Unified triquark equations

A. N. Kvinikhidze[1,2,*] and B. Blankleider[2,†]

[1]*Andrea Razmadze Mathematical Institute of Tbilisi State University, 6, Tamarashvili Str, 0186 Tbilisi, Georgia*

[2]*College of Science and Engineering, Flinders University, Bedford Park, South Australia 5042, Australia*

(Received 30 May 2023; accepted 24 August 2023; published 13 September 2023)

We derive covariant equations describing the three-quark bound state in terms of quark and diquark degrees of freedom. The equations are exact in the approximation where three-body forces are neglected. A feature of these equations is that they unify two often-used but seemingly unrelated approaches that model baryons as quark-diquark systems; namely, (i) the approach using Poincaré covariant quark + diquark Faddeev equations driven by a one-quark-exchange kernel [pioneered by Cahill *et al.*, Aust. J. Phys. **42**, 129 (1989) and Reinhardt, Phys. Lett. B **244**, 316 (1990)], and (ii) the approach using the quasipotential quark-diquark bound-state equation where the kernel consists of the lowest-order contribution from an underlying quark-quark potential [pioneered by Ebert *et al.*, Z. Phys. C **76**, 111 (1997)]. In particular, we show that each of these approaches corresponds to the unified equations with its kernel taken in different, nonoverlapping, approximations.

DOI: 10.1103/PhysRevD.108.054016

I. INTRODUCTION

The use of diquarks as effective degrees of freedom in describing hadrons has a long history, as evidenced by a number of reviews over the past thirty years [1–4]. Documented are different quark-diquark approaches for baryons, but to the best of our knowledge, no attempt has been made for their comparison on the basis of quantum field theory. In the present work, we would like to make such a comparison, demonstrating that two of the most often-used quark-diquark models of baryons, which have usually been considered as separate, unrelated models of baryons, are in fact two nonoverlapping parts of the same quark-diquark model.

The first of these models, proposed more than 30 years ago [5,6], is based on a description of three quarks using covariant Faddeev equations where the quark-quark t matrix is approximated by one or more diquark-pole terms (i.e., terms with a pole at the diquark mass, and with a residue that is expressed as an outer product of form factors Γ and $\bar{\Gamma}$ for the transition between the diquark and two free quarks). The resulting coupled set of bound-state equations is illustrated in Fig. 1. Sometimes referred to as Poincaré covariant quark + diquark Faddeev equations [4,7], and sometimes as quark-diquark Bethe-Salpeter equations [8,9], they have been used extensively over the years, see [7–18] for a representative selection of works.

The second model, proposed more that 25 years ago [19], is a relativistic description of the quark-diquark system using quasipotential equations (we will refer to it as the "quasipotential quark-diquark model"), which has likewise been often used over the years [19–29]. In this model one first constructs a quark-quark potential of the form

$$V_{qq} = V_{\text{gluon}} + V_{\text{conf}}, \tag{1}$$

where V_{gluon} is the quark-quark (qq) one-gluon-exchange potential and V_{conf} is a local confining potential, and then uses this to construct the quark-diquark potential which then forms the kernel of a relativistic quark-diquark quasi-potential equation for the baryon. Illustrated in Fig. 2, this bound-state equation again has the form of a Faddeev equation, but with a kernel corresponding to a single rescattering of two quarks via potential V_{qq} [specified in the diagram as quarks a and c scattering via a potential K_b ($= V_{qq}$) with quark b being a spectator].

In the following, we derive covariant triquark bound-state equations that are exact for the case where three-body forces are neglected. These equations are illustrated in Fig. 3, and have the form of Faddeev equations where the kernel consists of an infinite series involving successive numbers of quark exchanges between quark-diquark states. It is evident that the Poincaré covariant quark + diquark Faddeev equations of Refs. [5,6] correspond to keeping just

*sasha_kvinikhidze@hotmail.com
†boris.blankleider@flinders.edu.au

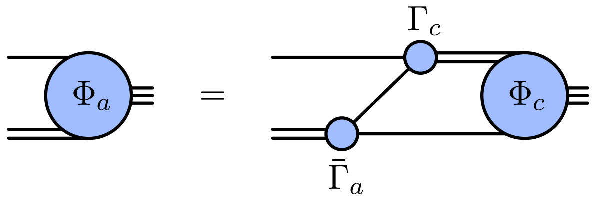

FIG. 1. Poincaré covariant quark + diquark Faddeev equations of Refs. [5,6]. The amplitudes Φ_a and Φ_c are Faddeev components coupling the baryon to quark (single line) and diquark (double-line) states. The equation kernel corresponds to one-quark exchange, with Γ_c and $\bar{\Gamma}_a$ being vertex functions describing the disintegration and formation of the diquark.

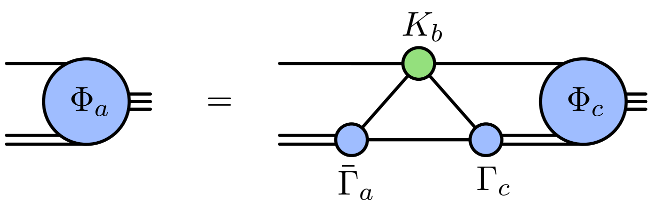

FIG. 2. Equations corresponding to the quasipotential quark-diquark model of Ref. [19]. Similar to Fig. 1, amplitudes Φ_a and Φ_c are Faddeev components coupling the baryon to quark-diquark states, with Γ_c and $\bar{\Gamma}_a$ being diquark vertex functions. However, the kernel of this equation involves a single scattering of two quarks (quarks a and c in this case) via a potential K_b.

the first term in the infinite series, and the quasipotential equations of Ref. [19] correspond to keeping just the second term in this series. As such, our triquark equations unify these two popular approaches for modeling baryons in terms of quark and diquark degrees of freedom. Moreover, it is evident that these two approaches should not be viewed as unrelated competing models of baryons, but rather, as different approximations of the same model. Indeed, any competition between these models at describing data needs to be assessed by comparing their kernels, as these are nonoverlapping terms appearing in the unified equations.

Ideally, the two approaches should be combined, with a kernel that is the sum of the first two terms of the infinite series illustrated in Fig. 3. Additionally, in light of the unification embodied in Fig. 3, all sorts of form factors (electromagnetic, axial-vector, pseudoscalar, etc.) should also be unified correspondingly. This can be done by gauging the equation of Fig. 3 [30], thereby obtaining contributions to the baryon form factors coming from both of the first two kernels in this figure. By contrast, the current situation is that the baryon form factors are being pursued intensively in each of the two approaches separately (just in the past few years, the Poincaré covariant quark + diquark Faddeev equations have been used to calculate such form factors in Refs. [31–38] and the quasipotential quark-diquark approach has been used to calculate them in Refs. [39–46]).

It is worth noting that analogous unified equations were derived for the tetraquark [47].

II. DERIVATION

A. Triquark equations for distinguishable quarks

For clarity of presentation, we first consider the case of three distinguishable quarks. To describe such a system where only pairwise interactions are taken into account, we follow the formulation of Faddeev [48]. Thus, assigning labels 1, 2 and 3 to the quarks, and using a notation where (abc) is a cyclic permutation of (123), the three-body ($3q$) kernel, K, is written as

$$K = \sum_a K_a, \tag{2}$$

where K_a is the kernel where quarks b and c are interacting while quark a is a spectator, as illustrated in Fig. 4.

The $3q$ bound-state wave function for distinguishable quarks is then

$$\Psi = G_0 K \Psi, \tag{3}$$

where G_0 is the fully disconnected part of the full $3q$ Green function G. The three-body kernels K_a can be used to define the Faddeev components Ψ_a as

$$\Psi_a = G_0 K_a \Psi, \tag{4}$$

so that

$$\Psi = \sum_a \Psi_a. \tag{5}$$

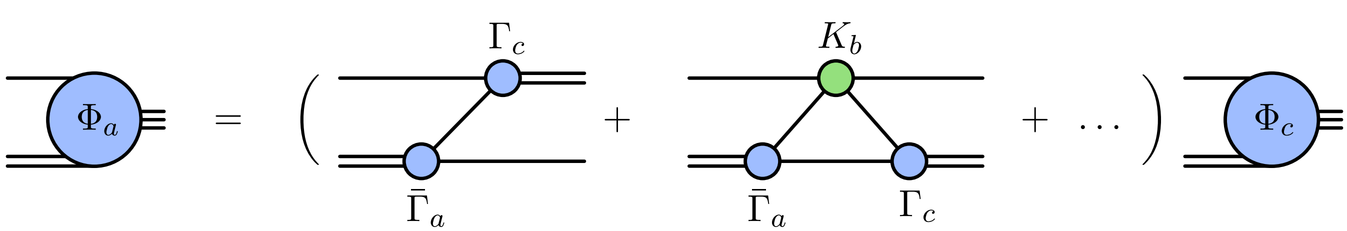

FIG. 3. Unified quark + diquark equations derived in this paper. The kernel of this equation is an infinite series whose first two terms, separately, correspond to the model of Refs. [5,6] as illustrated in Fig. 1, and the model of Ref. [19] as illustrated in Fig. 2, respectively.

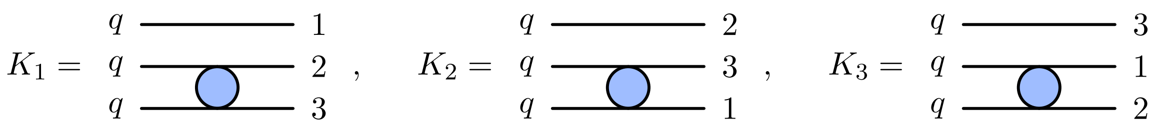

FIG. 4. Structure of the terms K_a ($a = 1, 2, 3$) making up the three-body kernel K where only two-body forces are included. The colored circles represent two-body kernels K_{bc} for the scattering of quarks b and c, as indicated.

From Eq. (3) follow Faddeev's equations for the components,

$$\Psi_a = \sum_b G_0 T_a \bar{\delta}_{ab} \Psi_b, \tag{6}$$

where $\bar{\delta}_{ab} = 1 - \delta_{ab}$ and T_a is the t matrix corresponding to kernel K_a, so that

$$T_a = K_a + K_a G_0 T_a. \tag{7}$$

Assuming that the qq interaction admits the creation of a diquark, the Green function G_a describing the scattering of quarks b and c will contain a corresponding pole at the diquark mass, so that one can write

$$G_a = G_a^P + G_a^R, \tag{8}$$

where G_a^P is the Green function's pole term while G_a^R is its regular part. Then, because

$$T_a = K_a + K_a G_a K_a, \tag{9}$$

the t matrix T_a can be written as

$$T_a = K_a + T_a^P + T_a^C, \tag{10}$$

where T_a^P is T_a's pole term, while the sum $K_a + T_a^C$ constitutes its regular part. It is important to note that there is no overcounting in this decomposition; that is, the terms K_a, T_a^P and T_a^C do not overlap. Note that in the case of unconfined quarks, the analytic structure of T_a would be represented by its pole part, T_a^P, its part with the $2q$ branch point, T_a^C, and the part K_a again with a branch point, but above the $2q$ mass.

We write Eq. (6) in matrix form as

$$\Psi = \mathcal{T}\Psi, \tag{11}$$

where Ψ is a column matrix of elements Ψ_a, and $\mathcal{T}$ is a square matrix whose (a, b)th element is $\mathcal{T}_{ab} = G_0 T_a \bar{\delta}_{ab}$. Similarly we write Eq. (10) in matrix form as

$$\mathcal{T} = \mathcal{K} + \mathcal{T}^P + \mathcal{T}^C, \tag{12}$$

where $\mathcal{K}_{ab} = G_0 K_a \bar{\delta}_{ab}$, $\mathcal{T}_{ab}^P = G_0 T_a^P \bar{\delta}_{ab}$, and $\mathcal{T}_{ab}^C = G_0 {T_a}^C \bar{\delta}_{ab}$. Equation (11) can then be recast as

$$\Psi = (1 - \mathcal{K} - \mathcal{T}^C)^{-1} \mathcal{T}^P \Psi. \tag{13}$$

Using the separable form of the pole term,

$$T_a^P = \Gamma_a D_a \bar{\Gamma}_a, \tag{14}$$

where Γ_a (similarly $\bar{\Gamma}_a$) and D_a are the diquark form factor and propagator, respectively, Eq. (13) implies that

$$\Phi_a = \sum_{bc} \bar{\Gamma}_a \bar{\delta}_{ab} [(1 - \mathcal{K} - \mathcal{T}^C)^{-1}]_{bc} G_0 \Gamma_c D_c \Phi_c, \tag{15}$$

where

$$\Phi_a = \sum_b \bar{\Gamma}_a \bar{\delta}_{ab} \Psi_b. \tag{16}$$

Expanding the inverse term in Eq. (15) as

$$(1 - \mathcal{K} - \mathcal{T}^C)^{-1} = 1 + \mathcal{K} + \mathcal{T}^C + \cdots, \tag{17}$$

we obtain

$$\Phi_a = \sum_{bc} \bar{\Gamma}_a \bar{\delta}_{ab} (\delta_{bc} + G_0 K_b \bar{\delta}_{bc} + \cdots) G_0 \Gamma_c D_c \Phi_c, \tag{18}$$

which is illustrated in Fig. 3.

It is apparent that the first two terms of this series correspond to the models of Refs. [5,6] and Ref. [19], respectively. Indeed, keeping just the first term in the series results in the bound-state equation

$$\Phi_a = \sum_b \bar{\Gamma}_a \bar{\delta}_{ab} G_0 \Gamma_b D_b \Phi_b \tag{19}$$

which is illustrated in Fig. 1 and coincides with the Poincaré covariant quark + diquark Faddeev equations of Refs. [5,6], and keeping just the second term in the series results in the bound-state equation

$$\Phi_a = \sum_{bc} \bar{\Gamma}_a \bar{\delta}_{ab} G_0 K_b \bar{\delta}_{bc} G_0 \Gamma_c D_c \Phi_c, \tag{20}$$

which is illustrated in Fig. 2 and coincides with the quasipotential quark-diquark equations of Ref. [19].

Although each of the approaches of Refs. [5,6] and Ref. [19], can be viewed as different approximations of the same unified equations, Eq. (18), the reality is that the quark-diquark picture of a baryon is described by a kernel that consists of at least the sum of the first two terms of the series in Eq. (18). This observation should clarify the true picture of quark-diquark dynamics in baryons.

B. Triquark equations for indistinguishable quarks

To take into account the antisymmetry of identical quarks, we first note that the Faddeev equations for distinguishable particles, Eq. (6), possess fully antisymmetric solutions (as well as symmetric ones) where the component wave functions have the symmetry properties

$$\begin{aligned} &P_{23}\Psi_1 = -\Psi_1, \qquad P_{12}\Psi_1 = -\Psi_2, \qquad P_{31}\Psi_1 = -\Psi_3,\\ &P_{31}\Psi_2 = -\Psi_2, \qquad P_{23}\Psi_2 = -\Psi_3, \qquad P_{12}\Psi_2 = -\Psi_1,\\ &P_{12}\Psi_3 = -\Psi_3, \qquad P_{31}\Psi_3 = -\Psi_1, \qquad P_{23}\Psi_3 = -\Psi_2, \end{aligned} \tag{21}$$

where P_{ab} is the operator that exchanges the quantum numbers of particles a and b. Choosing a solution with these symmetry properties, Eq. (6) for Ψ_1 reduces to

$$\Psi_1 = -G_0 T_1 P_{12}\Psi_1, \tag{22}$$

where T_1 results from antisymmetrizing the t matrix for distinguishable particles, T_1^d, using

$$T_1 = (1 - P_{23})T_1^d. \tag{23}$$

Equation (22) can be seen most easily by using Eq. (21):

$$\begin{aligned} \Psi_1 &= G_0 T_1^d(\Psi_2 + \Psi_3) = G_0 T_1^d(1 - P_{23})\Psi_2\\ &= G_0(1 - P_{23})T_1^d\Psi_2\\ &= -G_0(1 - P_{23})T_1^d P_{12}\Psi_1. \end{aligned} \tag{24}$$

We can then again express T_1 as

$$T_1 = K_1 + T_1^P + T_1^C, \tag{25}$$

where T_1^P and $K_1 + T_1^C$ are the pole and regular parts of T_1, but this time with all quantities antisymmetric under the interchange of quark 2 and 3's quantum numbers. Equation (22) can then be recast as

$$\Psi_1 = -[1 + (\mathcal{K}_1 + \mathcal{T}_1^C)P_{12}]^{-1}\mathcal{T}_1^P P_{12}\Psi_1, \tag{26}$$

where $\mathcal{K}_1 = G_0K_1$, $\mathcal{T}_1^P = G_0T_1^P$, and $\mathcal{T}_1^C = G_0T_1^C$. Using the separable form of the pole term,

$$T_1^P = \Gamma_1 D_1\bar{\Gamma}_1, \tag{27}$$

where the diquark form factors are now antisymmetric, $P_{23}\Gamma_1 = -\Gamma_1$ and $\bar{\Gamma}_1 P_{23} = -\bar{\Gamma}_1$, we obtain the equation for the Faddeev component

$$\Phi_1 = -\bar{\Gamma}_1 P_{12}[1 + G_0(K_1 + T_1^C)P_{12}]^{-1}G_0\Gamma_1 D_1\Phi_1, \tag{28}$$

where

$$\Phi_1 = \bar{\Gamma}_1 P_{12}\Psi_1. \tag{29}$$

Expanding the inverse term in Eq. (28) as

$$[1 + G_0(K_1 + T_1^C)P_{12}]^{-1} = 1 - G_0(K_1 + T_1^C)P_{12} + \cdots \tag{30}$$

leads to the final form of our unified equations for three identical quarks,

$$\Phi_1 = -\bar{\Gamma}_1 P_{12}[1 - G_0(K_1 + T_1^C)P_{12} + \cdots]G_0\Gamma_1 D_1\Phi_1. \tag{31}$$

Keeping only the first two terms of the series for the kernel, and making the further approximation, $T_1^C = 0$, leads to the equation

$$\Phi_1 = \bar{\Gamma}_1 P_{12}[1 + K_1 P_{12}]\Gamma_1 d_1\Phi_1 \tag{32}$$

which covers both approaches of Refs. [5,6] and Ref. [19].

III. DISCUSSION

We have derived covariant equations that describe the bound state of the triquark in terms of quark and diquark degrees of freedom. These equations are illustrated in Fig. 3, with exact expressions given for distinguishable quarks in Eq. (18), and for indistinguishable quarks in Eq. (31). An essential aspect of these equations is that they are exact in the approximation where only two-body forces are retained.

It is worth noting that our procedure leading to Eq. (15), and hence to Eqs. (18) and (31), is similar to the one used by Alt, Grassbeger, and Sandhas (AGS) to reduce three-particle Faddeev equations to that of coupled two-particle equations [49]; however, it differs from AGS in its details, and also in one essential way, namely, we have shown that the two-body matrix (in three-body space) T_a, can be decomposed into three mutually exclusive parts, as in Eq. (10), where the two-body kernel K_a appears explicitly (AGS and related prior works, decomposed two-body t matrices into two parts, a separable one, and the rest). It is just this decomposition of T_a into three parts involving K_a, which has led to the unification of previous works, as outlined above.

This unification is demonstrated explicitly for two of the most prominent and longest-used approaches in the literature, namely the one using the covariant quark + diquark Faddeev equations of Refs. [5,6], and the one using quasi-potential quark-diquark equations of Ref. [19]. In particular, the covariant quark + diquark Faddeev equations correspond to keeping just the first term of the kernel in our equations (the one-quark-exchange diagram in Fig. 3), and the quasipotential quark-diquark equations correspond to keeping just the second term of the kernel in our equations (the qq rescattering diagram in Fig. 3). It is noteworthy that our equations reveal that these two approaches, which have been pursued separately for more than 25 years in order to model not only bound states of baryons, but also various types of baryon form factors (electromagnetic, axial-vector, scalar, etc.), use equations with two, different, nonoverlapping, kernels. Our equations indicate that it is the sum of the first two terms in the kernel (at least) that should have been used instead. Although this is not an issue for cases where only one pair of quarks (out of three possible pairs) can form a diquark, in which case only the kernel of the quasipotential quark-diquark equations contributes [19,22], it may be a serious problem for other cases, like that of three identical quarks where both kernels contribute and therefore should be summed [29].

ACKNOWLEDGMENTS

A. N. K. was supported by the Shota Rustaveli National Science Foundation (Grant No. FR17-354).

[1] M. Anselmino, E. Predazzi, S. Ekelin, S. Fredriksson, and D. B. Lichtenberg, Diquarks, Rev. Mod. Phys. **65**, 1199 (1993).

[2] R. Alkofer and L. von Smekal, The infrared behavior of QCD Green's functions: Confinement, dynamical symmetry breaking, and hadrons as relativistic bound states, Phys. Rep. **353**, 281 (2001).

[3] G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, and C. S. Fischer, Baryons as relativistic three-quark bound states, Prog. Part. Nucl. Phys. **91**, 1 (2016).

[4] M. Y. Barabanov *et al.*, Diquark correlations in hadron physics: Origin, impact and evidence, Prog. Part. Nucl. Phys. **116**, 103835 (2021).

[5] R. T. Cahill, C. D. Roberts, and J. Praschifka, Baryon structure and QCD, Aust. J. Phys. **42**, 129 (1989).

[6] H. Reinhardt, Hadronization of quark flavor dynamics, Phys. Lett. B **244**, 316 (1990).

[7] C. Chen, L. Chang, C. D. Roberts, S. Wan, and D. J. Wilson, Spectrum of hadrons with strangeness, Few-Body Syst. **53**, 293 (2012).

[8] M. Oettel, L. Von Smekal, and R. Alkofer, Relativistic three quark bound states in separable two quark approximation, Comput. Phys. Commun. **144**, 63 (2002).

[9] G. Eichmann, C. S. Fischer, and H. Sanchis-Alepuz, Light baryons and their excitations, Phys. Rev. D **94**, 094033 (2016).

[10] M. Oettel, G. Hellstern, R. Alkofer, and H. Reinhardt, Octet and decuplet baryons in a covariant and confining diquark-quark model, Phys. Rev. C **58**, 2459 (1998).

[11] S. Ahlig, R. Alkofer, C. S. Fischer, M. Oettel, H. Reinhardt, and H. Weigel, Production processes as a tool to study parametrizations of quark confinement, Phys. Rev. D **64**, 014004 (2001).

[12] G. Eichmann, A. Krassnigg, M. Schwinzerl, and R. Alkofer, A covariant view on the nucleons' quark core, Ann. Phys. (Amsterdam) **323**, 2505 (2008).

[13] D. Nicmorus, G. Eichmann, A. Krassnigg, and R. Alkofer, Delta-baryon mass in a covariant Faddeev approach, Phys. Rev. D **80**, 054028 (2009).

[14] D. J. Wilson, I. C. Cloet, L. Chang, and C. D. Roberts, Nucleon and Roper electromagnetic elastic and transition form factors, Phys. Rev. C **85**, 025205 (2012).

[15] K.-l. Wang, Y.-x. Liu, L. Chang, C. D. Roberts, and S. M. Schmidt, Baryon and meson screening masses, Phys. Rev. D **87**, 074038 (2013).

[16] J. Segovia, C. D. Roberts, and S. M. Schmidt, Understanding the nucleon as a Borromean bound-state, Phys. Lett. B **750**, 100 (2015).

[17] C. Chen, G. I. Krein, C. D. Roberts, S. M. Schmidt, and J. Segovia, Spectrum and structure of octet and decuplet baryons and their positive-parity excitations, Phys. Rev. D **100**, 054009 (2019).

[18] L. Liu, C. Chen, and C. D. Roberts, Wave functions of $(I, J^\pi) = (\frac{1}{2}, \frac{3}{2}^\mp)$ baryons, Phys. Rev. D **107**, 014002 (2023).

[19] D. Ebert, R. N. Faustov, V. O. Galkin, A. P. Martynenko, and V. A. Saleev, Heavy baryons in the relativistic quark model, Z. Phys. C **76**, 111 (1997).

[20] D. Ebert, R. N. Faustov, V. O. Galkin, and A. P. Martynenko, Semileptonic decays of doubly heavy baryons in the relativistic quark model, Phys. Rev. D **70**, 014018 (2004); **77**, 079903(E) (2008).

[21] D. Ebert, R. N. Faustov, V. O. Galkin, and A. P. Martynenko, Properties of doubly heavy baryons in the relativistic quark model, Phys. At. Nucl. **68**, 784 (2005).

[22] D. Ebert, R. N. Faustov, and V. O. Galkin, Masses of heavy baryons in the relativistic quark model, Phys. Rev. D **72**, 034026 (2005).

[23] D. Ebert, R. N. Faustov, and V. O. Galkin, Masses of excited heavy baryons in the relativistic quark model, Phys. Lett. B **659**, 612 (2008).

[24] D. Ebert, R. N. Faustov, and V. O. Galkin, Excited heavy tetraquarks with hidden charm, Eur. Phys. J. C **58**, 399 (2008).

[25] D. Ebert, R. N. Faustov, and V. O. Galkin, Mass spectra of heavy baryons in the relativistic quark model, Phys. At. Nucl. **72**, 178 (2009).

[26] R. N. Faustov and V. O. Galkin, Strange baryon spectroscopy in the relativistic quark model, Phys. Rev. D **92**, 054005 (2015).

[27] R. N. Faustov and V. O. Galkin, Heavy baryon spectroscopy, EPJ Web Conf. **204**, 08001 (2019).

[28] R. N. Faustov and V. O. Galkin, Heavy baryon spectroscopy in the relativistic quark model, Particles **3**, 234 (2020).

[29] R. N. Faustov and V. O. Galkin, Triply heavy baryon spectroscopy in the relativistic quark model, Phys. Rev. D **105**, 014013 (2022).

[30] A. N. Kvinikhidze and B. Blankleider, Gauging of equations method. I. Electromagnetic currents of three distinguishable particles, Phys. Rev. C **60**, 044003 (1999); Gauging of equations method. II. Electromagnetic currents of three identical particles, Phys. Rev. C **60**, 044004 (1999).

[31] C. Chen, Y. Lu, D. Binosi, C. D. Roberts, J. Rodríguez-Quintero, and J. Segovia, Nucleon-to-Roper electromagnetic transition form factors at large Q^2, Phys. Rev. D **99**, 034013 (2019).

[32] Y. Lu, C. Chen, Z.-F. Cui, C. D. Roberts, S. M. Schmidt, J. Segovia, and H. S. Zong, Transition form factors: $\gamma^* + p \to \Delta(1232)$, $\Delta(1600)$, Phys. Rev. D **100**, 034001 (2019).

[33] Z.-F. Cui, C. Chen, D. Binosi, F. De Soto, C. D. Roberts, J. Rodríguez-Quintero, S. M. Schmidt, and J. Segovia, Nucleon elastic form factors at accessible large spacelike momenta, Phys. Rev. D **102**, 014043 (2020).

[34] C. Chen, C. S. Fischer, C. D. Roberts, and J. Segovia, Form factors of the nucleon axial current, Phys. Lett. B **815**, 136150 (2021).

[35] K. Raya, L. X. Gutiérrez-Guerrero, A. Bashir, L. Chang, Z.-F. Cui, Y. Lu, C. D. Roberts, and J. Segovia, Dynamical diquarks in the $\gamma^{(*)} p \to N(1535)\frac{1}{2}^-$ transition, Eur. Phys. J. A **57**, 266 (2021).

[36] C. Chen, C. S. Fischer, C. D. Roberts, and J. Segovia, Nucleon axial-vector and pseudoscalar form factors and PCAC relations, Phys. Rev. D **105**, 094022 (2022).

[37] C. Chen and C. D. Roberts, Nucleon axial form factor at large momentum transfers, Eur. Phys. J. A **58**, 206 (2022).

[38] P.-L. Yin, C. Chen, C. S. Fischer, and C. D. Roberts, Δ-baryon axialvector and pseudoscalar form factors, and associated PCAC relations, Eur. Phys. J. A **59**, 163 (2023).

[39] R. N. Faustov and V. O. Galkin, Semileptonic decays of Λ_c baryons in the relativistic quark model, Eur. Phys. J. C **76**, 628 (2016).

[40] R. N. Faustov and V. O. Galkin, Rare $\Lambda_b \to n l^+ l^-$ decays in the relativistic quark-diquark picture, Mod. Phys. Lett. A **32**, 1750125 (2017).

[41] R. N. Faustov and V. O. Galkin, Rare $\Lambda_b \to \Lambda l^+ l^-$ and $\Lambda_b \to \Lambda\gamma$ decays in the relativistic quark model, Phys. Rev. D **96**, 053006 (2017).

[42] R. N. Faustov and V. O. Galkin, Relativistic description of the Ξ_b baryon semileptonic decays, Phys. Rev. D **98**, 093006 (2018).

[43] R. N. Faustov and V. O. Galkin, Semileptonic Ξ_c baryon decays in the relativistic quark model, Eur. Phys. J. C **79**, 695 (2019).

[44] R. N. Faustov and V. O. Galkin, Semileptonic decays of heavy baryons in the relativistic quark model, Particles **3**, 208 (2020).

[45] V. O. Galkin and R. N. Faustov, Semileptonic decays of heavy baryons, Phys. Part. Nucl. **51**, 625 (2020).

[46] A. O. Davydov, R. N. Faustov, and V. O. Galkin, Rare radiative Ξb$- \to \Xi - \gamma$ decay in the relativistic quark model, Mod. Phys. Lett. A **37**, 2250158 (2022).

[47] A. N. Kvinikhidze and B. Blankleider, Unified tetraquark equations, Phys. Rev. D **107**, 094014 (2023).

[48] L. D. Faddeev, Scattering theory for a three particle system, Sov. Phys. JETP **12**, 1014 (1961), http://faddeev.com/wp-content/uploads/2017/06/Scattering-Theory-for-a-Three-Particle-System-.pdf.

[49] E. O. Alt, P. Grassberger, and W. Sandhas, Reduction of the three-particle collision problem to multichannel two-particle Lippmann-Schwinger equations, Nucl. Phys. **B2**, 167 (1967).