Impact of trans-Planckian quantum noise on the primordial gravitational wave spectrum

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We investigate the impact of stochastic quantum noise due to trans-Planckian effects on the primordial power spectrum for gravity waves during inflation. Given an energy scale Λ, expected to be close to the Planck scale m_{Pl} and larger than the Hubble scale H, this noise is described in terms of a source term in the evolution equation for comoving modes k which changes its amplitude growth from early times as long as the mode physical wavelength is smaller than Λ^{-1}. We model the source term as due to a gas of black holes in the trans-Planckian regime and the corresponding Hawking radiation. In fact, for energy scales larger than, or of the order of Λ, it is expected that trapped surfaces may form due to large energy densities. At later times the evolution then follows the standard sourceless evolution. We find that this mechanism still leads to a scale-invariant power spectrum of tensor perturbations, and in agreement with CMB data for $\Lambda/m_{Pl} \sim 10^{-2}$.

I. INTRODUCTION

Since its birth [1–3], cosmic inflation represents the most promising mechanism able to solve several puzzles of the very early universe cosmology such as the horizon problem for the CMB patches in the sky, flatness, monopole problems, etc.. It was then realized that it naturally provides a natural description for the formation of the primordial density and tensor perturbations. While the features of scalar perturbations are measured very precisely in CMB and large-scale structures and are in excellent agreement with the inflationary model, maybe the present smoking gun of the inflationary paradigm is the production of primordial gravitational waves. It is a peculiar outcome of inflation and, most importantly, it provides information about a very early stage of the evolution of the Universe. This is the reason why this stochastic background is getting more and more attention with many hopes toward the future space-interferometers (see for example [4–11]).

The issue we address in this paper is the choice of the vacuum state from which perturbations evolve. The standard choice is the "Euclidean vacuum" or Bunch-Davies (BD) vacuum. This vacuum has been adopted in the literature for many reasons. It is the state which minimizes the Hamiltonian of the theory, consistent with the uncertainty principle and with the instantaneous Minkowskian vacuum. It is also conjectured that the BD vacuum is an attractor solution for inflation [12,13]. Finally, this choice leads to a nearly scale-invariant power spectrum, as strongly preferred by observations.

Nevertheless, one can object that using an asymptotic Minkowskian configuration of the Universe has no fundamental underlying reasons and is a too strong condition. Indeed, even if the mode initial state is chosen as the lowest energy one (i.e. the particle state), for fields evolving in curved spacetimes a pure particle state can evolve into a mixed state since the Hamiltonian is time dependent. Moreover, taking the state at an initial conformal time $\tau \to -\infty$, is equivalent to considering modes k of zero wavelength and infinite energy. This means that mode evolution goes through a trans-Planckian regime [14–16], about which we have no robust theoretical clues. During that transition, departures from the standard treatment may arise and new phenomena can emerge, e.g., from generalizations of the uncertainty principle (GUP) [17–22]. A possible way out consists of moving the ultraviolet scale down to a more comfortable cut-off energy, somewhere in the range between the Planck scale m_{Pl} and the inflationary energy scale H, where classical general relativity can be safely used ([23–28]).

Whether we assume a BD vacuum or some initial time for perturbation evolution, a few questions remain unsolved. What are the behavior and dynamics of fluctuations in the trans-Planckian regime? What are the imprints of this early stage on their eventual amplitude at horizon crossing? Is inflation washing out their effects?

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In this paper, we suggest that the evolution of tensor (and scalar) perturbation in the high energy regime above some mass-energy scale Λ, can be effectively described by adding a *stochastic* source term with zero mean to the evolution equations, which is due to the interaction of modes with the underlying background of fluctuations due to quantum gravity nonlinear effects. These fluctuations are excited by trans-Planckian modes $k > a\Lambda$ which produce large perturbations of the metric, of order one, so their feedback on a single mode evolution cannot be neglected. The scale Λ, expected to be of the order of the Planck mass m_{Pl} or below, but larger than the Hubble scale H during inflation, is a free parameter defined by the condition that quantum gravity features are not negligible above this scale.

Since a full theory of quantum gravity is missing we have to describe a mode k evolution in the trans-Planckian regime in a phenomenological way. In particular, we consider a scenario that we call the "black hole (BH) gas", based on the idea that at trans-Planckian energy densities trapped surfaces can form producing an environment of BH's. Their feedback on a given trans-Planckian mode is described in terms of the corresponding emitted Hawking radiation, which couples to the mode evolution and acts as a source in the evolution equation. Once, due to the expansion of the background metric, the physical mode k/a falls below the scale Λ, the local induced perturbation of the metric becomes small and its evolution follows the standard behavior in the linear approximation and dictated by classical general relativity. In the following we will refer to the early trans-Planckian stage as that corresponding to mode energies larger than the scale Λ above which quantum gravity effects are not negligible.

The main result we obtain is that the resulting power spectrum is still scale invariant and, depending on the value of Λ, is smaller or larger than the standard result from tiny amounts up to large enhancement as Λ varies in the range $[H, m_{\text{Pl}}]$. This implies that the value of Λ can be constrained by CMB, mainly from Planck results [10], and is found to be of the order of $10^{-2} m_{\text{Pl}}$. We will discuss this point in the following.

On general grounds, we think that this approach can be also extended to scalar perturbations, with similar results. Scalar perturbations however are more sensitive to the details of the considered inflationary model, while tensor modes only depend upon initial conditions, possible quantum effects, as suggested here, and the value of the (almost) constant Hubble scale during inflation. In the following, we will concentrate on tensor modes and come back to the scalar part in the last section, which contains our conclusions and outlooks. We use natural units $c = \hbar = k_B = 1$.

II. EVOLVING TENSOR PERTURBATION FROM THE TRANS-PLANCKIAN REGIME WITH A STOCHASTIC SOURCE

We first shortly review the main aspects of the standard formalism describing cosmological tensor perturbation evolution during inflation ([2,29–38]). We start with the second-order Action

$$S = -\frac{1}{16\pi G} \sum_r \int d\tau d\mathbf{k} \, \frac{a(\tau)^2}{2} [\hat{h}'^r_{\mathbf{k}}(\tau)\hat{h}'^r_{-\mathbf{k}}(\tau)$$
$$- k^2 \hat{h}^r_{\mathbf{k}}(\tau)\hat{h}^r_{-\mathbf{k}}(\tau) + 32\pi G a(\tau)^2 \hat{\Pi}^r_{\mathbf{k}}(\tau)\hat{h}^r_{-\mathbf{k}}(\tau)], \quad (1)$$

with $\tau \in]-\infty, 0]$ the conformal time, $a(\tau)$ the scale factor and $\hat{h}^r_{\mathbf{k}}(\tau)$ the tensor perturbations quantum fields, which can be expanded in fundamental solutions as

$$\hat{h}^r_{\mathbf{k}}(\tau) = h_k(\tau)\hat{a}^r_{\mathbf{k}} + h^*_k(\tau)\hat{a}^{r\dagger}_{-\mathbf{k}}, \quad (2)$$

where $\hat{a}^r_{\mathbf{k}}$ and $\hat{a}^{r\dagger}_{-\mathbf{k}}$ satisfies canonical commutation relations. We recall that, while the quantum operator has to have the physical dimension of the inverse of the square of a mass ($[\hat{h}_{\mathbf{k}}] \approx [M]^{-2}$), the mode function h_k behaves like $[M]^{-1/2}$ since $[\hat{a}_{\mathbf{k}}] \approx [M]^{-3/2}$.

The source term $\hat{\Pi}^r_{\mathbf{k}}(\tau)$ vanishes at linear order in the standard case and thus the $h_k(\tau)$ evolution is dictated by a homogeneous differential equation. The main issue of this paper is to study the consequences of a stochastic source in a model which will be detailed in the following. Taking for the moment $\hat{\Pi}^r_{\mathbf{k}}(\tau) = 0$, variation of the action with respect to h_k leads to

$$h''_k + 2\mathcal{H}h'_k + k^2 h_k = 0, \quad (3)$$

and defining rescaled fields as

$$v_k(\tau) = a(\tau)h_k(\tau), \quad (4)$$

we obtain the well-known Mukhanov-Sasaki equation [39]

$$v''_k + \left(k^2 - \frac{a''}{a}\right)v_k = 0. \quad (5)$$

This equation is solved by a linear combination of the Hankel functions weighted by the Bogoliubov coefficients A_k and B_k

$$h_k(\tau) = \frac{A_k}{a(\tau)}\frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right) + \frac{B_k}{a(\tau)}\frac{e^{ik\tau}}{\sqrt{2k}}\left(1 + \frac{i}{k\tau}\right). \quad (6)$$

The coefficients A_k and B_k encode the natural ambiguity of the vacuum state in curved space-time due to the lack of timelike Killing vectors, and in order to characterize the solution one has to impose the initial condition for the mode functions. The standard choice is the "Euclidean vacuum" or Bunch-Davies vacuum with $A_k = 1$ and $B_k = 0$. which leads to a standard scale-invariant power spectrum, in agreement with observations

$$P'_{\rm BD} = (64\pi G)\frac{k^3}{2\pi^2}\langle 0|\hat{h}^\dagger_{\bf k}\hat{h}_{\bf k}|0\rangle$$

$$= (64\pi G)\frac{k^3}{2\pi^2}|h_k|^2_{\rm BD} = \frac{16H^2}{\pi m^2_{\rm Pl}}. \qquad (7)$$

Although the inflationary stage is not only a de Sitter space-time, since we only need a limited period with an exponential expansion, one can question whether using an asymptotic Minkowskian configuration of the Universe has fundamental underlying reasons. We stress again that taking the initial state at an initial time $\tau \to -\infty$, means that modes start at zero wavelength and with infinite energy. A possible way out has been considered in [23] where the author makes a different prescription introducing an initial time when the quantum fluctuations started evolving. This corresponds to recast creation and annihilation operators in terms of their value at a given time $\bar{\tau}_k$ [40], such that the vacuum definition is

$$\hat{a}_k(\bar{\tau}_k)|0,\bar{\tau}_k\rangle = 0, \qquad (8)$$

which in turn gives a relation between the Bogoliubov coefficients, A_k, and B_k. This is an example of what has been called α vacua ([40–46]).

Imposing the vacuum at a given $\bar{\tau}_k$, introduces a physical cutoff scale, $m_{\rm Pl} \geq \Lambda > H$, such that the mode evolution begins only once $k = a\Lambda$, i.e. in quasi–de Sitter

$$\bar{\tau}_k \simeq -\frac{\Lambda}{kH}. \qquad (9)$$

In this case one obtains the following expression for the dimensionless power spectrum [40],

$$P^t = \frac{16H^2}{\pi m^2_{\rm Pl}}\left[1 + \frac{H}{\Lambda}\sin\left(\frac{2\Lambda}{H}\right) + \cdots\right], \qquad (10)$$

thus an oscillating feature that is subdominant with respect to the standard value as long as $\Lambda \gg H$.

We now describe our model. Since we have not yet a full theory of gravity in its quantum regime, though there are many ideas and suggestions about it ([47–49]), we can only list a few considerations which will bring us to our proposal:

(i) let us consider a wave packet with mean comoving wave number k which is excited from the vacuum as a quantum massless fluctuation, distributed over a volume corresponding to its de Broglie physical length a/k. Using Einstein's equation $G_{ab} = 8\pi G T_{ab}$ and writing the metric as the background (homogeneous) Friedmann Robertson Lemaitre Walker term and the mode induced local perturbation δg we have at linear order

$$\delta G_{ab} \sim 8\pi G \delta T_{ab}. \qquad (11)$$

Computing the stress-energy tensor of the mode δT_{ab} it's easy to find for the perturbed metric δg

$$\delta g \sim 8\pi m^{-2}_{\rm Pl}\frac{k^2}{a(\tau)^2}, \qquad (12)$$

and for high $k/a(\tau) \sim \Lambda$ the value of δg can be quite large. This means that linear approximation would not be an appropriate one when modes experience the high energy scale Λ regime since nonlinear effects are crucial and the feedback of the perturbed metric cannot be neglected;

(ii) at energies larger than Λ fluctuations may give rise to metric perturbations δg corresponding to trapped surfaces [50], which is to say they can produce a black hole environment, which once produced evaporate in very short times. If we use the standard result for the evaporation time t_{ev} for a BH of mass M

$$t_{ev} = 5120\pi G^2 M^3 \qquad (13)$$

we find $t_{ev} \sim 10^4 t_{\rm Pl}$, with $t_{\rm Pl} = m^{-1}_{\rm Pl}$ the Planck time. As gravitational interactions are nonlinear, tensor fluctuations with a high k will interact with this environment and their feedback can be effectively described in terms of a nonhomogeneous differential equation with a source term. Again, once $k/a(\tau) < \Lambda$, the perturbed metric δg becomes negligible and we can treat the k mode evolution in the standard linear approximation neglecting the feedback of δg on the mode evolution;

(iii) on the other hand, once the perturbation physical wave number becomes sub-Planckian, and thus the induced local perturbation of the metric becomes small with respect to the homogeneous background metric, we can trust the standard linear evolution, i.e., the amplification till horizon crossing and the end of inflation. Yet, this evolution will keep the memory of the initial condition at the matching point given by $k/a = \Lambda$.

These points led us to consider the following model. We introduce a nonvanishing anisotropic stress tensor that will encode information about the chaotic environment from which each mode has to go through when it starts evolving. We generalize Eq. (3), by adding a source term acting on modes inside the horizon satisfying the condition $k/a > \Lambda$, or $\tau < \bar{\tau}_k$. This translates into a two-stage evolution

$$h''_k + 2\mathcal{H}h'_k + k^2 h_k = 16\pi G a^2\Pi_k \qquad \tau < \bar{\tau}_k \quad (14)$$

$$h''_k + 2\mathcal{H}h'_k + k^2 h_k = 0 \qquad \tau > \bar{\tau}_k, \qquad (15)$$

with the following matching conditions

$$\lim_{\tau \to \overline{\tau}_k^-} h_k(\tau) = \lim_{\tau \to \overline{\tau}_k^+} h_k(\tau) \tag{16}$$

$$\lim_{\tau \to \overline{\tau}_k^-} h_k'(\tau) = \lim_{\tau \to \overline{\tau}_k^+} h_k'(\tau). \tag{17}$$

At $\tau \to -\infty$ we assume the mode to be excited from the vacuum with the minimal possible energy, as in the Bunch-Davis case. Of course, this condition may be chosen differently, corresponding to some α vacuum. Yet, we are interested here in how the trans-Planckian regime changes mode evolution with respect to the standard case, so in order to make this comparison it is appropriate to use the standard choice $A_k = 1$, $B_k = 0$ for $\tau \to -\infty$. The role of a different choice for the initial condition has not been pursued here and may deserve further studies.

Notice that, differently than in the approach [40], $\overline{\tau}_k$ is now the time *where the source term switches off* and the evolution of the (now sub-Planckian) mode is the usual one. Notice too that *for all* comoving wave number k, modes in their initial evolution feel the effect of the source Π_k, as long as $\tau < \overline{\tau}_k$.

The shear source has to be quantized as the quantum metric fluctuations

$$\hat{\Pi}_{\mathbf{k}}^r(\tau) = \Pi_k(\tau)\hat{a}_{\mathbf{k}}^r + \Pi_k^*(\tau)\hat{a}_{-\mathbf{k}}^{r\dagger}, \tag{18}$$

and we suppose that Π_k is a stochastic incoherent source that satisfies the following relations ($\langle\ldots\rangle$ denotes average over the probability distribution)

$$\langle \Pi_k(\tau) \rangle = 0$$

$$\langle \Pi_k(\tau)\Pi_k^*(\tau') \rangle = \Lambda^6 \delta(\tau - \tau') \left| F\left(\frac{k}{a\Lambda}, \frac{\Lambda}{m_{\mathrm{Pl}}}\right) \right|^2. \tag{19}$$

The prefactor accounts for the dimensionality of Π_k and has been chosen as the correct power of the typical scale of the trans-Planckian phase, while F is an adimensional function accounting for the dependence of the source $\Pi_k(\tau)$ on k and on the relative value of Λ with respect to the Planck scale. The other dimensional relevant parameter, the Hubble scale during inflation, is fixed to $H/m_{\mathrm{Pl}} = 10^{-6}$.

Equations (14) and (15) have different solutions, which have to be matched at the transition time, $\overline{\tau}_k$. The homogeneous solution is given by Eq. (6), while in presence of the source $[\tau \leq \overline{\tau}(k)]$ we have

$$h_k(\tau) = \frac{16\pi G}{a(\tau)} \int_{-\infty}^{\tau} d\tau' a(\tau') G_k(\tau, \tau') \Pi_k(\tau'), \tag{20}$$

in terms of the Green's function, $G_k(\tau, \tau')$ (see Ref. [51]),

$$G_k(\tau, \tau') = \frac{e^{-ik(\tau+\tau')}}{2k^3\tau'^2} [e^{2ik\tau}(1 - ik\tau)(-i + k\tau')$$
$$+ e^{2ik\tau'}(1 + ik\tau)(i + k\tau')]\Theta(\tau - \tau'). \tag{21}$$

Using the matching conditions (16), (17), and making the average as in the Langevin approach to Brownian motion, we find the following expression for the Bogoliubov coefficients

$$\frac{A_k}{a(\overline{\tau}_k)} = e^{ik\overline{\tau}_k}[h(\overline{\tau}_k)(-1 + ik\overline{\tau}_k + k^2\overline{\tau}_k^2)$$
$$- h_k'(\overline{\tau}_k)(\overline{\tau}_k - ik\overline{\tau}_k^2)](\sqrt{2}k^{3/2}\overline{\tau}_k^2)^{-1}$$

$$\frac{B_k}{a(\overline{\tau}_k)} = e^{-ik\overline{\tau}_k}[h(\overline{\tau}_k)(-1 - ik\overline{\tau}_k + k^2\overline{\tau}_k^2)$$
$$- h_k'(\overline{\tau}_k)(\overline{\tau}_k + ik\overline{\tau}_k^2)](\sqrt{2}k^{3/2}\overline{\tau}_k^2)^{-1}. \tag{22}$$

Recalling that $h(\tau) \sim k^{-1/2}$ we see that they depend on the adimensional quantity $k\overline{\tau}_k = -\Lambda/H$, so that the power spectrum P^t is still predicted to be *scale invariant* (up to slow roll corrections). At the horizon crossing, $k \sim aH$

$$P^t = P_{\mathrm{BD}}^t \left[1 + |B_k|^2 \left(2 + \frac{2k\overline{\tau}_k + i}{i} e^{2ik\overline{\tau}_k} \right.\right.$$
$$\left.\left. - \frac{2k\overline{\tau}_k - i}{i} e^{-2ik\overline{\tau}_k} \right) \right], \tag{23}$$

where B_k can be easily found in terms of integral functions of the source Π_k and P_{BD}^t represents the standard contribution from the BD vacuum. Their expressions are though, quite involved and lengthy, and we do not report them explicitly for the sake of brevity.

A particular model for the shear source is specified by a choice of the adimensional function $F(k/(a\Lambda), \Lambda/m_{\mathrm{Pl}})$. We consider here a scenario that we name the "BH gas" model. As we mentioned, during the trans-Planckian phase, modes with $k > \Lambda$ experience quantum gravity effects which we describe in terms of interactions with the BH gas which are formed when trapped surfaces create. This background produces particles, in particular gravitational waves, through Hawking radiation, which act as a source for $h(\tau)$. Given a probability distribution of the BH's as a function of their mass M, $\xi(M)$, and approximating the Hawking emission spectrum with a Boltzmann shape we have

$$F\left(\frac{k}{a\Lambda}, \frac{\Lambda}{m_{\mathrm{Pl}}}\right) = \int_0^\infty \xi(M) \exp\left(-\frac{k}{a} \frac{8\pi M}{m_{\mathrm{Pl}}^2}\right) dM. \tag{24}$$

The distribution $\xi(M)$ depends upon Λ which represents the typical scale above which quantum effects are not negligible, and thus, a natural cutoff for black hole mass distribution. We use a simple exponential function, as we do not expect any particular feature in $\xi(M)$
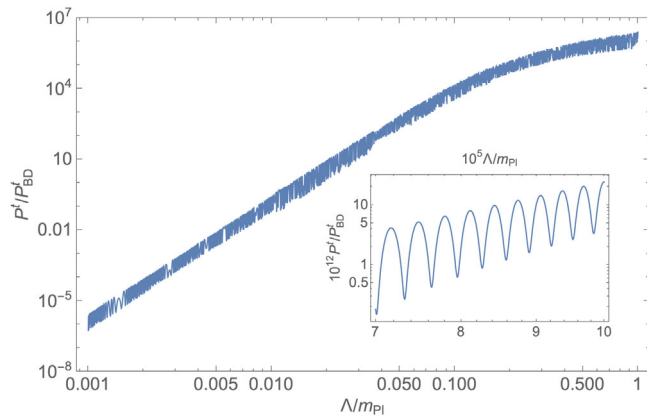
$$\xi(M)dM = \frac{1}{\Lambda}e^{-M/\Lambda}dM, \tag{25}$$

FIG. 1. The scale-invariant tensor power spectrum normalized to the standard result P_{BD}^t versus the ratio Λ/m_{Pl}. The zoom-in plot shows the oscillatory behavior in a narrow range for Λ/m_{Pl}. Only values of $\Lambda \sim 2 \times 10^{-2} m_{Pl}$ are in fair agreement with CMB data. See text.

and we finally obtain

$$F\left(\frac{k}{a\Lambda}, \frac{\Lambda}{m_{Pl}}\right) = \left(1 + \frac{k}{a\Lambda}\frac{8\pi\Lambda^2}{m_{Pl}^2}\right)^{-1}. \quad (26)$$

For $\Lambda \sim m_{Pl}$ the k-dependent term dominates since $k/(a\Lambda) > 1$, but it becomes less important as Λ decreases since its contribution is significant in a narrower τ interval.

We have numerically solved Eqs. (14), (15) using (26) and obtained the Bogoliubov coefficients A_k and B_k. The tensor power spectrum ratio P^t/P_{BD}^t for a given k (we recall that it is scale-invariant up to slow roll corrections as in the standard case) is shown in Fig. 1 as a function of the ratio Λ/m_{Pl}. Two features should be noticed. First of all the very rapid oscillatory behavior is due to the fact that P^t depends upon oscillating trigonometric functions of the variable Λ/H which in the selected range on the x-axis is very large (greater than 10^3) since we have chosen $H/m_{Pl} \simeq 10^{-6}$. In the figure, a zoom of the power spectrum ratio is reported in a narrow range and for very small values of Λ to appreciate more clearly this oscillatory behavior. For higher and more natural values of Λ, the oscillations are so rapid that it is hard to clearly distinguish them. We also see from the plot that the value of P^t/P_{BD}^t spans several orders of magnitude, from very small values up to a factor 10^6 if $\Lambda = m_{Pl}$. This is due to two combining effects, the interference term in Eq. (23) which produces a rapid oscillating term in the evolution of perturbation, and the overall source scale Λ^3 which weights its amplitude.

From the results shown in Fig. 1 we can obtain the value of Λ compatible with present experimental data. We first notice that the predicted power spectrum is of the order of P_{BD}^t when $H \sim 10^{13}$ GeV (the benchmark value we have used) for $\Lambda \sim 2 \times 10^{-2} m_{Pl}$, quite reasonable in the

framework of the theoretical model we have discussed, being close to the Planck scale.

Indeed, we have information on the tensor perturbation amplitude from CMB data which bound the tensor to scalar ratio $r_{0.05} < 0.12$ [10]. The question is thus, how our model affects scalar modes. Our point is that scalar perturbations too would experience the same behavior in the trans-Planckian regime we have described so far, up to a numerical factor related to helicity degrees of freedom by simply exploiting the equivalence principle. We thus, expect at a first glance that a similar behavior as in Fig. 1 would hold for scalar modes and that the tensor to scalar ratio would be *independent* of Λ. Scalar fluctuations depend upon the Hubble parameter ($H \sim 10^{13}$ GeV) and the features of the inflaton potential, namely the slow roll parameter ϵ, which is constrained in the relevant e-fold region since it is related to the scalar spectral index $n_s = 0.9649 \pm 0.0042$ [10]. Since the scalar perturbation amplitude is fixed by data as well as n_s, i.e., the values of H and ϵ, experimental results give $\Lambda \sim 2 \times 10^{-2} m_{Pl}$ so that the power spectrum agrees with the standard BD value. Yet, their dynamics described in this paper do not require an asymptotic Minkowsky asymptotic condition and accounts for trans-Planckian effects.

III. CONCLUSIONS AND OUTLOOKS

The picture of how perturbations are produced during inflation is at the same time simple and successful. It is simple because it relies on the very distinctive feature of inflation, i.e. a quasi de Sitter stage during which the event horizon is almost constant. It is successful because this feature naturally provides a scale invariant spectrum, in agreement with observations. What remains, perhaps, to be understood is the effects of the early stages of perturbation evolution, when they undergo a trans-Planckian regime where quantum gravity features may not be negligible and the linear approximation may fail due to the feedback of the induced local metric variation by a mode with a large k. This problem can be translated into the question of what is the initial condition for perturbation modes when they become "classical" from the point of view of gravitational interactions, i.e., when the wave number k is smaller than $a\Lambda$.

In this paper, we have considered the possibility that this initial condition, differently from the BD assumption of an asymptotic Minkowskian vacuum, is the outcome of an earlier stage where tensor perturbations are sourced by a shear term due to interactions with a stochastic background of excitations due to quantum gravity effects. In particular, we modeled these excitations as produced by Hawking radiation from a "BH gas," since at high trans-Planckian energies trapped surfaces and BH excitations may form.

Our result is that this model still predicts a scale invariant spectrum for tensor perturbation with an amplitude which

grows with the scale Λ from tiny values, for the unrealistic case of $\Lambda \sim H$ and due to interference effects, to a large enhancement for $\Lambda \sim m_{\mathrm{Pl}}$. From CMB constraints on the scalar perturbations, the scalar spectral index and the tensor-to-scalar ratio we find that $\Lambda/m_{\mathrm{Pl}} \sim 2 \times 10^{-2}$.

To conclude, we think it would be interesting to perform a likelihood analysis in our model of the values of H and Λ, using present and future data, since they both conspire to fix the scalar perturbations amplitude and are degenerate parameters, as well as the effects on non-Gaussian features

of the primordial spectrum in the scenario we have discussed [52,53].

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