quasiparticle model, as one of the QCD-inspired phenomenological models, has also been extensively studied and widely applied to the study of quark matter over the last two decades. In the case of a hot gluon gas, the equation of state with the effective mass agrees well with the lattice results even at temperatures where the QCD coupling constant is not small [34–37]. Motivated by the successful application of the hard-thermal-loop approximation at finite temperature, many authors have applied the hard-dense-loop approximation at finite density to investigate the properties of the strongly interacting cold quark matter [36–39]. Its original version keeps all the thermodynamic quantities in the same form as those of a free particle system [40–42]. The thermodynamic treatment in this model has been much discussed [43–45]. If the QCD coupling is treated as a real constant, the thermodynamic inconsistency problem can be removed by including an additional term in the thermodynamic potential density [39]. However, the actual QCD coupling is running, i.e., it depends on where the subtraction point is chosen in the renormalization scheme [46,47]. In high-energy physics, the subtraction point Λ is usually taken as the momentum transfer of the corresponding process, while in dense quark matter, it should be a function of the chemical potentials of the quarks through the subtraction point [48–50]. In this case, the additional term is determined by a path integral. Because the thermodynamic quantity is a state function, it should thus be independent of the chosen path. Otherwise, the thermodynamic inconsistency problem will appear [18,51]: the pressure at the minimum energy per baryon deviates from zero, which contradicts fundamental thermodynamics. In this work, we will show that the renormalization-group subtraction point should be generally taken as a function of the biquadratic summation of the respective chemical potential for quarks.

In this study, we focus on the thermodynamic inconsistency problem of a two-flavor quasiparticle model at zero temperature and finite chemical potential with an effective quark mass. Within the revised quasiparticle model, we are particularly interested in the stability windows and thermodynamic properties of the ud quark matter. Through our research, we seek to understand the behavior of quark matter under extreme conditions and its implications for cosmological and astrophysical phenomena including, in particular quark star structure and tidal deformability. The results for the mass-radius relation as well as the tidal deformability of quark stars indicate that

the recently observed massive pulsars with a mass of about two solar masses could be made of ud quark matter.

The rest of the paper is organized as follows. In Sec. II, we review the conventional treatment in the quasiparticle model with an effective quark mass and show the thermodynamic inconsistency in this treatment. Then in Sec. III, we limit the form of the renormalization subtraction point as a function of the chemical potentials and determine a medium-dependent effective bag constant which makes the thermodynamic treatment self-consistent. In Sec. IV, we present the numerical results for the calculations. Finally, a summary is given in Sec. V.

II. THERMODYNAMIC INCONSISTENCY IN THE CONVENTIONAL VERSION OF QUASIPARTICLE MODEL

Owing to the strong interaction between quarks, the quark masses are medium dependent, namely, the quark masses vary with the environment. The effective quark mass is obtained from the zero momentum limit of the quark dispersion relation, which follows the so-called hard-dense-loop approximation of the quark self-energy, giving [36,52]

$$m_i^* = \sqrt{\frac{2\alpha_s}{3\pi}}\mu_i \quad (i = u, d), \quad (1)$$

where μ_i represents the chemical potential for the i th flavor of quarks.

At zero temperature, the quasiparticle contribution to the thermodynamic potential density of the system is given by

$$\Omega_i = -\frac{g_i}{48\pi^2} \left(\mu_i k_{F,i} (2\mu_i^2 - 5m_i^{*2}) + 3m_i^{*4} \ln \frac{\mu_i + k_{F,i}}{m_i^*} \right), \quad (2)$$

where $k_{F,i} = \sqrt{\mu_i^2 - m_i^{*2}}$ is the Fermi momentum, and g_i is the statistical factor with $g_i = 6$ for quarks and $g_i = 2$ for electrons. In addition, the thermodynamic potential density of electrons is $\Omega_e = -\mu_e^4/12\pi^2$ with μ_e being the electron chemical potential.

By using the basic relations of standard thermodynamics $P = -\Omega$ and $E = \Omega + \mu n$ with Ω being the thermodynamic potential density of the system, n the number density of the particles, in Fig. 1 we plot the energy per baryon of ud quark matter as a function of the pressure without adding an effective bag constant.

As proved in Refs. [51,53,54] any consistent thermodynamic treatment must ensure that $\delta = P - n^2 \frac{d}{dn} \left(\frac{E}{n} \right) = 0$ at arbitrary density. In particular, the pressure at the minimum energy per baryon must be zero. From Fig. 1, however, it is evident that the energy minimum does not locate at the zero pressure point, as required by the thermodynamic consistency of phenomenological models. In other words, the

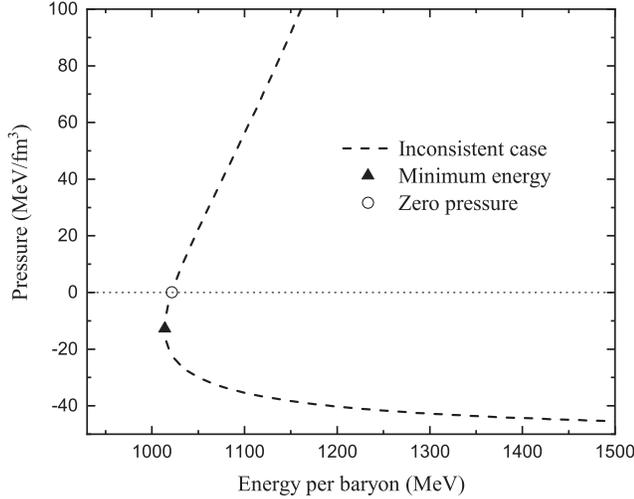


FIG. 1. Pressure versus energy per baryon of ud quark matter in the quasiparticle model without adding an effective bag constant (inconsistent case). It is clear that the energy minimum denoted by a triangle does not locate at the zero pressure point denoted by a circle.

quasiparticle model with a chemical potential-dependent quark mass but without adding an effective bag constant is thermodynamically inconsistent. We should emphasize that if one takes the strong coupling constant α_s as a free input parameter and also adds a medium-dependent effective bag constant to the thermodynamic potential density of the system, as was done in Refs. [38,55–58], the phenomenological models do not encounter thermodynamic inconsistency problems, but we note that the strong coupling runs with the energy scale. Hence, in the next section we will show that a thermodynamically determined effective bag constant with a running strong coupling constant can solve this problem properly.

III. SELF-CONSISTENT THERMODYNAMIC TREATMENT AT FINITE CHEMICAL POTENTIALS

The quasiparticle model is based on the idea that quarks can be treated as quasiparticles with an effective quark mass, which is determined by the quark self-energy and quark-quark interaction. The self-consistency of this model is maintained by adding a new term, which is a chemical potential dependent variable at zero temperature. In the two-flavor case, the total thermodynamic potential density of the system can be written as [38]

$$\Omega = \sum_i \Omega_i(\mu_i, m_i^*) + B^*(\{\mu_i\}), \quad (3)$$

where $B^*(\{\mu_i\}) \equiv B(\{\mu_i\}) + B_0$ is defined as an effective bag constant including the energy contributions both from the vacuum B_0 and medium effects $B(\{\mu_i\})$. Following the procedure proposed in Ref. [59], we have

$$\begin{aligned} n_i &= - \left. \frac{d\Omega}{d\mu_i} \right|_{\mu_{j \neq i}} \\ &= - \frac{\partial \Omega_i}{\partial \mu_i} - \underbrace{\left[\frac{\partial \Omega_i}{\partial m_i^*} \frac{\partial m_i^*}{\partial \mu_i} + \sum_{j=u,d} \frac{\partial \Omega_j}{\partial m_j^*} \frac{\partial m_j^*}{\partial \alpha_s} \frac{\partial \alpha_s}{\partial \Lambda} \frac{\partial \Lambda}{\partial \mu_i} + \frac{\partial B}{\partial \mu_i} \right]}_{=0} \end{aligned} \quad (4)$$

for the i th flavor quark in the two-flavor quasiparticle model. The derivative of the quasiparticle contribution of the thermodynamic potential density with respect to the effective quark mass m_i^* is given by

$$\frac{\partial \Omega_i}{\partial m_i^*} = \frac{g_i m_i^*}{4\pi^2} \left(\mu_i k_{F,i} - m_i^{*2} \ln \frac{\mu_i + k_{F,i}}{m_i^*} \right). \quad (5)$$

From the quasiparticle point of view, quarks are considered quasiparticles with an effective quark mass generated by medium effects. In this case, the quark number density should have the same form as in the standard statistical mechanics, with the current quark mass replaced by the effective mass, i.e.,

$$n_i = \frac{g_i}{6\pi^2} k_{F,i}^3. \quad (6)$$

This is, in fact, the first term on the right-hand side of the second equality of Eq. (4). By also considering the contribution from the down quark, we immediately get

$$dB = -\mathcal{F}d\mu_u - \mathcal{M}d\mu_d, \quad (7)$$

where

$$\mathcal{F}(\mu_u, \mu_d) = \frac{\partial \Omega_u}{\partial m_u^*} \frac{\partial m_u^*}{\partial \mu_u} + \sum_{j=u,d} \frac{\partial \Omega_j}{\partial m_j^*} \frac{\partial m_j^*}{\partial \alpha_s} \frac{\partial \alpha_s}{\partial \Lambda} \frac{\partial \Lambda}{\partial \mu_u}, \quad (8)$$

$$\mathcal{M}(\mu_u, \mu_d) = \frac{\partial \Omega_d}{\partial m_d^*} \frac{\partial m_d^*}{\partial \mu_d} + \sum_{j=u,d} \frac{\partial \Omega_j}{\partial m_j^*} \frac{\partial m_j^*}{\partial \alpha_s} \frac{\partial \alpha_s}{\partial \Lambda} \frac{\partial \Lambda}{\partial \mu_d}. \quad (9)$$

Because the thermodynamic quantity should be a function of the independent state variables, the integral of Eq. (7) should be path independent [60,61]. Mathematically, this can be achieved by imposing the following condition:

$$\frac{\partial \mathcal{F}(\mu_u, \mu_d)}{\partial \mu_d} = \frac{\partial \mathcal{M}(\mu_u, \mu_d)}{\partial \mu_u}. \quad (10)$$

Obviously, if the QCD coupling constant was assumed to be a pure constant as in Ref. [38], the above Cauchy conditions in Eq. (10) would always be satisfied. However, the QCD coupling is running with the quark chemical potentials. We thus need to find the condition that ensures the equality in Eq. (10). Substituting the explicit expressions for \mathcal{F} and \mathcal{M} in Eqs. (8) and (9) into Eq. (10) and simplifying the corresponding expression then gives

$$\mu_u^3 \frac{\partial \Lambda}{\partial \mu_d} = \mu_d^3 \frac{\partial \Lambda}{\partial \mu_u}. \quad (11)$$

Equation (11) is a quasilinear partial differential equation, and its general solution is an arbitrary function of $\mu_u^4 + \mu_d^4$, i.e., $\Lambda = f(\mu_u^4 + \mu_d^4)$. Since the subtraction point Λ carries the dimension of an energy, we take the simplest form

$$\Lambda = C \sqrt[4]{\frac{\mu_u^4 + \mu_d^4}{N_f}}, \quad (12)$$

where C is introduced as a dimensionless model parameter, and $N_f = 2$ is the number of flavors.

In this case, the bag constant B can be obtained by a path integral as

$$B = \int_{(0,0)}^{(\mu_u, \mu_d)} \mathcal{F}(\mu'_u, \mu'_d) d\mu'_u + \mathcal{M}(\mu'_u, \mu'_d) d\mu'_d. \quad (13)$$

IV. NUMERICAL RESULTS AND DISCUSSIONS

Considering a system composed of up, down quarks and electrons in β equilibrium achieved by the weak reactions $d \leftrightarrow u + e + \bar{\nu}_e$, one easily obtains the following equality:

$$\mu_u = \mu_d + \mu_e. \quad (14)$$

Neutrinos are assumed to enter and escape the system freely, so the chemical potential of neutrinos is zero. The baryon number density n_b is defined as

$$n_b = \frac{1}{3}(n_u + n_d). \quad (15)$$

The charge neutrality condition is also imposed on the system, i.e.,

$$\frac{2}{3}n_u - \frac{1}{3}n_d - n_e = 0. \quad (16)$$

The strong coupling constant α_s , which is related to the running coupling constant g by $\alpha_s = g^2/4\pi$, is, in fact, a function of the renormalization subtraction point Λ . In this work, we adopt the analytic QCD coupling constant proposed by Shirkov *et al.* at the one-loop level [62], which is written as

$$\alpha_s(\Lambda^2) = \frac{4\pi}{\beta_0} \left[\frac{1}{\ln(\Lambda^2/\Lambda_{\text{QCD}}^2)} + \frac{\Lambda_{\text{QCD}}^2}{\Lambda_{\text{QCD}}^2 - \Lambda^2} \right], \quad (17)$$

where $\beta_0 = 11 - 2N_f/3$, the QCD scale parameter $\Lambda_{\text{QCD}} = 147$ MeV [63], and $N_f = 2$ is the quark flavor considered in the system.

For a given baryon density n_b , the chemical potentials μ_i are obtained by solving a set of Eqs. (14)–(16). Because the

integrand in Eq. (13) is long and complicated, only numerical studies will be possible. Accordingly, by substituting Eq. (13) into Eq. (3), the energy density and the pressure of the system can then be derived:

$$E = \Omega + \sum_i \mu_i n_i, \quad P = -\Omega. \quad (18)$$

A. Parameter space and equation of state of quark matter

We note that the energy per baryon of ud quark matter should be greater than 930 MeV in order not to contradict the standard nuclear physics requirement. Consequently, the stable condition and the two solar masses of compact stars usually set strict constraints on the parameter space of phenomenological models. In Fig. 2, we show the parameter space for ud quark matter in the C versus $B_0^{1/4}$ plane, where the region above the red solid line corresponds to the allowed region for the ud quark matter. In contrast, the lower right shaded region below the red solid line is forbidden since the energy per baryon of ud quark matter is less than 930 MeV. In addition, the measured gravitational mass of PSR J0740 + 6620 [64] with a central value of $2.08M_\odot$ and a lower limit of $2.01M_\odot$ is also translated into the $C - B_0^{1/4}$ plane, indicated by the red dashed and blue dotted lines with star symbols. Therefore, in the allowed region between the red solid and dashed blue

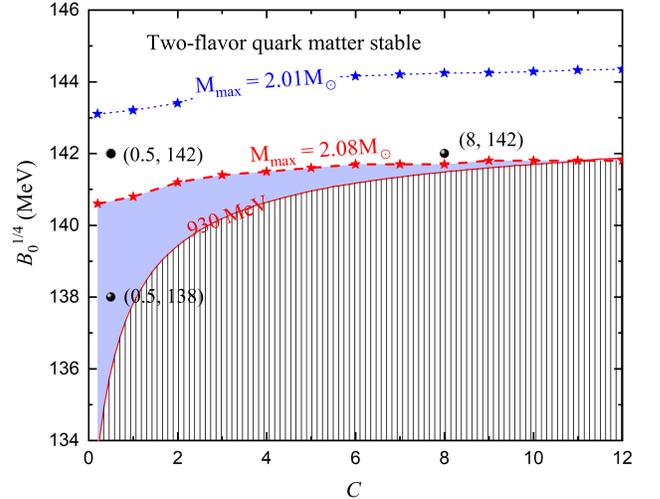


FIG. 2. Stability windows for ud quark matter in the $C - B_0^{1/4}$ plane. The down-right black shaded area is forbidden, where the energy per baryon of ud quark matter is less than 930 MeV. Meanwhile, the region between the red dashed and blue dotted lines with star symbols corresponds to the ud quark matter stable region with an equation of state that can support an ud quark star with the maximum mass in the range $2.01 \leq M_{\text{max}}/M_\odot \leq 2.08$. The selected typical model parameters are indicated with solid black dots.

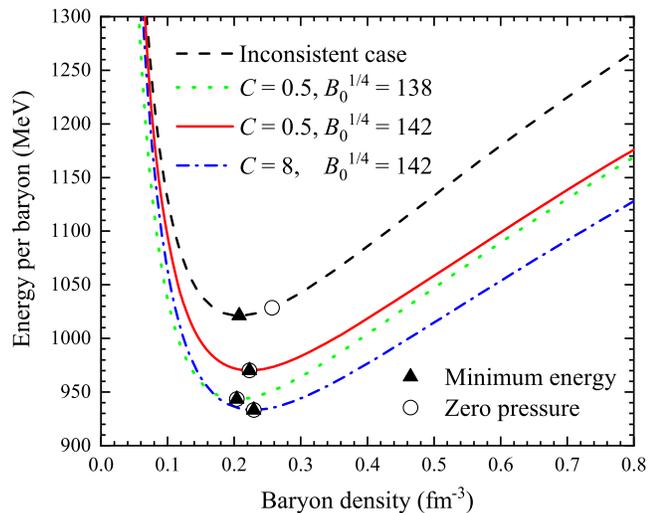


FIG. 3. Density behavior of the energy per baryon of ud quark matter with different values of C and B_0 . The result from the model without adding an effective bag constant is also shown for comparison. It is clear that in the revised quasiparticle model, the zero pressure points denoted by circles are exactly located at the minimum energy per baryon denoted by triangles, except for the inconsistent case denoted by the dashed line.

lines, the equation of state further meets the requirement that the maximum mass of ud quark stars can be as large as or even greater than $2.01M_\odot$, where M_\odot is the solar mass.

Considering the ud quark matter with the equations of state satisfying the requirement of two solar masses of ud quark stars, the parameters are set as $(C, B_0^{1/4}/\text{MeV}) = (0.5, 138), (0.5, 142), (8, 142)$, as indicated in Fig. 2 by dots. Figure 3 shows the energy per baryon of ud quark matter as a function of the baryon density with the selected parameter sets. It can be seen that for the revised quasiparticle model the energy minima lie exactly at the corresponding zero pressure points, thus satisfying the thermodynamic requirement of the model. However, for the model without adding the effective bag constant as shown by the black dashed line, the energy minimum does not coincide with the zero pressure point. Furthermore, comparing the three typical lines, we find that with the increase in B_0 (C), the minimum energy per baryon increases (decreases). This is because, with increasing B_0 (C), the thermodynamic potential density of the system Ω in Eq. (3) and in turn the energy density increases (decreases) according to Eq. (18).

In Fig. 4, we show the equations of state of ud quark matter in the revised quasiparticle model, and the result for the inconsistent case and $P = E$ are also shown for comparison. From left to right, the green dotted, blue dash-dotted, red solid, and black dashed lines correspond to the revised quasiparticle model with parameter sets $(C, B_0^{1/4}/\text{MeV}) = (0.5, 138), (8, 142), (0.5, 142)$ and the inconsistent case, respectively. One can see that the

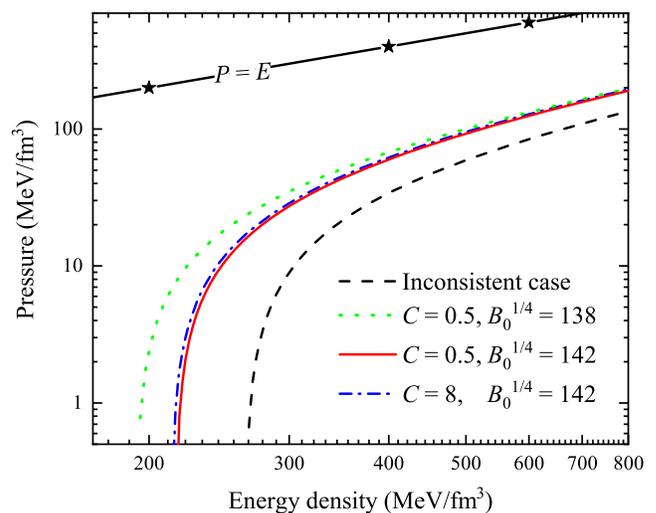


FIG. 4. Equations of state of cold quark matter in the revised quasiparticle model with different values of C and B_0 . For comparison, we also show the results for the inconsistent case and $P = E$. Conventions for line styles and colors are the same as in Fig. 3.

pressure increases with the energy density without any exception. In particular, as the bag constant B_0 increases, the line moves to higher values of the energy density, whereas for a larger value of C , the line moves to a lower value of the energy density. Furthermore, except for the black dashed line (inconsistent case), the other three typical lines, representing the results obtained in the revised quasiparticle model, almost coincide at large energy densities.

Figure 5 shows the effective bag constant B^* as a function of the baryon density for several parameter sets.

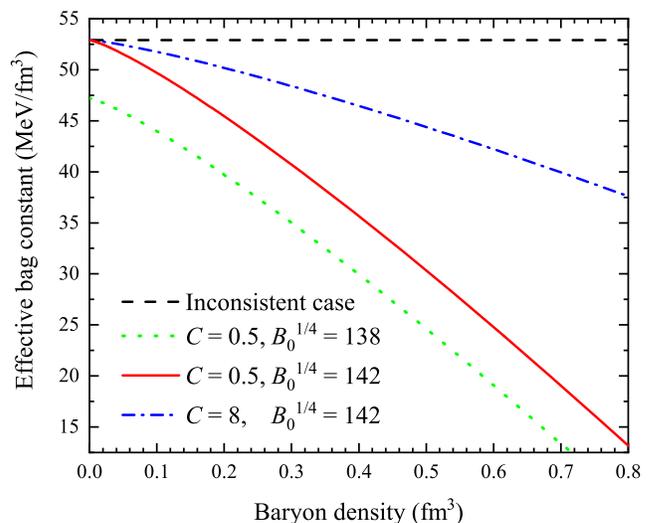


FIG. 5. Density behavior of the effective bag constant with different values of C and B_0 . Conventions for line styles and colors are the same as in Fig. 3.

For comparison, we also show results, indicated by the dashed line, for the inconsistent case without any correction to the energy density and pressure. For the revised quasiparticle model, as shown by the solid, dotted, and dash-dotted lines, the values decrease with increasing baryon density in all cases. This is consistent with the fact that color confinement becomes less important at sufficiently high densities.

B. Mass-radius relations and tidal deformability of quark stars

Quark stars are a class of hypothetical, theoretical compact stars composed of quark matter. Using the quark matter equation of state obtained from the revised two-flavor quasiparticle model shown in Eq. (18), one can solve the Tolman-Oppenheimer-Volkoff (TOV) equations to obtain the mass-radius relation for dense quark stars. The equilibrium structure of a static spherically symmetric quark star is determined by the TOV equation

$$\frac{dP(r)}{dr} = -\frac{GmE(1+P/E)(1+4\pi r^3 P/m)}{r^2(1-2Gm/r)}, \quad (19)$$

and the subsidiary condition

$$\frac{dm(r)}{dr} = 4\pi r^2 E, \quad (20)$$

where $G = 6.7 \times 10^{-45} \text{ MeV}^{-2}$ is the gravitational constant, r is the radial coordinate of the quark star, and m is the gravitational mass contained within the radius r .

Taking the equation of state shown in Fig. 3 as an input, the coupled TOV equations are solved numerically, and the obtained mass-radius relations of quark stars are shown in Fig. 6. As can be seen from Fig. 6 the inconsistent case supports a maximum mass of about $1.56M_\odot$, and the other curves show the maximum mass about 2 times the solar mass for the selected parameter sets in our model. Under the assumption that the ud quark matter is more stable than the uds quark matter, the observed two solar masses of compact stars [65,66] can be described as a $2M_\odot$ ud quark star.

Very recently, the determination of the gravitational mass of the compact star PSR J0740 + 6620 has been updated to $2.08^{+0.07}_{-0.07}M_\odot$ [64]. Moreover, two more massive compact stars, MSR J0740 + 6620 [67] and PSR J2215 + 5135 [68], have been measured to be $2.14 \pm_{0.09}^{0.10}M_\odot$ and $2.27^{+0.17}_{-0.15}M_\odot$ respectively. Figure 7 shows the maximum mass and radius of ud quark stars as functions of the bag constant $B_0^{1/4}$ at fixed $C = 0.1$. One can see that, both the maximum mass and the corresponding radius decrease with the increase in C . In particular, for the given equation of state with a small value of C , e.g., $C = 0.1$, the maximum mass of a quark star composed of ud quark matter has been

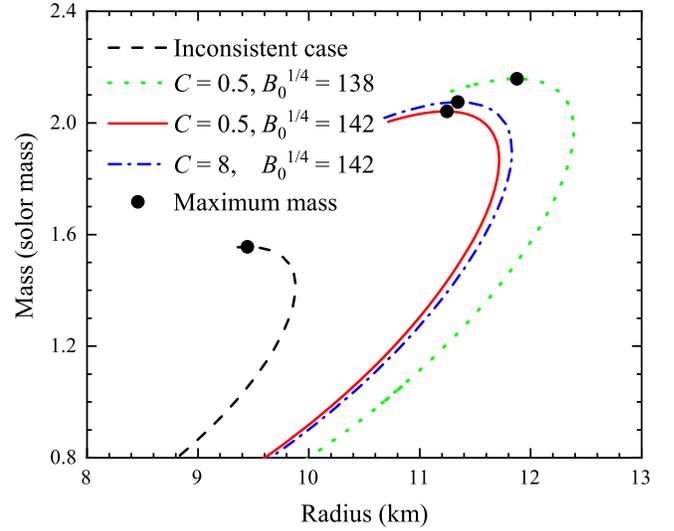


FIG. 6. Mass-radius relations for ud quark stars. The maximum masses are denoted by the solid circles. Conventions for line styles and colors are the same as in Fig. 3.

calculated to be up to $2.31M_\odot$. This result demonstrates the potential for ud quark stars to be more massive than neutron stars, which are limited to a maximum mass of about or greater than $2M_\odot$, suggesting that ud quark stars may be an important component of the astrophysical landscape [69].

We mention that, besides the mass-radius relation, owing to the recent detection of gravitational waves for event GW170817 [70] the measured tidal deformability also provides a stringent experimental constraint on the equation of state for compact stars [71–73]. Over recent years, many efforts have been made to constrain the properties of quark star matter [74–83], based on the improved estimate of tidal deformability $\tilde{\Lambda}_{1.4} = 190^{+390}_{-120}$ from the GW170817 event,

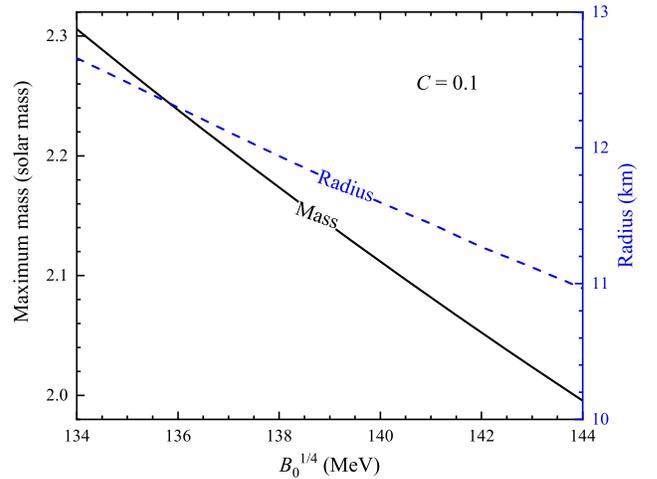


FIG. 7. Maximum mass and the corresponding radius of ud quark stars as a function of the bag constant B_0 for $C = 0.1$.

where $\tilde{\Lambda}_{1.4}$ is the dimensionless tidal deformability of a compact star with a mass of $1.4M_{\odot}$. The response of the quark star to the gravitational field is described by the tidal Love number k_2 , which depends on the quark star structure and consequently on the mass and the equation of state of quark matter. In general relativity, the dimensionless tidal deformability $\tilde{\Lambda}$ is related to the $l = 2$ tidal Love number k_2 as [70,84]

$$\tilde{\Lambda} = \frac{2}{3\beta^5} k_2, \quad (21)$$

where $\beta = GM/R$ and R are the compactness parameter and radius of the star, respectively, while the Love number k_2 can be expressed as [85]

$$\begin{aligned} k_2 = & \frac{8\beta^5}{5} (1 - 2\beta)^2 [2 + 2\beta(y_R - 1) - y_R] \\ & \times \{4\beta^3 [13 - 11y_R + \beta(3y_R - 2) + 2\beta^2(1 + y_R)] \\ & + 3(1 - 2\beta)^2 [2 - y_R + 2\beta(y_R - 1)] \ln(1 - 2\beta) \\ & + 2\beta[6 - 3y_R + 3\beta(5y_R - 8)]\}, \end{aligned} \quad (22)$$

where $y_R \equiv y(R)$, and $y(r)$ is determined by solving the following differential equation:

$$r \frac{dy(r)}{dr} + y^2(r) + y(r)F(r) + r^2 Q(r) = 0. \quad (23)$$

Here, $F(r)$ and $Q(r)$ are, respectively, defined as

$$F(r) \equiv \frac{1 - 4\pi r^2 [E(r) - P(r)] G}{f(r)} \quad (24)$$

and

$$\begin{aligned} Q(r) \equiv & \frac{4\pi}{f(r)} \left[5E(r)G + 9P(r)G + \frac{E(r) + P(r)}{V_s^2(r)} G - \frac{6}{4\pi r^2} \right] \\ & - 4 \left[\frac{m(r) + 4\pi r^3 P(r)}{r^2 f(r)} G \right]^2, \end{aligned} \quad (25)$$

where V_s^2 represents the squared sound velocity of quark matter, and $f(r)$ takes the form $f(r) = 1 - 2Gm(r)/r$.

Figure 8 shows the tidal deformability of ud quark stars as a function of the star mass, for the revised quasiparticle model with different parameter sets in Fig. 2. The vertical black solid line corresponds to the quark stars with a mass of $1.4M_{\odot}$, whereas the black shaded area between the two horizontal orange dotted lines indicates the range of tidal deformability $\tilde{\Lambda}_{1.4} = 190_{-120}^{+390}$, measured by the LIGO and Virgo Collaboration in the event GW170817 [70]. Obviously, the tidal deformability decreases rapidly with increasing quark star mass, indicating that lighter quark stars can be deformed easier than heavier ones, as expected. Furthermore, as can be noticed from the plots, with the

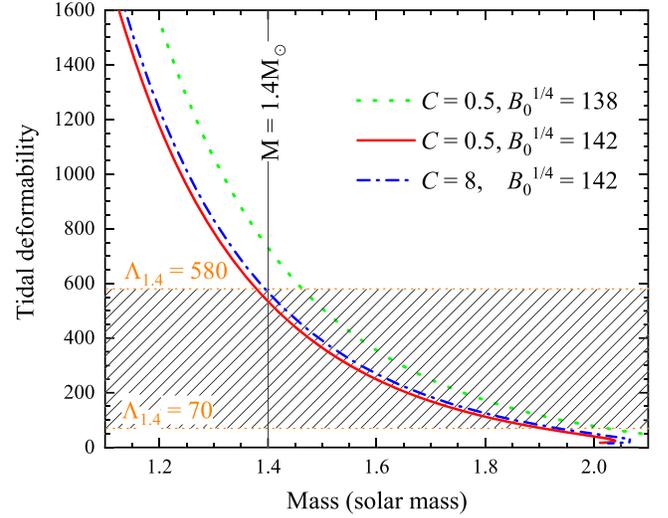


FIG. 8. Tidal deformability of ud quark stars as a function of the star mass for different selected parameter sets. The black shaded area represents the improved estimate of the tidal deformability $\tilde{\Lambda}_{1.4} = 190_{-120}^{+390}$ for GW170817.

increase in B_0 the intersection of the green dashed line and the black solid line moves toward lower values of tidal deformability. More specifically, except for the green dotted line, the red solid and blue dash-dotted lines intersect the vertical black solid line and are located in the black shaded area. This indicates that the equations of state for these two selected parameter sets can simultaneously satisfy the constraint of the tidal deformability $70 \leq \tilde{\Lambda}_{1.4} \leq 580$ measured for GW170817 and the astrophysical observations of the compact star PSR J0740 + 6620 with a mass of $2.08_{-0.07}^{+0.07} M_{\odot}$. In particular, we have verified that for the parameter sets located in the blue shaded area shown in Fig. 2, the corresponding equations of state satisfy the requirement that the massive compact stars compare with a mass about or even greater than 2 times the solar mass. While for the parameter sets taken from the region bounded by the red dashed line and the blue dotted line with star symbols, the equations of state simultaneously satisfy the constraint of the tidal deformability $70 \leq \tilde{\Lambda}_{1.4} \leq 580$ extracted from GW170817 and the astrophysical observations of the compact stars PSR J0740 + 6620 and PSR J1614-2230 with a mass of $2.08_{-0.07}^{+0.07} M_{\odot}$ and $1.97_{-0.04}^{+0.04} M_{\odot}$, respectively.

We close this section by noticing that the recently observed massive compact stars have already ruled out a large number of equations of state since these equations of state are too soft to support a compact star with a mass about 2 times the solar mass. However, there are still various ways to obtain a massive quark star. One way is to assume the presence of the pion superfluid phase in the quark star [86], since the speed of sound in the pion superfluid phase is much larger than that of normal quark matter and ordinary nuclear matter [87]. In fact, the

inclusion of the non-Newtonian gravity effect can also significantly enhance the mass of quark stars [73,88].

V. CONCLUSIONS

The quasiparticle model is an essential phenomenological model for studying the properties of strongly interacting quark matter. However, the model with a medium-dependent quark mass that accounts for the strong interactions can lead to thermodynamic inconsistency problems. In particular, thermodynamic consistency limits the functional form of the renormalization subtraction on the chemical potentials. In this work, we have shown that the renormalization subtraction point should be taken as a function of the summation of the biquadratic chemical potentials if the quark's current masses vanish in order to ensure full thermodynamic consistency. Taking the simplest form, we have studied the thermodynamic properties of ud quark matter and the mass-radius relation of compact stars in a revised thermodynamically consistent two-flavor quasiparticle model, considering the running of the QCD coupling constant. It is found that there is allowed parameter space for ud quark matter to be stable, and the corresponding equation of state can support the massive pulsars with a mass larger than 2 times the solar mass.

We have also computed the dimensionless tidal deformability of ud quark stars in the revised quasiparticle model. We found, however, that in order to further respect the upper limit of tidal deformability $\tilde{\Lambda}_{1,4} = 190_{-120}^{+390}$ measured in the binary star merger GW170817 event, the maximum mass of an ud quark star should not exceed $2.08M_{\odot}$, i.e., $M_{\max} \lesssim 2.08M_{\odot}$. Although the upper limit of the measured tidal deformability $\tilde{\Lambda}_{1,4} \leq 580$ extracted from the event GW170817 poses an upper limit on the allowed maximum mass of compact stars, our results show that there is still a range of parameter space for the revised two-flavor quasiparticle model to simultaneously satisfy the stable condition, the astrophysical observations of the massive compact star with a mass about or slightly larger than 2 times the solar mass, and the tidal deformability of quark stars in the range of $\tilde{\Lambda}_{1,4} = 190_{-120}^{+390}$. In addition, it would

be worth mentioning that if we relax the upper limit of the tidal deformability to $\tilde{\Lambda}_{1,4} \leq 800$ [89], the blue shaded area in Fig. 2 where the equations of state of ud quark matter enable the support of the massive compact stars with maximum masses in the range $2.01 \leq M_{\max}/M_{\odot} \leq 2.08$ and the tidal deformability $\tilde{\Lambda}_{1,4} \leq 800$ reported in Ref. [89], would also be allowed.

Consistent with the previous studies using different methods [30,31], our results on the maximum masses of compact stars in this work have revealed that there is a range of parameters that can support the existence of ud quark matter and are compatible with the observed massive pulsars with a mass about or greater than two solar masses. This has opened up new possibilities for understanding the physical properties of these incredibly dense stellar objects. It also provides insight into the potential implications of quark matter existing in the core of such massive pulsars. In this paper, only the bulk properties of the ud quark matter are considered. In fact, the finite size effect [90,91] should be taken into account when the system is finite, such as quark matter nuggets [92,93]. These effects can significantly alter the physical properties of a system and can, therefore, have an important impact on the results of any simulations or experiments performed. For example, a finite system may exhibit unique behavior due to the presence of surface effects or the effects of a limited number of particles. Therefore, the extension of the model to include the surface and curvature terms would be highly relevant in the near future.

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