

B^+ decay to $K^+\eta\eta$ with $(\eta\eta)$ from the $D\bar{D}(3720)$ bound statePedro C. S. Brandão,^{1,2,*} Jing Song,^{3,2,†} Luciano M. Abreu^{1,4,‡} and E. Oset^{1,2,§}¹*Instituto de Física, Universidade Federal da Bahia, Campus Ondina, Salvador, Bahia 40170-115, Brazil*²*Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC
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We search for a B decay mode where one can find a peak for a $D\bar{D}$ bound state predicted in effective theories and in lattice QCD calculations, which has also been claimed from some reactions that show an accumulated strength in $D\bar{D}$ production at threshold. We find a good candidate in the $B^+ \rightarrow K^+\eta\eta$ reaction, by looking at the $\eta\eta$ mass distribution. The reaction proceeds via a first step in which one has the $B^+ \rightarrow D_s^{*+}\bar{D}^0$ reaction followed by D_s^{*+} decay to D^0K^+ and a posterior fusion of $D^0\bar{D}^0$ to $\eta\eta$, implemented through a triangle diagram that allows the $D^0\bar{D}^0$ to be virtual and to produce the bound state. The choice of $\eta\eta$ to see the peak is based on results of calculations that find the $\eta\eta$ among the light pseudoscalar channels with stronger coupling to the $D\bar{D}$ bound state. We find a neat peak around the predicted mass of that state in the $\eta\eta$ mass distribution, with an integrated branching ratio for $B^+ \rightarrow K^+(D\bar{D}, \text{bound}); (D\bar{D}, \text{bound}) \rightarrow \eta\eta$ of the order of 1.5×10^{-4} , a large number for hadronic B decays, which should motivate its experimental search.

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I. INTRODUCTION

The search for hadronic states in the charm sector and the description of their structure is attracting much attention recently as evidenced by the large number of review papers devoted to the subject [1–10]. We mention two examples: the state $X(3872)$ couples strongly to $D^*\bar{D}$, and it is a subject of debate concerning its nature as a $D^*\bar{D}$ molecule or a compact tetraquark state [11]; and the $T_{cc}(3875)$ [12,13], coupling strongly to DD^* , is also thought to be a DD^* molecule state, but other opinions have also been given (see list of references in [14]). Taking advantage of this wave of enthusiasm on this subject we want to come back to a recurrent problem, the possible existence of a $D\bar{D}$ bound state, proposing a method to find it experimentally. The state was predicted studying the meson-meson interaction in the charm sector in [15] and was bound about 20 MeV. The state was confirmed in posterior theoretical

studies [16,17]. More recently it was also found in lattice calculations [18].

Several works have tried to see experimental evidence for its existence. Since a $D\bar{D}$ bound state cannot decay into meson states containing $c\bar{c}$, evidence for its existence has been searched for in the $D\bar{D}$ distribution close to the threshold in several reactions. In [19] support for its existence was found in the $e^+e^- \rightarrow D\bar{D}$ reaction looking at the $D\bar{D}$ spectrum close to the threshold. An updated experimental work for this reaction was done in [20] and, again, support for the $D\bar{D}$ state from this reaction and $\gamma\gamma \rightarrow D\bar{D}$ was claimed in [21,22]. A more refined theoretical work of these two latter reactions was done in [23] claiming evidence for this bound state. In [24] three reactions were proposed to observe this bound state, but they have not yet been implemented. In [25,26] it was suggested to be found in the $\psi(3770)$ radiative decay, $\psi(3770) \rightarrow \gamma D^0\bar{D}^0$, and in [27] in the $B^+ \rightarrow D^0\bar{D}^0K^+$ and $B^0 \rightarrow D^0\bar{D}^0K^0$ decays, looking in both cases for the $D^0\bar{D}^0$ mass distribution close to threshold.

In the present work we propose a different reaction, the $B^+ \rightarrow K^+\eta\eta$, looking for the $\eta\eta$ invariant mass distribution where a peak is being expected. The reason to propose this reaction is twofold. Among the light pairs of light pseudoscalar mesons into which this state could decay, the $\eta\eta$ channel stands as one of the most important. On the other hand, in the PDG [28] one finds that the reaction $B^+ \rightarrow D_s^{*+}\bar{D}^0$ has a very large branching fraction for a B^+ decay,

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of the order of 10^{-2} . It might seem that this decay has nothing to do with the $K^+\eta\eta$ decay, but we will show that a triangle diagram with $B^+ \rightarrow D_s^{*+}\bar{D}^0$ followed by $D_s^{*+} \rightarrow D^0K^+$, and fusion of \bar{D}^0D^0 to produce the $D\bar{D}$ bound final state, with its posterior decay to $\eta\eta$, has a reasonable large branching fraction which would make this decay easily accessible.

The $D\bar{D}$ bound state with isospin $I = 0$ looks now more acceptable after the discovery of the $T_{cc}(3875)$, but from the theoretical point of view, it resembles very much the $f_0(980)$, which couples strongly to $K\bar{K}$, that has been obtained within the chiral unitary approach [29–35]. Since a general rule is that the binding of states becomes larger when going from lighter to heavier quarks with the same configuration [36], the existence of the $D\bar{D}$ bound state seems unavoidable, and with this conviction we propose the new reaction with the $B^+ \rightarrow K^+\eta\eta$ decay which is accessible by the LHCb and Belle Collaborations.

II. FORMALISM

The idea is to find an efficient mechanism to produce $\eta\eta$ at the end. For this purpose it is not necessary to produce $\eta\eta$ in a first step in a B decay. Instead, the idea is to produce $D\bar{D}$ since this is the main component of the $D\bar{D}$ bound state and $\eta\eta$ is only one decay channel. Yet, it is convenient to have three particles in the final state because this allows one to have different values for the invariant mass of any pair; in particular, we can have an $\eta\eta$ invariant mass distribution and observe the peak of the $D\bar{D}$ bound state. The idea is then to produce one particle and $D\bar{D}$. Then the $D\bar{D}$ can interact, producing the $D\bar{D}$ bound state. One way to accomplish it is to produce $D_s^{*+}\bar{D}^0$, let D_s^{*+} decay to K^+D^0 , and then we have the pair $D^0\bar{D}^0$ to interact and proceed via $D\bar{D} \rightarrow \eta\eta$.

The choice of the first step is most welcome since the process proceeds via the most Cabibbo favored mode for a B decay, with external emission, as shown in Fig. 1 for the complex conjugate $B^- \rightarrow D_s^{*-}D^0$ reaction. This favors a

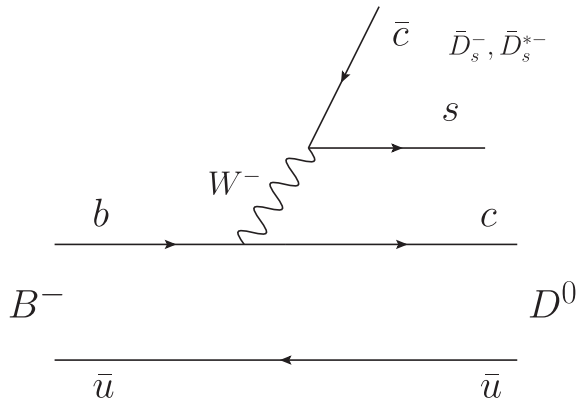


FIG. 1. Cabibbo favored process for a B decay, with external emission at the quark level.

large rate of this decay mode, and one finds the branching fraction [28],

$$\text{Br}[B^+ \rightarrow D_s^{*+}\bar{D}^0] = (7.6 \pm 1.6) \times 10^{-3}. \quad (1)$$

This is a big rate for a B decay, which necessarily involves a suppressed Cabibbo transition $b \rightarrow c$. The next step after the $D_s^{*+}\bar{D}^0$ production is to allow the D_s^{*+} decay to D^0K^+ (virtually) and then proceed with the $D^0\bar{D}^0$ transition to $\eta\eta$, where the peak of the bound state would show up. This process is depicted in Fig. 2, through a triangle diagram, which, however, does not develop a triangle singularity [37], since $D_s^{*+} \rightarrow D^0K^+$ is kinematically forbidden and one cannot place the three intermediate particles on shell [38].

We take the meson masses from the PDG [28],

$$\begin{aligned} m_{B^+} &= 5279.34 \text{ MeV}, & m_\eta &= 547.862 \text{ MeV}, \\ m_{D^0} &= 1864.84 \text{ MeV}, & m_{K^+} &= 493.677 \text{ MeV}, \quad \text{and} \\ M_{D_s^{*+}} &= 2112.2 \text{ MeV}. \end{aligned} \quad (2)$$

A. B^+ decay to $D_s^{*+}\bar{D}^0$

In the diagram of Fig. 2 we have a vertex $D_s^{*+} \rightarrow K^+D^0$ which one can obtain from a standard Lagrangian, the $D^0\bar{D}^0 \rightarrow \eta\eta$ scattering amplitude that one takes from [24] and the $B^+ \rightarrow D_s^{*+}\bar{D}^0$ transition, determined from the experiment as described below.

The $B^+ \rightarrow D_s^{*+}\bar{D}^0$ vertex has the typical structure of a vector coupling to two pseudoscalars as follows:

$$t_1 = C\epsilon^\mu(P+q)_\mu, \quad (3)$$

where ϵ^μ is the polarization vector of the D_s^{*+} and C is the coupling constant.

The $B^+ \rightarrow D_s^{*+}\bar{D}^0$ width is given by

$$\Gamma[B^+ \rightarrow D_s^{*+}\bar{D}^0] = \frac{1}{8\pi} \frac{1}{m_B^2} \sum_{\text{pol}} |t_1|^2 q, \quad (4)$$

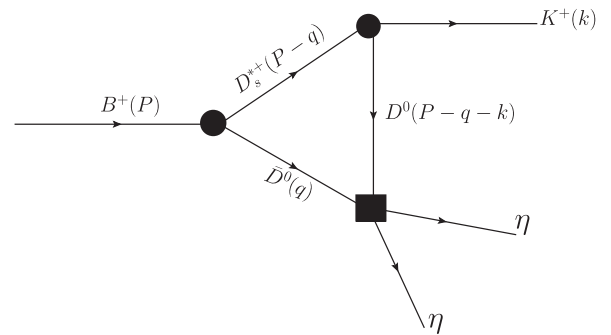


FIG. 2. Mechanism producing $\eta\eta$ through the rescattering of $D^0\bar{D}^0$.

with

$$q = \frac{\lambda^{1/2}(m_B^2, m_{\bar{D}^0}^2, m_{D_s^*}^2)}{2m_B}. \quad (5)$$

After some algebra, we obtain

$$\begin{aligned} \sum_{\text{pol}} |t_1|^2 &= C^2 \sum_{\text{pol}} \epsilon^\mu(P+q)_\mu \epsilon^\nu(P+q)_\nu \\ &= 4C^2 \left(\frac{m_B}{m_{D_s^*}} \right)^2 \vec{q}^2, \end{aligned} \quad (6)$$

then the branching fraction can be written as

$$\text{Br}[B^+ \rightarrow D_s^{*+} \bar{D}^0] = \frac{\Gamma}{\Gamma_B} = \frac{1}{\Gamma_B} \frac{1}{2\pi} C^2 \frac{q^3}{m_{D_s^*}^2}, \quad (7)$$

and using Eq. (1) we find

$$\frac{C^2}{\Gamma_B} = \frac{2\pi m_{D_s^*}^2}{q^3} (7.6 \pm 1.6) \times 10^{-3} = 0.00537 \text{ MeV}^{-1}. \quad (8)$$

B. Loop evaluation

We now evaluate the amplitude for the $K^+\eta\eta$ triangle diagram of Fig. 2. We construct the $D_s^{*+} \rightarrow D^0 K^+$ vertex by using the effective Lagrangian

$$\mathcal{L}_{VPP} = -ig \langle [P, \partial_\mu P] V^\mu \rangle, \quad (9)$$

where $g = \frac{m_v}{2f_\pi}$, $m_v = 800 \text{ MeV}$, $f_\pi = 93 \text{ MeV}$, and P, V are the $q\bar{q}$ matrices with u, d, s quarks, written in terms of pseudoscalar (P) or vector mesons (V) as

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & \frac{-\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & K^0 & D^- \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + \sqrt{\frac{2}{3}}\eta' & D_s^- \\ D^0 & D^+ & D_s^+ & \eta_c \end{pmatrix}, \quad (10)$$

$$V_\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & \frac{-\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & J/\Psi \end{pmatrix}_\mu, \quad (11)$$

where we have taken the ordinary $\eta - \eta'$ mixing of Ref. [39]. The symbol $\langle \dots \rangle$ in Eq. (9) denotes the trace in $SU(4)$. Note, however, that by using Eqs. (9)–(11) one is only making use of the $q\bar{q}$ character of the meson [40].

We find for this vertex

$$-it_2 = -ig\epsilon^\mu[(2k - P + q)_\mu], \quad (12)$$

where the momenta are chosen according to Fig. 2. Then, the loop amplitude is given by

$$\begin{aligned} -it_L &= \int \frac{d^4 q}{(2\pi)^4} (-i) C \epsilon^\mu[(P+q)_\mu] g(-i) \epsilon^\nu[(2k - P + q)_\nu] \\ &\quad \times (-i) t_{D^0 \bar{D}^0, \eta\eta}(M_{\text{inv}}(\eta\eta)) \times \frac{i}{q^2 - m_{D^0}^2 + i\epsilon} \\ &\quad \times \frac{i}{(P-q)^2 - m_{D_s^*}^2 + i\epsilon} \times \frac{i}{(P-q-k)^2 - m_{D^0}^2 + i\epsilon}, \end{aligned} \quad (13)$$

where $M_{\text{inv}}(\eta\eta)$ is the invariant mass of the $\eta\eta$ system. By doing the sum over polarizations for the vector meson D_s^* we get

$$\begin{aligned} &\sum \epsilon^\mu[(P+q)_\mu] \epsilon^\nu[(2k - P + q)_\nu] \\ &= \left[-g^{\mu\nu} + \frac{(P-q)^\mu (P-q)^\nu}{m_{D_s^*}^2} \right] (P+q)_\mu (2k - P + q)_\nu \\ &= m_B^2 - q^2 - 2Pk - 2kq \\ &\quad + \frac{1}{m_{D_s^*}^2} [(m_B^2 - q^2)(2Pk - 2kq - m_B^2 - q^2 + 2Pq)]. \end{aligned} \quad (14)$$

We perform the q^0 integration analytically using Cauchy's residues. For this purpose we use

$$\begin{aligned} &\frac{1}{q^2 - m_{D^0}^2 + i\epsilon} \\ &= \frac{1}{2w_D(\mathbf{q})} \left(\frac{1}{q^0 - w_D(\mathbf{q}) + i\epsilon} - \frac{1}{q^0 + w_D(\mathbf{q}) - i\epsilon} \right), \end{aligned} \quad (15)$$

and keep only the positive energy part because we are dealing with heavy particles that propagate mostly with positive energy, as one can induce from the diagram of Fig. 2. We do this choice for the three propagators. The Cauchy integration picks up the pole $q^0 = w_{D^0}(\mathbf{q})$. As a consequence, we obtain the loop amplitude

$$\begin{aligned}
t_L = & \int \frac{d^3 q}{(2\pi)^3} Cg[-m_{D^0}^2 - m_K^2 + M_{\text{inv}}^2(\eta\eta) - 2w_K(\mathbf{k})w_D(\mathbf{q}) + 2\vec{k} \cdot \vec{q}] \\
& + \frac{1}{m_{D_s^*}^2} (m_B^2 - m_{D^0}^2)(m_K^2 - m_{D^0}^2 - M_{\text{inv}}^2(\eta\eta) - 2w_K(\mathbf{k})w_D(\mathbf{q}) + 2\vec{k} \cdot \vec{q} + 2m_B w_D(\mathbf{q})) \\
& \times t_{D^0 \bar{D}^0, \eta\eta}(M_{\text{inv}}(\eta\eta)) \times \frac{1}{2w_D(\mathbf{q})} \times \frac{1}{2w_{D_s^*}(\mathbf{q})} \times \frac{1}{2w_D(\mathbf{k} + \mathbf{q})} \\
& \times \frac{1}{m_B - w_D(\mathbf{q}) - w_{D_s^*}(\mathbf{q}) + i\epsilon} \times \frac{1}{m_B - w_D(\mathbf{q}) - w_K(\mathbf{k}) - w_D(\mathbf{k} + \mathbf{q}) + i\epsilon} F_{\text{HQS}} \Theta(q_{\text{max}} - |\vec{q}^*|), \quad (16)
\end{aligned}$$

with

$$k = \frac{\lambda^{1/2}(m_B^2, m_K^2, M_{\text{inv}}^2(\eta\eta))}{2m_B}, \quad (17)$$

where we have used

$$\begin{aligned}
(P - k)^2 &= M_{\text{inv}}^2(\eta\eta), \\
2Pk &= m_B^2 + m_K^2 - M_{\text{inv}}^2(\eta\eta), \\
2kq &= 2w_K(\mathbf{k})w_D(\mathbf{q}) - 2\vec{k} \cdot \vec{q}, \\
2Pq &= 2m_B w_D(\mathbf{q}). \quad (18)
\end{aligned}$$

In Eq. (16) we have the factor F_{HQS} given by

$$F_{\text{HQS}} = \frac{m_{D_s^*}}{m_{K^*}}, \quad (19)$$

which stems from consideration of heavy quark spin symmetry [41] to obtain the correct width of the $D^* \rightarrow D\pi$ decay. On the other hand, the factor $\Theta(q_{\text{max}} - |\vec{q}^*|)$ comes from the way that we regularize the loops in our t amplitudes with the cutoff method. Indeed, in Ref. [42] it was found that the T matrix evaluated in the form $T = [1 - VG]^{-1}V$ with the G function regularized with a cutoff, q_{max} , was equivalent to starting with a separable potential

$$V(\mathbf{q}, \mathbf{q}') = \Theta(q_{\text{max}} - |\mathbf{q}|)\Theta(q_{\text{max}} - |\mathbf{q}'|)V, \quad (20)$$

with

$$\begin{aligned}
G &= \int_{|\mathbf{q}| < q_{\text{max}}} \frac{d^3 \mathbf{q}}{(2\pi)^3} \frac{w_1(\mathbf{q}) + w_2(\mathbf{q})}{2w_1(\mathbf{q})w_2(\mathbf{q})} \\
&\times \frac{1}{s - (w_1(\mathbf{q}) + w_2(\mathbf{q}))^2 + i\epsilon} \quad (21)
\end{aligned}$$

with $w_i(\mathbf{q}) = \sqrt{m_i^2 + \vec{q}^2}$. We take $q_{\text{max}} = 750$ MeV as suited in the study of Ref. [43]. Equation (20) automatically implies that the t matrix has the structure

$$T(\mathbf{q}, \mathbf{q}') = t\Theta(q_{\text{max}} - |\mathbf{q}|)\Theta(q_{\text{max}} - |\mathbf{q}'|). \quad (22)$$

Since q_{max} regulates the $D\bar{D}$ loops in their rest frame, we must take the factor $\Theta(q_{\text{max}} - |\vec{q}^*|)$ where \vec{q}^* is the \bar{D}^0 momentum in the rest frame of $\eta\eta$ given by [38]

$$\vec{q}^* = \left[\left(\frac{E_R}{M_{\text{inv}}(\eta\eta)} - 1 \right) \frac{\vec{q} \cdot \vec{k}}{k^2} + \frac{w_{D^0}(\mathbf{q})}{M_{\text{inv}}(\eta\eta)} \right] \vec{k} + \vec{q}, \quad (23)$$

with

$$E_R = \sqrt{M_{\text{inv}}^2(\eta\eta) + \vec{k}^2}. \quad (24)$$

Now the integral only depends on $|\vec{k}|$, and hence on $M_{\text{inv}}(\eta\eta)$.

The matrix elements for the $D\bar{D} \rightarrow j$ transition are obtained using the Bethe-Salpeter equation $T = [1 - VG]^{-1}V$ in coupled channels in Ref. [24], but since the couplings of the state to the $D\bar{D}$ bound state are calculated there, we directly take the $t_{D^0 \bar{D}^0, \eta\eta}$ transition amplitude from this reference and write it with a Breit-Wigner form as

$$t_{D^0 \bar{D}^0, \eta\eta}(M_{\text{inv}}(\eta\eta)) = \frac{g_{D^0 \bar{D}^0} g_{\eta\eta}}{M_{\text{inv}}^2(\eta\eta) - m_{D^0 \bar{D}^0}^2 + im_{D^0 \bar{D}^0} \Gamma_{D^0 \bar{D}^0}}, \quad (25)$$

with the relevant quantities given by [24]

$$\begin{aligned}
g_{D^0 \bar{D}^0} &= (5962 + i1695) \text{ MeV}, \\
g_{\eta\eta} &= (1023 + i242) \text{ MeV}, \\
m_{D\bar{D}}|_b &= 3722 \text{ MeV (for the bound state)}, \\
\Gamma_{D\bar{D}}|_b &= 36 \text{ MeV}. \quad (26)
\end{aligned}$$

Finally, the differential mass distribution for the $\eta\eta$ system is given by

$$\frac{d\Gamma}{dM_{\text{inv}}(\eta\eta)} = \frac{1}{(2\pi)^3} \frac{1}{4m_B^2} k \tilde{P}_\eta |t_L|^2, \quad (27)$$

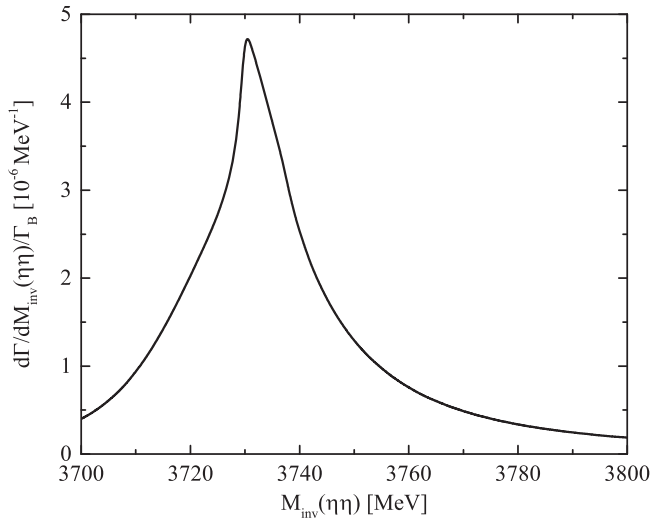


FIG. 3. Results of R_T as a function of $M_{\text{inv}}(\eta\eta)$ invariant mass distribution.

where \tilde{P}_η is the momentum of η in the $\eta\eta$ rest frame,

$$\tilde{P}_\eta = \frac{\lambda^{1/2}(M_{\text{inv}}^2(\eta\eta), m_\eta^2, m_\eta^2)}{2M_{\text{inv}}(\eta\eta)}. \quad (28)$$

III. RESULTS

In Fig. 3 we show the results of $R_T = \frac{1}{\Gamma_B} \frac{d\Gamma}{dM_{\text{inv}}}$. We make use of the value of the ratio $\frac{C^2}{\Gamma_B}$ from Eq. (8); hence, we can predict not only the shape of the mass distribution but also its strength. We indeed find in Fig. 3 a neat peak around the mass of the $D\bar{D}$ bound state with the width predicted in Ref. [24].

To find the feasibility of this experiment we integrate R_T over $M_{\text{inv}}(\eta\eta)$ to get the strength of the peak as $\frac{1}{\Gamma_B} \int \frac{d\Gamma}{dM_{\text{inv}}(\eta\eta)} dM_{\text{inv}}(\eta\eta)$, and we obtain the following value for the branching ratio of the reaction $B^+ \rightarrow K^+ D\bar{D}|_b; D\bar{D}|_b \rightarrow \eta\eta$, where $D\bar{D}|_b$ means the $D\bar{D}$ bound state,

$$\mathcal{B}[B^+ \rightarrow K^+ D\bar{D}|_b; D\bar{D}|_b \rightarrow \eta\eta] = 1.47 \times 10^{-4}. \quad (29)$$

Taking into account that most of the hadronic branching fractions reported in the PDG are of the order of 10^{-4} or smaller, with some branching ratios of the order of 10^{-7} , this branching ratio is relatively big and could easily be observed in experiments. This result can only encourage experimental teams to perform this measurement that would show for the first time the peak associated with the $D\bar{D}$ bound state.

IV. CONCLUSIONS

We have studied the reaction $B^+ \rightarrow K^+\eta\eta$ with the aim of finding a peak in the $\eta\eta$ mass distribution corresponding

to a $D\bar{D}$ bound state that has been predicted by several theoretical frameworks, in lattice QCD simulations, and has also been claimed to exist from the observation of a concentration of strength around the $D\bar{D}$ threshold in reactions producing $D\bar{D}$ in the final state.

To maximize the chances of observation we have selected a reaction that in a first step produces a $D\bar{D}$ which is allowed to interact and produce the $\eta\eta$ at the end. The reaction chosen is $B^+ \rightarrow D_s^{*+} \bar{D}^0$, which has a large branching fraction for a B hadronic decay, of the order of 10^{-2} . The D_s^{*+} decays to $D^0 K^+$ and the $D^0 \bar{D}^0$ interact and produce the $\eta\eta$. Technically the combined process is evaluated by means of a triangle diagram where the $D\bar{D}$ are virtual, a necessary condition to produce the $D\bar{D}$ bound state. The choice of $\eta\eta$ being produced by the $D^0 \bar{D}^0$ interaction is motivated because the $D\bar{D}$ bound state only decays in light meson pairs, where the $c\bar{c}$ quarks have been annihilated. From previous calculations one knows that the $\eta\eta$ channel is one of the light pseudoscalar channels that couples most strongly to the $D\bar{D}$ bound state.

With this promising scenario we have evaluated the $\eta\eta$ mass distribution for the $B^+ \rightarrow K^+\eta\eta$ decay, and we have found indeed a clear peak around the predicted mass of the $D\bar{D}$ bound state. Then we have integrated the mass distribution and found a branching fraction for $B^+ \rightarrow K^+ (D\bar{D}, \text{bound}); (D\bar{D}, \text{bound}) \rightarrow \eta\eta$ of the order of 1.5×10^{-4} . This is a relatively large branching fraction for a B decay, which should encourage its search to finally find a peak for this much searched for state.

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Note added.—While sending the paper to the Inspire web, a similar paper [44] appeared there dealing with a similar reaction, $B^- \rightarrow K^- \eta_c$ decay. While the reaction is also promising, the method and formalism used in [44] are different, and no absolute rate is predicted. This latter reaction is also studied in [45] and branching ratios of the order of 10^{-4} are predicted.

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