Veltman criteria in the beyond standard model effective field theory of a complex scalar triplet

Jaydeb Das[1,*] and Nilanjana Kumar[2,†]

[1]*Department of Physics and Astrophysics, University of Delhi, Delhi-110007, India*
[2]*Centre For Cosmology and Science Popularization (CCSP), SGT University,*
Gurugram, Haryana-122006, India

The Higgs mass is not protected by any symmetry in the Standard Model. Hence, the self-energy corrections to the Higgs mass become large due to the quadratic divergence terms. The Veltman condition (V.C.) ensures that the coefficient of the quadratic divergent term either vanishes or becomes negligible. The nonobservation of new physics has pushed the new physics scale to be larger than 1 TeV, making it impossible to satisfy the Veltman condition in the Standard Model without a very large fine-tuning. Many attempts are made to satisfy the V.C. in beyond the Standard Model theories, but the V.C. is hard to achieve at a very large scale (Λ). Alternatively, it is possible that the new physics appears much above the electroweak scale, and the effect of the new physics is observed in terms of the Wilson coefficients of the Standard Model effective field theory (SMEFT) operators. The V.C. can be addressed in the SMEFT framework. In this paper, some specific new physics scenarios are considered at a very large scale. Below that scale, the effect of the new physics is observed as beyond the SMEFT. We particularly study the type-II seesaw model with the complex scalar triplet ($Y = 1$) in the context of V.C. We found that this particular model is the minimal model to generate all SMEFT operators that appear in V.C. and satisfies V.C. We also examine the model parameter dependence of the Wilson coefficients in detail and show how the cancellation of the Wilson coefficients is highly dependent on some specific values of the model parameters.

I. INTRODUCTION

The smallness of the observed Higgs mass is confirmed by the experiments [1,2] at the Large Hadron Collider (LHC). However, in the Standard Model (SM) of particle physics, the scalar mass (mass of Higgs boson) is not protected by any symmetry. Hence, if the SM is valid up to a large scale, such as the Planck scale, then the Higgs mass suffers from quadratic divergence ($\sim \Lambda^2$). To ensure that the mass of the Higgs boson is small, one has to consider a very large fine-tuning in the SM. A way to ensure that the Higgs mass does not get large correction at a higher scale is coined as the "Veltman condition" (V.C.) [3]. The V.C. checks if the sum of all quadratically divergent terms coming from the self-energy diagrams of the Higgs boson is either zero or very small. However, experiments such as

the LHC is pushing the new physics (NP) scale towards >1 TeV, thus the Veltman condition is not possible to satisfy in the SM, as it demands Λ to be less than 760 GeV [4].

Simple extensions of SM have been studied in the literature [4–12], where the V.C. is valid but only in some regions of parameter space. Overall, there are two main concerns in these models: (1) These theories encounter different problems at a large scale, such as the potential becoming unstable, leading to the invalidity of the theory beyond that scale. (2) The nonobservation of the beyond the Standard Model (BSM) particles has pushed their masses above TeV scale [13].

One may assume that the SM is valid up to a certain scale (Λ) and above that scale, some unknown symmetry appears to protect the Higgs mass, then the Higgs mass can be stabilized and the fine-tuning problem can be addressed. For example, in the composite Higgs scenario [14], where the Higgs is dissolved in higher degrees of freedom above the symmetry breaking scale or in supersymmetric theories [15], where the bosonic and fermionic degrees of freedom cancel out exactly—the Higgs mass is maintained to be finite and small. These theories also cannot avoid a certain amount of fine-tuning [16,17] coming from several

*jaydebphysics@gmail.com
†nilanjana.kumar@gmail.com

sources. However, experiments are yet to confirm the existence of these theories.

These observations raise the question that what if the new physics lies at a very large scale. In such a scenario, SM can emerge as an effective field theory (SMEFT) [18] by integrating out the dynamics of the larger theory. The information of the heavy particles appearing in the loop is absorbed in the higher dimensional operators in the effective field theory (EFT) and the theory is invariant under SM symmetries. Reference [19] has shown that the V.C. can be satisfied in the SMEFT framework by including the higher dimensional operators and their Wilson coefficients. Only a few of the operators are relevant to the V.C. and they play a major role in satisfying the V.C.

In this paper, we take one step forward and ask these questions: What if the theory at a very high scale (Λ) includes specific type of BSM scenarios? How do these affect the V.C.? We adopt the beyond Standard Model effective field theory (BSM-EFT) [20] approach, which has been studied previously in Refs. [20–27]. In BSM-EFT the Lagrangian becomes invariant under the particular BSM model in consideration. The motivation to study the V.C. in the BSM-EFT framework is twofold. (1) We can specifically check how many SMEFT operators are allowed by the model. (2) As the Wilson coefficients can be expressed in terms of the model parameters, the sign of the W.C., which is crucial to obtain the V.C., comes naturally. For calculation of the SMEFT operators we choose WARSAW basis [28,29].

We begin with simple BSM scenarios (in BSM-SMEFT) such as scalar singlets, doublets, and triplets (real or complex) and found that the V.C. can be satisfied in all these models. We also found that the complex scalar triplet model with $Y = 1$ is the minimal model (with only of one type of BSM particle) where it is possible to generate all the SMEFT operators that contributes to the V.C. In Sec. IV we discuss more on this.[1] Moreover, this particular model is well motivated in literature from other aspects as well: (1) Neutrino mass generation through the seesaw mechanism [30], (2) type-II Leptogenesis scenario [31], and (3) enhancement of the $h \to \gamma\gamma$ branching ratio [32] etc., among many others [33–35].

In Sec. II, we show how the V.C. depends on the SMEFT operators in the WARSAW basis [28]. In Sec. III, we discuss some specific models in the BSM-EFT scenarios, and express the Wilson coefficients of $Y = 1$ complex scalar triplet model in terms of the model parameters. In Sec. IV, we show how the V.C. is achieved by the exact cancellation of the Wilson coefficients at different scale.

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[1]Types I and II seesaw models do not generate all SMEFT operators to satisfy the V.C. Moreover a recent study [24] has also shown that these models are also not favored from the fact that the radiative electroweak symmetry breaking cannot be triggered even at the Planck scale.

We also interpret the result in terms of the model parameter space. Then in Sec. V, we conclude.

II. SMEFT OPERATORS AND VELTMAN CONDITION

The physical mass of the Higgs in the Standard Model can be written in terms of the bare mass term $m_h(0)$ and the higher-order self-energy corrections:

$$
\begin{aligned}
m_h^2 &= m_h^2(0) + \delta m_h^2 \\
&= m_h^2(0) + \text{Log Div Term} \\
&\quad + \text{Quadratic Div Term} + \text{Finite terms}, \quad (1)
\end{aligned}
$$

where the assumption is that the SM is valid up to the scale Λ and the correction terms are coming from the loop diagrams involving scalars, fermions, and bosons in the loop. The $d = 4$ potential in the and the Standard Model in terms of Higgs doublet (H) is

$$
V(H) = -m_H^2 H^\dagger H + \lambda (H^\dagger H)^2. \quad (2)
$$

This leads to the correction to the Higgs mass and the quadratic divergent contribution can be expressed as

$$
(\delta m_h^2)_{\text{SM}} = \frac{\Lambda^2}{16\pi^2} \left(6\lambda + \frac{9}{4} g_W^2 + \frac{3}{4} g_Y^2 - 6 y_t^2 \right), \quad (3)
$$

where, g_Y and g_W are the $U(1)_Y$ and $SU(2)_L$ gauge couplings, respectively, and $y_t = \sqrt{2} m_t / v_H$ is the top quark Yukawa coupling with $\langle H \rangle = v_H / \sqrt{2}$. Here we neglect the couplings of the lighter quarks and Λ is the cutoff scale. The V.C. demands that $\delta m_h^2 \sim 0$ or at least controllably small. With the observed Higgs mass at 125 GeV, the condition to make $\delta m_h \sim 0$ demands $\Lambda < 760$ GeV, which is already ruled out by LHC. One way to solve this problem is to introduce new particles, which can contribute in the loops and soften the fine-tuning by ensuring exact cancellation or partial as we have already discussed in the introduction.

A popular way to address this problem is to consider the effects of the higher dimensional operators in the EFT framework. Let us assume that the NP exists at a very high scale Λ. The effect of NP can be integrated out at Λ and this will effectively give us SM, plus some effective operators involving only the SM fields. This is known as the SMEFT [18]. The Lagrangian, which incorporates dimension six SMEFT operators in addition to the Standard Model dimension four operators, can be expressed as

$$
\mathcal{L} = \sum_i \mathcal{C}_{4i} \mathcal{Q}_{4i} + \frac{1}{\Lambda^2} \sum_i \mathcal{C}_{6i} \mathcal{Q}_{6i}. \quad (4)
$$

In contrast to $\mathcal{C}_{4i}$, which is the only function of the parameters linked to the degrees of freedom in the Standard Model, $\mathcal{C}_{6i}$ are the Wilson coefficients, which are functions of the integrated out dynamics at Λ. These operators can be expanded at any choice of basis, for example, HISZ basis [36,37], Warsaw basis [28,29], SILH basis [38] etc. The set of dimension six operators that involves Higgs in the Warsaw basis are

$$\mathcal{Q}_H = (H^\dagger H)^3, \qquad \mathcal{Q}_{HD} = (H^\dagger \mathcal{D}_\mu H)^*(H^\dagger \mathcal{D}^\mu H), \qquad \mathcal{Q}_{H\square} = (H^\dagger H)\square(H^\dagger H)$$
$$\mathcal{Q}_{HB} = (H^\dagger H)B_{\mu\nu}B^{\mu\nu}, \qquad \mathcal{Q}_{HW} = (H^\dagger H)W^a_{\mu\nu}W^{a,\mu\nu}, \qquad \mathcal{Q}_{GG} = (H^\dagger H)G^A_{\mu\nu}G^{A,\mu\nu}$$
$$\mathcal{Q}_{HWB} = (H^\dagger \tau_a H)B^{\mu\nu}W^a_{\mu\nu}. \tag{5}$$

It can be shown that the last operator, $\mathcal{Q}_{HWB}$ does not contribute Higgs self-energy correction [19]. The first operator, $\mathcal{Q}_H$ will also not contribute at the one-loop level as the Higgs does not develop a vacuum expectation value at Λ. There can be the appearance of the operators involving the gluons of the form $\mathcal{Q}_{GG} = (H^\dagger H)G^A_{\mu\nu}G^{A,\mu\nu}$. However, while considering BSM-EFT framework with heavy scalars, this operator does not contribute as scalars do not carry any color charge. Note that these operators can be written in any basis, for example Ref. [19] chose the HISZ basis. We choose the Warsaw basis because it is self-consistent at one loop [29,39] and easier to check the running of the Wilson coefficients in the Warsaw basis.

The correction to the Higgs mass from the higher order terms in the Lagrangian is given by

$$(\delta m_h^2)_{\text{total}} = \frac{\Lambda^2}{16\pi^2}\sum_i f_i(\mathcal{C}_{4i}, \mathcal{C}_{6i}) + \frac{\Lambda^2}{(16\pi^2)^2}\sum_i g_i(\mathcal{C}_{4i}, \mathcal{C}_{6i}). \tag{6}$$

Here f_i and g_i are one-loop and two-loop corrections to the Higgs mass. The V.C., $\delta m_h^2 \sim 0$ translates into

$$f(\mathcal{C}_{4i}, \mathcal{C}_{6i}), \qquad g(\mathcal{C}_{4i}, \mathcal{C}_{6i}) \sim 0 \tag{7}$$

if loop contributions are considered separately. The coefficients, $\mathcal{C}_{4i}$ and $\mathcal{C}_{6i}$ are functions of Λ and the BSM model parameters. Hence Eq. (6) can be written in terms of the SM and higher dimension operators contribution as

$$(\delta m_h^2)_{\text{total}} \equiv (\delta m_h^2)_{\text{SM}}(f_i(\mathcal{C}_{4i}), g_i(\mathcal{C}_{4i}))$$
$$+ (\delta m_h^2)_{\text{HO}}(f_i(\mathcal{C}_{6i}), g_i(\mathcal{C}_{6i})). \tag{8}$$

Also, it has been shown in Ref. [19] that, at $d \geq 8$, the SMEFT operators are not able to produce any Λ^6 divergence, which will produce any effective Λ^2 divergence while calculating the self-energy correction of Higgs mass. There are studies in the literature, where the V.C. in terms of the EFT has been studied in detail [19,40,41]. In particular it has been shown in Ref. [19] that it is possible to satisfy the V.C. for appropriate values and sign of the Wilson coefficients at large Λ.

III. BSM-EFT WITH COMPLEX SCALAR TRIPLET

In the above section, we saw that only four operators in the WARSAW basis are involved in the V.C. Now, we assume that the new physics at a large scale follow certain symmetries of a BSM model which effectively produces the SM as an EFT. It has been already shown in literature how some BSM extensions [4–12] address the V.C. In this paper we consider them to appear at a large scale and dictate the underlying symmetry of the EFT.

In this BSM-EFT framework, these four operators may or may not be possible to generate at one loop, depending on the underlying symmetry of the model at scale Λ. In Table I, we present if these four operators can be generated at one loop in some simple BSM-EFT cases with additional scalar(s) or not.[2] For the calculation, we have implemented the Lagrangian of each model in CoDEx [42,43] and generated the Wilson coefficients.[3]

Among all popular models, we have found that BSM-EFT with a complex scalar triplet is the minimal model where all four Wilson coefficients are generated at one loop. In other models, the number of operators is less than four except for the model with a complex scalar triplet and the additional doublet. In two Higgs doublet model (2HDM) scenario and real scalar singlet + triplet model, only three operators can be generated, whereas, in the complex scalar singlet model, only two operators are generated. The real scalar singlet model generates only one operator. All this models, with four or less number of operators, satisfy the V.C. But, it is intuitive to see that the parameter space of the W.C.s is more constrained in a model that generates less than four of the EFT operators. Hence, we chose to study the complex scalar triplet model in detail as the minimal model (with only one type of BSM particle) along with other motivations as mentioned in the Introduction.

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[2]Note that we are not checking nonscalar extensions of SM because the sign of the top-loop contribution (dominant contribution) or rather fermionic contribution is opposite to the other diagrams with a gauge boson or a scalar in the loop. Therefore, V.C. is hard to solve by adding nonscalar particles such as vectorlike quarks or fermions, additional gauge bosons etc.

[3]We have also cross checked our result with Matchmakereft [44].

TABLE I.   SMEFT operators in WARSAW basis in different BSM-EFT scenarios.

Model	Quantum No	$\mathcal{Q}_{HD}$	$\mathcal{Q}_{HB}$	$\mathcal{Q}_{HW}$	$\mathcal{Q}_{H\square}$
Real scalar singlet	(1,1,0)	✓	✗	✗	✗
Real scalar triplet	(1,3,0)	✗	✗	✓	✓
Complex scalar triplet	(1,3,1)	✓	✓	✓	✓
Complex scalar doublets (2HDM)	$(1, 2, \pm 1/2)$	✓	✗	✓	✓
Real scalar singlet +	(1,1,0)	✓	✗	✓	✓
Real scalar triplet	(1,3,0)				
Complex scalar triplet +	(1,3,1)	✓	✓	✓	✓
Complex scalar doublet	(1,2,1/2)				

Let us consider that, beyond the scale Λ, there exists a heavy complex triplet, Δ, with weak hypercharge $Y = 1$. The most general renormalizable tree-level scalar potential of such a model is given by

$$
\begin{aligned}
V(H, \Delta) =\ & -m_H^2 (H^\dagger H) + M^2 \mathrm{Tr}[\Delta^\dagger \Delta] \\
& + (\mu_\Delta H^T i\sigma^2 \Delta^+ H + \text{H.c.}) + \lambda (H^\dagger H)^2 \\
& + \lambda_1 (H^\dagger H)\mathrm{Tr}[\Delta^\dagger \Delta] + \lambda_2 (\mathrm{Tr}[\Delta^\dagger \Delta])^2 \\
& + \lambda_3 \mathrm{Tr}[(\Delta^\dagger \Delta)^2] + \lambda_4 (H^\dagger \Delta \Delta^\dagger H).
\end{aligned}
\tag{9}
$$

The extra Yukawa term for neutrino mass generation is

$$
\mathcal{L}_Y = y_\Delta \ell^T i C i\sigma^2 \Delta \ell + \text{H.c.}
\tag{10}
$$

Here the trilinear coupling μ_Δ can be taken as positive by absorbing its phase into H and Δ. The total Lagrangian is

$$
\mathcal{L} = \mathcal{L}_Y - V(H, \Delta).
\tag{11}
$$

The details of this model is summarized in the "model description" section of the Appendix.

We explicitly show the expansion of the dimension six operators [as listed in Eq. (5)], in the "calculation" section of the Appendix. The Higgs mass correction in terms of W.C.s is the WARSAW basis[4] given by

$$
\begin{aligned}
(\delta m_h^2)_{\text{BSM}} =\ & \frac{\Lambda^2}{16\pi^2} (-3\mathcal{C}_{HD} + 12\mathcal{C}_{H\square} + 9\mathcal{C}_{HW} + 3\mathcal{C}_{HB}) \\
& + \frac{\Lambda^2}{(16\pi^2)^2} \left(54\mathcal{C}_H - \frac{9}{2}(g_Y^2 + 3g_W^2)\mathcal{C}_{HD} \right. \\
& \left. + 108 g_W^2 \mathcal{C}_{HW} \right).
\end{aligned}
\tag{12}
$$

The total correction to the Higgs mass is

$$
\delta m_h^2 = (\delta m_h^2)_{\text{SM}} + (\delta m_h^2)_{\text{BSM}}.
\tag{13}
$$

[4]Please check to the Appendix for the result in a different basis.

The Wilson coefficients appearing in one-loop contribution in $(\delta m_h^2)_{\text{BSM}}$ can be expressed in terms of the model parameters as

$$
\begin{aligned}
\mathcal{C}_{HD} =\ & -\frac{g_Y^4}{320\pi^2} + \frac{4\mu_\Delta^2}{M^2} - \frac{\lambda_4^2}{24\pi^2} + \frac{11 g_Y^2 \mu_\Delta^2}{24\pi^2 M^2} - \frac{8\mu_\Delta^4}{3\pi^2 M^4} \\
& + \frac{\lambda_4 \mu_\Delta^2}{6\pi^2 M^2} + \frac{3\lambda \mu_\Delta^2}{8\pi^2 M^2},
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
\mathcal{C}_{H\square} =\ & -\frac{g_W^4}{1920\pi^2} + \frac{2\mu_\Delta^2}{M^2} - \frac{\lambda_1^2}{16\pi^2} - \frac{\lambda_1 \lambda_4}{16\pi^2} - \frac{\lambda_4^2}{192\pi^2} - \frac{g_W^2 \mu_\Delta^2}{96\pi^2 M^2} \\
& + \frac{11 g_Y^2 \mu_\Delta^2}{96\pi^2 M^2} - \frac{49\mu_\Delta^4}{12\pi^2 M^4} + \frac{\lambda_1 \mu_\Delta^2}{8\pi^2 M^2} + \frac{\lambda_4 \mu_\Delta^2}{48\pi^2 M^2} + \frac{3\lambda \mu_\Delta^2}{4\pi^2 M^2},
\end{aligned}
\tag{15}
$$

$$
\mathcal{C}_{HB} = \frac{g_Y^2 \lambda_1}{32\pi^2} + \frac{g_Y^2 \lambda_4}{64\pi^2} + \frac{11 g_Y^2 \mu_\Delta^2}{64\pi^2 M^2},
\tag{16}
$$

$$
\mathcal{C}_{HW} = \frac{g_W^2 \lambda_1}{48\pi^2} + \frac{g_W^2 \lambda_4}{96\pi^2} + \frac{25 g_W^2 \mu_\Delta^2}{192\pi^2 M^2}.
\tag{17}
$$

Here, M is the mass of the heavy triplet. For the theory to be valid, it is sufficient to assume that M is greater than Λ. We assume the order of magnitude to be the same for M and Λ in our calculation as a limiting scenario. For $M \gg \Lambda$, the W.C.s will obtain smaller values.

IV. RESULT

In order to satisfy the Veltman condition, we consider the one-loop correction to the Higgs mass (δm_h^2) and fix two benchmark scenarios at $\Lambda = 100$ TeV and $\Lambda = 10^6$ TeV. Following that, we figure out the model parameter space of λ_1 and λ_4, for which the quadratic divergence in δm_h^2 cancels out exactly. The SM input parameters, g_W, y_t, g_Y, and λ, are determined at Λ by solving the two-loop renormalized group equations. λ_1 and λ_4 are varied in such a way that the Wilson coefficients obey the perturbative limit and the running of the Wilson coefficients from Λ to the electroweak scale are smooth. The values of the tree level couplings (λ and μ_H) also shift due to the higher-dimensional operators. The parameter λ cannot be more

FIG. 1.   Variation of λ_1 and λ_4 with μ_Δ at two benchmark values of Λ. The V.C. is satisfied over the lines. The blue points show the allowed region when the full theory with the triplet is bounded from below and satisfies perturbative unitarity conditions.

than $O(1)$ and this puts an upper limit on the quantity $\frac{\mu_\Delta^2}{2M^2} < O(1)$, where $\mu_\Delta = \frac{\sqrt{2}v_\Delta M^2}{v_H^2}$ in the limit of large scalar triplet mass. Also, precision measurements have set the value of the ρ parameter to be in the range 1.00038 ± 0.00020 [45,46]. This constrains the vacuum expectation value of the triplet (Δ) to be less than 2.56 GeV [47].

In Fig. 1 we show the parameter space of λ_1 and λ_4 that satisfies the V.C. We have found that both positive and negative values of λ_1 and λ_4 satisfy the V.C. The green line represents the highest possible value of μ_Δ, which comes from the constraint $\frac{\mu_\Delta^2}{2M^2} \sim O(1)$. The nature of these plots is highly dependent on the values of μ_Δ, because the Wilson coefficients have $(\mu_\Delta/M)^2$ and $(\mu_\Delta/M)^4$ dependence with additional suppression of $1/16\pi^2$. The sign of the W.C.s come naturally from the fact that they are determined in terms of the model parameters λ_1, λ_4, and μ_Δ, which we allow to vary freely with the above mentioned constraints. We have found that the V.C. is satisfied even if Λ is very large (10^6 TeV). It is also essential to check whether the full theory with the triplet scalar is well behaved above Λ. For that we have considered the vacuum stability, bounded from below and unitarity conditions [48,49] and the blue points in Fig. 1 satisfy these conditions. We have found that the positive values of λ_1 and λ_4 are largely preferred for the full theory with the triplet to be well behaved. Hence the V.C. can be satisfied in a constrained region where the full theory with the triplet obeys stability and unitarity condition. The λ_1 and λ_4 parameter space remains mostly unchanged at large Λ because the Wilson coefficients do not change much with Λ (shown later). Although, a slight variation in the parameter space is present due to the running of SM parameters. The cancellation in the V.C. is dependent on the precision of the input parameters, which is also the source of a negligible amount of fine-tuning.

In Fig. 2, we show the variation of the Wilson coefficients with the model parameter λ_4 at 100 TeV. The corresponding value(s) of λ_1 can be inferred form Fig. 1. The Wilson coefficients show similar behavior at the other benchmark scenario. For Wilson coefficients $\mathcal{C}_{HD}$ and $\mathcal{C}_{H\square}$, negative values are more preferred, whereas, for $\mathcal{C}_{HW}$ and $\mathcal{C}_{HB}$, both positive and negative values are allowed.[5] However, when λ_4 is negative, almost all coefficients are negative, except for some values of the $\mathcal{C}_{HD}$ and $\mathcal{C}_{H\square}$. Again, when λ_4 is positive, $\mathcal{C}_{HW}$ and $\mathcal{C}_{HB}$ are always positive but $\mathcal{C}_{HD}$ and $\mathcal{C}_{H\square}$ are mostly negative except for some values as shown in Fig. 2. Thus, it is visible that the cancellation among the Wilson coefficients is not *ad hoc*, but is controlled by the model parameters. We have checked the V.C. by considering the two-loop contributions to the Higgs mass correction as well but due to the extra suppression by $(1/16\pi^2)$, the effect is not visible.

We have also checked the running of the Wilson coefficients from the effective scale Λ to the electroweak scale. We show the running of the Wilson coefficients in Fig. 3 for a particular choice of the model parameters, $\lambda_1 = 4.0$ and $\lambda_4 = 40$. This particular choice of parameter represents the maximum possible value of these parameters. We found that the values of these W.C.s do not change much and also the sign of the W.C.s do not change in the running. The conclusion remains the same for other allowed values of λ_1 and λ_4 that satisfy the V.C. The values of W.C.s ($C_i(1\text{ TeV})^2/\Lambda^2$) are highly constrained at the EW scale [50] from various low energy experiments. The values of the W.C.s that satisfy the V.C., as shown in Figs. 2 and 3, lie within the current experimental bounds.

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[5]The sign of the W.C. is different in different SMEFT basis. We list the transformation rules of the W.C.s in the Appendix.
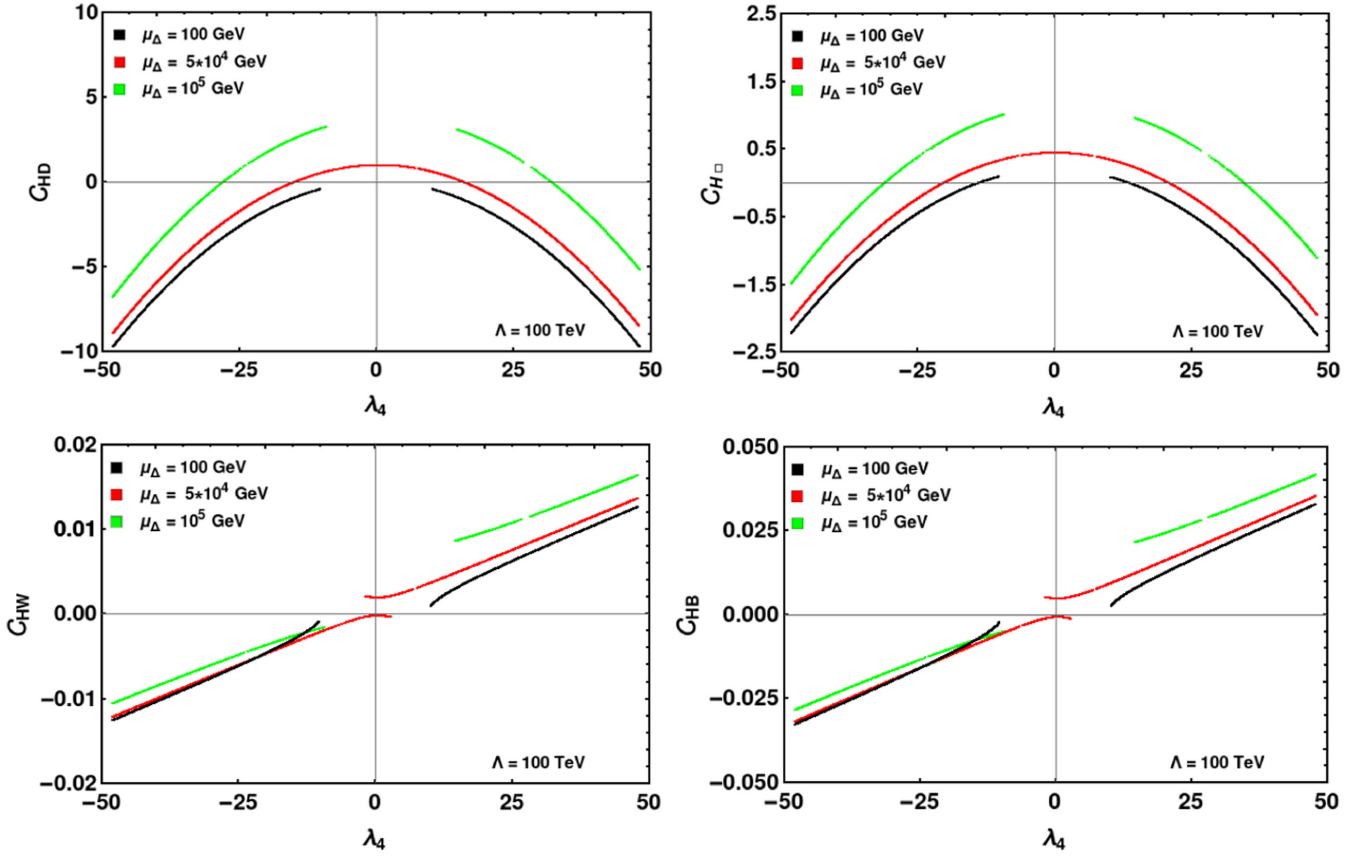
FIG. 2. Variation of the Wilson coefficients with model parameter λ_4 when the V.C. is satisfied.
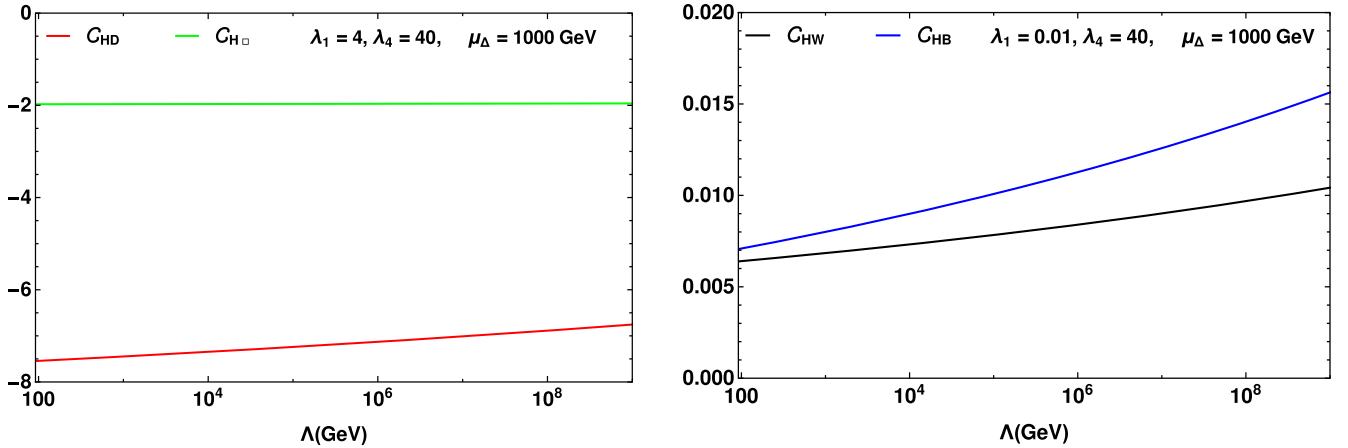

FIG. 3. Running of the Wilson coefficients from $\Lambda = 10^6$ TeV to the cutoff scale for one set of model parameters. We have kept the value of μ_Δ to be fixed at 1 TeV.

V. CONCLUSION

The Veltman condition cannot be satisfied within the framework of the Standard Model because of significant quadratic divergences to the Higgs self-energy correction if the cutoff scale Λ is ~1 TeV or higher. In addition to the dimension four operators from the Standard Model, we have also included dimension six operators whose contributions

to the Higgs mass correction results from integrating out the heavy triplet scalar with hypercharge one. We show how the quadratic divergence of the Higgs self-energy vanishes in this particular model due to the cancellation among the SM parameters and the Wilson coefficients.

We have shown the relevant SMEFT operators that contribute to the V.C. and found agreement with

Ref. [19]. The W.C. of the operators are expressed in terms of the model parameters of the complex scalar triplet model. Hence, in this study the sign of the Wilson coefficients are not *ad hoc*, it is driven by the larger theory, which is a heavy triplet scalar in our case. We found that this model generates all four operators that appear in the V.C., allowing more parameter space to the W.C.s compared to the other models, where the number of operators is less than four. However, the values of the Wilson coefficients will be different in every model, as it is controlled by the specific model parameters.

To achieve the Veltman condition, it should be noted that the contributions from two particular dimension six operators $\mathcal{Q}_{HD}$ and $\mathcal{Q}_{H\square}$ play a dominating role in canceling out the quadratic divergences. We show the parameter space where the V.C. is satisfied for both positive and negative values of the model parameters. We found that the V.C. can be satisfied in a constrained region where the full theory with the triplet obeys stability and unitarity condition as well. We have observed that for energy scales $\Lambda = 100$ and 10^6 TeV, the cancellation is almost similar. This is because the value of the W.C.s does not change much with Λ and the insignificant amount of change appears due to the running of the SM parameters. If we introduce some relaxation in the V.C. by allowing some amount of fine-tuning, then the model parameter space will surely enlarge, but it will get narrower with the increasing values of Λ. Thus, the Veltman condition can be easily satisfied in the framework of effective field theory, when a complex scalar triplet exists at a very large scale.

The study of this model as an effective field theory can also be useful to revisit the type II leptogenesis scenario, where it will be possible to generate specific dimension six terms that are dictated by the model.

ACKNOWLEDGMENTS

APPENDIX

1. Model description

In the type-II seesaw model, the scalar sector is extended by a complex scalar triplet(Δ) with hypercharge 1, in addition to the Higgs doublet (H). Explicitly the fields can be written as

$$H(1, 2, +1/2) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix},$$

$$\Delta(1, 3, +1) = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}. \quad \text{(A1)}$$

The numbers in the parentheses represent the charges of $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group of the SM. The neutral components are

$$\phi^0 = \frac{v_H + h + i\phi_3}{\sqrt{2}}, \qquad \Delta^0 = \frac{v_\Delta + \delta + i\xi}{\sqrt{2}}. \quad \text{(A2)}$$

The kinetic terms corresponding to the scalar fields are given as

$$\mathcal{L}_{\text{kin}} \supset (D^\mu H)^\dagger D_\mu H + \text{Tr}[(D^\mu \Delta)^\dagger (D_\mu \Delta)], \quad \text{(A3)}$$

with the covariant derivatives

$$D_\mu H = \partial_\mu H - i\frac{g_Y}{2} W_\mu^a \sigma^a H - i\frac{g_W}{2} B_\mu H,$$

$$D_\mu \Delta = \partial_\mu \Delta - i\frac{g_Y}{2} \text{Tr}[W_\mu^a \sigma^a, \Delta] - i\frac{g_W}{2} B_\mu \Delta. \quad \text{(A4)}$$

Here σ^a ($a = 1, 2, 3$) are the Pauli spin matrices and g_W and g_Y are the gauge couplings associated with $SU(2)_L$ and $U(1)_Y$ gauge group, respectively.

2. Calculation

The dimension six SMEFT operators that contribute Higgs mass correction either at the one-loop or the two-loop level can be written up to a total derivative as

$$\mathcal{Q}_{HD} = (H^+ \mathcal{D}_\mu H)^* (H^+ \mathcal{D}^\mu H) \supset (\partial_\mu H^\dagger) H H^\dagger (\partial^\mu H^\dagger) + \left[\frac{g_W^2}{4} \sigma^a \sigma^b H^+ W_\mu^a H H^\dagger W^{\mu b} H + \frac{g_Y^2}{4} H^\dagger B_\mu H H^\dagger B^\mu H \right],$$

$$\mathcal{Q}_{H\square} = (H^+ H)\square(H^+ H) = -\partial_\mu(H^\dagger H)\partial^\mu(H^\dagger H),$$

$$\mathcal{Q}_{HW} = (H^+ H) W_{\mu\nu}^a W^{a,\mu\nu} \supset 2H^\dagger [\sigma^a(\partial_\mu W_\nu^a)\sigma^b(\partial^\mu W^{\nu b}) - \sigma^a(\partial_\mu W_\nu^a)\sigma^b(\partial^\nu W^{\mu b})]H$$

$$+ g_W^2 \sigma^a f_{abc} \sigma^p f_{pqr} H^\dagger W_\mu^b W_\nu^c W^{\mu q} W^{\nu r} H,$$

$$\mathcal{Q}_{HB} = (H^+ H) B_{\mu\nu} B^{\mu\nu} \supset 2H^\dagger [\partial_\mu B_\nu \partial^\mu B^\nu - \partial_\mu B_\nu \partial^\nu B^\mu] H,$$
$$\mathcal{Q}_H = (H^\dagger H)^3. \tag{A5}$$

This result is given in the WARSAW basis. Note that only momentum dependent vertices can generate quartic divergence at the one-loop level. Possible Feynman diagrams originating from these terms are similar to Ref. [19].

The authors in Ref. [19] has performed the calculation in the HISZ basis. The transformations between the HISZ and WARSAW bases are given by

$$\mathcal{Q}_{HD} = \mathcal{O}_{\phi,1}, \qquad \mathcal{Q}_{H\square} = -2\mathcal{O}_{\phi,2}, \qquad \mathcal{Q}_H = 3\mathcal{O}_{\phi,3}, \qquad \mathcal{Q}_{GG} = \mathcal{O}_{GG},$$
$$\mathcal{Q}_{HW} = \frac{-4}{g_W^2} \mathcal{O}_{WW}, \qquad \mathcal{Q}_{HB} = \frac{-4}{g_Y^2} \mathcal{O}_{BB}, \qquad \mathcal{Q}_{HWB} = \frac{-4}{g_W g_Y} \mathcal{O}_{BW}. \tag{A6}$$

A complete list of transformations can be obtained from Ref. [37]. The relevant operators in the HISZ basis, $\{\mathcal{O}_{\phi,1}, \mathcal{O}_{\phi,2}, \mathcal{O}_{\phi,3}, \mathcal{O}_{GG}, \mathcal{O}_{WW}, \mathcal{O}_{BB}, \mathcal{O}_{BW}\}$ are given in Refs. [19,37]. It is worth noting that the sign of the coefficients of these SMEFT operators is not the same in both bases.

We would also like to mention that the result obtained in Eqs. (12) and (13) of Ref. [19] can be mapped exactly to our result of δm_h^2, subject to the fact that no operator in the WARSAW basis transforms to the operator $\mathcal{O}_{\phi,4}$ [51] in the HISZ basis and there is no contribution of the gluonic operator in our model. Due to the above mentioned reasons the parameter space of Wilson coefficients found in [19] (HISZ basis) and this paper (WARSAW basis) are different.

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