AdS wormholes from Ricci-flat/AdS correspondence

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We discuss the wormholes in general dimensions by studying the Einstein-phantom scalar field with and without the cosmological constant. Solving anti–de Sitter (AdS) wormholes in general dimensions is hard due to the nonlinear nature of the theory. In this work, we implement the AdS/Ricci-flat correspondence, extended to include the axion field (the phantom scalar field), to construct AdS wormholes. Wormholes of the Ellis-Bronnikov class are discussed in general dimensions.

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I. INTRODUCTION

The wormhole is thought to be the tool for fast interstellar travel, for it connects different parts of spacetimes via the throat [1,2]. Traversable wormholes have been studied in four dimensions and higher dimensions [3]. The most intuitive wormhole is the Ellis wormhole from the theory $\mathcal{S} = \frac{1}{16\pi G}\int \mathrm{d}x^4\sqrt{-g}(R + \frac{1}{2}(\partial\chi)^2)$. The singularity in Schwarzschild spacetime hinges the exploration of particle theories in gravitational fields as it is not geodesically complete. The Ellis wormhole was constructed in the effort to remove the problematic singularities in Schwarzschild spacetimes [4–6]. It was first discussed in four dimensions where it was illustrated as a drainhole. The Ellis wormhole in higher dimensions was later studied in different settings [7–9]. Having the general ansatz in a higher dimension, the general Ellis-Bronnikov class wormhole was also constructed [10–12]. Specific solutions of wormholes with a bare cosmological constant in four dimensions were found under various conditions [13,14].

However, wormholes with a bare cosmological constant in higher dimensions are yet to be constructed [15]. Such anti–de Sitter (AdS) wormholes play a crucial role since AdS/CFT correspondence becomes a rich field of study [16–19]. In the context of AdS/CFT correspondence, the dimension of the conformal field theory on the boundary is of codimension 1 to the AdS gravity in the bulk with generic dimensions. Thus looking for AdS wormhole solutions in a general dimension is of particular interest. In this paper, we use the Ricci-flat/AdS correspondence to construct the solutions of wormholes with a bare cosmological constant in general dimensions from Ellis-Bronnikov wormhole solutions.

The AdS/Ricci-flat correspondence relates the solutions in asymptotically AdS space on a torus and asymptotically flat space on a sphere [20,21]. Kuluza-Klein (KK) reduction was used to demonstrate the validity of this correspondence. KK reduction comes from the string compactification. It reduces higher dimensional theories to lower dimensional ones, keeping the additional dimensions compactified [22]. Previous works show that this correspondence was used to construct new solutions in supergravity theories [23], where a matter field was added to the AdS theories on torus. The correspondence interchanges the matter field with the Ricci scalar of the sphere in the pure gravity theory.

In this paper, we consider the action for the AdS wormhole is given by

$$\mathcal{S} = \frac{1}{16\pi G}\int \mathrm{d}x^n\sqrt{-g}\left(R - 2\Lambda + \frac{1}{2}(\partial\chi)^2\right), \qquad (1)$$

where χ is the phantom scalar field. In this paper we add matter fields, the phantom field χ, to both theories in the correspondence. We found that the matter fields descend down after the KK reduction calculation for both theories. So the AdS/Ricci-flat correspondence interchanges the phantom field in the Ellis-Bronnikov wormhole with the matter field in the AdS wormhole. Such an AdS wormhole needs to violate the null energy condition [24,25]. The null energy condition is believed to be satisfied in general relativity. However, the Ellis wormhole is opened via the addition of a ghost field, or what we call an axion. So the criterion for a solution to be a wormhole is that the solution violates the null energy condition. The solution constructed from AdS/Ricci-flat correspondence does satisfy this requirement.

The organization of this paper is as follows. In Sec. II, we describe the correspondence of two theories and discuss the identification of parameters. In Sec. III, we find the solutions of wormholes with an axion field, namely the Ellis wormhole, and then we use the correspondence map to construct the solutions of AdS Ellis wormholes.

In Sec. IV, we further the discussion in the last section to include the generic solution of the Ellis-Bronnikov wormhole. We see that in four dimensions, this generic solution reduces back to the form in previous sections. In Sec. V, we find the solution of the AdS wormhole in the general dimension by applying AdS/Ricci-flat correspondence. We show this solution reduces back to the AdS Ellis wormhole via coordinate transformations. In Sec. VI, we demonstrate that the AdS wormhole solution violates the null energy condition so that it indeed satisfies the criterion for a wormhole to be traversable. In the appendixes, detailed computations of the KK reduction and the Ellis-Bronnikov wormhole are presented.

II. CORRESPONDENCE VIA KALUZA-KLEIN REDUCTION

In this section, we extend the AdS/Ricci-flat correspondence to include an additional axion field [20,23]. The inclusion of the axion field allows us to later explore the connections between wormholes with a cosmological constant and wormholes in flat space, thus gaining access to exact results of the otherwise impregnable problems. Let us start by introducing the correspondence. One of the theories is the $\hat{D}$-dimensional Einstein gravity with an axion field (phantom field) χ,

$$\hat{S} = \frac{1}{16\pi\hat{G}} \int \mathrm{d}^{\hat{D}} x \hat{\mathcal{L}}_{\hat{D}}, \tag{2}$$

where the Lagrangian density $\hat{\mathcal{L}}$ is simply the Einstein-Hilbert one

$$\hat{\mathcal{L}}_{\hat{D}} = \sqrt{-\hat{g}}(\hat{R} + (\partial\hat{\chi})^2). \tag{3}$$

The correspondence is obtained by performing spherical reduction for this theory and then torus reduction for the other one. For spherical reduction, the ansatz is given by

$$\mathrm{d}\hat{s}^2_{\hat{D}} = e^{2\alpha\phi_1}\mathrm{d}s^2_d + e^{2\beta\phi_2}\mathrm{d}\Omega^2_n, \tag{4}$$

where $\hat{D} = d + n$. This reduction is what we call "diagonal" since it does not mix any off-diagonal elements in the metric tensor. One can regard the reduction as removing the fiber part of the bundle, keeping only the theory on the base manifold. This we can do as the dimension of the fibers are extremely small as compared to the base,

$$\mathcal{L}_d = \ell^n \sqrt{-g} X^{\hat{D}-2}\bigg(R + (\hat{D}-1)(\hat{D}-2)(\partial X)^2 X^{-2} + \frac{n(n-1)}{\ell^2} + \frac{1}{2}(\partial\chi)^2\bigg), \tag{5}$$

where the term $n(n-1)/\ell^2$ is the curvature of the n-sphere. This theory has Newton's constant $G_d = \ell^{-n}\hat{G}$. The other theory we consider in the correspondence is

$$\tilde{S} = \frac{1}{16\pi\tilde{G}} \int \mathrm{d}^{\tilde{D}} \tilde{\mathcal{L}}_{\tilde{D}},$$
$$\tilde{\mathcal{L}}_{\tilde{D}} = \sqrt{-\tilde{g}}\bigg(\tilde{R} - 2\Lambda + \frac{1}{2}(\partial\tilde{\chi})^2\bigg). \tag{6}$$

For the torus reduction, we use the metric ansatz,

$$\mathrm{d}\tilde{s}^2_{\tilde{D}} = \mathrm{d}\tilde{s}^2_d + Y^2 \mathrm{d}s^2_{\mathcal{T}^Q}, \tag{7}$$

where $\tilde{D} = d + Q$. In the reduction, the field χ is directly reduced from the higher dimension to the low dimension as a scalar field [22],

$$\mathcal{L}'_d = \mathcal{V}_Q \sqrt{-g} Y^Q \bigg(R + Q(Q-1)(\partial Y)^2 Y^{-2} + \frac{\tilde{n}(\tilde{n}-1)}{\tilde{\ell}^2} + \frac{1}{2}(\partial\chi)^2\bigg), \tag{8}$$

where $\tilde{n} = -(\tilde{D}-2)$ and the parameter $\tilde{\ell}$ is related to the bare cosmological constant Λ by

$$\Lambda = -\frac{(\tilde{D}-1)(\tilde{D}-2)}{2\tilde{\ell}}. \tag{9}$$

$\mathcal{V}_Q$ is the volume of the torus, giving us Newton's constant $\tilde{G}_d = \mathcal{V}_Q^{-1}\tilde{G}$. Thus we found that the two theories match at a lower dimension, provided that we identify the parameter in the following way:

$$X = Y^{-1}, \qquad \ell = \tilde{\ell}, \qquad \ell^n \tilde{G} = \mathcal{V}_Q \hat{G}. \tag{10}$$

The dimensions of the two theories are related by the two equivalent maps,

$$n \leftrightarrow \tilde{n} \equiv -(\tilde{D}-2) \Leftrightarrow Q \leftrightarrow -(\hat{D}-2). \tag{11}$$

We note that the mapping of the dimension parameters are represented by the $\leftrightarrow$ rather than equality. We observe the negative sign in the mapping for Q. Thus we are mapping one theory with positive dimensions to another one with negative dimensions. The second map implies that we get inverted power for parameters X and Y. Implementing this correspondence, we are able to find exact solutions for one theory by the dictionary as we demonstrate in the next section, providing that we know the solution for the other one.

III. MAPPING OF THE SOLUTIONS OF ELLIS WORMHOLE TO AdS ELLIS WORMHOLE IN GENERAL DIMENSIONS

In this section, we demonstrate that some special wormholes in higher dimensions with a cosmological constant could be solved exactly using the mapping we obtained in the previous section. We first present the exact solution of the well-known Ellis-Bronnikov wormholes in higher dimensions, i.e., where we set $f(r) = 1$ in the metric of the wormholes [7]. Then we apply the dictionary to extract the exact solution of a special class of wormholes with a cosmological constant by comparing the overall factor in the reduced Lagrangian from the two wormhole theories. We shall start with the wormhole in Ricci-flat theory and find the AdS wormhole solution by the dictionary. Recall that we have for the Ellis wormhole the action

$$S = \frac{1}{16\pi G}\int \mathrm{d}^n x\sqrt{-g}(R - (\nabla\chi)^2). \tag{12}$$

We are working with the metric ansatz as for the hat theory in the previous section which we dimensionally reduced on sphere. For the $\hat{D} = p + 2 + n$ dimensional theory, the metric becomes

$$\mathrm{d}\hat{s}^2_{\hat{D}} = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + \rho^2(r)\mathrm{d}\Omega_n^2 = \frac{\rho^2}{\ell^2}(\mathrm{d}s_d^2 + \ell^2\mathrm{d}\Omega_n^2). \tag{13}$$

Note that $p = 0$ and $\hat{D} = 2 + n$. The functions $f(r)$ and $\rho(r)$ satisfy the equations of motion [7],

$$\begin{aligned} -\frac{1}{2}\chi'^2 &= (\hat{D}-2)\frac{\rho'}{\rho}\left(\frac{f'}{f} + (\hat{D}-3)\frac{\rho'}{\rho}\right) \\ &\quad - (\hat{D}-2)(\hat{D}-3)\frac{1}{f\rho^2}, \\ \frac{1}{2}\chi'^2 &= (\hat{D}-2)\frac{\rho''}{\rho}, \\ \chi' &= \frac{C}{f\rho^{\hat{D}-2}}. \end{aligned} \tag{14}$$

We find the integration constant $C^2 = (\hat{D}-2)\times(\hat{D}-3)a^{2(\hat{D}-3)}$. Thus we find the solution to the equations of motion as

$$\begin{aligned} f(r) &\equiv 1, \\ \rho'(r) &= \sqrt{1 - \left(\frac{a}{\rho}\right)^{2(\hat{D}-3)}}, \\ \chi(r) &= \sqrt{2(\hat{D}-2)(\hat{D}-3)}a^{\hat{D}-3}\int\frac{\mathrm{d}r}{\rho(r)^{\hat{D}-2}}. \end{aligned} \tag{15}$$

We have set $f(r) = 1$ for this particular class of wormhole solutions to better illustrate the structure of the dual theory. Now, in order to see the wormhole structure of this theory, we note that

$$\begin{aligned} \frac{\mathrm{d}\rho}{\mathrm{d}r} &= \sqrt{1 - (a/\rho)^{2(\hat{D}-3)}}, \\ \mathrm{d}r &= \frac{\mathrm{d}\rho}{\sqrt{1 - (a/\rho)^{2(\hat{D}-3)}}}, \\ \mathrm{d}r^2 &= \frac{\mathrm{d}\rho^2}{1 - (a/\rho)^{2(\hat{D}-3)}}. \end{aligned} \tag{16}$$

This we can do as it is a mere coordinate transformation. After swapping the variables ρ and r, we can rewrite the $\mathrm{d}\hat{S}^2$ metric as

$$\mathrm{d}\hat{s}^2 = -\mathrm{d}t^2 + \frac{\mathrm{d}\rho^2}{1 - (a/\rho)^{2(\hat{D}-3)}} + \rho^2\mathrm{d}\Omega_n^2. \tag{17}$$

The metric of the dimensionally reduced theory is given by

$$\mathrm{d}s_2^2 = \frac{1}{X^2}\left(-f\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f}\right) = \frac{\ell^2}{\rho^2}\left(-f\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f}\right). \tag{18}$$

So apply the mapping, we find that

$$\mathrm{d}\tilde{s}_2^2 = Y^2\left(-\tilde{f}\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\tilde{f}}\right) = \frac{\tilde{\ell}^2}{\rho^2}\left(-\tilde{f}\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\tilde{f}}\right). \tag{19}$$

From here, we recover the full tilde theory by lifting the reduced metric back to higher dimensions. Then the $\tilde{D} = Q + 2$ dimensional metric becomes

$$\mathrm{d}\tilde{s}^2_{\tilde{D}} = \frac{\tilde{\ell}^2}{\rho^2}\left(-\tilde{f}\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\tilde{f}} + \mathrm{d}y^j\mathrm{d}y^j\right), \tag{20}$$

where the functions $\tilde{f}(r)$ and $\rho(r)$ should satisfy the equations

$$\begin{aligned} \frac{1}{2}\chi'^2 &= Q\frac{\rho'}{\rho}\left(\frac{\tilde{f}'}{\tilde{f}} - (Q+1)\frac{\rho'}{\rho}\right) + Q(Q+1)\frac{1}{\tilde{f}\rho^2}, \\ -\frac{1}{2}\chi'^2 &= Q\frac{\rho''}{\rho}, \\ \chi' &= \frac{C\rho^q}{\tilde{f}}, \end{aligned} \tag{21}$$

where we used the identification we discussed in the previous section,

$$Q \leftrightarrow -(\hat{D}-2). \tag{22}$$

Again we can express r in terms of ρ instead,

$$\frac{\mathrm{d}\rho}{\mathrm{d}r} = \sqrt{1 - (a/\rho)^{-2(Q+1)}} \Rightarrow \mathrm{d}r = \frac{\mathrm{d}\rho}{\sqrt{1 - (\rho/a)^{2(Q+1)}}}. \tag{23}$$

Putting this back to the $\tilde{D} = Q + 2$ dimensional metric, we can rewrite it as

$$\mathrm{d}\tilde{s}^2_{\tilde{D}} = \frac{\ell^2}{\rho^2}\left(-\mathrm{d}t^2 + \frac{\mathrm{d}\rho^2}{1-(\rho/a)^{2(q+1)}} + \mathrm{d}y^j\mathrm{d}y^j\right) \tag{24}$$

and

$$\chi(\rho) = \sqrt{2Q(Q+1)}a^{-(Q+1)}\int\frac{\rho^Q\mathrm{d}\rho}{\sqrt{1-(\frac{\rho}{a})^{2(Q+1)}}} = \sqrt{\frac{2Q}{Q+1}}\arcsin\frac{\rho^{Q+1}}{a^{Q+1}}. \tag{25}$$

We show that in general the solution obtained via the correspondence is the wormhole solution in Sec. VI.

IV. ELLIS-BRONNIKOV WORMHOLE IN GENERAL DIMENSIONS

Now we proceed to the exact solution of the AdS wormhole in general dimensions. For the n-dimensional wormhole in Ricci flat spacetime, it was found that with specific metric ansatz [10], one can obtain the generic solution:

$$\mathrm{d}s^2 = -F(r)^{-2}\mathrm{d}t^2 + F(r)^{2/(n-3)}G(r)^{-(n-4)/(n-3)} \times (\mathrm{d}r^2 + G(r)\gamma_{ij}(z)\mathrm{d}z^i\mathrm{d}z^j), \tag{26}$$

where the functions $F(r)$ and $G(r)$ satisfy

$$0 = \frac{F(r)''}{F(r)} - \frac{F(r)'}{F(r)} + \frac{F(r)'}{F(r)}\frac{G(r)'}{G(r)}, \tag{27}$$

$$0 = \frac{F(r)'^2}{F^2(r)} - \frac{1}{4}\frac{G(r)'^2}{G^2(r)} + \frac{(n-3)^2}{G(r)} - \frac{(n-3)}{2(n-2)}\frac{C^2}{G^2(r)}, \tag{28}$$

and we also have a master equation for $G(r)$ from $E^r_r + E^i_i = 0$,

$$G(r)'' - 2(n-3)^2 = 0. \tag{29}$$

The general solutions of these equations, in the case of the Ellis-Bronnikov class solution of this metric, are given by

$$\mathrm{d}s^2 = -\frac{1}{F(\bar{r})^2}\mathrm{d}\bar{t}^2 + F(\bar{r})^{\frac{2}{n-3}}G(\bar{r})^{-\frac{n-4}{n-3}}(\mathrm{d}\bar{r}^2 + G(\bar{r})\mathrm{d}\Omega^2),$$
$$F(\bar{r}) = F_0\exp\left(\beta\arctan\left(\frac{\bar{r}}{R}\right)\right),$$
$$G(\bar{r}) = (n-3)^2(\bar{r}^2 + R^2), \tag{30}$$

where we have relabeled the original coordinates t and r as $\bar{t}$ and $\bar{r}$. Putting all the pieces together, we can write the metric explicitly as

$$\mathrm{d}s^2 = -e^{-2\beta\arctan(\bar{r}/R)}\frac{\mathrm{d}\bar{t}^2}{F_0^2} + e^{2\beta\arctan(\bar{r}/R)/(n-3)}F_0^{2/(n-3)} \times ((n-3)^2(\bar{r}^2+R^2))^{-(n-4)/(n-3)} \times (\mathrm{d}\bar{r}^2 + (n-3)^2(\bar{r}^2+R^2)\mathrm{d}\Omega^2). \tag{31}$$

We arrive at the following metric after redefining the variables $\bar{t}$ and $\bar{r}$ and defining $U(r)$ and $V(r)$ as in Appendix B,

$$\mathrm{d}s^2 = -e^{-2\beta U(r)}\mathrm{d}t^2 + e^{2\beta U(r)/(n-3)}V(r)^{1/(n-3)} \times \left(\frac{\mathrm{d}r^2}{V(r)} + r^2\mathrm{d}\Omega^2\right). \tag{32}$$

This solution reduces to the desired wormhole solution that we derived in the previous section in the $f \equiv 1$ case. To see this, we first take the following coordinate transformation:

$$r = x\left(1 - \frac{M^2}{16x^{2(n-3)}}\right)^{1/(n-3)} \tag{33}$$

so the metric can be further rewritten in the form

$$\mathrm{d}s^2 = -e^{-2\beta\hat{U}(x)}\mathrm{d}t^2 + e^{2\beta\hat{U}(x)/(n-3)}\left(1 + \frac{M^2}{16x^{2(n-3)}}\right)^{2/(n-3)} \times (\mathrm{d}x^2 + x^2\mathrm{d}\Omega^2). \tag{34}$$

We can check explicitly, in four dimensions, that this metric indeed coincides with the Ellis wormhole with $f \equiv 1$, or equivalently, $\beta = 0$. To see this, suppose we set $n = 4$, and let $a \equiv \frac{M^2}{4}$; we define another new coordinate ρ as the following:

$$\rho^2 \equiv x^2\left(1 + \frac{M^2}{16x^{2(\hat{D}-3)}}\right)^{2(\hat{D}-3)}. \tag{35}$$

The metric (34) thus becomes

$$\mathrm{d}\hat{s}^2 = -\mathrm{d}t^2 + \frac{\mathrm{d}\rho^2}{1 - \frac{M^2}{4}/\rho^{2(\hat{D}-3)}} + \rho^2\mathrm{d}\Omega^2.$$

We have shown that this solution is equivalent to Eq. (17).

V. AdS WORMHOLE IN GENERAL DIMENSION

Finally, we proceed to use the mapping to compute the solutions of the tilde theory. We already have the hat theory given as

$$
\begin{aligned}
\mathrm{d}\hat{s}^2 &= -\frac{\mathrm{d}t^2}{F(r)^2} + F(r)^{2/(\hat{D}-3)} G(r)^{-(\hat{D}-4)/(\hat{D}-3)} (\mathrm{d}r^2 + G(r)\mathrm{d}\Omega^2) = \frac{\rho(r)^2}{\ell^2}(\mathrm{d}s_2^2 + \ell^2 \mathrm{d}\Omega^2),\\
\rho(r) &= F(r)^{1/(\hat{D}-3)} G(r)^{1/2(\hat{D}-3)},\\
\mathrm{d}s_2^2 &= \frac{\ell^2}{\rho(r)^2}\left(-\frac{1}{F(r)^2}\mathrm{d}t^2 + F(r)^{2/(\hat{D}-3)} G(r)^{-(\hat{D}-4)/(\hat{D}-3)}\mathrm{d}r^2\right),\\
X^2 &= \frac{\rho(r)^2}{\ell^2} = \frac{1}{\ell^2} F(r)^{2/(\hat{D}-3)} G(r)^{1/(\hat{D}-3)},\\
F(r) &= F_0 e^{\beta U(r)}, \qquad G(r) = r^{2(\hat{D}-3)} V(r), \qquad U(r) = \arctan\left(\frac{2r^{\hat{D}-3}}{M}\right), \qquad V(r) = 1 + \frac{M^2}{4r^{2(\hat{D}-3)}}.
\end{aligned} \tag{36}
$$

Apply the mapping:

$$
\begin{aligned}
Y^2 &= \frac{1}{X^2} = \ell^2 \tilde{F}(r)^{2/(Q+1)} \tilde{G}(r)^{1/(Q+1)},\\
\tilde{F}(r) &= F_0 e^{\beta\tilde{U}(r)}, \qquad \tilde{G}(r) = \frac{\tilde{U}(r)}{r^{2(Q+1)}}, \qquad \tilde{U}(r) = \arctan\left(\frac{2}{\tilde{M} r^{Q+1}}\right), \qquad \tilde{V}(r) = 1 + \frac{\tilde{M}^2 r^{2(Q+1)}}{4}.
\end{aligned} \tag{37}
$$

Thus the $\tilde{D} = 2 + Q$ dimensional metric becomes

$$
\mathrm{d}\tilde{s}^2 = Y^2\left(-\frac{\mathrm{d}t^2}{\tilde{F}(r)^2} + \tilde{\tilde{F}}(r)^{-2/(Q+1)} \tilde{G}(r)^{-(Q+2)/(Q+1)}\mathrm{d}r^2 + \mathrm{d}y^j \mathrm{d}y^j\right). \tag{38}
$$

Last, we write the tilde theory explicitly,

$$
\mathrm{d}\tilde{s}^2 = \ell^2\left(-e^{-2\beta\tilde{U}(r)/(Q+1)} \frac{\tilde{V}(r)^{1/(Q+1)}}{r^2}\mathrm{d}t^2 + \frac{r^{2(Q+1)}}{\tilde{V}(r)}\mathrm{d}r^2 + e^{2\beta\tilde{U}/(Q+1)} \frac{\tilde{V}^{1/(Q+1)}}{r^2}\mathrm{d}y^j \mathrm{d}y^j\right). \tag{39}
$$

This is the desired wormhole solution in general dimensions. We notice that the dimension parameter Q ranges from negative infinity to positive infinity. Now we do similar coordinate transformation as for the hat theory in the previous section. Take

$$
r^2 = x^2\left(1 - \frac{M^2}{16}x^{2(Q+1)}\right)^{-\frac{2}{Q+1}}. \tag{40}
$$

Then the metric after this transformation becomes

$$
\mathrm{d}\tilde{s}^2 = \frac{\ell^2}{x^2}\left(1 + \frac{M^2}{16}x^{2(Q+1)}\right)^{\frac{2}{Q+1}}\left(-e^{-2\beta\tilde{U}/(Q+1)}\mathrm{d}t^2 + \frac{\mathrm{d}x^2}{\left(1 + \frac{M^2}{16}x^{2(Q+1)}\right)^{\frac{2}{Q+1}}} + e^{2\beta\tilde{U}/(Q+1)}\mathrm{d}y^i \mathrm{d}y^i\right). \tag{41}
$$

A further coordinate transformation

$$
\rho^2 = x^2\left(1 + \frac{M^2}{16}x^{2(Q+1)}\right)^{-\frac{2}{Q+1}} \tag{42}
$$

gives us the desired form of solution

$$
\mathrm{d}\tilde{s}^2_{\tilde{D}} = \frac{\ell^2}{\rho^2}\left(-e^{-2\beta\tilde{U}/(Q+1)}\mathrm{d}t^2 + \frac{\mathrm{d}\rho^2}{1 - \rho^{2(Q+1)}/a} + e^{2\beta\tilde{U}/(Q+1)}\mathrm{d}y^i \mathrm{d}y^i\right). \tag{43}
$$

We discuss criteria [1] for qualifying a wormhole for our solution obtained above. The metric solution satisfies Einstein field equations everywhere. And the throat of the metric solution is at $\rho = a^{-2(Q+1)}$. Furthermore, the solution does not have a horizon by inspecting its form. In fact, this theory could be schematically written in the form as a worhmholelike Eq. (2.1) in Ref. [16]. And it meets the definition of the spacetime wormhole which is defined as "connected geometries whose boundaries have more than one compact connected component [26]."

In particular, for the case $\beta = 0$, the solution takes the form

$$d\tilde{s}^2_{\tilde{D}} = \frac{\ell^2}{\rho^2}\left(-dt^2 + \frac{d\rho^2}{1-\rho^{2(Q+1)}/a} + dy^i dy^i\right). \tag{44}$$

This is the same as the solution (24) we found in Sec. III. In the next section, we discuss the necessary condition for our wormhole solution to be traversable.

VI. NULL ENERGY CONDITION

It is known that in order for a solution to be a traversable wormhole, it necessarily needs to violate the null energy condition [24]

$$T_{\mu\nu}n^\mu n^\nu \geq 0. \tag{45}$$

To demonstrate that our solution indeed satisfies this condition, we use the null vector

$$n_\mu = (g_{tt}, \sqrt{g_{tt}g_{\rho\rho}}, 0, 0), \tag{46}$$

whose covariant counterpart is given by

$$n^\mu = (1, \sqrt{g_{tt}/g_{\rho\rho}}, 0, 0). \tag{47}$$

The null energy condition takes the form

$$T_{\mu\nu}n^\mu n^\nu = T^t_t n^t n_t + T^\rho_\rho n^\rho n_\rho = -g_{tt}(-T^t_t + T^\rho_\rho). \tag{48}$$

We found that

$$\begin{aligned} T_{\mu\nu}n^\mu n^\nu &= T_{tt} - g_{tt}T^\rho_\rho \\ &= -\frac{a + Q\rho^{2+2Q} + a\ell^2\Lambda}{a\rho^2} \\ &\quad + \frac{\rho^2}{\ell^2}\left(\frac{1}{\rho^2} + \frac{a\ell^2\Lambda}{a\rho^2 - \rho^{4+2Q}}\right) \\ &= \frac{1}{\ell^2} - \frac{1}{\rho^2} - \frac{\ell^2\Lambda}{\rho^2} - \frac{Q\rho^{2Q}}{a} + \frac{a\Lambda}{a-\rho^{2+2Q}} < 0. \end{aligned} \tag{49}$$

This theory violates the null energy condition and thus indeed is a traversable wormhole.

VII. CONCLUSION AND SUMMARY

The wormholes with a cosmological constant in higher dimensions are in general hard to solve directly. We found the general exact solution of wormholes with a cosmological constant in higher dimensions. This was achieved by implementing the mapping method that we learned from the Kaluza-Klein reduction of two different theories which were identified after dimensionally reducing to the same lower theory. We also studied the property of such a solution and demonstrated that they are indeed good wormholes. It is of further interest if one could try to find a correspondence more general that maps different theories with gauge fields involved, thus solving more profound problems that were otherwise impossible to tackle with.

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APPENDIX A: KALUZA-KLEIN REDUCTION

In this appendix, we focus on deriving the reduced Lagrangian in a lower dimension. In Sec. II, we have the metric ansatz (4) for spherical reduction,

$$d\hat{s}^2_{\hat{D}} = e^{2\alpha\phi_1}ds^2_d + e^{2\beta\phi_2}d\Omega^2_n. \tag{A1}$$

We choose the following vielbien for this reduction:

$$\hat{E}^a = e^{\alpha\phi_1}E^a, \qquad \hat{E}^i = e^{\beta\phi_2}E^i. \tag{A2}$$

Upon taking the exterior derivative,

$$\begin{aligned} d\hat{E}^a &= -\hat{\omega}^a{}_b \wedge \hat{E}^b - \hat{\omega}^a{}_j \wedge \hat{E}^j \\ &= d(e^{\alpha\phi_1}E^a) = d(e^{\alpha\phi_1}) \wedge E^a + e^{\alpha\phi_1}dE^a \\ &= \alpha e^{\alpha\phi_1}\partial_b\phi_1 E^b \wedge E^a + e^{\alpha\phi_1}(-\omega^a_b \wedge E^b) \\ &= \alpha e^{-\alpha\phi_1}\partial_b\phi_1 \hat{E}^b \wedge \hat{E}^a - \omega^a_b \wedge \hat{E}^b, \end{aligned} \tag{A3}$$

where we used Cartan's first structural equation in the first line and the fact that $\omega^a_j = 0$ for our diagonal reduction ansatz in the third line. So we can read off the spin connection 1-form as

$$\hat{\omega}^a{}_b = \omega^a{}_b + \alpha e^{-\alpha\phi_1}(\partial_b\phi_1\hat{E}^a - \partial^a\phi_1\hat{E}^c\delta_{cb}). \tag{A4}$$

We obtain all the spin connections in a similar way and list them here:

$$\hat{\omega}^a{}_b = \omega^a{}_b + \alpha e^{-\alpha\phi_1}(\partial_b\phi_1\hat{\mathrm{E}}^a - \partial^a\phi_1\hat{\mathrm{E}}^c\delta_{cb}),$$
$$\hat{\omega}^i{}_j = \omega^i{}_j,$$
$$\hat{\omega}^i{}_c = \beta e^{-\alpha\phi_1}\partial_c\phi_2\hat{\mathrm{E}}^i. \tag{A5}$$

The curvature 2-forms $\hat{\Omega}$'s can be computed by using the spin connection 1-forms we found. To proceed, we recall the definition

$$\hat{\Omega}^a{}_b = \mathrm{d}\hat{\omega}^a{}_b + \hat{\omega}^a{}_c \wedge \hat{\omega}^c{}_b + \hat{\omega}^a{}_j \wedge \hat{\omega}^j{}_b. \tag{A6}$$

The first term is given by

$$\begin{aligned}\mathrm{d}\hat{\omega}^a{}_b &= \alpha(-\alpha e^{-\alpha\phi_1})(\mathrm{d}\phi_1) \wedge (\partial_b\phi_1\hat{\mathrm{E}}^a - \partial^a\phi_1\hat{\mathrm{E}}^c\delta_{cb}) + \mathrm{d}\omega^a{}_b + \alpha e^{-\alpha\phi_1}\mathrm{d}(\partial_b\phi_1\hat{\mathrm{E}}^a - \partial^a\phi_1\hat{\mathrm{E}}^c\delta_{cb})\\ &= \mathrm{d}\omega^a{}_b - \alpha^2 e^{-2\alpha\phi_1}\partial_c\phi_1\hat{\mathrm{E}}^c \wedge (\partial_b\phi_1\hat{\mathrm{E}}^a - \partial^a\phi_1\hat{\mathrm{E}}^d\delta_{db}) + \alpha e^{-2\alpha\phi_1}(\partial_c\partial_b\phi_1\hat{\mathrm{E}}^c \wedge \hat{\mathrm{E}}^a - \partial_c\partial^a\phi_1\hat{\mathrm{E}}^c \wedge \hat{\mathrm{E}}^d\delta_{db})\\ &\quad + \alpha e^{-\alpha\phi_1}(\partial_b\phi_1\mathrm{d}\hat{\mathrm{E}}^a - \partial^a\phi_1\mathrm{d}\hat{\mathrm{E}}^d\delta_{db}).\end{aligned} \tag{A7}$$

Note that the terms in the last brackets involve exterior derivatives of $\hat{\mathrm{E}}$'s, and we want to show in detail the computation of these terms:

$$\begin{aligned}\alpha e^{-\alpha\phi_1}\partial_b\phi_1\mathrm{d}\hat{\mathrm{E}}^a &= \alpha e^{-\alpha\phi_1}\partial_b\phi_1(-\alpha e^{-\alpha\phi_1}\partial_c\phi_1\hat{\mathrm{E}}^a \wedge \hat{\mathrm{E}}^c - \omega^a{}_c \wedge \hat{\mathrm{E}}^c)\\ &= \alpha^2 e^{-2\alpha\phi_1}\partial_b\phi_1\partial_c\phi_1\hat{\mathrm{E}}^c \wedge \hat{\mathrm{E}}^a - \alpha e^{-\alpha\phi_1}\partial_b\phi_1\omega^a{}_c \wedge \hat{\mathrm{E}}^c.\end{aligned} \tag{A8}$$

Plug Eq. (A8) back into Eq. (A7), and we find that

$$\begin{aligned}\mathrm{d}\hat{\omega}^a{}_b &= \mathrm{d}\omega^a{}_b - \alpha^2 e^{-2\alpha\phi_1}\partial_c\phi_1\partial_b\phi_1\hat{\mathrm{E}}^c \wedge \hat{\mathrm{E}}^a + \alpha^2 e^{-2\alpha\phi_1}\partial_c\phi_1\partial^a\phi_1\hat{\mathrm{E}}^c \wedge \hat{\mathrm{E}}^d\delta_{db} + \alpha e^{-2\alpha\phi_1}(\partial_c\partial_b\phi_1\hat{\mathrm{E}}^c \wedge \hat{\mathrm{E}}^a - \partial_c\partial^a\phi_1\hat{\mathrm{E}}^c \wedge \hat{\mathrm{E}}^d\delta_{db})\\ &\quad + \alpha^2 e^{-2\alpha\phi_1}\partial_b\phi_1\partial_c\phi_1\hat{\mathrm{E}}^c \wedge \hat{\mathrm{E}}^a - \alpha e^{-\alpha\phi_1}\partial_b\phi_1\omega^a{}_c \wedge \hat{\mathrm{E}}^c - \alpha^2 e^{-2\alpha\phi_1}\partial^a\phi_1\partial_c\phi_1\hat{\mathrm{E}}^c \wedge \hat{\mathrm{E}}^d\delta_{db} + \alpha e^{-\alpha\phi_1}\partial^a\phi_1\omega^d{}_f \wedge \hat{\mathrm{E}}^f\delta_{db}\\ &= \mathrm{d}\omega^a{}_b - \alpha e^{-2\alpha\phi_1}(\partial_c\partial_b\phi_1\hat{\mathrm{E}}^a + \omega^a{}_{cf}\partial_b\phi_1\hat{\mathrm{E}}^f) \wedge \hat{\mathrm{E}}^c + \alpha e^{-2\alpha\phi_1}(\partial_c\partial^a\phi_1\hat{\mathrm{E}}^d + \omega^d{}_{cf}\partial^a\phi_1\hat{\mathrm{E}}^f)\delta_{db} \wedge \hat{\mathrm{E}}^c\\ &= \mathrm{d}\omega^a{}_b + \alpha e^{-2\alpha\phi_1}(\nabla_c\nabla^a\phi_1\hat{\mathrm{E}}^d\delta_{db} - \nabla_b\nabla_c\phi_1\hat{\mathrm{E}}^a) \wedge \hat{\mathrm{E}}^c,\end{aligned} \tag{A9}$$

where in the last line, we used the definition

$$\begin{aligned}\nabla_a V^b &\equiv \partial_a V^b + \omega_a{}^b{}_c V^c,\\ \nabla_a V_b &\equiv \partial_a V_b - \omega_a{}^c{}_b V_c,\end{aligned} \tag{A10}$$

where the $\omega_a{}^c{}_b$ comes from the definition

$$\omega^a{}_b \equiv \omega^a{}_{bc}\mathrm{E}^c. \tag{A11}$$

Then we can compute the curvature 2-form $\hat{\Omega}^a{}_b$,

$$\begin{aligned}\hat{\Omega}^a{}_b &= (\mathrm{d}\omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b) + \alpha e^{-2\alpha\phi_1}(\nabla_c\nabla^a\phi_1\hat{\mathrm{E}}^d\delta_{db} - \nabla_b\nabla_c\phi_1\hat{\mathrm{E}}^a) \wedge \hat{\mathrm{E}}^c + \alpha^2 e^{-2\alpha\phi_1}(\partial_c\phi_1\hat{\mathrm{E}}^a - \partial^a\phi_1\hat{\mathrm{E}}^d\delta_{dc})\\ &\quad \wedge (\partial_b\phi_1\hat{\mathrm{E}}^c - \partial^c\phi_1\hat{\mathrm{E}}^f\delta_{fb}) + \alpha e^{-\alpha\phi_1}[\omega^a{}_c \wedge (\partial_b\phi_1\hat{\mathrm{E}}^c - \partial^c\phi_1\hat{\mathrm{E}}^f\delta_{fb}) + (\partial_c\phi_1\hat{\mathrm{E}}^a - \partial^a\phi_1\hat{\mathrm{E}}^d\delta_{dc}) \wedge \omega^c{}_b]\\ &\quad - \beta^2 e^{-2\alpha\phi_1}\partial_a\phi_2\hat{\mathrm{E}}^k \wedge \partial_b\phi_2\hat{\mathrm{E}}^j\delta_{kj}\\ &= \Omega^a{}_b + \alpha e^{-2\alpha\phi_1}(\nabla_c\nabla^a\phi_1\hat{\mathrm{E}}^d\delta_{db} - \nabla_b\nabla_c\phi_1\hat{\mathrm{E}}^a) \wedge \hat{\mathrm{E}}^c + \alpha^2 e^{-2\alpha\phi_1}(\partial_c\phi_1\hat{\mathrm{E}}^a - \partial^a\phi_1\hat{\mathrm{E}}^d\delta_{dc}) \wedge (\partial_b\phi_1\hat{\mathrm{E}}^c - \partial^c\phi_1\hat{\mathrm{E}}^f\delta_{fb}).\end{aligned} \tag{A12}$$

By noting that $\hat{\mathrm{E}}^k \wedge \hat{\mathrm{E}}^j\delta_{kj} = \hat{\mathrm{E}}^j \wedge \hat{\mathrm{E}}^j = 0$, we could further simplify the curvature 2-form. We list here all the curvature 2-forms:

$$\begin{aligned}\hat{\Omega}^a{}_b &= \Omega^a{}_b + \alpha e^{-2\alpha\phi_1}(\nabla_c\nabla^a\phi_1\hat{\mathrm{E}}^d\delta_{db} - \nabla_b\nabla_c\phi_1\hat{\mathrm{E}}^a) \wedge \hat{\mathrm{E}}^c - \alpha^2 e^{-2\alpha\phi_1}(\partial\phi_1)^2\hat{\mathrm{E}}^a \wedge \hat{\mathrm{E}}^f\delta_{fb}\\ &\quad + \alpha^2 e^{-2\alpha\phi_1}\partial_c\phi_1(\partial_b\phi_1\hat{\mathrm{E}}^a - \partial^a\phi_1\hat{\mathrm{E}}^f\delta_{fb}) \wedge \hat{\mathrm{E}}^c,\\ \hat{\Omega}^i{}_j &= \Omega^i{}_j - \beta^2 e^{-2\alpha\phi_1}(\partial\phi_2)^2\hat{\mathrm{E}}^i \wedge \hat{\mathrm{E}}^k\delta_{kj},\\ \hat{\Omega}^i{}_b &= -\alpha\beta e^{-2\alpha\phi_1}(\partial_c\phi_1\partial_b\phi_2\hat{\mathrm{E}}^c + \partial_c\phi_2\partial_b\phi_1\hat{\mathrm{E}}^c - \partial_a\phi_2\partial^a\phi_1\hat{\mathrm{E}}^c\delta_{cb}) \wedge \hat{\mathrm{E}}^i + \beta e^{-2\alpha\phi_1}(\beta\partial_b\partial_c\phi_2 + \nabla_b\nabla_c\phi_2)\hat{\mathrm{E}}^c \wedge \hat{\mathrm{E}}^i.\end{aligned} \tag{A13}$$

Now we use the following equation to read off the Riemann curvature:

$$\hat{\Omega}^A{}_B = \frac{1}{2}\hat{R}^A{}_{BMN}\hat{\mathrm{E}}^M \wedge \hat{\mathrm{E}}^N, \tag{A14}$$

where $A, B, M, N \in \{1, \ldots, d, d+1, \ldots, d+n\}$. We first look at

$$\begin{aligned}\hat{\Omega}^a{}_b &= \frac{1}{2}\hat{R}^a{}_{bMN}\hat{\mathrm{E}}^M \wedge \hat{\mathrm{E}}^N \\ &= \frac{1}{2}\hat{R}^a{}_{bcd}\hat{\mathrm{E}}^c \wedge \hat{\mathrm{E}}^d + \frac{1}{2}\hat{R}^a{}_{bij}\hat{\mathrm{E}}^i \wedge \hat{\mathrm{E}}^j + \frac{1}{2}\hat{R}^a{}_{bcj}\hat{\mathrm{E}}^c \wedge \hat{\mathrm{E}}^j,\end{aligned} \tag{A15}$$

compare this with Eq. (A13), and note that

$$\begin{aligned}\Omega^a_b &= \frac{1}{2}R^a_{bMN}\mathrm{E}^M \wedge \mathrm{E}^N = \frac{1}{2}R^a_{bcd}\mathrm{E}^c \wedge \mathrm{E}^d \\ &= e^{-2\alpha\phi_1}\frac{1}{2}R^a_{bcd}\hat{\mathrm{E}}^c \wedge \hat{\mathrm{E}}^d.\end{aligned} \tag{A16}$$

We can read off Riemann curvatures,

$$\begin{aligned}\hat{R}^a{}_{bcd} &= e^{-2\alpha\phi_1}R^a{}_{bcd} - \alpha^2 e^{-2\alpha\phi_1}(\partial\phi_1)^2(\delta^a{}_c\delta_{bd} - \delta^a{}_d\delta_{bc}) + \alpha e^{-2\alpha\phi_1}(\nabla_b\nabla_c\phi_1\delta^a{}_d - \nabla_b\nabla_d\phi_1\delta^a{}_c \\ &\quad - \nabla^a\nabla_c\phi_1\delta_{bd} + \nabla^a\nabla_d\phi_1\delta_{bc}) - \alpha^2 e^{-2\alpha\phi_1}(\partial_b\phi_1\partial_c\phi_1\delta^a{}_d - \partial_b\phi_1\partial_d\phi_1\delta^a{}_c - \partial^a\phi_1\partial_c\phi_1\delta_{bd} + \partial^a\phi_1\partial_d\phi_1\delta_{bc}), \\ \hat{R}^i{}_{bjd} &= \alpha\beta e^{-2\alpha\phi_1}(2\partial_d\phi_1\partial_b\phi_2\delta^i{}_j + 2\partial_d\phi_2\partial_b\phi_1\delta^i{}_j - 2\partial_a\phi_1\partial^a\phi_2\delta_{bd}\delta^i{}_j) - 2\beta e^{-2\alpha\phi_1}(\beta\partial_b\phi_1\partial_d\phi_2 + \nabla_b\nabla_d\phi_2)\delta^i{}_j, \\ \hat{R}^i{}_{jkl} &= e^{-2\beta\phi_2}R^i{}_{jkl} - \beta^2 e^{-2\alpha\phi_1}(\partial\phi_2)^2(\delta^i{}_k\delta_{jl} - \delta^i{}_l\delta_{jk}),\end{aligned} \tag{A17}$$

where we antisymmetrized the indices to ensure the antisymmetry of the Riemann curvature tensor. Finally, we contract Riemann curvatures,

$$\begin{aligned}\hat{R}_{bd} &= \hat{R}^a{}_{\mathrm{bad}} + \hat{R}^i{}_{bid} \\ &= e^{-2\alpha\phi_1}R_{bd} - (d-1)\alpha^2 e^{-2\alpha\phi_1}(\partial\phi_1)^2\delta_{bd} - \alpha e^{-2\alpha\phi_1}((d-2)\nabla_b\nabla_d\phi_1 + \Box\phi_1\delta_{bd}) + \alpha^2 e^{-2\alpha\phi_1}((d-2)\partial_b\phi_1\partial_d\phi_1 \\ &\quad + (\partial\phi_1)^2\delta_{bd}) + 2n\alpha\beta e^{-2\alpha\phi_1}(\partial_d\phi_1\partial_b\phi_2 + \partial_d\phi_2\partial_b\phi_1 - \partial_a\phi_1\partial^a\phi_2\delta_{bd}) - 2n\beta e^{-2\alpha\phi_1}(\beta\partial_b\phi_2\partial_d\phi_2 + \nabla_b\nabla_d\phi_2), \\ \hat{R}_{jl} &= e^{-2\beta\phi_2}R_{jl} - (n-1)\beta^2 e^{-2\alpha\phi_1}(\partial\phi_2)^2\delta_{jl}.\end{aligned} \tag{A18}$$

Then use $\hat{R} = \hat{R}^b{}_b + \hat{R}^j{}_j$. Eventually, we find that

$$\begin{aligned}\hat{R} &= e^{-2\alpha\phi_1}R + e^{-2\beta\phi_2}R_\Omega - n(n-1)\beta^2 e^{-2\alpha\phi_1}(\nabla\phi_2)^2 - d(d-1)\alpha^2 e^{-2\alpha\phi_1}(\nabla\phi_1)^2 + 2(d-1)\alpha e^{-2\alpha\phi_1}(\Box\phi_1 - \alpha(\nabla\phi_1)^2) \\ &\quad - 2n\beta e^{-2\alpha\phi_1}(\beta(\nabla\phi_2)^2 + \Box\phi_2) + (4n - 2nd)\alpha\beta e^{-2\alpha\phi_1}\nabla_a\phi_1\nabla^a\phi_2.\end{aligned} \tag{A19}$$

Now note the metric determinant in a lower dimension given by

$$\sqrt{-\hat{g}} = \sqrt{-(e^{2\alpha\phi_1})^d(e^{2\beta\phi_2})^n g} = e^{d\alpha\phi_1 + n\beta\phi_2}\sqrt{-g}. \tag{A20}$$

Collecting the pieces above, we find the reduced Lagrangian,

$$\begin{aligned}\mathcal{L}_d &= \sqrt{-\hat{g}}\hat{R} = \sqrt{-g}[e^{-2\alpha\phi_1}R + e^{-2\beta\phi_2}R_\Omega + 2(d-1)\alpha e^{-2\alpha\phi_1}(\Box\phi_1 - \alpha(\nabla\phi_1)^2) - 2n\beta e^{-2\alpha\phi_1}(\beta(\nabla\phi_2)^2 + \Box\phi_2) \\ &\quad + 2n(2-d)\alpha\beta e^{-2\alpha\phi_1}\nabla_a\phi_1\nabla^a\phi_2 - d(d-1)\alpha^2 e^{-2\alpha\phi_1}(\nabla\phi_1)^2 - n(n-1)\beta^2 e^{-2\alpha\phi_1}(\nabla\phi_2)^2]e^{d\alpha\phi_1 + n\beta\phi_2} \\ &= \sqrt{-g}[e^{-2\alpha\phi_1}(R + 2(d-1)\alpha(\Box\phi_1 - \alpha(\nabla\phi_1)^2) - 2n\beta(\beta(\nabla\phi_2)^2 + \Box\phi_2) - d(d-1)\alpha^2(\nabla\phi_1)^2 \\ &\quad - n(n-1)\beta^2(\nabla\phi_2)^2 + 2n(d-2)\alpha\beta\nabla_a\phi_1\nabla^a\phi_2) + e^{-2\beta\phi_2}R_\Omega]e^{d\alpha\phi_1 + n\beta\phi_2}.\end{aligned} \tag{A21}$$

Setting

$$\phi_1 = \frac{1}{\alpha}\ln X, \qquad \phi_2 = \frac{1}{\beta}\ln(X\ell). \tag{A22}$$

We arrive at the reduced Lagrangian,

$$\mathcal{L}_d = \ell^n\sqrt{-g}X^{\hat{D}-2}\bigg(R + (\hat{D}-1)(\hat{D}-2)(\partial X)^2X^{-2} + \frac{n(n-1)}{\ell^2}\bigg), \tag{A23}$$

where $\hat{D} = d + n$ and the last term is the curvature of the n-sphere.

APPENDIX B: SOLVING THE HIGHER DIMENSIONAL WORMHOLES

Here we derive in detail the solution of Ellis-Bronnikove wormholes. We start with the action

$$S = \int \mathrm{d}^n x\sqrt{-g}\bigg(R + \frac{1}{2}(\partial\phi)^2\bigg). \tag{B1}$$

The equations of motion are given by

$$0 = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \frac{1}{2}\left(\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}(\partial\phi)^2\right) \equiv E_{\mu\nu},$$
$$0 = \Box\phi \equiv \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi). \tag{B2}$$

Assume that $\phi = \phi(r)$, and use the metric ansatz (26), and then we find from the last equation that

$$\frac{\mathrm{d}\phi}{\mathrm{d}r} = \frac{C}{G(r)}. \tag{B3}$$

Now we use the equation

$$E^r{}_r + E^i{}_i = 0. \tag{B4}$$

Then we conclude that the master equation for $G(r)$ is

$$G'' - 2(n-3)^2 = 0. \tag{B5}$$

Then for $E^t{}_t$ and $E^r{}_r$, we use *Mathematica* with mathematical induction and find the equation of motion in a generic dimension to be

$$E^t{}_t = F(r)^{\frac{-2}{n-3}}G(r)^{\frac{n-4}{n-3}}\left(\frac{n-2}{8(n-3)}\left(8\frac{F''}{F} + 8\frac{F'G'}{FG} - 4\frac{F'^2}{F^2} - \frac{G'^2}{G^2} + \frac{4(n-3)^2}{G}\right) - \frac{1}{4}\frac{C^2}{G^2}\right),$$
$$E^r{}_r = F(r)^{\frac{-2}{n-3}}G(r)^{\frac{n-4}{n-3}}\left(\frac{n-2}{8(n-3)}\left(-4\frac{F'^2}{F^2} + \frac{G'^2}{G^2} - \frac{4(n-3)^2}{G}\right) + \frac{1}{4}\frac{C^2}{G^2}\right). \tag{B6}$$

Since we have $E_{\mu\nu} =$, then $E^t{}_t = 0$ and $E^r{}_r = 0$, so

$$\frac{1}{4}\frac{C^2}{G^2} = \frac{n-2}{8(n-3)}\left(8\frac{F''}{F} + 8\frac{F'G'}{FG} - 4\frac{F'^2}{F^2} - \frac{G'^2}{G^2} + \frac{4(n-3)^2}{G}\right). \tag{B7}$$

Substitute this back into $E^r{}_r$,

$$E^r{}_r = F(r)^{\frac{-2}{n-3}}G(r)^{\frac{n-4}{n-3}}\left(\frac{n-2}{8(n-3)}\left(-4\frac{F'^2}{F^2} + \frac{G'^2}{G^2} - \frac{4(n-3)^2}{G} + 8\frac{F''}{F} + 8\frac{F'G'}{FG} - 4\frac{F'^2}{F^2} - \frac{G'^2}{G^2} + \frac{4(n-3)^2}{G}\right)\right),$$
$$0 = F(r)^{\frac{-2}{n-3}}G(r)^{\frac{n-4}{n-3}}\left(\frac{n-2}{8(n-3)}\left(8\frac{F''}{F} + 8\frac{F'G'}{FG} - 8\frac{F'^2}{F^2}\right)\right),$$
$$0 = \left(\frac{F''}{F} + \frac{F'G'}{FG} - \frac{F'^2}{F^2}\right). \tag{B8}$$

Also from $E^r{}_r = 0$, we find that

$$0 = F(r)^{\frac{-2}{n-3}}G(r)^{\frac{n-4}{n-3}}\left(\frac{n-2}{8(n-3)}\left(-4\frac{F'^2}{F^2} + \frac{G'^2}{G^2} - \frac{4(n-3)^2}{G}\right) + \frac{1}{4}\frac{C^2}{G^2}\right),$$
$$0 = \frac{F'^2}{F^2} - \frac{1}{4}\frac{G'^2}{G^2} + \frac{(n-3)^2}{G} - \frac{n-3}{2(n-2)}\frac{C^2}{G^2}. \tag{B9}$$

We completed the derivation of Eqs. (27) and (28). Now we derive the Ellis-Bronnikov class solution of this metric; in the case that $G(r)$ has no real roots, its solution for (B5) can be written as

$$G(r) = (n-3)^2 r^2 + G_0, \tag{B10}$$

then

$$G' = 2(n-3)^2 r,$$

and Eq. (B8) becomes

$$\frac{F''}{F} - \frac{F'^2}{F^2} + \frac{2(n-3)^2 r}{(n-3)^2 r^2 + G_0}\frac{F'}{F} = 0. \tag{B11}$$

The solution of $F(r)$ is given by

$$F(r) = F_0 \exp\left(\beta \arctan\left(\frac{r}{R}\right)\right), \tag{B12}$$

where we defined the constant G_0 as

$$G_0 = (n-3)^2 R^2. \tag{B13}$$

F_0 and β are integration constants; in particular, we absorbed a constant $\frac{1}{R}$ in front of the arctan into the constant β.

Now going back to Eq. (28), we find the constant C to be

$$C^2 = 2R^2(n-2)(n-3)^3(1+\beta^2). \tag{B14}$$

Use Eq. (B3), $\phi' = \frac{C}{G}$, and we find the axion field to be

$$\phi(r) = \phi_0 \pm \sqrt{\frac{2(n-2)(1+\beta^2)}{n-3}} \arctan\left(\frac{r}{R}\right). \tag{B15}$$

A coordinate change can put the metric into a better form. For convenience, let us simply relabel the original coordinates t and r as $\bar{t}$ and $\bar{r}$, and then

$$\begin{aligned} \mathrm{d}s^2 &= -\frac{1}{F(\bar{r})^2}\mathrm{d}\bar{t}^2 + F(\bar{r})^{\frac{2}{n-3}} G(\bar{r})^{-\frac{n-4}{n-3}}(\mathrm{d}\bar{r}^2 + G(\bar{r})\mathrm{d}\Omega^2), \\ F(\bar{r}) &= F_0 \exp\left(\beta \arctan\left(\frac{\bar{r}}{R}\right)\right), \\ G(\bar{r}) &= (n-3)^2(\bar{r}^2 + R^2). \end{aligned} \tag{B16}$$

We can write the metric explicitly as

$$\begin{aligned} \mathrm{d}s^2 = &-e^{-2\beta \arctan(\bar{r}/R)}\frac{\mathrm{d}\bar{t}^2}{F_0^2} + e^{2\beta \arctan(\bar{r}/R)/(n-3)} \\ &\times ((n-3)^2(\bar{r}^2+R^2))^{-(n-4)/(n-3)} \\ &\times (\mathrm{d}\bar{r}^2 + (n-3)^2(\bar{r}^2+R^2)\mathrm{d}\Omega^2). \end{aligned} \tag{B17}$$

Now we express $\bar{t}$ and $\bar{r}$ in terms of the new coordinates t and r,

$$\bar{t} = F_0 t, \qquad \bar{r} = \frac{2Rr^{n-3}}{M}, \qquad M = 2(n-3)RF_0. \tag{B18}$$

Defining

$$U(r) \coloneqq \arctan\left(\frac{2r^{n-3}}{M}\right), \tag{B19}$$

$$V(r) \coloneqq 1 + \frac{M^2}{4r^{2(n-3)}}, \tag{B20}$$

we can write the axion fields as

$$\phi(r) = \phi_0 \pm \sqrt{\frac{2(n-2)(1+\beta^2)}{n-3}} U(r). \tag{B21}$$

Also we have

$$\begin{aligned} V(r)^{1/(n-3)} r^2 &= \left(\left(1 + \frac{M^2}{4r^{2(n-3)}}\right) r^{2(n-3)}\right)^{1/(n-3)} \\ &= (r^{2(n-3)} + (n-3)^2 R^2)^{1/(n-3)} \\ &= G(\bar{r})^{-(n-4)/(n-3)} G(\bar{r}) \end{aligned} \tag{B22}$$

and

$$\begin{aligned} &((n-3)^2(\bar{r}^2+R^2))^{-\frac{n-4}{n-3}}\mathrm{d}\bar{r}^2 \\ &\quad = ((n-3)^2R^2 + r^{2(n-3)})^{-\frac{n-4}{n-3}} \frac{4(n-3)^2R^2}{M^2} r^{2(n-4)}\mathrm{d}r^2 \\ &\quad = V(r)^{1/(n-3)} \frac{\mathrm{d}r^2}{V(r)}. \end{aligned} \tag{B23}$$

The metric is now written as

$$\begin{aligned} \mathrm{d}s^2 = &-e^{-2\beta U(r)}\mathrm{d}t^2 + e^{2\beta U(r)/(n-3)} V(r)^{1/(n-3)} \\ &\times \left(\frac{\mathrm{d}r^2}{V(r)} + r^2 \mathrm{d}\Omega^2\right). \end{aligned} \tag{B24}$$

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