Cosmological constraints on unimodular gravity models with diffusion

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A discrete space-time structure lying at about the Planck scale may become manifest in the form of very small violations of the conservation of the matter energy-momentum tensor. In order to include such kind of violations, forbidden within the general relativity framework, the theory of unimodular gravity seems as the simplest option to describe the gravitational interaction. In the cosmological context, a direct consequence of such violation of energy conservation might be heuristically viewed a "diffusion process of matter (both dark and ordinary)" into an effective dark energy term in Einstein's equations, which leads under natural assumptions to an adequate estimate for the value of the cosmological constant. Previous works have also indicated that these kinds of models might offer a natural scenario to alleviate the Hubble tension. In this work, we consider a simple model for the cosmological history including a late time occurrence of such energy violation and study the modifications of the predictions for the anisotropy and polarization of the cosmic microwave background (CMB). We compare the model's predictions with recent data from the CMB, supernovae type Ia, cosmic chronometers and baryon acoustic oscillations. The results show the potential of this type of model to alleviate the Hubble tension.

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I. INTRODUCTION

Modern cosmology relies on Einstein's equation, and thus, on the strict conservation of energy momentum tensor. This is a feature that seems essentially untouchable. However, a deeper look at the issue reveals that conservation of energy momentum is unlikely to be an exact feature of nature [1]. When taking into account the quantum nature of matter, it becomes rather apparent that our gravitational theory must also undergo some modifications which are likely to include at least small departures from exact conservation laws. However, these changes might be larger in early cosmological times and could have cumulative effects with empirically relevant consequences.

Recently, a group involving some of us [2], used the fact that in unimodular gravity (UG) the strict conservation of the energy momentum tensor is not required for the consistency of the theory, in contrast with what occurs in general relativity (GR). The theory might be said to reduce the general invariance under diffeomorphisms, which is characteristic of GR to a more restricted invariance under four-volume preserving diffeomorphisms.[1] Concretely [2] studied within that scheme—i.e. violation

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[1]Strictly speaking, the invariance under diffeomorphisms is automatic for any theory that relies on tensor fields defined over a manifold. The point, as discussed in [3], is whether or not, all geometrical entities entering the theory are related to (or derived from) the fundamental degrees of freedom represented in the formalism, which, in theories of gravitation, correspond to the space-time metric, or equivalent structures. When one contemplates the presence of some additional nondynamical structure, connected to space-time, but which is not assumed to behave in the appropriate manner under diffeomorphisms, one encounters the situations that are known as "violations of full diffeomorphism invariance." In the case under consideration, such structures could be either the distribution of spontaneous collapse events, called upon in addressing the conceptual problems of quantum theory as discussed in [1], or the entities that characterize an hypothetical discreteness of space-time.

of energy momentum conservation together with unimodular gravity—the idea that a violation of the conservation of the energy momentum tensor might have important and observable consequences on the value of the effective cosmological constant. Of course, the idea under consideration contemplates a violation that is too small to be directly detectable, as no direct observation of any departure from strict energy conservation has been observed at this stage. Further studies [4,5] considered the idea that such violations might also result from defects in the fabric of space-time, and its ultimate origin might lie at the level of the quantum gravitational description thereof. That analysis resulted on an attractive account for the nature and magnitude of the cosmological constant [4,5] which, without the use of any kind of fine-tuning, leads to an estimate for the amount of the dark energy which agrees with those emerging from astronomical and cosmological observations. In that proposal an effective cosmological constant represents the accumulated effect of extremely small departures of such conservation with the dominant part of the effect arising around the electroweak scale [yielding a value $\Lambda \sim m_p^2 (E_{ew}/m_p)^7$ which can easily be seen to be of correct order of magnitude]. The perspective provided by this work motivated the investigation of the possible role of the fundamental granularity of Planckian physics as the primordial cause of inhomogeneities ultimately observed at the CMB. This led to an alternative paradigm to the standard inflationary one where inhomogeneities at the CMB are directly linked with Planckian discreteness during a primordial inflationary era [6].

In some recent work [7,8] the issue of the so-called H_0 tension—i.e. the incompatibility at more than 4σ between the value of H_0 obtained in the context of the standard cosmological model by the use of latest CMB data [9] and what is obtained in studies based on type Ia supernovae (SnIa) explosions together with local distance calibrations (CC) [10,11]—has been considered from the present perspective. In the proposal for the generation of a cosmological constant in the context of UG gravity, the conditions that resulted in the relatively large violations of the energy-momentum tensor conservation are tied to regimes in which large space-time curvatures are involved. The proposal to deal with the H_0 tension is meant to be an extension of such an account, which thus must involve situations where similar high curvature conditions prevail in the post-recombination epoch (where the effective cosmological constant would have to change by a nontrivial amount). At late cosmological times the only situations where such strong gravitational fields can be reasonably assumed to arise correspond to the late time dynamics of black holes (BHs), particularly highly rotating ones, which might lose up to $1/3$ of their mass if slowed down via a diffusion mechanism without violating the second law of thermodynamics.[2]

[2]We assume that the second law of thermodynamics holds even in those extreme circumstances.

Such preliminary study, which assumed a simple expression for the violation of the energy-momentum tensor, concluded that the overall plausibility of the model appears to be supported by observational data [8]. That analysis focused on the effects on the value of the acoustic angular scale of the CMB and the comparison with the corresponding recent observational data, and therefore was able to consider just the changes in the global geometry of the Universe introduced by the UG model. A similar analysis which considered different assumptions for the variation of the cosmological constant was performed in [12]. Also in [13], the authors consider a cosmological model with sign switching cosmological constant and show that this model can alleviate some tensions that are related to the so-called Hubble tension. In the present paper we perform a more complete and strict (methodologically speaking) analysis of the viability of the scenario studied in [8]. Concretely we study a relatively simple model parametrized by three quantities: one characterizing the magnitude of the violation of energy conservation involved, and the other two characterizing the cosmological period during which the violation is assumed to have taken place. We analyze the model by simultaneously considering the anisotropies and polarization of the CMB, involving therefore the growth of the structure of the Universe, in addition to the other data related to the global geometry of the Universe.

The physical source of the relatively large violation of the conservation of energy momentum is thought to be related to the dragging effect on the rotation of black holes, presumably arising from some fundamental granularity of space-time. Thus, in order to extract a formula for the *energy-momentum violation current* $\mathbf{J}_a \equiv \nabla^b \mathbf{T}_{ab}$ and produce quantitative calculations, one would need to model the cosmological density of black holes as a function of their mass, their angular momentum and cosmic time $n = f(M, J, t)$ in combination with the proposed form of the effective dissipation rate described in [7]. With that expression at hand, we could directly insert it into the equations of UG, and study the resulting modifications of the late time cosmology, allowing us to check whether or not the corresponding predictions can account well for all the relevant observations. Unfortunately, at this stage, our knowledge about $f(M, J, t)$ is just too vague to carry such analysis. There are only some known bounds on the proportion of matter that might be in the form of primordial black holes which unfortunately seem to refer mostly to monochromatic contributions [14] rather than constraints on something like the function $f(M, J, t)$ or even the momentum integrated function,

$$F(M, t) = \int_0^{M^2} f(M, J, t) dJ$$

(the upper bound resulting from limiting consideration of up to the extremal case). Beyond that, we have recently learned [15–23] that the range of masses of black hole is very wide, involving in the lower end objects as light as a few solar masses and at the other extreme objects as large as 10^9 solar masses, with increasing evidence for the existence of many objects in the 60–100 solar mass range. It is fair to say that we have very little understanding regarding the process by which all but the lighter black holes formed and also very limited knowledge of the form that even $F(M,t)$ might take at intermediate scales (say 10^2–10^5 solar mass range).

Despite our ignorance concerning the details of the current characterizing the violation of energy-momentum conservation in the general cosmological context, it is useful to perform a first analysis using recent cosmological data, such as the ones provided by the CMB, baryon acoustic oscillations (BAOs), SnIa and CC, to test the viability of cosmological models that are motivated by UG to explain the cosmological evolution including the growth of perturbations. However, it follows from the previous discussions that some assumptions, related to the violation of the energy-momentum conservation, will have to be included in order to describe the behavior of the dark energy component of the Universe as well as the baryonic and dark matter ones.

The organization of this manuscript is as follows: In Sec. II we offer a brief discussion of the general theoretical ideas underpinning this work. Section III is devoted to specify the concrete cosmological model based on these ideas and to characterize its effective parametrization as well as the ensuing evolution of the various contributions to the energy density budget as a function of cosmic time. We also discuss in this section the effects of the modified cosmic evolution introduced by our model on the CMB spectrum. The details of the analysis method used in this work to perform the statistical analyses (including the observational datasets) are discussed in Sec. IV. Results of the statistical analysis of the modified cosmological model motivated by unimodular gravity with CMB, BAO, SnIa and CC data are shown in Sec. V. We end in Sec. VI with a brief discussion of the results, the overall viability of the model, and of the general theoretical scenario.

II. THEORETICAL SCENARIO

A novel and rather successful scheme to account for the nature and magnitude of the cosmological constant, yielding in a natural way its correct order of magnitude, was proposed in [4,5]. This was a particularly encouraging result, particularly when contrasted with standard estimates that are typically wrong by 120 orders of magnitude. The ingredients underlining the proposal are:

(1) The generic idea that quantum gravity implied some sort of space-time granularity.

(2) The result of [24] where it was showed that the granularity cannot select a preferential reference frame without generating (presently) detectable violations to Lorentz invariance at low energies.

(3) The idea that the two considerations above might be reconciled if the effects of granularity become relevant only in the presence of strong gravitational fields or high curvature.

A basic assumption is that the interactions with granularity need to be of a *relational* nature in the sense that the relevant affected degrees of freedom must be capable (via their excitations) of selecting the local preferential reference frame with respect to which the fundamental scale can have an operational meaning. This suggested the rather natural assumption that the probes which could be sensitive to such granularity needed to possess both a scale (and thus break scale invariance to relate to the curvature scales) and an intrinsic direction to determine the directionality of the effects under consideration. The natural candidate was thus identified as spinning and massive degrees of freedom. The effect would have to be modulated by curvature (to avoid the no-go result of item 2 above). A mean-field perspective suggests taking the scalar curvature $\mathbf{R}$ which (linked to the trace of the energy-momentum tensor via Einstein's equations) is a natural order parameter of the violations of scale invariance for the gravitational sources involved. This led us to propose that, at the level of the effective characterization of matter in terms of particles, the effect would take the form of a deviation from the standard geodesic equation given by

$$u^\mu \nabla_\mu u^\nu = \alpha \frac{m}{m_{\rm p}^2} \mathrm{sign}(s \cdot \xi) \mathbf{R} s^\nu, \tag{1}$$

where $\alpha > 0$ is a dimensionless constant, m is the particle's mass, m_P is Planck's mass, s^μ is the particle's spin, and ξ^μ is a preferred timelike vector field characterizing the state of motion of the matter source. The analysis of this hypothesis in the cosmological context in terms of standard kinetic theory led to an effective violation of energy momentum conservation given by [4,5]

$$\mathbf{J}_\nu \equiv 8\pi G \nabla^\mu \mathbf{T}_{\mu\nu} = 4\pi\alpha \frac{T}{m_p^2} R[8\pi G \Sigma_i |s_i| \mathbf{T}^{(i)}] \xi_\mu; \tag{2}$$

where T is the temperature of the cosmological plasma, the sum involves contributions of the different species i in the standard model of particle physics, $\mathbf{T}^{(i)}$ is trace of the energy momentum tensor of the species i, and $|s_i|$ the spin of that species.

Moreover, given that violations of the energy-momentum conservation are inconsistent in the context of GR, the proposal needs to be formulated in a context of a gravity theory that allows for such violations [5,8]. In fact, the theory known as unimodular gravity permits a specific kind of violations and is therefore ideal to describe those types of effects expected to arise from a fundamental space-time granularity. The relatively recent interest in this theory, which was initially considered by Einstein himself, can be traced back to works such as [25,26]. At this point it is worthwhile to briefly describe the theory of unimodular gravity. The action can be written in a coordinate independent form, using, say, abstract index notation (see for instance [3]):

$$S = \int [R\epsilon_{abcd} + \lambda(\epsilon_{abcd} - \varepsilon_{abcd}) + \mathcal{L}_{\mathrm{matt}}\epsilon_{abcd}], \quad (3)$$

where ε_{abcd} is a fiduciary four-volume element and ϵ_{abcd} is the four-volume element associated to the metric g_{ab}, while $\lambda(x)$ is a Lagrange multiplier function. Using coordinates x^μ adapted to the fiduciary volume element, one is led to the simple relation $\epsilon_{abcd} = \sqrt{-g}\varepsilon_{abcd}$ with $g = \det g_{\mu\nu}$. The theory is said to be invariant under a restricted class of diffeomorphisms, i.e. those that are four-volume preserving.[3] The resulting equations of motion for the metric are

$$G_{ab}(x) + \lambda(x)g_{ab}(x) = 8\pi G T_{ab}(x) \quad (4)$$

while the variation with respect to λ leads to the constraint $\epsilon_{abcd} = \varepsilon_{abcd}$. The value of the Lagrange multiplier $\lambda(x)$ can be determined by taking the trace of the previous equation. This leads to

$$\lambda(x) = \frac{1}{4}(8\pi G T + R) \quad (5)$$

which upon substitution result in the standard form of the equations of unimodular gravity, namely,

$$R_{ab} - \frac{1}{4}g_{ab}R = 8\pi G\left(T_{ab} - \frac{1}{4}g_{ab}T\right). \quad (6)$$

As noted, a central feature of this theory, which makes it useful for our purposes, is that, in contrast with general relativity, it does not require the conservation law $\nabla^a T_{ab} = 0$, for its self-consistency. Taking the divergence of Eq. (4) and making use of the Bianchi identity, we find

$$\nabla_b \lambda(x) = 8\pi G \nabla^a T_{ab}(x). \quad (7)$$

In fact, one might define the "energy momentum non-conservation current" as $J_a \equiv 8\pi G \nabla^b T_{ab} \neq 0$ which, provided the integrability condition $dJ = 0$ is satisfied,[4] can be integrated to yield

$$R_{ab} - \frac{1}{2}g_{ab}R + g_{ab}\left(\lambda_{-\infty} + \int J\right) = 8\pi G T_{ab} \quad (8)$$

which coincides with Einstein's equation with a term that acts like an "effective dark energy component" which need not be a constant. In the cosmological setting, the role of this term—just as it occurs with the usual cosmological constant term for the range of values that are relevant in our Universe—is entirely negligible except for the very late times, where every other contribution to the Universe's energy budget has to be diluted to an extreme level, and thus such term can become dominant.

Calculating the contribution to dark energy in terms of Eq. (2) using the standard cosmological history of our Universe,[5] it was found that the effective cosmological constant was given approximately by

$$\Lambda \approx 16\pi\alpha\sqrt{\frac{5\pi^3}{g^*}}\frac{m_t^4 T_{\mathrm{EW}}^3}{\hbar^2 m_p^2}\epsilon(T_{\mathrm{EW}}), \quad (9)$$

where m_t is the mass of the heaviest particle with non-vanishing spin (here the top quark), T_{EW} is the temperature of the electroweak transition, g^* is the degeneracy factor and ϵ is a factor that takes into account the running of the top quark mass. For standard values of the cosmological parameters, the previous prediction coincides with the observed value of the cosmological dark energy in order of magnitude when the dimensionless parameter α is assumed to be of order 1.

It became quite clear after subsequent analysis that, apart from the generation of a cosmological constant, the conditions for other effects to be observable were simply too extreme to be accessible to testing in situations to which we have access at present in our Universe. For instance, simple dimensional considerations imply violation of Lorentz invariance parametrized in dangerous[6] dimension 5 operators modulated by $\mathbf{R}$ such as

$$\lambda\bar{\psi}\gamma_\mu\psi\xi^\mu\frac{\mathbf{R}}{m_p}, \quad (10)$$

for fermion fields ψ, a dimensionless λ. According to Kostelecky [27] the strongest bound on such operators is

$$\frac{\lambda\mathbf{R}}{m_p^2} \approx \lambda\frac{\rho}{m_p^4} < 10^{-12}. \quad (11)$$

With neutron star densities estimated to reach scales of about $\rho_{\mathrm{n}} \approx 10^{-80}m_p^4$, it is easy to see that not even under

[3]Here, as is common in these discussions it is assumed that only the dynamical elements are subject to the action of the diffeomorphisms.

[4]The integrability condition is a direct consequence of the volume-preserving diffeomorphism invariance of the action principle [2].

[5]This is justified because it can be shown that the resulting changes in the dynamics of the background are completely negligible during the period in which the effective cosmological constant is generated, and its effects only become relevant at the very late cosmological times when the dilution of all other forms of energies make the dark energy the dominant component.

[6]These are terms that might appear in the effective matter Lagrangian, which have the potential to generate violations of Lorentz invariance that are large enough to enter in conflict with empirical bounds.

such extreme conditions, empirically significant direct manifestations of the previous fundamental matter-diffusion type of effect can take place in matter configurations as presently known.

However, similar effects could be important in the context of nonfundamental matter forms; for instance, in the case of black holes produced by the gravitational collapse of fundamental matter fields which later become (for the purpose of outside observers) a seemingly empty region but with a (classically) unbounded internal curvature. Of course, black holes are not simple pointlike particles and the applicability of Eq. (2) to such macroscopic objects might not seem directly justified. Nevertheless, the fact that black holes are intimately connected with regimes of arbitrarily high curvatures—expected to reach Planck scale curvatures inside where only a full quantum gravity description could be truly relied on—naturally opens the door to the consideration of the phenomenological possibility that they might be affected in a special way by our hypothetical space-time granularity. This suggests, as discussed in [7], that black holes could be subject to some kind of *effective friction* related to the motion of the "frames they drag" with them and those associated to the matter that was connected to the space-time structure of their respective environment.

This led to two interconnected effects responsible for a translational and a rotational friction, respectively. An early analysis carried out in [7] showed clearly that the one that seems more likely to have any important energetic relevance was the second effect, in which (assuming, conservatively, that the second law would not be violated by the type of effects considered) up to 30% of the mass of a highly rotating black hole could be lost as a result of the rotational friction. Moreover, the same analysis indicates that in the context of unimodular gravity the rotational friction could be a significant source of a change in the effective cosmological constant.

Even when the basic theoretic ingredients for the generalization of (2) to the case of black holes are available from the analysis of [7], an accurate account of the number density of black holes as a function of their mass angular momentum and cosmic time $n = f(M, J, t)$ for quantitative estimates. Such a formula presumably would have to incorporate the formation of new black holes as the result of standard gravitational collapse during cosmic evolutions, the effects of black hole mergers, as well as any possible contribution of primordial black holes. With $n = f(M, J, t)$, one would be able to compute both the modification of that distribution due to the novel friction hypothesis as well as (most importantly for this work) its exact contribution to the evolution of the dark energy density during late cosmological dynamics. A project is under way to try to construct reasonable models for $f(M, J, t)$ taking into account hypothetical primordial distribution, simplified models of merger histories, and direct bounds on abundances of black holes at certain mass scales.

Unfortunately that task will not be completed in the very near future to the level where the form of f will be narrow enough so that we could think of simulating the effects with a single $f(M, J, t)$. That means that even in the best of circumstances we will have a multiparametric characterization of the possible $f(M, J, t)$ s and the studies of their cosmological effects will have to be performed in conjunction with data analysis of many other sources such as the one we will contemplate here.

In view of this, one can consider rather simple models to describe the evolution of the cosmological constant over time [8]. As this study is motivated by the so called "H_0 tension" we focus attention on the result of the effect we have described that might have taken place from the time of decoupling until today. We therefore study the overall feasibility of the model in the light of the available cosmological data.

Some preliminary results were obtained by setting the current value of Hubble parameter, H_0, to the SnIa estimation [10] and considering for the range of values of the model parameters that would be roughly compatible with the observational value of θ, the angular acoustic scale of the CMB. This gave encouraging results in the light of the H_0 tension, as the fitting of the model parameters did not require a large change in either the age of the Universe or the present amount of matter (both baryonic and dark components). We note that this preliminary analysis only studied global geometric quantities like θ, however a change in the evolution of cosmic history introduced by this kind of models would also affect the growth of perturbations that are the seeds of the present cosmic structure. In order to include consideration of such aspects, a more complete analysis, like the one presented in the present paper, which also includes the prediction of the anisotropies and polarization of the CMB, as well as geometric quantities such as the luminosity or angular diameter distance, is necessary.

In this work, we consider some relatively richer model which will be carefully described in Sec. III. The friction-like effect will be characterized by three parameters: two of them are used to describe the period in which the effect of "dissipation of black hole rotational energy" takes place, and the last one characterizes the overall magnitude of the corresponding change in the cosmological constant. At the level of analysis that is possible, given our lack of knowledge regarding $f(M, J, t)$, it makes sense to simplify the treatment and model directly the time dependence of the effective energy transfer to dark energy to study its effects in the dynamics of the scale factor. These modifications of the background evolution are then inserted into the standard analysis of structure growth starting from the usual form of the quasi-flat-scale invariant primordial spectrum usually taken to emerge during the inflationary epoch.

This paper aims at dealing with two crucial aspects: first, the constraining of the parameters of the theory with the current data using a complete and appropriate statistical analysis; next, the examination and interpretation of the results obtained in light of other relevant constraints. In particular, one has the usual nucleosynthesis constraint setting the overall contribution to the Universe's current energy budget appearing in the form of baryonic matter at 4%–5%, and on the other the high precision data emerging from the CMB, providing bounds on the relative abundances of dark matter to baryonic matter at the last scattering surface. On the other hand, as the model calls for anomalous reduction of the corresponding energy densities at late times, we must ensure compatibility with the data from nearby galaxies and galaxy clusters constraining the present abundance of baryonic matter as well as bounds on nonluminous matter (which might include some baryonic components). In other words, the proposed mechanism, if it is to be considered as viable, should not result in too large of a reduction in baryonic or dark matter components today so as to generate a conflict with the relevant observations. As we will see the best fits obtained in this work avoid that "danger" by a rather large margin.

III. THE MATTER DENSITIES IN THE UNIMODULAR MODEL

Let us consider a homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker (FLRW) Universe, and introduce our modifications at an effective level in the sector of both matter (including baryonic and dark matter) and dark energy. In the context of the UG theory, the modified conservation equation (7), when applied to the energy momentum of a pressureless fluid (taken as representing the matter content of the Universe after the radiation dominated epoch), can be expressed as

$$\dot{\rho}_M + 3\frac{\dot{a}}{a}\rho_M = -\dot{\rho}_\Lambda, \tag{12}$$

where we have denoted $\rho_\Lambda \equiv \frac{\lambda}{8\pi G}$. Therefore, provided an expression for $\rho_\Lambda(t)$ we can integrate the latter equation from the beginning of the radiation era ($t_{\rm rad}$) up to an arbitrary later time, t, to obtain an expression for $\rho_M(t)$. As discussed previously, even with a phenomenological model for the diffusive process in black holes such as that described in the Appendix (and originally in [7]), the form of the energy-momentum violation current cannot be well described given our present lack of knowledge of the black hole densities in the late Universe. However, we can postulate a simple expression for ρ_Λ in order to analyze the generic viability of a UG model with diffusion as follows:

$$\rho_\Lambda(a) = \rho_\Lambda(t_{\rm rad}) + f(a)\Delta\rho. \tag{13}$$

At $t_{\rm rad}$, the corresponding scale factor is referred to as $a_{\rm rad}$. Assuming $f(a_{\rm rad}) = 0$ and $f(a_0) = 1$, the current (i.e today) value of the dark energy density reads

$$\rho_\Lambda(a_0) = \rho_\Lambda(t_{\rm rad}) + \Delta\rho = \rho_\Lambda^0, \tag{14}$$

where ρ_Λ^0 is the current value of the dark energy density. The final expression for $\rho_\Lambda(a)$ yields

$$\rho_\Lambda(a) = \rho_\Lambda^0 + \Delta\rho[f(a) - 1]. \tag{15}$$

For the sake of simplicity we assume $f(a)$ to be constant in two time intervals, while shows a linear behavior in the interval of interest as follows:

$$f(a) = \begin{cases} 0 & a \in (a_{\rm rad}, a^* - \delta/2), \\ \frac{a - a^* + \delta/2}{\delta} & a \in (a^* - \delta/2, a^* + \delta/2) \\ 1 & a \in (a^* + \delta/2, a_0). \end{cases} \tag{16}$$

In this way ρ_Λ is constant during the Universe evolution, except for the time interval for which the value of the scale factor a is $(a^* - \delta/2,\ a^* + \delta/2)$ where it changes linearly in a. Moreover, the value of the cosmological constant is not always the same, namely $\rho_\Lambda = \rho_\Lambda^0 - \Delta\rho$ when $a \in (a_{\rm rad}, a^* - \delta/2)$ and $\rho_\Lambda = \rho_\Lambda^0$ when $a \in (a^* + \delta/2, a_0)$. We should consider of course $a_{\rm rad} < a^* - \delta/2$ and $a^* + \delta/2 < a_0$. In this way, $\Delta\rho$ accounts for the change in the value of the cosmological constant with respect to its present value ρ_Λ^0, and $(a^* - \delta,\ a^* + \delta)$ refers to the time interval in which the energy-momentum tensor is not conserved. Consequently, we can obtain the expression for the total matter density $\rho_M(t)$ as a function of the scale factor integrating Eq. (12) and assuming Eqs. (15) and (16):

$$\rho_M(t) = \begin{cases} \frac{a_{\rm rad}^3 \rho_M(t_{\rm rad})}{a(t)^3} & a \in (a_{\rm rad}, a^* - \delta/2), \\ \frac{a_{\rm rad}^3 \rho_M(t_{\rm rad})}{a(t)^3} - \frac{\Delta\rho}{4\delta}\left[a - \frac{(a^* - \delta/2)^4}{a(t)^3}\right] & a \in (a^* - \delta/2, a^* + \delta/2), \\ \frac{a_{\rm rad}^3 \rho_M(t_{\rm rad})}{a(t)^3} - \frac{\Delta\rho}{a(t)^3}\left[a^{*3} + \frac{(a^* \delta^2)}{4}\right] & a \in (a^* + \delta/2, a_0). \end{cases} \tag{17}$$

We now proceed to analyze the observational predictions of this model and how the CMB spectra are modified with respect to the standard model as the a^*, δ, $\Delta\rho$ parameters vary. For this, we recall that baryons and dark matter have different physical interactions with other particles during the formation of neutral hydrogen and the following Universe evolution, and therefore in principle they should be described separately. Let us propose the following expression for the baryon, $\rho_B(t)$, and dark matter, $\rho_{\rm DM}(t)$, densities which follow from Eq. (17):

$$\rho_B(a) = \frac{a_{\rm rad}^3 \rho_B(t_{\rm rad})}{a(t)^3} + \alpha F(a, \Delta\rho, a^*, \delta) \tag{18}$$

$$\rho_{\rm DM}(a) = \frac{a_{\rm rad}^3 \rho_{\rm DM}(t_{\rm rad})}{a(t)^3} + (1 - \alpha) F(a, \Delta\rho, a^*, \delta), \tag{19}$$

where

$$F(a,\Delta\rho,a^*,\delta) = \begin{cases} 0 & a\in(a_{\rm rad},a^*-\delta/2),\\ -\frac{\Delta\rho}{4\delta}\left[a-\frac{(a^*-\delta/2)^4}{a(t)^3}\right] & a\in(a^*-\delta/2,a^*+\delta/2),\\ -\frac{\Delta\rho}{a(t)^3}\left[a^{*3}+\frac{(a^*\delta^2)}{4}\right] & a\in(a^*+\delta/2,a_0). \end{cases} \quad (20)$$

In order to obtain expressions for α, $a_{\rm rad}$, $\rho_B(t_{\rm rad})$, $\rho_{\rm DM}(t_{\rm rad})$ in terms of the present values of the baryon density parameter $\Omega_B = \frac{\rho_B(t_0)}{\rho_{\rm crit}}$ and the dark matter density parameter $\Omega_{\rm DM} = \frac{\rho_{\rm DM}(t_0)}{\rho_{\rm crit}}$ where $\rho_{\rm crit}$ is the present critical density, we first define the present fraction of baryon to dark matter density as follows:

$$\beta = \frac{\rho_B(t_0)}{\rho_{\rm DM}(t_0)}. \quad (21)$$

Next, we assume that the fraction of baryon to dark matter at the beginning of the radiation era is equal to its present value[7]

$$\beta = \frac{\rho_B(t_0)}{\rho_{\rm DM}(t_0)} = \frac{\rho_B(t_{\rm rad})}{\rho_{\rm DM}(t_{\rm rad})}. \quad (22)$$

With a bit of algebra we obtain from Eqs. (21) and (22) that

$$\alpha = \frac{\beta}{1+\beta} = \frac{\rho_B(t_0)}{\rho_B(t_0)+\rho_{\rm DM}(t_0)} = \frac{\Omega_B}{\Omega_B+\Omega_{\rm DM}}. \quad (23)$$

It is important to note that Ω_B and $\Omega_{\rm DM}$ are different from Ω_B^Λ and $\Omega_{\rm DM}^\Lambda$ the current baryonic and dark matter densities in units of the critical density defined in the standard cosmological model.

Therefore, recalling that $\Omega_B = \frac{\rho_B(a_0)}{\rho_{\rm crit}}$ and $\Omega_{\rm DM} = \frac{\rho_{\rm DM}(a_0)}{\rho_{\rm crit}}$ we obtain the expressions for $\rho_B(a)$ and $\rho_{\rm DM}(a)$ in terms of Ω_B and $\Omega_{\rm DM}$:

$$\rho_B(a) = \begin{cases} \frac{\Omega_B\rho_{\rm crit}+\frac{\Omega_B}{\Omega_B+\Omega_{\rm DM}}\Delta\rho(a^{*3}+\frac{a^*\delta^2}{4})}{a^3} & a\in(a_{\rm rad},a^*-\delta/2)\\ \frac{\Omega_B\rho_{\rm crit}+\frac{\Omega_B}{\Omega_B+\Omega_{\rm DM}}\Delta\rho(a^{*3}+\frac{a^*\delta^2}{4})}{a^3} - \frac{\Omega_B}{\Omega_B+\Omega_{\rm DM}}\frac{\Delta\rho}{4\delta}\left[a-\frac{(a^*-\delta/2)^4}{a^3}\right] & a\in(a^*-\delta/2,a^*+\delta/2),\\ \frac{\Omega_B\rho_{\rm crit}}{a^3} & a\in(a^*+\delta/2,a_0) \end{cases} \quad (24)$$

$$\rho_{\rm DM}(a) = \begin{cases} \frac{\Omega_{\rm DM}\rho_{\rm crit}+\frac{\Omega_{\rm DM}}{\Omega_B+\Omega_{\rm DM}}\Delta\rho(a^{*3}+\frac{a^*\delta^2}{4})}{a^3} & a\in(a_{\rm rad},a^*-\delta/2)\\ \frac{\Omega_{\rm DM}\rho_{\rm crit}+\frac{\Omega_{\rm DM}}{\Omega_B+\Omega_{\rm DM}}\Delta\rho(a^{*3}+\frac{a^*\delta^2}{4})}{a^3} - \frac{\Omega_{\rm DM}}{\Omega_B+\Omega_{\rm DM}}\frac{\Delta\rho}{4\delta}\left[a-\frac{(a^*-\delta/2)^4}{a^3}\right] & a\in(a^*-\delta/2,a^*+\delta/2),\\ \frac{\Omega_{\rm DM}\rho_{\rm crit}}{a^3} & a\in(a^*+\delta/2,a_0). \end{cases} \quad (25)$$

On the other hand, we point out that the baryon and dark matter fractions that must be considered for the nucleosynthesis predictions are

$$\Omega_B^{\rm early} = \Omega_B + \frac{\Omega_B}{\Omega_B+\Omega_{\rm DM}}\frac{\Delta\rho}{\rho_{\rm crit}}\left(a^{*3}+\frac{a^*\delta^2}{4}\right) \quad (26)$$

$$\Omega_{\rm DM}^{\rm early} = \Omega_{\rm DM} + \frac{\Omega_{\rm DM}}{\Omega_B+\Omega_{\rm DM}}\frac{\Delta\rho}{\rho_{\rm crit}}\left(a^{*3}+\frac{a^*\delta^2}{4}\right). \quad (27)$$

Note that, except for the interval $(a^*-\frac{\delta}{2},\ a^*+\frac{\delta}{2})$, the baryon and dark matter densities behave in the same way as the ones of the ΛCDM model. Moreover, when $a<a^*-\frac{\delta}{2}$, the density parameters are equal to $\Omega_B^{\rm early}$ for baryons and $\Omega_{\rm DM}^{\rm early}$ for dark matter, while for $a>a^*+\frac{\delta}{2}$, Ω_B and $\Omega_{\rm DM}$ are the ones defined above, and correspond to the "late time values." On the other hand, the evolution of the baryonic and dark matter densities during the interval $(a^*-\frac{\delta}{2}, a^*+\frac{\delta}{2})$ is different from the usual ΛCDM ones as can be inferred

[7]As a rather straightforward motivation for this simplifying assumption we might adopt the view that most black holes in the Universe (or at least those that play a substantial role in the process at hand) originate ultimately in a population of primordial black holes, which in turn were formed well before the spatial distributions of dark and baryonic matter differed substantially. Thus, the great majority of matter in the form of black holes can be viewed as traceable to such components which would therefore naturally contribute in proportion to their cosmic abundances. The result is that, when energy in black holes is lost as a result of our hypothetical aforementioned "effective friction," the proportion of baryonic and dark matter would remain essentially unchanged in the whole process.

from Eqs. (24) and (25), which reflect what should be expected for the time interval during which the energy-momentum conservation is violated.

Let us stress that we are considering the simplifying assumption that both components of matter (baryonic and dark) are at the end equally affected by the energy diffusion under consideration. The expressions we have obtained for the evolution of the baryon and dark matter and dark energy densities in our unimodular model with diffusion will result in a modification of the Friedmann equation as follows:

$$H^2(a) = \frac{8\pi G}{3}[\rho_R(a) + \rho_B(a) + \rho_{\rm DM}(a) + \rho_\Lambda(a)] - \frac{k}{a^2}, \tag{28}$$

where $\rho_B(a)$, $\rho_{\rm DM}(a)$ and $\rho_\Lambda(a)$ are described by Eqs. (24), (25), and (13) respectively and $\rho_R(a)$ is the radiation density which is not modified in our model. The change in the Hubble factor results in changes in other relevant physical quantities for CMB physics such as $z_{\rm eq}$, i.e., the redshift at which the density of matter and radiation are equal, z_{LS}, which is the redshift of last scattering, $\theta(z_{LS})$, which is the angular diameter distance at last scattering and l_D, the diffusion damping length. Also, for baryon acoustic oscillations (BAOs) physics we have to consider the modification in $z_{\rm drag}$, i.e., the redshift at which baryons decouple from photons.[8] In turn, $z_{\rm drag}$ and l_D are not only affected by the modification in the Hubble factor, but also depend on the speed of sound, which in turn depends on ρ_B. However, we have checked that the variations of the latter quantities are of order 0.07% for $\frac{\Delta\rho h^2}{\rho_{\rm crit}} = 0.002$ and therefore the most important effect introduced by our model is the change in the Friedmann equation.

A. Considerations for the CMB anisotropy and polarization spectra

We implement the model discussed in the previous section in the public code for anisotropies in the microwave background [28], changing the expressions of ρ_B, $\rho_{\rm DM}$ and ρ_Λ to the ones described in Eqs. (13), (16), (24), and (25), both in the background evolution and in the calculation of the growth of perturbations. We did not consider the linear perturbation equations of unimodular gravity, because we assume that the corresponding modification in the observable quantities would be very tiny compared to the change introduced by the background quantities. In principle, and as noted by an anonymous referee, it would be desirable to analyze the general problem (background and perturbations) in a rigorous way, if only to confirm the natural expectations that no significant changes arise for the situation at hand.

Next, in order to compare the behavior of the class of models analyzed in this paper, we assume as a reference a fiducial model, namely a ΛCDM one, with the cosmological parameters fixed to the best-fit values of the Planck collaboration (2018) [9]. Also, we define

$$\Delta\rho_\Lambda = \frac{8\pi G}{3}\frac{\Delta\rho}{100^2} = \frac{\Delta\rho h^2}{\rho_{\rm crit}} \tag{29}$$

which we use instead of $\Delta\rho$ to analyze the effects of assuming the unimodular models in the CMB anisotropy and polarization spectrum.[9]

In Fig. 1 we show the unimodular behavior as the parameters a^*, δ, $\Delta\rho_\Lambda$ vary, exploring the impact of one parameter at a time and leaving the other two fixed (indicated at the top of each column), while for the cosmological parameters we set the ΛCDM best-fit model reported by the Planck collaboration (2018) [9]. Our choice of the cosmological parameters for the fiducial and unimodular model results in that the physics of the late Universe ($a > a^* + \delta/2$) is the same in both models [see Eqs. (24), (25), (13), and (16)].[10]

We note that increasing $\Delta\rho_\Lambda$ results in a decrease in the height of the Doppler peaks and a corresponding increase in the valleys. Also, we note that this produces, in addition, a very small shift in the locations of the peaks. These effects are very similar to the ones that occur if the value of $\Omega_{\rm DM}h^2$ is changed, so we can anticipate a degeneration between $\Omega_{\rm DM}h^2$ and $\Delta\rho_\Lambda$, in the statistical analysis and parameter constraints. Moreover, it is useful to recall that a decrease in the value of Ω_B has the effect of decreasing the height of the odd peaks and enhancing the height of the even peaks. The latter is relevant, because we also note that the decrease (due to the increase in $\Delta\rho_\Lambda$) in the Doppler peaks is larger in the even than in the odd ones. The reason for this is that modifying $\Delta\rho$ affects both $\rho_{\rm DM}(t)$ and $\rho_B(t)$ in a proportional way and therefore the effect that is shown in Fig. 1 is a combination of the change in both densities through the variation in $\Delta\rho$. Therefore, we also expect a degeneration between Ω_B and $\Delta\rho_\Lambda$, in the statistical analysis. We further note that, as a^* moves away from 1 in the model, the stage of the Universe during which its physical parameters are

[8] Since there are far more photons than baryons, after photon decoupling the photons continued to drag baryons with them slightly longer into the Compton drag epoch. The redshift at which the baryon velocity decouples from the photons is called $z_{\rm drag}$.

[9] In this way, the difference between the values of the cosmological constant at early and late times can be expressed as a density parameter, in a similar way, to the baryon and dark matter density parameters.

[10] Here we mean that both the behavior of the energy densities and the corresponding energy parameters are the same in this period($a > a^* + \delta/2$). Note that as a result the unimodular model will not necessary fit BBN constraint, but the point of this section is to study the behavior of the CMB spectra as the unimodular parameters are changed in order to understand the results of the statistical analysis of the next section.

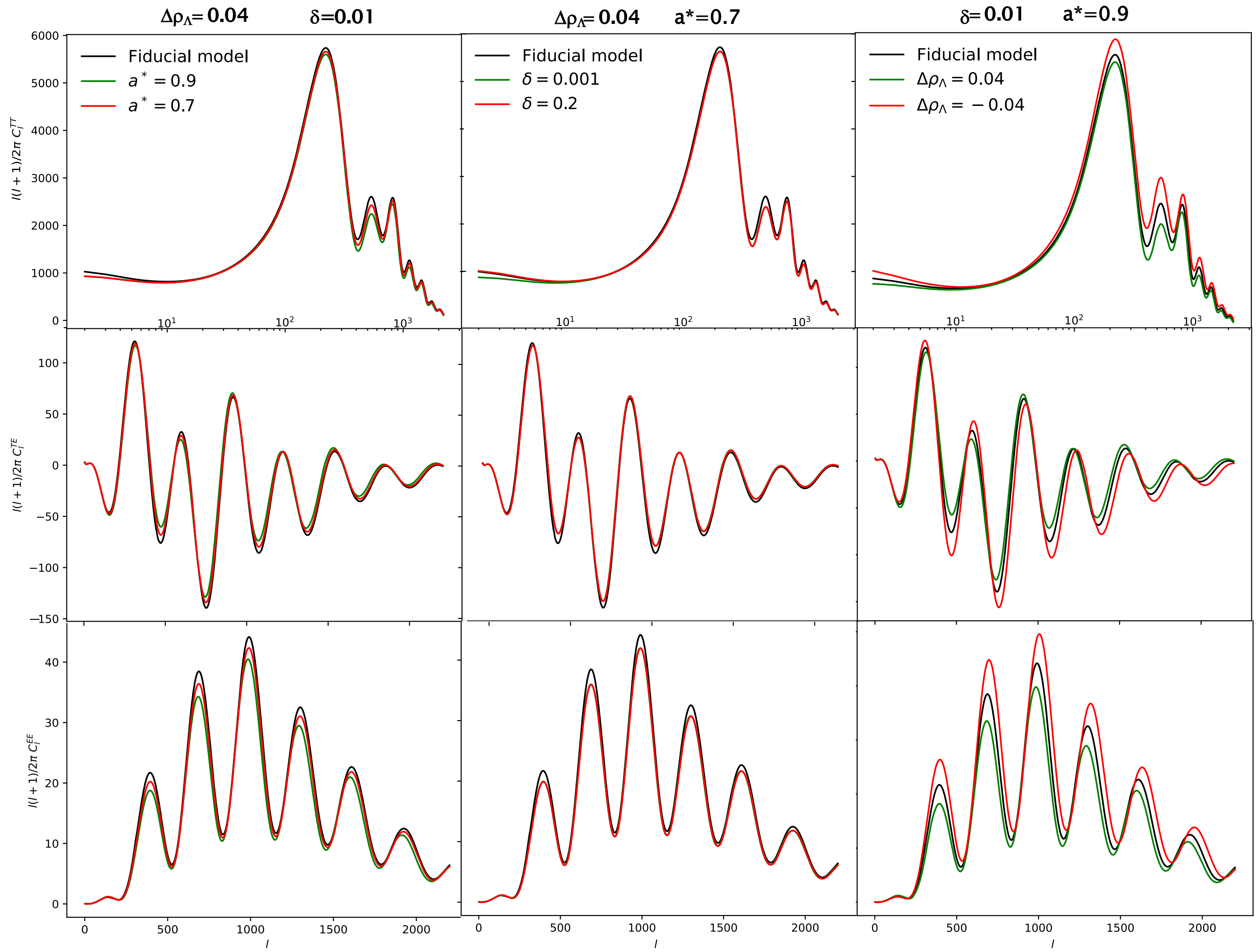

FIG. 1. The CMB anisotropy and polarization spectrum for different combinations of the unimodular parameter. The two fixed parameters are indicated at the top of the column, while the legend of each column indicates the third parameter with two explored values. Top: temperature autocorrelation function (C_l^{TT}). Middle: temperature E-mode cross-correlation function (C_l^{TE}). Bottom: E-model autocorrelation function (C_l^{EE}).

different from the fiducial model is enlarged, resulting in a decrease in the peaks and valleys of the spectrum. Moreover, a change in δ affects only the lower multipoles of the CMB spectra which is usually attributed to the integrated Sachs-Wolff (ISW) effect if a standard model of inflation is assumed. Moreover, it is well known that the ISW effect scales with the amount of dark energy [29]. We recall that the dark energy in our model behaves as $\frac{1}{\delta}$ [see Eqs. (15) and (16)] and this explains why, for greater values of δ, the departure from the standard behavior is less than for lower values. Likewise, the low multipoles are also modified by a change in $\Delta\rho_\Lambda$ and a^*, and this can be explained since a change in these parameters also affects the amount of dark energy. Finally, due to the lower sensitivity of the spectra to changes in a^* and δ, we expect the degeneration of these parameters with the usual cosmological ones to be small or almost negligible.

IV. METHODOLOGY OF THE ANALYSIS

The observational predictions of the previous section are now compared with cosmological data, so as to obtain the constraints of the free parameters of the unimodular model. To do this, we use a Monte Carlo Markov chain exploration of the parameters space using the available package CosmoMC [30]. We consider an extended dataset comprising cosmic microwave background measurements, through the Planck (2018) likelihoods [31],[11] the CMB lensing reconstruction power spectrum [31,32], the baryon acoustic oscillation measurements from 6dFGS [33], SDSS-MGS

[11]We use Plik likelihood "TT, TE, EE + lowE" by combination of temperature TT, polarization EE and their cross-correlation TE power spectra over the range $\ell \in [30, 2508]$, the low-ℓ temperature Commander likelihood, and the low-ℓ SimAll EE likelihood.

[34], and BOSS DR12 [35] surveys, the type Ia SNe Pantheon compilation [36] and cosmic chronometer measurements of the expansion rate $H(z)$ from the relative ages of passively evolving galaxies [37–42]. We also consider a Gaussian prior for the SNe Ia absolute magnitude M, in order to consider the calibration given by the current H_0 local measurements [43], (-19.2435 ± 0.0373) mag, as suggested in [44]. This approach prevents double counting of low redshift supernovae and avoids assuming a value of the deceleration parameter, considering M constrained by the local calibration of SNeI, which is not included otherwise. Let we stress that several approaches have been considered in the literature to address the so-called "H_0 tension." The initial approaches started by fixing the value of H_0, or imposing a prior on it in the statistical analysis [45,46], but the method proved to be statistically inadequate [47–50] as it attempts to combine two datasets that are intrinsically incompatible when considered in the context of the ΛCDM model, i.e. CMB and SNeIa, and limits the parametric space of the analysis in regions not determined by the CMB data alone. Thus, it has been shown that it is statistically more acceptable to consider a prior on the supernova calibration parameter instead [44]. Of course, it is crucial to test if this incompatibility of supernovae and CMB datasets still holds in the context of the unimodular model and this will be analyzed in the Results section.

In our analysis, we vary the usual cosmological parameters, namely, the physical baryon density $\Omega_B h^2$, the physical cold dark matter density $\Omega_{\rm DM} h^2$, the ratio between the sound horizon and the angular diameter distance at decoupling θ, the optical depth τ, the primordial amplitude A_s, and the spectral index n_s. We also vary the unimodular model parameters $\Delta\rho_\Lambda$, a^* and δ and the nuisance foreground parameters [51]. We assume large flat priors for the free parameters, varying the unimodular ones between $\Delta\rho_\Lambda \in [-0.2; 0.2]$, $a^* \in [0.02; 1]$ and $\delta \in [0.02; 1]$. We consider purely adiabatic initial conditions. The sum of neutrino masses is fixed to 0.06 eV, and we limit the analysis to scalar perturbations with $k^* = 0.05$ Mpc.

We choose to perform two statistical analyses that we describe as follows: (i) the *main* analysis, which we simply call "unimodular parameters free," in which we let both the cosmological and the parameters of the unimodular model free to vary, (ii) a *speculative* analysis, which we refer to as "unimodular parameters fixed" where we fix the value of the parameters of the unimodular model to arbitrary values and let the other cosmological parameters vary. This second choice is due to the fact that, as mentioned in Sec. III, a degeneration between $\Delta\rho_\Lambda$ and the cosmological parameters $\Omega_{\rm DM}$, Ω_B and H_0 is expected and will be shown next in the Results section. Such degeneration can allow this model to predict higher H_0 values, and here we want to show the cost of this achievement and the ability of this model to alleviate the Hubble tension whether future data may show better sensitivity to the constraint of its parameters.

V. RESULTS

We present the results of our statistical analyses in Table I and Figs. 2–4. Note that we also included the results of a ΛCDM model analysis using the same dataset for comparison. About our main analysis, i.e. when the unimodular parameters are free to vary, we found an agreement within 1σ with the ΛCDM model cosmological parameters, as shown in the second column of the table and with gray curves in the plots. At the same time, both $\Delta\rho_\Lambda$ and δ are well constrained (see left panel of Fig. 4) while just a lower bound on a^* can be established. Here we stress that the data clearly indicate that the anomalous behavior in the energy densities must take place much later than the formation of neutral hydrogen ($a_{\rm rec} \sim 10^{-3}$).

TABLE I. Analysis constraints for the models parameters using the data set Planck(2018) + lensing + BAO + Pantheon + CC + M prior. Quoted intervals correspond to 68% C.L. intervals, whereas quoted upper/lower limits correspond to 95% C.L. upper/lower limits.

	ΛCDM		Unimodular parameters free		Unimodular parameters fixed	
Parameter	Mean value and 68% confidence levels	Best fit	Mean value and 68% confidence levels	Best fit	Mean value and 68% confidence levels	Best fit
$100\Omega_B h^2$	2.250 ± 0.013	2.254	2.242 ± 0.027	2.257	2.123 ± 0.012	2.132
$\Omega_{\rm DM} h^2$	0.1186 ± 0.0009	0.1191	0.1181 ± 0.0017	0.1189	0.1140 ± 0.0009	0.1134
τ	0.058 ± 0.007	0.053	0.056 ± 0.007	0.060	0.053 ± 0.007	0.052
$\ln(10^{10} A_s)$	3.050 ± 0.015	3.040	3.045 ± 0.015	3.055	3.058 ± 0.014	3.056
n_s	0.9684 ± 0.0036	0.9688	0.9700 ± 0.0056	0.966	0.984 ± 0.004	0.982
$\Delta\rho_\Lambda$	⋯	⋯	0.0010 ± 0.0040	−0.0003	Fixed to 0.09	⋯
a^*	⋯	⋯	>0.4	0.8	Fixed to 0.4	⋯
δ	⋯	⋯	<0.65	0.22	Fixed to 0.02	⋯
H_0 [Km/s/Mpc]	68.02 ± 0.40	67.88	68.51 ± 0.57	68.24	71.18 ± 0.42	71.43
$100\Omega_B^{\rm early} h^2$	⋯	⋯	$2.245^{+0.038}_{-0.035}$	2.255	2.173 ± 0.024	2.182
$\Omega_{\rm DM}^{\rm early} h^2$	⋯	⋯	0.1190 ± 0.0025	0.1195	0.1174 ± 0.0018	0.1168

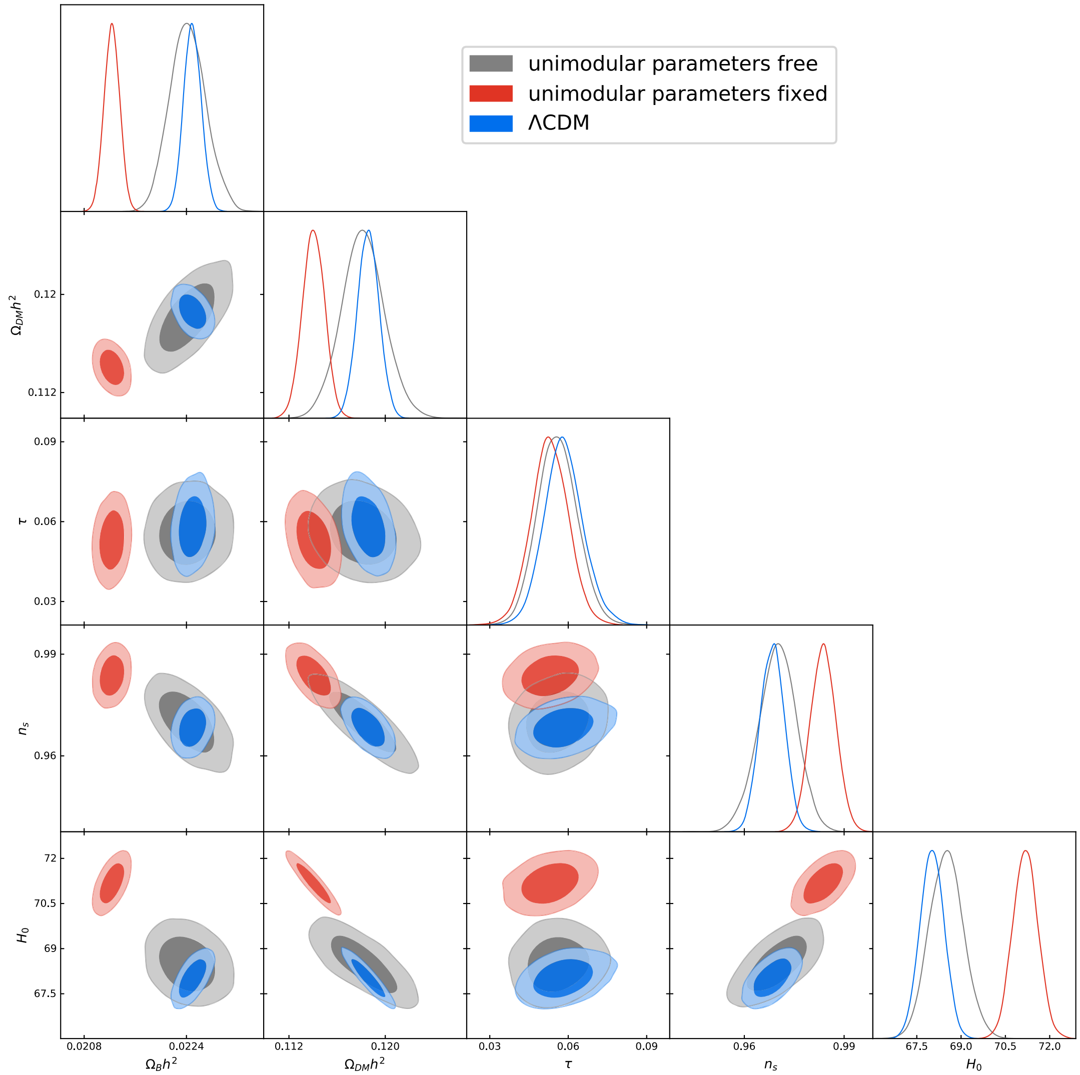

FIG. 2. Constraints on model cosmological parameters considering the case of all parameters free to vary (gray line), setting theory unimodular parameters to arbitrary values (red line), compared with the standard cosmological model (blue line).

The H_0 value of this analyses is compatible with that of the ΛCDM, although it should be noted that slightly higher values of H_0 are allowed. As expected (see discussion in Sec. III A), $\Delta\rho_\Lambda$ shows degeneracy with the matter density values and a weak correlation with H_0, as shown in the right panel of Fig. 4. On the other hand, the a^* and δ parameters show no degeneracy with the cosmological parameters, which can be explained by the low sensitivity of the CMB spectrum on these parameters (see discussion in Sec. III A). On the other hand, we recall that in our model the cosmological constant takes two different fixed values over different epochs of standard cosmological evolution [see Eqs. (15) and (16)]. Our results for $\Delta\rho_\Lambda$ show that the difference between them is small, namely of order 0.33%. Finally, it is worth mentioning that the $\chi^2_{\rm min}$ of this model is comparable to that of the standard model, but has three more parameters in the theory. On the other hand, one of the parameters of the unimodular model is not well constrained and this deserves further investigation with the next generation of data that may be more sensitive to the modifications to the usual theory analyzed here.

Given the difficulty in constraining the a^* parameter, and the apparent lack of degeneracy of δ with cosmological parameters, we now focus on a model where both a^* and δ parameters are fixed at the smallest values allowed by our first analysis, 0.4 and 0.02, respectively. Besides, the value of $\Delta\rho_\Lambda$ is set at a large positive arbitrary value which lies within the 1σ confidence interval obtained in the first analysis (i.e where all parameters varying freely). This allows us to perform a second speculative analysis, shown in red in Figs. 2 and 3 and in the last columns of the Table I. We found that, for this choice of unimodular parameter values, the H_0 is shifted into higher values with respect to the ΛCDM model, in agreement within 1.9σ with the latest value of Riess *et al.* [10] obtained from local measurements. This is an important result of the present work,

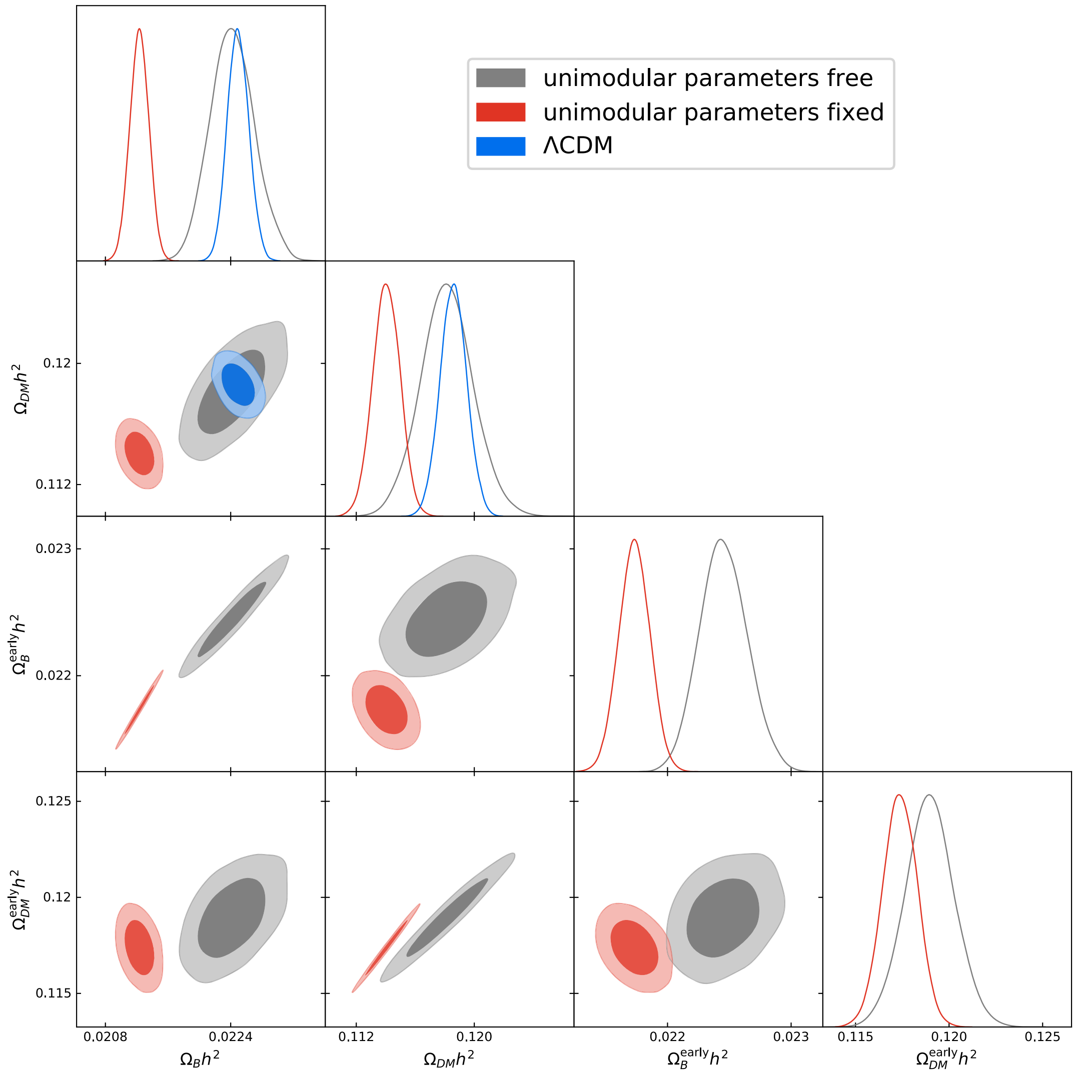

FIG. 3. Constraints on model density parameters ($\Omega_i h^2$) considering the case of cosmological and unimodular theory parameters free to vary (gray line), setting theory unimodular parameters to arbitrary values ($\Delta\rho_\Lambda = 0.09$, $a^* = 0.04$, $\delta = 0.02$) while the cosmological parameters are free to vary (red line), compared with the standard cosmological model (blue line).

because it provides a guide for the construction of models that describe the density of black holes, which are, according to our model, the scenario where the violation of the conservation of the tensor-energy moment occurs and, therefore, gives rise to the distinctive behavior of the model. Our results show that higher values of $\Delta\rho$ shift H_0 to values that further alleviate the Hubble tension and, therefore, the models that describe the energy diffusion in black holes (which are at present under construction) should, in order to fully solve the issue, point toward these values. We recall that, in the present work, we assumed a simple model for the behavior of ρ_Λ, which is supposed to arise from an energy diffusion from matter (both dark and baryonic) to dark energy catalyzed by black holes. Likewise the predicted values of n_s are also not in agreement with those of the ΛCDM model, but still consistent with the predictions of standard inflationary models. About the constraints for $\Omega_B h^2$ and $\Omega_{\rm DM} h^2$, there is also not agreement within 2σ with the ΛCDM model. However, as discussed in Sec. III, the behavior of these quantities is not the same in both models and, therefore, we do not expect to obtain the same estimation. Indeed, for the model to be viable it is necessary that the predicted values of the baryon density in the early Universe $\Omega_b^{\rm early} h^2$ are consistent with the nucleosynthesis constraint $\Omega_b h^2 = (0.021, 0.024)$ and this is verified by the results shown in Table I.

Finally, we use the deviance information criterion (DIC) [52] in order to test whether the complexity of the UG model is statistically supported by the data with respect to the vanilla ΛCDM one. The DIC has been proven to be a useful tool to test the average performance of a model with a penalty given by the Bayesian complexity [53–55] (see Refs. [56–58] and references inside for some applications

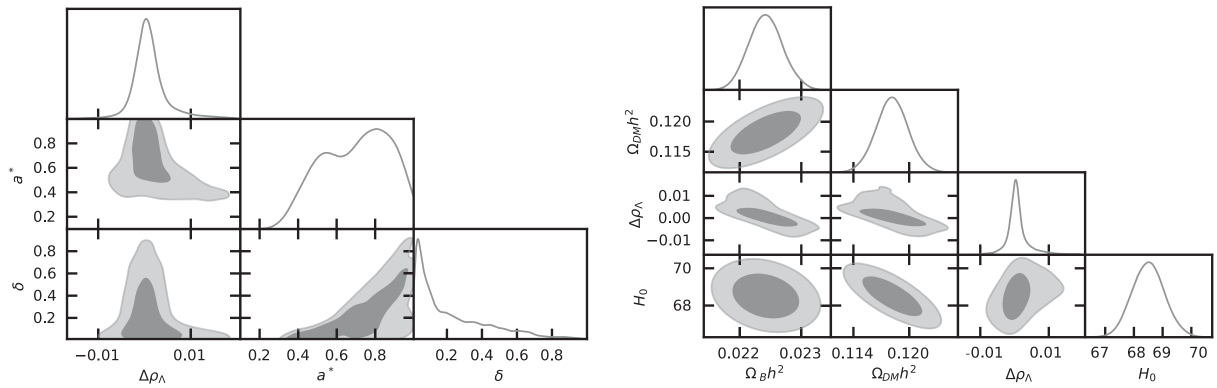

FIG. 4. Constraints on model unimodular parameters.

of DIC in cosmology). The DIC value is defined, for the selected model $\mathcal{M}$, as

$$\mathrm{DIC}_{\mathcal{M}} \equiv -2\overline{\ln \mathcal{L}(\theta)} + 2p_D, \tag{30}$$

where the first term is the posterior mean of $\mathcal{L}(\theta)$, i.e. the likelihood of the data given the model parameters θ, and the second term is the Bayesian complexity $p_D = -2\overline{\ln \mathcal{L}(\theta)} + 2\ln \mathcal{L}(\tilde{\theta})$, where the tilde refers to the chosen estimator.[12] We considered following [59] the following scale to determine the performance of our model in that test: $\Delta\mathrm{DIC} = 10/5/1$ as indicating, respectively, strong/moderate/null preference for the reference model (if the value of $\Delta\mathrm{DIC}$ is negative, it would indicate a preference for the model under consideration). As we obtained $\Delta\mathrm{DIC} = 3.6$, we conclude that there is no evidence, according to this criteria, to support the analyzed UG model over the standard ΛCDM one. This indicates that the current data are not sensitive enough to detect the full complexity of the model. However, we stress that even using this strict measure the model is not discarded in comparison to ΛCDM.

VI. SUMMARY AND CONCLUSIONS

In this work, we perform a methodologically proper analysis on a general idea involving late time violation of energy-momentum conservation, resulting from a kind of granularity of space-time whose ultimate origin lies in quantum gravitational features. For this, it is necessary to formulate the cosmological model in the context of unimodular gravity, a modification of GR that allows such violations (under certain conditions that hold automatically in cosmology). This idea was applied to the very early Universe to offer an account of the nature and magnitude of the cosmological constant, a fact that serves as a strong motivation to seek a resolution of the H_0 tension on similar grounds [2,5].

The analysis carried out in this work must however be considered as preliminary since the basic idea is that the effect would be linked to a kind of effective friction affecting black holes in the epoch between last scattering and the present. However, we lack, at this time, a clear picture of the black hole abundances as a function of their mass angular momentum and time in the relevant period. This forces us to use a rather simple and rudimentary model of the effect in terms of an anomalous evolution of the energy budget during the relevant epochs.

We analyze the model using a Boltzmann solver code to take into account both background and perturbation evolution. By comparison with a selected set of data, such as CMB, BAO, SnIa and CC, we are able to constrain, to the best of the current ability, the free parameters of the theory. In addition, we note the sensitivity limits of the data on constraining model complexity and discuss in depth the degeneracies between parameters. In summary, we are only able to constrain one unimodular parameter, $\Delta\rho_\Lambda$, while we can at most find upper/lower bounds for the other two, a^* and δ. We do not notice significant changes on the constraint of cosmological parameters compared to the standard cosmological model.

It is however a noteworthy fact that our results show that the model does not spontaneously reduce the tension on H_0, producing only a small shift in the value of the parameter. However, it must be emphasized that the potential of the model is not fully expressed as the data proved unable to fully constrain the unimodular parameters. In fact, looking at the results of our speculative analysis (in which we fix the values of the three unimodular parameters at arbitrarily chosen values), we see the H_0 tension being relaxed, resulting in the value of $H_0 = 71.6$ Km/s/Mpc at 1σ. This implies very small modification of most other cosmological parameters and without leading to a problematic depletion of the dark matter and baryonic components (dark and bright) which are of course required at late times to account for the present

[12]We choose to use the best fit as estimator, so that the DIC can be rewritten as $\mathrm{DIC}_{\mathcal{M}} = 2\ln \mathcal{L}(\theta) - 4\overline{\ln \mathcal{L}(\theta)}$. We obtain the mean likelihood from the output chains of the MCMC analysis, and the best-fit likelihood via the BOBYQA algorithm implemented in CosmoMC for likelihood maximization.

features of galaxies and galactic clusters. More specifically, the present day energy budget resulting from the model is the following ranges (the following values are at 68% confidence level): $\Omega_B = (0.0412, 0.0543)$, $\Omega_{\rm DM} = (0.2438, 0.2594)$, $\Omega_\Lambda = (0.69, 0.71)$ when all parameters are free to vary and $\Omega_B = (0.0453, 0.0458)$, $\Omega_{\rm DM} = (0.2429, 0.2467)$, $\Omega_\Lambda = (0.73, 0.74)$ when the unimodular parameters are fixed which can be compared with the standard ΛCDM values given by $\Omega_B = (0.0505, 0.0574)$, $\Omega_{\rm DM} = (0.2513, 0.2613)$, $\Omega_\Lambda = (0.69, 0.70)$.

In our view this clearly illustrates the potential of the proposal to fully resolve the H_0 tension, once a more clear picture of the detail form of the function $f(M, J, t)$ is known (or realistically characterized in terms of a suitable set of parameters) allowing a repetition on the analysis carried out in this work with better modeling of the relevant black hole abundances.

Finally, since the predictions of the unimodular model for the B modes are different from the ones of the standard model, future CMB polarization data should provide more strict constraints on the unimodular model.

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APPENDIX: EFFECTS OF THE FRICTION DRIVEN FORCE IN BLACK HOLES

In this Appendix we discuss the effects of the force driven by the friction that arises when the effects of granularity of space-times are considered in black holes. In this regard, general considerations led us to postulate a rotational friction term of the form

$$u^\mu \nabla_\mu s^\nu = \bar{\alpha}_{\rm bh} \frac{M}{m_{\rm p}^2} {\rm sign}(s\cdot\xi) \tilde{\mathbf{R}}(s\cdot s) u^\nu - \bar{\beta}_{\rm bh} \frac{M}{m_{\rm p}^2} \tilde{\mathbf{R}}_{\rm BH} s^\nu, \tag{A1}$$

where M is the black hole's mass, $\bar{\alpha}_{\rm bh}$ is connected to the translational friction term analogous to the one affecting particles [as in Eq. (1)] and the second term takes into account that, in contrast to elementary particles, the spin of the black hole might change not only in orientation but also in magnitude. Thus, $\bar{\beta}_{\rm bh}$ is a new dimensionless parameter characterizing the "intrinsic" spin diffusion term. In [7] two options for the natural order of magnitude of these effective parameters were considered, involving *"high"* and *"low"* suppression levels. In order to assess the magnitude of the effect under consideration, we compare the order of magnitude or the resulting anomalous force, which we denote by F, with that of the *standard* gravitational force between two black holes at the moment of closest approach in a black hole coalescence (BHC) event which we denote by $F_{\rm BHC}$. The estimate of $F_{\rm BHC}$ is based on the consideration of two equal mass BHs and the use of the Newtonian expression, evaluated when the separation between both is of the order of the Schwarzschild radius, and the anomalous force is estimated for the case that the two black holes are extremal so $s = (M/m_p)^2$ (the magnitude of the black hole's spin measured in natural units). The order of magnitude estimate for such a quantity turns out to be given by $|F_{\rm BHC}| \sim GM^2/(2GM)^2 = m_p^2/4$. It is worth emphasizing that while $F_{\rm BHC}$ is taken to represent the *standard gravitational interaction*, the hypothetical anomalous force F we are considering would be also, ultimately, of gravitational origin, although, in this case, tied intrinsically with the granular aspects, of space-time which we take to characterize its underlying fundamental quantum gravity origin. As an analogy, one might consider comparing an electromagnetic force of between two magnets with the friction force that affects their motion on a surface, which is, of course, also of electromagnetic origin. The results are the following [7], in the case involving a high suppression level,

$$\boxed{\bar{\alpha}_{\rm bh} = \bar{\alpha}_{\rm bh}^{01} \frac{m_{\rm p}}{M} \Rightarrow \left|\frac{F}{F_{\rm BHC}}\right| \le 4\bar{\alpha}_{\rm bh}^{01} 10^{-6} \left(\frac{M}{M_\odot}\right)^3}, \tag{A2}$$

and for the second possibility involving a low level of suppression,

$$\boxed{\bar{\alpha}_{\rm bh} = \bar{\alpha}_{\rm bh}^{02} \sqrt{\frac{m_{\rm p}}{M}} \Rightarrow \left|\frac{F}{F_{\rm BHC}}\right| \le 4\bar{\alpha}_{\rm bh}^{02} 10^{13} \left(\frac{M}{M_\odot}\right)^{\frac{7}{2}}}. \tag{A3}$$

These two possibilities might be seen as derived from a $1/\sqrt{N}$ suppression of some stochastic origin with the number of "area quanta" $N \approx M^2/m_{\rm p}^2$ or the number of "energy quanta" $N \approx M/m_{\rm p}$ involved, respectively.

The quantity $\tilde{\mathbf{R}}_{\rm BH}$ represents an appropriate measure of the mean local curvature in the surroundings of the black hole. Such a quantity could be for instance something like an "averaged value" of the local Kretschman scalar $\sqrt{R_{abcd}R^{abcd}}$, in the region occupied by the black hole.

For simplicity in [8] we explored the simple case in which that quantity is taken to be a simple estimate of curvature $\tilde{\mathbf{R}}_{\rm BH} = 1/M^2$. Once this choice is made, and taking natural order of magnitude estimates for the parameters $\bar{\alpha}_{\rm bh}$ and $\bar{\beta}_{\rm bh}$, the remainder of the analysis would in principle be quite direct but heavily dependent on aspects of astrophysics and cosmology on which our knowledge is still quite incipient.

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