Holographic correlators of semiclassical states in defect CFTs

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We set up the computation of correlation functions for operators that are dual to semiclassical string states in strongly coupled defect conformal field theories (dCFTs). In the dCFT that is dual to the D3-D5 probe-brane system, we calculate the correlation function of two heavy operators perturbatively, in powers of the conformal ratio. We find that the leading term agrees with the prediction of the operator product expansion (OPE). In the case of two heavy Berenstein-Maldacena-Nastase operators, we find agreement in subleading orders as well.

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I. INTRODUCTION

Boundary and defect conformal field theories (CFTs) are currently attracting increased attention,[1] mainly because they bridge the gap between idealistic, highly symmetric models and real-world systems. These are generally characterized by finite sizes: impurities, domain walls, defects, and boundaries separate regions with different properties and break many of the underlying symmetries [1]. Naturally, boundaries and defects permeate all branches of physics, from high-energy and particle physics, to condensed matter, statistical, even gravity, and mathematical physics.

Deforming the gauge/string duality [2] by inserting probe branes on its string theory side [3,4] has provided us with more and more realistic holographic models (AdS/dCFT correspondence) which are in principle solvable at strong coupling by string theory. Probe branes break many symmetries and supersymmetries of holographic theories, yet there is a single property that we would still like to keep. This property is planar integrability [5,6]. Integrability has the power of bridging the two opposing ends of holographic dualities (which are generally disconnected due to the weak/strong coupling dilemma), endowing holography with a genuine nonperturbative capacity [7].

Integrability methods were introduced in the AdS/dCFT correspondence in 2015 [8], sparking a wide range of weak-coupling computations at tree level [9–12] and one-loop order [13,14]. Asymptotic all-loop results appeared in [15–19]. Classical string integrability was shown in [20,21]. While the majority of works so far concerns the D3-D5 probe-brane system, more integrable setups are currently known,[2] such as the D3-D7 [25–27] and the D2-D4 probe-brane system [28–30].

The D3-D5 system consists of a probe D5-brane embedded in the $\mathrm{AdS}_5 \times \mathrm{S}^5$ background which is generated by N D3-branes. The D5-brane wraps an $\mathrm{AdS}_4 \times \mathrm{S}^2$ geometry which is supported by k units of Abelian flux through S^2. The flux forces k of the D3-branes to terminate on one side of the D5-brane. On the dual gauge theory side, we encounter a 4-dimensional dCFT. Two copies of $\mathcal{N} = 4$ super Yang-Mills (SYM) theory with different gauge groups, $SU(N-k)$ and $SU(N)$ are separated by a codimension-1 defect [31]. Two-point functions in this theory have been studied in [32–34].

However, apart from the early supergravity calculations of one-point functions in [35,36], the systematic computation of correlators in strongly coupled dCFTs with strings is still missing.[3] The aim of the present paper is to fill this gap. First we address an important open problem in AdS/dCFT, that is the computation of two-point functions at strong coupling. Second, we provide a systematic framework for the calculation of correlators involving operators dual to semiclassical

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[1]See e.g. the review [1].

[2]See the review articles [22–24] for more.

[3]On the other hand, interesting results have been obtained with supersymmetric localization [37–40]. Note also the holographic computations [41–46] of correlation functions between open strings and finite-size branes such as giant gravitons.

string states in strongly coupled AdS/dCFT. We emphasize that even though our method is illustrated in a specific dCFT (the D3-D5 system), it can be implemented in any dCFT with a gravity dual.

In Sec. II we compute the three-point function of two heavy Berenstein-Maldacena-Nastase (BMN) operators and a light BPS scalar at strong coupling in $SO(3)\times SO(3)$ symmetric $\mathcal{N}=4$ SYM. In Sec. III we revisit the computation of the one-point function of a BPS operator in strongly coupled D3-D5 dCFT. In Sec. IV we set up the computation of correlators for operators that are described by semiclassical worldsheets in the presence of a defect brane. To illustrate our method, we compute the two-point function of two heavy operators in strongly coupled D3-D5 dCFT. In Sec. V we compare our findings for the two-point function to the prediction of the OPE. We report complete agreement between the leading term of our strong coupling results and the leading term of the OPE for an arbitrary choice of heavy operators. For the case of two BMN operators, agreement is shown up to next-to-next-to-leading (NNLO) order.

II. THREE-POINT FUNCTION

Three and higher-point correlators can be computed in strongly coupled AdS/CFT when one of the operators is dual to a supergravity mode, based on a method that was developed in [47] and applied to $\mathcal{N}=4$ SYM by [48,49]. Let $\mathcal{W}$ be a nonlocal operator of $\mathcal{N}=4$ SYM (e.g. a Wilson loop or a product of local operators) that is dual to a classical string worldsheet and $\mathcal{O}_I(y)$ a local operator of $\mathcal{N}=4$ SYM that is dual to the scalar supergravity field $\phi_I(y,w)$. Defining

$$\langle\mathcal{O}_I(y)\rangle_{\mathcal{W}} \equiv \frac{\langle\mathcal{W}\mathcal{O}_I(y)\rangle_{\mathcal{N}=4}}{\langle\mathcal{W}\rangle_{\mathcal{N}=4}}, \tag{1}$$

the correlator can be computed at strong coupling from

$$\langle\mathcal{O}_I(y)\rangle_{\mathcal{W}} = \lim_{w\to 0}\left[\frac{\pi}{w^{\Delta_I}}\sqrt{\frac{2}{\Delta_I-1}}\times\left\langle\phi_I(y,w)\cdot\frac{1}{Z_{\rm str}}\int D\mathbb{X}e^{-S_{\rm str}[\mathbb{X}]}\right\rangle_{\rm bulk}\right]. \tag{2}$$

$S_{\rm str}[\mathbb{X}]$ is the classical string action

$$S_{\rm str} = -\frac{T_2}{2}\int d^2\sigma\sqrt{-\gamma}\gamma^{ab}\partial_a\mathbb{X}^M\partial_b\mathbb{X}^N g_{MN}+\ldots \tag{3}$$

$T_2\equiv(2\pi\alpha')^{-1}$ is the string tension and $\mathbb{X}$ are the embedding coordinates of the string worldsheet in $\mathrm{AdS}_5\times\mathrm{S}^5$. Also $\lambda=\ell^4/\alpha'^2$ for the 't Hooft coupling $\lambda\equiv g_{\rm YM}^2N$.

The string action $S_{\rm str}$ depends indirectly on the bulk supergravity modes ϕ_I via a disturbance that is induced on the fields of type IIB supergravity by a local operator insertion. The relevant perturbations are

$$g_{MN} = \hat{g}_{MN}+\delta g_{MN} \tag{4}$$

$$C_{MNPQ} = \hat{C}_{MNPQ}+\delta C_{MNPQ}, \tag{5}$$

where g_{MN} is the graviton and C_{MNPQ} is the 4-form Ramond-Ramond (RR) potential of type IIB supergravity. The corresponding background solution consists of the $\mathrm{AdS}_5\times\mathrm{S}^5$ metric $\hat{g}_{MN}$ (A1) and the 4-form potential $\hat{C}_{MNPQ}$ (A3). Both perturbations in (4)–(5) can be expressed as linear combinations of the bulk modes ϕ_I and their derivatives:

$$\delta g_{MN} = V^I_{MN}\cdot\phi_I, \qquad \delta C_{MNPQ} = v^I_{MNPQ}\cdot\phi_I, \tag{6}$$

where V^I_{MN} and v^I_{MNPQ} are differential operators which depend on the target-space coordinates $\mathbb{X}$.

In the strong coupling regime ($\lambda\to\infty$), the path integral in (2) is dominated by a saddle point which corresponds to classical solutions $\mathbb{X}_{\rm cl}$, so that

$$\langle\mathcal{O}_I(y)\rangle_{\mathcal{W}} = -\frac{1}{4\ell^2}\sqrt{\frac{2\lambda}{\Delta_I-1}}\int d^2\sigma\partial_a\mathbb{X}^M\partial^a\mathbb{X}^N \times V^I_{MN}(\mathbb{X},\partial_x,\partial_z)\mathcal{G}_{\Delta_I}(x,z;y)+\ldots, \tag{7}$$

in the conformal gauge, $\gamma_{ab}=\mathrm{diag}(-,+)$. The boundary limit $\mathcal{G}_{\Delta_I}$ of the bulk-to-bulk propagator (A4) is (A6).

Now take $\mathcal{O}_I$ to be a chiral primary operator (CPO) of $\mathcal{N}=4$ SYM with length L.[4] Then the vertex operators that appear in (6) are given by (B3)–(B5). It follows that the string correlator (1), (7) simplifies significantly in the $y_i\to\infty$ limit [48]:

$$\langle\mathcal{O}^{\rm CPO}_I(y)\rangle_{\mathcal{W}} = -\frac{L\sqrt{2(L-1)\lambda}}{4\pi^2\mathcal{N}_L y^{2L}}\int d^2\sigma Y_I(x_\mu) \times\{-z^{L-2}\partial_a\mathbb{X}^i\partial^a\mathbb{X}^i+z^{L-2}\partial_a\mathbb{X}^z\partial^a\mathbb{X}^z + z^L\ell^{-2}\partial_a\mathbb{X}^\mu\partial^a\mathbb{X}^\nu\hat{g}_{\mu\nu}\},\quad i=0,\ldots,3, \tag{8}$$

where $\mathcal{N}_L$ is defined in (B6). Taking the operator $\mathcal{W}$ to be $\mathcal{W}\equiv\mathcal{O}_1^\dagger\mathcal{O}_2$, where $\mathcal{O}_\mathfrak{i}$ ($\mathfrak{i}=1,2$) is a BMN chiral primary of length $L_\mathfrak{i}$,

$$\mathcal{O}_\mathfrak{i} = \frac{1}{\sqrt{L_\mathfrak{i}}}\left(\frac{4\pi^2}{\lambda}\right)^{\frac{L_\mathfrak{i}}{2}}\mathrm{tr}[Z^{L_\mathfrak{i}}], \qquad Z\equiv\Phi_1+i\Phi_2, \tag{9}$$

and $L=L_1-L_2$ is small,[5] the classical string solution that is holographically dual to $\mathcal{W}$ is given by [50,51]

[4] See appendix B for the definition of CPOs.
[5] So that $\mathcal{O}_1\approx\mathcal{O}_2$ and $\mathcal{O}^{\rm CPO}_I$ is a light operator.

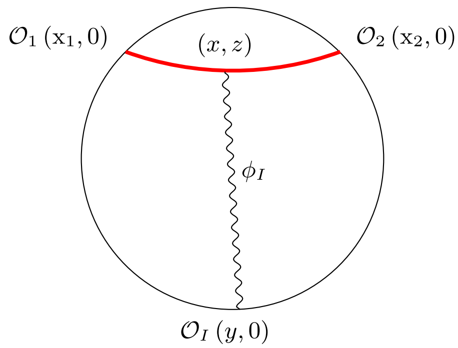

FIG. 1. Heavy-heavy-light (HHL) correlator.

$$x_3 = \bar{\mathrm{x}} + R\tanh\omega\tau, \qquad z = \frac{R}{\cosh\omega\tau} \tag{10}$$

$$\psi = 0, \qquad \varphi = i\omega\tau, \qquad \theta = \frac{\pi}{2}, \tag{11}$$

where $\omega = L_2/\sqrt{\lambda}$ and the 5-sphere parametrization can be found in (A2). The operators $\mathcal{O}_{1,2}$ are located at the points $\mathrm{x}_{1,2}$ on the x_3 axis and a small distance from each other. In other words, $R = \mathrm{x}_{12}/2$ is also small:

$$R = \frac{|\mathrm{x}_1 - \mathrm{x}_2|}{2} = \frac{\mathrm{x}_{12}}{2}, \qquad \bar{\mathrm{x}} = \frac{\mathrm{x}_1 + \mathrm{x}_2}{2}. \tag{12}$$

The three-point function is depicted in Fig. 1. The red line represents the string worldsheet (heavy state) and the curly line represents the CPO (light state).

Plugging (10)–(12) into (8) we obtain, for $y_i \to \infty$:

$$\langle \mathcal{O}_I^{\mathrm{CPO}}(y)\rangle_{\mathcal{W}}^{\mathrm{BMN}} = \frac{(-1)^{L/2}\ell^2}{2^{L+\frac{3}{2}}N}\cdot L_2\sqrt{L(L+1)(L+2)} \times B\left(\frac{L}{2}+1,\frac{1}{2}\right)\cdot\frac{\mathrm{x}_{12}^L}{y^{2L}}. \tag{13}$$

For reasons that will become apparent in Sec. V, we have used the $SO(3)\times SO(3)$ invariant spherical harmonics (given by $\mathfrak{C}_{L/2}$ in (31) below), for which $L = 2j$.

Inserting (13) and the (generic CFT) 2-point function,

$$\langle \mathcal{W}(\mathrm{x}_1,\mathrm{x}_2)\rangle_{\mathcal{N}=4} = \langle \mathcal{O}_1^\dagger(\mathrm{x}_1)\mathcal{O}_2(\mathrm{x}_2)\rangle_{\mathcal{N}=4} = \frac{\delta_{12}}{\mathrm{x}_{12}^{L_1+L_2}} \tag{14}$$

into the definition (1), we may compare the result with the generic form of three-point functions in CFTs and extract the HHL structure constant

$$C_{12}^I = \frac{(-1)^{L/2}L_2}{2^{L+\frac{3}{2}}N}\sqrt{L(L+1)(L+2)}\cdot B\left(\frac{L}{2}+1,\frac{1}{2}\right). \tag{15}$$

The structure constant (15) is protected from receiving quantum corrections [52]. Interesting further works on holographic three-point functions (e.g. calculations involving twist operators and conserved currents) can be found in [53–58].

III. ONE-POINT FUNCTION

We now revisit the computation of one-point functions in strongly-coupled dCFTs. We focus on the D3-D5 system. The action of the probe D5-brane is the sum of the Dirac-Born-Infeld (DBI) and the Wess-Zumino (WZ) term:

$$S_{\mathrm{D5}} = -\frac{T_5}{g_s}\int\left[d^6\zeta\sqrt{\det\left(G_{ab}+2\pi\alpha' F_{ab}\right)} + 2\pi\alpha' F\wedge C\right],$$
$$G_{ab} \equiv \partial_a\mathbb{Y}^M\partial_b\mathbb{Y}^N g_{MN}, \tag{16}$$

where $T_5 \equiv (2\pi)^{-5}\alpha'^{-3}$ is the D5-brane tension, $g_s = g_{\mathrm{YM}}^2/4\pi$ is the string coupling, G_{ab} is the pullback of the IIB graviton field g_{MN} (4) on the 5-brane, F_{ab} is the field strength of the world volume gauge field and C is the 4-form RR potential (5). For the embedding coordinates $\mathbb{Y}$ we set

$$h_{ab} \equiv \partial_a\mathbb{Y}^M\partial_b\mathbb{Y}^N\hat{g}_{MN} + 2\pi\alpha' F_{ab}, \qquad h \equiv \det h_{ab}. \tag{17}$$

As it turns out, the D5-brane wraps an $\mathrm{AdS}_4\times\mathrm{S}^2$ geometry that is parametrized by [4]

$$y_3 = \kappa\cdot w, \qquad \kappa \equiv \frac{\pi k}{\sqrt{\lambda}} \equiv \tan\alpha, \qquad \tilde{\psi} = 0, \tag{18}$$

where k are the units of magnetic flux through S^2:

$$\int_{\mathrm{S}^2}\frac{F}{2\pi} = k, \qquad F = \frac{k}{2}\cdot d\cos\tilde{\theta}\wedge d\tilde{\varphi}, \tag{19}$$

(y,w) are the AdS_5 coordinates in (A1) and the S^5 coordinates in (A2) carry a tilde. The world volume coordinates of the D5-brane are $(\zeta_0,\ldots,\zeta_5) = (y_0,y_1,y_2,w,\tilde{\theta},\tilde{\varphi})$.

One-point functions of dCFT operators that are dual to a supergravity mode ϕ_I can be computed in strongly coupled AdS/dCFT by the recipe (2):

$$\langle\mathcal{O}_I(x)\rangle_{\mathrm{D5}} = \lim_{z\to 0}\left[\frac{\pi}{z^{\Delta_I}}\sqrt{\frac{2}{\Delta_I-1}}\cdot\left\langle\phi_I(x,z) \times\frac{1}{Z_{\mathrm{D5}}}\int D\mathbb{Y}e^{-S_{\mathrm{D5}}[\mathbb{Y}]}\right\rangle_{\mathrm{bulk}}\right], \tag{20}$$

where Δ_I is the scaling dimension of ϕ_I and S_{D5} is given by (16). By means of (4)–(6), we obtain [35,59,60]:

$$\langle \mathcal{O}_I(x)\rangle_{\mathrm{D5}} = -\frac{\pi}{z^{\Delta_I}}\sqrt{\frac{2}{\Delta_I - 1}}\cdot\frac{T_5}{g_s}\int d^6\zeta(\delta\mathcal{L}_{\mathrm{DBI}} + \delta\mathcal{L}_{\mathrm{WZ}}) \cdot \mathcal{G}_{\Delta_I}(y, w; x). \tag{21}$$

Again, $\mathcal{G}_{\Delta_I}(y, w; x)$ is the boundary limit (A6) of the bulk-to-bulk propagator and

$$\delta\mathcal{L}_{\mathrm{DBI}} \equiv \sqrt{h}h^{ab}\partial_a\mathbb{Y}^M\partial_b\mathbb{Y}^N V^I_{MN}(\mathbb{Y}, \partial_y, \partial_w) \tag{22}$$

$$\delta\mathcal{L}_{\mathrm{WZ}} \equiv 2\pi\alpha'(F \wedge v^I(\mathbb{Y}, \partial_y, \partial_w)). \tag{23}$$

Take $\mathcal{O}_I$ to be a CPO of D3-D5 dCFT with length L, situated at the point $x_0 = x_1 = x_2 = 0$, $x_3 > 0$. Of all the CPOs of $\mathcal{N} = 4$ SYM, only those which share the $SO(3) \times SO(3)$ global symmetry of the defect are expected to have nontrivial one-point functions. The spherical harmonics depend on a single quantum number j which is related to the length of the associated CPO via $L = 2j$ (see e.g. [35]). The one-point function is depicted in Fig. 2. The blue line represents the world volume of the D5-brane and the curly line is the CPO.

Inserting the vertex operators (B3)–(B5) and the D5-brane parametrization (18)–(19) into the one-point function formula (21)–(23), we are led to

$$\langle \mathcal{O}^{\mathrm{CPO}}_I(x_3)\rangle_{\mathrm{D5}} = \frac{\mathcal{C}_I}{x_3^L}. \tag{24}$$

The one-point function structure constant $\mathcal{C}_I$ reads [35]:

$$\mathcal{C}_I = \frac{(-1)^{L/2}\sqrt{\lambda}}{\pi^{3/2}}\sqrt{\frac{L+2}{2L(L+1)}}\cdot\frac{\Gamma(L+\frac{1}{2})}{\Gamma(L)}\mathcal{I}_{L-2,L+\frac{1}{2}}, \tag{25}$$

for even $L = 2j$ and nonnegative integer $j = 0, 1, \ldots$ The analytic computation of the integral $\mathcal{I}_{a,b}(\kappa)$, for various values of a, b can be found in appendix C.

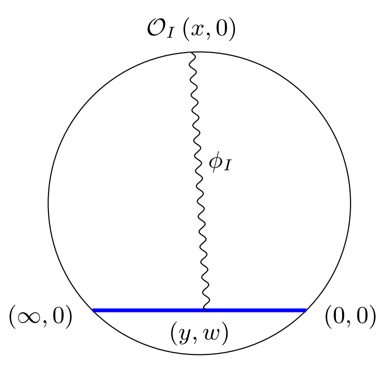

FIG. 2. One-point function of a CPO in defect CFT.

IV. TWO-POINT FUNCTION

Two and higher-point correlators can still be computed to leading order in strongly coupled AdS/dCFT by the recipe (2), (20). Let $\mathcal{W}$ be a nonlocal operator of $\mathcal{N} = 4$ SYM that is dual to a classical string worldsheet. Suppose that there is a probe D5-brane in the bulk of $\mathrm{AdS}_5 \times \mathrm{S}^5$ which interacts with the semiclassical string state via a scalar type IIB supergravity mode ϕ_I whose scaling dimension is Δ_I and its mass is m. The ratio of the correlator $\langle\mathcal{W}\rangle_{\mathrm{D5}}$ in D5-brane deformed $\mathcal{N} = 4$ SYM over its value $\langle\mathcal{W}\rangle_{\mathcal{N}=4}$ in pure $\mathcal{N} = 4$ SYM will be given at strong coupling by [47]:

$$\langle\tilde{\mathcal{W}}\rangle_{\mathrm{D5}} \equiv \frac{\langle\mathcal{W}\rangle_{\mathrm{D5}}}{\langle\mathcal{W}\rangle_{\mathcal{N}=4}} = \left\langle \frac{1}{Z_{\mathrm{str}}}\int D\mathbb{X}e^{-S_{\mathrm{str}}[\mathbb{X}]} \times \frac{1}{Z_{\mathrm{D5}}}\int D\mathbb{Y}e^{-S_{\mathrm{D5}}[\mathbb{Y}]}\right\rangle_{\mathrm{bulk}}. \tag{26}$$

This formula should include all possible virtual states that can be exchanged between the string and the brane (CPOs and non-protected heavy string states). Only the former are taken into account in (26). To determine the contribution of the latter, we should find the minimal surface $\mathcal{A}$ that terminates on the brane. However, such states are exponentially suppressed as $e^{-\sqrt{\lambda}\mathcal{A}}$ compared to the CPOs [61]. Moreover, their contribution to the two-point correlators that we are considering below will be extremely suppressed due to the large anomalous dimensions these operators acquire at strong coupling.

In the strong coupling regime ($\lambda \to \infty$) both path integrals in (26) will be dominated by their saddle points (corresponding to classical solutions $\mathbb{X}_{\mathrm{cl}}$, $\mathbb{Y}_{\mathrm{cl}}$).[6] The defect correlator of the operator $\mathcal{W}$ that is dual to an $\mathrm{AdS}_5 \times \mathrm{S}^5$ semiclassical string state becomes:

$$\langle\tilde{\mathcal{W}}\rangle_{\mathrm{D5}} = 1 + \frac{T_2 T_5}{2g_s}\int d^2\sigma d^6\zeta\{\delta\mathcal{L}_{\mathrm{str}}(\sigma, x, z) \times \delta\mathcal{L}_{\mathrm{D5}}(\zeta, y, w)G_{\Delta_I}(x, z; y, w)\}, \tag{27}$$

where G_{Δ_I} is the bulk-to-bulk propagator of a scalar field (mass m, scaling dimension Δ_I) in AdS_5 and

$$\delta\mathcal{L}_{\mathrm{str}} = \partial_a\mathbb{X}^M\partial^a\mathbb{X}^N V^I_{MN}(\mathbb{X}, \partial_x, \partial_z) + \ldots \tag{28}$$

[6]The saddle point of the worldsheet connects all the boundary points at which the operators are inserted. In principle there might exist other saddle points consisting of several disconnected worldsheets, some of which would directly join a boundary insertion to the defect. Each of these bulk-boundary contributions is essentially proportional to the one-point function and comes with an extra $1/N$ factor. Because the connected contribution (c.f. Fig. 3) scales as $1/N$ (as we will see below), it will dominate over any disconnected saddle point in the large-N limit.

$$\delta\mathcal{L}_{\mathrm{D5}} = \sqrt{h}h^{ab}\partial_a\mathbb{Y}^M\partial_b\mathbb{Y}^N V^I_{MN}(\mathbb{Y},\partial_y,\partial_w) + 2\pi\alpha'(F\wedge v^I(\mathbb{Y},\partial_y,\partial_w)). \tag{29}$$

Let the supergravity mode ϕ_I be dual to a CPO $\mathcal{O}_I^{\mathrm{CPO}}$ of the D3-D5 dCFT with length $L=2j$. For simplicity let us also assume that the string worldsheet lies very close to the AdS boundary, that is $z\to 0$. The near-boundary expansion of the bulk-to-bulk propagator is

$$G_L(x,z;y,w) = \frac{L-1}{2\pi^2}\cdot\left\{1+\frac{L\Lambda_w z^2}{(L-1)K_w^2}+\mathcal{O}(z^4)\right\}\cdot\left(\frac{zw}{K_w}\right)^L, \tag{30}$$

where $K_w\equiv w^2+(x-y)^2$ and $\Lambda_w\equiv 2w^2-(L-1)(x-y)^2$ (since $\nu=L-2$). We first compute the integrand by applying the vertex operators (B3)–(B5) on the propagator (30). We find

$$\int d^6\zeta\,\delta\mathcal{L}_{\mathrm{D5}}G_L = -\frac{16\pi^{1/2}\mathfrak{C}_{L/2}\ell^6 L(L-1)}{\mathcal{N}_L}\times\sum_{n=0}^{\infty}\mathfrak{F}_n\cdot\frac{z^{L+2n}}{x_3^{L+2n}},\qquad \mathfrak{C}_{L/2} = \left(-\frac{1}{2}\right)^{\frac{L}{2}}\sqrt{\frac{L+2}{2L+2}}, \tag{31}$$

where the first two coefficients read, for $L=2j$:

$$\mathfrak{F}_0 = \frac{\Gamma(2j+\frac{1}{2})}{\Gamma(2j+2)}\cdot\mathcal{I}_{2j-2,2j+\frac{1}{2}} = \frac{\sqrt{\pi}\kappa^{2j+1}}{2j(2j+1)}\cdot\left\{1+\frac{j(2j+1)}{2(2j-1)}\cdot\frac{1}{\kappa^2}+\ldots\right\} \tag{32}$$

$$\begin{aligned}\mathfrak{F}_1 &= \frac{-1}{2(2j-1)}\Bigg[\frac{\Gamma(2j+\frac{1}{2})}{\Gamma(2j+2)}\cdot\mathcal{I}_{2j-2,2j+\frac{1}{2}} + 2(2j+2)\cdot\frac{\Gamma(2j+\frac{3}{2})}{\Gamma(2j+3)}[2\kappa\cdot\mathcal{I}_{2j-1,2j+\frac{3}{2}} + (2j-1)\cdot\mathcal{I}_{2j-2,2j+\frac{3}{2}}] - 2(2j+2)(2j+3)\cdot\frac{\Gamma(2j+\frac{5}{2})}{\Gamma(2j+4)}\cdot\mathcal{I}_{2j,2j+\frac{5}{2}}\Bigg] \\ &= -\frac{\sqrt{\pi}\kappa^{2j+1}}{4(2j-1)}\cdot\left\{1+\frac{j(2j+1)}{2(2j-1)}\cdot\frac{1}{\kappa^2}+\ldots\right\}.\end{aligned} \tag{33}$$

We have also computed $\mathfrak{F}_2$ but it is far too lengthy to be included here. The integrals $\mathcal{I}_{a,b}$ are known as power series of $\kappa\to\infty$ (see appendix C). Note however that the ratios of the coefficients $\mathfrak{F}_n$ depend only on j:

$$\frac{\mathfrak{F}_1}{\mathfrak{F}_0} = -\frac{j(2j+1)}{2(2j-1)},\qquad \frac{\mathfrak{F}_2}{\mathfrak{F}_0} = \frac{(j+1)(2j+1)(2j+3)}{16(2j-1)}. \tag{34}$$

To obtain the value of the correlation function (27) we must also integrate over the string worldsheet coordinates. The integrand is again obtained by applying the vertex operators (B3) on the D5-brane integral (31):

$$\delta\mathcal{L}_{\mathrm{str}}(\sigma,x,z)\int d^6\zeta\,\delta\mathcal{L}_{\mathrm{D5}}(\zeta,y,w)G_L(x,z;y,w). \tag{35}$$

Putting together the two contributions, we obtain the general form of the defect correlator (26):

$$\begin{aligned}\langle\tilde{\mathcal{W}}\rangle_{\mathrm{D5}} = 1 &+ \frac{(-1)^L(L+2)\lambda}{16N\pi^{5/2}}\int_0^{2\pi}\int_{-\infty}^{+\infty}d\sigma d\tau\cdot\sum_{n=0}^{\infty}\mathfrak{F}_n\cdot\frac{z^{L+2n}}{x_3^{L+2n}}\{[(L^2+L+4n)(\partial_a\mathbb{X}^i\partial^a\mathbb{X}^i) \\ &- (L^2+(8n+1)L+8n^2)(\partial_a\mathbb{X}^z\partial^a\mathbb{X}^z)]z^{-2} - L(L+1)(\ell^{-2}\partial_a\mathbb{X}^\mu\partial^a\mathbb{X}^\nu\hat{g}_{\mu\nu}) \\ &+ 4(L+2n)(L+2n+1)(\partial_a\mathbb{X}^3\partial^a\mathbb{X}^z)x_3^{-1}z^{-1} - 2(L+2n)(L+2n+1)(\partial_a\mathbb{X}^3\partial^a\mathbb{X}^3)x_3^{-2}\}.\end{aligned} \tag{36}$$

For arbitrary heavy semiclassical operators, the leading term ($n=0$) in the correlator (36) factorizes into the product of the correlator (8) and the one-point function (24) as follows (for $y_i\to\infty$ and $x_3=\mathrm{x}_2=\mathrm{const}$):

$$\langle\tilde{\mathcal{W}}\rangle_{\mathrm{D5}} = 1+\langle\mathcal{O}_I^{\mathrm{CPO}}(\mathrm{x}_2)\rangle_{\mathrm{D5}}\langle\mathcal{O}_I^{\mathrm{CPO}}(y)\rangle_{\mathcal{W}}y^{2L}+\ldots \tag{37}$$

Non-protected operators that are exchanged between the heavy states and the D5-brane acquire very large dimensions at strong coupling and contribute only to subleading orders. For two heavy operators $\mathcal{O}_{1,2}$, (37) becomes:

$$\frac{\langle\mathcal{O}_1(\mathrm{x}_1)\mathcal{O}_2(\mathrm{x}_2)\rangle_{\mathrm{D5}}}{\langle\mathcal{O}_1(\mathrm{x}_1)\mathcal{O}_2(\mathrm{x}_2)\rangle_{\mathcal{N}=4}} = 1+2^L\mathcal{C}_I\mathrm{C}^I_{12}\xi^j+\mathcal{O}(\xi^{j+1}), \tag{38}$$

where $\mathcal{C}_I$ is the one-point function structure constant (25) and C^I_{12} is the HHL structure constant. Moreover $L=L_1-L_2=2j$ ($L_{1,2}$ are the lengths of the operators) and

$$\xi \equiv \frac{\mathrm{x}_{12}^2}{4\mathrm{x}_1\mathrm{x}_2} \equiv \frac{v^2}{1-v^2}, \qquad \mathrm{x}_{12} \equiv |\mathrm{x}_1 - \mathrm{x}_2|, \tag{39}$$

defines the conformal ratios. We will see in Sec. V below that the leading order behavior (38) is in complete agreement with the OPE. For two BMN chiral primary operators (9), agreement will also be shown for the subleading terms. Let us first compute their two-point function.

Take $\mathcal{W}$ to be the operator $\mathcal{W} \equiv \mathcal{O}_1^\dagger \mathcal{O}_2$, where $\mathcal{O}_\mathfrak{i}$ ($\mathfrak{i} = 1, 2$) are BMN chiral primaries (9) that are located at the points $\mathrm{x}_{1,2}$ on the x_3 axis and a small distance x_{12} from each other (see Fig. 3). As we have already mentioned in Sec. II, when $L = L_1 - L_2$ is small, $\mathcal{W}$ is holographically dual to the classical (pointlike) string solution (10)–(12) with $R = \mathrm{x}_{12}/2 \to 0$. In addition, the two heavy operators $\mathcal{O}_{1,2}$ are nearly equal ($\mathcal{O}_1 \approx \mathcal{O}_2$) and $\mathcal{O}_I^{\mathrm{CPO}}$ is a light operator. Using the identification (12) we may write the conformal ratios ξ and v as

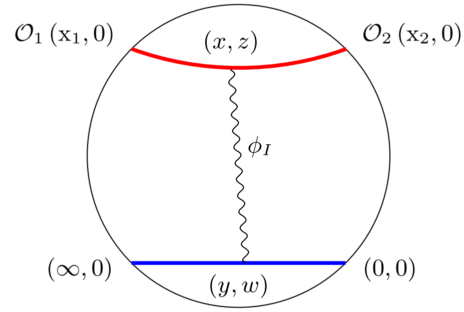

FIG. 3. Two-point function of BMN operators in D3-D5.

$$\xi \equiv \frac{\mathrm{x}_{12}^2}{4\mathrm{x}_1\mathrm{x}_2} = \frac{R^2}{\bar{\mathrm{x}}^2 - R^2} \Rightarrow \frac{R^2}{\bar{\mathrm{x}}^2} = \frac{\xi}{\xi+1} \equiv v^2. \tag{40}$$

Inserting the ansatz (10)–(12) into the formula (36) for the defect two-point function $\langle \tilde{\mathcal{W}} \rangle_{\mathrm{D5}}$ we arrive at

$$\frac{\langle \mathcal{O}_1^\dagger(\mathrm{x}_1)\mathcal{O}_2(\mathrm{x}_2)\rangle_{\mathrm{D5}}}{\langle \mathcal{O}_1^\dagger(\mathrm{x}_1)\mathcal{O}_2(\mathrm{x}_2)\rangle_{\mathcal{N}=4}} = 1 - \frac{(-1)^L(L+2)\omega\lambda}{8N\pi^{3/2}} \sum_{n=0}^{\infty} \mathfrak{F}_n \cdot \{2[L^2 + 2n + L - v^{-2}(L+2n)(L+2n+1)]\mathcal{J}_{\frac{L}{2}+n,L+2n+2} + 8n(L+n)[2v^{-1}\mathcal{J}_{\frac{L}{2}+n-1,L+2n+1} - (v^{-2}-1)\mathcal{J}_{\frac{L}{2}+n-1,L+2n+2}]\}, \tag{41}$$

where the integrals $\mathcal{J}_{a,b}$ are known as power series of $v \to 0$ (see appendix C). Plugging (C7)–(C9) into the two-point function (41) we find, for $L = 2j$:

$$\frac{\langle \mathcal{O}_1^\dagger(\mathrm{x}_1)\mathcal{O}_2(\mathrm{x}_2)\rangle_{\mathrm{D5}}}{\langle \mathcal{O}_1^\dagger(\mathrm{x}_1)\mathcal{O}_2(\mathrm{x}_2)\rangle_{\mathcal{N}=4}} = 1 + \frac{2j^2(j+1)L_2\sqrt{\lambda}}{N\pi^{3/2}} B(j, 1/2) \times \mathfrak{F}_0 \xi^j \left\{1 + \frac{2j}{(2j+1)}\frac{\mathfrak{F}_1}{\mathfrak{F}_0}\xi + \frac{4j(j+1)}{(2j+1)(2j+3)}\frac{\mathfrak{F}_2}{\mathfrak{F}_0}\xi^2 + \ldots\right\}. \tag{42}$$

Taking into account the ratios (34), our finding (checked up to NNLO) is in perfect agreement with our expectations from the operator product expansion (OPE) as we show right below. It is quite straightforward to obtain the two-point function to any subsequent perturbative order. Complete agreement with the OPE is expected.

V. OPERATOR PRODUCT EXPANSION

In the present section we show that the leading-order defect two-point function (38) of two arbitrary heavy operators and the NNLO defect two-point function of two BMN chiral primaries (42) agree with the OPE. The bulk channel OPE reads:

$$\mathcal{O}_1(\mathrm{x}_1)\mathcal{O}_2(\mathrm{x}_2) = \frac{\delta_{12}}{\mathrm{x}_{12}^{\Delta_1+\Delta_2}} + \sum_I \frac{\mathrm{C}_{12}^I}{\mathrm{x}_{12}^{\Delta_1+\Delta_2-\Delta_I}} \cdot C[\mathrm{x}_1 - \mathrm{x}_2, \partial_{\mathrm{x}_2}]\mathcal{O}_I(\mathrm{x}_2), \tag{43}$$

where C is a differential operator, $\Delta_{1,2}$, Δ_I are the dimensions of the operators $\mathcal{O}_{1,2}$, $\mathcal{O}_I$, and C_{12}^I their CFT three-point function. Inserting (43) into the general formula for the defect two-point function

$$\langle \mathcal{O}_1(\mathrm{z}_1, \mathbf{x}_1)\mathcal{O}_2(\mathrm{z}_2, \mathbf{x}_2)\rangle = \frac{f_{12}(\xi)}{|\mathrm{z}_1|^{\Delta_1}|\mathrm{z}_2|^{\Delta_2}} \tag{44}$$

and using the generic form of one-point functions (24),[7]

$$f_{12}(\xi) = (4\xi)^{-\frac{\Delta_1+\Delta_2}{2}}\left[\delta_{12} + \sum_I 2^{\Delta_I}\mathrm{C}_I\mathrm{C}_{12}^I \times F_{\mathrm{bulk}}(\Delta_I, \Delta_1 - \Delta_2, \xi)\right], \qquad \xi \equiv \frac{\mathrm{x}_{12}^2}{4\mathrm{z}_1\mathrm{z}_2}. \tag{45}$$

The bulk conformal blocks F_{bulk} have been determined in [62,63] from the expression $C[\mathrm{x}_1 - \mathrm{x}_2, \partial_{\mathrm{x}_2}]\mathrm{x}_2^{-\Delta_I}$:

[7]We have also set $\Delta_I = L$. The defect is located at $\mathrm{z} = 0$ and $\mathrm{x}_\mathfrak{i} = (\mathrm{z}_\mathfrak{i}, \mathbf{x}_\mathfrak{i})$, for $\mathfrak{i} = 1, 2$. The extra factor 2^{Δ_I} in (45) compensates for the missing $2x_3$ in the denominator of (24), cf. [62,63].

$$F_{\mathrm{bulk}}(\Delta_I,\delta\Delta,\xi)=\xi^{\frac{\Delta_I}{2}}{}_2F_1\left(\frac{\Delta_I+\delta\Delta}{2},\frac{\Delta_I-\delta\Delta}{2},\Delta_I-1;-\xi\right), \tag{46}$$

for $\delta\Delta\equiv\Delta_1-\Delta_2$. Dividing the (generic) dCFT two-point function (44) by the (generic) CFT two-point function (14) (for $\Delta_{1,2}=L_{1,2}$) we are led to

$$\frac{\langle\mathcal{O}_1(\mathrm{x}_1)\mathcal{O}_2(\mathrm{x}_2)\rangle_{\mathrm{D5}}}{\langle\mathcal{O}_1(\mathrm{x}_1)\mathcal{O}_2(\mathrm{x}_2)\rangle_{\mathcal{N}=4}}=\xi^{\frac{L_1+L_2}{2}}\cdot\frac{f_{12}(\xi)}{\delta_{12}}. \tag{47}$$

Plugging (45)–(46) into (47) and concentrating on the contribution of a single protected primary operator of dimension $\Delta_I=L=2j$, we get

$$\frac{\langle\mathcal{O}_1(\mathrm{x}_1)\mathcal{O}_2(\mathrm{x}_2)\rangle_{\mathrm{D5}}}{\langle\mathcal{O}_1(\mathrm{x}_1)\mathcal{O}_2(\mathrm{x}_2)\rangle_{\mathcal{N}=4}}=1+2^L\mathcal{C}_I\mathrm{C}_{12}^I\xi^j\times{}_2F_1(j,j,2j-1;-\xi), \tag{48}$$

so that by expanding the hypergeometric around $\xi=0$,

$$\frac{\langle\mathcal{O}_1(\mathrm{x}_1)\mathcal{O}_2(\mathrm{x}_2)\rangle_{\mathrm{D5}}}{\langle\mathcal{O}_1(\mathrm{x}_1)\mathcal{O}_2(\mathrm{x}_2)\rangle_{\mathcal{N}=4}}=1+2^L\mathcal{C}_I\mathrm{C}_{12}^I\xi^j\times\left\{1-\frac{j^2}{2j-1}\cdot\xi+\frac{j(j+1)^2}{4(2j-1)}\cdot\xi^2+\dots\right\}, \tag{49}$$

where $\mathcal{C}_I$ is the one-point function structure constant (25) and C_{12}^I is the generic three-point function structure constant. Comparing (49) with the strong coupling expansion (38) for the leading-order defect correlator of two arbitrary heavy operators and (42) for the NNLO defect correlator of two BMN chiral primaries [so that $\mathrm{C}_{12}^I=\mathcal{C}_{12}^I$ is the structure constant (15)], we find complete agreement. In the case of two arbitrary heavy states, it would be interesting to verify the agreement of the subleading terms in (38). To this end, an integral representation of the bulk-to-bulk propagator or even the Mellin transform of the amplitude could be useful [64,65].

The agreement of the leading-order correlator (38) of two arbitrary heavy operators and the OPE (49) implies that the value of the defect two-point function at strong coupling (38) will agree with its value at weak coupling whenever the heavy state is dual to a protected operator [e.g. for the correlator (42)]. This is guaranteed by the fact that the three-point function structure constant C_{12}^I is protected and, in the large κ limit [see (18)], the one-point function structure constant $\mathcal{C}_I$ agrees between weak and strong coupling. Obviously, agreement is no longer expected to hold for non-protected operators.

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APPENDIX A: CONVENTIONS

The equations of motion of type IIB supergravity afford a solution [66] which consists of the $\mathrm{AdS}_5\times\mathrm{S}^5$ metric,

$$ds^2=\frac{\ell^2}{z^2}(dx_0^2+dx_1^2+dx_2^2+dx_3^2+dz^2)+\ell^2d\Omega_5^2, \tag{A1}$$

written out here in the Poincaré coordinate system and Euclidean time. The line element of the unit 5-sphere $d\Omega_5$ takes the following $SO(3)\times SO(3)$ symmetric form:

$$d\Omega_5^2=d\psi^2+\cos^2\psi(d\theta^2+\sin^2\theta d\varphi^2)+\sin^2\psi(d\vartheta^2+\sin^2\vartheta d\chi^2), \tag{A2}$$

where $\psi\in[0,\pi/2]$, $\theta,\vartheta\in[0,\pi]$, $\varphi,\chi\in[0,2\pi)$. The solution (A1) is supported by a 4-form RR potential $\hat{C}$. The corresponding field strength $\hat{F}=d\hat{C}$ reads:

$$\hat{F}_{mnpqr}=\varepsilon_{mnpqr},\qquad \hat{F}_{\mu\nu\rho\sigma\tau}=\varepsilon_{\mu\nu\rho\sigma\tau}. \tag{A3}$$

The bulk-to-bulk propagator of a massive scalar field (mass m, scaling dimension Δ) in AdS_5 is given by

$$G_\Delta(x,z;y,w)=\frac{\Gamma(\Delta)\eta^\Delta}{2^{\Delta+1}\pi^2\Gamma(\Delta-1)}\times{}_2F_1\left(\frac{\Delta}{2},\frac{\Delta+1}{2},\nu+1,\eta^2\right), \tag{A4}$$

where we have defined,

$$\eta\equiv\frac{2zw}{z^2+w^2+(x-y)^2},\qquad \nu\equiv\sqrt{4+m^2\ell^2}. \tag{A5}$$

The asymptotic value of the propagator (A4) near the AdS boundary ($w=0$) becomes, for $K_z\equiv z^2+(x-y)^2$:

$$\mathcal{G}_\Delta(x,z;y)\equiv\lim_{w\to0}\frac{G_\Delta(x,z;y,w)}{w^\Delta}=\frac{\Delta-1}{2\pi^2}\cdot\frac{z^\Delta}{K_z^\Delta}. \tag{A6}$$

APPENDIX B: CHIRAL PRIMARY OPERATORS

The CPOs of $\mathcal{N}=4$ SYM are given by symmetrized single-trace products of the theory's six scalar fields:

$$\mathcal{O}_I^{\mathrm{CPO}}(x) = \frac{1}{\sqrt{L}}\left(\frac{8\pi^2}{\lambda}\right)^{\frac{L}{2}}\Psi_I^{\mu_1\ldots\mu_L}\mathrm{tr}[\Phi_{\mu_1}\ldots\Phi_{\mu_L}], \quad (\mathrm{B1})$$

where $\Psi_I^{\mu_1\ldots\mu_L}$ are traceless symmetric tensors of $SO(6)$. The tensors $\Psi_I^{\mu_1\ldots\mu_L}$ define the S^5 spherical harmonics:

$$Y_I(x_\mu) \equiv \Psi_I^{\mu_1\ldots\mu_L}x_{\mu_1}\ldots x_{\mu_L},$$
$$\Psi_I^{\mu_1\ldots\mu_L}\Psi_J^{\mu_1\ldots\mu_L} = \delta_{IJ}, \qquad \sum_{\mu=4}^{9}x_\mu^2 = 1, \quad (\mathrm{B2})$$

where I, J are the corresponding quantum numbers. The overall factor in front of the CPOs (B1) ensures that their 2-point functions are normalized to unity [52].

The scalar supergravity modes $s_I(x_m)$ that are dual to the CPOs (B1) have been identified [52,66]. They are linear combinations of the scalar modes of the metric and the RR potential with $m^2\ell^2 = L(L-4)$ and $\nu = L-2$. The perturbation (4)–(5) can be expressed in terms of the modes $s(x_m,x_\mu) \equiv s_I(x_m)Y_I(x_\mu)$ so that the vertex operators that show up in (6) are given by:

$$V_{mn}^I = \frac{2}{\mathcal{N}_L}\frac{1}{L+1}Y_I[2\ell^2\nabla_m\nabla_n - L(L-1)\hat{g}_{mn}] \quad (\mathrm{B3})$$

$$V_{\mu\nu}^I = \frac{2L}{\mathcal{N}_L}Y_I\hat{g}_{\mu\nu}, \quad v_{mnpq}^I = \frac{\ell}{\mathcal{N}_L}\sqrt{\hat{g}_{\mathrm{AdS}}}\epsilon_{mnpqr}\nabla^r Y_I \quad (\mathrm{B4})$$

$$v_{\mu\nu\rho\sigma}^I = -\frac{\ell}{\mathcal{N}_L}\sqrt{\hat{g}_{\mathrm{s}}}\epsilon_{\mu\nu\rho\sigma\tau}Y_I\nabla^\tau, \quad (\mathrm{B5})$$

where the Latin indices (m, n, p, q, r) refer to the AdS_5 and the Greek indices (μ, ν, ρ, σ, τ) to the S^5 coordinates. The normalization factor $\mathcal{N}_L$ is defined as

$$\mathcal{N}_L^2 = \frac{N^2L(L-1)}{2^{L-3}\pi^2(L+1)^2}. \quad (\mathrm{B6})$$

APPENDIX C: INTEGRALS

The integrals $\mathcal{I}_{a,b}$ are defined as follows:

$$\mathcal{I}_{a,b}(\kappa) \equiv \int_0^\infty du\frac{u^a}{[u^2+(1-\kappa u)^2]^b}, \qquad b > \frac{1}{2}. \quad (\mathrm{C1})$$

For $j > 1/2$ and $\kappa \to \infty$, we find:

$$\mathcal{I}_{2j-2,2j+\frac{1}{2}} = \kappa^{2j+1}B\left(2j,\frac{1}{2}\right) \times\left\{1+\left[\frac{3}{2}+\frac{(2j-3)(j-1)}{2(2j-1)}\right]\frac{1}{\kappa^2}+\ldots\right\} \quad (\mathrm{C2})$$

$$\mathcal{I}_{2j-1,2j+\frac{3}{2}} = \kappa^{2j+2}B\left(2j+1,\frac{1}{2}\right) \times\left\{1+\left[\frac{3}{2}+\frac{(2j-1)(j-1)}{4j}\right]\frac{1}{\kappa^2}+\ldots\right\} \quad (\mathrm{C3})$$

$$\mathcal{I}_{2j-2,2j+\frac{3}{2}} = \kappa^{2j+3}B\left(2j+1,\frac{1}{2}\right) \times\left\{1+\left[\frac{5}{2}+\frac{(2j-3)(j-1)}{4j}\right]\frac{1}{\kappa^2}+\ldots\right\} \quad (\mathrm{C4})$$

$$\mathcal{I}_{2j,2j+\frac{5}{2}} = \kappa^{2j+3}B\left(2j+2,\frac{1}{2}\right) \times\left\{1+\left[\frac{3}{2}+\frac{j(2j-1)}{2(2j+1)}\right]\frac{1}{\kappa^2}+\ldots\right\}. \quad (\mathrm{C5})$$

The integrals $\mathcal{J}_{a,b}$ are defined as:

$$\mathcal{J}_{a,b} \equiv \int_{-\infty}^{+\infty}\frac{v^b\mathrm{sech}^{2a+2}s\cdot ds}{(1+v\tanh s)^b}, \quad (\mathrm{C6})$$

so that for $j,n = 0,1,2,\ldots$ and $v\to 0$ they are given by:

$$\mathcal{J}_{j+n,2j+2n+2} = \frac{\Gamma(\frac{1}{2})}{\Gamma(j+n+\frac{3}{2})} \times\sum_{m=j+n+1}^{\infty}\frac{\Gamma(m)}{\Gamma(m-j-n)}\cdot v^{2m}, \times j+n = 0,1,\ldots \quad (\mathrm{C7})$$

$$\mathcal{J}_{j+n-1,2j+2n+1} = \frac{\Gamma(\frac{1}{2})}{(j+n)\Gamma(j+n+\frac{1}{2})} \times\sum_{m=j+n+1}^{\infty}\frac{\Gamma(m)}{\Gamma(m-j-n)}\cdot v^{2m-1}, \times j+n = 1,2,\ldots \quad (\mathrm{C8})$$

$$\mathcal{J}_{j+n-1,2j+2n+2} = \frac{\Gamma(\frac{3}{2})}{(j+n)\Gamma(j+n+\frac{3}{2})}\sum_{m=j+n+1}^{\infty} \times(2m-1)\frac{\Gamma(m)}{\Gamma(m-j-n)}\cdot v^{2m}, \times j+n = 1,2,\ldots \quad (\mathrm{C9})$$

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