Nuclear effects in extracting $\sin^2\theta_W$ and a probe for short-range correlations

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We investigate the neutral-current neutrino-nucleon deep inelastic scattering with particular emphasis on short-range correlation (SRC) and the European Muon Collaboration (EMC) effect, as well as their impact on the weak-mixing angle $\sin^2\theta_W$ determination. The ratios of relevant structure functions are presented where the nuclei are chosen as carbon, iron, and lead. One kind of universal modification function is proposed which would provide a nontrivial test of SRC universality on the platform of neutrino-nucleon deep inelastic scattering. In addition, we study the impact of "SRC-driven" nuclear effects on the extraction of $\sin^2\theta_W$ which is naturally associated with the renowned NuTeV anomaly. The results indicate that these effects may account for a substantial fraction of the NuTeV anomaly and considerably affect the value of extracted $\sin^2\theta_W$.

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I. INTRODUCTION

Neutrino-nucleon scattering provides one of the most precise platforms for the weak neutral-current. The high statistics measurements of neutrino deep inelastic scattering (DIS) on heavy nuclear targets have attracted lots of attention due to their importance in global fits of parton distribution functions (PDFs). Besides, the data on ν and $\bar{\nu}$, contrary to charged leptons, give direct access to both the weak-mixing angle θ_W and the Z^0 coupling to (anti)neutrinos.

Owing to the weak nature of neutrino interactions, the use of heavy nuclear targets is unavoidable in neutrino DIS experiments, and this complicates the extraction of relevant observables because of the nuclear effects. The original idea of having nuclear effects in PDFs was driven by data in DIS measurements performed by the European Muon Collaboration (EMC) and subsequently conformed by other experiments [1–6]. They found a reduction of the cross section per nucleon in nucleus A compared with deuteron in the valence quark dominated regime $0.3 < x < 0.7$, here x is the Bjorken variable. This phenomenon is referred to as the EMC effect. One should note that there is no consensus on the exact nature of EMC effect until now.

Recently, the possible connection between the EMC effect and short-range correlation (SRC) has been investigated substantially [7–19]. The two-nucleon SRC is defined experimentally as having small center-of-mass momentum and large relative momentum, it describes the probability that two nucleons are close in coordinate space, as a result of nontrivial nucleon-nucleon interactions in the nucleus. One can refer to this nicely written review [20] for more details. The neutrino-nucleon DIS is an ideal platform for testing nucleon structures and SRC interpretation of the EMC effect, in this work, we will study the neutral-current neutrino-nucleon DIS where the nuclei A are selected to be ^{12}C, ^{56}Fe, and ^{208}Pb. The structure functions $F_{2(\mathrm{NC})}^{\nu A}(x, Q^2)$ and $xF_{3(\mathrm{NC})}^{\nu A}(x, Q^2)$ are calculated with consideration of nuclear PDFs (NPDFs) in terms of EPPS21 parametrization, and we choose CT18ANLO as free nucleon baseline [21,22]. One kind of universal modification functions was proposed in this process which can be viewed as nontrivial tests of SRC universality on the platform of neutrino-nucleon scattering. Here universality means the partonic structure from the correlated nucleon-nucleon SRC pair is same for all kinds of nuclei.

Compared to charged lepton-nucleon scattering, data on neutrino-nucleon are in short supply, and the understanding

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of nuclear effects in them are therefore pretty limited. In addition to testing the SRC universality, there is an even more important issue, we explore the way in which the "SRC-driven" nuclear effects modifies the extraction of weak-mixing angle $\sin^2\theta_W$, which is a key parameter in the electroweak sector of the Standard Model (SM) [23–25]. The precise determination of this angle is among the fundamental works in particle physics and it had been accurately measured by collider experiments. Nevertheless, the NuTeV Collaboration reported an anomalously large weak-mixing angle $\sin^2\theta_W = 0.2277 \pm 0.0013(\mathrm{stat}) \pm 0.0009(\mathrm{syst})$ [26,27]. There is a three-sigma discrepancy between this result and global analysis of other data $\sin^2\theta_W = 0.2227 \pm 0.0004$. This is the renowned NuTeV anomaly, which has not been fully understood yet. Since this anomaly came up, a lot of detailed data analyzing works followed, the results raise a deep question as to whether the neutrino DIS data could be combined with the charged-lepton DIS data to get better NPDFs [28–33].

This situation requires resolution. Historically, the precise measurement of θ_W is closely related to new physics (NP) [34–36], many mechanisms based on NP were proposed to explain the cause for this anomaly [37,38]. Meanwhile, a number of works that attempted to interpret the NuTeV result within the context of the SM have been suggested, most of them have potential to attenuate the anomaly [39–49]. These works largely focused on nucleon charge symmetry violating (CSV) effects, finite distributions of strange sea quark, as well as nuclear corrections such as Fermi motion and binding and the isovector EMC effect. If one or more contributions mentioned above are as large as expected in the references, it undoubtedly will be a milestone discovery concerning fundamental QCD effects in nuclei. In spite of these remarkable efforts, effects from the SRCs of the bound nucleon have not been investigated in relation to the NuTeV anomaly. These effects are potentially essential since they are widely accepted as one of the leading approaches for explaining the EMC effect.

It is known that the Paschos-Wolfenstein (PW) relation which was deduced for the isoscalar nucleon $R_A^- = (\sigma_{\mathrm{NC}}^{\nu A} - \sigma_{\mathrm{NC}}^{\bar{\nu}A})/(\sigma_{\mathrm{CC}}^{\nu A} - \sigma_{\mathrm{CC}}^{\bar{\nu}A}) = 1/2 - \sin^2\theta_W$ [50], was used for the determination of $\sin^2\theta_W$ in NuTeV experiment. Here, $\sigma_{\mathrm{NC}}^{\nu A}$ and $\sigma_{\mathrm{CC}}^{\nu A}$ are the deep inelastic cross sections for neutral-current (NC) and charged-current (CC) neutrino-nucleon interactions, and A represents the target. In this paper, motivated by the correlation between the EMC effect and the SRC scale factor, we derive a modified PW relation. Then, we discuss its impact on the extraction of $\sin^2\theta_W$.

The rest of this paper is organized as follows: In Sec. II, the formalism and results of structure functions in NC neutrino-nucleon DIS will be briefly reviewed, and the proposal of one kind of universal modification function is discussed. In Sec. III we study the "SRC-driven" nuclear corrections of the PW relation and their possible effects on the extraction of $\sin^2\theta_W$. We conclude in Sec. IV.

II. UNIVERSAL MODIFICATION FUNCTION IN NC NEUTRINO-NUCLEON DIS

As illustrated in Fig. 1, a high energy neutrino interacts with a nucleon through the exchange of a neutral Z^0 boson, producing a corresponding neutrino and hadron in the final states. These processes can be analyzed by the following Lorentz invariants: the Bjorken scaling variable $x \equiv Q^2/(2p\cdot q)$; the inelasticity $y \equiv (2p\cdot k)/(2p\cdot q)$; the negative squared four momentum transfer $Q^2 \equiv -q^2$.

The cross section for NC (anti)neutrino interactions with the nucleon in nucleus A is given by

$$\begin{aligned}\frac{d\sigma_{\mathrm{NC}}^{\nu A}}{dxdy} &= \frac{G_F^2 s}{2\pi(1+Q^2/M_Z^2)^2}\bigg[F_{1(\mathrm{NC})}^{\nu A} xy^2 \\ &\quad + F_{2(\mathrm{NC})}^{\nu A}(1-y) + F_{3(\mathrm{NC})}^{\nu A} xy\left(1-\frac{y}{2}\right)\bigg], \\ \frac{d\sigma_{\mathrm{NC}}^{\bar{\nu}A}}{dxdy} &= \frac{G_F^2 s}{2\pi(1+Q^2/M_Z^2)^2}\bigg[F_{1(\mathrm{NC})}^{\bar{\nu}A} xy^2 \\ &\quad + F_{2(\mathrm{NC})}^{\bar{\nu}A}(1-y) - F_{3(\mathrm{NC})}^{\bar{\nu}A} xy\left(1-\frac{y}{2}\right)\bigg]. \end{aligned} \tag{1}$$

Here G_F is the Fermi coupling constant and s is the square of the center-of-mass energy. The expressions above can be reduced by Callan-Gross relation $F_{2(\mathrm{NC})}^{\nu A} = 2xF_{1(\mathrm{NC})}^{\nu A}$, $F_{2(\mathrm{NC})}^{\bar{\nu}A} = 2xF_{1(\mathrm{NC})}^{\bar{\nu}A}$ [51]. In the above expression, we have omitted the explicit dependence on x and Q^2 for brevity. The nuclear structure functions for a nucleus with mass number A and atomic number Z can be decomposed into two parts

$$\begin{cases} F_{2(\mathrm{NC})}^{\nu A} = \frac{Z}{A}F_{2(\mathrm{NC})}^{\nu p/A} + \frac{A-Z}{A}F_{2(\mathrm{NC})}^{\nu n/A}, \\ F_{2(\mathrm{NC})}^{\bar{\nu}A} = \frac{Z}{A}F_{2(\mathrm{NC})}^{\bar{\nu}p/A} + \frac{A-Z}{A}F_{2(\mathrm{NC})}^{\bar{\nu}n/A}, \end{cases} \tag{2}$$

$$\begin{cases} xF_{3(\mathrm{NC})}^{\nu A} = \frac{Z}{A}xF_{3(\mathrm{NC})}^{\nu p/A} + \frac{A-Z}{A}xF_{3(\mathrm{NC})}^{\nu n/A}, \\ xF_{3(\mathrm{NC})}^{\bar{\nu}A} = \frac{Z}{A}xF_{3(\mathrm{NC})}^{\bar{\nu}p/A} + \frac{A-Z}{A}xF_{3(\mathrm{NC})}^{\bar{\nu}n/A}. \end{cases} \tag{3}$$

which can be further expressed in terms of NPDFs:

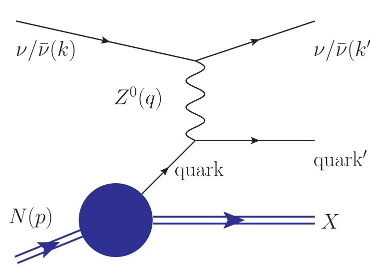

FIG. 1. A schematic diagram for neutral-current neutrino-nucleon DIS. The process was carried out through the exchange of an electroweak gauge boson Z^0.

$$\begin{aligned}F^{\nu A}_{2(\mathrm{NC})} &= F^{\bar\nu A}_{2(\mathrm{NC})}\\ &= \frac{Z}{A}2x[(u_L^2+u_R^2)(u_p^{A+}+c_p^{A+})\\ &\quad+(d_L^2+d_R^2)(d_p^{A+}+s_p^{A+})]\\ &\quad+\frac{A-Z}{A}2x[(u_L^2+u_R^2)(u_n^{A+}+c_n^{A+})\\ &\quad+(d_L^2+d_R^2)(d_n^{A+}+s_n^{A+})]. \end{aligned}\tag{4}$$

Here the left- and right-handed couplings for a quark are expressed as $u_L = \frac{1}{2}-\frac{2}{3}\sin^2\theta_W$, $u_R = -\frac{2}{3}\sin^2\theta_W$ and $d_L = -\frac{1}{2}+\frac{1}{3}\sin^2\theta_W$, $d_R = \frac{1}{3}\sin^2\theta_W$. We also define $q^{A\pm} \equiv q^A \pm \bar{q}^A$.

The size of partonic CSV correction to NuTeV anomaly has been estimated to be significant [48,49]. Nevertheless, since this correction has been relatively well studied and we are only interested in the SRC-induced nuclear corrections, the isospin symmetry $u_p^A(\bar u_p^A) = d_n^A(\bar d_n^A)$, $d_p^A(\bar d_p^A) = u_n^A(\bar u_n^A)$, $s_p^A(\bar s_p^A) = s_n^A(\bar s_n^A)$, $c_p^A(\bar c_p^A) = c_n^A(\bar c_n^A)$ will be utilized in this paper. Therefore, the expression for the structure function in Eq. (4) simplifies to

$$\begin{aligned}F^{\nu A}_{2(\mathrm{NC})} &= \frac{Z}{A}2x[(u_L^2+u_R^2)u_p^{A+}+(d_L^2+d_R^2)d_p^{A+}]\\ &\quad+\frac{A-Z}{A}2x[(u_L^2+u_R^2)d_p^{A+}+(d_L^2+d_R^2)u_p^{A+}]\\ &\quad+2x[(u_L^2+u_R^2)(c_p^{A+})+(d_L^2+d_R^2)s_p^{A+}]. \end{aligned}\tag{5}$$

Similarly,

$$\begin{aligned}xF^{\nu A}_{3(\mathrm{NC})} &= xF^{\bar\nu A}_{3(\mathrm{NC})}\\ &= \frac{Z}{A}2x[(u_L^2-u_R^2)u_p^{A-}+(d_L^2-d_R^2)d_p^{A-}]\\ &\quad+\frac{A-Z}{A}2x[(u_L^2-u_R^2)d_p^{A-}+(d_L^2-d_R^2)u_p^{A-}]\\ &\quad+2x[(u_L^2-u_R^2)c_p^{A-}+(d_L^2-d_R^2)s_p^{A-}]. \end{aligned}\tag{6}$$

The NPDF $f_i^{p/A}(x,Q^2)$ can be defined relative to the free proton PDF $f_i^p(x,Q^2)$ as [21]

$$f_i^{p/A}(x,Q^2) = R_i^A(x,Q^2)f_i^p(x,Q^2),\tag{7}$$

where i denotes the types of partons and $R_i^A(x,Q^2)$ refers to nuclear modification factor. The free proton baseline is CT18ANLO [22].

We define the ratios in line with Ref. [16]

$$\begin{aligned}R(F^{\nu A}_{2(\mathrm{NC})};x,Q^2) &= F^{\nu A}_{2(\mathrm{NC})}/F^{\nu N}_{2(\mathrm{NC})},\\ R(xF^{\nu A}_{3(\mathrm{NC})};x,Q^2) &= (xF^{\nu A}_{3(\mathrm{NC})})/(xF^{\nu N}_{3(\mathrm{NC})}). \end{aligned}\tag{8}$$

Here $F^{\nu N}_{2(\mathrm{NC})}$ and $xF^{\nu N}_{3(\mathrm{NC})}$ have the same expressions of $F^{\nu A}_{2(\mathrm{NC})}$ and $xF^{\nu A}_{3(\mathrm{NC})}$, just with the NPDFs in nucleon

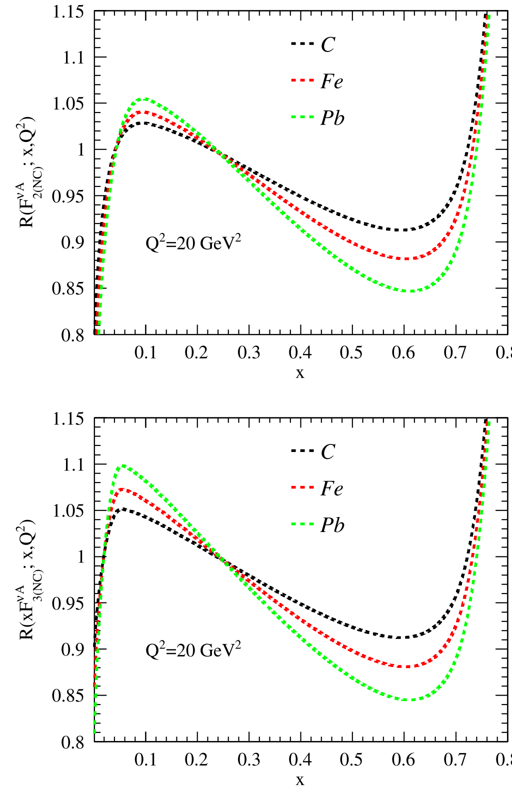

FIG. 2. The ratios $R(F^{\nu A}_{2(\mathrm{NC})};x,Q^2)$ and $R(xF^{\nu A}_{3(\mathrm{NC})};x,Q^2)$ as functions of x with $Q^2 = 20\ \mathrm{GeV}^2$. The black, red, and green lines correspond to ^{12}C, ^{56}Fe, and ^{208}Pb, respectively.

replaced by PDFs in free proton. With Eqs. (7) and (8), we depict the dependence of $R(F^{\nu A}_{2(\mathrm{NC})};x,Q^2)$ and $R(xF^{\nu A}_{3(\mathrm{NC})};x,Q^2)$ on x in Fig. 2. The Q^2 is fixed to $20\ \mathrm{GeV}^2$, which is attainable in many neutrino-nucleon scattering experiments. A comparison will be conducted between Figs. 2 and 3 that appears subsequently.

Motivated by the amazing linear correlation between the EMC effect and the SRC scale factor which has received enormous attention in recent years, we parameterize the u and d quark distributions in the EMC region as that for the structure function in Refs. [52,53] by assuming that all nuclear modifications originate from the nucleon-nucleon SRCs,

$$\begin{aligned}u_v^{p/A}(x,Q^2) &= \frac{1}{Z}[Zu_v^p(x,Q^2)+n_{\mathrm{src}}^A\delta u_v^p(x,Q^2)],\\ d_v^{p/A}(x,Q^2) &= \frac{1}{Z}[Zd_v^p(x,Q^2)+n_{\mathrm{src}}^A\delta d_v^p(x,Q^2)], \end{aligned}\tag{9}$$

where n_{src}^A represents number of nucleon-nucleon pairs in nucleus A, notably the subscript v in u_v^p and d_v^p means distributions of valence quark since experimental results pointed to the EMC effect being due to a change in the valence quark distributions [54,55]. δu_v^p and δd_v^p represent

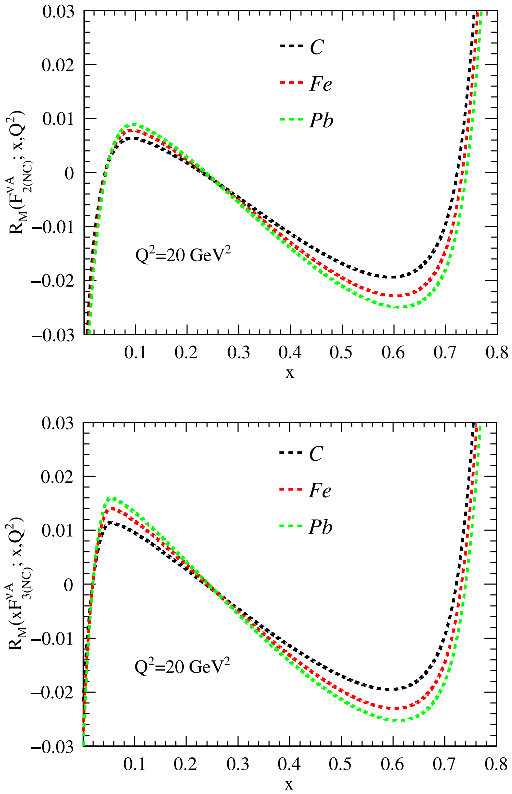

FIG. 3. The ratios $R_M(F_{2(\mathrm{NC})}^{\nu A}; x, Q^2)$ and $R_M(xF_{3(\mathrm{NC})}^{\nu A}; x, Q^2)$ present the universality of SRC contributions with EPPS21 parametrization.

the difference between u and d valence quark distributions in the SRC pair and in the free proton, respectively.

We then rearrange equation Eq. (9) with the aid of parametrization in Eq. (7):

$$\delta u_v^p(x, Q^2)/u_v^p(x, Q^2) = (R_{u_v}^A(x, Q^2) - 1)/(n_{\mathrm{src}}^A/Z),$$
$$\delta d_v^p(x, Q^2)/d_v^p(x, Q^2) = (R_{d_v}^A(x, Q^2) - 1)/(n_{\mathrm{src}}^A/Z). \quad (10)$$

Because δu_v^p and δd_v^p are assumed to be nucleus-independent, our model predicts that the left-hand side of Eq. (10) should be a universal function, here universal means they are the same for all kinds of nuclei. This indicates that the nucleus-dependent quantities on the right-hand side of Eq. (10) combine to give a nucleus-independent result. This universality of SRC can be illustrated more specifically by introducing one kind of universal modification functions. From Eq. (10), the universality indicates that if one has two different kinds of nuclei A and B:

$$\frac{R_{u_v}^A - 1}{n_{\mathrm{src}}^A/Z_A} = \frac{R_{u_v}^B - 1}{n_{\mathrm{src}}^B/Z_B}, \quad (11)$$

here the explicit dependence on x and Q^2 are omitted for brevity. Define $a_2^A = (n_{\mathrm{src}}^A/A)/(n_{\mathrm{src}}^d/2)$, which is the SRC scale factor of nucleus A respect to that of deuteron, it can be measured through the nuclear structure functions at $x > 1.5$ region [56,57]. Equation (11) can be reexpressed as

$$\frac{R_{u_v}^A - 1}{a_2^A}\frac{2Z_A}{A_A} = \frac{R_{u_v}^B - 1}{a_2^B}\frac{2Z_B}{A_B}. \quad (12)$$

Similarly, for the valence d quark

$$\frac{R_{d_v}^A - 1}{a_2^A}\frac{2Z_A}{A_A} = \frac{R_{d_v}^B - 1}{a_2^B}\frac{2Z_B}{A_B}. \quad (13)$$

Therefore, one can parametrize the valence quark distribution in nucleus A in terms of that in B,

$$R_{u_v}^A = \frac{Z_B}{A_B}\frac{A_A}{Z_A}\frac{a_2^A}{a_2^B}(R_{u_v}^B - 1) + 1,$$
$$R_{d_v}^A = \frac{Z_B}{A_B}\frac{A_A}{Z_A}\frac{a_2^A}{a_2^B}(R_{d_v}^B - 1) + 1. \quad (14)$$

In the EMC region, the NPDFs of valence u and d quarks dominate the structure function $F_{2(\mathrm{NC})}^{\nu A}$, the distributions of sea partons can be dropped if we only concerned about the universality of SRC for the time being. Thus, the structure function of $F_{2(\mathrm{NC})}^{\nu A}$ further simplifies to

$$\begin{aligned} F_{2(\mathrm{NC})}^{\nu A} &= \frac{Z}{A}2x[(u_L^2 + u_R^2)u_v^A + (d_L^2 + d_R^2)d_v^A] \\ &\quad + \frac{A-Z}{A}2x[(u_L^2 + u_R^2)d_v^A + (d_L^2 + d_R^2)u_v^A] \\ &= \frac{Z}{A}2x[(u_L^2 + u_R^2)R_{u_v}^A u_v^p + (d_L^2 + d_R^2)R_{d_v}^A d_v^p] \\ &\quad + \frac{A-Z}{A}2x[(u_L^2 + u_R^2)R_{d_v}^A d_v^p + (d_L^2 + d_R^2)R_{u_v}^A u_v^p]. \end{aligned} \quad (15)$$

Bringing Eq. (14) into Eq. (15), the $F_{2(\mathrm{NC})}^{\nu A}$ can be reconstructed in terms of $R_{u_v}^B$ and $R_{d_v}^B$. We do not show the explicit expression here, since it is rather lengthy and the step is straightforward. Bringing this newly obtained expression into Eq. (8). After some algebra, one can readily find

$$\frac{2Z_A}{A_A}\frac{R(F_{2(\mathrm{NC})}^{\nu A}; x, Q^2) - 1}{a_2^A} = \frac{2Z_B}{A_B}\frac{R(F_{2(\mathrm{NC})}^{\nu B}; x, Q^2) - 1}{a_2^B}. \quad (16)$$

The equation for $xF_{3(\mathrm{NC})}^{\nu A}$ can be also derived in an analogous way. These results indicate that the universality of SRC can be illustrated more specifically by introducing one kind of universal modification functions

$$R_M(F_{2(\mathrm{NC})}^{\nu A};x,Q^2)=\frac{2Z}{A}\frac{R(F_{2(\mathrm{NC})}^{\nu A};x,Q^2)-1}{a_2^A},$$
$$R_M(xF_{3(\mathrm{NC})}^{\nu A};x,Q^2)=\frac{2Z}{A}\frac{R(xF_{3(\mathrm{NC})}^{\nu A};x,Q^2)-1}{a_2^A}. \quad (17)$$

The role of R_M is normalizing the ratios defined in Eq. (8) by the respective SRC scale factors. The universal modification functions are plotted in Fig. 3, it can be seen that the ratios of different nuclei tend to shrink substantially compared with those in Fig. 2. This universality has been investigated in the CC neutrino-nucleon DIS previously [16], our result in the NC process here supports the theoretical assumption raised in Eq. (9) and indicates that in the valence quark dominated regime $0.3 < x < 0.7$, the EMC effect is mainly caused by SRC pairs. Figure 3 is a nontrivial test of SRC universality, providing a new clue to understand how the relatively long-range nuclear interaction influences the short-distance parton structure inside the nucleon.

III. MODIFIED PW RELATION AND ITS IMPACT ON $\sin^2\theta_W$ DETERMINATION

The NuTeV target is mainly the iron nuclei, nuclear corrections should be carefully taken into account for a precise determination of $\sin^2\theta_W$. In this section, we derive a modified PW relation which has taken the SRC of nucleons into account. Then, we discuss a possible nuclear modification factor which could change the extracted $\sin^2\theta_W$ value.

The differential cross sections of charged-current (anti) neutrino-nucleon DIS expressed in terms of NPDFs are

$$\begin{aligned}\frac{d\sigma_{\mathrm{CC}}^{\nu A}}{dxdy}&=\frac{G_F^2 s}{\pi(1+Q^2/M_W^2)^2}x\\&\quad\times[d^A+s^A+(1-y)^2(\bar{u}^A+\bar{c}^A)],\\\frac{d\sigma_{\mathrm{CC}}^{\bar{\nu} A}}{dxdy}&=\frac{G_F^2 s}{\pi(1+Q^2/M_W^2)^2}x\\&\quad\times[\bar{d}^A+\bar{s}^A+(1-y)^2(u^A+c^A)].\end{aligned} \quad (18)$$

For the NC neutrino-nucleon scattering, the differential cross sections are

$$\begin{aligned}\frac{d\sigma_{\mathrm{NC}}^{\nu A}}{dxdy}&=\frac{G_F^2 s}{\pi(1+Q^2/M_Z^2)^2}x\{[u_L^2+u_R^2(1-y)^2](u^A+c^A)+[u_R^2+u_L^2(1-y)^2](\bar{u}^A+\bar{c}^A)\\&\quad+[d_L^2+d_R^2(1-y)^2](d^A+s^A)+[d_R^2+d_L^2(1-y)^2](\bar{d}^A+\bar{s}^A)\},\\\frac{d\sigma_{\mathrm{NC}}^{\bar{\nu} A}}{dxdy}&=\frac{G_F^2 s}{\pi(1+Q^2/M_Z^2)^2}x\{[u_R^2+u_L^2(1-y)^2](u^A+c^A)+[u_L^2+u_R^2(1-y)^2](\bar{u}^A+\bar{c}^A)\\&\quad+[d_R^2+d_L^2(1-y)^2](d^A+s^A)+[d_L^2+d_R^2(1-y)^2](\bar{d}^A+\bar{s}^A)\}.\end{aligned} \quad (19)$$

Utilizing Eqs. (18) and (19), we obtain the PW relation in the form of parton distributions [42],

$$\begin{aligned}R_A^-&=(1-(1-y)^2)\\&\quad\times\frac{(u_L^2-u_R^2)(u_v^A+c_v^A)+(d_L^2-d_R^2)(d_v^A+s_v^A)}{d_v^A+s_v^A-(1-y)^2(u_v^A+c_v^A)}.\end{aligned} \quad (20)$$

The valence quark NPDFs are defined by $q_v^A\equiv q^A-\bar{q}^A$. We can parameterize u_v^A and d_v^A in the EMC region as we did in Eq. (9), therefore for a certain nucleus with mass number A, atomic number Z and neutron number N, it's valence u and d quark distributions would be decomposed into contributions from unmodified protons and neutrons and np SRC pairs with modified quark distributions:

$$\begin{aligned}u_v^A&=\frac{1}{A}[(Z-n_{\mathrm{src}}^A)u_v^p+(N-n_{\mathrm{src}}^A)u_v^n+n_{\mathrm{src}}^A(\tilde{u}_v^p+\tilde{u}_v^n)]\\&=\frac{1}{A}[Zu_v^p+Nd_v^p+n_{\mathrm{src}}^A(\delta u_v^p+\delta u_v^n)].\end{aligned} \quad (21)$$

Here the partonic charge symmetry between free proton and neutron $u_v^n=d_v^p$ has been utilized. $\tilde{u}_v^p$ and $\tilde{u}_v^n$ are the modified distributions for protons and neutrons in SRC pairs and $\delta u_v^p\equiv\tilde{u}_v^p-u_v^p$ (similarly for δu_v^n). It is analogous for d_v^A,

$$d_v^A=\frac{1}{A}[Zd_v^p+Nu_v^p+n_{\mathrm{src}}^A(\delta d_v^p+\delta d_v^n)]. \quad (22)$$

In nucleus A, the isospin symmetry $u_v^{p/A}=d_v^{n/A}$ and $d_v^{p/A}=u_v^{n/A}$ restricts:

$$\delta d_v^n=\frac{N}{Z}\delta u_v^p,\qquad \delta u_v^n=\frac{N}{Z}\delta d_v^p. \quad (23)$$

This makes Eqs. (21) and (22) into

$$\begin{aligned}u_v^A&=\frac{Zu_v^p+Nd_v^p}{A}+\frac{n_{\mathrm{src}}^A}{A}\left(\delta u_v^p+\frac{N}{Z}\delta d_v^p\right),\\d_v^A&=\frac{Zd_v^p+Nu_v^p}{A}+\frac{n_{\mathrm{src}}^A}{A}\left(\delta d_v^p+\frac{N}{Z}\delta u_v^p\right).\end{aligned} \quad (24)$$

Next, we define neutron excess constant ϵ_n as well as Δ^+, Δ^- by

$$\epsilon_n \equiv \frac{N-Z}{A}, \qquad \Delta^+ \equiv \delta d_v^p + \delta u_v^p, \qquad \Delta^- \equiv \delta d_v^p - \delta u_v^p. \tag{25}$$

Substituting Eqs. (24) and (25) together with the coupling constants into Eq. (20), we can reexpress R_A^-:

$$\begin{aligned} R_A^- = {} & [y(y-2)(A(1+\epsilon_n)[s_v^A - c_v^A + (3u_v^p + 3d_v^p + 2s_v^A + 4c_v^A)\cos 2\theta_W + 2\epsilon_n(d_v^p - u_v^p)\sin^2\theta_W] \\ & + n_{\mathrm{src}}^A[4\epsilon_n\Delta^-\sin^2\theta_W + 6\Delta^+\cos 2\theta_W])]/[6(A(1+\epsilon_n)[y(y-2)(u_v^p + d_v^p) - (2-2y+y^2)\epsilon_n(d_v^p - u_v^p) \\ & - 2s_v^A + 2(1-y)^2 c_v^A] + 2n_{\mathrm{src}}^A[(-2+2y-y^2)\epsilon_n\Delta^- + y(y-2)\Delta^+])]. \end{aligned} \tag{26}$$

The neutron excess effects, i.e., the ϵ_n terms, have been taken into account in the NuTeV analysis [26], with the assumption that the target is composed of free nucleons. The "valence" strange and charm distributions are very tiny, if not negligible [27,48]. These sources of corrections would not be discussed in this work, since we are interested in finding out the influence of the "SRC-driven" nuclear effects, i.e., $\Delta^\pm$ related terms. We reduce R_A^- in Eq. (26) by considering that these terms are small, retain only the leading power corrections,

$$\begin{aligned} R_A^- = {} & \frac{1}{2} - \sin^2\theta_W - (s_v^A - c_v^A)\frac{n_{\mathrm{src}}^A}{A}\Delta^+ \frac{y(y-2)(y(y-2) + 2(3+y(y-2))\cos 2\theta_W)}{3(y(y-2)(u_v^p + d_v^p))^2} \\ & + \epsilon_n \frac{n_{\mathrm{src}}^A}{A}\Delta^- \frac{y(y-2) + 2(3+y(y-2))\cos 2\theta_W}{3y(y-2)\left(u_v^p + d_v^p + 2\frac{n_{\mathrm{src}}^A}{A}\Delta^+\right)} + \mathcal{O}(\text{other corrections}). \end{aligned} \tag{27}$$

The first term is the PW relation [50]. The second and third terms are corrections caused by $\Delta^\pm$, here "other corrections" means the higher corrections of $\Delta^\pm$ and corrections of ϵ_n, s_v^A, c_v^A which are not related to $\Delta^\pm$. We note reader that the u_v^p and d_v^p are PDFs of free proton.

In order to explore whether the $\Delta^\pm$ corrections could explain, or at least partially explain the NuTeV anomaly, we combine Eqs. (7) and (9) to obtain estimates of $n_{\mathrm{src}}^A\Delta^\pm$,

$$\begin{aligned} n_{\mathrm{src}}^A\Delta^+ &= Z[(R_{d_v}^A - 1)d_v^p + (R_{u_v}^A - 1)u_v^p], \\ n_{\mathrm{src}}^A\Delta^- &= Z[(R_{d_v}^A - 1)d_v^p - (R_{u_v}^A - 1)u_v^p]. \end{aligned} \tag{28}$$

The contribution of second term in Eq. (27) is almost zero since it is proportional to $(s_v^A - c_v^A)$ and the contribution of third term is related to neutron excess constant.

In the NuTeV measurements, 97% of the data is contained within $1\ \mathrm{GeV}^2 < Q^2 < 140\ \mathrm{GeV}^2$, $0.01 < x < 0.75$. The average $Q^2 = 25.6\ \mathrm{GeV}^2$ and $E_\nu = 120$ GeV for ν events as well as $Q^2 = 15.4\ \mathrm{GeV}^2$ and $E_{\bar\nu} = 112$ GeV for $\bar\nu$ events. In our estimation, we have adopted $Q^2 = 20\ \mathrm{GeV}^2$, $E = 116$ GeV. Figure 4 shows the shape of Δ^- correction term as a function of x. As one can see, the magnitude of this correction is strongly dependent on the neutron excess constant which is $0/12$, $4/56$ and $44/208$ for ^{12}C, ^{56}Fe, and ^{208}Pb, respectively. Another essential feature illustrated in this figure is that the functions change their signs at $x \approx 0.25$ where the transition from antishadowing region to the EMC region takes place. The Δ^- correction term of ^{56}Fe implies a resulting decrease in the NuTeV value of $\sin^2\theta_W$ and becomes comparable to the NuTeV deviation $(0.2227 - 0.2277 = -0.0050)$.

Alternatively, we can rewrite Eq. (27) to make it explicitly dependent on the SRC scale factor a_2^A of different nucleus A. The universality of SRC indicates that Δ^+, Δ^- are the same for all kinds of nuclei. Taking the EPPS21 parametrization result of quark NPDFs for carbon (^{12}C) as input, we can obtain $n_{\mathrm{src}}^{^{12}\mathrm{C}}\Delta^+$ and $n_{\mathrm{src}}^{^{12}\mathrm{C}}\Delta^-$ from Eq. (28). Therefore, the power corrections of PW relation can be written down in terms of a_2^A as:

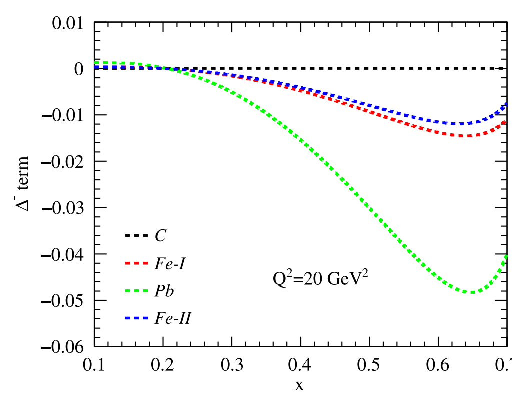

FIG. 4. The Δ^- correction term is evaluated at $Q^2 = 20\ \mathrm{GeV}^2$. The black, red, and green dashed lines correspond to ^{12}C, ^{56}Fe, and ^{208}Pb obtained from Eq. (27), respectively. The blue dashed line of ^{56}Fe labeled as *Fe–II* is plotted according to Eq. (29).

$$R_A^- = \frac{1}{2} - \sin^2\theta_W + \epsilon_n \frac{a_2^A}{a_2^{^{12}C}} \frac{n_{src}^{^{12}C}}{12} \Delta^- \times \frac{y(y-2) + 2(3 + y(y-2))\cos 2\theta_W}{3y(y-2)\left(u_v^p + d_v^p + 2\frac{a_2^A}{a_2^{^{12}C}}\frac{n_{src}^{^{12}C}}{12}\Delta^+\right)} + \mathcal{O}(\text{other corrections}). \tag{29}$$

We set the nucleus to be iron (^{56}Fe) and the reference values of the SRC scale factors $a_2^{^{56}Fe} = 4.80$, $a_2^{^{12}C} = 4.49$ are taken [10]. The resulting shape of power correction term obtained from Eq. (29) is plotted as a blue dotted line in Fig. 4. One can see that the difference between the two lines of iron, obtained from Eqs. (27) and (29), respectively, is relatively small. This consistency shows a nontrivial test of the theoretical assumption in our paper.

We then proceed to simply average the curve of ^{56}Fe over x from 0.05 to 0.7 to investigate the order of magnitude effects of Δ^- term on extracting $\sin^2\theta_W$. It is plotted in Fig. 5. The Δ^- term depends on the momentum transfer significantly, which is approximately as same as the NuTeV deviation at $Q^2 = 20$ GeV^2. This kind of simple average could overestimate the contributions from large x region, since much of the data in NuTeV came from $x \leq 0.2$ where our assumptions in Eqs. (21) and (22) would not be very suitable.

Since our results shown in Figs. 4 and 5 are mainly derived from parametrizations in EPPS21 and CT18ANLO as well as "SRC-driven" nuclear effects assumption, there is little model dependence in our conclusion that the Δ^- term has a effect of reducing the NuTeV result for $\sin^2\theta_W$. It is important to remember that NuTeV does not measure directly R_A^-, but rather measures ratios of experimental candidates within kinematic criteria and compares them to Monte Carlo simulations [27]. The average $Q^2 \approx 20$ GeV^2 was obtained from Monte Carlo simulation, in fact the actual kinematics of the selected events is poorly known.

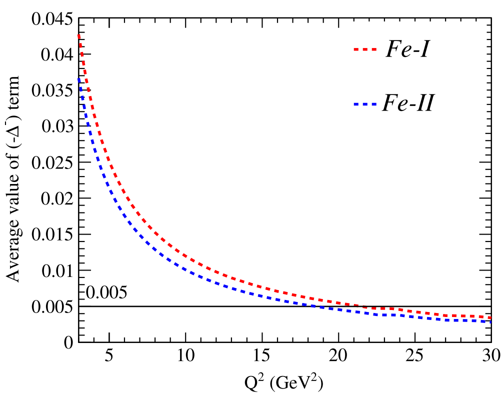

FIG. 5. The negative value of Δ^- term averaged on x is plotted. The red (Fe–I) and blue (Fe–II) dashed lines are obtained from Eqs. (27) and (29), respectively. The black solid line refers to the NuTeV deviation for convenience of comparison.

Therefore our results are not directly applicable to the NuTeV data, but only indicate some general features of "SRC-driven" nuclear effects on the extraction of weak-mixing angle.

Previous works indicate the CSV correction may explain roughly half of the NuTeV discrepancy with the SM and the correction from strange quark asymmetry has a significant uncertainty. Other studies of nuclear corrections to the R_A^- include Fermi motion and nuclear shadowing effects [58,59]. Each experiment requires a specific analysis according to the relevant experimental conditions. Principally, all of the corrections mentioned above should be formally incorporated into a reanalysis of the NuTeV data, with good control over various systematic uncertainties, before we could claim that there is no longer any significant discrepancy between the predictions of the SM and the NuTeV data.

The future high luminosity EIC would allow for a series of precision extractions of $\sin^2\theta_W$ [60]. Besides, the proposed LHC Forward Physics Facility (FPF) is estimated to about 3% precision on measuring $\sin^2\theta_W$ at $Q \approx 10$ GeV [61]. Other future experiments include DUNE, Moller, IsoDAR, MESA-P2, etc. [62–65], which would shed new light on the longstanding NuTeV anomaly.

IV. SUMMARY

In summary, we have studied neutral-current neutrino-nucleon DIS with particular interest in the relationship between SRC and the EMC effect. The ratios of structure functions $F_{2(NC)}^{\nu A}(x, Q^2)$ and $xF_{3(NC)}^{\nu A}(x, Q^2)$ are presented to illustrate that the EMC effect in different nuclei can be described by the abundance of SRC pairs and the proposed modification functions in this work are in fact universal.

In addition, we have derived a modified PW relation for nuclei motivated by the correlation between the EMC effect and the SRC scale factor. Taking advantage of this relation, we found the "SRC-driven" nuclear effects may account for a substantial fraction of the NuTeV anomaly. This conclusion may have fundamental consequences for our understanding of nucleon structure. Apart from the importance mentioned above, we think the idea of investigating correlation between EMC and SRC effects with modified PW relation R_A^- on the platform of NuTeV and other future neutrino-nucleon scattering experiments in itself is pretty stimulating.

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[1] J. J. Aubert *et al.* (European Muon Collaboration), Phys. Lett. **123B**, 275 (1983).
[2] M. Arneodo *et al.* (European Muon Collaboration), Phys. Lett. B **211**, 493 (1988).
[3] M. Arneodo *et al.* (European Muon Collaboration), Nucl. Phys. **B333**, 1 (1990).
[4] D. Allasia *et al.* (New Muon (NMC) Collaboration), Phys. Lett. B **249**, 366 (1990).
[5] J. Gomez, R. G. Arnold, P. E. Bosted, C. C. Chang, A. T. Katramatou, G. G. Petratos, A. A. Rahbar, S. E. Rock, A. F. Sill, Z. M. Szalata *et al.*, Phys. Rev. D **49**, 4348 (1994).
[6] J. Seely, A. Daniel, D. Gaskell, J. Arrington, N. Fomin, P. Solvignon, R. Asaturyan, F. Benmokhtar, W. Boeglin, B. Boillat *et al.*, Phys. Rev. Lett. **103**, 202301 (2009).
[7] L. B. Weinstein, E. Piasetzky, D. W. Higinbotham, J. Gomez, O. Hen, and R. Shneor, Phys. Rev. Lett. **106**, 052301 (2011).
[8] K. S. Egiyan *et al.* (CLAS Collaboration), Phys. Rev. Lett. **96**, 082501 (2006).
[9] O. Hen, M. Sargsian, L. B. Weinstein, E. Piasetzky, H. Hakobyan, D. W. Higinbotham, M. Braverman, W. K. Brooks, S. Gilad, K. P. Adhikari *et al.*, Science **346**, 614 (2014).
[10] CLAS Collaboration, Nature (London) **566**, 354 (2019).
[11] CLAS Collaboration, Nature (London) **560**, 617 (2018).
[12] J. W. Chen, W. Detmold, J. E. Lynn, and A. Schwenk, Phys. Rev. Lett. **119**, 262502 (2017).
[13] J. Xu and F. Yuan, Phys. Lett. B **801**, 135187 (2020).
[14] Y. Hatta, M. Strikman, J. Xu, and F. Yuan, Phys. Lett. B **803**, 135321 (2020).
[15] J. E. Lynn, D. Lonardoni, J. Carlson, J. W. Chen, W. Detmold, S. Gandolfi, and A. Schwenk, J. Phys. G **47**, 045109 (2020).
[16] F. Huang, J. Xu, and X. H. Yang, Phys. Rev. D **104**, 033002 (2021).
[17] S. M. Hu, Y. S. Luan, and J. Xu, Chin. Phys. C **47**, 093102 (2023).
[18] V. Bertone, R. Gauld, and J. Rojo, J. High Energy Phys. 01 (2019) 217.
[19] R. Wang, N. N. Ma, and T. F. Wang, Chin. Phys. C **47**, 044103 (2023).
[20] O. Hen, G. A. Miller, E. Piasetzky, and L. B. Weinstein, Rev. Mod. Phys. **89**, 045002 (2017).
[21] K. J. Eskola, P. Paakkinen, H. Paukkunen, and C. A. Salgado, Eur. Phys. J. C **82**, 413 (2022).
[22] T. J. Hou, J. Gao, T. J. Hobbs, K. Xie, S. Dulat, M. Guzzi, J. Huston, P. Nadolsky, J. Pumplin, C. Schmidt *et al.*, Phys. Rev. D **103**, 014013 (2021).
[23] K. S. Kumar, S. Mantry, W. J. Marciano, and P. A. Souder, Annu. Rev. Nucl. Part. Sci. **63**, 237 (2013).
[24] C. Marquet, S. Y. Wei, and B. W. Xiao, Phys. Lett. B **802**, 135253 (2020).
[25] S.-y. Wei, Phys. Lett. B **817**, 136356 (2021).
[26] G. P. Zeller *et al.* (NuTeV Collaboration), Phys. Rev. Lett. **88**, 091802 (2002); **90**, 239902(E) (2003).
[27] G. P. Zeller *et al.* (NuTeV Collaboration), Phys. Rev. D **65**, 111103 (2002); **67**, 119902(E) (2003).
[28] M. Tzanov *et al.* (NuTeV Collaboration), Phys. Rev. D **74**, 012008 (2006).
[29] I. Schienbein, J. Y. Yu, C. Keppel, J. G. Morfin, F. Olness, and J. F. Owens, Phys. Rev. D **77**, 054013 (2008).
[30] I. Schienbein, J. Y. Yu, K. Kovarik, C. Keppel, J. G. Morfin, F. Olness, and J. F. Owens, Phys. Rev. D **80**, 094004 (2009).
[31] M. Hirai, S. Kumano, and K. Saito, AIP Conf. Proc. **1189**, 269 (2009).
[32] K. Kovarik, I. Schienbein, F. I. Olness, J. Y. Yu, C. Keppel, J. G. Morfin, J. F. Owens, and T. Stavreva, Phys. Rev. Lett. **106**, 122301 (2011).
[33] H. Paukkunen and C. A. Salgado, J. High Energy Phys. 07 (2010) 032.
[34] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).
[35] H. Georgi, H. R. Quinn, and S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974).
[36] W. J. Marciano and J. L. Rosner, Phys. Rev. Lett. **65**, 2963 (1990); **68**, 898(E) (1992).
[37] S. Davidson, S. Forte, P. Gambino, N. Rius, and A. Strumia, J. High Energy Phys. 02 (2002) 037.
[38] A. Kurylov, M. J. Ramsey-Musolf, and S. Su, Nucl. Phys. **B667**, 321 (2003).
[39] J. T. Londergan and A. W. Thomas, Phys. Rev. D **67**, 111901 (2003).
[40] A. D. Martin, R. G. Roberts, W. J. Stirling, and R. S. Thorne, Eur. Phys. J. C **35**, 325 (2004).
[41] S. Kovalenko, I. Schmidt, and J. J. Yang, Phys. Lett. B **546**, 68 (2002).
[42] S. Kumano, Phys. Rev. D **66**, 111301 (2002).
[43] M. Gluck, P. Jimenez-Delgado, and E. Reya, Phys. Rev. Lett. **95**, 022002 (2005).
[44] S. A. Kulagin and R. Petti, Phys. Rev. D **76**, 094023 (2007).
[45] R. D. Ball *et al.* (NNPDF Collaboration), Nucl. Phys. **B823**, 195 (2009).
[46] I. C. Cloet, W. Bentz, and A. W. Thomas, Phys. Rev. Lett. **102**, 252301 (2009).
[47] W. Yang and X. Yang, Phys. Rev. D **106**, 093003 (2022).
[48] W. Bentz, I. C. Cloet, J. T. Londergan, and A. W. Thomas, Phys. Lett. B **693**, 462 (2010).
[49] J. T. Londergan and A. W. Thomas, Prog. Part. Nucl. Phys. **41**, 49 (1998).
[50] E. A. Paschos and L. Wolfenstein, Phys. Rev. D **7**, 91 (1973).
[51] C. G. Callan, Jr. and D. J. Gross, Phys. Rev. Lett. **22**, 156 (1969).
[52] L. L. Frankfurt, M. I. Strikman, D. B. Day, and M. Sargsian, Phys. Rev. C **48**, 2451 (1993).
[53] E. P. Segarra, A. Schmidt, T. Kutz, D. W. Higinbotham, E. Piasetzky, M. Strikman, L. B. Weinstein, and O. Hen, Phys. Rev. Lett. **124**, 092002 (2020).
[54] D. M. Alde, H. W. Baer, T. A. Carey, G. T. Garvey, A. Klein, C. Lee, M. J. Leitch, J. W. Lillberg, P. L. McGaughey, C. S. Mishra *et al.*, Phys. Rev. Lett. **64**, 2479 (1990).
[55] G. F. Bertsch, L. Frankfurt, and M. Strikman, Science **259**, 773 (1993).
[56] N. Fomin, J. Arrington, R. Asaturyan, F. Benmokhtar, W. Boeglin, P. Bosted, A. Bruell, M. H. S. Bukhari, M. E. Christy, E. Chudakov *et al.*, Phys. Rev. Lett. **108**, 092502 (2012).

[57] J. Arrington, A. Daniel, D. Day, N. Fomin, D. Gaskell, and P. Solvignon, Phys. Rev. C **86**, 065204 (2012).
[58] S. A. Kulagin, Phys. Rev. D **67**, 091301 (2003).
[59] S. J. Brodsky, I. Schmidt, and J. J. Yang, Phys. Rev. D **70**, 116003 (2004).
[60] R. Abdul Khalek, A. Accardi, J. Adam, D. Adamiak, W. Akers, M. Albaladejo, A. Al-bataineh, M. G. Alexeev, F. Ameli, P. Antonioli *et al.*, Nucl. Phys. **A1026**, 122447 (2022).
[61] R. Mammen Abraham, S. Foroughi-Abari, F. Kling, and Y. D. Tsai, arXiv:2301.10254.
[62] A. de Gouvea, P. A. N. Machado, Y. F. Perez-Gonzalez, and Z. Tabrizi, Phys. Rev. Lett. **125**, 051803 (2020).
[63] P. L. Anthony *et al.* (SLAC E158 Collaboration), Phys. Rev. Lett. **95**, 081601 (2005).
[64] J. Alonso, C. A. Argüelles, A. Bungau, J. M. Conrad, B. Dutta, Y. D. Kim, E. Marzec, D. Mishins, S. H. Seo, M. Shaevitz *et al.*, Phys. Rev. D **105**, 052009 (2022).
[65] N. Berger, K. Aulenbacher, S. Baunack, D. Becker, J. Diefenbach, M. Gericke, K. Gerz, R. Herbertz, K. Kumar, F. Maas *et al.*, J. Univ. Sci. Tech. China **46**, 481 (2016).