Cabibbo angle anomalies and oblique corrections: The remarkable role of the vectorlike quark doublet

Benedetta Belfatto[1,2] and Sokratis Trifinopoulos[3]

[1]*SISSA International School for Advanced Studies, Via Bonomea 265, 34136, Trieste, Italy*
[2]*INFN—Sezione di Trieste, Via Bonomea 265, 34136, Trieste, Italy*
[3]*Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts, 02139, USA*

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The combination of precise determinations of V_{us} and V_{ud} hints toward a violation of the Cabibbo-Kobayashi-Maskawa matrix first row unitarity at about 3σ level. Conversely, the recent measurement of the W-boson mass by the CDF Collaboration exhibits significant tension with the Standard Model prediction, intensifying the conflict that may arise in models addressing the unitarity violation. We demonstrate that one vectorlike $SU(2)_L$ doublet with mass of a few TeV mixing with light quarks and the top quark can simultaneously account for the two anomalies, without conflicting with flavor-changing phenomena and electroweak observables. Moreover, another tension in the value of the Cabibbo angle is also reported at the 3σ level, between two determinations of V_{us} obtained from semileptonic $K\ell 3$ and leptonic $K\mu 2$ kaon decays. We show that the vectorlike doublet can be at the origin of this discrepancy and the substantial positive shift in the W-boson mass. This unique feature of the vectorlike quark doublet may render it a crucial puzzle piece in new physics scenarios addressing the Cabibbo-Kobayashi-Maskawa matrix unitarity problem. The model can be potentially probed in future colliders.

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I. INTRODUCTION

The Standard Model (SM) of particle physics has unambiguously proven to be extremely resilient against a vast plethora of high-precision measurements designed to tests its limits. Among the last remaining puzzles are the so-called Cabibbo angle anomalies, the tensions between three different determinations of the Cabibbo angle. Meanwhile, the W-boson mass m_W as measured by the CDF Collaboration significantly departs from the SM prediction [1]. This set of anomalous observations is quite recent, and it is important to scrutinize their validity and consistency both at the experimental as well as theoretical level.

As regards the Cabibbo angle θ_C, recent calculations of short-distance radiative corrections in β decays led to an improved determination of $|V_{ud}| = \cos\theta_C$. Additionally, experimental data on kaon decays combined with recent theoretical and lattice computations provide a precise determination of $|V_{us}| = \sin\theta_C$ and $|V_{us}/V_{ud}| = \tan\theta_C$. These three determinations are in tension between each other, and two anomalies arise. The first Cabibbo angle anomaly (CAA1) can be identified as the deficit in the Cabibbo-Kobayashi-Maskawa (CKM) matrix first row unitarity relation when confronting the value of $|V_{ud}|$ from β decays with the value of $|V_{us}|$ from kaon decays. The second Cabibbo angle anomaly (CAA2) stems from the two different measurements of $|V_{us}|$, i.e., the direct determination of $|V_{us}|$ provided by semileptonic $K\ell 3$ kaon decays, and the determination of the ratio $|V_{us}/V_{ud}|$ obtained from leptonic $K\mu 2$ and $\pi\mu 2$ decay rates. The significance of each anomaly individually amounts to approximately 3σ.

These anomalies, if confirmed with future data, would indicate the presence of physics beyond the SM. Various models that include mediators at the TeV scale have been suggested as possible solutions to CAA1 [2–22]. However, in general, explanations for the CKM unitarity can be in conflict with the new experimental development for m_W, which points toward an enhancement with respect to the SM. For example, before the CDF-II result it was suggested in Ref. [2] that a solution to CAA1 by means of modification of the Fermi constant G_F with respect to the muon decay constant G_μ also implies a deficit of m_W. Reversely, in Ref. [23], it is shown that the models that predict a positive shift in the W mass may also predict a huge violation of CKM unitarity, much larger than the one indicated by the current anomaly.

In this work we show that taking into consideration the CDF-II result, the vectorlike quark charged as a doublet of

$SU(2)_L$ emerges as a favored candidate mediator for the Cabibbo angle anomalies. In particular, it is known that the mixing with the first generation SM quarks can consistently resolve the CAA1 at tree level [8,21], while at one-loop level they can generate a positive contribution to m_W via their coupling to the top [24–26]. We find that these two solutions are compatible with each other while all the other low-energy constraints are satisfied, even in the case of a large positive shift in m_W as required by CDF-II. This is a nontrivial result since it is not the case for other mediators that could in principle satisfy both anomalies as in the case of vectorlike quark singlets. In fact, although it was shown that an up-type singlet mixing with the first generation can be a solution to the CAA1 [2,8,9,20,21,27] and while its mixing with the top can shift the m_W prediction [22,24–26], we explicitly show that this species cannot be the common origin of the anomalies.

Intriguingly, it was shown in Ref. [8] that a vectorlike quark doublet coupling predominantly to the up and strange quarks can be the cause of the tension between the $K\ell 3$ and $K\mu 2/\pi\mu 2$ determinations of V_{us}. We reexamine this solution by assuming a large mixing of the extra doublet also with the top quark. We study the relevant phenomenology and analyze the parameter space. We find that the vectorlike doublet can remarkably be the common explanation of the CAA2 together with the m_W discrepancy.[1] Additionally, in both scenarios we notice that the different magnitudes of the mixing parameters give rise to an interesting pattern of flavor hierarchies for the newly introduced Yukawa couplings. Furthermore, the resulting flavor texture implies contributions to certain channels, e.g., top quark decays, that constitute unique signatures of the model. We thus identify the most promising discovery avenues and assess the sensitivity of future experiments with special emphasis on the high-energy frontier.

The paper is organized as follows. First we summarize and update the current situation of the Cabibbo angle anomalies in Sec. II, considering the most recent reanalysis of electromagnetic radiative corrections in $K\ell 3$ decays [28,29] as well as the most recent developments in superallowed and free neutron β decays. In Sec. III we present the model of the vectorlike quark doublet and provide explicitly the mixing matrices with the SM quarks. We also present the anomalous observables in terms of the model parameters and list all the other relevant constraints. We further report in the Appendix the updated bounds from the most relevant flavor-changing processes. Subsequently, in Sec. IV we present the results of the phenomenological analysis, charting the relevant parameter space and evaluating the improvement over the SM. We also discuss other possible one-particle scenarios with respect to a combined explanation, laying special emphasis on the nontrivial case of the vectorlike quark singlets. Finally, in Sec. V we conclude and discuss briefly the future prospects.

[1] However, we confirm that the conclusion of Ref. [8], namely that the stringent constraints from flavor-changing phenomena forbid the simultaneous explanation of both Cabibbo angle anomalies, still holds regardless of the inclusion of the new mixing with the top. In order to avoid those constraints, at least two generations of vectorlike quarks are required.

II. STATUS OF CABIBBO ANGLE ANOMALIES

The SM predicts the unitarity of V_{CKM}, which for the first row implies

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1. \quad (1)$$

Since the contribution of $|V_{ub}|^2 \sim 1.6 \times 10^{-5}$ is too small, the condition in Eq. (1) reduces to unitarity of Cabibbo mixing. In particular, three different kinds of information can be extracted on the Cabibbo angle: $|V_{us}|_A = \sin\theta_C$, $|V_{us}/V_{ud}|_B = \tan\theta_C$, $|V_{ud}|_C = \cos\theta_C$. In this section we list the three independent determinations for the $|V_{us}|$ and $|V_{ud}|$ CKM matrix elements that drive the anomalies.

(1) Determination A: One precise determination of $|V_{us}|$ stems from semileptonic $K\ell 3$ decays $K \to \pi\ell\nu$ ($K_L e3$, $K_S e3$, $K^\pm e3$, $K^\pm \mu 3$, $K_L \mu 3$, $K_S \mu 3$). Recently, a reanalysis of electromagnetic radiative corrections was performed [28,29], obtaining results in agreement with previous chiral perturbation theory calculations [30] but with reduced uncertainties. After including updated values of experimental inputs, phase-space factors, radiative and isospin-breaking corrections, the result is $f_+(0)|V_{us}| = 0.21634(38)$ [29]. Using the average of four-flavor lattice QCD calculations for the vector form factor $f_+(0) = 0.9698(17)$ as reported by FLAG 2021 [31] gives

$$|V_{us}|_{\mathrm{A}} = 0.22308(55). \quad (2)$$

The element $|V_{us}|$ can also be determined from semileptonic hyperon decays $|V_{us}| = 0.2250(27)$ [32] and from hadronic τ decays $|V_{us}| = 0.2221(13)$ [33], which however present quite large uncertainties and therefore are not included in the present analysis.

(2) Determination B: The ratio $|V_{us}/V_{ud}|$ can be independently determined from the ratio of the kaon and pion leptonic decay rates $K\mu 2$ and $\pi\mu 2$, i.e., $K \to \mu\nu(\gamma)$ and $\pi \to \mu\nu(\gamma)$ [34] which after including electroweak radiative corrections [35–37] yields $f_K|V_{ud}|/(f_\pi|V_{us}|) = 0.27600(37)$ [38–40]. Using the four-flavor average lattice QCD calculations for the decay constants ratio $f_K/f_\pi = 1.1932(21)$ [31], the result is

$$|V_{us}|/|V_{ud}|_B = 0.23131(51). \quad (3)$$

(3) Determination C: $|V_{ud}|$ can be obtained from β decays. The most precise determination is obtained from superallowed 0^+–0^+ nuclear β decays, pure Fermi transitions which are sensitive only to the vector coupling $G_V = G_F|V_{ud}|$. The master formula is [41,42]

$$|V_{ud}|^2_{0^+-0^+} = \frac{K}{2G_F^2 \mathcal{F}t(1+\Delta_R^V)} = \frac{2984.432(3)\ \mathrm{s}}{\mathcal{F}t(1+\Delta_R^V)}, \quad (4)$$

where $K = 2\pi^3 \ln 2/m_e^5 = 8120.27624(1) \times 10^{-10}\ \mathrm{s/GeV}^4$, $G_F = G_\mu = 1.1663787(6) \times 10^{-5}\ \mathrm{GeV}^{-2}$ is the Fermi constant determined from the muon decay [43], Δ_R^V is the transition-independent short-distance radiative correction, $\mathcal{F}t = ft(1+\delta_R')(1+\delta_{\mathrm{NS}} - \delta_C)$ is the "corrected" $\mathcal{F}t$ value obtained from the ft value (which depends on the transition energy, the half-life of the parent state, and the relevant branching ratio) after including the transition-dependent part of radiative corrections (δ_R', δ_{NS}) and isospin-symmetry-breaking correction δ_C. The $\mathcal{F}t$ value was recently updated in Ref. [42], averaging the corrected ft values of 15 superallowed 0^+–0^+ nuclear transitions and obtaining $\mathcal{F}t = 3072.24(1.85)$ s. Compared with the previous average, the new result is almost unchanged in the central value, but the uncertainty is increased by a factor of 2.6 due to new contributions in nuclear structure corrections δ_{NS} [44,45], which now dominate the uncertainty. The short-distance radiative correction Δ_R^V was recently calculated with reduced theory uncertainty, and found to be $\Delta_R^V = 0.02467(22)$ [44,46], which is significantly larger than the previous determination $\Delta_R^V = 0.02361(38)$ [47]. This shift was confirmed by other recent studies [48–51]. We use the result $\Delta_R^V = 0.02467(22)$ [44,46]. Then, Eq. (4) yields $|V_{ud}|_{0^+-0^+} = 0.97367(31)$.

One can determine $|V_{ud}|$ also from free neutron β decay. The master formula gives (for a review see for example Refs. [52,53])

$$|V_{ud}|_n^2 = \frac{K/\ln 2}{G_F^2 \mathcal{F}_n \tau_n (1+3g_A^2)(1+\Delta_R^V)} = \frac{5024.5(6)\ \mathrm{s}}{\tau_n(1+3g_A^2)(1+\Delta_R^V)}, \quad (5)$$

where $\mathcal{F}_n = f_n(1+\delta_R')$ with $f_n = 1.6887(2)$, and $\delta_R' = 0.014902(2)$ is the long-distance QED correction [54], τ_n the neutron lifetime, and $g_A = G_A/G_V$, where G_A is the axial-vector coupling. The neutron lifetime τ_n can be obtained from bottle experiments counting survived ultracold neutrons stored in traps. The average of eight results [55–62] (with rescaled uncertainty) reads as $\tau_n = 878.4 \pm 0.5$ s.[2] Regarding the axial-vector coupling, the average quoted by the Particle Data Group (PDG) is $g_A = -1.2754 \pm 0.0013$ [40], with inflated uncertainty because of the tension between different results, which combined with bottle lifetime in Eq. (5) would give $|V_{ud}|_{n,\mathrm{PDG}} = 0.97436(88)$. However, the latest experiments measuring parity-violating β-asymmetry parameter A from polarized neutrons [65–67] have produced the most precise results, in very good agreement between each other, yielding an average of $g_A = -1.27624(50)$. They show some tension with old results and with one of the recent results obtained measuring the electron-antineutrino angular correlation (a coefficient) from the recoil spectrum of protons, $g_A = -1.2677(28)$ [68] by the aSPECT experiment. Nevertheless there is agreement with the other recent measurement of the a coefficient by the aCORN experiment $g_A = -1.2796(62)$ [69].

Using the average of the three results from A asymmetry together with the "bottle" lifetime $\tau_n = 878.4 \pm 0.5$ s [see Eq. (5)] gives $|V_{ud}|_n = 0.97383(44)$.[3] By combining the results from superallowed β decays and free neutron decay we receive

$$|V_{ud}|_C = 0.97372(26). \quad (6)$$

We also mention that the value of $|V_{ud}|$ can also be extracted from the small branching ratio ($\sim 10^{-8}$) measured by the PIBETA experiment of $\pi^+ \to \pi^0 e^+ \nu_e$ [71], which gives $|V_{ud}^\pi| = 0.9739(29)$ [72]. Although this decay has the cleanest theoretical prediction, the experimental uncertainty is rather large. Another determination is obtained also from mirror decays: $|V_{ud}^\pi| = 0.9739(10)$ [50], about 3 times less precise than superallowed β decays.

The three determinations are not in agreement with each other within the context of the SM. The anomalies become apparent, if we translate by means of unitarity the three determinations into values of $|V_{us}|$ (or, correspondingly, of $|V_{ud}|$):

$$|V_{us}|_A = 0.22308(55),$$
$$|V_{us}|_B = 0.22536(47),$$
$$|V_{us}|_C = 0.2277(11). \quad (7)$$

Interestingly, there is a 3.7σ discrepancy between determination A and C. Taking a conservative average, without reducing the error, between determination A and B (obtained from kaon physics) $|V_{us}|_{A+B} = 0.22440(51)$ still yields a 2.7σ discrepancy, which we call CAA1. It can be encoded as a deficit of the CKM first row unitarity condition by the parameter

[2] The neutron lifetime τ_n can be experimentally obtained with two different methods, namely bottle experiments and beam experiments, which count protons produced in β decay. However, there is more than 4σ tension between the results of the two methods. The average of beam experiments [63,64] gives $\tau_n^{\mathrm{beam}} = 888.0 \pm 2.0$ s. The average including bottle and beam experiments with rescaled uncertainty would give 878.6 ± 0.6 s (see also footnote 3).

[3] As noted in Ref. [70], the combination of Eqs. (4) and (5) provides a precise prediction of the neutron lifetime, which is independent of $1+\Delta_R^V$. In fact, the relation $\tau_n^{\mathrm{SM}} = 5172.3(3.2)/(1+3g_A^2)$ is obtained, which using $g_A = -1.27624(50)$ gives $\tau_n^{\mathrm{SM}} = 878.7 \pm 0.8$ s, in perfect agreement with the "bottle" lifetime $\tau_n = 878.4 \pm 0.5$ s.

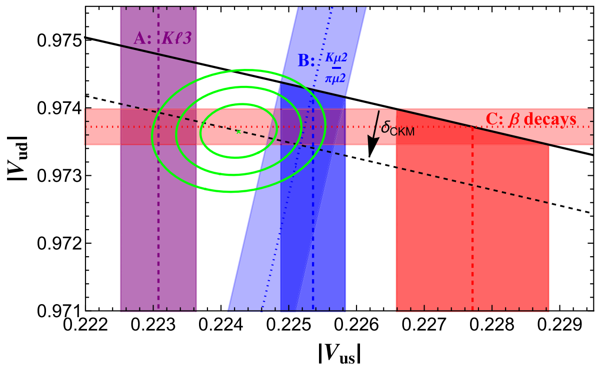

FIG. 1. Independent determinations of $|V_{us}|$ obtained from semileptonic $K\ell 3$ decays (purple), $|V_{us}|/|V_{ud}|$ from leptonic $K\mu 2/\pi\mu 2$ decays (blue), and $|V_{ud}|$ from superallowed $0^+ - 0^+$ and free neutron β decays (red) (see text for details). The corresponding projections on the V_{us} axis are shown using the CKM first row unitarity condition (1) as depicted by the black solid line. A χ^2 fit with two parameters V_{us} and V_{ud} is performed; the green curves show 1σ, 2σ, and 3σ contours ($\chi^2_{\rm min} + 2.3, +6.18, +11.83$) around the best-fit point. The dashed black line corresponds to the violation of unitarity as encoded by the parameter $\delta_{\rm CKM}$.

$$\delta_{\rm CKM} \equiv 1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2. \qquad (8)$$

By performing a χ^2 fit with two parameters V_{us} and V_{ud}, using Eqs. (2), (3), and (6) we obtain $\delta_{\rm CKM} \approx 1.7 \times 10^{-3}$. Finally, CAA2 refers to the 3.1σ discrepancy between the determinations A and B obtained from kaon physics. We illustrate the three determinations in Fig. 1.

III. VECTORLIKE QUARK DOUBLET

A. Model

In addition to the three SM chiral families of fermions, additional vectorlike generations can exist with the left- and right-handed components in the same representations of the SM. Vectorlike fermions are a motivated extension of the SM particle spectrum. They appear in models of grand unification [73–77], play fundamental roles in models with interfamily symmetries which explain the origin of fermion mass hierarchies and mixings [78–81], or solve the strong *CP* problem in models with the axion [82,83] or without it (Nelson-Barr type) [84–88], and they emerge in models addressing the electroweak hierarchy problem in which the light Higgs arises as a pseudogoldstone boson of a global symmetry [89–93].

In the following we consider the inclusion of a vectorlike weak-doublet of quarks $Q_{L,R} = (T, B)_{L,R}$ in the same representation of $SU(3) \times SU(2)_L \times U(1)_Y$ as standard left-handed quarks, that is with the SM quantum numbers $(\mathbf{3}, \mathbf{2})_{1/6}$. The Yukawa sector is augmented by the following couplings and mass terms:

$$\mathcal{L}_{\rm Y} \supset Y_{uij}\bar{q}_{Li}\tilde{\varphi}u_{Rj} + Y_{dij}\bar{q}_{Li}\varphi d_{Rj} + h_{ui}\bar{Q}_L\tilde{\varphi}u_{Ri} + h_{di}\bar{Q}_L\varphi d_{Ri} + M_Q\bar{Q}_L Q_R + \text{H.c.}, \qquad (9)$$

where φ is the Higgs doublet and i, $j = 1, 2, 3$ are the family indexes. We have defined the quark basis in which the mixed mass terms of the type $\mu_Q\bar{q}_L Q_R$ are rotated away, using the fact that the four species of left-handed doublets have identical quantum numbers.[4] After substituting the Higgs vacuum expectation value $\langle\phi\rangle = v_w = 174$ GeV, the mass matrices of up-type and down-type quarks read as

$$\mathcal{M}_u = \begin{pmatrix} Y_u v_w & 0 \\ \hat{h}_u v_w & M_Q \end{pmatrix}, \qquad \mathcal{M}_d = \begin{pmatrix} Y_d v_w & 0 \\ \hat{h}_d v_w & M_Q \end{pmatrix}, \qquad (10)$$

where $Y_{d(u)}$ are the 3×3 SM Yukawa matrices and $\hat{h}_u$ and $\hat{h}_d$ are row vectors $\hat{h}_d = (h_{d1}, h_{d2}, h_{d3})$, $\hat{h}_u = (h_{u1}, h_{u2}, h_{u3})$.

The mass matrices can be diagonalized via biunitary transformations $U^\dagger_{uL}\mathcal{M}_u U_{uR} = \text{diag}(y_u v_w, y_c v_w, y_t v_w, M_{T'})$ and $U^\dagger_{dL}\mathcal{M}_d U_{dR} = \text{diag}(y_d v_w, y_s v_w, y_b v_w, M_{B'})$. The initial states are related to mass eigenstates as

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ B \end{pmatrix}_{L,R} = U_{dL,R}\begin{pmatrix} d \\ s \\ b \\ B' \end{pmatrix}_{L,R},$$
$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ T \end{pmatrix}_{L,R} = U_{uL,R}\begin{pmatrix} u \\ c \\ t \\ T' \end{pmatrix}_{L,R}. \qquad (11)$$

The unitary matrices $U_{dL,R}$ can be found using the relations $U^\dagger_{dL}\mathcal{M}_d\mathcal{M}^\dagger_d U_{dL} = U^\dagger_{dR}\mathcal{M}^\dagger_d\mathcal{M}_d U_{dR} = \mathcal{M}^2_{d,\rm diag}$ and similarly for the up sector.

As regards the left-handed rotations $U_{d(u)L}$, the extra elements describe the mixing of SM quarks with the vectorlike doublet. These mixings can be parametrized by sines of very small angles $s_{Li} \approx y_i|h_i|v_w^2/M_Q^2$, proportional to the SM Yukawa couplings y_i and suppressed by the ratio v_w^2/M_Q^2. By rotating the first three generations, we can choose the basis in which the Yukawa submatrix of up quarks $\hat{U}^\dagger_{uL}Y_u\hat{U}_{uR}$ is diagonal. Then, the unitary matrices $U_{d(u)L}$ can be parametrized as

[4] In some predictive models for fermion masses and mixings, the SM Yukawa terms can be forbidden by some symmetry (e.g., flavor symmetry or Peccei-Quinn symmetry) and emerge after integrating out the heavy states as, e.g., in Refs. [94–97] (so called "universal" seesaw mechanism [98–100]). In the context of supersymmetric models with flavor symmetry this mechanism can give a natural realization of the minimal flavor violation scenario [101–103].

$$U_{dL} = \begin{pmatrix} U_{L1d} & U_{L1s} & U_{L1b} & U_{L1B'} \\ U_{L2d} & U_{L2s} & U_{L2b} & U_{L2B'} \\ U_{L3d} & U_{L3s} & U_{L3b} & U_{L3B'} \\ U_{LBd} & U_{LBs} & U_{LBb} & U_{LBB'} \end{pmatrix} \approx \begin{pmatrix} & & & 0 \\ & \hat{U}_{dL} & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$U_{uL} \approx \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{y_t h_t^* v_W^2}{M_Q^2} \\ 0 & 0 & -\frac{y_t h_t v_W^2}{M_Q^2} & 1 \end{pmatrix}, \tag{12}$$

where we introduced the Yukawa couplings $(h_u, h_c, h_t) = \hat{h}_u \hat{U}_{uR}$ in this weak basis.

As regards the right-handed sector, we can rotate the first three generations and choose the basis in which $\hat{U}^\dagger_{dR} Y^\dagger_d Y_d \hat{U}_{dR}$ is diagonal. Then, for couplings $\lesssim \mathcal{O}(1)$, the mixings of SM quarks with the vectorlike doublet are determined by the small parameter v_w/M_Q, and we can write U_{dR} as

$$U_{dR} = \begin{pmatrix} U_{R1d} & U_{R1s} & U_{R1b} & U_{R1B'} \\ U_{R2d} & U_{R2s} & U_{R2b} & U_{R2B'} \\ U_{R3d} & U_{R3s} & U_{R3b} & U_{R3B'} \\ U_{RBd} & U_{RBs} & U_{RBb} & U_{RBB'} \end{pmatrix}$$
$$\approx \begin{pmatrix} 1 & 0 & 0 & \frac{h_d^* v_w}{M_Q} \\ 0 & 1 & 0 & \frac{h_s^* v_w}{M_Q} \\ 0 & 0 & 1 & \frac{h_b^* v_w}{M_Q} \\ -\frac{h_d v_w}{M_Q} & -\frac{h_s v_w}{M_Q} & -\frac{h_b v_w}{M_Q} & 1 \end{pmatrix} + \mathcal{O}\left(\frac{v_w^2}{M_Q^2}\right), \tag{13}$$

where we introduced $(h_d, h_s, h_b) = \hat{h}_d \hat{U}_{dR}$. Similar expressions hold for the rotations of up quarks U_{uR}. As for the mass eigenvalues, we have

$$\frac{M_{T'}^2}{M_Q^2} = 1 + (h_t^2 + h_c^2 + h_u^2)\frac{v_w^2}{M_Q^2} + \mathcal{O}\left(\frac{v_w^4}{M_Q^4}\right), \tag{14}$$

and similarly for the down sector.

Next, we discuss the modifications to the gauge interactions induced by the presence of the vectorlike quark. The charged-current Lagrangian can be written as

$$\mathcal{L}_{cc} \supset -\frac{g}{\sqrt{2}} W_\mu^+ \left[(\bar{u}_L \quad \bar{c}_L \quad \bar{t}_L \quad \bar{T}'_L)\gamma^\mu V_L \begin{pmatrix} d_L \\ s_L \\ b_L \\ B'_L \end{pmatrix} + (\bar{u}_R \quad \bar{c}_R \quad \bar{t}_R \quad \bar{T}'_R)\gamma^\mu V_R \begin{pmatrix} d_R \\ s_R \\ b_R \\ B'_R \end{pmatrix} \right] + \text{H.c.}, \tag{15}$$

where the right-handed current originates from the term $-\frac{g}{\sqrt{2}} W_\mu^+ \bar{T}_R \gamma^\mu B_R$ after performing the rotation to the mass eigenbasis [see Eq. (11)].

Since the four species are $SU(2)_L$ doublets, the mixing matrix for the left-handed sector is a unitary matrix. From Eq. (12) we have

$$V_L = U_{uL}^\dagger U_{dL} = \begin{pmatrix} V_{Lud} & V_{Lus} & V_{Lub} & V_{LuB'} \\ V_{Lcd} & V_{Lcs} & V_{Lcb} & V_{LcB'} \\ V_{Ltd} & V_{Lts} & V_{Ltb} & V_{LtB'} \\ V_{LT'd} & V_{LT's} & V_{LT'b} & V_{LT'B'} \end{pmatrix} \approx \begin{pmatrix} V_{Lud} & V_{Lus} & V_{Lub} & 0 \\ V_{Lcd} & V_{Lcs} & V_{Lcb} & 0 \\ V_{Ltd} & V_{Lts} & V_{Ltb} & -y_t h_t^* \frac{v_w^2}{M_Q^2} + y_b h_b^* \frac{v_w^2}{M_Q^2} \\ V_{Ltd} y_t h_t \frac{v_w^2}{M_q^2} & V_{Lts} y_t h_t \frac{v_w^2}{M_Q^2} & y_t h_t \frac{v_w^2}{M_Q^2} V_{Ltb} - y_b h_b \frac{v_w^2}{M_Q^2} & 1 \end{pmatrix}, \tag{16}$$

where the upper left 3×3 submatrix is given by $\hat{U}_{dL}$ and corresponds to the CKM matrix in the limit of decoupled new physics. Then, because of the suppression of the extra mixings, it still holds the unitarity relation for the first row,

$$|V_{Lud}|^2 + |V_{Lus}|^2 + |V_{Lub}|^2 = 1 + \mathcal{O}(y_{u(d)}^2 |h_{u(d)}|^2 v_w^4 / M_Q^4), \tag{17}$$

where $|V_{Lub}|$ is also indeed irrelevant.

Since Q_R is a $SU(2)_L$ doublet mixing with the right-handed singlet quarks, a charged-current coupling with the W boson is generated also in the right-handed sector [see Eq. (15)] with a nonunitary mixing matrix given by

$$V_R = U_{uR}^\dagger \mathrm{diag}(0,0,0,1) U_{dR} = \begin{pmatrix} V_{Rud} & V_{Rus} & V_{Rub} & V_{RuB'} \\ V_{Rcd} & V_{Rcs} & V_{Rcb} & V_{RcB'} \\ V_{Rtd} & V_{Rts} & V_{Rtb} & V_{RtB'} \\ V_{RT'd} & V_{RT's} & V_{RT'b} & V_{RT'B'} \end{pmatrix} \approx \begin{pmatrix} h_u^* h_d \frac{v_w^2}{M_q^2} & h_u^* h_s \frac{v_w^2}{M_q^2} & h_u^* h_b \frac{v_w^2}{M_q^2} & -h_u^* \frac{v_w}{M_q} \\ h_c^* h_d \frac{v_w^2}{M_q^2} & h_c^* h_s \frac{v_w^2}{M_q^2} & h_c^* h_b \frac{v_w^2}{M_q^2} & -h_c^* \frac{v_w}{M_q} \\ h_t^* h_d \frac{v_w^2}{M_q^2} & h_t^* h_s \frac{v_w^2}{M_q^2} & h_t^* h_b \frac{v_w^2}{M_q^2} & -h_t^* \frac{v_w}{M_q} \\ -h_d \frac{v_w}{M_q} & -h_s \frac{v_w}{M_q} & -h_b \frac{v_w}{M_q} & 1 \end{pmatrix}. \quad (18)$$

As a consequence, the weak interaction of SM quarks with the W boson loses its pure $V - A$ character. In fact, by denoting as $\hat{V}_L$ and $\hat{V}_R$ the 3×3 submatrices of V_L and V_R describing the mixing between SM quarks, from Eq. (15) the couplings change as

$$\mathcal{L}_{cc} \supset -\frac{g}{2\sqrt{2}} W_\mu^+ \overline{(uct)}[\gamma^\mu(\hat{V}_L + \hat{V}_R) - \gamma^\mu\gamma^5(\hat{V}_L - \hat{V}_R)] \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \quad (19)$$

Additionally, the mixing of SM quarks with the vectorlike quarks induces couplings with Higgs and Z boson which are flavor-nondiagonal in the mass basis and originate flavor-changing phenomena. In particular, left-handed couplings remain diagonal at tree level as in the SM, while in the right-handed sector additional couplings proportional to weak isospin appear:

$$\begin{aligned} \mathcal{L}_{\rm nc} \supset & -\frac{g}{\cos\theta_W} Z^\mu (\, \bar{u}_R \quad \bar{c}_R \quad \bar{t}_R \quad \overline{T'}_R \,) \\ & \times \gamma^\mu \left(\frac{1}{2} K_{uR} - \frac{2}{3}\sin^2\theta_w \mathbf{1}\right) \begin{pmatrix} u_R \\ c_R \\ t_R \\ T'_R \end{pmatrix} \\ & - \frac{g}{\cos\theta_W} Z^\mu (\, \bar{d}_R \quad \bar{s}_R \quad \bar{b}_R \quad \overline{B'}_R \,) \\ & \times \gamma^\mu \left(-\frac{1}{2} K_{dR} + \frac{1}{3}\sin^2\theta_w \mathbf{1}\right) \begin{pmatrix} d_R \\ s_R \\ b_R \\ B'_R \end{pmatrix}, \end{aligned} \quad (20)$$

where

$$\begin{aligned} K_{uR} &= U_{uR}^\dagger \mathrm{diag}(0,0,0,1) U_{uR}, \\ K_{dR} &= U_{dR}^\dagger \mathrm{diag}(0,0,0,1) U_{dR}. \end{aligned} \quad (21)$$

As far as the interactions with the radial Higgs H are concerned, the mass matrices are not proportional to the Yukawa matrices, and flavor-nondiagonal couplings are also generated. The relevant Lagrangian reads as

$$\mathcal{L}_H \supset \frac{1}{\sqrt{2}} H (\bar{d}_L \;\; \bar{s}_L \;\; \bar{b}_L \;\; \overline{B'}_L) U_{dL}^\dagger \begin{pmatrix} Y_d & 0 \\ h_d & 0 \end{pmatrix} U_{dR} \begin{pmatrix} d_R \\ s_R \\ b_R \\ B'_R \end{pmatrix} \quad (22)$$

and similarly for up-type quarks.

B. Observables

1. Cabibbo angle anomalies

The value of V_{ud} obtained from β decays is determined by the vector coupling $G_V = G_F|V_{ud}|$. Also semileptonic kaon decays $K\ell 3$ determine the weak vector coupling, while leptonic decays $K\mu 2$ and $\pi\mu 2$ depend on the axial-vector current. As stated in the previous section, in the scenario with vectorlike doublet vector and axial-vector couplings are not equal as in the SM. Therefore, the three determinations correspond to different couplings [8]. From Eqs. (15) and (18) we have

$$\begin{aligned} |V_{us}|_A &= |V_{Lus} + V_{Rus}| = 0.22308(55), \\ |V_{us}|/|V_{ud}|_B &= \frac{|V_{Lus} - V_{Rus}|}{|V_{Lud} - V_{Rud}|} = 0.23131(51), \\ |V_{ud}|_C &= |V_{Lud} + V_{Rud}| = 0.97372(26). \end{aligned} \quad (23)$$

This system can be solved with real parameters,

$$\begin{aligned} V_{Rud} &= U_{RTu}^* U_{RBd} \approx h_u^* h_d \frac{v_w^2}{M_Q^2} = -0.78(27) \times 10^{-3}, \\ V_{Rus} &= U_{RTu}^* U_{RBs} \approx h_u^* h_s \frac{v_w^2}{M_Q^2} = -1.26(38) \times 10^{-3}, \end{aligned} \quad (24)$$

and $V_{Lud} = 0.97450(8)$, $V_{Lus} = 0.22434(36)$, using the unitarity of the V_L matrix.

Consequently, the mixing V_{Rud} in the right-handed current can explain the apparent deficit in CKM unitarity when confronting determination from β decays with the other determinations from kaon decays, while V_{Rus} would explain the gap between the determinations from semi-leptonic kaon decays $K\ell 3$ and leptonic decays $K\mu 2/\pi\mu 2$ (see Fig. 2). This is the special property of the contributions generated by the vectorlike quark doublet which, by generating right-handed currents, induces a difference in

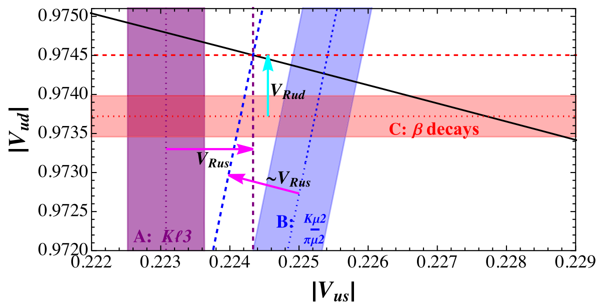

FIG. 2. Effect of the mixing of the vectorlike quark doublet with the SM quarks on the three independent determinations of $|V_{us}|$ and $|V_{ud}|$. Semileptonic $K\ell 3$ decays (purple) provide the vector coupling $|V_{Lus} + V_{Rus}|$, β decays (red) the vector coupling $|V_{Lud} + V_{Rud}|$, while leptonic decays $K \to \mu\nu/\pi \to \mu\nu$ (blue) provide the axial-vector couplings $|V_{Lus} - V_{Rus}|/|V_{Lud} - V_{Rud}|$. The black solid line depicts the CKM first row unitarity condition in Eq. (1). The dashed lines represent the elements V_{Lud}, V_{Lus}, and V_{Lus}/V_{Lud}, while the arrows represent the solutions to CAA1 (cyan) and CAA2 (magenta) [see Eq. (24)].

vector and axial-vector couplings for each transition. From Eq. (23), it also results that in order to have $h_i \lesssim 1$, it should be $M_Q \lesssim 6$ TeV for the mass of the vectorlike species.

2. W-boson mass

The recent measurement of the mass of the W boson published by the CDF Collaboration [1] $m_{W,\mathrm{CDFII}} = 80.4335 \pm 0.0094$ GeV exhibits a discrepancy of around 6.6σ from the SM expectation $m_{W,\mathrm{SM}} = 80.360 \pm 0.006$ GeV [40]. There is also a 3.7σ discrepancy with the average of the other measurements $m_{W,\mathrm{old}} = 80.377 \pm 0.012$ GeV [40]. In Ref. [104] an average of all measurements including the CDF-II result has been computed:

$$m_{W,\mathrm{exp}} = 80.413 \pm 0.015 \text{ GeV}, \tag{25}$$

showing 3.3σ tension with the SM prediction.

The vectorlike quark doublet induces radiative corrections to gauge boson propagators (oblique corrections) via one loop diagrams, which can be parametrized by the electroweak oblique parameters T, S, and U [105]. The shift of the W mass in terms of the oblique parameters reads as [105]

$$\begin{aligned}\delta m_W^2 &= m_W^2 - m_{W,\mathrm{SM}}^2 \\ &= c^2 m_Z^2 \alpha \left[\frac{c^2}{c^2 - s^2} T - \frac{1}{2(c^2 - s^2)} S + \frac{1}{4s^2} U\right],\end{aligned} \tag{26}$$

where m_Z is the Z-boson mass, $m_Z = 91.1882(20)$ GeV, $s = \sin\theta_W$, $c = \cos\theta_W$ with θ_W the Weinberg angle, $s^2 = 0.23122(4)$, and α is the fine structure constant $\alpha(m_Z) = 1/127.951(9)$. We report in Appendix B the general expressions of T, S, and U in the presence of vectorlike quarks as derived in Ref. [106]. In the scenario with the vectorlike doublet, taking into account the matrices V_L, V_R, and $K_{d(u)R}$ in Eqs. (16), (18), and (21), and the mass splitting given in Eq. (14), after subtracting the SM effect of the top quark, the contribution to the T parameter results in

$$\begin{aligned}T \approx{}& \frac{3}{16\pi^2} \frac{2\pi v_w^2}{s^2 c^2 m_Z^2}\left[y_t^2 |h_t|^2 \frac{v_w^2}{M_Q^2}\left(-3 + 2\ln\frac{M_Q^2}{m_t^2}\right)\right. \\ &\left. + \frac{2}{3}\frac{v_w^2}{M_Q^2}\left(\sum_{\alpha=u,c,t} |h_\alpha|^2 - \sum_{\beta=d,s,b} |h_\beta|^2\right)^2 + \mathcal{O}\left(\frac{v_w^4}{M_Q^4}\right)\right].\end{aligned} \tag{27}$$

For the S and U parameters we have

$$\begin{aligned}S \approx{}& \frac{3}{18\pi}\frac{v_w^2}{M_Q^2}\left[\sum_{\alpha=u,c,t} |h_\alpha|^2 \left(-10 + 4\ln\frac{M_Q^2}{m_\alpha^2}\right)\right. \\ &\left. + \sum_{\beta=d,s,b} |h_\beta|^2 \left(-6 + 2\ln\frac{M_Q^2}{m_\beta^2}\right)\right] + \mathcal{O}\left(\frac{v_w^4}{M_Q^4}\right),\end{aligned} \tag{28}$$

$$\begin{aligned}U \approx{}& \frac{1}{2\pi}\frac{v_w^2}{M_Q^2}[(|h_u|^2 + |h_c|^2 + |h_t|^2 + |h_d|^2 + |h_s|^2 + |h_b|^2) \\ &+ 4.2\mathrm{Re}(V_{Lud} h_u h_d^*)] + \mathcal{O}\left(\frac{v_w^4}{M_Q^4}\right).\end{aligned} \tag{29}$$

The contribution of the parameter U is negligible in this scenario. Since the coupling with the top h_t is larger than the couplings to lighter quarks, the T parameter produces the main contribution to δm_W^2. Then, the shift in the W mass is mostly originated by the weak isospin-breaking effect,

$$\begin{aligned}\frac{\delta m_W^2}{m_{W,\mathrm{SM}}^2} \approx{}& \frac{c^2}{c^2 - s^2}\frac{3}{16\pi^2}\frac{v_w^2}{M_Q^2}\left[y_t^2 |h_t|^2 \left(-3 + 2\ln\frac{M_Q^2}{m_t^2}\right)\right. \\ &\left. + \frac{2}{3}(|h_d| + |h_s|^2 + |h_b|^2 - |h_u|^2 - |h_c|^2 - |h_t|^2)^2\right].\end{aligned} \tag{30}$$

In the scenario of one vectorlike doublet with mass $M_Q = 2$ TeV coupling only to the top, the m_W value of Eq. (25) would be explained with a Yukawa coupling of $h_t \approx 1.0 \pm 0.1$, corresponding to a mass splitting of 7 GeV. Assuming the CDF II result, we would get $h_t \approx 1.1 \pm 0.1$.

C. Low-energy constraints

In the following, we mention the most stringent constraints from flavor-changing phenomena and then present the ones from flavor-conserving observables and processes involving the third family. In Table I we summarize the most important constraints on the model parameters.

TABLE I. Summary of most relevant experimental bounds on the mixing of the vectorlike quark doublet with the SM quarks.

Process	Constraint
$\Gamma(Z \to \text{hadrons})$, $\Gamma(Z)$	$[\lvert U_{RTu}\rvert^2 + \lvert U_{RTc}\rvert^2 + 0.5\sum_{q=d,s,b}\lvert U_{RBq}\rvert^2] \lesssim 5\times 10^{-3}$
$Q_W(Cs)$	$-0.0022 < (\lvert U_{RTu}\rvert^2 - 1.12\lvert U_{RBd}\rvert^2) < 0.0066$
$Q_W(p)$	$\lvert -\lvert U_{RTu}\rvert^2 + \frac{1}{2}\lvert U_{RBd}\rvert^2\rvert < 0.0045$
$t \to uH$, $t \to cH$	$\lvert U^*_{RTu,c}U_{RTt}\rvert m_t/v_w \lesssim 0.08$
$t \to uZ$, $t \to cZ$	$\lvert U^*_{RTu,c}U_{RTt}\rvert \lesssim 0.01$
$K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \mu^+\mu^-$, $K^0 - \bar{K}^0$	$\lvert U^*_{RBs}U_{RBd}\rvert \lesssim 5\times 10^{-6}$
$B^0 - \bar{B}^0$, $B^0 \to \mu^+\mu^-$	$\lvert U^*_{RBd}U_{RBb}\rvert < 1.8\times 10^{-4}$
$B^0_s - \bar{B}^0_s$, $B^0_s \to \mu^+\mu^-$	$\lvert U^*_{RBd}U_{RBb}\rvert < 3.3\times 10^{-4}$
$D^0 - \bar{D}^0$	$\lvert U^*_{RTu}U_{RTc}\rvert < 1.3\times 10^{-4}$

1. Flavor-changing neutral currents

Flavor-changing neutral currents (FCNCs) are rare processes within the SM because they appear only at loop level and receive additional suppression due to the Glashow-Iliopoulos-Maiani mechanism [107–109]. The mixing of SM quarks with the vectorlike quarks, which generates flavor-nondiagonal couplings with the Higgs and the Z boson, is thus strongly constrained by FCNCs (e.g., see also Refs. [110,111]).

The most stringent constraints come from kaons ($K^0 - \bar{K}^0$ mixing, $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \mu^+\mu^-$, etc.), and we report the leading ones in Appendix A in more detail. Considered together with flavor-diagonal constraints, it was shown in Ref. [8] that one vectorlike doublet cannot actually accommodate both Cabibbo angle anomalies simultaneously. In particular, these processes impose a bound on the product of the mixing elements U_{RBd}, U_{RBs} in Eq. (13). Depending on the phase $\mathrm{Arg}(U^*_{RBs}U_{RBd})$, assuming $M_Q = 2$ TeV for the mass of the vectorlike doublet we receive the estimate

$$|U^*_{RBs}U_{RBd}| \approx |h^*_s h_d|\frac{v_w^2}{M_Q^2} \lesssim (0.6\text{–}5.2)\times 10^{-6}, \tag{31}$$

and this constraint becomes even more stringent for larger values of the mass of the extra doublet. On the other hand, according to Eq. (24) a solution to the anomalies would require

$$|U_{RTu}|^2 U^*_{RBs}U_{RBd} \approx 1.0\times 10^{-6}. \tag{32}$$

The less stringent upper bound in Eq. (31) applied to this relation leads to $|U_{RTu}| \approx |h_u|v_w/M_Q \gtrsim 0.3$. Such a large mixing with the heavy species contradicts data on Z decay rate into hadrons, which imply $|U_{RTu}| < 0.08$, as we will see in Sec. III C 3. Moreover, such a mixing also implies a large Yukawa coupling $h_u \approx 1.8(M_Q/1\ \text{TeV})$, at the verge of loss of perturbativity. If the vectorlike doublet couples predominantly to either the down or the strange quark, while the other coupling is suppressed, flavor-changing effects can be avoided. Following this observation, in Sec. IVA, we are addressing only one Cabibbo angle anomaly at a time assuming that some other new physics would be necessary to explain the other.

Mixing in the neutral $B^0_{d(s)}$-mesons system and flavor-changing $B_{d(s)}$ decays imposes constraints on the mixing $|U^*_{RBd(s)}U_{RBb}|$. We receive the approximate bounds $|U^*_{RBd}U_{RBb}| \approx |h^*_d h_b|v_w^2/M_Q^2 < (0.4\text{–}1.7)\times 10^{-4}$ and $|U^*_{RBs}U_{RBb}| \approx |h^*_s h_b|v_w^2/M_Q^2 < 8\times 10^{-7}\text{–}6.4\times 10^{-4}$. Regarding the up-quarks sector, we estimate from the neutral D-mesons system that $|U^*_{RTu}U_{RTc}| \approx |h^*_u h_c|v_w^2/M_Q^2 < 1.0\times 10^{-4}$ for the mass of the vectorlike doublet $M_Q = 2$ TeV. We will assume that the couplings to charm and bottom are small enough to respect these limits.

2. Top decays

Since couplings of the vectorlike doublet with both top quark and light quarks are different from zero, flavor-changing top decays $t \to Hu$, $t \to Zu$ are induced. The experimental limits on these decays are [112,113]

$$\mathrm{Br}(t \to uZ)_{\text{exp}} < 6.6\times 10^{-5},$$
$$\mathrm{Br}(t \to cZ)_{\text{exp}} < 1.2\times 10^{-4}, \tag{33}$$

$$\mathrm{Br}(t \to uH)_{\text{exp}} < 6.9\times 10^{-4},$$
$$\mathrm{Br}(t \to cH)_{\text{exp}} < 9.4\times 10^{-4}. \tag{34}$$

The SM predictions for these flavor-changing decays are well below experimental bounds, $\mathrm{Br}(t \to uZ)_{\text{SM}} \sim 10^{-16}$, $\mathrm{Br}(t \to uH)_{\text{SM}} \sim 10^{-17}$, $\mathrm{Br}(t \to cZ)_{\text{SM}} \sim 10^{-14}$, and $\mathrm{Br}(t \to cH)_{\text{SM}} \sim 10^{-15}$ [114]. In our scenario the Lagrangian includes the interaction terms

$$\mathcal{L}_{\rm top} \supset -\frac{g}{2\cos\theta_W} U^*_{RTu} U_{RTt} Z^\mu \bar{u}_R \gamma_\mu t_R - \frac{1}{\sqrt{2}} U_{LTt} h_u^* H \bar{u}_R t_L + \text{H.c.}, \tag{35}$$

(and similarly for charm, however we neglect the couplings with second generation in our case). The predicted branching ratios are

$$\mathrm{Br}(t\to uZ)_{\rm NP} \approx \frac{1}{2|V_{tb}|^2} |U^*_{RTu} U_{RTt}|^2 \left(1-\frac{m_Z^2}{m_t^2}\right)^2 \cdot \left(1+2\frac{m_Z^2}{m_t^2}\right)\left(1-\frac{m_W^2}{m_t^2}\right)^{-2}\left(1+2\frac{m_W^2}{m_t^2}\right)^{-1}, \tag{36}$$

$$\mathrm{Br}(t\to uH)_{\rm NP} \approx \frac{1}{\Gamma_t}\frac{1}{64\pi} |U_{LTt} h_u^*|^2 m_t \left(1-\frac{m_H^2}{m_t^2}\right)^2, \tag{37}$$

using the rate $\Gamma_t \approx \Gamma(t\to bW^+) = \frac{G_F}{8\pi\sqrt{2}} |V_{tb}|^2 m_t^3 (1-\frac{m_W^2}{m_t^2})^2 \times (1+2\frac{m_W^2}{m_t^2})$. The resulting bounds are

$$|U^*_{RTu} U_{RTt}| \approx |h_u^* h_t| \frac{v_w^2}{M_Q^2} \lesssim 0.01, \qquad |U_{LTt} h_u^*| \approx y_t |h_u^* h_t| \frac{v_w^2}{M_Q^2} \lesssim 0.08. \tag{38}$$

We also report here the future prospects for these decay channels in high luminosity LHC (HL-LHC) and the hadron-hadron Future Circular Collider (FCC-hh). The center of mass energy and integrated luminosity for HL-LHC are 14 TeV and 3 ab^{-1} [115,116] and for FCC-hh 100 TeV and 30 ab^{-1} [115,117]. For the FCC-hh we give both the limits assuming conservative 10% systematics as well as optimistic 0% in the parenthesis:

$$\begin{aligned} \mathrm{Br}(t\to uZ)_{\rm HL-LHC} &< 4.08(2.34)\times 10^{-5}, \\ \mathrm{Br}(t\to cZ)_{\rm HL-LHC} &< 6.65(3.13)\times 10^{-5}, \\ \mathrm{Br}(t\to uH)_{\rm HL-LHC} &< 2.4\times 10^{-4}, \\ \mathrm{Br}(t\to cH)_{\rm HL-LHC} &< 2\times 10^{-4}, \end{aligned} \tag{39}$$

$$\begin{aligned} \mathrm{Br}(t\to uZ)_{\rm FCC-hh} &< 2.17(0.069)\times 10^{-5}, \\ \mathrm{Br}(t\to cZ)_{\rm FCC-hh} &< 3.54(0.089)\times 10^{-5}, \\ \mathrm{Br}(t\to uH)_{\rm FCC-hh} &< 2.3(0.73)\times 10^{-5}, \\ \mathrm{Br}(t\to cH)_{\rm FCC-hh} &< 3(0.96)\times 10^{-5}. \end{aligned} \tag{40}$$

3. Flavor-conserving processes

First, we consider the total decay rate of the Z boson as well as the partial decay rate into hadrons. The experimental measurements yield [40,118]

$$\Gamma(Z)_{\rm exp} = 2.4955 \pm 0.0023\ \text{GeV}, \qquad \Gamma(Z\to \text{hadr})_{\rm exp} = 1.7448 \pm 0.0026\ \text{GeV}, \tag{41}$$

while the corresponding SM predictions are $\Gamma(Z)_{\rm SM} = 2.4941 \pm 0.0009$ GeV and $\Gamma(Z\to\text{hadr})_{\rm SM} = 1.74097 \pm 0.00085$ GeV [40].

Mixing with the heavy doublet changes the prediction as

$$\begin{aligned} \Gamma(Z\to\text{had}) - \Gamma(Z\to\text{had})_{\rm SM} &= \Gamma(Z) - \Gamma(Z)_{\rm SM} \\ &\approx \frac{G_F M_Z^3}{\sqrt{2}\pi}\Big[-\frac{2}{3}\sin^2\theta_W(|U_{RTu}|^2 + |U_{RTc}|^2) \\ &\quad -\frac{1}{3}\sin^2\theta_W(|U_{RBd}|^2 + |U_{RBs}|^2 + |U_{RBb}|^2)\Big], \end{aligned} \tag{42}$$

which means that the predicted decay rate is lower than the SM expectation. At 2σ C.L. we obtain the limit

$$|U_{RTu}|^2 + |U_{RTc}|^2 + \frac{1}{2}(|U_{RBd}|^2 + |U_{RBs}|^2 + |U_{RBb}|^2) \lesssim 5\times 10^{-3}. \tag{43}$$

Another set of flavor-conserving constraints originate from parity violating effects at low-energy electron-hadron processes with Z-boson exchange. The interaction Lagrangian can be written as

$$\mathcal{L}_{e-\rm had} = \frac{G_F}{\sqrt{2}} \sum_q (g_{AV}^{eq} \bar{e}\gamma_\mu\gamma^5 e \bar{q}\gamma^\mu q + g_{VA}^{eq} \bar{e}\gamma_\mu e \bar{q}\gamma^\mu\gamma^5 q). \tag{44}$$

Measurements of atomic parity violation provide the determination of nuclear weak charges $Q_W^{Z,N}$ [40]:

$$Q_W^{Z,N} = -2[Z(g_{AV}^{ep} + 0.00005) + N(g_{AV}^{en} + 0.00006)]\left(1-\frac{\alpha}{2\pi}\right), \tag{45}$$

where Z and N are the numbers of protons and neutrons in the nucleus, $g_{AV}^{ep} = 2g_{AV}^{eu} + g_{AV}^{ed}$, $g_{AV}^{en} = g_{AV}^{eu} + 2g_{AV}^{ed}$, and α is the fine structure constant, $\alpha^{-1} \approx 137.036$. After including higher order corrections, the values of the neutral current parameters are $g_{AV,\rm SM}^{eu} = -0.1887$ and $g_{AV,\rm SM}^{ed} = 0.3419$ [40]. The most precise measurement of atomic parity violation is in cesium [40]:

$$Q_W^{55,78}(Cs)_{\rm exp} = -72.82 \pm 0.42, \tag{46}$$

corresponding to $55g_{AV}^{ep} + 78g_{AV}^{en} = 36.46 \pm 0.21$, while the SM prediction is $Q_W^{55,78}(Cs)_{\rm SM} = -73.24 \pm 0.01$ (i.e., $55g_{AV,\rm SM}^{ep} + 78g_{AV,\rm SM}^{en} = 36.66$) [40]. The new physics contribution to the weak charge of cesium in our scenario is given by

$$\Delta Q_W(Cs) = Q_W^{55,78}(Cs) - Q_W^{55,78}(Cs)_{\rm SM} \approx -2\left(-94\frac{1}{2}|U_{RTu}|^2 + 105.5|U_{RBd}|^2\right). \quad (47)$$

The parity violating asymmetry in $e^- p \to e^- p$ elastic scattering was determined by the $Q_{\rm weak}$ collaboration [119], resulting in the experimental determination of the weak charge of the proton $Q_W(p)_{\rm exp} = 0.0719 \pm 0.0045$ [40], which corresponds to the coupling

$$g^{ep}_{AV,\rm exp} = -0.0356 \pm 0.0023. \quad (48)$$

In the SM, we have $Q_W(p)_{\rm SM} = 0.0709 \pm 0.0002$ (i.e., $g^{ep}_{AV,\rm SM} = -0.0355$) in agreement with the experimental result. The new contribution to g^{ep}_{AV} due to the presence of the vectorlike doublet is

$$\Delta g^{ep}_{AV} = g^{ep}_{AV} - g^{ep}_{AV,\rm SM} \approx -|U_{RTu}|^2 + \frac{1}{2}|U_{RBd}|^2. \quad (49)$$

IV. DISCUSSION

In order to alleviate the flavor-changing constraints discussed in Sec. III C 1, in this work we consider two separate scenarios, based on whether the vectorlike doublet couples predominantly to the down or the strange quark. Consequently, the two scenarios aim toward an explanation of either CAA1 or CAA2 respectively, together with the CDF-II result. In the following, we present the two scenarios, reporting in the parentheses at the beginning of the paragraph the relevant new physics couplings.[5]

A. Vectorlike quark doublet

For CAA1 + $\boldsymbol{m_W}$ $(\boldsymbol{h_u}, \boldsymbol{h_d}, \boldsymbol{h_t}, \boldsymbol{M_Q})$, we perform a χ^2 fit using the parameters U_{RTu}, U_{RBd}, U_{RTt}, V_{Lus}, including the Cabibbo angle determinations of Eqs. (2), (3), and (6), the W-mass global value of Eq. (25), and the observables in Sec. III C 3 $\Gamma(Z \to \text{had})$, $Q_W(p)$, $Q_W(Cs)$.[6] In this scenario, the best-fit point results in

$$U_{RTu} = \pm 0.035, \qquad U_{RBd} = \mp 0.019,$$
$$|U_{RTt}| = 0.084, \qquad V_{Lus} = 0.22452, \quad (50)$$

where U_{RTt} is determined for $M_Q = 2$ TeV. The SM fit assuming CKM unitarity yields in the minimum $\chi^2_{\rm SM} = 31.4$ with $V_{us} = 0.2247$ ($\chi^2_{\rm SM}/\text{dof} = \chi^2_{\rm SM}/6 = 5.2$). We obtain an improvement of $\chi^2_{\rm SM} - \chi^2_{\rm min} = 17.2$ which is independent of the mass of the vectorlike doublet. The χ^2 per degree of freedom results in $\chi^2_{\rm min}/\text{dof} = 4.7$. The remaining discrepancy is mostly due to the difference between the determinations of the Cabibbo angle from $K\ell 3$ and $K\mu 2/\pi\mu 2$ decays, which we are not aiming at explaining in this scenario. E.g., in the presence of another vectorlike doublet coupling with u and s quarks, we would have $\chi^2_{\rm SM} - \chi^2_{\rm min} = 27.7$, $\chi^2_{\rm min} = 3.7$. A mild unsettled "tension" would appear from the expectation of the Z-boson decay rate, which would be $\sim 1.7\sigma$ lower than the experimental determination. This means that an increase in sensitivity would signal the presence of vectorlike quarks exhibiting large mixing with light generations. The improvement over the SM would also be larger assuming a less conservative value for m_W. In fact, the SM fit would considerably worsen ($\chi^2_{\rm SM}/\text{dof} = 10.7$ using the CDF II result [1]), while in the new physics scenario the goodness of the fit would be unchanged ($\chi^2_{\rm min}/\text{dof} = 4.7$, with $h_t = 1.1$ in the best-fit point for $M_Q = 2$ TeV).

We present the results of the analysis in the U_{RTu}-U_{RBd} and U_{RTu}-U_{RTt} planes in Figs. 3a and 3b respectively, marginalizing over the other variables. Limits from LHC exclude vectorlike doublets coupling to the top for masses lower than ~ 1.37 TeV [120], so Fig. 3b is given for a benchmark mass $M_Q = 2$ TeV. The plot in Fig. 3a does not depend on the mass of the vectorlike doublet, although information on the mass and the couplings resides in the relations $U_{RTu} \approx -h_u v_w/M_Q$, $U_{RBd} \approx -h_d v_w/M_Q$. As already mentioned in Sec. III B 1 if we require $h_u, h_d \lesssim 1$ then the vectorlike quark needs to be lighter than ~6 TeV in order to account for the anomaly. An analogous plot can be obtained in the second quadrant.

We show the 1σ and 2σ (green and light-green regions) confidence intervals of the total fit ($\chi^2_{\rm min} + 1$, $\chi^2_{\rm min} + 4$). Moreover, the constraints at 2σ CL are displayed for the Z boson decay to hadrons (yellow), the weak charge of the proton (red), the atomic parity violation in ${}^{133}_{78}Cs$ (orange), and the current LHC limits for $t \to uZ$ (solid magenta) and $t \to uH$ (solid purple). We also exhibit the future prospects from FCC-hh on the decay $t \to uZ$ under the two assumptions of 0% (dot-dashed magenta) and 5% (dashed magenta) systematic errors [115] ($\text{Br}(t \to uZ)_{\rm FCC-hh} < 6.86 \times 10^{-7}$ and $\text{Br}(t \to uZ)_{\rm FCC-hh} < 1.1 \times 10^{-5}$ respectively). The prospects on $t \to uH$ are always subleading and thus are omitted.

For comparison, we also perform the fit including only the anomalies, namely Eqs. (2), (3), (6), and (25) without including other experimental constraints and depict the 1σ region ($\chi^2_{\rm min} + 1$) in the plots (blue bands). As it can be inferred from Fig. 3b, the coupling of the vectorlike doublet

[5] We use real parameters for simplicity.

[6] We perform the analysis in the approximation $U_{RT(B)\alpha} \approx -h_\alpha v_w/M_Q$, fixing the mass of the extra doublet at $M_Q = 2$ TeV, using as well the approximated expressions in Sec. III B 2, therefore neglecting terms of order higher than v_w^3/M_Q^3. We believe that further precision is not necessary for the purpose of this paper, taking also into account that the mass of the vectorlike species is unknown.

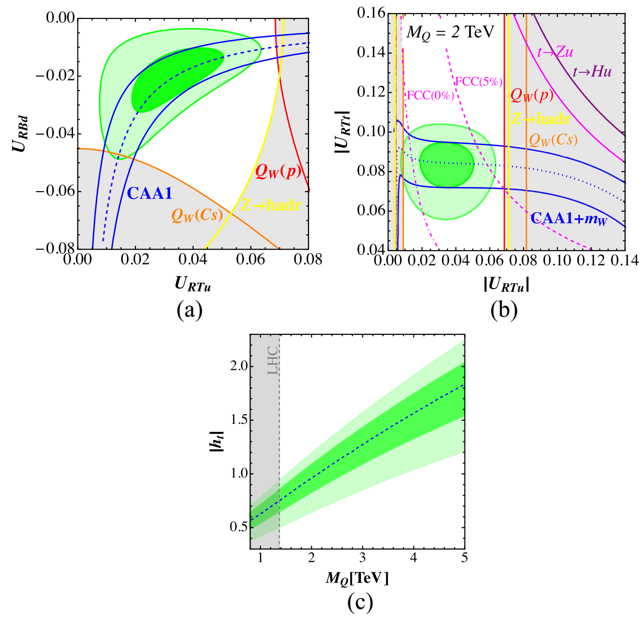

FIG. 3. Parameter space in the scenario with one vectorlike quark doublet coupling with up, down, and top quarks [see Eq. (13)]. 1σ and 2σ preferred regions of the mixing parameters are indicated (green and lighter green), $(\chi^2_{\rm min}+1, \chi^2_{\rm min}+4)$. We also show the region excluded at 2σ by experimental bounds (gray) and 1σ interval which would explain the CAA1 and the m_W-mass shift (blue band), without including other constraints. In Fig. 3b, $M_Q = 2$ TeV is assumed for the mass of the vectorlike doublet, and we indicate the experimental bounds on the left side using the conservative value $|V_{RTd}| = 0.4\times10^{-3}$ (gray region on the left side).

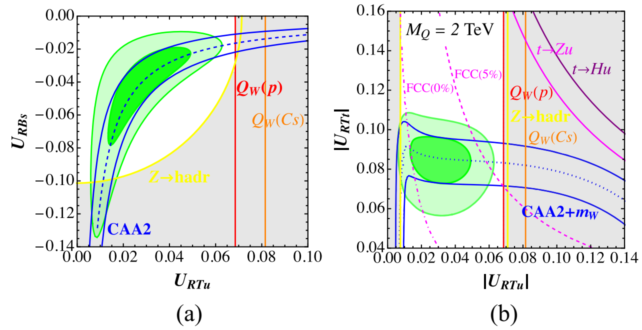

FIG. 4. Parameter space in the scenario with one vectorlike quark doublet coupling with up, strange, and top quarks [see Eq. (13)]. 1σ and 2σ preferred regions of the mixing parameters are indicated (green and lighter green), $(\chi^2_{\rm min}+1, \chi^2_{\rm min}+4)$. We also show the region excluded at 2σ by experimental bounds (gray) and 1σ interval which would explain the CAA2 and the m_W-mass shift (blue band), without including other constraints. In Fig. 4b, $M_Q = 2$ TeV is assumed for the mass of the vectorlike doublet, and we indicate the experimental bounds on the left side using the conservative value $|V_{Rus}| = 0.75\times10^{-3}$ (gray region on the left side).

to the top is crucial in resolving the m_W anomaly. On the other hand, the mixing with the first generation $V_{Rud} = U^*_{RTu}U_{RBd} = -0.68(27)\times10^{-3}$ (blue band in Fig. 3a) corrects the unitarity relation. Since in the $U_{RTu} - U_{RBd}$ plane the coupling to the top is marginalized over, the plot in Fig. 3a is effectively determined by CAA1, and the m_W value does not change this evaluation. In Fig. 3c we plot the result of the fit for h_t as a function of M_Q, marginalizing over the other parameters and imposing $h_u, h_d \le 1$. It follows that we can have $h_t \lesssim 1$ for vectorlike quark masses below 3 TeV.

In the CAA2 + $\boldsymbol{m_W}$ $(\boldsymbol{h_u}, \boldsymbol{h_s}, \boldsymbol{h_t}, \boldsymbol{M_Q})$ scenario the χ^2 fit is performed using the parameters U_{RTu}, U_{RBs}, U_{RTt}, V_{Lus} and including the same observables as before [i.e., determinations of the Cabibbo angle, m_W, $\Gamma(Z\to{\rm had})$, $Q_W(p)$, and $Q_W(Cs)$]. The best-fit point is found to be

$$U_{RTu} = \pm0.031, \qquad U_{RBs} = \mp0.035,$$
$$|U_{RTt}| = 0.085, \qquad V_{Lus} = 0.22457, \tag{51}$$

where again U_{RTt} is determined for $M_Q = 2$ TeV. The improvement over the SM is slightly better in this case, $\chi^2_{\rm SM} - \chi^2_{\rm min} = 20.0$, with $\chi^2_{\rm min}/{\rm dof} = 4.7$. Also in this case there is a residual tension mostly due to CAA1 which we are not addressing in this scenario. This anomaly can be explained by a completely different source, or by another doublet with large mixing with the d quark or by vectorlike singlets mixing with the first generation. In Figs. 4a and 4b we illustrate the results of the fit on the $U_{RTu} - U_{RBs}$ and $U_{RTu} - U_{RTt}$ planes, respectively, marginalizing over the other variables. The 1σ and 2σ regions of the fit $(\chi^2_{\rm min}+1, \chi^2_{\rm min}+4)$ as well as the current and future constraints are presented with the same colors as in Fig. 3. The plot in Fig. 4a does not change with the mass of the vectorlike quark, but from the relations $U_{RTu} \approx -h_u v_w/M_Q$, $U_{RBs} \approx -h_d v_w/M_Q$ we arrive as before at the upper bound of $M_Q \lesssim 6$ TeV when $h_u, h_s \lesssim 1$.

We also perform the stand-alone fit of CAA2 and m_W and exhibit the 1σ region in the plots (blue bands). The mixing with up and strange quarks obtained from this fit $V_{Rus} = U^*_{RTu}U_{RBs} \approx h^*_u h_s v_w^2/M_Q^2 = -1.13(38)\times10^{-3}$ (see Fig. 4a) induces the difference between the vector coupling from semileptonic $K\ell3$ decays and the axial-vector coupling from leptonic $K\mu2/\pi\mu2$ decays. Furthermore, we infer from 4b that the mixing with the top is also in this case the main source of the modification of the T parameter and m_W prediction. By plotting this coupling against the mass of the vectorlike doublet we obtain a similar plot as in Fig. 3c; therefore it is omitted.

Summarizing, one vectorlike doublet with mass of few TeV can explain the new measurement of m_W together with either the tension between the determinations from kaon decays or the deficit in the CKM unitarity. At high energies a unique prediction of the model is the enhancement of the top decay rate to the up quark and a Z or Higgs boson.

Although the prospects in HL-LHC are only slightly improving over the current bound (and hence are not shown in the plot), it is noteworthy that most of the parameter space of the model can be explored in the FCC-hh. As a matter of fact, according to the optimistic scenario with respect to the systematics, the whole 1σ region will be covered.

B. Vectorlike quark singlets

The unitarity anomaly in the first row of CKM is also explained by vectorlike quark $SU(2)_L$ singlets, i.e., the down-type $D_{L,R}$ with SM quantum numbers $(\mathbf{3},\mathbf{1})_{-1/3}$ or the up-type $U_{L,R}$ with $(\mathbf{3},\mathbf{1})_{2/3}$. The Yukawa couplings and mass terms for the up-type singlet are

$$\mathcal{L}_{\rm Y} \supset +Y_{uij}\bar{q}_{Li}\tilde{\varphi}u_{Rj} + z_i\tilde{\varphi}\bar{q}_{Li}U_R + M_U\bar{U}_L U_R + \text{H.c.}, \quad (52)$$

while for the down-type

$$\mathcal{L}_{\rm Y} \supset +Y_{dij}\bar{q}_{Li}\varphi d_{Rj} + w_i\varphi\bar{q}_{Li}D_R + M_D\bar{D}_L D_R + \text{H.c.}, \quad (53)$$

where $i,\ j = 1,\ 2,\ 3$ are the family indexes. The mass matrices can be diagonalized via biunitary transformations $U_{UL}^\dagger\tilde{\mathcal{M}}_u U_{UR} = \text{diag}(y_u v_w, y_c v_w, y_t v_w, M_{U'})$ and analogously for down type. We can choose the basis in which the Yukawa submatrix of up-type quarks $\hat{U}_{UL}^\dagger Y_u \hat{U}_{UR}$ is diagonal and define $(z_u, z_c, z_t)^T = \hat{U}_{UL}^\dagger(z_1, z_2, z_3)^T$ (and vice versa for down-type).

Using the results in Ref. [106] (also reported in Appendix B), the shift of the oblique parameters in the presence of the up-type singlet is given by

$$T_U \approx \frac{3}{16\pi}\frac{v_w^2}{s^2c^2m_Z^2}\frac{v_w^2}{M_U^2}\left[y_t^2|z_t|^2\left(\ln\frac{M_U^2}{m_t^2}-1\right)\right. \\ \left. +\frac{1}{2}(|z_u|^2+|z_c|^2+|z_t|^2)^2\right], \quad (54)$$

$$S_U \approx \frac{3}{2\pi}\left[\sum_{\alpha=u,c,t}\frac{v_w^2}{M_U^2}|z_\alpha|^2\left(-\frac{5}{9}+\frac{1}{3}\ln\frac{M_U^2}{m_\alpha^2}\right)\right. \\ \left. -\frac{1}{9}\sum_{\beta=d,s,b}|V_{U\beta}|^2\ln\frac{M_U^2}{m_\beta^2}\right], \quad (55)$$

$$U_U \approx \frac{3}{2\pi}\left[\sum_{\alpha=u,c,t}\frac{v_w^2}{M_U^2}|z_\alpha|^2\left(\frac{5}{9}-\frac{1}{3}\ln\frac{M_U^2}{m_\alpha^2}\right)\right. \\ \left. +\frac{1}{3}\sum_{\beta=d,s,b}|V_{U\beta}|^2\ln\frac{M_U^2}{m_\beta^2}\right], \quad (56)$$

while for the down-type

$$T_D \approx \frac{3}{16\pi}\frac{v_w^2}{s^2c^2m_Z^2}\frac{v_w^2}{M_D^2}\left[-y_t^2|w_b|^2\ln\frac{M_D^2}{m_t^2}\right. \\ \left. +\frac{1}{2}(|w_d|^2+|w_s|^2+|w_b|^2)^2\right], \quad (57)$$

$$S_D \approx \frac{3}{2\pi}\left[\sum_{\beta=d,s,b}\frac{v_w^2}{M_D^2}|w_\beta|^2\left(-\frac{5}{9}+\frac{1}{3}\ln\frac{M_D^2}{m_\beta^2}\right)\right. \\ \left. +\frac{1}{9}\sum_{\alpha=u,c,t}|V_{\alpha D}|^2\ln\frac{M_D^2}{m_\alpha^2}\right], \quad (58)$$

$$U_D \approx \frac{3}{2\pi}\left[\sum_{\beta=d,s,b}\frac{v_w^2}{M_D^2}|w_\beta|^2\left(\frac{5}{9}-\frac{1}{3}\ln\frac{M_D^2}{m_\beta^2}\right)\right. \\ \left. +\frac{1}{3}\sum_{\alpha=u,c,t}|V_{\alpha D}|^2\ln\frac{M_D^2}{m_\alpha^2}\right], \quad (59)$$

where $V_{U\beta}$, $V_{\alpha D}$ are the extra elements of the mixing matrices in left-handed charged-current interactions with W boson in the mass basis. Then, the m_W prediction is modified according to Eq. (26). Also in this case the main contribution comes from the T parameter. To address the CAA1 with the down-type singlet, a mixing with the first generation $|w_d|v_w/M_D \sim 0.04$ is needed [2,8]. However, in the allowed range of values for the Yukawa couplings, a positive m_W shift cannot be generated. In fact, flavor-changing kaon processes impose strict constraints on w_s, while the mixing $w_b v_w/M_D$ is bounded by the Z decay rate into hadrons, which gives $|w_b|v_w/M_D < 0.03$. In addition, the constraints from flavor-changing B-meson decays and neutral B-meson systems imply $|w_b^* w_d|v_w^2/M_D^2 < 2\times 10^{-4}$. These constraints, combined with limits from the D^0 meson system, set an upper limit on the mass of the down-type vectorlike quark at around 1.5 TeV [8]. Thus, we obtain an approximate upper limit of $|w_b| \lesssim 0.05$, which cannot induce the positive contribution to m_W.

In the case of the up-type singlet, the Lagrangian for the charged-current interaction reads as

$$\mathcal{L}_{\rm cc} = -\frac{g}{\sqrt{2}}W_\mu^+(\bar{u}_L\ \ \bar{c}_L\ \ \bar{t}_L\ \ \overline{U'}_L)\gamma^\mu V\begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \text{H.c.}, \quad (60)$$

where V is a 4×3 matrix and the upper left 3×3 submatrix corresponds to the CKM matrix in the limit of decoupled new physics. The elements of the fourth row are

$$V_{U'd} \approx \frac{z_u^* v_w}{M_U},$$
$$V_{U's} \approx \frac{z_u^* v_w}{M_U} V_{us} + \frac{z_c^* v_w}{M_U} V_{cs} + \frac{z_t^* v_w}{M_U} V_{ts},$$
$$V_{U'b} \approx \frac{z_t^* v_w}{M_U}. \tag{61}$$

For the first row it holds that

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |U_{LUu}|^2 \approx 1 - \frac{|h_u|^2 v_w^2}{M_U^2}, \tag{62}$$

and thus the anomaly can be resolved if the mixing with the first family is $|U_{LUu}|^2 = 1.68(55) \times 10^{-3}$, that is $|U_{LUu}| \approx 0.041$.

However, the mixing of SM quarks with the vectorlike quarks induces nonstandard couplings of the Z boson with the left-handed up quarks because of different weak isospin couplings. These couplings are constrained by limits on FCNCs ($\Delta F = 2$ transitions, B and D meson decays) as well as flavor-conserving processes, e.g., Z-boson decay. We list the most important constraints in Appendix A, and a more detailed analysis can be found in Refs. [8,9,20,27]. As a result, although a combination of couplings and relative phases can be found to explain the anomalies separately, a simultaneous explanation can be in strong contrast to experimental bounds.

In fact, the coupling with the second generation z_c is strongly constrained by a neutral D-mesons system. By requiring that a new physics contribution onto the mass splitting cannot exceed the experimental value $\Delta M_D = 6.56(76) \times 10^{-15}$ GeV [40] at 2σ CL, one gets the limit

$$|U_{LUu}^* U_{LUc}| \approx |z_u^* z_c| v_w^2 / M_U^2$$
$$< 1.3 \times 10^{-4} [1 + (M_U / 3.1\ \text{TeV})^2]^{-1/2}. \tag{63}$$

However, the coupling with the charm cannot vanish, since it will turn out to be necessary to interfere with other contributions in the $K^0 - \bar{K}^0$ system [see Eq. (A20)] to satisfy constraints on CP violation (see also Ref. [20]). Taking into account the bound on z_c from the D-mesons mass difference, $B^0 - \bar{B}^0$ mixing gives a bound which is approximately

$$(V_{U'b}^* V_{U'd})^2 \frac{1}{4} \frac{M_U^2}{m_w^2} \lesssim (V_{tb}^* V_{td}) S_0(m_t^2/m_W^2) \Delta_{B_d}, \tag{64}$$

where $S_0(m_t^2/m_W^2)$ is the Inami-Lim function [see Eq. (A14)] [121]. Requiring that the fraction of new physics contribution to the mixing mass is at most 30% of the SM contribution ($\Delta_{B_d} = 0.3$) [122] one gets

$$|V_{U'b}^* V_{U'd}| \approx |z_u^* z_t| v_w^2 / M_U^2 \lesssim 1.1 \times 10^{-3} \left[\frac{1\ \text{TeV}}{M_U}\right]. \tag{65}$$

When $|z_u| v_w / M_U = 0.04$ is set for CKM unitarity, the constraints imposed by $B^0 - \bar{B}^0$ system and CP violation in $K^0 - \bar{K}^0$ system cannot be simultaneously satisfied by any choice of value and phase of z_t, unless z_c is turned on, without violating the constraint from $D^0 - \bar{D}^0$. Assuming the relative phase of z_u and z_c is chosen to compensate the other contributions in ϵ_K, the limit in Eq. (65) translates into the bound $|z_t| \lesssim 0.15$. The necessary m_W enhancement would require instead a coupling as large as $z_t = 0.67(10)$ for $M_U = 1$ TeV ($z_t = 1.08(15)$ for $M_U = 2$ TeV). As a consequence, it seems hard to justify both the apparent CKM unitarity deficit and the m_W mass shift.

In the following, we analyze the $z_u - z_t$ parameter space in more detail and illustrate the result in Fig. 5. Limits from LHC exclude up-type vectorlike singlets coupling to the top for masses lower than ~1.3 TeV [120], so we assume a vectorlike quark mass of $M_U = 1.3$ TeV (left plot) and $M_U = 2$ TeV (right plot). We perform a χ^2 fit of the Cabibbo angle determinations in Eqs. (2), (3), and (6), and the m_W mass (25). We illustrate the 1σ and 2σ intervals (blue and lighter blue regions) of the parameters z_u and z_t obtained from the fit ($\chi_{\min} + 1$, $\chi_{\min} + 4$). We indicate the constraints from neutral-meson systems (see Appendix A for details), i.e., $K^0 - \bar{K}^0$: ϵ_K (red) and Δm_K (cyan), $B_d^0 - \bar{B}_d^0$ (purple), $B_s^0 - \bar{B}_s^0$ (orange), $D^0 - \bar{D}^0$ (magenta), Z-boson decay into hadrons (yellow), and $t \to Zu$ branching ratio (brown). We fix the other parameters $|z_c|$, $\mathrm{Arg}(z_u^* z_c)$, $\mathrm{Arg}(z_u^* z_t)$ at convenient values. In particular, the phase of z_t is selected to reduce the contribution in the B_d^0-mesons system, and the phase and value of the coupling z_c is set in order to compensate for the CP-violating effect in $K^0 - \bar{K}^0$.

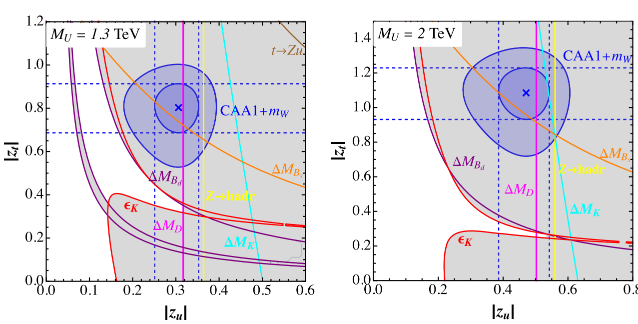

FIG. 5. Parameter space in the scenario with one up-type vectorlike singlet for the couplings with up and top quarks, assuming $M = 1.3$ TeV (left) and $M = 2$ TeV (right) for the mass of the vectorlike singlet. The 1σ and 2σ confidence intervals (blue regions) of the parameters z_u and z_t obtained from the fit of Cabibbo angle determinations and m_W are shown ($\chi_{\min} + 1$, $\chi_{\min} + 4$). We set $U_{LUc} \approx 0.003$, $\mathrm{Arg}(h_u^* h_c) = -3.0$, and $\mathrm{Arg}(h_u^* h_t) = 2.8$. We see that the region of interest is excluded at 2σ by experimental bounds (gray).

As can be seen by the projections, allowed regions can be found to solve CAA1 or m_W. However, the limit set by $B_d^0 - \bar{B}_d^0$ is robust against variations of the other parameters and still excludes the preferred region for the combined explanation. Moreover, constraints become more stringent with increasing mass, and in any case the mass of the up-type vectorlike singlet cannot exceed about ~2.5 TeV [8] and still accommodate CAA1.

In any case, vectorlike weak singlets cannot be the source of the CAA2. It is possible to picture a scenario presenting the vectorlike doublet together with the up-type and down-type weak singlets, assembling a sort of "complete vectorlike family" in order to find a complete explanation of all the anomalies. For instance, the vector-like doublet could couple with u, s, and t quarks, generating the CAA2 and a shift of the W-boson mass. The vectorlike singlets coupling with the first generation would be the cause of the CKM unitarity deficit. In this case, the couplings of the extra doublet with d and c quarks would be further suppressed, due to the presence of additional mixed left-right contributions in neutral-meson systems. The prediction of the Z-boson decay rate would be $\sim 2\sigma$ lower than the experimental determination. Then, if such scenario is at the origin of the anomalies, the presence of the vectorlike quarks would be detected by experiments of increased precision in Z-boson physics.

C. Other mediators

In the following we list the other possible mediators that have been discussed in the context of the Cabibbo angle anomalies and comment on their compatibility with the CDF-II result.

(i) In the case of a *singly-charged scalar singlet* [15,16,18], the solution to CAA1 is based solely on the modification of G_F, which is in turn a consequence of the tree-level contribution to the muon decay rate. As already discussed in the Introduction, this effect is correlated with a negative shift in the W-boson mass [2,17,23] and is thus disfavored.

(ii) Regarding *vectorlike leptons* [7,11,14], the new physics effects generated by their presence modify the W-boson couplings and thus the β decays directly. However, also in this case, G_F is affected and worsens the tension in m_W.

(iii) The *vector boson singlet* [123] is a mediator that can in principle modify the m_Z via mixing with the Z boson and that could translate into a positive shift in the prediction of m_W [124,125], but it is found to be incompatible with CAA1 as a one-particle solution (at least when the flavor-diagonal couplings are nonzero).[7]

(iv) The *vector boson* $SU(2)_L$ *triplet* [10] can alleviate the tension in CAA1 by modifying the muon decay rate at tree level, but as previously for the scalar singlet, the W-boson mass is decreased [10].

(v) The *vector boson* $SU(2)_R$ *triplet* (W_R) [3,126] is the only field of this list that can generate right-handed currents necessary to resolve CAA2 (and possibly also the CDF-II anomaly). Nevertheless, the gauge boson needs to be relatively light, a scenario which is excluded in the minimal left-right symmetric model.

(vi) The *leptoquark* [127,128] fields can induce tree-level contributions to β decays. However, they are excluded not only by flavor-changing low-energy bounds but also by direct searches at colliders.

[7]However, in the context of horizontal gauge symmetries, in Ref. [2] the shift in the muon decay constant is induced by flavor-changing gauge bosons related to a family symmetry in the left-handed lepton sector.

V. CONCLUSION

In this paper we have demonstrated that a vectorlike quark doublet provides a simple extension to the Standard Model to accommodate the Cabibbo angle anomalies and the measurement of the W-boson mass by CDF-II. In fact, tree-level mixing with light quarks induces right-handed charged currents, which can be the reason behind the former, while the latter is due to the mixing with the top quark, which can produce a sizeable loop-level contribution to the oblique T parameter. The scenario is consistent with the absence of deviations from the Standard Model so far observed in other low- and high-p_T observables. In particular, one generation of vectorlike doublet can account for either the violation of CKM unitarity or the reconciliation of the $K\ell 3$ and $K\mu 2/\pi\mu 2$ determinations of $|V_{us}|$, together with the m_W measurement. No fine-tuned cancellations between diagrams are required.

The first scenario is realized when the vectorlike doublet predominantly couples to up, down, and top quarks. The unitarity deficit can be induced by a right-handed mixing between up and down quarks $h_u h_d v_w^2/M_Q^2 \sim -0.8 \times 10^{-3}$. Then, a flavor texture emerges for the Yukawa couplings with the vectorlike doublet. Couplings with the first generation are of order $h_u \sim h_d \gtrsim 0.3$. For the other couplings, it should be $h_s \lesssim \mathcal{O}(10^{-3})$, $h_c \lesssim \mathcal{O}(10^{-2})$, and $h_b \lesssim \mathcal{O}(10^{-1})$ in order to comply with the stringent constraints from flavor-changing phenomena. Finally, mixing with the top quark is less constrained and $h_t \approx 1$ suffices to generate the m_W anomaly for a mass of $M_Q = 2$ TeV. We notice that couplings of that size between the Higgs and TeV scale fermions can considerably reduce the instability scale [129] and potentially provide an argument for the dynamical selection of the electroweak scale [130].

In the second scenario, a vectorlike doublet couples to the up, strange, and top quarks. The right-handed mixing $h_u h_s v_w^2/M_Q^2 \sim -1.3 \times 10^{-3}$ suitably modifies the vector and axial-vector couplings, and thus, the determinations obtained from semileptonic and leptonic decays, respectively. A similar texture for the Yukawa couplings is required for this scenario, with $h_u \sim h_s \approx 0.3$, $h_t \approx 1$, and similar suppressions for the other couplings as in the previous case. In both cases, the mass of the vectorlike quark should be in the few TeV range, namely, for $|h_t| \lesssim 1$ it must be $M_Q \lesssim 3$ TeV, potentially making it accessible by direct searches in future colliders.

All three discrepancies can be addressed if the fermion sector would be extended by two generations of vectorlike doublets that each couples to the up, the top, and either the down or the strange. Alternatively, one can envision a nonminimal scenario in which the first Cabibbo angle anomaly is resolved by some other mechanism and the second by a vectorlike doublet coupling predominantly to the second and the third generation. Additionally, the vectorlike doublet would not only induce a positive shift in m_W, but also compensate for the adverse effect of a modification of the Fermi constant possibly induced by the other mechanism. This is a unique feature of the model featuring the vectorlike doublet not shared by other one-particle mediator models.

In order to settle the CKM unitary puzzle, improved experimental inputs for neutron decay time [131,132], g_A parameter [133–135], and pion β decay [136,137] are expected in the foreseeable future. The leading hadronic uncertainties at both superallowed $0^+ - 0^+$ and neutron decays can be reduced by lattice QCD calculations, which improve the estimation of γW box diagrams (and are executable with the state of the art techniques) [138]. Furthermore, $K_{\ell 3}$ decays can be measured at experiments such as LHCb [139]. For the W-boson mass anomaly on the other hand, a confirmation of the CDF-II result from the LHC experiments would be of utmost importance.

Finally, disentangling the new physics contributions specifically due to the vectorlike quark can become feasible both at the high-precision as well as the high-intensity frontiers. For example, the measurement of $K_{\mu 3}/K_{\mu 2}$ at NA62 proposed in Ref. [72] can distinguish the presence of right-handed charged currents involving strange quarks predicted in this model. Another probe at low energies can be offered by the P2 [140] and MOLLER [141] experiments, which will perform precise measurements of the Weinberg angle and that would imply improvement of the bounds from atomic parity violation [142]. On the other hand, future colliders can potentially offer the possibility of testing the model at high energies. The smoking-gun signature of the model is the channel $t \to uZ$, where FCC-hh is expected to provide sufficient sensitivity. The LEP bounds on Z-boson couplings can also be significantly improved at future e^+e^- colliders such as FCC-ee [143].

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APPENDIX A: BOUNDS FROM FLAVOR-CHANGING NEUTRAL CURRENT PROCESSES

We summarize here and update some of the most stringent constraints from flavor-changing phenomena previously analyzed within this framework in Ref. [8].

1. $K^+ \to \pi^+ \nu\bar{\nu}$

The decay $K^+ \to \pi^+ \nu\bar{\nu}$ is identified as one of the golden modes, since long-distance contributions are negligibly small. The effective interaction originates from Z-penguin and box diagrams, and it is given by [144]

$$\begin{aligned}\mathcal{L}(K \to \pi\nu\bar{\nu})_{\rm SM} &= -\frac{4G_F}{\sqrt{2}}\frac{\alpha(M_Z)}{2\pi\sin^2\theta_{\rm W}}\sum_{\ell=e,\mu,\tau}[V_{cs}^* V_{cd} X^\ell(x_c) \\ &\quad + V_{ts}^* V_{td} X^\ell(x_t)](\overline{s_L}\gamma^\mu d_L)(\overline{\nu_{\ell L}}\gamma_\mu \nu_{\ell L}) \\ &= -\frac{4G_F}{\sqrt{2}}\mathcal{F}_K(\overline{s_L}\gamma^\mu d_L)\sum_{\ell=e,\mu,\tau}(\overline{\nu_{\ell L}}\gamma_\mu \nu_{\ell L}).\end{aligned} \tag{A1}$$

$X(x_a)$ are the relevant Inami-Lim function including QCD and electroweak corrections, with $x_a = m_a^2/M_W^2$, $a = c$, t. The experimental measurement for the branching ratio is [40]

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\rm exp} = 1.14^{+0.40}_{-0.33} \times 10^{-10}, \tag{A2}$$

which is compatible with the SM prediction $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\rm SM} \approx 0.81 \times 10^{-10}$. With increasing experimental precision, any deviation in this channel would point toward new physics. The mixings with the vectorlike doublet induces at tree level the operator

$$\begin{aligned}&\mathcal{L}(K \to \pi\nu\bar{\nu})_{\rm NP} \\ &\quad = \frac{4G_F}{\sqrt{2}}\frac{1}{2}(U_{RBs}^* U_{RBd})(\bar{s}_R\gamma^\mu d_R)\sum_{e,\mu,\tau}(\bar{\nu}_{\ell \rm L}\gamma_\mu \nu_{\ell \rm L}),\end{aligned} \tag{A3}$$

which generates the total branching ratio

$$\begin{aligned}&\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu}) \\ &\quad \approx \mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\rm SM}\left|\frac{-\frac{1}{2}U_{RBs}^* U_{RBd}}{\mathcal{F}_K} + 1\right|^2,\end{aligned} \tag{A4}$$

where $\mathcal{F}_K \approx (-3.7 + i1.1) \times 10^{-6}$ is defined as in Ref. [8]. We use the experimental limit in Eq. (A2) at 2σ to obtain an upper bound

$$\left|-\frac{\frac{1}{2}U^*_{RBs}U_{RBd}}{\mathcal{F}_K}+1\right|<1.5, \tag{A5}$$

which, depending on the phase $\mathrm{Arg}(U^*_{RDs}U_{RDd})$, gives a limit on the couplings with the vectorlike doublet:

$$|U^*_{RBs}U_{RBd}|<(0.4\text{–}2.0)\times 10^{-5}. \tag{A6}$$

2. $K_\mathrm{L}\to\mu^+\mu^-$

The rare decay $K_\mathrm{L}\to\mu^+\mu^-$ is a CP-conserving decay, and its short-distance contribution is generated by Z-mediated penguin and box diagrams. The effective Lagrangian in the SM can be written as [144]

$$\begin{aligned}\mathcal{L}(K\to\mu^+\mu^-)_{\mathrm{SM,SD}} &= \frac{G_F}{\sqrt{2}}\frac{\alpha(M_Z)}{2\pi\sin^2\theta_\mathrm{W}}(V^*_{cs}V_{cd}Y(x_c)+V^*_{ts}V_{td}Y(x_t))\cdot\\ &\quad\cdot(\bar{s}\gamma^\mu\gamma_5 d)(\bar{\mu}\gamma_\mu\gamma_5\mu)+\mathrm{H.c.}\\ &=\frac{G_F}{\sqrt{2}}\mathcal{F}_{L2}(\bar{s}\gamma^\mu\gamma_5 d)(\bar{\mu}\gamma_\mu\gamma_5\mu)+\mathrm{H.c.},\end{aligned} \tag{A7}$$

where $Y(x_a)$, $x_a=m_a^2/M_W^2$, are the relevant Inami-Lim functions including QCD and electroweak corrections. The SM prediction for the short-distance contribution is calculated to be $\mathrm{Br}(K_\mathrm{L}\to\mu^+\mu^-)_{\mathrm{SM,SD}}\approx 0.9\times 10^{-9}$ [145]. However, this decay is dominated by a long-distance contribution from a two-photon intermediate state which almost saturates the observed rate $\mathrm{Br}(K_\mathrm{L}\to\mu^+\mu^-)_{\mathrm{exp}}=(6.84\pm 0.11)\times 10^{-9}$ [40]. An upper bound on the short-distance contribution is estimated in Ref. [146] as

$$\mathrm{Br}(K_\mathrm{L}\to\mu^+\mu^-)_{\mathrm{SD}}<2.5\times 10^{-9}. \tag{A8}$$

The vectorlike doublet induces the decay at tree level:

$$\begin{aligned}\mathcal{L}(K\to\mu^+\mu^-)_{\mathrm{NP}}=\frac{G_F}{2\sqrt{2}}U^*_{RBs}U_{RBd}(\bar{s}\gamma_\mu\gamma_5 d)(\bar{\mu}\gamma^\mu\gamma_5\mu)\\ +\mathrm{H.c.}\end{aligned} \tag{A9}$$

Then we can define the branching ratio given by the amplitude of the short-distance contribution as

$$\begin{aligned}&\mathrm{Br}(K_\mathrm{L}\to\mu^+\mu^-)_{\mathrm{SD}}\\ &=\mathrm{Br}(K_\mathrm{L}\to\mu^+\mu^-)_{\mathrm{SM,SD}}\left[1+\frac{\mathrm{Re}(U^*_{RBs}U_{RBd})}{2\mathrm{Re}(\mathcal{F}_{L2})}\right]^2,\end{aligned} \tag{A10}$$

where $\mathcal{F}_{L2}\approx(-2.1+i0.74)\times 10^{-6}$ is defined as in Ref. [8]. By using the upper bound in Eq. (A8) on the branching ratio we get

$$\left|1+\frac{\mathrm{Re}(U^*_{RBs}U_{RBd})}{2\mathrm{Re}(\mathcal{F}_{L2})}\right|<1.7, \tag{A11}$$

which results in the approximate limit

$$-0.3\times 10^{-5}<\mathrm{Re}(U^*_{RBs}U_{RBd})<1.1\times 10^{-5}. \tag{A12}$$

3. $K^0-\bar{K}^0$ mixing

In the SM the short-distance contribution to the transition $K^0\leftrightarrow\bar{K}^0$ arises from weak box diagrams. The two relevant observables describing the mixing are the mass splitting $\Delta M_K=m_{K_L}-m_{K_S}$ and the CP-violating parameter ϵ_K. They are primarily described by the off diagonal term M_{12} of the mass matrix of neutral kaons, $M^K_{12}=-\langle K^0|\mathcal{L}_{\Delta S=2}|\bar{K}^0\rangle/(2m_{K^0})$, which in the SM is [147]

$$\begin{aligned}M^K_{12,\mathrm{SM}}=m_{K^0}f_K^2\hat{B}_K\frac{G_F^2m_W^2}{12\pi^2}(&\eta_1(V_{cs}V^*_{cd})^2S_0(x_c)\\ &+\eta_2(V_{ts}V^*_{td})^2S_0(x_t)\\ &+2\eta_3(V_{cs}V^*_{cd})(V_{ts}V^*_{td})S_0(x_c,x_t)),\end{aligned} \tag{A13}$$

where $x_a=m_a^2/m_W^2$, f_K is the kaon decay constant, which can be estimated in lattice QCD to be $f_K=155.7(0.7)$ MeV [31]; $m_{K^0}=497.611\pm 0.013$ MeV is the neutral kaon mass; and the factors $\eta_1=1.87\pm 0.76$ [148], $\eta_2=0.5765\pm 0.0065$ [149], and $\eta_3=0.496\pm 0.047$ [150] describe short-distance QCD effects. The factor $\hat{B}_K$ is the correction to the vacuum insertion approximation, which is calculated in lattice QCD $\hat{B}_K=0.7625(97)$ [31]. The Inami-Lim functions are [121]

$$S_0(x)=x\left(\frac{4-11x+x^2}{4(1-x)^2}-\frac{3x^2\ln x}{2(1-x)^3}\right), \tag{A14}$$

$$\begin{aligned}S_0(x_j,x_k)=x_jx_k\Bigg[&\left(\frac{1}{4}-\frac{3}{2(x_j-1)}-\frac{3}{4(x_j-1)^2}\right)\frac{\log x_j}{x_j-x_k}\\ &+(x_j\leftrightarrow x_k)-\frac{3}{4(x_j-1)(x_k-1)}\Bigg].\end{aligned} \tag{A15}$$

The modulus and the imaginary part of the mixing mass M^K_{12} describe short-distance contributions in the mass splitting and CP-violation in $\bar{K}^0\leftrightarrow K^0$ transitions [144]

$$\Delta M_K\approx 2|M^K_{12}|+\Delta m_{K,\mathrm{LD}},\qquad |\epsilon_K|\approx\frac{|\mathrm{Im}M^K_{12}|}{\sqrt{2}\Delta M_K}, \tag{A16}$$

(using the phase choice $CP|K^0\rangle=-|\bar{K}^0\rangle$, in the standard parametrization of V_{CKM}). $\Delta m_{K,\mathrm{LD}}$ is the long-distance contribution which is difficult to evaluate [151,152]. However the short-distance contribution gives the dominant contribution to the experimental determinations [40]

$$\Delta M_{K,\exp} = (3.484 \pm 0.006) \times 10^{-15}\ \text{GeV},$$
$$|\epsilon_K|_{\exp} = (2.228 \pm 0.011) \times 10^{-3}. \tag{A17}$$

The new contribution from the vectorlike doublet to the mixing mass term of neutral-meson systems includes relevant right-handed currents at both the tree and loop level but also chirality-mixing enhanced contributions at the loop level. It is given by

$$M^K_{12,\mathrm{NP}} \approx \frac{1}{3} m_{K^0} f_K^2 0.43 \Bigg\{ \frac{G_F}{\sqrt{2}} (U^*_{RBd} U_{RBs})^2 + \frac{G_F^2}{4\pi^2} \Bigg[\frac{1}{2} M_Q^2 (U^*_{RBd} U_{RBs})^2 + -3.1 \frac{m^2_{K^0}}{(m_d + m_s)^2} (U^*_{RBd} U_{RBs})(V^*_{Ltd} V_{Lts}) \times m_W^2 f(x_Q, x_t) \Bigg] \Bigg\}, \tag{A18}$$

where $f(x_Q, x_t) \approx x_t \ln(x_Q)/4$, and we used the numerical coefficients calculated in Refs. [153,154]. Bounds on the new physics contribution can be estimated as $|M^K_{12,\mathrm{NP}}| < |M^K_{12,\mathrm{SM}}|\Delta_K$, $|\mathrm{Im} M^K_{12,NP}| < |\mathrm{Im} M^K_{12,\mathrm{SM}}|\Delta_{\epsilon_K}$. Setting $\Delta_K = 1$ and using the results in Ref. [122] at 95% C.L., (which approximately corresponds to $\Delta_{\epsilon_K} = 0.3$) we obtain

$$|U^*_{RBs} U_{RBd}| < 6 \times 10^{-7}\text{–}4 \times 10^{-4}, \tag{A19}$$

depending on the relative phase of the couplings and on the mass of the heavy doublet. In fact, the limit in (A19) is computed for $M_Q \approx 2$ TeV, but the limit on the mixing elements in (13) $|U^*_{RDs} U_{RDd}|$ becomes stronger with increasing mass M_Q [8].

In the scenario with extra up-type quark, box diagrams with U' quark running in the loop give the contribution

$$M^K_{12,\mathrm{NP}} = \frac{1}{3} m_{K^0} f_K^2 \frac{G_F^2 m_W^2}{4\pi^2} 0.43((V^*_{U's} V_{U'd})^2 S_0(x_{U'}) + 2(V^*_{U's} V_{U'd})(V^*_{cs} V_{cd}) S_0(x_c, x_{U'}) + 2(V^*_{U's} V_{U'd})(V^*_{ts} V_{td}) S_0(x_t, x_{U'})), \tag{A20}$$

with the same definitions as before.

4. Neutral B mesons

In a neutral B-mesons system long-distance contributions are estimated to be small. The dominant short-distance contribution to the $B^0_d - \bar{B}^0_d$ mixing in the SM is given by

$$\Delta M_{B_d,\mathrm{SM}} = 2|M^B_{12,\mathrm{SM}}| = m_{B_d} f^2_{B_d} B_{B_d} \eta_B \frac{G_F^2 m_W^2}{6\pi^2} |(V_{tb} V^*_{td})^2| S_0(x_t), \tag{A21}$$

where $M^{B_d}_{12,\mathrm{SM}} = -\langle B^0_d | \mathcal{L}_{\Delta B_d = 2} | \bar{B}^0_d \rangle / (2 m_{B^0_d}) \eta_B$ is the QCD factor $\eta_B = 0.551$ [144], and B_{B_d} is the correction factor to the vacuum-insertion approximation. Analogously, the expression for the $B^0_s - \bar{B}^0_s$ system can be obtained by substituting $d \to s$. Lattice QCD calculations yield $f_{B_d} \sqrt{\hat{B}_{B_d}} = 210.6(5.5)$ MeV and $f_{B_s} \sqrt{\hat{B}_{B_s}} = 256.1(5.7)$ MeV [31]. The experimental result is [40]

$$\Delta M_{B_d,\exp} = (3.334 \pm 0.013) \times 10^{-13}\ \text{GeV},$$
$$\Delta M_{B_s,\exp} = (1.1693 \pm 0.0004) \times 10^{-11}\ \text{GeV}. \tag{A22}$$

The additional contribution due to the presence of the vectorlike doublet is

$$M^{B_d}_{12,\mathrm{NP}} \approx \frac{1}{3} m_{B_d} f^2_{B_d} 0.80 \Bigg\{ \frac{G_F}{\sqrt{2}} (U^*_{RBd} U_{RBb})^2 + \frac{G_F^2}{4\pi^2} \Bigg[\frac{1}{2} M_Q^2 (U^*_{RBd} U_{RBb})^2 - 3.37 (U^*_{RBd} U_{RBb})(V^*_{Ltd} V_{Ltb}) m_W^2 f(x_Q, x_t) \Bigg] \Bigg\} \tag{A23}$$

where $f(x_Q, x_t) \approx x_t \ln(x_Q)/4$, $f_{B_d} = 190.0(1.3)$ MeV [31] and we used the numerical results in Refs. [154,155], and similarly for B_s,

$$M^{B_s}_{12,\mathrm{NP}} \approx \frac{1}{3} m_{B_s} f^2_{B_s} 0.79 \Bigg\{ \frac{G_F}{\sqrt{2}} (U^*_{RBs} U_{RBb})^2 + \frac{G_F^2}{4\pi^2} \Bigg[\frac{1}{2} M_Q^2 (U^*_{RBs} U_{RBb})^2 + - 3.14 (U^*_{RBs} U_{RBb})(V^*_{Lts} V_{Ltb}) m_W^2 f(x_Q, x_t) \Bigg] \Bigg\} \tag{A24}$$

with $f_{B_s} = 230.3(1.3)$ MeV [31] and using the results in Refs. [154,155]. We can use the constraints obtained in Ref. [122] at 95% C.L. in order to limit the contribution of new physics. These bounds approximately yield a constraint $\Delta M^{\mathrm{new}}_{B_{d(s)}} < \Delta M^{\mathrm{SM}}_{B_{d(s)}} \Delta_{B_{d(s)}}$ with $\Delta_{B_d} = 0.3$, $\Delta_{B_s} = 0.2$. For $M_q \approx 2$ TeV we obtain $|U^*_{RBd} U_{RBb}| \lesssim (1.6\text{–}3.9) \times 10^{-4}$ and $|U^*_{RBs} U_{RBb}| \lesssim (0.6\text{–}1.6) \times 10^{-3}$, which however is stronger for $M > 2$ TeV.

Also flavor-changing B decays give a limit on the product $|U^*_{RBd(s)} U_{RBb}|$. In particular, the most constraining processes are the decays $B^0_{d(s)} \to \mu^+\mu^-$, for which the experimental upper limits are $\mathrm{Br}(B_d^0 \to \mu^+\mu^-)_{\rm exp} < 2.1 \times 10^{-10}$ at 95% C.L. [156] and $\mathrm{Br}(B_s^0 \to \mu^+\mu^-)_{\rm exp} = (3.01 \pm 0.35) \times 10^{-9}$ [40]. We obtain the following bounds $|U^*_{RBd} U_{RBb}| < (0.4\text{–}2.0) \times 10^{-4}$, $|U^*_{RBs} U_{RBb}| < 8 \times 10^{-7}\text{–}8.3 \times 10^{-4}$, with the range determined by the phase.

By combining the results above, we can get the approximate bounds

$$|U^*_{RBd} U_{RBb}| < (0.4\text{–}1.7) \times 10^{-4},$$
$$|U^*_{RBs} U_{RBb}| < 8 \times 10^{-7}\text{–}6.4 \times 10^{-4}. \tag{A25}$$

In the scenario with vectorlike up-type quark, the extra contribution is

$$M_{12,\rm NP}^{B_s} \approx \frac{1}{3} m_{B_d} f_{B_d}^2 \frac{G_F^2 m_W^2}{4\pi^2} 0.80((V^*_{U'b} V_{U'd})^2 S_0(x_{U'}) + 2(V^*_{U'b} V_{U'd})(V^*_{tb} V_{td}) S_0(x_t, x_{U'})), \tag{A26}$$

and analogously for $d \to s$.

5. D^0-$\bar{D}^0$ mixing

The measured value of the mass difference in the $D^0 - \bar{D}^0$ system is [40]

$$\Delta M_{D,\rm exp} = (6.6 \pm 0.8) \times 10^{-15}\ \mathrm{GeV}. \tag{A27}$$

The short-distance contribution in the SM from box and penguin diagrams is estimated to contribute in a very small amount $|M_{12}^D| \sim 10^{-17}\text{–}10^{-16}$ GeV [147,157]. Long-distance effects are expected to be large but they are difficult to compute [147,158]. Then, new physics can be the main source of the mass difference ΔM_D in the $D^0 - \bar{D}^0$ system.

The mixing mass induced by the vectorlike doublet reads as

$$M_{12,\rm NP}^D \approx \frac{1}{3} f_D^2 M_{D^0} \frac{G_F}{\sqrt{2}} (U^*_{RTu} U_{RTc})^2 \left(1 + \frac{G_F M_Q^2}{4\sqrt{2}\pi^2}\right), \tag{A28}$$

where $f_D = 212.0(0.7)$ MeV [31] and left-right contributions are subdominant. New physics can be the dominant contribution to the mass difference ΔM_D in the $D^0 - \bar{D}^0$ system. Then, we can obtain an estimate by requiring that the contribution $\Delta M_{D\rm new} = 2|M_{12,\rm NP}^D|$ does not exceed the experimental value in Eq. (A27) at 2σ,

$$|U^*_{RTu} U_{RTc}| < 1.0 \times 10^{-4} \left[\frac{f_Q(2\ \mathrm{TeV})}{f_Q(M_Q)}\right]^{1/2}, \tag{A29}$$

where, as in Ref. [8]

$$f_Q(M) \approx 1 + \left(\frac{M}{2.2\ \mathrm{TeV}}\right)^2. \tag{A30}$$

As regards the contribution of an up-type vectorlike singlet, we have

$$M_{12,\rm NP}^D \approx \frac{1}{3} f_D^2 M_{D^0} \frac{G_F}{\sqrt{2}} (U^*_{LUu} U_{LUc})^2 \left(1 + \frac{G_F M_{t'}^2}{8\sqrt{2}\pi^2}\right)$$
$$\approx \frac{1}{3} f_D^2 M_{D^0} \frac{G_F}{\sqrt{2}} (U^*_{LUu} U_{LUc})^2 \left[1 + \left(\frac{M_{t'}}{3.1\ \mathrm{TeV}}\right)^2\right]. \tag{A31}$$

APPENDIX B: OBLIQUE PARAMETERS WITH VECTORLIKE QUARKS

The expressions for the oblique-correction parameters in the presence of vectorlike quarks were computed in Ref. [106]. We report the general expressions of T, S, and U for the reader's convenience. The contribution to the parameter T reads as

$$T = \frac{3}{16\pi \sin^2\theta_w \cos^2\theta_w} \left\{ \sum_{\alpha,i} [(|V_{L\alpha i}|^2 + |V_{R\alpha i}|^2)\theta_+(x_\alpha, x_i) + 2\mathrm{Re}(V_{L\alpha i} V^*_{R\alpha i})\theta_-(x_\alpha, x_i)]. - \sum_{\beta<\alpha} [(|K_{uL\alpha\beta}|^2 + |K_{uR\alpha\beta}|^2)\theta_+(x_\alpha, x_\beta) + 2\mathrm{Re}(K_{uL\alpha\beta} K^*_{uR\alpha\beta})\theta_-(x_\alpha, x_\beta)]. - \sum_{j<i} [(|K_{dLij}|^2 + |K_{dRij}|^2)\theta_+(x_i, x_j) + 2\mathrm{Re}(K_{dLij} K^*_{dRij})\theta_-(x_i, x_j)] \right\}, \tag{B1}$$

where $x_i = m_i^2/m_Z^2$, $K_{d(u)L}$ are defined analogously to $K_{d(u)R}$ in Eq. (20). In this Appendix we adopt the same convention of Ref. [106] of using greek letters to denote up-type quarks and latin ones to denote down-type. The functions $\theta_{+/-}$ are

$$\theta_+(x_i, x_j) = x_i + x_j - \frac{2x_i x_j}{x_i - x_j} \ln\frac{x_i}{x_j},$$
$$\theta_-(x_i, x_j) = 2\sqrt{x_i x_j}\left(\frac{x_i + x_j}{x_i - x_j} \ln\frac{x_i}{x_j} - 2\right). \tag{B2}$$

These functions are symmetric under interchange of the arguments, and they are zero for $x_i = x_j$. In the limit $x_i \gg x_j$, it holds that $\theta_+(x_i, x_j) \to x_i$, $\theta_-(x_i, x_j) \to 0$. In our scenario with the vectorlike doublet, the matrices $K_{u(d)L}$ correspond to the 4×4 identity matrix, while V_L, V_R, and $K_{d(u)R}$ are described in Eqs. (16), (18), and (21), respectively. The mass splitting is given in Eq. (14). The matrices are defined in an analogous way for vectorlike singlets, in which case $V_R = 0$, $K_{u(d)R} = \mathbf{1}$.

The general result for the contribution to the parameter U is [106]

$$\begin{aligned} U = -\frac{N_c}{2\pi}\Bigg\{ & \sum_{\alpha,i}[(|V_{L\alpha i}|^2 + |V_{R\alpha i}|^2)\chi_+(x_\alpha, x_i) + 2\mathrm{Re}(V_{L\alpha i}V^*_{R\alpha i})\chi_-(x_\alpha, x_i)] \\ & - \sum_{\beta<\alpha}[(|K_{uL\alpha\beta}|^2 + |K_{uR\alpha\beta}|^2)\chi_+(x_\alpha, x_\beta) + 2\mathrm{Re}(K_{uL\alpha\beta}K^*_{uR\alpha\beta})\chi_-(x_\alpha, x_\beta)] \\ & - \sum_{j<i}[(|K_{dLij}|^2 + |K_{dRij}|^2)\chi_+(x_i, x_j) + 2\mathrm{Re}(K_{dLij}K^*_{dRij})\chi_-(x_i, x_j)]\Bigg\}, \end{aligned} \tag{B3}$$

where

$$\begin{aligned} \chi_+(x_i, x_j) &= \frac{5(x_i^2 + x_j^2) - 22x_ix_j}{9(x_i - x_j)^2} + \frac{3x_ix_j(x_i + x_j) - x_i^3 - x_j^3}{3(x_i - x_j)^3}\ln\frac{x_i}{x_j}, \\ \chi_-(x_i, x_j) &= -\sqrt{x_ix_j}\left(\frac{x_i + x_j}{6x_ix_j} - \frac{x_i + x_j}{(x_i - x_j)^2} + \frac{2x_ix_j}{(x_i - x_j)^3}\ln\frac{x_i}{x_j}\right). \end{aligned} \tag{B4}$$

Also, in this case the functions are symmetric under interchange of the variables and $\chi_\pm(x, x) = 0$. As regards the parameter S, the result is [106]

$$\begin{aligned} S = -\frac{N_c}{2\pi}\Bigg\{ & \sum_{\alpha,i}[(|V_{L\alpha i}|^2 + |V_{R\alpha i}|^2)\psi_+(x_\alpha, x_i) + 2\mathrm{Re}(V_{L\alpha i}V^*_{R\alpha i})\psi_-(x_\alpha, x_i)] \\ & - \sum_{\beta<\alpha}[(|K_{uL\alpha\beta}|^2 + |K_{uR\alpha\beta}|^2)\chi_+(x_\alpha, x_\beta) + 2\mathrm{Re}(K_{uL\alpha\beta}K^*_{uR\alpha\beta})\chi_-(x_\alpha, x_\beta)] \\ & - \sum_{j<i}[(|K_{dLij}|^2 + |K_{dRij}|^2)\chi_+(x_i, x_j) + 2\mathrm{Re}(K_{dLij}K^*_{dRij})\chi_-(x_i, x_j)]\Bigg\}, \end{aligned} \tag{B5}$$

where

$$\psi_+(x_\alpha, x_i) = \frac{1}{3} - \frac{1}{9}\ln\frac{x_\alpha}{x_i}, \qquad \psi_-(x_\alpha, x_i) = -\frac{x_\alpha + x_i}{6\sqrt{x_\alpha x_i}}. \tag{B6}$$

These functions do not vanish for $x_\alpha = x_i$, but $\psi_+(x, x) = -\psi_-(x, x)$ and $\psi_+(x_\alpha, x_i)$ is not symmetric under interchange of the arguments.

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