Supersymmetry with Lorentz symmetry violation

I. T. Drummond[*]
Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, United Kingdom

(Received 28 June 2023; accepted 21 July 2023; published 8 August 2023)

We study two (massless free field) models, a photon/photino model with a vector gauge field and a Majorana spinor field, and a Wess-Zumino model. They each exhibit Lorentz symmetry violation but retain, in an appropriate way, the supersymmetry correspondence between the particles of the two fields. In relation to the photon field the Lorentz symmetry violation is of a simple but nontrivial kind that implies birefringence. In relation to the spinor field the Lorentz violation is produced by a modification of the Majorana equation that is a simplified version of more general investigations of Lorentz symmetry violation of the Dirac equation. In the case of the Wess-Zumino model we retain the same violation of Lorentz symmetry for the Majorana field and adjust the propagation of the scalar particles so that they exhibit a corresponding birefringence. The advantages of the models are that they are straightforward to investigate completely and both retain the basic aspect of supersymmetry namely the one-to-one correspondance between bosons and fermions. As a result of this bottom-up approach it is then possible to construct conserved supersymmetry charges and investigate their algebraic properties. To some extent these are similar to those encountered in the case of Lorentz invariance. However, there are differences and in particular nonlocal terms appear in the commutation relations of the supersymmetry charges and fields of the models. We examine carefully the rather intricate nature of the limit back to Lorentz invariance.

DOI: 10.1103/PhysRevD.108.045005

I. INTRODUCTION

Generally, and for good reasons, supersymmetry is viewed as fitting into the standard scheme of Lorentz-invariant field theory. However there are also good reasons for investigating the possibility that Lorentz symmetry might be violated and in particular at high energy [1–4]. One of the attractions of supersymmetry is that it can modify the high-energy behavior of field theories rendering them strongly renormalizable or even finite. Hence, the possibility of retaining supersymmetry in a context in which Lorentz symmetry is violated is to be taken seriously. There have been a number of proposals formulating models that retain supersymmetry while admitting Lorentz symmetry violation (LSV) [5–9]. A way of understanding some of the of issues involved is to adopt a distinction between intrinsic and extrinsic Lorentz symmetry violation. An example is LSV in deep inelastic scattering of electrons on hadrons [10–13]. If we assume that the electron beam is as it is usually understood to be then LSV effects might be due either to a modification of the Dirac equation describing the quarks making up the hadrons (intrinsic LSV), or to a distortion of the spacetime relationship between the quarks, perfectly standard in their own frame that differs however from that of the elctron beam (extrinsic LSV) (see the discussions in Refs. [7,13]). Of course both types of LSV together with additional spin modification of the electron beam may be present.

We regard the case of intrinsic LSV as of particular interest and propose two models to illustrate this. The first comprises a vector field, the "photon," and a massless Majorana spinor field, the "photino." The Lorentz symmetry breaking is unequivocal, both particles exhibit birefringence in the form of a double light cone. The parameters in the model can be adjusted so that the dispersion relations for the photon and the photino conform appropriately with one another. This makes it possible to make a pairing, for each light cone, between a photon state of given 4-momentum and a photino state with the same 4-momentum. We are then able to construct conserved supersymmetry charges that convert one type of particle into the other, thus justifying the photon/photino nomenclature. However although the supersymmetry charges are obtained from locally conserved currents they are more limited in scope than the standard supersymmetry charges of the fully Lorentz invariant model. For example, they cannot connect states on distinct light cones.

[*]itd@damtp.cam.ac.uk

The second model is a simplified Wess-Zumino model [14] in which we retain the massless Majorana particle and introduce scalar and pseudoscalar particles that travel on lightcones that match the birefringent light cones of the Majorana particle. In a manner similar to that of the photon/photino model we find conserved supersymmetry charges that are again of restricted scope. In both models this feature complicates the approach to Lorentz symmetry where the full supersymmetry should be recovered. We examine this limiting procedure in some detail.

II. PHOTON LAGRANGIAN

On setting up the photon/photino model we introduce, on the basis of discussions of Lorentz symmetry breaking in Refs. [4,15–17] for the vector field of the photon, $A_\mu(x)$, the Lagrangian

$$\mathcal{L}_P(x) = -\frac{1}{4}F^{\mu\nu}(x)F_{\mu\nu}(x) + \frac{1}{8}C^{\mu\nu\lambda\tau}F_{\mu\nu}(x)F_{\lambda\tau}(x). \quad (1)$$

Here, $F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$, and $C^{\mu\nu\lambda\tau}$ is a tensor with the algebraic symmetries of a Weyl tensor, namely

$$C^{\mu\nu\lambda\tau} = C^{\lambda\tau\mu\nu} = -C^{\nu\mu\lambda\tau}, \quad (2)$$

together with a zero-trace condition

$$\eta_{\mu\lambda}C^{\mu\nu\lambda\tau} = 0, \quad (3)$$

and a Bianchi identity

$$C^{\mu\nu\lambda\tau} + C^{\mu\lambda\tau\nu} + C^{\mu\tau\mu\lambda} = 0. \quad (4)$$

Here $\eta_{\mu\lambda}$ is the diagonal metric tensor with entries $(1, -1, -1, -1)$. This framework covers all possible types of (intrinsic) Lorentz symmetry violation for the photon field. From the Lagrangian $\mathcal{L}_P(x)$ we obtain the equation of motion

$$\partial_\mu G^{\mu\nu}(x) = 0, \quad (5)$$

where

$$G^{\mu\nu} = -\frac{\partial\mathcal{L}_P}{\partial(\partial_\mu A_\nu(x))} = F^{\mu\nu}(x) - C^{\mu\nu\lambda\tau}\partial_\lambda A_\tau(x). \quad (6)$$

Petrov [18] identified a classification of the possible versions of $C^{\mu\nu\lambda\tau}$. The Petrov classification is based on the principal null vectors of the Weyl tensor [19]. In the general case there are four for a given Weyl tensor. However they may coincide in various ways and this is the basis of the Petrov scheme. In our model we select the simplest case, class N, in which they all coincide. If we denote this principal null vector by l_μ we can include it in a Penrose null tetrad [19] comprising vectors $l_\mu, n_\mu, m_\mu, \bar{m}_\mu$, that satisfy

$$l^2 = n^2 = m^2 = \bar{m}^2 = l.m = l.\bar{m} = n.m = n.\bar{m} = 0, \quad (7)$$

together with

$$l.n = -m.\bar{m} = 1. \quad (8)$$

Explicitly, we choose $l^\mu = (1/\sqrt{2}, 0, 0, 1/\sqrt{2})$, $n^\mu = (1/\sqrt{2}, 0, 0, -1/\sqrt{2})$, $m^\mu = (0, 1/\sqrt{2}, i/\sqrt{2}, 0)$, $\bar{m} = (0, 1/\sqrt{2}, -i/\sqrt{2}, 0)$. We can complete the construction of the photon Lagrangian by setting (see [19])

$$C^{\mu\nu\lambda\tau} = \kappa(A^{\mu\nu}A^{\lambda\tau} + \bar{A}^{\mu\nu}\bar{A}^{\lambda\tau}), \quad (9)$$

where

$$A^{\mu\nu} = l^\mu m^\nu - l^\nu m^\mu \quad (10)$$

and

$$\bar{A}^{\mu\nu} = l^\mu \bar{m}^\nu - l^\nu \bar{m}^\mu. \quad (11)$$

An important property of the bivector $A^{\mu\nu}$ is self-duality, that is

$$\frac{i}{2}\epsilon^{\mu\nu\lambda\tau}A_{\lambda\tau} = A^{\mu\nu}, \quad (12)$$

while the bivector $\bar{A}^{\mu\nu}$ is antiself-dual, that is

$$\frac{i}{2}\epsilon^{\mu\nu\lambda\tau}\bar{A}_{\lambda\tau} = -\bar{A}^{\mu\nu}. \quad (13)$$

It is useful to note that

$$l_\mu A^{\mu\nu} = l_\mu \bar{A}^{\mu\nu} = 0, \quad (14)$$

and

$$A^{\mu\nu}\bar{A}_{\mu\lambda} = -l^\nu l_\lambda. \quad (15)$$

A. Photon dynamics

Because of the presence of the Lorentz symmetry violating term in Eq. (1) the model has unconventional features, we present the quantization of the photon field in some detail. Following Refs. [16,20,21] we adopt the well known Gupta-Bleuler method adapted to our new circumstances. The first step is to replace the Lagrangian in Eq. (1) with

$$\mathcal{L}_{\mathrm{GB}}(x) = -\frac{1}{2}\partial_\mu A_\nu \partial^\mu A^\nu + \frac{1}{2}C^{\mu\nu\lambda\tau}\partial_\mu A_\nu \partial_\lambda A_\tau. \quad (16)$$

This is equivalent to the original version (up to total derivative terms) for fields obeying the gauge constraint $\partial.A(x) = 0$. The equation of motion is

$$\partial_\mu \left(\frac{\partial \mathcal{L}_{\mathrm{GB}}}{\partial(\partial_\mu A_\nu)} \right) = 0. \tag{17}$$

This becomes

$$-\partial^2 A^\nu + \partial_\mu C^{\mu\nu\lambda\tau} \partial_\lambda A_\tau = 0. \tag{18}$$

The Hamiltonian formulation of the photon dynamics requires the construction of the field $\Pi^\mu(x)$ canonically conjugate to $A_\mu(x)$. This is given by

$$\Pi^\nu(x) = \frac{\partial \mathcal{L}_P}{\partial(\partial_0 A_\nu(x))} = -K^{\nu\tau} \dot{A}_\tau(x) + \sum_{k=1}^{3} C^{0\nu k\tau} \partial_k A_\tau(x), \tag{19}$$

where

$$K^{\nu\tau} = \eta^{\nu\tau} - C^{0\nu 0\tau}. \tag{20}$$

B. Plane-wave solutions

Plane-wave solutions have the form

$$A^\nu(x) = \varepsilon^\nu e^{-ip.x}. \tag{21}$$

From Eq. (18) we have

$$p^2 \varepsilon^\nu - p_\mu p_\lambda C^{\mu\nu\lambda\tau} \varepsilon_\tau = 0. \tag{22}$$

The gauge condition is

$$p.\varepsilon = 0. \tag{23}$$

The wave equation becomes

$$p^2 \varepsilon^\nu - \kappa(p_\mu A^{\mu\nu} p_\lambda A^{\lambda\tau} + p_\mu \bar{A}^{\mu\nu} p_\lambda \bar{A}^{\lambda\tau})\varepsilon_\tau = 0. \tag{24}$$

There are four positive energy solutions. Two are "unphysical," comprising a gauge solution, $\varepsilon_G^\nu = p^\nu$ and a complementary solution $\varepsilon_C^\nu = l^\nu$. Both these solutions require $p^2 = 0$ which implies that the mass-shell cone and the light cone on which they propagate are the standard cones associated with the metric $\eta^{\mu\nu}$. There are also two "physical" solutions for which $p^2 \neq 0$. It is immediately obvious from Eq. (24) any such solution has the form

$$\varepsilon^\nu = \alpha p_\mu A^{\mu\nu} + \bar{\alpha} p_\mu \bar{A}^{\mu\nu}, \tag{25}$$

for some values of α and $\bar{\alpha}$. On substituting this form into Eq. (24) we obtain

$$p^2 \alpha + \kappa(l.p)^2 \bar{\alpha} = 0, \tag{26}$$

$$p^2 \bar{\alpha} + \kappa(l.p)^2 \alpha = 0. \tag{27}$$

To obtain a nontrivial solution we require

$$(p^2)^2 - \kappa^2 (l.p)^4 = 0. \tag{28}$$

That is

$$p^2 \pm \kappa(l.p)^2 = 0. \tag{29}$$

For simplicity of presentation we assume that $\kappa > 0$. It is then obvious that the two mass-shell cones are nested, the "−" cone lying in the interior of the "+" cone, except where they touch along a common generator parallel to l^μ. Indeed they share this generator with the standard cone $p^2 = 0$ (appropriate to the unphysical solutions) which is nested between the physical "±" cones. However it should be noted that when $\kappa = 2$ the "−" cone acquires a generator parallel to the 0-axis and the "+" cone acquires a generator along the (negative) 3-axis. We impose the constraint $\kappa < 2$. We comment on the significance of this constraint later. When $\kappa < 0$ the "+" and "−" cones interchange roles in the nesting structure. For this reason we impose also the corresponding constraint $\kappa > -2$. Ultimately then we have (see also Ref. [22])

$$-2 < \kappa < 2. \tag{30}$$

Subject to this restriction we can identify four positive energy solutions. The negative energy solutions are obtained by complex conjugation. From Eq. (29) we find the allowed momenta $p_\pm^\mu = (E_\pm, \mathbf{p}) = (E_\pm, p_1, p_2, p_3)$, where

$$E_\pm = \frac{1}{1 \pm \kappa/2} \left(\pm(\kappa/2) p_3 + \sqrt{(1 \pm \kappa/2)(p_1^2 + p_2^2) + p_3^2} \right). \tag{31}$$

Taking account of the relevant mass-shell conditions, the solutions to Eq. (27) are $\alpha_\pm = \pm\bar{\alpha}_\pm$. The physical solutions, conveniently normalized, are then

$$\varepsilon_+^\mu(p_+) = (l.p_+) e_1^\mu - (e_1.p_+) l^\mu, \tag{32}$$

and

$$\varepsilon_-^\mu(p_-) = (l.p_-) e_2^\mu - (e_2.p_-) l^\mu, \tag{33}$$

where $e_1^\mu = (0, 1, 0, 0)$ and $e_2^\mu = (0, 0, 1, 0)$.

C. Light cone structure

It is useful to examine the light cone behavior of the photons since this governs the causal structure of the model. Consider the "+" mode. Define a transform of the momentum $p_+ \to \hat{p}_+$ where

$$\hat{p}^\mu_+ = p^\mu_+ + \frac{\kappa}{2} l^\mu (l.p_+). \quad (34)$$

This mapping is a null shear in momentum space. It is immediately obvious from Eq. (29) that

$$\hat{p}^2_+ = p^2_+ + \kappa(l.p_+)^2 = 0. \quad (35)$$

The null shear maps the "+" mass-shell cone onto the standard null cone. Similarly in spacetime introduce $x \to \hat{x}_+$, where

$$\hat{x}^\mu_+ = x^\mu - \frac{\kappa}{2} l^\mu (l.x). \quad (36)$$

This is a null shear (of the opposite sign) in spacetime. We have immediately

$$\hat{x}_+.\hat{p}_+ = x.p_+, \quad (37)$$

and therefore the phase factor $f = \exp\{-ix.p_+\} = \exp\{-i\hat{x}_+.\hat{p}_+\}$ satisfies the wave equation

$$\hat{\partial}^2_+ f = -\hat{p}^2_+ f = 0. \quad (38)$$

Here we have set

$$\hat{\partial}_{+\mu} = \frac{\partial}{\partial \hat{x}^\mu_+}. \quad (39)$$

It follows that in terms of the coordinates $\hat{x}^\mu_+$ the "+" light cone is $\hat{x}^2_+ = 0$, which yields

$$x^2 - \kappa(l.x)^2 = \hat{x}^2_+ = 0. \quad (40)$$

A similar discussion for the "−" mode leads to the transformation

$$p^\mu_- \to \hat{p}^\mu_- = p^\mu_- - \frac{\kappa}{2} l^\mu (l.p_-), \quad (41)$$

and

$$x^\mu \to \hat{x}^\mu_- = x^\mu + \frac{\kappa}{2} l^\mu (l.x), \quad (42)$$

and the conclusion that the corresponding light cone is

$$\hat{x}^2_- = x^2 + \kappa(l.x)^2 = 0. \quad (43)$$

The light cone structure therefore is similar, to that of the mass shells in momentum space, except that now the interior cone, the slow cone, is the "+" cone and the exterior cone, the fast cone, is the "−" cone.

A geometrical understanding of the need for a restriction on the range of κ can be obtained by noting that when $\kappa = 2$ the "+" cone tilts so that it acquires a generator along the time axis. When $\kappa > 2$ the positive time "+" cone lies entirely within the region $x_3 > 0$ at which point the coordinate system represented by $x^\mu = (t, \mathbf{x}) = (t, x_1, x_2, x_3)$ is no longer appropriate for describing the causal evolution of the model. This picture can be developed further by noting that observers (each associated with a reference frame that has $\eta^{\mu\nu}$ as a metric) have coordinate systems related by Lorentz transformations. One such transformation is $L(\psi)$; a boost along the (negative) 3-axis of velocity $v = \tanh\psi$. It is easy to check that under such a transformation

$$l^\mu \to L^\mu{}_\nu(\psi) l^\nu = e^\psi l^\mu. \quad (44)$$

The description of the model in the new frame is unchanged provided we make the replacement $\kappa \to \kappa' = e^{2\psi}\kappa$. Even if κ lies within the acceptable range a sufficiently powerful boost will shift κ' out of this range. A boost in the opposite direction can be of any strength and will cause a shift eventually to observers who, because of limitations of measurement accuracy, are unable to detect the Lorentz symmetry violation. The constraint on κ can be reinterpreted as a constraint on the allowed reference frames. Observers may not be boosted to the point at which they can overtake particles moving on the slow light cone. We develop this point further in Sec. VI.

D. Overlaps of the photon wave functions

In order to control the normalization of wave functions it is necessary to define a scalar product or overlap between them. We follow Ref. [20] and define the overlap of two solutions, $A^\mu(x)$ and $B^\mu(x)$, of Eq. (18) to be (A, B) where

$$(A, B) = -i \int d^3\mathbf{x} (A^*_\nu(x)\partial^0 B^\nu(x) - B_\nu(x)\partial^0 A^{\nu*}(x) \\ -C^{0\nu\lambda\tau}(A^*_\nu(x)\partial_\lambda B_\tau(x) - B_\nu(x)\partial_\lambda A^*_\tau(x))). \quad (45)$$

It is easy to verify that it is independent of time. We denote the wave functions as

$$\begin{aligned}
\Psi_{G\mu}(\mathbf{p}, x) &= p_\mu e^{-ip.x}, \\
\Psi_{C\mu}(\mathbf{p}, x) &= l_\mu e^{-ip.x}, \\
\Psi_{+\mu}(\mathbf{p}, x) &= ((l.p_+)e_{1\mu} - (e_1.p_+)l_\mu)e^{-ip_+.x} = \varepsilon_{+\mu}(p_+)e^{-ip_+.x}, \\
\Psi_{-\mu}(\mathbf{p}, x) &= ((l.p_-)e_{2\mu} - (e_2.p_-)l_\mu)e^{-ip_-.x} = \varepsilon_{-\mu}(p_-)e^{-ip_-.x}.
\end{aligned} \quad (46)$$

The overlaps for the unphysical wave functions are

$$(\Psi_G(\mathbf{p}),\Psi_G(\mathbf{p}')) = (\Psi_C(\mathbf{p}),\Psi_C(\mathbf{p}')) = 0, \tag{47}$$

$$(\Psi_G(\mathbf{p}),\Psi_C(\mathbf{p}')) = -2E(l.p)(2\pi)^3\delta^3(\mathbf{p}-\mathbf{p}'). \tag{48}$$

The overlaps between unphysical and physical wave functions all vanish. The nonvanishing overlaps for the physical wave functions are

$$\begin{aligned}(\Psi_+(\mathbf{p}),\Psi_+(\mathbf{p}')) &= (2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{p}')(E_+-p_3)^2 \\ &\quad\times\left(E_+ + \frac{\kappa}{2}(E_+-p_3)\right), \\ (\Psi_-(\mathbf{p}),\Psi_-(\mathbf{p}')) &= (2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{p}')(E_--p_3)^2 \\ &\quad\times\left(E_- - \frac{\kappa}{2}(E_--p_3)\right). \end{aligned}\tag{49}$$

The structure of the overlaps of the unphysical wave functions is equivalent to that of the standard Gupta-Bleuler formalism. That is, they comprise a subspace, in this case spanned by two sets of zero norm wave functions, that contains both positive and negative norm wave functions. Ultimately this is why the corresponding excitations do not contribute to the values of physical, that is gauge invariant, quantities. This is particularly significant here because the unphysical light cone is distinct from the birefringent structure of the physical light cones and any unphysical signal will be transported in manner distinct from the physical signals.

E. Quantization of the photon field

The quantization of the photon field is achieved by imposing the (equal-time) commutation relations

$$[\Pi^\nu(x), A_\lambda(x')] = -i\delta^\nu_\lambda\delta^{(3)}(\mathbf{x}-\mathbf{x}'). \tag{50}$$

The field $\Pi^\nu(x)$ is defined in Eq. (19), see also [20,21].

A convenient way of exploiting the canonical commutation relations is through the identity

$$[(f,A), A_\mu(x)] = f^*_\mu(x), \tag{51}$$

where $f_\mu(x)$ is any arbitrary photon wave function and $A_\mu(x)$ is the quantum photon field. We can obtain this result by noting that from the definition in Eq. (45) we obtain

$$(f,A) = i\int d^3\mathbf{x}' f^*_\nu(x')\Pi^\nu(x') + \mathcal{R}, \tag{52}$$

where the remainder term $\mathcal{R}$ commutes (at equal times) with $A_\mu(x)$. The identity follows.

The quantum field $A_\mu(x)$ can be separated into a number of terms. They are

$$A_\mu(x) = A_{+\mu}(x) + A_{-\mu}(x) + A_{U\mu}(x), \tag{53}$$

where

$$\begin{aligned}A_{\pm\mu}(x) = \int d^3\mathbf{p}\frac{1}{\mathcal{N}_\pm(\mathbf{p})}[a_\pm(\mathbf{p})\Psi_{\pm\mu}(\mathbf{p},x) \\ + a^\dagger_\pm(\mathbf{p})\Psi^*_{\pm\mu}(\mathbf{p},x)],\end{aligned}\tag{54}$$

and $A_{U\mu}(x)$ contains the unphysical mode contributions. We include a normalizing factor $1/\mathcal{N}_\pm(\mathbf{p})$ in Eq. (54) in order to permit the imposition of the nonvanishing commutation relations in the form

$$[a_\pm(\mathbf{p}), a^\dagger_\pm(\mathbf{p}')] = (2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{p}'). \tag{55}$$

It follows, for example, that

$$[a_+(\mathbf{p}), A_\mu(x)] = \frac{1}{\mathcal{N}_+(\mathbf{p})}\Psi^*_{+\mu}(\mathbf{p},x). \tag{56}$$

We also have from the overlap calculation

$$\begin{aligned}(\Psi_+(\mathbf{p}), A) &= \frac{1}{\mathcal{N}_+(\mathbf{p})}(E_+-p_3)^2 \\ &\quad\times\left(E_+ + \frac{\kappa}{2}(E_+-p_3)\right)a_+(\mathbf{p}).\end{aligned}\tag{57}$$

If we choose $f_\mu(x) = \Psi_{+\mu}(\mathbf{p},x)$ in the identity Eq. (51) then using Eqs. (56) and (57) we find

$$\begin{aligned}[(\Psi_+(\mathbf{p}),A), A_{+\mu}(x)] &= \frac{1}{(\mathcal{N}_+(\mathbf{p}))^2}(E_+-p_3)^2 \\ &\quad\times\left(E_+ + \frac{\kappa}{2}(E_+-p_3)\right)\Psi^*_{+\mu}(\mathbf{p},x).\end{aligned}\tag{58}$$

It follows that

$$\mathcal{N}_+(\mathbf{p}) = \sqrt{(E_+-p_3)^2\left(E_+ + \frac{\kappa}{2}(E_+-p_3)\right)}. \tag{59}$$

By a parallel discussion we can show that

$$\mathcal{N}_-(\mathbf{p}) = \sqrt{(E_--p_3)^2\left(E_- - \frac{\kappa}{2}(E_--p_3)\right)}. \tag{60}$$

F. Photon energy-momentum tensor

The energy-momentum tensor for photons in the Gupta-Bleuler formalism can be computed along conventional lines in the form

$$\Theta^{\mu\nu}_{\rm GB}(x) = \frac{\partial \mathcal{L}_{\rm GB}}{\partial(\partial_\mu A_\lambda)}\partial^\nu A_\lambda - \eta^{\mu\nu}\mathcal{L}_{\rm GB}. \tag{61}$$

If we define

$$G^{\mu\nu} = -\frac{\partial \mathcal{L}_{\rm GB}}{\partial(\partial_\mu A_\nu)}, \tag{62}$$

then the Lagrangian is

$$\mathcal{L}_{\rm GB} = \frac{1}{4}G^{\mu\nu}F_{\mu\nu}, \tag{63}$$

and the equation of motion is

$$\partial_\mu G^{\mu\nu} = 0. \tag{64}$$

The energy-momentum tensor becomes

$$\Theta^{\mu\nu}_{\rm GB}(x) = -G^{\mu\lambda}(x)\partial^\nu A_\lambda(x) + \frac{1}{4}\eta^{\mu\nu}G^{\lambda\tau}(x)F_{\lambda\tau}(x). \tag{65}$$

It follows readily that

$$\partial_\mu \Theta^{\mu\nu}_{\rm GB}(x) = 0. \tag{66}$$

This version of the energy-momentum tensor, just as in the Lorentz symmetric case, is unsatisfactory as a physical quantity because it is not gauge invariant. The remedy is the same also see [23]. We introduce a correction

$$\Theta^{\mu\nu}_C(x) = G^{\mu\lambda}\partial_\lambda A^\nu(x). \tag{67}$$

The physical energy-momentum tensor is then

$$\Theta^{\mu\nu}(x) = \Theta^{\mu\nu}_{\rm GB}(x) + \Theta^{\mu\nu}_C(x). \tag{68}$$

It is conserved

$$\partial_\mu \Theta^{\mu\nu}(x) = 0. \tag{69}$$

The 4-momentum operator is

$$P^\nu = \int d^3\mathbf{x}\Theta^{0\nu}(x). \tag{70}$$

In terms of mode operators we have

$$P^\nu = P^\nu_+ + P^\nu_-, \tag{71}$$

where

$$P^\nu_\pm = \frac{1}{2}\int \frac{d^3\mathbf{p}}{(2\pi)^3}(a^\dagger_\pm(\mathbf{p})a_\pm(\mathbf{p}) + a_\pm(\mathbf{p})a^\dagger_\pm(\mathbf{p}))p^\nu_\pm. \tag{72}$$

In the Lorentz symmetric case the procedure we have adopted in constructing $\Theta^{\mu\nu}(x)$ also renders it symmetrical. This is not true in the presence of Lorentz symmetry violation, the reason being that the generators of Lorentz transformations $L^{\mu\nu}$ are computed as

$$L^{\mu\nu} = \int d^3\mathbf{x}(x^\mu\Theta^{0\nu}(x) - x^\nu\Theta^{0\mu}(x)). \tag{73}$$

Now

$$\partial_\lambda(x^\mu\Theta^{\lambda\nu}(x) - x^\nu\Theta^{\lambda}(x)) = \Theta^{\mu\nu}(x) - \Theta^{\nu\mu}(x). \tag{74}$$

The absence of symmetry for $\Theta^{\mu\nu}(x)$ then implies that the generators $L^{\mu\nu}$ are not time independent which is the case in our model.

III. MAJORANA SPINOR FIELD

The Lorentz-invariant Lagrangian, $\mathcal{L}_M(x)$ for the Majorana field $\psi(x)$, is

$$\mathcal{L}_M(x) = \frac{i}{2}\bar{\psi}(x)\gamma^\mu\partial_\mu\psi(x). \tag{75}$$

Here γ^μ are the standard Dirac matrices appropriate to the metric $\eta^{\mu\nu}$. We will follow [24] and adopt a chiral representation. The Majorana field satisfies the massless Dirac equation,

$$\gamma.\partial\psi(x) = 0. \tag{76}$$

The charge-conjugation transformation is $\psi \to \psi_C$ where

$$\psi_C = C(\bar{\psi})^T. \tag{77}$$

The γ^μ obey the conditions

$$C\gamma^\mu C^{-1} = -(\gamma^\mu)^T, \tag{78}$$

and

$$\begin{aligned} C^T &= C^\dagger = -C, \\ C^2 &= -1. \end{aligned} \tag{79}$$

These properties are satisfied by the representation

$$C = \begin{pmatrix} i\sigma^2 & 0 \\ 0 & -i\sigma^2 \end{pmatrix}. \tag{80}$$

These properties imply that $\psi_C(x)$ also satisfies Eq. (76). We are then free to impose the Majorana condition $\psi(x) = \psi_C(x)$ thus reducing the Majorana field to the two independent components appropriate for a photino field.

A. Lorentz symmetry violation and the Majorana field

In order to introduce Lorentz symmetry violation into the evolution of the Majorana field we follow the work of a number of authors [24–26] who have considered the implications of introducing Lorentz symmetry violation by means of generalisations of the Dirac equation. In constructing our model we require only a simplified version of that approach applied to the Majorana equation. We adopt then the modified Majorana Lagrangian $\mathcal{L}_M$ where

$$\mathcal{L}_M(x) = \frac{i}{2}\bar{\psi}(x)\Gamma^\mu\partial_\mu\psi(x), \tag{81}$$

where

$$\Gamma^\mu = \gamma^\mu + \frac{1}{2}T^\mu{}_{\alpha\beta}\sigma^{\alpha\beta}, \tag{82}$$

and $\sigma^{\alpha\beta} = (i/2)[\gamma^\alpha, \gamma^\beta]$. The modified Majorana equation is

$$\Gamma.\partial\psi(x) = 0. \tag{83}$$

It is easy to check that $\psi_C(x)$ is also a solution of Eq. (83) so we can indeed impose the condition $\psi_C(x) = \psi(x)$ on the solutions. The field $\pi(x)$ canonically conjugate to $\psi(x)$ is

$$\pi(x) = i\bar{\psi}(x)\Gamma^0, \tag{84}$$

and the canonical equal time anticommutation relation is

$$\{\psi_\alpha(x), \pi_\beta(x')\} = i\delta_{\alpha\beta}\delta^{(3)}(\mathbf{x} - \mathbf{x}'), \tag{85}$$

or more succinctly

$$\{\psi(x), \bar{\psi}(x')\}\Gamma^0 = \delta^{(3)}(\mathbf{x} - \mathbf{x}'). \tag{86}$$

B. Plane waves and dispersion relation

Crucial to constructing a supersymmetric model is arranging for a concordance between the photon and photino dispersion relations [5]. To investigate this we need the plane-wave solutions of Eq. (83). These have the form

$$\psi(\mathbf{p}, x) = ue^{-ip.x}, \tag{87}$$

where $p = (E, \mathbf{p})$ with $E > 0$ and $u(\mathbf{p})$ is spinor with components (u_1, u_2, u_3, u_4). Equation (83) implies

$$Mu = 0, \tag{88}$$

where

$$M = \Gamma.p = \gamma.p + \frac{1}{2}T_{\alpha\beta}\sigma^{\alpha\beta}, \tag{89}$$

and

$$T_{\alpha\beta} = p_\mu T^\mu{}_{\alpha\beta}. \tag{90}$$

Following the reasoning in [24–26] we introduce the dual tensor

$$\tilde{T}^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\lambda\tau}T_{\lambda\tau}, \tag{91}$$

together with

$$T^{(\pm)}_{\alpha\beta} = \frac{1}{2}(T_{\alpha\beta} \pm i\tilde{T}_{\alpha\beta}), \tag{92}$$

the self-dual and antiself-dual parts of $T_{\alpha\beta}$. It then follows (see [24–26]) that $\Delta(\mathbf{p}) = \det M$ is given by

$$\Delta(\mathbf{p}) = (p^2)^2 + (T^{(+)})^2(T^{(-)})^2 + 8V^{(+)}.V^{(-)}, \tag{93}$$

where

$$V^{(\pm)}_\nu = p^\mu T^{(\pm)}_{\mu\nu}. \tag{94}$$

The dispersion relation we require is

$$\Delta(p) = 0. \tag{95}$$

In order to fix the model precisely we must choose a specific form for $T^{(\pm)\mu}_{\alpha\beta}$. In doing so it helps to recall that the photon model involved the null vector l_μ and the self- and antiself-dual tensors $A_{\alpha\beta}$ and $\bar{A}_{\alpha\beta}$. These suggest themselves as candidates for the chiral structure we seek. Our initial proposal is then

$$T^{(+)\mu}_{\alpha\beta}(\theta) = \xi e^{i\theta}l^\mu A_{\alpha\beta}, \tag{96}$$

together with the complex conjugate relation, θ being real,

$$T^{(-)\mu}_{\alpha\beta}(\theta) = \xi e^{-i\theta}l^\mu \bar{A}_{\alpha\beta}. \tag{97}$$

In turn this yields

$$T^\mu_{\alpha\beta}(\theta) = \xi l^\mu(e^{i\theta}A_{\alpha\beta} + e^{-i\theta}\bar{A}_{\alpha\beta}). \tag{98}$$

If we make the replacements $m \to m' = e^{i\theta}m$ and $\bar{m} \to \bar{m}' = e^{-i\theta}\bar{m}$ in the choice of Penrose tetrad in the Majorana field we see that relative to the photon tetrad this represents a clockwise rotation in the (1, 2)-plane about the 3-axis. However the extra generality represented by the angle θ is spurious. If we define

$$\Gamma^\mu(\theta) = \gamma^\mu + \frac{1}{2}T^\mu_{\alpha\beta}(\theta)\sigma^{\alpha\beta}, \quad (99)$$

then we can easily show that

$$\Gamma^\mu(\theta) = X(\theta)\Gamma^\mu X(\theta), \quad (100)$$

where $\Gamma^\mu = \Gamma^\mu(\theta = 0)$ and

$$X(\theta) = e^{i\theta/2}\frac{1+\gamma_5}{2} + e^{-i\theta/2}\frac{1-\gamma_5}{2}. \quad (101)$$

The Majorana Lagrangian becomes

$$\mathcal{L}_M(x) = \frac{i}{2}\bar{\psi}(x)X(\theta)\Gamma.\partial X(\theta)\psi(x). \quad (102)$$

By means of the field transformations $X(\theta)\psi(x) \to \psi(x)$ and (consistently) $\bar{\psi}(x)X(\theta) \to \bar{\psi}(x)$ we have

$$\mathcal{L}_M(x) = \frac{i}{2}\bar{\psi}(x)\Gamma.\partial\psi(x). \quad (103)$$

The implication is that we can choose any value of θ without changing the model. For convenience we then choose $\theta = 0$ and replace Eq. (98) with

$$T^\mu_{\alpha\beta} = \xi l^\mu(A_{\alpha\beta} + \bar{A}_{\alpha\beta}). \quad (104)$$

For our model then we easily see that $T^{(\pm)2} = 0$ and find for the dispersion relation

$$(p^2)^2 - 8\xi^2(l.p)^4 = 0. \quad (105)$$

This coincides with the result in Eq. (28) when $\kappa^2 = 8\xi^2$. There are then two possibilities

$$\kappa = \pm 2\sqrt{2}\xi. \quad (106)$$

In either case the birefringent mass-shell cone structure of the Majorana field corresponds exactly with that of the birefringent photons though with a differing matching of states in the two cases. We make the choice $\xi = \kappa/(2\sqrt{2})$ (implying $\xi > 0$) for simplicity of exposition. We will however deal with the case $\xi = -\kappa/(2\sqrt{2})$ later when considering the limiting case of Lorentz invariance. There are a number of approaches to deriving the expression for $\Delta(p)$ but it will reemerge straightforwardly when we examine the explicit form of the spinor wave functions.

To obtain explicit plane-wave solutions we follow Refs. [24–26] and adopt the chiral representation for the γ^μ, namely

$$\gamma^\mu = \begin{pmatrix} 0 & \bar{\sigma}^\mu \\ \sigma^\mu & 0 \end{pmatrix}, \quad (107)$$

with $\sigma^\mu = (\mathbf{1}, \sigma^1, \sigma^2, \sigma^3)$ and $\bar{\sigma}^\mu = (\mathbf{1}, -\sigma^1, -\sigma^2, -\sigma^3)$, σ^k $k = 1, 2, 3$ being the standard Pauli matrices and

$$\gamma_5 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}. \quad (108)$$

It is a little simpler to deal with the modified version of Eq. (88)

$$\tilde{M}u = 0, \quad (109)$$

where

$$\tilde{M} = \gamma.pM = p^2 + \frac{1}{2}T_{\alpha\beta}\gamma.p\sigma^{\alpha\beta}. \quad (110)$$

Using the well-known identity

$$\gamma^\lambda\sigma^{\alpha\beta} = i(\eta^{\lambda\alpha}\gamma^\beta - \eta^{\lambda\beta}\gamma^\alpha) - \epsilon^{\lambda\alpha\beta\tau}\gamma_\tau\gamma_5, \quad (111)$$

$\tilde{M}$ can be put in the form

$$\tilde{M} = \begin{pmatrix} p^2 & 2iV^{(-)}.\bar{\sigma} \\ 2iV^{(+)}.\sigma & p^2 \end{pmatrix}, \quad (112)$$

where $V^{(\pm)}_\mu$ are defined in Eq. (94). We then obtain the equations for the spinor u

$$\begin{aligned} p^2u_1 - i2\sqrt{2}\xi(l.p)(\bar{m}.p)u_3 &= 0, \\ p^2u_2 + i2\sqrt{2}\xi(l.p)^2u_3 &= 0, \\ -i2\sqrt{2}\xi(l.p)^2u_2 + p^2u_3 &= 0, \\ -i2\sqrt{2}\xi(l.p)(m.p)u_2 + p^2u_4 &= 0. \end{aligned} \quad (113)$$

In order to yield a nontrivial solution the first and last of these equations show that u_2 and u_3 cannot both vanish. The other two equations therefore require that

$$\det\begin{pmatrix} p^2 & i2\sqrt{2}\xi(l.p)^2 \\ -i2\sqrt{2}\xi(l.p)^2 & p^2 \end{pmatrix} = 0. \quad (114)$$

That is, of course, identical to the dispersion relation from Eq. (105) and, on imposing the relation $\kappa = 2\sqrt{2}\xi$ (which we will asume from here on), the same as that from Eq. (28). The mass-shell "±" cones are identical between photon and photino. The plane-wave solutions are

$$\psi_\pm(\mathbf{p}, x) = u_\pm(p_\pm)e^{-ip_\pm.x}, \quad (115)$$

where

$$u_\pm(p_\pm) = \begin{pmatrix} -(\bar{m}.p_\pm) \\ (l.p_\pm) \\ \mp i(l.p_\pm) \\ \mp i(m.p_\pm) \end{pmatrix}. \tag{116}$$

The positive-energy plane waves are therefore

$$\psi_\pm(\mathbf{p},x) = u_\pm(p_\pm)e^{-ip_\pm.x}. \tag{117}$$

The negative energy plane waves are the charge-conjugate wave functions

$$\psi_{\pm C}(\mathbf{p},x) = C(\bar{\psi}_\pm(\mathbf{p},x))^T = u_{\pm C}(p_\pm)e^{ip_\pm.x}, \tag{118}$$

where

$$u_{\pm C}(p_\pm) = C(\bar{u}_\pm(p_\pm))^T = \mp\, iu_\pm(p_\pm). \tag{119}$$

For general 4-vector q we write

$$u_\pm(q) = \begin{pmatrix} -(\bar{m}.q) \\ (l.q) \\ \mp i(l.q) \\ \mp i(m.q) \end{pmatrix}. \tag{120}$$

It is easily checked, in the chiral representation for γ-matrices, that

$$u_\pm(q) = \gamma.q u_\pm^{(0)}, \tag{121}$$

where

$$u_\pm^{(0)} = \begin{pmatrix} \mp i/\sqrt{2} \\ 0 \\ 0 \\ 1/\sqrt{2} \end{pmatrix}. \tag{122}$$

It also obvious that

$$u_\pm(q+sl) = u_\pm(q), \tag{123}$$

for any value of the scalar s. We have then

$$u_\pm(p_\pm) = \gamma.p_\pm u_\pm^{(0)} = \gamma.\hat{p}_\pm u_\pm^{(0)}, \tag{124}$$

with the result

$$\gamma.\hat{p}_\pm u_\pm(p_\pm) = (\gamma.\hat{p}_\pm)^2 u_\pm^{(0)} = 0. \tag{125}$$

The Majorana wave functions can be put in the form

$$\psi_\pm(\mathbf{p},x) = i\gamma.\hat{\partial}_\pm u_\pm^{(0)} e^{-ip_\pm.x} \tag{126}$$

and

$$\psi_{\pm C}(\mathbf{p},x) = C(\bar{\psi}_\pm(\mathbf{p},x))^T = -i\gamma.\hat{\partial}_\pm u_{\pm C}^{(0)} e^{ip_\pm.x}, \tag{127}$$

where

$$u_{\pm C}^{(0)} = C(\bar{u}_\pm^{(0)})^T. \tag{128}$$

We then have the result

$$\gamma.\hat{\partial}_\pm \psi_\pm(\mathbf{p},x) = \hat{\partial}_\pm^2 \psi_\pm(\mathbf{p},x) = 0. \tag{129}$$

The complete Majorana field comprises a superposition of these plane-wave solutions. It can be split into two parts $\psi_\pm(x)$ each associated in the obvious way with the "±" light cones. We can write

$$\psi(x) = \psi_+(x) + \psi_-(x), \tag{130}$$

where

$$\gamma.\hat{\partial}_\pm \psi_\pm(x) = 0. \tag{131}$$

C. Overlaps of the Majorana wave functions

If $\psi(x)$ and $\phi(x)$ are Majorana wave functions then the current $J^\mu(x) = \bar{\phi}(x)\Gamma^\mu\psi(x)$ is conserved

$$\partial_\mu J^\mu(x) = 0. \tag{132}$$

It is then possible to define a time-independent overlap (ϕ,ψ),

$$(\phi,\psi) = \int d^3\mathbf{x}\bar{\phi}(x)\Gamma^0\psi(x). \tag{133}$$

The nonvanishing overlaps between the plane-wave solutions are easily computed as

$$(\psi_\pm(\mathbf{p}),\psi_\pm(\mathbf{p}')) = (2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{p}')2(E_\pm - p_3) \times \left(E_\pm \pm \frac{\kappa}{2}(E_\pm - p_3)\right). \tag{134}$$

The same holds true $(\psi \to \psi_C)$ for the charge conjugate wave functions.

D. Quantization of the Majorana field

We introduce mode operators $b_\pm(\mathbf{p}), b_\pm^\dagger(\mathbf{p})$ for the Majorana field by expanding $\psi_\pm(x)$ in the form

$$\psi_\pm(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{N_\pm(\mathbf{p})}(b_\pm(\mathbf{p})e^{-ip_\pm.x} \mp ib_\pm^\dagger(\mathbf{p})e^{-ip_\pm.x})u_\pm(p_\pm). \tag{135}$$

The factor $\mp i$ in the second term of the integrand renders the field even under charge conjugation.The normalizing factor $N_{\pm}(\mathbf{p})$ is chosen so that the nonvanishing anti-commutation relations

$$\{b_{\pm}(\mathbf{p}), b^{\dagger}_{\pm}(\mathbf{p}')\} = (2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{p}'), \qquad (136)$$

are consistent with the canonical anticommutation relations in Eq. (86). It is easily checked that

$$N_{\pm}(\mathbf{p}) = \sqrt{2(E_{\pm}-p_3)(E_{\pm}\pm(\kappa/2)(E_{\pm}-p_3))}. \qquad (137)$$

E. Energy-momentum tensor for the Majorana field

The energy-momentum tensor for the Majorana field in our model is $\Theta^{\mu\nu}_M(x)$ where

$$\Theta^{\mu\nu}_M(x) = \frac{i}{2}\bar{\psi}(x)\Gamma^{\mu}\partial^{\nu}\psi(x) - \eta^{\mu\nu}\mathcal{L}_M(x). \qquad (138)$$

When $\psi(x)$ satisfies the equations of motion the Lagrangian contribution vanishes and

$$\partial_{\mu}\Theta^{\mu\nu}_M(x) = 0. \qquad (139)$$

The 4-momentum P^{ν} is then given by

$$P^{\nu} = \int d^3\mathbf{x}\Theta^{0\nu}_M(x), \qquad (140)$$

and is independent of time. Expressed in terms of mode operators we have

$$P^{\nu} = P^{\nu}_{+} + P^{\nu}_{-}, \qquad (141)$$

where

$$P^{\nu}_{\pm} = \frac{1}{2}\int\frac{d^3\mathbf{p}}{(2\pi)^3}p^{\nu}_{\pm}(b^{\dagger}_{\pm}(\mathbf{p})b_{\pm}(\mathbf{p}) - b_{\pm}(\mathbf{p})b^{\dagger}_{\pm}(\mathbf{p})). \qquad (142)$$

It is immediately obvious that the vacuum contributions of the Majorana spinors to the 4-momentum cancel the corresponding contributions of the gauge fields. This must be necessarily the case if supersymmetry is to be maintained in the Lorentz symmetry breaking model.

IV. SUPERSYMMETRY CHARGES

The photon/photino model is potentially supersymmetric, even in the presence of Lorentz symmetry breaking, because it is possible to align the birefringent mass-shell cones and light cones of the photons and photinos. The crucial stage in completing the model is the construction of supersymmetry charges. We show that this is indeed possible although with a somewhat unconventional approach forced on us by the birefringence that expresses the Lorentz symmetry violation in the model. The conventional procedure is to derive a conserved Noether current from a symmetry of the Lagrangian. We reverse the procedure and postulate a current that we show to be conserved. Subsequently, we explore the algebra of conserved charges and their efficacy in connecting photon and photino states.

Guided by the conventional form of the current we postulate currents, one for each mass-shell cone, $\hat{\mathcal{J}}^{\mu}_{\pm\pm}(x)$ where

$$\hat{\mathcal{J}}^{\mu}_{\pm\pm}(x) = \hat{F}_{\pm\lambda\tau}(x)\sigma^{\lambda\tau}\gamma^{\mu}\psi_{\pm}(x). \qquad (143)$$

Here, we are using

$$\hat{F}_{\pm\mu\nu}(x) = \hat{\partial}_{\pm\mu}\hat{A}_{\pm\nu} - \hat{\partial}_{\pm\nu}\hat{A}_{\pm\mu}(x), \qquad (144)$$

where [recall that $l.A_{\pm}(x) = 0$]

$$\hat{A}_{\pm\mu}(x) = \left(\delta^{\lambda}_{\mu} \pm \frac{\kappa}{2}l_{\mu}l^{\lambda}\right)A_{\pm\lambda}(x) = A_{\pm\mu}(x). \qquad (145)$$

It is then easy to see that

$$\hat{\partial}_{\pm}.\hat{A}_{\pm}(x) = \partial.A_{\pm}(x) = 0, \qquad (146)$$

and that

$$\hat{\partial}^{\mu}_{\pm}\hat{F}_{\pm\mu\nu}(x) = \hat{\partial}^2_{\pm}A_{\pm\nu}(x) = 0. \qquad (147)$$

We have also the Bianchi identity

$$\hat{\partial}_{\pm\lambda}\hat{F}_{\pm\mu\nu}(x) + \hat{\partial}_{\pm\mu}\hat{F}_{\pm\nu\lambda}(x) + \hat{\partial}_{\pm\nu}\hat{F}_{\pm\lambda\mu}(x) = 0. \qquad (148)$$

Using the identity in Eq. (111) and the Bianchi identity we see by a standard argument that

$$\hat{\partial}_{\pm\mu}\hat{F}_{\pm\lambda\tau}(x)\sigma^{\lambda\tau}\gamma^{\mu} = 0. \qquad (149)$$

Also from Eq. (126) we have

$$\gamma.\hat{\partial}_{\pm}\psi_{\pm}(x) = 0. \qquad (150)$$

It follows immediately that

$$\hat{\partial}_{\pm\mu}\hat{\mathcal{J}}^{\mu}_{\pm\pm}(x) = 0. \qquad (151)$$

Introducing

$$J^{\mu}_{\pm\pm}(x) = \hat{\mathcal{J}}^{\mu}_{\pm\pm}(x) \pm \frac{\kappa}{2}l^{\mu}l.\hat{J}_{\pm\pm}(x), \qquad (152)$$

we find

$$\partial_\mu J^\mu_{\pm\pm}(x) = 0. \tag{153}$$

It is worth noting that we can also introduce the currents $\hat{J}^\mu_{\pm\mp}(x)$, where

$$\hat{J}^\mu_{\pm\mp}(x) = \hat{F}_{\pm\lambda\tau}(x)\sigma^{\lambda\tau}\gamma^\mu\psi_\mp(x). \tag{154}$$

However these currents, for the present choice of $\xi = +\kappa/(2\sqrt{2})$, are not conserved. For example we can easily show that

$$\hat{\partial}_{+\mu}\hat{J}^\mu_{+-}(x) = \kappa\hat{F}_{+\lambda\tau}(x)\sigma^{\lambda\tau}l.\gamma l.\partial\psi_-(x) \neq 0. \tag{155}$$

The corresponding supersymmetry charges are therefore not independent of time and reflect the presence of Lorentz symmetry violation in the model when $\xi = +\kappa/(2\sqrt{2})$. Suitably interpreted however when $\xi = -\kappa/(2\sqrt{2})$, they do provide conserved charges because of the interchange of photino mass shells that occurs in this case. We consider this possibility later.

The conserved supersymmetry charges with which we are concerned are given by

$$Q_{\pm\pm} = \int d^3\mathbf{x} J^0_{\pm\pm}(x). \tag{156}$$

When expressed in terms of the mode operators we have

$$Q_{++} = 2\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{\sqrt{(E_+ - p_3)}} \times (a_+(\mathbf{p})b_+^\dagger(\mathbf{p}) - ia_+^\dagger(\mathbf{p})b_+(\mathbf{p}))u_+(p_+), \tag{157}$$

and

$$Q_{--} = 2\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{\sqrt{(E_- - p_3)}} \times (ia_-(\mathbf{p})b_-^\dagger(\mathbf{p}) - a_-^\dagger(\mathbf{p})b_-(\mathbf{p}))u_+(p_-). \tag{158}$$

We draw attention to the apparently anomalous factor $u_+(p_-)$ in the integrand in Eq. (158). This factor can be reexpressed as $\gamma_5 u_-(p_-)$ which leaves it more seemingly natural but requires the explicit presence of γ_5. We leave Eq. (158) as it stands. We define also the conjugate charges

$$\bar{Q}_{\pm\pm} = Q^\dagger_{\pm\pm}\gamma^0. \tag{159}$$

We have immediately

$$\{Q_{\pm\pm}, \bar{Q}_{\mp\mp}\} = 0. \tag{160}$$

The nonvanishing anticommutators are

$$\{Q_{\pm\pm}, \bar{Q}_{\pm\pm}\} = 8\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{(E_\pm - p_3)}(a_\pm^\dagger(\mathbf{p})a_\pm(\mathbf{p}) + b_\pm^\dagger(\mathbf{p})b_\pm(\mathbf{p}))u_+(p_\pm)\bar{u}_+(p_\pm), \tag{161}$$

using the result (which can be checked by taking the trace with a complete basis of γ-matrices)

$$u_+(p_\pm)\bar{u}_+(p_\pm) = \frac{1}{2}[(E_\pm - p_3)\hat{p}_\pm.\gamma + \sqrt{2}\varepsilon_+^\lambda(p_\pm)\hat{p}_\pm^\tau\sigma_{\lambda\tau}]. \tag{162}$$

We can then show that

$$\{Q_{\pm\pm}, \bar{Q}_{\pm\pm}\} = 4\int \frac{d^3\mathbf{p}}{(2\pi)^3}(a_\pm^\dagger(\mathbf{p})a_\pm(\mathbf{p}) + b_\pm^\dagger(\mathbf{p})b_\pm(\mathbf{p})) \times \left[\hat{p}_\pm.\gamma + \sqrt{2}\frac{\varepsilon_+^\lambda(p_\pm)\hat{p}_\pm^\tau\sigma_{\lambda\tau}}{(E_\pm - p_3)}\right]. \tag{163}$$

The first term in the integrand in Eq. (163) yields a contribution to the anticommutator of the supersymmetry charges of the form

$$\{Q_{\pm\pm}, \bar{Q}_{\pm\pm}\} = 4\hat{P}_\pm.\gamma + \cdots, \tag{164}$$

where $\hat{P}_\pm$ is the appropriately modified 4-momentum operator for the photon/photino system. This term is a contribution to the anticommutator similar to the standard result for the Lorentz symmetric case and to which it reduces when $\xi \to 0+$. The remaining term seems to stand in the way of reproducing the standard result in the limit $\xi \to 0$. However an examination of the corresponding limit $\xi \to 0-$ provides the appropriate canceling contributions.

A. Interchange of mass-shell cones

The result of setting $\xi = -\kappa/(2\sqrt{2})$ is not only directly interesting but is crucial in understanding the limit $\kappa \to 0$ when Lorentz invariance is restored. We retain the $\pm$ identification of the mass-shells established by the photon field. The reversal of sign for ξ interchanges the mass shells for the photino field with the outcome that

$$\psi_\mp(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{N_\pm(\mathbf{p})} \times (b_\mp(\mathbf{p})e^{-ip_\pm.x} \pm ib_\mp^\dagger(\mathbf{p})e^{ip_\pm.x})u_\mp(p_\pm). \tag{165}$$

The time independent supersymmetry charges Q_{+-} and Q_{-+} can be constructed from the now conserved currents $J^\mu_{+-}(x)$ and $J^\mu_{-+}(x)$, where

$$J^\mu_{\pm\mp}(x) = \hat{J}^\mu_{\pm\mp}(x) \pm (\kappa/2)l^\mu l.\hat{J}_{\pm\mp}(x), \tag{166}$$

and

$$\hat{J}^{\mu}_{\pm\mp}(x) = \hat{F}_{\pm\lambda\tau}(x)\sigma^{\lambda\tau}\gamma^{\mu}\psi_{\mp}(x). \tag{167}$$

The supersymmetry charges take the form

$$Q_{+-} = 2\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{\sqrt{(E_+ - p_3)}} \times (a_+(\mathbf{p})b_-^{\dagger}(\mathbf{p}) + ia_+^{\dagger}(\mathbf{p})b_-(\mathbf{p}))u_-(p_+). \tag{168}$$

Similarly, we have

$$Q_{-+} = 2\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{\sqrt{(E_- - p_3)}} \times (ia_-(\mathbf{p})b_+^{\dagger}(\mathbf{p}) + a_-^{\dagger}(\mathbf{p})b_+(\mathbf{p}))u_-(p_-). \tag{169}$$

Following the scheme of previous calculations we find the nonvanishing anticommutators to be

$$\{Q_{\pm\mp}, \bar{Q}_{\pm\mp}\} = 4\int \frac{d^3\mathbf{p}}{(2\pi)^3}(a_{\pm}^{\dagger}(\mathbf{p})a_{\pm}(\mathbf{p}) + b_{\mp}^{\dagger}(\mathbf{p})b_{\mp}(\mathbf{p})) \times \left[\hat{p}_{\pm}.\gamma - \sqrt{2}\frac{\varepsilon_+^{\lambda}(p_{\pm})\hat{p}_{\pm}^{\tau}\sigma_{\lambda\tau}}{(E_{\pm} - p_3)}\right]. \tag{170}$$

Note the change of sign for the second term in square brackets relative to the corresponding term in Eq. (163).

B. Lorentz invariant limit

Lorentz invariance is achieved in the model by setting $\xi = 0$. We have then that $p_{\pm}$ and $\hat{p}_{\pm}$ all reduce to a common value of p where $p^2 = 0$. We have also in this limit $\mathcal{N}_+(\mathbf{p}) = \mathcal{N}_-(\mathbf{p}) = \mathcal{N}(\mathbf{p}) = \sqrt{(E - p_3)^2 E}$ and $N_+(\mathbf{p}) = N_-(\mathbf{p}) = N(\mathbf{p}) = \sqrt{2(E - p_3)E}$. In that case the supersymmetry charges, all of which are constant, take the form

$$\begin{aligned}
Q_{++} &= 2\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{\sqrt{(E - p_3)}}(a_+(\mathbf{p})b_+^{\dagger}(\mathbf{p}) - ia_+^{\dagger}(\mathbf{p})b_+(\mathbf{p}))u_+(p),\\
Q_{--} &= 2\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{\sqrt{(E - p_3)}}(ia_-(\mathbf{p})b_-^{\dagger}(\mathbf{p}) - a_-^{\dagger}(\mathbf{p})b_-(\mathbf{p}))u_+(p),\\
Q_{+-} &= 2\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{\sqrt{(E - p_3)}}(a_+(\mathbf{p})b_-^{\dagger}(\mathbf{p}) + ia_+^{\dagger}(\mathbf{p})b_-(\mathbf{p}))u_-(p),\\
Q_{-+} &= 2\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{\sqrt{(E - p_3)}}(ia_-(\mathbf{p})b_+^{\dagger}(\mathbf{p}) + a_-^{\dagger}(\mathbf{p})b_+(\mathbf{p}))u_-(p).
\end{aligned} \tag{171}$$

The full list of anticommutation relations is given in Appendix A. The diagonal relations can be read off from Eqs. (163) and (170) by setting $\kappa = 0$. We have

$$\{Q_{\pm\pm}, \bar{Q}_{\pm\pm}\} = 4\int \frac{d^3\mathbf{p}}{(2\pi)^3}\left[p.\gamma + \sqrt{2}\frac{\varepsilon_+^{\lambda}(p)p^{\tau}\sigma_{\lambda\tau}}{E - p_3}\right] \times (a_{\pm}^{\dagger}(\mathbf{p})a_{\pm}(\mathbf{p}) + b_{\pm}^{\dagger}(\mathbf{p})b_{\pm}(\mathbf{p})), \tag{172}$$

and

$$\{Q_{\pm\mp}, \bar{Q}_{\pm\mp}\} = 4\int \frac{d^3\mathbf{p}}{(2\pi)^3}\left[p.\gamma - \sqrt{2}\frac{\varepsilon_+^{\lambda}(p)p^{\tau}\sigma_{\lambda\tau}}{E - p_3}\right] \times (a_{\pm}^{\dagger}(\mathbf{p})a_{\pm}(\mathbf{p}) + b_{\mp}^{\dagger}(\mathbf{p})b_{\mp}(\mathbf{p})). \tag{173}$$

Of the off-diagonal anticommutators some are directly zero. The others as can be seen from Appendix A, yield contributions that cancel in pairs. It follows that if we set

$$Q = Q_{++} + Q_{--} + Q_{+-} + Q_{-+}, \tag{174}$$

then we find

$$\{Q, \bar{Q}\} = 8\int \frac{d^3\mathbf{p}}{(2\pi)^3}p.\gamma(a_+^{\dagger}(\mathbf{p})a_+(\mathbf{p}) + a_-^{\dagger}(\mathbf{p})a_-(\mathbf{p}) + b_+^{\dagger}(\mathbf{p})b_+(\mathbf{p}) + b_-^{\dagger}(\mathbf{p})b_-(\mathbf{p})) = 8\gamma.P, \tag{175}$$

where P^{μ} is the complete 4-momentum operator for the model in the Lorentz invariant limit. This demonstrates that in the Lorentz invariant case, we can recover the complete constant supersymmetric charge with the correct anticommutation relation. However it is evident that to achieve this outcome it is necessary to include contributions from both limits $\xi \to 0\pm$.

C. Action of supersymmetry charges on modes and fields

We return to the broken symmetry case with $\xi = +\kappa/(2\sqrt{2})$. The action of the supersymmetry charges on mode operators can be read off from their definition in Eqs. (157) and (158). We have

$$
\begin{aligned}
[Q_{++}, a_+(\mathbf{p})] &= \frac{2i}{\sqrt{(E_+ - p_3)}} b_+(\mathbf{p}) u_+(p_+), \\
[Q_{++}, a_+^\dagger(\mathbf{p})] &= \frac{2}{\sqrt{(E_+ - p_3)}} b_+^\dagger(\mathbf{p}) u_+(p_+), \\
\{b_+(\mathbf{p}), \bar{Q}_{++}\} &= \frac{2i}{\sqrt{(E_+ - p_3)}} a_+(\mathbf{p}) \bar{u}_+(p_+), \\
\{b_+^\dagger(\mathbf{p}), \bar{Q}_{++}\} &= \frac{2}{\sqrt{(E_+ - p_3)}} a_+^\dagger(\mathbf{p}) \bar{u}_+(p_+),
\end{aligned} \tag{176}
$$

and

$$
\begin{aligned}
[Q_{--}, a_-(\mathbf{p})] &= \frac{2}{\sqrt{(E_- - p_3)}} b_-(\mathbf{p}) u_+(p_-), \\
[Q_{--}, a_-^\dagger(\mathbf{p})] &= \frac{2i}{\sqrt{(E_- - p_3)}} b_-^\dagger(\mathbf{p}) u_+(p_-), \\
\{b_-(\mathbf{p}), \bar{Q}_{--}\} &= \frac{-2}{\sqrt{(E_- - p_3)}} a_-(\mathbf{p}) \bar{u}_+(p_-), \\
\{b_-^\dagger(\mathbf{p}), \bar{Q}_{--}\} &= \frac{-2i}{\sqrt{(E_- - p_3)}} a_-^\dagger(\mathbf{p}) \bar{u}_+(p_-).
\end{aligned} \tag{177}
$$

The (anti)commutators therefore convert the photon/photino mode operators correctly into the corresponding photino/photon mode operators. The action of the supersymmetry charges on fields can be deduced immediately.

1. Anticommutators for Majorana fields

From Eqs. (135) and (137) we have

$$
\begin{aligned}
\{\psi_+(x), \bar{Q}_{++}\} = i\sqrt{2} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{\mathcal{N}_+(\mathbf{p})} \\
\times (a_+(\mathbf{p}) e^{-ip_+.x} - a_+^\dagger(\mathbf{p}) e^{ip_+.x}) u_+(p_+) \\
\times \bar{u}_+(p_+).
\end{aligned} \tag{178}
$$

We have used the result

$$
N_+(\mathbf{p})(E_+ - p_3)^{1/2} = \sqrt{2}\mathcal{N}_+(\mathbf{p}). \tag{179}
$$

From Eq. (162) we conclude that right-hand side of Eq. (178) becomes

$$
\begin{aligned}
-\hat{\partial}_{+\tau} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{\mathcal{N}_+(\mathbf{p})} (a_+(\mathbf{p}) e^{-ip_+.x} + a_+^\dagger(\mathbf{p}) e^{ip_+.x}) \\
\times [\gamma^\tau (l.p_+) + \varepsilon_{+\lambda}(p_+) \sigma^{\lambda\tau}].
\end{aligned}
$$

Expressed in terms of fields Eq. (178) takes the form

$$
\{\psi_+(x), \bar{Q}_{++}\} = \frac{1}{2} \hat{F}_{+\lambda\tau} \sigma^{\lambda\tau} - \gamma.\hat{\partial}_+ l.\partial\phi_+(x), \tag{180}
$$

where

$$
\phi_+(x) = i \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{\mathcal{N}_+(\mathbf{p})} (a_+(\mathbf{p}) e^{-ip_+.x} - a_+^\dagger(\mathbf{p}) e^{ip_+.x}). \tag{181}
$$

Similarly we have

$$
\begin{aligned}
\{\psi_-(x), \bar{Q}_{--}\} = -\sqrt{2} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{\mathcal{N}_-(\mathbf{p})} \\
\times (a_-(\mathbf{p}) e^{-ip_-.x} - a_-^\dagger e^{ip_-.x}) u_-(p_-) \bar{u}_+(p_-).
\end{aligned} \tag{182}
$$

Making use of the identities

$$
\gamma_5 \sigma^{\lambda\tau} = \frac{i}{2} \epsilon^{\lambda\tau\alpha\beta} \sigma_{\alpha\beta}, \tag{183}
$$

and

$$
\varepsilon_{+\lambda}(p_-) \hat{p}_{-\tau} \gamma_5 \sigma^{\lambda\tau} = i \hat{p}_{-\lambda} \varepsilon_{-\tau}(p_-) \sigma^{\lambda\tau}, \tag{184}
$$

we can show that

$$
\{\psi_-(x), \bar{Q}_{--}\} = \frac{1}{2} \hat{F}_{-\lambda\tau} \sigma^{\lambda\tau} - i\gamma_5 \gamma.\hat{\partial}_- l.\partial\phi_-(x), \tag{185}
$$

where

$$
\phi_-(x) = i \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{\mathcal{N}_-(\mathbf{p})} (a_-(\mathbf{p}) e^{-ip_-.x} - a_-^\dagger(\mathbf{p}) e^{ip_-.x}). \tag{186}
$$

These anticommutation relations for the supersymmetry charges involve the newly introduced additional fields $\phi_\pm(x)$. Since they are built from the mode operators $a_\pm(\mathbf{p}), a_\pm^\dagger(\mathbf{p})$ they involve the same degrees of freedom as the original vector fields.

2. Commutators for the photon fields

The commutation relations with the photon field yield

$$
\begin{aligned}
[Q_{++}, A_{+\mu}(x)] = 2i \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{N_+(\mathbf{p})} \\
\times (b_+(\mathbf{p}) e^{-ip_+.x} - i b_+^\dagger(\mathbf{p}) e^{ip_+.x}) \\
\times \left(e_{1\mu} - \frac{e_1.p_+}{l.p_+} l_\mu \right) u_+(p_+),
\end{aligned} \tag{187}
$$

which can be put in the form

$$
[Q_{++}, A_{+\mu}(x)] = 2i\psi_+(x) e_{1\mu} + 2l_\mu (e_1.\partial)\Omega_+(x), \tag{188}
$$

where the additional field $\Omega_+(x)$ depends on the photino mode operators in the form

$$\Omega_+(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{N_+(\mathbf{p})} \frac{1}{l.p_+} \times (b_+(\mathbf{p})e^{-ip_+.x} + ib_+^\dagger(\mathbf{p})e^{ip_+.x})u_+(p_+), \quad (189)$$

and $\Omega_+(x)$ satisfies

$$il.\partial\Omega_+(x) = \psi_+(x). \quad (190)$$

We also have

$$[Q_{--}, A_{-\mu}(x)] = 2\int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{N_-(\mathbf{p})} \times (b_-(\mathbf{p})e^{-ip_-.x} + ib_-^\dagger(\mathbf{p})e^{ip_-.x}) \times \left(e_{2\mu} - \frac{e_2.p_-}{l.p_-} l_\mu\right) u_-(p_-), \quad (191)$$

leading to

$$[Q_{--}, A_{-\mu}(x)] = 2\psi_-(x)e_{2\mu} - 2il_\mu(e_2.\partial)\Omega_-(x), \quad (192)$$

where

$$\Omega_-(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{N_-(\mathbf{p})} \frac{1}{l.p_-} \times (b_-(\mathbf{p})e^{-ip_-.x} - ib_-^\dagger(\mathbf{p})e^{ip_-.x})u_-(p_-), \quad (193)$$

and $\Omega_-(x)$ satisfies

$$il.\partial\Omega_-(x) = \psi_-(x). \quad (194)$$

It is interesting to note that the solutions of Eqs. (190) and (194) can be expressed in the form

$$\Omega_\pm(x) = i\int_0^\infty ds e^{-\epsilon s}\psi_\pm(x + sl). \quad (195)$$

The limit $\epsilon \to 0+$ is assumed. The integration over s creates the factor $1/(l.p_\pm)$ in the integrands on the right-hand sides of Eqs. (189) and (193). The point here is that although, in this form, the right-hand side of Eq. (195) can be expressed directly in terms of the Majorana field, the result is not local but requires an integration over a line of points emerging from x in the lightlike direction l. A similar factor appeared in the integrand of the right side of Eq. (163) for the anticommutators of $Q_\pm$. This suggests that a nonlocal structure is intrinsic to the supersymmetry algebra that can be formulated in the presence of Lorentz symmetry breaking (of the type we have considered). This may help to explain why there seems to be no obvious Noether method for constructing the supersymmetry charges in our model. In this context we should recall that while this is not true for an extrinsic Lorentz symmetry violation our model has a violation that is intrinsic.

Finally, we note that the fields $\phi_\pm(x)$ and $\Omega_\pm(X)$ are related through the commutation relations

$$[Q_{\pm\pm}, \phi_\pm] = -\Omega_\pm(x), \quad (196)$$

and

$$\{\Omega_+(x), \bar{Q}_{++}\} = i\int_0^\infty ds e^{-\epsilon s}\{\psi_+(x + sl), \bar{Q}_{++}\}, \quad (197)$$

and similarly

$$\{\Omega_-(x), \bar{Q}_{--}\} = i\int_0^\infty ds e^{-\epsilon s}\{\psi_-(x + sl), \bar{Q}_{--}\}. \quad (198)$$

Again we note the nonlocal character of these results.

D. Algebra of supersymmetric charges and fields in the Lorentz invariant limit

We again consider the limit of Lorentz symmetry. The four supersymmetry charges are given in Eq. (171). The nonvanishing anticommutators of the charges with the fields become

$$\{\psi_+(x), \bar{Q}_{++}\} = \frac{1}{2}F_{+\lambda\tau}\sigma^{\lambda\tau} - \gamma.\partial l.\partial\phi_+(x),$$
$$\{\psi_+(x), \bar{Q}_{-+}\} = \frac{1}{2}F_{-\lambda\tau}\sigma^{\lambda\tau} + i\gamma_5\gamma.\partial l.\partial\phi_-(x),$$
$$\{\psi_-(x), \bar{Q}_{--}\} = \frac{1}{2}F_{-\lambda\tau}\sigma^{\lambda\tau} - i\gamma_5\gamma.\partial l.\partial\phi_-(x),$$
$$\{\psi_-(x), \bar{Q}_{+-}\} = \frac{1}{2}F_{+\lambda\tau}\sigma^{\lambda\tau} + \gamma.\partial l.\partial\phi_+(x). \quad (199)$$

Using the definition in Eq. (174) for the total supersymmetry charge Q we can deduce that

$$\{\psi(x), \bar{Q}\} = F_{\lambda\tau}\sigma^{\lambda\tau}, \quad (200)$$

where, of course

$$\psi(x) = \psi_+(x) + \psi_-(x), \quad (201)$$

and

$$F_{\lambda\tau} = F_{+\lambda\tau} + F_{-\lambda\tau}(x). \quad (202)$$

This is exactly what we expect for this Majorana field anticommutator in the Lorentz symmetric case.

The nonvanishing commutators for the photon fields become in the Lorentz invariant limit is most conveniently considered in two stages. In the first stage we find directly

$$[Q_{++}, A_{+\mu}(x)] = 2i\psi_+(x)e_{1\mu} + 2il_\mu e_1.\partial \int_0^\infty dse^{-\epsilon s}\psi_+(x+sl),$$
$$[Q_{+-}, A_{+\mu}(x)] = -2i\psi_-(x)e_{1\mu} - 2il_\mu e_1.\partial \int_0^\infty dse^{-\epsilon s}\psi_-(x+sl),$$
$$[Q_{--}, A_{-\mu}(x)] = 2\gamma_5\psi_-(x)e_{2\mu} + 2l_\mu\gamma_5 e_2.\partial \int_0^\infty dse^{-\epsilon s}\psi_-(x+sl),$$
$$[Q_{-+}, A_{-\mu}(x)] = -2\gamma_5\psi_+(x)e_{2\mu} - 2l_\mu\gamma_5 e_2.\partial \int_0^\infty dse^{-\epsilon s}\psi_+(x+sl). \quad (203)$$

These equations, together with the related vanishing anticommutators, imply that

$$[Q, A_\mu(x)] = 2(ie_{1\mu} - \gamma_5 e_{2\mu})(\psi_+(x) - \psi_-(x)) + 2l_\mu(ie_1 - \gamma_5 e_2).\partial \int_0^\infty dse^{-\epsilon s} \times (\psi_+(x+sl) - \psi_-(x+sl)), \quad (204)$$

where Q is the total supersymmetry charge [see Eq. (174)] and $A_\mu(x) = A_{+\mu}(x) + A_{-\mu}(x)$. We now make use of the identity

$$\gamma_\mu u_+(p) = 2p_\mu u_+^{(0)} - l_\mu u_{n+}(p) + (ie_{1\mu} - \gamma_5 e_{2\mu})u_+(p), \quad (205)$$

where

$$u_{n+}(p) = \gamma.pn.\gamma u_+^{(0)}. \quad (206)$$

From Eq. (205) we then obtain

$$\gamma_\mu\psi_+(x) = 2\partial_\mu\psi_+^{(0)}(x) - l_\mu\psi_{n+}(x) + (ie_{1\mu} - \gamma_5 e_{2\mu})\psi_+(x). \quad (207)$$

We have for convenience, introduced

$$\psi_+^{(0)}(x) = i\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{N(\mathbf{p})}(b_+(\mathbf{p})e^{-ip.x} + ib_+^\dagger(\mathbf{p})e^{ip.x})u_+^{(0)}, \quad (208)$$

and

$$\psi_{n+}(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{N(\mathbf{p})}(b_+(\mathbf{p})e^{-ip.x} - ib_+^\dagger(\mathbf{p})e^{ip.x})u_{n+}(p). \quad (209)$$

By multiplying the terms in Eq. (205) by γ_5 we obtain

$$-\gamma_\mu u_-(p) = -2p_\mu u_-^{(0)} + l_\mu u_{n-}(p) + (e_{1\mu} - \gamma_5 e_{2\mu})u_-(p), \quad (210)$$

where $u_-^{(0)} = -\gamma_5 u_+^{(0)}$ and $u_{n-}(p) = -\gamma_5 u_{n+}(p)$. This leads to

$$-\gamma_\mu\psi_-(x) = -2i\partial_\mu\psi_-^{(0)}(x) + l_\mu\psi_{n-}(x) + (ie_{1\mu} - \gamma_5 e_{2\mu})\psi_-(x), \quad (211)$$

where

$$\psi_-^{(0)}(x) = i\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{N(\mathbf{p})}(b_-(\mathbf{p})e^{-ip.x} - ib_-^\dagger(\mathbf{p})e^{ip.x})u_-^{(0)}, \quad (212)$$

and

$$\psi_{n-}(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{N(\mathbf{p})}(b_-(\mathbf{p})e^{-ip.x} + ib_-^\dagger(\mathbf{p})e^{ip.x})u_{n-}(p). \quad (213)$$

Subtracting Eq. (211) from Eq. (207) we obtain

$$\gamma_\mu\psi(x) = 2i\partial_\mu\psi^{(0)}(x) - l_\mu\psi_n(x) + (ie_{1\mu} - \gamma_5 e_{2\mu})(\psi_+(x) - \psi_-(x)), \quad (214)$$

where $\psi^{(0)} = \psi_+^{(0)}(x) + \psi_-^{(0)}(x)$ etc. We can use Eq. (214) to eliminate $(\psi_+(x) - \psi_-(x))$ from Eq. (204) with the result

$$[Q, A_\mu(x)] = 2\gamma_\mu\psi(x) - 4\partial_\mu\psi^{(0)}(x) + 2l_\mu\psi_n(x) + 2l_\mu\partial^\alpha \int_0^\infty dse^{-\epsilon s}(\gamma_\alpha\psi(x+sl) - 2\partial_\alpha\psi^{(0)}(x) + l_\alpha\psi_n(x+sl)). \quad (215)$$

We note that $\gamma.\partial\psi(x) = \partial^2\psi^{(0)}(x) = 0$ and

$$l.\partial \int dse^{-\epsilon s}\psi_n(x+sl) = -\psi_n(x), \quad (216)$$

we find

$$[Q, A_\mu(x)] = 2\gamma_\mu\psi(x) - 4\partial_\mu\psi^{(0)}(x). \quad (217)$$

The outcome is exactly what we expect for the Lorentz symmetric model apart from the derivative term on the right-hand side of Eq. (217). This can be accommodated by incorporating an appropriate gauge transformation of $A_\mu(x)$ and in any case does not affect the result for $[Q, F_{\mu\nu}(x)]$. In fact we could have approached the whole analysis in the Lorentz symmetric case along the lines we have set out above without reference to Lorentz symmetry breaking. It would of course seem a rather roundabout approach. The key point is that when we have Lorentz symmetry there are four conserved supersymmetry charges that may be combined to make up the total supersymmetry charge, whereas there are only two when we violate Lorentz symmetry in the manner of our model.

V. WESS-ZUMINO MODEL

It is interesting to compare the results of the photon/photino model with similar birefringent phenomena that can be obtained in a simple non-interacting Wess-Zumino model [14] with the Majorana photino field and two scalar fields. We retain the Majorana Lagrangian of Eq. (103). We introduce a scalar field $\Phi_+(x)$ and a pseudoscalar field $\Phi_-(x)$ with a Lagrangian $\mathcal{L}_S(x)$, where

$$\mathcal{L}_S(x) = \frac{1}{2}((\partial + (\kappa/2)ll.\partial)\Phi_+(x))^2 + \frac{1}{2}((\partial - (\kappa/2)ll.\partial)\Phi_-(x))^2. \qquad (218)$$

The fields $\Phi_\pm(x)$ satisfy the equations of motion

$$(\partial^2 \pm \kappa(l.\partial)^2)\Phi_\pm(x) = 0. \qquad (219)$$

The plane waves satisfying these wave equations are

$$\Phi_\pm(x) = e^{-ip_\pm.x}, \qquad (220)$$

where

$$p_\pm^2 \pm \kappa(l.p_\pm)^2 = 0. \qquad (221)$$

It is obvious that these dispersion relations for the scalar particles associated with these plane waves yield the same "±" light cones as for the photons and the photinos.

The quantum fields can be expanded in terms of these wave functions in the form

$$\Phi_\pm(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^2}\frac{1}{\mathcal{N}_{S\pm}(\mathbf{p})}(c_\pm(\mathbf{p})e^{-ip_\pm.x} + c_\pm^\dagger(\mathbf{p})e^{ip_\pm.x}), \qquad (222)$$

where

$$[c_\pm(\mathbf{p}), c_\pm^\dagger(\mathbf{p}')] = (2\pi)^3\delta(\mathbf{p}-\mathbf{p}'), \qquad (223)$$

and $\mathcal{N}_{S\pm}(\mathbf{p})$ is chosen to guarantee the canonical commutation relations, that is

$$\mathcal{N}_{S\pm} = \sqrt{2(E_\pm \pm (\kappa/2)(E_\pm - p_3))}. \qquad (224)$$

A. Supersymmetry charges with scalar fields

Because of the conformity of the scalar and photino light cones we can realize the associated supersymmetry by means of conserved currents $J^\mu_{S\pm\pm}(x)$ that give rise to conserved supersymmetry charges $Q_{S\pm\pm}$, where

$$Q_{S\pm\pm} = \int d^3\mathbf{x} J^0_{S\pm\pm}(x). \qquad (225)$$

First we note the modified currents

$$\hat{J}^\mu_{S++}(x) = i\hat{\partial}_{+\lambda}\Phi_+(x)\gamma^\lambda\gamma^\mu\psi_+(x), \qquad (226)$$

and

$$\hat{J}^\mu_{S--}(x) = \gamma_5\hat{\partial}_{-\lambda}\Phi_-(x)\gamma^\lambda\gamma^\mu\psi_-(x), \qquad (227)$$

satisfy

$$\hat{\partial}_{\pm\mu}\hat{J}^\mu_{S\pm\pm}(x) = 0. \qquad (228)$$

The factor γ_5 in Eq. (227) is for future convenience. It relates to the fact that $\Phi_+(x)$ and $\Phi_-(x)$ have opposite parities. We then set

$$J^\mu_{S\pm\pm}(x) = \hat{J}^\mu_{S\pm\pm}(x) \pm (\kappa/2)l^\mu l.\hat{J}_{S\pm\pm}(x). \qquad (229)$$

It follow from the equations of motion, that

$$\partial_\mu J^\mu_{S\pm\pm}(x) = 0. \qquad (230)$$

The associated supersymmetry charges are then also conserved. We find

$$Q_{S++} = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{\sqrt{(E_+ - p_3)}} \times (-ic_+(\mathbf{p})b_+^\dagger(\mathbf{p}) - c_+^\dagger(\mathbf{p})b_+(\mathbf{p}))u_+(p_+), \qquad (231)$$

$$Q_{S--} = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{\sqrt{(E_- - p_3)}} \times (c_-(\mathbf{p})b_-^\dagger(\mathbf{p}) + ic_-^\dagger(\mathbf{p})b_-(\mathbf{p}))u_+(p_-). \qquad (232)$$

These have the same form (up to a normalization) as the corresponding charges linking photons and photinos. The nonvanishing anticommutation relations are

$$\{Q_{S\pm\pm}, \bar{Q}_{S\pm\pm}\} = 2\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{(E_\pm - p_3)}(c_\pm^\dagger(\mathbf{p})c_\pm(\mathbf{p}) + b_\pm^\dagger(\mathbf{p})b_\pm(\mathbf{p}))u_+(p_\pm)\bar{u}_+(p_\pm). \quad (233)$$

Making use of the identity in Eq. (162) we have

$$\{Q_{S++}, \bar{Q}_{S++}\} = \int \frac{d^3\mathbf{p}}{(2\pi)^3}(c_+^\dagger(\mathbf{p})c_+(\mathbf{p}) + b_+^\dagger(\mathbf{p})b_+(\mathbf{p})) \times \left[\gamma.\hat{p}_+ + \sqrt{2}\frac{\varepsilon_+^\lambda(p_+)\hat{p}_+^\tau\sigma_{\lambda\tau}}{(E_+ - p_3)}\right], \quad (234)$$

and

$$\{Q_{S--}, \bar{Q}_{S--}\} = \int \frac{d^3\mathbf{p}}{(2\pi)^3}(c_-^\dagger(\mathbf{p})c_-(\mathbf{p}) + b_-^\dagger(\mathbf{p})b_-(\mathbf{p})) \times \left[\gamma.\hat{p}_- + \sqrt{2}\frac{\varepsilon_+^\lambda(p_-)\hat{p}_-^\tau\sigma_{\lambda\tau}}{(E_- - p_3)}\right]. \quad (235)$$

B. Commutation relations of supersymmetry charges with fields

We have for the mode operators

$$[Q_{S++}, c_+(\mathbf{p})] = \frac{1}{\sqrt{(E_+ - p_3)}}b_+(\mathbf{p})u_+(p_+),$$
$$[Q_{S++}, c_+^\dagger(\mathbf{p})] = \frac{-i}{\sqrt{(E_+ - p_3)}}b_+^\dagger(\mathbf{p})u_+(p_+),$$
$$\{b_+(\mathbf{p}), \bar{Q}_{S++}\} = \frac{-1}{\sqrt{(E_+ - p_3)}}c_+(\mathbf{p})\bar{u}_+(p_+),$$
$$\{b_+^\dagger(\mathbf{p}), \bar{Q}_{S++}\} = \frac{i}{\sqrt{(E_+ - p_3)}}c_+^\dagger(\mathbf{p})\bar{u}_+(p_+), \quad (236)$$

and

$$[Q_{S--}, c_-(\mathbf{p})] = \frac{-i}{\sqrt{(E_- - p_3)}}b_-(\mathbf{p})u_+(p_-),$$
$$[Q_{S--}, c_-^\dagger(\mathbf{p})] = \frac{1}{\sqrt{(E_- - p_3)}}b_-^\dagger(\mathbf{p})u_+(p_-),$$
$$\{b_-(\mathbf{p}), \bar{Q}_{S--}\} = \frac{-i}{\sqrt{(E_- - p_3)}}c_-(\mathbf{p})\bar{u}_+(p_-),$$
$$\{b_-^\dagger(\mathbf{p}), \bar{Q}_{S--}\} = \frac{1}{\sqrt{(E_- - p_3)}}c_-^\dagger(\mathbf{p})\bar{u}_+(p_-), \quad (237)$$

giving rise to the results

$$[Q_{S++}, \Phi_+(x)] = \psi_+(x),$$
$$\{\psi_+(x), \bar{Q}_{S++}\} = \frac{-i}{2}\left[\gamma.\hat{\partial}_+\Phi_+(x) + e_{1\lambda}\hat{\partial}_\tau\sigma^{\lambda\tau}\Phi_+(x) + e_1.\partial l_\lambda\hat{\partial}_{+\tau}\sigma^{\lambda\tau}\int_0^\infty ds e^{-\epsilon s}\Phi_+(x+sl)\right], \quad (238)$$

and

$$[Q_{S--}, \Phi_-(x)] = -i\gamma_5\psi_-(x),$$
$$\{\psi_-(x), \bar{Q}_{S--}\} = \frac{1}{2}\gamma_5\left[\gamma.\hat{\partial}_-\Phi_-(x) + e_{1\lambda}\hat{\partial}_{-\tau}\sigma^{\lambda\tau}\Phi_-(x) + e_1.\partial l_\lambda\hat{\partial}_{-\tau}\sigma^{\lambda\tau}\int_0^\infty ds e^{-\epsilon s}\Phi_-(x+sl)\right]. \quad (239)$$

These results are similar in character to those for the photon/photino model and reveal the presence of nonlocal terms in the anticommutators of the conserved supersymmetry charges with the photino fields. The commutators with the scalar fields are in this case purely local and yield the photino fields without any further contributions.

C. Interchange mass-shell cones in Wess-Zumino model

Just as for the photon/photino model we can interchange the mass-shell cones of the Majorana particles by setting $\xi = -\kappa/(2\sqrt{2})$. This leads in a straightforward way to conserved supercurrents $J^\mu_{S\pm\mp}(x) = \hat{J}_{S\pm\mp}(x) \pm (\kappa/2)l^\mu l.\hat{J}^\mu_{S\pm\mp}(x)$, where

$$\hat{J}^\mu_{S+-}(x) = i\hat{\partial}_{+\lambda}\Phi_+(x)\gamma^\lambda\gamma^\mu\psi_-(x), \quad (240)$$

and

$$\hat{J}^\mu_{S-+}(x) = \gamma_5\hat{\partial}_{-\lambda}\Phi_-(x)\gamma^\lambda\gamma^\mu\psi_+(x). \quad (241)$$

Here $\psi_\mp(x)$ has the form given by Eq. (165). Computing the now conserved supercharges in the usual way we find

$$Q_{S+-} = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{\sqrt{(E_+ - p_3)}} \times (ic_+(\mathbf{p})b_-^\dagger(\mathbf{p}) - c_+^\dagger(\mathbf{p})b_-(\mathbf{p}))u_-(p_+), \quad (242)$$

$$Q_{S-+} = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{\sqrt{(E_- - p_3)}} \times (-c_-(\mathbf{p})b_+^\dagger(\mathbf{p}) + ic_-^\dagger(\mathbf{p})b_+(\mathbf{p}))u_-(p_-). \quad (243)$$

We have, following the same pattern of argument as Sec. IVA

$$\{Q_{S\pm\mp},\bar{Q}_{S\pm\mp}\} = \int\frac{d^3\mathbf{p}}{(2\pi)^3}\left[\gamma.\hat{p}_\pm - \frac{\sqrt{2}\varepsilon_{+\lambda}(p_\pm)\hat{p}_{\pm\tau}\sigma^{\lambda\tau}}{E_\pm - p_3}\right] \times (c^\dagger_\pm(\mathbf{p})c_\pm(\mathbf{p}) + b^\dagger_\mp(\mathbf{p})b_\mp(\mathbf{p})). \quad (244)$$

The nonvanishing (anti)commutation relations of these charges are

$$[Q_{S+-},\Phi_+(x)] = \psi_-(x),$$
$$\{\psi_-(x),\bar{Q}_{S+-}\} = \frac{-i}{2}\left[\gamma.\hat{\partial}_+\Phi_+(x) - e_{1\lambda}\hat{\partial}_{+\tau}\sigma^{\lambda\tau}\Phi_+(x) - e_1.\partial l_\lambda\hat{\partial}_{+\tau}\sigma^{\lambda\tau}\int_0^\infty ds e^{-\epsilon s}\Phi_+(x+sl)\right], \quad (245)$$

and

$$[Q_{-+},\Phi_-(x)] = -i\gamma_5\psi_+(x),$$
$$\{\psi_+(x),\bar{Q}_{S-+}\} = \frac{1}{2}\gamma_5\left[\gamma.\hat{\partial}_-\Phi_-(x) - e_{1\lambda}\hat{\partial}_{-\tau}\sigma^{\lambda\tau}\Phi_-(x) - e_1.\partial l_\lambda\hat{\partial}_{-\tau}\sigma^{\lambda\tau}\int_0^\infty ds e^{-\epsilon s}\Phi_-(x+sl)\right]. \quad (246)$$

The results in Eqs. (245) and (246), although a little simpler than the corresponding results for the photon/photino model, have in common the feature of involving nonlocal terms with integrals over a line starting at x and parallel to the null 4-vector l.

D. Lorentz-invariant limit for the Wess-Zumino model

In the same way as for the photon/photino model we can examine the limit of Lorentz invariance for the Wess-Zumino model. Recall that when $\kappa \to 0\pm$ the 4-momenta satisfy $p_\pm = \hat{p}_\pm = p$ where $p^2 = 0$ and all of the supercharges are conserved. The form of the of the supersymmetry charges is listed in Appendix B together with the complete set of anticommutators. We again find that the off-diagonal anticommutators cancel in pairs. We may therefore define an overall supersymmetry charge $Q_S = Q_{S++} + Q_{S--} + Q_{S+-} + Q_{S-+}$. It is easy to establish that

$$\{Q_S,\bar{Q}_S\} = 2\gamma.P, \quad (247)$$

where P^μ is the 4-momentum operator for the fields in the model.

From Eqs. (238), (239), (245), and (246) we can, in the Lorentz invariant limit, obtain the results

$$[Q_{S++},\Phi_+(x)] = \psi_+(x),$$
$$[Q_{S--},\Phi_-(x)] = -i\gamma_5\psi_-(x),$$
$$[Q_{S+-},\Phi_+(x)] = \psi_-(x),$$
$$[Q_{S-+},\Phi_-(x)] = -i\gamma_5\psi_+(x). \quad (248)$$

These may be combined appropriately to yield

$$[Q_S,\Phi_+(x) \pm i\Phi_-(x)] = (1\pm\gamma_5)\psi(x). \quad (249)$$

We can also extract the results

$$\{\psi_+(x),\bar{Q}_{S++}\} = -\frac{i}{2}\gamma.\partial\Phi_+(x) + \cdots,$$
$$\{\psi_-(x),\bar{Q}_{S--}\} = \frac{1}{2}\gamma_5\gamma.\partial\Phi_-(x) + \cdots,$$
$$\{\psi_-(x),\bar{Q}_{S+-}\} = -\frac{i}{2}\gamma.\partial\Phi_+(x) + \cdots,$$
$$\{\psi_+(x),\bar{Q}_{S-+}\} = \frac{1}{2}\gamma_5\gamma.\partial\Phi_-(x) + \cdots. \quad (250)$$

The ellipses in Eq. (250) indicate terms that cancel from the final result. In particular these cancelling terms include the nonlocal contributions. Finally, we obtain the purely local result

$$\{\psi(x),\bar{Q}_S\} = -i\gamma.\partial\Phi_+(x) + \gamma_5\gamma.\partial\Phi_-(x). \quad (251)$$

This result may be reexpressed as

$$\{(1\pm\gamma_5)\psi(x),\bar{Q}_S\} = \pm i(1\pm\gamma_5)\gamma.\partial(\Phi_+(x) \pm i\Phi_-(x)), \quad (252)$$

which is the standard result for the Lorentz invariant Wess-Zumino model.

VI. CONCLUSIONS

We have argued for a distinction between an extrinsic breaking of Lorentz symmetry that can be “removed” by an appropriate coordinate transformation and an intrinsic breaking that cannot be so removed. In the extrinsic case such a model can retain in full the original supersymmetry in a “disguised" form but with essentially the same formal algebraic structure. This does not mean that the violation of Lorentz invariance is illusory but merely that the allowed observer frames may differ from the coordinate frames natural to the model under investigation. In the intrinsic case the origin of Lorentz symmetry violation is irremovable by a change of coordinates. A clear case is the presence of birefringence. It is then less obvious *a priori* that any supersymmetry can be retained. However, we have shown from an examination of two simple models, a photon/photino model and a Wess-Zumino model (both exhibiting birefringence), that some supersymmetry can indeed be retained.

In the photon/photino model the Lorentz symmetry violation for the photons is provided by coupling the vector field to a Weyl tensor background field. The Weyl tensor is of class N in the Petrov classification scheme. This is the simplest case. The result is a birefringent form of photon propagation. The photino field is a (massless) Majorana field coupled to a Lorentz symmetry violating background bivector field related to the (Petrov class N) Weyl field controlling photon propagation. By adjusting the strength of the coupling it is possible to align the birefringence of the photinos with that of the photons. This means that for every photon of spatial momentum $\mathbf{p}$ there is an appropriate companion photino with the same momentum and energy. This is the basis of the supersymmetry in the model. In the Wess-Zumino model we retain the Majorana field and adjust the scalar field propagation so that it exhibits birefringence that matches appropriately the propagation of the Majorana field.

Consideration of the double light cone structure that governs the propagation of both photons and photinos or scalar particles and Majorana particles, shows that there are limitations on the ensemble of allowed observational reference frames. No boost to a reference frame is acceptable if it requires the observer to travel faster than particles on the slower light cone. This constraint can be re-expressed as a bound on the magnitude of the coupling to the Lorentz violating background fields. When this bound is broken the (positive time) slow cone tilts over so far that the time axis of the observer's coordinate system no longer lies within it thus rendering the coordinates inappropriate for describing the unitary evolution of the model. Of course in order to genuinely observe the system there has to be an interrogating interaction between model and observer. In that case an observer breaking the slow cone speed barrier will generate an associated shower of Čerenkov radiation (see Refs. [27,28]) thus paying a speeding penalty by ceasing to be an observer and effectively becoming part of the model. This Čerenkov phenomenon can of course be detected by observers that are obeying the speed limit. These considerations apply more generally to any model with multiple light cones.

If we accept the speeding restriction on our observation frame we can proceed to study the equations of motion of the model and quantize it in a conventional way. We use a slightly modified version of the Gupta-Bleuler method to quantize the vector field $A_\mu(x)$ of the photon resulting in a breakup into three parts, $A_{+\mu}(x)$, associated with the slow cone, $A_{-\mu}(x)$ associated with the fast cone and $A_{U\mu}(x)$ which comprises a pure derivative gauge field and a conjugate zero norm term. They do not contribute to matrix elements of physical observables. Interestingly the light cone appropriate for the propagation of $A_{U\mu}(x)$, is the standard cone that is invariant under the Lorentz transformations that connect the coordinate systems of the set of observers. For this additional reason it is essential that $A_{U\mu}(x)$ is not involved in the construction of observables which relate to the energies and momenta of photons and photinos traveling on the fast and slow light cones.

The Majorana equation appropriate to the photino field $\psi(x)$ is modified in a way that breaks Lorentz invariance. The resulting birefringence, makes it possible to view it as the sum of two parts, $\psi_+(x)$ associated with the slow cone and $\psi_-(x)$ associated with the fast cone. From $A_{+\mu}(x)$ and $\psi_+(x)$ we can construct a (spinor-valued) current $J_{++\mu}(x)$ that is conserved and which gives rise to a constant supersymmetry charge Q_{++}. Similarly a supersymmetry charge Q_{--} can be obtained from a conserved current $J_{--\mu}(x)$ constructed from $A_{-\mu}(x)$ and $\psi_-(x)$. The nonzero anticommutation relations satisfied by the two charges each have two contributions [see Eq. (163)]. One part has the form $\gamma.\hat{P}_\pm$ and is analogous to the standard result but with the momentum operator P_μ replaced by $\hat{P}_{\pm\mu}$. The second part has a more elaborate form that we argue later has indications of nonlocality. In the Lorentz invariant limit ($\kappa = 0$) the first part reduces to the standard form but the second part remains. The full understanding of this limit requires an examination of the regime in which $\xi = -\kappa/2\sqrt{2}$. In this case the fast and slow light cones for photinos interchange and the conserved supersymmetry currents are $J_{+-\mu}(x)$ and $J_{-+\mu}(x)$ [see Eq. (170)]. When $\kappa = 0$ all four currents are conserved and it is then possible to build a total supersymmetry charge that has the standard anticommutation relations (see Sec. IV B). It is evident then that the limit back to Lorentz invariance is not straightforward and issues related to the complexity of the algebra of conserved charges reappear when their effect on the photon and photino fields are considered.

The various (anti)commutation relations with the mode operators is straightforward and the conversion $a_\pm(\mathbf{p}) \leftrightarrow b_\pm(\mathbf{p})$ is as expected [see Eqs. (176) and (177)]. The (anti)commutation relations of $Q_{\pm\pm}$ with the dynamical fields is rather less conventional and involves the introduction of the modified fields $\phi_\pm(x)$ and $\Omega_\pm(x)$ that have a nonlocal relationship with $A_{\pm\mu}(x)$ and $\psi_\pm(x)$ even though they are constructed from the same sets of mode operators. We speculate that this nonlocal property is related to the apparent impossibility of constructing in our photon/photino model, conserved supersymmetry currents by means of the Noether method. Nevertheless, if we accept these complexities and limitations of the model we can argue that even in this model with intrinsic Lorentz symmetry violation it is possible to identify a remaining supersymmetry structure.

The Wess-Zumino model with Lorentz symmetry breaking exhibits the same features as the photon/photino model. There is the same reduced number of conserved supersymmetry currents and charges. The (anti)commutation relations with the fields again show nonlocal outcomes. The limit back to Lorentz symmetry yields a second set of conserved charges that permit the construction of a complete conserved charge Q_S with the appropriate algebraic properties for the Lorentz invariant Wess-Zumino model. The nonlocal terms again cancel in a satisfactory manner.

Of course a major limitation of our models is that the fields and their associated particles are noninteracting. It would therefore be of interest to generalise the models so that the fields do have interactions such as a non-Abelian gauge invariance. The task would, in particular, be to find a method of identifying $A_{\pm a\mu}(x)$, $\psi_{\pm a}(x)$, and $\Phi_{\pm a}(x)$ (a being the group multiplet label). This would presumably be achieved perturbatively and would intersect with the related task of confirming the renormalization properties of the interacting model. It would also be interesting to generalise the nature of the Lorentz symmetry violation for the photon/photino model to other Petrov classes. In this context it should be noted that Petrov class D is the only other example of the Weyl tensor giving rise to a dispersion relation that factorises into two lightcones. The other classes, I, II, and III of Weyl tensor yield dispersion relations that are intrinsically quartic in the photon momentum [17]. Since a product structure for the dispersion relation holds generally for modified Dirac equations [29] this may prevent the aligning of the photon and photino dispersion relations and the achieving of supersymmetry in the photon/photino model in those cases.

Finally, we remark that there is an intriguing parallelism of supersymmetry structure between our models and the $N = 1/2$ supersymmetry proposed by Seiberg [30] in the context on noncommutative spacetime geometry. A major difference between the models is that the Seiberg model is based on the standard chiral structure of supersymmetry and retains Lorentz invariance whereas in the models analyzed here the violation of Lorentz invariance replaces chiral structure with charge-conjugation symmetry.

ACKNOWLEDGMENTS

We acknowledge many helpful comments and suggestions by G. M. Shore and J. M. Drummond. This work has been partially supported the United Kingdom Technologies Facilities Council Grant No. ST/T000694/1.

APPENDIX A: ANTICOMMUTATORS FOR SUPERSYMMETRY CHARGES IN LORENTZ SYMMETRY LIMIT

We list here the anticommutators, in the Lorentz symmetry limit, for the supersymmetry charges of the photon/photino model. The 4-momentum p satisfies $p^2 = 0$. We have made the contractions $a_\pm(\mathbf{p}) \to a_\pm$ etc. for compactness:

$$\{Q_{++}, \bar{Q}_{++}\} = 8 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{E - p_3} (a_+^\dagger a_+ + b_+^\dagger b_+) u_+(p) \bar{u}_+(p),$$

$$\{Q_{++}, \bar{Q}_{--}\} = 0,$$

$$\{Q_{++}, \bar{Q}_{+-}\} = 4 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{E - p_3} (b_+^\dagger b_- + b_+ b_-^\dagger) u_+(p) \bar{u}_-(p),$$

$$\{Q_{++}, \bar{Q}_{-+}\} = -4i \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{E - p_3} (a_+^\dagger a_- + a_+ a_-^\dagger) u_+(p) \bar{u}_-(p),$$

$$\{Q_{--}, \bar{Q}_{--}\} = 8 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{E - p_3} (a_-^\dagger a_- + b_-^\dagger b_-) u_+(p) \bar{u}_+(p),$$

$$\{Q_{--}, \bar{Q}_{++}\} = 0,$$

$$\{Q_{--}, \bar{Q}_{+-}\} = 4i \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{E - p_3} (a_-^\dagger a_+ + a_- a_+^\dagger) u_+(p) \bar{u}_-(p),$$

$$\{Q_{--}, \bar{Q}_{-+}\} = 4 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{E - p_3} (b_-^\dagger b_+ + b_- b_+^\dagger) u_+(p) \bar{u}_-(p),$$

$$\{Q_{+-}, \bar{Q}_{+-}\} = 8 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{E - p_3} (a_+^\dagger a_+ + b_-^\dagger b_-) u_-(p) \bar{u}_+(p),$$

$$\{Q_{+-}, \bar{Q}_{-+}\} = 0,$$

$$\{Q_{+-}, \bar{Q}_{++}\} = 4 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{E - p_3} (b_-^\dagger b_+ + b_- b_+^\dagger) u_-(p) \bar{u}_+(p),$$

$$\{Q_{+-},\bar{Q}_{--}\} = -4i\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{E-p_3}(a_+^\dagger a_- + a_+ a_-^\dagger)u_-(p)\bar{u}_+(p),$$
$$\{Q_{-+},\bar{Q}_{-+}\} = 8\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{E-p_3}(a_-^\dagger a_- + b_+^\dagger b_+)u_-(p)\bar{u}_-(p),$$
$$\{Q_{-+},\bar{Q}_{+-}\} = 0,$$
$$\{Q_{-+},\bar{Q}_{++}\} = 4i\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{E-P_3}(a_- a_+^\dagger + a_-^\dagger a_+)u_-(p)\bar{u}_+(p),$$
$$\{Q_{-+},\bar{Q}_{--}\} = 4\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{E-p_3}(b_+^\dagger b_- + b_+ b_-^\dagger)u_-(p)\bar{u}_+(p),$$

The canceling pairs of off-diagonal contributions are obvious from the above list.

APPENDIX B: LORENTZ SYMMETRIC LIMIT FOR THE WESS-ZUMINO MODEL

Using the same notational conventions as in Appendix A we have, in the limit of Lorentz invariance,

$$Q_{S++} = \int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{\sqrt{E-p_3}}(-ic_+ b_+^\dagger - c_+^\dagger b_+)u_+(p),$$
$$Q_{S--} = \int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{\sqrt{E-p_3}}(c_- b_-^\dagger + ic_-^\dagger b_-)u_+(p),$$
$$Q_{S+-} = \int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{\sqrt{E-p_3}}(ic_+ b_-^\dagger - c_+^\dagger b_-)u_-(p),$$
$$Q_{S-+} = \int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{\sqrt{E-p_3}}(-c_- b_+^\dagger + ic_-^\dagger b_+)u_-(p). \tag{B1}$$

The anticommutation relations for these charges are

$$\{Q_{S++},\bar{Q}_{S++}\} = \int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{2}{E-p_3}(c_+^\dagger c_+ + b_+^\dagger b_+)u_+(p)\bar{u}_+(p),$$
$$\{Q_{S++},\bar{Q}_{S--}\} = 0,$$
$$\{Q_{S++},\bar{Q}_{S+-}\} = \int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{E-p_3}(-b_+^\dagger b_- + b_-^\dagger b_+)u_+(p)\bar{u}_-(p),$$
$$\{Q_{S++},\bar{Q}_{S-+}\} = \int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{i}{E-p_3}(c_+^\dagger c_- + c_+^\dagger c_-)u_+(p)\bar{u}_-(p),$$
$$\{Q_{S--},\bar{Q}_{S++}\} = 0,$$
$$\{Q_{S--},\bar{Q}_{S--}\} = \int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{2}{E-p_3}(c_-^\dagger c_- + b_-^\dagger b_-)u_+(p)\bar{u}_+(p),$$
$$\{Q_{S--},\bar{Q}_{S+-}\} = \int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{-i}{E-p_3}(c_+^\dagger c_- + c_-^\dagger c_+)u_+(p)\bar{u}_-(p),$$
$$\{Q_{S--},\bar{Q}_{S-+}\} = \int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{E-p_3}(-b_-^\dagger b_+ + b_+^\dagger b_-)u_+(p)\bar{u}_-(p),$$
$$\{Q_{S+-},\bar{Q}_{S++}\} = \int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{E-p_3}(-b_-^\dagger b_+ + b_+^\dagger b_-)u_-(p)\bar{u}_+(p),$$
$$\{Q_{S+-},\bar{Q}_{S--}\} = \int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{i}{E-p_3}(c_-^\dagger c_+ + b_+^\dagger b_-)u_-(p)\bar{u}_+(p),$$

$$\{Q_{S+-}, \bar{Q}_{S+-}\} = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{2}{E - p_3} (c_+^\dagger c_+ + b_-^\dagger b_-) u_-(p) \bar{u}_-(p),$$
$$\{Q_{S+-}, \bar{Q}_{S-+}\} = 0,$$
$$\{Q_{S-+}, \bar{Q}_{S++}\} = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{-i}{E - p_3} (c_+^\dagger c_- + c_-^\dagger c_+) u_-(p) \bar{u}_+(p),$$
$$\{Q_{S-+}, \bar{Q}_{S--}\} = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{E - p_3} (-b_+^\dagger b_- + b_-^\dagger b_+) u_-(p) \bar{u}_+(p),$$
$$\{Q_{S-+}, \bar{Q}_{S+-}\} = 0,$$
$$\{Q_{S-+}, \bar{Q}_{S-+}\} = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{2}{E - p_3} (c_-^\dagger c_- + b_+^\dagger b_+) u_-(p) \bar{u}_-(p). \tag{B2}$$

By inspection it is clear that the off-diagonal anticommutators cancel in the sum.

[1] A. R. Vieira and N. Sherrill, Lorentz-violating running of coupling constants, in *Eighth Meeting on CPT and Lorentz Symmetry* (World Scientific, Singapore, 2020), pp. 98–101.

[2] A. Held, J. M. Pawlowski, M. Schiffer, F. Versteegen, S. P. de Alwis, and A. Eichhorn, Asymptotic safety, string theory and weak gravity conjecture, Phys. Lett. B **798**, 134991 (2019).

[3] A. Platania, A. Eichhorn, and M. Schiffer, Lorentz invariance violations in the interplay of quantum gravity with matter, Phys. Rev. D **102**, 026007 (2020).

[4] I. T. Drummond, Lorentz symmetry violation in QCD and the frustration of asymptotic freedom, Phys. Rev. D **101**, 085004 (2020).

[5] M. S. Berger and V. A. Kostelecký, Supersymmetry and Lorentz violation, Phys. Rev. D **65**, 091701 (2002).

[6] S. G. Nibbelink and M. Pospelov, Lorentz Violation in Supersymmetric Field Theories, Phys. Rev. Lett. **94**, 081601 (2005).

[7] D. Colladay and P. McDonald, Vector superfields and Lorentz violation, Phys. Rev. D **83**, 025021 (2011).

[8] J. R. Nascimento, C. F. Farias, A. C. Lehun, and A. Yu. Petrov, Superfield supersymmetric aetherlike Lorentz-breaking models, Phys. Rev. D **86**, 065035 (2012).

[9] J. R. Nascimento, C. F. Farias, A. C. Lehum, and A. Yu. Petrov, Supergauge theories in aether superspace, Phys. Rev. D **88**, 045022 (2013).

[10] E. Lunghi, V. A. Kostelecký, and A. R. Vieira, Lorentz violation and deep inelastic scattering, Phys. Lett. B **769**, 272 (2017).

[11] Enrico Lunghi and Nathan Sherrill, Lorentz violation and the electron-ion collider, Phys. Rev. D **98**, 115018 (2018).

[12] N. Sherrill, V. A. Kostelecký, E. Lunghi, and A. R. Vieira, Lorentz and *CPT* violation in partons, J. High Energy Phys. 04 (2020) 143.

[13] ZEUS Collaboration, Search for effective Lorentz and *CPT* violation using ZEUS data, Phys. Rev. D **107**, 092008 (2023).

[14] J. Wess and B. Zumino, A Lagrangian model invariant under supergauge transformations, Phys. Lett. **49B**, 52 (1974).

[15] D. Colladay and V. A. Kostalecký, Lorentz-violating extension of the Standard Model, Phys. Rev. D **58**, 116002 (1998).

[16] M. Schreck, Quantum field theory based on birefringent modified Maxwell theory, Phys. Rev. D **89**, 085013 (2014).

[17] I. T. Drummond, Bimetric QED, Phys. Rev. D **95**, 025006 (2017).

[18] A. Z. Petrov, Classification of spaces defined by gravitational fields, Uch. Zapiski Kazan Gos. Univ. **144**, 55 (1954).

[19] R. Penrose and W. Rindler, *Spinors and Spacetime 2: Spinor and Twistor Methods in Spacetime Geometry* (Cambrige University Press, Cambridge, England, 1986).

[20] D. Colladay, P. McDonald, and R. Potting, Gupta-Bleuler photon quantization in the sme, Phys. Rev. D **89**, 085014 (2014).

[21] D. Colladay, P. McDonald, J. Noordmans, and R. Potting, Covariant quantisation of *CPT*-violating photons, Phys. Rev. D **95**, 025025 (2017).

[22] F. R. Klinkhammer and M. Schreck, New two-sided bound on the isotropic Lorentz-violating parameter of modified Maxwell theory, Phys. Rev. D **78**, 085026 (2008).

[23] Claude Itzykson and Jean-Bernard Zuber, *Quantum Field Theory* (McGraw-Hill International Book Company, New York, 1980), pp. 22–25.

[24] V. A. Kostelecký and M. Mewes, Fermions with Lorentz-violating operators of arbitrary dimension, Phys. Rev. D **88**, 096006 (2013).

[25] V. A. Kostelecký and R. Lehnert, Stability and Lorentz and *CPT* violation, Phys. Rev. D **63**, 065008 (2001).
[26] J. A. A. S. Reis and M. Schreck, Formal developments for Lorentz-violating Dirac fermions and neutrinos, Symmetry **11**, 1197 (2019).
[27] R. Lehnert and R. Potting, Čerenkov effect in Lorentz-violating vacua, Phys. Rev. D **70**, 125010 (2004).
[28] M. Schreck, Vacuum Čerenkov radiation for Lorentz-violating fermions, Phys. Rev. D **96**, 095026 (2017).
[29] D. Colladay, Factoring the dispersion relation in the presence of Lorentz violation, J. Phys. A **43**, 275202 (2010).
[30] N. Seiberg, Noncommutative superspace, $N = 1/2$ supersymmetry, field theory and string theory, J. High Energy Phys. 06 (2003) 010.