Higher states of the B_c meson family

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In this work, we study higher B_c mesons to the $L = S, P, D, F, G$ multiplets using the Cornell potential model, which takes account of the screening effect. The calculated mass spectra of B_c states are in reasonable agreement with the present experimental data. Based on the spectroscopy, partial widths of all allowed radiative transitions and strong decays of each state are also evaluated by applying our numerical wave functions. Comparing our results with the former results, we point out the difference among various models and derive new conclusions obtained in this paper. Our theoretical results are valuable for searching more B_c mesons in experiments.

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I. INTRODUCTION

In the past few decades, significant progress has been achieved in exploring B_c mesons experimentally [1–3] and theoretically [1–5]. In 1998, the ground state of the B_c meson family was first observed by the CDF Collaboration at Fermilab [5], with its mass $M_{B_c} = 6400 \pm 390 \pm 130$ MeV. There was no reported evidence for the excited B_c states until 2014. The ATLAS Collaboration reported a structure with the mass of $6842 \pm 4 \pm 5$ MeV [1], which is regarded as the $B_c^*(2S)$ state in Ref. [1]. However, the mass of this state is lower than the experimental mass of $B_c(2^1S_0)$ state in Particle Data Group (PDG) and the theoretical mass of $B_c(2^1S_0)$ state in other literature, this assignment seems unreasonable. The excited $B_c(2^1S_0)$ state has been observed in the $B_c^+\pi^+\pi^-$ invariant mass spectrum by the LHCb and CMS collaborations and the masses are determined to be $6871.7 \pm 1.3 \pm 0.3$ MeV and $6870.6 \pm 1.4 \pm 0.3$ MeV in 2019 [2,3], respectively. So far, different experimental groups have successively observed S-wave B_c mesons. The PDG only included two B_c mesons in the summary table [6]. In general, the experimental information about the higher states of B_c meson is still scarce, which is different from the status of charmonium and bottomonium. Exploring the higher states of B_c meson will be helpful for constructing the B_c meson family.

When studying the higher states of charmed/charmed-strange meson, charmonium, and charmed baryon, the role of the coupled-channel effect was found to be important. There are typical examples including $D_{s0}(2317)$ [7], $D_{s1}(2460)$ [8], $X(3872)$ [9,10], and $\Lambda_c(2940)$ [11], where their low mass puzzles can be understood well when the coupled-channel effect is introduced. Borrowing the experience mentioned above, we have the reason to believe that the coupled-channel effect cannot be ignored when focusing on the higher states of B_c meson.

For reflecting the coupled-channel effect generally, we introduce the screening effect [12] in the potential model adopted in this work and present the mass spectrum for higher states of B_c meson. The detail can be found in the next section. Accompanied with the obtained mass values of these discussed B_c mesons, we can also numerically get the corresponding spatial wave functions for the higher states of B_c meson, which can be applied to the calculation of two-body Okuba–Zweig–Iizuka (OZI) allowed strong decays and radiative decays of them. The information of mass spectrum, strong decays and radiative decays may

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form a general ideal for the spectroscopy of B_c meson. We hope that the present study can not only reflect the coupled-channel effect in the B_c meson spectroscopy, but also provide theoretical hints to experimental search for higher B_c states in the future.

The structure of this paper is as follows. In Sec. II, we adopt the Cornell potential model including the screening effect to study the mass spectrum of B_c mesons. In Sec. III, we present the detailed study of the OZI-allowed two-body strong decays and radiative decays of the discussed B_c mesons. We analysis decay behaviors of higher excited states of B_c mesons in Sec. IV. The paper ends with a conclusion.

II. MASS SPECTRUM

B_c mesons are composed of two heavy quarks. The heavier mass of the heavy quarks makes that the velocity of the quark in the B_c meson system is relatively small. Thus, we can regard the B_c meson system as a typical nonrelativistic system. The mass spectrum of the B_c meson system can be obtained by solving the Schrödinger equation [13,14], where the Hamiltonian is

$$H|\psi\rangle = (H_0 + V)|\psi\rangle = E|\psi\rangle. \tag{1}$$

Under the nonrelativistic approximation, H_0 denotes

$$H_0 = \sum_{i=1}^{2}\left(m_i + \frac{p^2}{2m_i}\right), \tag{2}$$

here, m_1 and m_2 are the masses of b and c quarks, respectively. We use the Cornell potential as a starting point [15], i.e.,

$$s(r) = br + c, \tag{3}$$

$$G(r) = -\frac{4\alpha_s}{3r}, \tag{4}$$

where $s(r)$ and $G(r)$ are the linear and Coulomb potentials, respectively, and the parameter c denotes the scaling parameter [16]. When including the screening effect [12], the linear potential becomes

$$s(r)' = \frac{b(1 - e^{-\mu r})}{\mu} + c, \tag{5}$$

where μ is a screening parameter.

For the spin-dependent term, we refer to it given in the GI model [17–21]. The additional screening potential makes that the spin correlation term related to the linear potential also has a corresponding transformation. Thus, we have

$$V = H^{\mathrm{conf}} + H^{\mathrm{cont}} + H^{\mathrm{so}} + H^{\mathrm{ten}}. \tag{6}$$

In Eq. (6), $H^{\mathrm{conf}} = s(r)' + G(r)$ contains the screening potential and Coulomb-like interaction. The color contact interaction is expressed as

$$H^{\mathrm{cont}} = \frac{32\pi\alpha_s}{9m_1m_2}\left(\frac{\sigma}{\pi^{\frac{1}{2}}}\right)^3 e^{-\sigma^2 r^2}\boldsymbol{S}_1\cdot\boldsymbol{S}_2. \tag{7}$$

$$H^{\mathrm{so}} = H^{\mathrm{so(cm)}} + H^{\mathrm{so}(tp)}, \tag{8}$$

which is the spin-orbit interaction, where

$$H^{\mathrm{so(cm)}} = \frac{4\alpha_s}{3}\frac{1}{r^3}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)^2 \boldsymbol{L}\cdot\boldsymbol{S}_{1(2)}, \tag{9}$$

$$\begin{aligned} H^{\mathrm{so}(tp)} &= -\frac{1}{2r}\frac{\partial H^{\mathrm{conf}}}{\partial r}\left(\frac{\boldsymbol{S}_1}{m_1^2} + \frac{\boldsymbol{S}_2}{m_2^2}\right)\cdot\boldsymbol{L} \\ &= -\frac{1}{2r}\left(\frac{4\alpha_s}{3}\frac{1}{r^2} + be^{-\mu r}\right)\left(\frac{1}{m_1^2} + \frac{1}{m_2^2}\right)\boldsymbol{L}\cdot\boldsymbol{S}_{1(2)} \end{aligned} \tag{10}$$

is the Thomas precession term with the screening effect. Additionally, we define

$$H^{\mathrm{ten}} = \frac{4}{3}\frac{\alpha_s}{m_1m_2}\frac{1}{r^3}\left(\frac{3\boldsymbol{S}_1\cdot\boldsymbol{r}\boldsymbol{S}_2\cdot\boldsymbol{r}}{\boldsymbol{r}^2} - \boldsymbol{S}_1\cdot\boldsymbol{S}_2\right), \tag{11}$$

which depicts the color tensor interaction,

$$\boldsymbol{T} = \frac{\boldsymbol{S}_1\cdot\boldsymbol{r}\boldsymbol{S}_2\cdot\boldsymbol{r}}{\boldsymbol{r}^2} - \frac{1}{3}\boldsymbol{S}_1\cdot\boldsymbol{S}_2, \tag{12}$$

$$\langle T\rangle = \begin{cases} -\frac{L}{6(2L+3)} & J = L+1 \\ \frac{1}{6} & J = L \\ -\frac{(L+1)}{6(2L-1)} & J = L-1 \end{cases}, \tag{13}$$

where $\boldsymbol{T}$ is the tensor operator, $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ are the spins of the quarks contained by the meson, and $\boldsymbol{L}$ is the orbital angular momentum [22].

The S-L coupling causes the mixture of spin singlet and spin triplet states. The Hamiltonian corresponding to the S-L term can be sperated as the symmetric and antisymmetric parts. For the B_c mesons, the results of the symmetric part are zero, we only need to consider the antisymmetric part

$$H_{\mathrm{anti}} = \frac{1}{4}\left[\left(\frac{4\alpha_s}{3r^3} - \frac{be^{-\mu r}}{r}\right)\left(\frac{1}{m_1^2} - \frac{1}{m_2^2}\right)(\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{L}\right]. \tag{14}$$

The mixture of states denotes as

$$L' = {}^1L_J\cos\theta + {}^3L_J\sin\theta, \tag{15}$$

$$L = -{}^1L_J\sin\theta + {}^3L_J\cos\theta, \tag{16}$$

here, θ is the mixing angle.

The mass spectrum and the wave function of the B_c mesons can be obtained by solving the energy eigenvalue and eigenvector of the Hamiltonian in Eq. (1) with the simple harmonic oscillator (SHO) base expanding method. In configuration and momentum space, SHO wave functions have explicit form respectively

$$\Psi_{nLM_L}(\boldsymbol{r}) = R_{nL}(r,\beta)Y_{LM_L}(\Omega_r), \tag{17}$$

$$\Psi_{nLM_L}(\boldsymbol{p}) = R_{nL}(p,\beta)Y_{LM_L}(\Omega_r), \tag{18}$$

where

$$R_{nL}(r,\beta)=\beta^{\frac{3}{2}}\sqrt{\frac{2n!}{\Gamma(n+L+\frac{3}{2})}}(\beta r)^L e^{\frac{-r^2\beta^2}{2}} L_n^{L+\frac{1}{2}}(\beta^2 r^2), \tag{19}$$

$$R_{nL}(p,\beta)=\frac{(-1)^n(-i)^L}{\beta^{\frac{3}{2}}} e^{-\frac{p^2}{2\beta^2}}\sqrt{\frac{2n!}{\Gamma(n+L+\frac{3}{2})}}\left(\frac{p}{\beta}\right)^L \times L_n^{L+\frac{1}{2}}\left(\frac{p^2}{\beta^2}\right), \tag{20}$$

where $Y_{LM_L}(\Omega_r)$ is a spherical harmonic function, R_{nL} ($n = 0, 1, 2, 3, \ldots$) is a radial wave function, and $L_n^{L+\frac{1}{2}}(x)$ denotes a Laguerre polynomial.

We considered the Landau full in our calculation. To avert the condition, we gave a cutoff distance r_c for solving the Schrödinger equation. r_c is determined by fitting the spectrum and the value of it is listed in Table II. The detail for this r_c is introduced in Ref [23].

In order to obtain the mass spectra of B_c mesons, we should fix the correspondent parameters involved in the adopted potential model by fitting the experimental data. Here, we select the reported B_c mesons, charmonia, bottomonia, and charmed/charmed-strange and bottom/bottom-strange mesons in the experiment [6], which are listed in Table I. By performing the χ^2 fitting defined as

$$\chi^2 = \sum_i \frac{(\mathrm{Th}_i - \mathrm{Exp}_i)^2}{\mathrm{Error}_i{}^2}, \tag{21}$$

finally these input parameters can be obtained,[1] where Th, Exp, and Error represent the theoretical, experimental data, and fitting error, respectively.

Using the parameters in Table II, we obtain the mass spectra of B_c mesons as shown in Table III and give the masses of these S-wave, P-wave, D-wave, F-wave, and G-wave states. The calculated S-, P-, D-, F-, and G-wave states are considered as $n + L \leq 6$ (where n is the radial quantum number and $n = 1$ is the ground state). In addition, the mixing angles for some mixtures of states are also given in Table III. Of course, we notice that there

TABLE I. The experimental mass values and fitted results of heavy flavor mesons in this work (unit: MeV). The experimental data of B_c mesons are derived from PDG [6], and the other mesons are also derived from PDG [6]. Here, Exp and Th represent the experimental data and theoretical data, respectively.

Meson	State	Exp	Th
B_c	1^1S_0	$6274.47 \pm 0.27 \pm 0.17$	6269
	2^1S_0	6871.2 ± 1.0	6886
$c\bar{c}$	1^1S_0	2983.9 ± 0.5	3007
	2^1S_0	3637.6 ± 1.2	3645
	1^3S_1	3096.9 ± 0.006	3100
	2^3S_1	3686.097 ± 0.01	3686
	1^1P_1	3525.38 ± 0.11	3522
	1^3P_0	3414.71 ± 0.3	3406
	1^3P_1	3510.67 ± 0.05	3515
	1^3P_2	3556.17 ± 0.07	3540
$b\bar{b}$	1^1S_0	9399 ± 2.3	9372
	2^1S_0	9999 ± 3.5	10025
	1^3S_1	9460.3 ± 0.26	9414
	2^3S_1	10023.2 ± 0.31	10037
	1^1P_1	9899.3 ± 0.8	9933
	1^3P_0	$9859.4 \pm 0.42 \pm 0.31$	9888
	1^3P_1	$9892.8 \pm 0.26 \pm 0.31$	9928
	1^3P_2	$9912.2 \pm 0.26 \pm 0.31$	9950
D	1^1S_0	1864.84 ± 0.5	1889
	2^1S_0	2564 ± 20	2583
	1^3S_1	2006.85 ± 0.05	2007
	1^3P_2	2460.7 ± 0.4	2438
D_s	1^1S_0	1969 ± 1.4	1995
	1^3S_1	2112.2 ± 0.4	2111
	2^3S_1	2708.3 ± 4	2736
	1^3P_1	2459.6 ± 0.9	2538
	1^3P_2	2569.1 ± 0.8	2544
	1^3D_1	$2859 \pm 12 \pm 24$	2866
	1^3D_3	$2860.5 \pm 26 \pm 6.5$	2828
B	1^1S_0	5279.25 ± 0.26	5272
	1^3S_1	5324.7 ± 0.21	5319
	1^3P_2	5737.2 ± 0.7	5719
B_s	1^1S_0	5366.84 ± 0.14	5275
	1^3S_1	5415.8 ± 1.5	5321
	1^3P_2	5839.92 ± 0.14	5751

TABLE II. The fitted parameters in the potential model adopted in this work.

Parameter	Value	Parameter	Value
m_b	5.368 GeV	m_u, m_d	0.606 GeV
m_c	1.984 GeV	m_s	0.780 GeV
α_s	0.3930	σ	1.842 GeV
b	0.2312 GeV^2	c	-1.1711 GeV
μ	0.0690 GeV	r_c	0.3599 GeV^{-1}

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[1] In the χ^2 fitting, we should find the minimum value of χ^2, where these parameters can be fixed. In this work, χ^2/d.o.f. is 5656, where d.o.f. is the degree of freedom which is 24. In Table II, the obtained parameters in the potential model are given.

TABLE III. Predicted masses (MeV) of B_c states compared with other model predictions and data. The mixing angles between $B_c(n^3L_J)$ and $B_c(n^1L_J)$ obtained in this work are also presented.

State	Ours	LZ [24]	ZVR [25]	SJSCP [26]	MBV [27]	EQ [28]	EFG [29]	GI [30]	KLT [31]
3^1S_0	7261	7239	7240	7306	7308	7244	7193	7250	···
4^1S_0	7551	7540	7550	7684	7713	7562	···	···	···
5^1S_0	7790	7805	···	8025	8097	···	···	···	···
6^1S_0	7994	8046	···	8340	8469	···	···	···	···
1^3S_1	6322	6326	6340	6321	6357	6337	6332	6338	6317
2^3S_1	6907	6890	6900	6900	6897	6899	6881	6887	6902
3^3S_1	7275	7252	7280	7338	7333	7280	7235	7272	···
4^3S_1	7561	7550	7580	7714	7734	7594	···	···	
5^3S_1	7798	7813	···	8054	8115	···	···	···	···
6^3S_1	8001	8054	···	8368	8484	···	···	···	···
$1P_1'$	6761	6776	6740	···	6734	6736	6749	6750	6729
$1P_1$	6770	6757	6730	···	6686	6730	6734	6741	6717
θ_{1p}	−24.3°	35.5°	···	···	···	···	20.4°	22.4°	···
$2P_1'$	7156	7150	7150	···	7173	7142	7145	7150	7124
$2P_1$	7164	7134	7140	···	7137	7135	7126	7145	7113
θ_{2p}	−28.4°	38.0°	···	···	···	···	23.2°	18.9°	···
$3P_1'$	7458	7458	7470	···	7572	···	···	···	···
$3P_1$	7466	7441	7460	···	7546	···	···	···	···
θ_{3p}	−30.2°	39.7°	···	···	···	···	···	···	···
$4P_1'$	7708	7727	7740	···	7942	···	···	···	···
$4P_1$	7715	7710	7740	···	7943	···	···	···	···
θ_{4p}	−31.0°	39.7°	···	···	···	···	···	···	···
$5P_1'$	7921	···	···	···	···	···	···	···	···
$5P_1$	7927	···	···	···	···	···	···	···	···
θ_{5p}	−31.6°	···	···	···	···	···	···	···	···
1^3P_0	6712	6714	6680	6686	6638	6700	6699	6706	6683
1^3P_2	6783	6787	6760	6712	6737	6747	6762	6768	6743
2^3P_0	7118	7107	7100	7146	7084	7108	7091	7122	7088
2^3P_2	7175	7160	7160	7173	7175	7153	7156	7164	7134
3^3P_0	7427	7420	7430	7536	7492	···	···	···	···
3^3P_2	7476	7464	7480	7565	7575	···	···	···	···
4^3P_0	7682	7693	7710	7885	7970	···	···	···	···
4^3P_2	7724	7732	7760	7915	7970	···	···	···	···
5^3P_0	7899	···	···	···	···	···	···	···	···
5^3P_2	7936	···	···	···	···	···	···	···	···
$1D_2'$	7046	7032	7030	···	7003	7012	7079	7036	7124
$1D_2$	7037	7024	7020	···	6974	7009	7077	7041	7113
θ_{1D}	−41.7°	45.0°	···	···	···	−35.9°	···	44.5°	···
$2D_2'$	7365	7347	7360	···	7408	···	···	···	···
$2D_2$	7360	7343	7360	···	7385	···	···	···	···
θ_{2D}	−42.6°	45.0°	···	···	···	···	···	···	···
$3D_2'$	7627	7623	7650	···	7783	···	···	···	···
$3D_2$	77623	7620	7650	···	7781	···	···	···	···
θ_{3D}	43.6°	45.0°	···	···	···	···	···	···	···
$4D_2'$	7849	···	···	···	···	···	···	···	···
$4D_2$	7846	···	···	···	···	···	···	···	···
θ_{4D}	−44.6°	···	···	···	···	···	···	···	···
1^3D_1	7037	7020	7010	6998	6973	7012	7072	7025	7088
1^3D_3	7042	7030	7040	6990	7004	7005	7081	7045	7134
2^3D_1	7357	7336	7350	7403	7377	···	···	···	···
2^3D_3	7364	7348	7370	7399	7410	···	···	···	···
3^3D_1	7619	7611	7640	7762	7761	···	···	···	···
3^3D_3	7627	7625	7660	7761	7796	···	···	···	···
4^3D_1	7842	···	···	···	···	···	···	···	···

(Table continued)

TABLE III. *(Continued)*

State	Ours	LZ [24]	ZVR [25]	SJSCP [26]	MBV [27]	EQ [28]	EFG [29]	GI [30]	KLT [31]
4^3D_3	7850	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯
$1F_3'$	7261	7240	7250	⋯	⋯	⋯	⋯	7266	⋯
$1F_3$	7248	7224	7240	⋯	⋯	⋯	⋯	7276	⋯
θ_{1F}	−41.8°	41.4°	⋯	⋯	⋯	⋯	⋯	41.4°	⋯
$2F_3'$	7537	7525	7550	⋯	⋯	⋯	⋯	7571	⋯
$2F_3$	7526	7508	7540	⋯	⋯	⋯	⋯	7563	⋯
θ_{2F}	−41.2°	43.4°	⋯	⋯	⋯	⋯	⋯	⋯	⋯
$3F_3'$	7770	7779	7810	⋯	⋯	⋯	⋯	⋯	⋯
$3F_3$	7762	7768	7800	⋯	⋯	⋯	⋯	⋯	⋯
θ_{3F}	−41.3°	42.4°	⋯	⋯	⋯	⋯	⋯	⋯	⋯
1^3F_2	7258	7235	7240	7234	⋯	⋯	⋯	7269	⋯
1^3F_4	7249	7227	7250	7244	⋯	⋯	⋯	7271	⋯
2^3F_2	7533	7518	7540	7607	⋯	⋯	⋯	7565	⋯
2^3F_4	7528	7514	7550	7617	⋯	⋯	⋯	7568	⋯
3^3F_2	7767	7730	7800	7946	⋯	⋯	⋯	⋯	⋯
3^3F_4	7764	7771	7810	7956	⋯	⋯	⋯	⋯	⋯
$1G_4'$	7443	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯
$1G_4$	7427	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯
θ_{1G}	−41.5°	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯
$2G_4'$	7687	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯
$2G_4$	7675	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯
θ_{2G}	−41.5°	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯
1^3G_3	7441	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯
1^3G_5	7428	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯
2^3G_3	7686	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯
2^3G_5	7675	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯

were former theoretical studies of the B_c mass spectrum. Thus, we also make the comparison of our results with other theoretical results from Refs. [24–31].

It is found that the masses of the low-lying $1S$-, $1P$-, $1D$-, and $1F$-wave B_c states predicted in this work are compatible with the other potential model predictions. For the higher mass states, such as $5S$- and $4P$-wave states, the masses predicted by us are smaller than those predicted with the nonrelativistic quark model in Ref. [24]. The strong screening effect for the large angular momentum and larger average distance between quark pair leads to the mass for the higher exited states in this work lower than other potential models.

The states with quantum numbers ${}^1S_0$ and ${}^3S_1$ are the partner of B_c mesons. The ground state $B_c(1S)$ and radial excited state $B_c(2S)$ have been established in the experiment, and their measured average masses are $6274.47 \pm 0.27 \pm 0.17$ MeV and 6871.2 ± 1.0 MeV [32], respectively, which are in agreement with our theoretical values in Table I. We find that our predict mass for the 1^3S_1 state is 6322 MeV, which matches the result of Lattice QCD of 6331 ± 10 MeV [33]. For the 2^3S_1 state, we give its mass 6907 MeV. The mass for the 3^1S_0 is about 7216 MeV. The hyperfine mass splitting between the spin-singlet and spin-triplet states $\Delta m(nS) = m[B_c(n^3S_1)] - m[B_c(n^1S_0)]$, which is caused by the spin-dependent interaction and often be used to test various potential models. For the $1S$ state, $\Delta m(1S) \simeq 60$ MeV, which is very close to 55 MeV predicted in Ref. [24]. The predicted masses for the other higher S-wave states compared with other works are also given in Table III.

For the P-wave states of B_c mesons, we should consider the mixture for the n^1P_1 and n^3P_1 states, which have the same radial quantum number n. By including the mixture of the n^1P_1 and n^3P_1 states, the physical states P_1' and P_1 can be formed. The masses of $1P_1'$ and $1P_1$ states are 6761 and 6770 MeV, respectively, where the mixing angle $\theta_{1p} = -24.3°$ is obtained. The masses of $1P_1'$ and $1P_1$ states are very close to those predicted values in Ref. [24]. The masses of $B_c(2P)$ and $B_c(2P')$ state are also consistent with the other predictions with potential models [24,25,27]. The predicted masses of $3P$ and $4P$ states are smaller approximately 100–200 MeV than the predictions in Ref. [27], which is resulted in the screening effect. The masses of $5P$ are also calculated.

Except $B_c(4D)$, the absolute mixing angle of nD and nF states consist with Ref. [24]. In addition, the G-wave mass of the B_c mesons also be calculated, there are few work about the mass of $B_c(1G)$ and $B_c(2G)$ states. The masses of $1G$ and $1G'$ states are located at a range of 7427–7443 MeV according to our estimates.

We hope that this work can help for the research of higher states of B_c mesons in the future. After providing the information of mass spectrum for the discussed higher states of the B_c meson family, we should further study their decay behaviors as illustrated in the following section.

III. THEORETICAL MODELS OF DECAY BEHAVIORS

Next, we give the necessary formulas for calculating two-body OZI-allowed strong decays and radiative transitions.

A. Two-body OZI-allowed strong decays

The quark pair creation (QPC) model was first proposed by Micu and has been widely used to calculate the strong two-body decay allowed by OZI rule after further development [34–39]. For a decay process $A \to B + C$, the transition matrix is defined by [40–44]

$$\langle BC|\mathcal{T}|A\rangle = \delta^3(\boldsymbol{P_B}+\boldsymbol{P_C})\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}, \tag{22}$$

the amplitude $\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}(\mathbf{P})$ can be derived. $\mathcal{T}$ can be expressed as

$$\begin{aligned}\mathcal{T} = &-3\gamma\sum_m \langle 1m;1-m|00\rangle \int d\boldsymbol{p}_3 d\boldsymbol{p}_4 \delta^3(\boldsymbol{p}_3+\boldsymbol{p}_4)\\ &\times \mathcal{Y}_{1m}\left(\frac{\boldsymbol{p}_3-\boldsymbol{p}_4}{2}\right)\chi^{34}_{1,-m}\phi^{34}_0(\omega^{34}_0)_{ij}b^\dagger_{3i}(\boldsymbol{p}_3)d^\dagger_{4j}(\boldsymbol{p}_4),\end{aligned} \tag{23}$$

which is the transition operator and it can describe the creation of a quark-antiquark pair from vacuum. Where $b_3^\dagger(d_4^\dagger)$ is quark (antiquark) creation operator. χ, ϕ and ω denote the spin, flavor, and color wave functions, respectively. γ is a dimensionless constant depicting the generation rate of a quark-antiquark pair from a vacuum. $\mathcal{Y}_{\ell m}(p) = |p|^\ell Y_{\ell m}(p)$ is a solid spherical harmonic function. Using the Jacobi-Wick formula, the $\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}(\mathbf{P})$ is converted into the partial wave amplitudes $\mathcal{M}^{JL}$, and it can be expressed as

$$\begin{aligned}\mathcal{M}^{JL}(\mathbf{P}) = &\frac{\sqrt{4\pi(2L+1)}}{2J_A+1}\sum_{M_{J_B}M_{J_C}}\langle L0;JM_{J_A}|J_AM_{J_A}\rangle\\ &\times\langle J_BM_{J_B};J_CM_{J_C}|J_AM_{J_A}\rangle\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}.\end{aligned} \tag{24}$$

Then the strong decay partial width for a given decay mode of $A \to B + C$ reads as

$$\Gamma = \frac{\pi}{4}\frac{|P_E|}{m_A^2}\sum_{J,L}|\mathcal{M}^{JL}(\boldsymbol{P})|^2, \tag{25}$$

where m_A is the mass of the initial meson A.

In addition, the meson wave function is defined as a mock state, i.e.,

$$\begin{aligned}|A(n^{2S+1}L_{JM_J})(\boldsymbol{p}_A)\rangle = &\sqrt{2E}\sum_{M_S,M_L}\langle LM_LSM_S|JM_J\rangle\chi^A_{SM_S}\\ &\times\phi^A\omega^A\int d\boldsymbol{p}_1d\boldsymbol{p}_2\delta^3(\boldsymbol{p}_A-\boldsymbol{p}_1-\boldsymbol{p}_2)\\ &\times\Psi^A_{nLM_L}(\boldsymbol{p}_1,\boldsymbol{p}_2)|q_1(\boldsymbol{p}_1)\bar{q}_2(\boldsymbol{p}_2)\rangle,\end{aligned} \tag{26}$$

where the spatial wave function $\psi_{nLM_L}(p)$ of the meson is obtained by solving Eq. (1).

We can filter out OZI-allowed two-body strong decay channels of these discussed B_c mesons. For these strong decays with a pair of strange quarks from the vacuum, the strength of strange pair creation has the relation $\gamma_s = \gamma/\sqrt{3}$, where $\gamma = 6.947$ is taken from Ref. [24]. The decay widths of the two-body strong decay are calculated and the results are collected in Sec. IV. The corresponding branching ratios of the decay widths of these discussed B_c mesons are also given.

B. Radiative transitions

The partial width for the $E1$ transitions of B_c mesons in the nonrelativistic quark model is given by Ref. [30]

$$\Gamma(i\to f+\gamma) = \frac{4}{3}\langle e_Q\rangle^2\alpha\omega^3C_{fi}\delta_{SS'}|\langle f|r|i\rangle|^2, \tag{27}$$

where

$$C_{fi} = \mathrm{Max}(L,L')(2J'+1)\begin{Bmatrix}L' & J' & S\\ J & L & 1\end{Bmatrix}^2, \tag{28}$$

$$\langle e_Q\rangle = \frac{m_be_c - m_ce_{\bar{b}}}{m_b+m_c}, \tag{29}$$

where the quantum numbers with prime and without prime denote final state and initial state, respectively. m_c and m_b are the masses of the quarks. The charge of the quark is $e_b = -1/3$, $e_c = 2/3$. ω is the energy of the photon. The following equation can be obtained by the conservation of energy and momentum

$$M_i = \sqrt{M_f^2+\omega^2}+\omega. \tag{30}$$

Here the radial wave function $R_{nL}(r)$ is same with one used in the strong decay.

IV. ANALYSIS OF EXCITED STATES OF B_c MESONS

In this section, we will discuss the radiative transition (here, we neglect the smaller M1 transitions and only consider E1 transitions) and the OZI-allowed two-body strong decay of B_c mesons. In the final states of strong decay, we consider $1P$ or $1P'$ mixed of the resonances

$D_1(2430)$, $D_1(2420)$, $D_{s1}(2536)$, $B_1(5721)$, and $B_{s1}(5830)$ [45,46]. In our calculation of the strong decays, we neglect the value of Br less than one percent.

A. *S*-wave states

The details of S-wave decay for the B_c mesons are given in Tables IV–VI and Table XV. For completeness we give a brief overview of these decays. For the $B_c(1S)$ and $B_c(2S)$ states below the BD threshold, we consider the $E1$ transitions of the $B_c(2S)$. In our calculation, $\Gamma_{B_c(2^1S_0)\to 1P_1\gamma} \simeq 1$ keV, $\Gamma_{B_c(2^1S_0)\to 1P_1'\gamma} \simeq 8$ keV. The $E1$ transitions of the S-wave states $B_c^*(2S)$ are also discussed. The $E1$ transitions of S-wave states of B_c mesons are also discussed in other literature for establish these missing B_c states in PDG. Our predicted partial width $\Gamma_{B_c(2^3S_1)\to 1P_1\gamma} \simeq 4.2$ keV, our result is very close to 4.7 keV predicted in Ref. [30]. The total radiative decay width of $B_c(2^3S_1)$ is about 16 keV.

TABLE IV. Partial widths and branching ratios of OZI-allowed strong decay for $3^1S_0 - 6^1S_0$ states of B_c mesons. The width results are in units of MeV.

State	Channels	This work	Br (%)	[24]	Br (%)
3^1S_0	B^*D	173	100	161	100
	Total	173	100	161	100
4^1S_0	B^*D^*	66	50.3	104	54
	BD^*	40.8	31.1	34.9	18.3
	B^*D	18.4	14	0.14	0.1
	$B_s^*D_s^*$	4.01	3.06	15.5	8.1
	$B_sD_s^*$	1.28	0.976	5.8	3.1
	$B_s^*D_s$	0.64	0.49	6.7	3.5
	$BD_0^*(2300)$	⋯	⋯	24	12.6
	Total	131	100	191	100
5^1S_0	$B^*D_1'(2420)$	47.3	24.4	70.9	17.2
	$BD_2^*(2460)$	42.5	21.9	48.2	11.7
	$B_2^*(5747)D^*$	39.9	20.5	56.5	13.7
	B^*D^*	31.9	16.4	2.28	0.6
	BD^*	10.5	5.46	1.5	0.4
	$B_2^*(5747)D$	8.98	4.63	27.6	6.7
	$B_1(5721)D^*$	5.88	⋯	6.2	1.5
	$B^*D_1(2430)$	3.99	2.06	12.3	3.0
	B^*D	0.789	0.406	24.5	5.9
	$B^*D_2^*(2460)$	0.707	0.364	25.7	6.2
	$BD_0^*(2300)$	0.584	0.3	23.5	5.7
	$B_sD_s^*$	0.218	0.112	4.65	1.1
	$B(1^3P_0)D$	⋯	⋯	18.6	4.5
	$B_s^*D_s^*$	⋯	⋯	5.75	1.4
	Total	194	100	413	100
6^1S_0	$B_2^*(5747)D$	15.8	30.4	4.85	1.3
	$B^*D(2550)$	10.1	19.4	⋯	⋯
	B^*D^*	9.02	17.4	24.3	6.7
	$B^*D_2^*(2460)$	6.78	13.1	23.5	6.5
	$B^*D_1(2430)$	1.59	3.07	41.4	11.4
	BD^*	1.53	2.95	24.3	6.7
	$B^*D_1'(2420)$	1.37	2.64	46.8	13
	$B_2^*(5747)D^*$	0.968	1.87	20.6	5.7
	$B_1(5721)D^*$	0.831	1.6	24.7	6.8
	$B_s^*D_{s1}(2536)$	0.624	1.2	4.14	1.1
	B^*D	0.617	1.19	44.4	12
	$B_{s1}(5830)D_s^*$	0.541	1.46	0.03	0.01
	$B_sD_{s0}^*(2317)$	0.429	0.825	6.62	1.8
	$BD_0^*(2300)$	0.35	0.673	13.2	3.6
	$BD_2^*(2460)$	0.148	0.285	28.9	8
	$B(1P')D^*$	⋯	⋯	28.3	7.8
	$B(1^3P_0)D$	⋯	⋯	11.3	3.1
	Total	52	100	361	100

TABLE V. Partial widths and branching ratios of OZI-allowed strong decay for $3^3S_1 - 5^3S_1$ states of B_c mesons. The width results are in units of MeV.

State	Channels	This work	Br (%)	[24]	Br (%)
3^3S_1	B^*D	108	75.8	105	79
	BD	34.4	24.2	28	21
	Total	142	100	133	100
4^3S_1	B^*D^*	85.1	67.6	112	66
	BD^*	25.5	20.2	17.0	10
	B^*D	8.23	6.56	0.41	0.2
	$B_s^*D_s^*$	5.08	4.03	26.9	16
	BD	0.673	0.537	4.53	2.7
	$B_s^*D_s$	0.614	0.499	5.29	3.1
	B_sD_s	0.574	0.456	2.81	1.6
	$B_sD_s^*$	0.488	0.38	1.83	1.1
	Total	126	100	171	100
5^3S_1	$B_2^*(5747)D^*$	64.3	34.1	96	24
	B^*D^*	32.7	17.3	0.19	0.05
	$B^*D_1(2430)$	20.6	10.9	6.8	1.7
	$BD_2^*(2460)$	20.5	10.9	17.34	4.3
	$B^*D_1'(2420)$	12.3	6.52	32.9	8.2
	$B^*D_2^*(2460)$	11	5.77	69.87	17
	$B_2^*(5747)D$	6.69	3.54	16.1	4
	BD^*	5.38	2.85	2.65	0.7
	$B_1(5721)D^*$	3.78	1.9	5.19	1.3
	$B_1(5721)D$	3.77	1.9	13.34	3.3
	$BD_1(2430)$	3.25	1.72	0.63	0.2
	$BD_1'(2420)$	2.62	1.39	0.89	0.2
	BD	0.491	0.26	15.81	3.9
	$B^*D_0^*(2300)$	0.408	0.215	18.32	4.6
	$B(1P')D^*$	⋯	⋯	53.93	13.4
	$B(1P')D$	⋯	⋯	18.96	4.7
	B^*D	⋯	⋯	20.18	5
	Total	189	100	401	100

Comparing the calculation of the strong decay $B_c(3^1S_0) \to B^*D$ predicted in [24] with this work, the value of the decay width is estimated to be $\Gamma_{\text{total}} \sim 173$ MeV, which is larger than the result 161 MeV in Ref. [24]. This process is the important finial state to determine the $B_c(3^1S_0)$ state in future experiments. The two-body decay information of 4^1S_0, 5^1S_0 and 6^1S_0 state can also be obtained in Table IV. Ratios of BD^*, B^*D^*, and B^*D for the 4^1S_0 state are 0.31, 0.5, and 0.14, respectively. $B^*D_1'(2420)$ is the dominant decay mode of

TABLE VI. Partial widths and branching ratios of OZI-allowed strong decay for 6^3S_1 state of B_c mesons. The width results are in units of MeV.

State	Channels	This work	Br (%)	[24]	Br (%)
6^3S_1	B^*D^*	7.65	16.2	37.9	10.2
	$B_2^*(5747)D$	7.56	16	1.09	0.3
	$B^*D(2550)$	5.45	11.6	⋯	⋯
	$B^*D_2^*(2460)$	5.03	10.7	39.9	10.7
	$B_1(5721)D$	4.56	9.67	11.1	3
	$BD_1'(2420)$	3.98	8.44	9.37	2.5
	$BD_1(2430)$	3.76	7.97	16.6	4.4
	BD	1.85	3.92	17.6	4.7
	$B^*D_1(2430)$	1.58	3.35	27.9	7.5
	$B^*D_1'(2420)$	1.05	2.23	30.5	8.2
	B^*D	1.01	2.14	31	8.3
	$B_1(5721)D^*$	0.757	1.61	17	4.6
	BD^*	0.477	1.01	19.1	5.1
	$B_s^*D_s^*$	0.473	1	2.96	0.8
	$B^*D_0^*(2300)$	0.428	0.91	12.9	3.5
	$BD_2^*(2460)$	0.389	0.825	14.1	3.8
	$B_s^*D_{s1}(2536)$	⋯	⋯	3.06	0.8
	$B_2^*(5747)D^*$	⋯	⋯	34	9.1
	$B(1P')D^*$	⋯	⋯	21.2	5.7
	$B(1^3P_0)D^*$	⋯	⋯	10.9	3
	Total	48	100	372	100

5^1S_0, and $BD_2^*(2460)$ and $B_2^*(5747)D^*$ are its important final state. For the 6^1S_0 state, the dominant decay channel is $B_2^*(5747)D$. The total decay width of $4^1S_0, 5^1S_0$, and 6^1S_0 are 131, 194, and 52 MeV, respectively. The B^*D and BD channels are the dominant decay modes of 3^3S_1. The total width of 3^3S_1 state is 142 MeV, larger than the predicted result of ∼133 MeV in Ref. [24]. B^*D^* is the largest decay mode of 4^3S_1. B^*D and BD^* have important contribution to the total width of 4^3S_1 state. 5^3S_1 has a broad width of ∼189 MeV. 6^3S_1 has the dominant decay channel B^*D^*. Other details for the higher S-wave states $B_c(n^1S_0)$ and $B_c(n^3S_1)$ are given in Tables IV–VI, respectively.

B. *P*-wave states

The decay behaviors of $B_c(nP)$ with $n = 1, 2, 3, 4, 5$ are given in Tables VII, VIII, and XV. The $1P$-wave of B_c states have no OZI-alowed two body strong decay channel and the mainly decay models are radiative decays. We have calculated the partial decay widths for the $E1$ transitions $1P \to 1S\gamma$, $2P \to 1S\gamma$, $2P \to 2S\gamma$, and $2P \to 1D\gamma$. Combine with the processes $B_c(2^1S_0) \to 1P_1\gamma$ and $B_c(2^1S_0) \to 1P_1'\gamma$, $B_c(2^1S_0) \to B_c\gamma\gamma$ and $B_c\gamma$ final channel for $B_c(1P_1)$ and $B_c(1P_1')$ maybe helpful to search the $B_c(1P_1)$ and $B_c(1P_1')$, i.e., the $B_c(1P_1)$ and $B_c(1P_1')$ states might be first found in the $B_c\gamma$ and $B_c\gamma\gamma$ final states via their radiative transitions. In Table XV, we can see that the $B_c(1^3P_0)$ and $B_c(1^3P_2)$ states dominantly decay into $B_c^*\gamma$

TABLE VII. Partial widths and branching ratios of OZI-allowed strong decay for the mixture of the 3^1P_1 and 3^3P_1 states -5^1P_1 and 5^3P_1 states. The width results are in units of MeV.

State	Channels	This work	Br (%)	[24]	Br (%)
$3P_1'$	B^*D^*	134	66.8	129	69.4
	BD^*	62.1	30.9	32	17.2
	$B_s^*D_s$	2.86	1.43	11.1	6
	B^*D	1.66	1.827	13.6	7.3
	Total	201	100	185	100
$3\mathrm{P}_1$	B^*D^*	164	61.8	145	65.8
	BD^*	55.8	21	62	28.1
	B^*D	45.2	17	9.3	4.3
	$B_s^*D_s$	0.227	0.09	4.0	1.8
	Total	265	100	220	100
$4P_1'$	$B_2^*(5747)D$	39.9	43.8	6.55	4.6
	B^*D^*	29.1	32	6.55	4.6
	BD^*	16.6	32.9	11.9	8.4
	B^*D	2.98	3.27	41.6	29.1
	$B_sD_s^*$	0.552	0.607	6.2	4.3
	$B_s^*D_s^*$	0.211	0.232	9.09	6.3
	$B(1^3P_2)D$	⋯	⋯	36.6	25.6
	$BD_0^*(2300)$	⋯	⋯	13.6	9.5
	$B(1^3P_0)D$	⋯	⋯	10.4	7.3
	$B_sD_{s0}^*(2317)$	⋯	⋯	4.75	3.3
	Total	91	100	143	100
$4\mathrm{P}_1$	B^*D^*	21.9	54.4	0.86	0.7
	$BD_1(2430)$	7.28	18.1	23.03	18.3
	BD^*	4.63	11.5	3.7	2.9
	$B_s^*D_s$	1.79	4.45	4.4	3.5
	B^*D	1.51	3.75	24.5	19.4
	$B_sD_s^*$	0.63	1.57	6.78	5.4
	$B_s^*D_s^*$	0.366	0.909	6.66	5.3
	$BD_1'(2420)$	⋯	⋯	15.32	12.2
	$B_2^*(5747)D$	0.314	0.78	15	11.9
	$B(1^3P_0)D^*$	⋯	⋯	11.8	9.4
	$B^*D_0^*(2300)$	⋯	⋯	10.02	8.0
	$B_1(5721)D$	⋯	⋯	3.32	2.6
	Total	40.2	100	126	100
$5\mathrm{P}_1'$	$B^*D(2550)$	24.1	30.2	⋯	⋯
	$B_2^*(5747)D$	14.3	17.9	⋯	⋯
	$B^*D_2^*(2460)$	8.5	10.6	⋯	⋯
	B^*D^*	6.61	8.27	⋯	⋯
	B^*D	5.51	6.9	⋯	⋯
	$B_2^*(5747)D$	5.39	6.75	⋯	⋯
	BD^*	4.25	5.32	⋯	⋯
	$BD_2^*(2460)$	3.43	4.29	⋯	⋯
	$B^*D_1'(2420)$	3.21	4.02	⋯	⋯
	$B_1(5721)D$	1.37	1.71	⋯	⋯
	Total	79.9	100	⋯	⋯
$5\mathrm{P}_1$	$B_2^*(5747)D$	16.7	26.3	⋯	⋯
	$B^*D_2^*(2460)$	12.7	20	⋯	⋯
	$B^*D(2550)$	10.8	17	⋯	⋯
	B^*D^*	5.87	9.25	⋯	⋯
	$B_1(5721)D$	3.68	5.8	⋯	⋯

(Table continued)

TABLE VII. *(Continued)*

State	Channels	This work	Br (%)	[24]	Br (%)
	$B^*D_1'(2420)$	2.78	4.38	⋯	⋯
	$BD_2^*(2460)$	2.69	4.24	⋯	⋯
	BD^*	1.52	2.39	⋯	⋯
	$BD_1(2430)$	0.926	1.46	⋯	⋯
	$B_s^*D_s$	0.756	1.19	⋯	⋯
	Total	63.5	100	⋯	⋯

TABLE VIII. Partial widths and branching ratios of OZI-allowed strong decay for $2^3P_2-5^3P_2$-wave B_c states. The width results are in units of MeV.

State	Channels	This work	Br (%)	[24]	Br (%)
$2^3\mathrm{P}_2$	BD	4.01	100	⋯	⋯
	Total	4.01	100	⋯	⋯
$3^3\mathrm{P}_0$	B^*D^*	211	78.2	255	93
	BD	58.4	21.6	9.6	3.5
	B_sD_s	0.892	0.33	9.7	3.5
	Total	270	100	274	100
$3^3\mathrm{P}_2$	B^*D^*	135	84.4	146	74
	BD^*	17.2	10.7	3.4	1.7
	BD	4.76	2.97	22	11.1
	$B_s^*D_s$	1.76	1.1	7.8	4
	B_sD_s	0.942	0.588	2.7	1.4
	B^*D	0.437	0.273	16	8.1
	Total	160	100	198	100
$4^3\mathrm{P}_0$	B^*D^*	73.5	86.6	14	26.4
	BD	8.61	10.1	13.6	25.6
	B_sD_s	2.77	3.26	7.16	13.5
	$B(1P')D$	⋯	⋯	7.66	14.4
	$B^*D_0^*(2300)$	⋯	⋯	5.5	10.4
	$B_s^*D_s^*$	⋯	⋯	4.6	8.7
	Total	84.9	100	53	100
$4^3\mathrm{P}_2$	B^*D^*	33.4	42	7.82	4.1
	$B_2^*(5747)D$	17.3	21.7	20.2	10.6
	$BD_1'(2420)$	12.4	15.6	13.1	6.9
	BD	6.59	8.28	21.76	11.4
	B^*D	4.36	5.48	30.1	15.8
	$B_1(5721)D$	3.27	4.11	6.95	3.6
	$BD_1(2430)$	0.787	0.989	6.61	3.5
	$B_s^*D_s^*$	0.501	0.629	11.1	5.8
	$B_sD_s^*$	0.4	0.498	2.34	1.2
	$B(1P')D$	⋯	⋯	27.7	14.5
	$B^*D_0^*(2300)$	⋯	⋯	10.1	5.3
	$B^*D_1(2430)$	⋯	⋯	9.22	4.8
	$B(1^3P_0)D^*$	⋯	⋯	8.8	4.6
	Total	79.6	100	190	100
$5^3\mathrm{P}_0$	$B^*D_2^*(2460)$	32.1	35.5	⋯	⋯
	B^*D^*	22.5	24.9	⋯	⋯
	$BD(2550)$	16.4	18.1	⋯	⋯
	$B_2^*(5747)D^*$	9.98	11	⋯	⋯
	$B^*D_1(2430)$	3.55	3.93	⋯	⋯
	$B_1(5721)D^*$	1.93	2.13	⋯	⋯
	B_sD_s	1.22	1.34	⋯	⋯
	Total	90.4	100	⋯	⋯
$5^3\mathrm{P}_2$	$B^*D_2^*(2460)$	13.4	18.8	⋯	⋯
	$B^*D(2550)$	9.33	13.3	⋯	⋯
	BD	6.84	9.32	⋯	⋯
	B^*D	6.55	8.8	⋯	⋯
	B^*D^*	5.15	7.62	⋯	⋯
	$B_2^*(5747)D$	5.4	7.58	⋯	⋯
	$BD_1(2430)$	4.8	6.73	⋯	⋯
	$BD_1'(2420)$	4.47	6.27	⋯	⋯
	$B_2^*(5747)D^*$	4.1	5.75	⋯	⋯
	$B^*D_1(2430)$	3.17	4.45	⋯	⋯
	$BD_2^*(2460)$	2.42	3.4	⋯	⋯
	$B_1(5721)D$	1.56	2.19	⋯	⋯
	$B^*D_1'(2420)$	1.22	1.71	⋯	⋯
	$B_s^*D_s^*$	1.11	1.56	⋯	⋯
	Total	71.3	100	⋯	⋯

final state with decay widths 60 keV and 90 keV, respectively. The approximate width of the $B_c(2P_1)\to 2^1S_0\gamma$, $B_c(2P_1)\to 2^3S_1\gamma$ and $B_c(2^3P_2)\to 1^3D_3\gamma$ decay processes are 14 keV, 38 keV, and 10 keV respectively. $B_c(2^3P_0)$ dominantly decays into $B_c^*(2S)\gamma$ final state. The details of E1 transitions for $B_c(nP)$ states can be found in Table XV.

As shown in Table VIII, $B_c(2^3P_2)$ only has one two body strong decay mode BD, its width is about 4 MeV, which may be helpful for experiment to search this B_c state. For the $3P_1'$ and $3P_1$ states, B^*D^* and BD^* are all their main decay modes, which have the total width of 201 MeV and 265 MeV. The values of the total width for these two states are larger than the results in Ref. [24] and the decay channels are similar. Compared to the results in Ref. [24], the total widths for the 4^3P_2, $4P_1'$ and $4P_1$ are smaller. But the total width for the 4^3P_0 is larger and our total width is of ~84.9 MeV. We also calculate the decay width for the $5P$-wave B_c states.

Except $B_c(2^3P_2)$, $B_c(5P_1')$, $B_c(5P_1)$ and $B_c(5^3P_2)$, B^*D^* final channel is the dominant decay mode for the other open-charmed decays of $B_c(nP)$ such as $B_c(3P_1')$, $B_c(3P_1)$, $B_c(4P_1')$, $B_c(4P_1)$, $B_c(3^3P_0)$, $B_c(3^3P_2)$, $B_c(4^3P_0)$, $B_c(4^3P_2)$, and $B_c(5^3P_0)$. B^*D^* mode will play an important role in the discovery for the higher $B_c(nP)$ states. The other OZI allowed strong decay channels can be found in Table VII and VIII.

C. *D*-wave states

Partial widths and branching ratios of the OZI-allowed strong decay and radiative transition for D-wave B_c states are shown in Tables IX–XII, XV and XVII. For the

TABLE IX. Partial widths and branching ratios of OZI-allowed strong decay for the mixture of the 2^1D_2 and 2^3D_2 states and 3^1D_2 and 3^3D_2 states. The width results are in units of MeV.

State	Channels	This work	Br (%)	[24]	Br (%)
2D$'_2$	BD^*	138	50.6	66.8	40.7
	B^*D	76.5	28.1	57.1	34.7
	B^*D^*	57.9	21.2	40.4	24.6
	Total	272	100	164	100
2D$_2$	B^*D^*	45.3	47.2	12.3	9
	B^*D	36.1	37.6	38.2	27
	BD^*	14.6	15.2	89	64
	Total	96.1	100	139	100
3D$'_2$	B^*D	24.6	37.6	45.8	34.6
	B^*D^*	18.6	28.6	21.1	16
	BD^*	15.2	23.4	20.6	15.6
	$B_1(5721)D$	3.73	5.71	⋯	⋯
	$B_s^*D_s^*$	1.4	2.11	9.07	6.8
	$B_2^*(5747)D$	0.735	1.14	⋯	⋯
	$B_sD_s^*$	0.619	0.947	6.33	4.8
	$B_s^*D_s$	0.221	0.338	2.25	1.7
	$BD_0^*(2300)$	⋯	⋯	14.4	10.9
	$B(1^3P_0)D$	⋯	⋯	12.1	9.1
	Total	65.4	100	132	100
3D$_2$	$B_2^*(5747)D$	93.3	76.6	⋯	⋯
	B^*D^*	18.9	15.5	22.1	19
	$B_1(5721)D$	2.77	2.27	2.82	2.5
	BD^*	2.16	1.77	13.8	12
	$B_s^*D_s$	2.07	1.7	3.89	3.4
	$B_s^*D_s^*$	1.35	1.11	11.6	10
	$B_sD_s^*$	1.14	0.936	6.46	5.7
	B^*D	0.14	0.115	38.9	34.2
	$B^*D_0^*(2300)$	⋯	⋯	13.6	12
	Total	122	100	114	100

$B_c(1^3D_3)$ state, its main decay mode is $B_c(1^3D_3) \to B_c(1^3P_2)\gamma$, which has a branching fraction to be ∼100%. So this state may decay via the radiative decay chain $B_c(1^3D_3) \to B_c(1^3P_2)\gamma \to B_c(1^3S_1)\gamma\gamma$. Compared with the predicted partial width in Ref. [24], the value of radiative transition $1D_2 \to 1P_1\gamma$ is 7 times smaller. Our calculated partial widths of the dominant radiative decay $1D_2 \to 1^3P_2\gamma$ is roughly consistent with the value 8.7 keV given by Ref. [24].

In the strong decay case, BD^* is dominant decay mode of $2D'$ state, which have the width and branching ratio 138 MeV and 0.51, respectively. For the $2D$ state, B^*D^* and B^*D are its main decay modes, which have the width of 45 MeV and 36 MeV. $B_c(3D'_2)$ are governed by the B^*D mode with a branching fraction

$$Br[B_c(3D'_2) \to B^*D] \simeq 38\%.$$

The $B_c(3^3D_1)$ and $B_c(3^3D_3)$ states all mainly decay into B^*D^* channel, too. The corresponding strong decay width are

TABLE X. Partial widths and branching ratios of OZI-allowed strong decay for the mixture of the 4^1D_2 and 4^3D_2 states. The width results are in units of MeV.

State	Channels	This work	Br (%)
4D$'_2$	$B_2^*(5747)D^*$	27	22.3
	$B^*D_2^*(2460)$	21.7	17.9
	B^*D	17.3	14.4
	$B^*D'_1(2420)$	15.6	12.9
	$BD_2^*(2460)$	12.1	10
	$B^*D_1(2430)$	7.88	6.51
	$B_2^*(5747)$D	5.05	4.17
	B^*D^*	4.63	3.83
	$B_1(5721)D^*$	2.95	2.44
	$B_1(5721)$D	2.35	1.94
	BD^*	1.85	1.53
	Total	121	100
4D$_2$	$B^*D_2^*(2460)$	32.3	20.3
	$B_2^*(5747)D^*$	23.7	17.7
	$B_2^*(5747)$D	22.9	17.2
	$BD_2^*(2460)$	18.3	13.7
	$B^*D'_1(2420)$	10.6	7.95
	$B^*D_1(2430)$	8.49	6.37
	B^*D^*	4.61	3.46
	$B_1(5721)$D	2.21	1.66
	B^*D	1.89	1.42
	$B_1(5721)D^*$	1.25	0.938
	Total	133	100

$$\Gamma[B_c(3^3D_1) \to B^*D^*] \simeq 14\ \text{MeV},$$
$$\Gamma[B_c(3^3D_3) \to B^*D^*] \simeq 44\ \text{MeV}.$$

Furthermore, the mass of $3D$ and $3D'$ lie on the threshold of $B_s^*D_s^*$. And $B_sD_s^*$, $B_s^*D_s$, $B_s^*D_s^*$ channels have some contribution for the decays of $3D$ and $3D'$. It is found that the $B_c(3D'_2)$ state has a relatively narrow width of ∼65 MeV, which is half of that in Ref. [24]. $4D$ and $4D'$ have more strong decay channels for the higher mass and have the total width of about 133 MeV and 121 MeV, respectively. Branching ratios of main B_c decay modes BD^*, B^*D^*, and B^*D for 2^3D_1 state are calculated to be 55%, 22%, and 16%, respectively. One can see that B^*D^* is the leading decay channel and the following B^*D and BD are the next-to-leading decay modes for the $B_c(2^3D_3)$. $B_c(3^3D_1)$ and $B_c(3^3D_3)$ dominantly decay to B^*D^* final channel, and BD are their next-to-leading decay mode.

The higher 4^3D_1 and 4^3D_3 states of B_c mesons are also studied in present work and the corresponding strong decay widths are listed in Tables IX–XII. There are more strong decay channels for this two 4D-wave B_c states. The total widths of 4^3D_1 and 4^3D_3 are 130 and 184 MeV, respectively.

D. *F*-wave states

In the following, we will focus on the higher nF-wave B_c states with $n = 1, 2, 3$. The theoretical mass values of

TABLE XI. Partial widths and branching ratios of OZI-allowed strong decay for $2^3D_1 - 3^3D_3$-wave B_c states. The width results are in units of MeV.

State	Channels	This work	Br (%)	[24]	Br (%)
$2^3\mathrm{D}_1$	BD^*	47.5	54.7	50.1	87
	B^*D^*	19	21.9	0.48	0.8
	B^*D	13.5	15.6	6.24	10.9
	BD	4.2	4.8	0.55	1.0
	B_sD_s	2.85	3.28	0.18	0.3
	Total	87.3	100	57	100
$2^3\mathrm{D}_3$	B^*D^*	140	61	87	46.1
	B^*D	42.3	18.4	50.8	26.9
	BD	39.5	17.2	41.6	22.1
	BD^*	7.73	3.37	9.29	4.9
	Total	230	100	189	100
$3^3\mathrm{D}_1$	B^*D^*	13.9	45.3	19.5	21.9
	BD^*	7.41	24.1	0.48	0.5
	$B_1(5721)D$	3.46	11.3	0.76	0.9
	$B_s^*D_s^*$	2.24	7.27	16.5	18.5
	B_sD_s	1.29	4.2	2.27	2.5
	$B_s^*D_s$	0.797	2.6	3.16	3.5
	BD	0.782	2.6	25.2	28.2
	$B_2^*(5747)D$	0.328	1.07	...	...
	B^*D	0.28	0.912	5.65	6.3
	$B_sD_s^*$	0.142	0.463	1.82	2.0
	Total	30.7	100	89	100
$3^3\mathrm{D}_3$	B^*D^*	43.9	55.9	18.4	16.4
	BD	14.8	18.9	19.3	17
	B^*D	14.2	18.1	29.7	26.5
	$B_1(5721)D$	1.25	1.59	4.62	4.1
	BD^*	1.64	2.09	20.8	18.6
	$B_2^*(5747)D$	1.08	1.38	...	...
	$B_s^*D_s^*$	0.824	1.05	6.6	5.9
	$B_sD_s^*$	0.628	0.803	2.94	2.6
	B_sDs	...	...	1.45	1.3
	$B^*D_0^*(2300)$	...	...	8.14	7.3
	Total	83.9	100	112	100

TABLE XII. Partial widths and branching ratios of OZI-allowed strong decay for $4^3D_1 - 4^3D_3$-wave B_c states. The width results are in units of MeV.

State	Channels	This work	Br (%)
$4^3\mathrm{D}_1$	$B_2^*(5747)D^*$	30.1	30.5
	$BD(2550)$	20	20.3
	$B^*D_2^*(2460)$	16.3	16.4
	$B_2^*(5747)\mathrm{D}$	6.81	6.9
	$B^*D_1(2430)$	5.59	5.66
	$B^*D_1'(2420)$	3.05	3.09
	BD	3.02	3.06
	B^*D^*	2.94	2.98
	$B_1(5721)\mathrm{D}$	2.77	2.8
	$BD_2^*(2460)$	1.86	1.88
	BD^*	1.16	1.17
	$B_s^*D_s^*$	1.12	1.13
	$BD_1'(2420)$	1.09	1.1
	$B_1(5721)D^*$	1.06	1.07
	Total	98.8	100
$4^3\mathrm{D}_3$	$B^*D_2^*(2460)$	47.5	30.6
	$B_2^*(5747)D^*$	34.1	22.2
	B^*D	10.7	6.77
	BD	9.86	6.35
	B^*D^*	8.37	5.39
	$B^*D_1(2430)$	7.8	5.03
	$BD_2^*(2460)$	6.87	4.43
	$BD_1(2430)$	6.37	4.11
	$BD_1'(2420)$	5.58	3.6
	$B^*D_1'(2420)$	4.81	3.1
	$B_2^*(5747)\mathrm{D}$	4.74	3.05
	BD^*	3.25	2.09
	$B_1(5721)D^*$	1.61	1.03
	$B_s^*D_s^*$	1.29	0.831
	Total	155	100

F-wave B_c states are presented in Table III. From our predictions of the decay properties for these F-wave states, it is found that the $B_c(1F_3')$ state decay via radiative transitions $B_c(1F_3') \to 1^3D_3\gamma$, $B_c(1F_3') \to 1D_2'\gamma$ and $B_c(1F_3') \to 1D_2\gamma$. Our predicted partial width

$$\Gamma[B_c(1F_3') \to 1^3D_3\gamma] \simeq 4\ \text{keV},$$

$$\Gamma[B_c(1F_3') \to 1D_2'\gamma] \simeq 1\ \text{keV},$$

$$\Gamma[B_c(1F_3') \to 1D_2\gamma] \simeq 73\ \text{keV},$$

our total radiative decay width is very close to 82 keV predicted in Ref. [30]. Other radiative transition properties can be found in Tables XV and XVII. the $B_c(1F_3')$ state also has strong decay $B_c(1F_3') \to B^*D$, the predicted partial width

$$\Gamma[B_c(1F_3') \to B^*D] \simeq 0.426\ \text{MeV}.$$

The other $1F$ states $B_c(1F_3)$ has similar decay channels. B^*D^* is the leading decay channel of $B_c(2F)$ and $B_c(2F')$ and the branching ratio is larger than 0.5. B^*D and BD^* are the next-to-leading decay channels of $B_c(2F)$ and $B_c(2F')$, respectively. The total widths of $B_c(2F)$ and $B_c(2F')$ are 127 MeV and 133 MeV, respectively. B^*D^* has the similar contribution for the $B_c(3F)$ and $B_c(3F')$ states and these two states have the total width of 135 MeV and 155 MeV. These two states have different leading decay channel, which are $B^*D_1'(2420)$ and $BD_2^*(2460)$.

$B_c(1^3F_4)$ only has two strong decay channels BD and B^*D, which contribute 78% and 22% for the total width. The results for the total width in this work and the results in Ref. [24] are all small, which are 0.99 MeV and 0.88 MeV, respectively. Similar with $B_c(2F)$ and $B_c(2F')$, B^*D^* is also the leading decay channel of $B_c(2^3F_2)$ and $B_c(2^3F_4)$. The total width of this two states are 124 MeV and 153 MeV, respectively. $B_c(3^3F_2)$ dominantly decays to

TABLE XIII. Partial widths and branching ratios of OZI-allowed strong decay for the mixture of the 1^1F_3 and 1^3F_3 states -3^1F_3 and 3^3F_3 states. The width results are in units of MeV.

State	Channels	This work	Br (%)	[24]	Br (%)
$1F'_3$	B^*D	0.426	100	15.1	100
	Total	0.426	100	15.1	100
$1F_3$	B^*D	27.6	100	8.53	100
	Total	27.6	100	8.53	100
$2F'_3$	B^*D^*	71.4	53.6	80.3	45
	B^*D	53.2	39.9	45.2	25
	BD^*	5.7	4.28	41.0	23
	$B_sD_s^*$	1.98	1.49	4.53	3
	$B_s^*D_s$	1.04	0.78	7.19	4
	Total	133	100	178	100
$2F_3$	B^*D^*	69.2	54.2	90.2	52
	BD^*	49.1	38.5	30.2	17
	B^*D	6.1	4.79	43.9	25
	$B_s^*D_s$	2.75	2.16	7.78	4.5
	$B_sD_s^*$	⋯	⋯	2.57	1.5
	Total	127	100	175	100
$3F'_3$	$BD_2^*(2460)$	53.9	34.8	27.1	8.9
	B^*D	27.2	17.6	33.6	11
	$B^*(5747)D^*$	20.2	13	36.7	12
	$B^*(5747)D$	18.2	11.8	36.7	12
	B^*D^*	16.1	10.4	59.9	19.6
	$B^*D_1(2430)$	6.62	4.28	⋯	⋯
	$B^*D'_1(2420)$	4.59	2.96	38.9	12.8
	$BD_1(2430)$	2.48	1.6	38.9	12.8
	$B_1(5721)D$	1.93	1.25	⋯	⋯
	$B_sD_s^*$	1	0.646	1.69	0.6
	$B_s^*D_s^*$	0.961	0.621	4.85	1.6
	$B_1(5721)D^*$	0.794	0.513	7.75	2.5
	BD^*	0.544	0.351	34.4	11.3
	$B(1P')D^*$	⋯	⋯	30.4	10
	$B_sD_{s0}^*(2317)$	⋯	⋯	3.03	1.0
	$B_s(1^3P_0)D_s$	⋯	⋯	1.38	0.45
	Total	155	100	305	100
$3F_3$	$B^*D'_1(2420)$	33.5	24.8	57.4	17.5
	$B_2^*(5747)D$	28.9	21.4	27.6	8.4
	BD^*	16.9	12.5	31.3	9.5
	$B^*D_1(2430)$	16.1	11.9	⋯	⋯
	B^*D^*	14.7	10.9	63.5	19.3
	$B_1(5721)D^*$	7.79	5.77	2.3	0.7
	B^*D	6.51	4.82	39.9	12
	$BD_1(2430)$	3.69	2.73	8.23	2.5
	$B_1(5721)D$	2.46	1.82	2.26	0.7
	$B_2^*(5747)D^*$	1.75	1.3	⋯	⋯
	$BD_2^*(2460)$	1.37	1.01	34.2	10
	$BD_1(2430)$	⋯	⋯	16.6	5.1
	$B(1P')D^*$	⋯	⋯	8.69	2.6
	$B(1^3P_0)D^*$	⋯	⋯	8.06	2.5
	$B(1P')D$	⋯	⋯	6.25	1.9
	$B_s^*D_s^*$	⋯	⋯	3.63	1.1
	$B_s^*D_s$	⋯	⋯	2.64	0.8
	$B_sD_s^*$	⋯	⋯	2.02	0.6
	$B^*D_0^*(2300)$	⋯	⋯	1.73	0.53
	Total	135	100	329	100

B^*D^* final channel, and $B^*D_1(2430)$ is its next-to-leading decay mode. $B_2^*(5747)$ and BD are the important decay channels. The total width for this state is about 109 MeV. For the $B_c(3^3F_4)$ state, B^*D, B^*D^*, BD and $B_2^*(5747)D^*$ have the widths of 14 MeV, 12 MeV, 10 MeV, and 23 MeV, respectively, which are the main decay modes. Compared to Ref. [24], our results have smaller decay width for the $3F_3$, $3F'_2$, and 3^3F_4 states.

There are many strong decay channels for these higher mass F-wave states. Other detail of decay information can be seen in Tables XIII–XIV.

TABLE XIV. Partial widths and branching ratios of OZI-allowed strong decay for $1^3F_2 - 3^3F_4$-wave B_c states. The width results are in units of MeV.

State	Channels	This work	Br (%)	[24]	Br (%)
1^3F_2	BD	54.7	77.8	61.9	85
	B^*D	15.6	22.2	11.1	15
	Total	70.3	100	73	100
1^3F_4	BD	0.815	82.3	0.85	97
	B^*D	0.112	11.3	0.03	3
	Total	0.991	100	0.88	100
2^3F_2	B^*D^*	102	82.3	151	68
	BD	13.8	11.1	45.1	20.2
	B^*D	2.83	2.29	19.2	8.6
	BD^*	2.77	2.24	0.39	0.2
	$B_s^*D_s$	1.11	0.907	3.63	1.6
	$B_sD_s^*$	0.735	0.593	3.17	1.4
	B_sD_s	0.713	0.575	0.68	0.3
	Total	124	100	223	100
2^3F_4	B^*D^*	75.4	49.4	57	43
	B^*D	29.1	19	20.9	16
	BD^*	28	18.3	37.7	29
	BD	19.1	12.5	8	6
	B_sD_s	0.7	0.458	4.48	3.4
	$B_s^*D_s$	0.474	0.31	3.26	2.5
	Total	153	100	131	100
3^3F_2	B^*D^*	27.5	25.3	72	31.5
	$B^*D_1(2430)$	15.6	14.3	⋯	⋯
	$B_2^*(5747)D$	10.7	9.83	12.1	5.3
	BD	10.3	9.46	32.1	14
	$BD'_1(2420)$	8.75	8.04	⋯	⋯
	$B^*D'_1(2420)$	8.13	7.47	⋯	⋯

(Table continued)

TABLE XIV. *(Continued)*

State	Channels	This work	Br (%)	[24]	Br (%)
	$BD_1(2430)$	6.76	6.21	2.3	1
	$BD_2^*(2460)$	5.73	5.26	···	···
	$B_1(5721)D$	4.28	3.93	5.03	2.2
	$B_1(5721)D^*$	4	3.67	···	···
	B^*D	3.33	3.06	16.1	7
	$B_2^*(5747)D^*$	2.37	2.18	···	···
	$B_s^*D_s^*$	0.79	0.73	5.25	2.3
	$B^*D_0^*(2300)$	···	···	9.14	4
	Total	109	100	228	100
$3^3\mathrm{F}_4$	$B_2^*(5747)D^*$	22.9	23.8	···	···
	B^*D	14.4	14.8	8.9	5.0
	B^*D^*	12.2	12.8	50.9	28.7
	BD	10.7	11.1	2.82	1.6
	$B_2^*(5747)D$	10.6	11	12.3	6.9
	BD^*	10.4	10.6	20.2	11.4
	$BD_1'(2420)$	8.36	8.67	9.19	5.2
	$B_1(5721)D$	2.28	2.37	1.85	1.04
	$B_s^*D_s^*$	1.5	1.54	9.19	5.2
	$B^*D_1'(2420)$	1.07	1.11	2.83	1.6
	$B_1(5721)D^*$	0.628	0.651	3.03	1.7
	$BD_1(2430)$	0.528	0.548	11.2	6.3
	$BD_2^*(2460)$	0.496	0.514	11.2	6.3
	B_sD_s	0.257	0.26	3.2	1.8
	$B_s^*D_s$	0.136	0.135	3.2	1.8
	$B^*D_1(2430)$	0.123	0.128	8.42	4.7
	BD^*	···	···	34.4	11.3
	$B(1P')D$	···	···	19.4	10.9
	$B^*D_0^*(2300)$	···	···	1.91	1.1
	Total	96.4	100	177	100

E. *G*-wave states

In this subsection, we sweepingly discuss the decay features of G-wave B_c states.

The decay properties are presented in Tables XVI and XVIII. It is found that all G-wave B_c states decay via radiative transition and strong transition.

The $B_c(1G')$ mainly decays into BD^*, B^*D^* and B^*D with the branching ratios 59%, 24% and 18% respectively. The corresponding partner $B_c(1G)$ dominantly decay to B^*D, which has the width and branching ratio 88.3 MeV and 0.87.

For the $B_c(2G)$ and $B_c(2G')$, B^*D^* has the similar width, BD^* and B^*D are their two important decay channels. The total widths of these two states are 156 MeV and 67 MeV, respectively.

We predict that the main decay channels of $B_c(1^3G_3)$ are BD, B^*D, and BD^* with corresponding partial widths 64 MeV, 43 MeV, and 20 MeV, respectively. For the $B_c(1^3G_5)$ state, BD and B^*D^* are important decay modes. It is found that this state has a relatively narrow width of ~45 MeV. Furthermore, the radiative transitions have negligible contribution for all these states.

The $B_c(2^3G_3)$ dominantly decays into B^*D^*, and BD channel is the next-to-leading decay channel. $B_c(2^3G_5)$ dominantly decays into B^*D^*, BD^*, and B^*D channels. It has the total width of 54 MeV.

We give our predictions of the $E1$ transitions and strong decays of $B_c(nG)$ ($n = 1$, 2). Combining these $E1$ transitions with their strong decays, we found that the branching fractions of the $E1$ transitions are so small which can be be ignored.

TABLE XV. Partial widths of the E1 dominant radiative transitions for the $2S$-, $1P$-, $1D$-, and $1F$-wave B_c states.

		E_γ (MeV)				Γ_{E1} (keV)			
	Final state	Ours	Ref. [24]	Ref. [31]	Ref. [30]	Ours	Ref. [24]	Ref. [31]	Ref. [30]
2^1S_0	$1P_1'$	110.1	96	138	104	7.56	6.38	15.9	6.1
	$1P_1$	101.2	114	150	113	1.2	5.33	1.9	1.3
2^3S_1	1^3P_2	122.8	102	159	118	7.28	6.98	14.8	5.7
	$1P_1'$	144.4	115	173	136	1.03	1.56	1.0	0.7
	$1P_1$	135.6	133	185	144	4.2	4.62	12.8	4.7
	1^3P_0	192.2	174	219	179	3.45	3.48	7.7	2.9
1^3P_2	1^3S_1	445.3	445	426	416	90.04	87	102.9	83
2^3P_2	1^3D_3	131.7	129	127	118	9.78	14	10.9	6.8
	$1D_2'$	127.8	127	118	122	0.66	0.93	0.5	0.6
	$1D_2$	136.7	135	133	127	1.02	1.1	1.5	0.7
	1^3D_1	136.7	139	126	135	0.112	0.13	0.1	0.1
	2^3S_1	263	265	232	272	49.37	50	49.4	55
	1^3S_1	802.3	785	817	778	30.35	52	25.8	14
$1P_1'$	1^3S_1	424.7	433	412	399	13.75	40	8.1	11
	1^1S_0	469.5	484	476	462	76.5	74	131.1	80
$2P_1'$	$1D_2'$	109	117	108	113	0.96	1.05	3.5	5.5
	$1D_2$	118	125	123	123	7.0	0.03	2.5	1.3
	1^3D_1	118	129	116	121	0.47	1.27	0.3	0.2
	2^3S_1	244.7	255	222	258	8.8	25	5.9	5.5

(Table continued)

TABLE XV. *(Continued)*

		E_γ (MeV)				Γ_{E1} (keV)			
	Final state	Ours	Ref. [24]	Ref. [31]	Ref. [30]	Ours	Ref. [24]	Ref. [31]	Ref. [30]
	1^3S_1	785	777	807	769	4.17	26	2.5	0.6
	2^1S_0	278	274	257	289	43	36	58.0	52
	1^1S_0	827.6	825	871	825	24.42	44	131.1	19
$1P_1$	1^3S_1	433	416	400	391	71.52	70	77.8	60
	1^1S_0	477.8	468	464	454	16.5	35	11.6	13
$2P_1$	$1D'_2$	117	101	97	108	6.4	0.006	1.2	0.8
	$1D_2$	125.8	109	112	103	0.748	0.84	3.9	3.6
	1^3D_1	125.8	113	105	116	1.94	1.45	1.6	1.6
	2^3S_1	252.4	240	211	253	38.48	34.0	32.1	45.0
	1^3S_1	792.5	762	796	761	14.67	40	15.3	5.4
	2^1S_0	286	258	246	284	13.62	19	8.1	5.7
	1^1S_0	834.7	811	860	820	7.32	25	3.1	2.1
1^3P_0	1^3S_1	378.6	377	366	358	59.58	96	65.3	55
2^3P_0	1^3D_1	80.5	86	80	93	3.24	5.6	3.2	4.2
	2^3S_1	208	214	186	231	33.49	53	25.5	42
	1^3S_1	751.5	738	771	741	3.19	41	16.1	1
1^3D_3	1^3P_2	254	239	264	272	65.98	67	76.9	78
$1D'_2$	1^3P_2	258	241	273	263	7.57	8.3	6.8	8.8
	$1P'_1$	279	253	287	280	16.3	41	46	63
	$1P_1$	271	271	301	289	50.45	0.39	25	7
$1D_2$	1^3P_2	249.4	233	258	268	8.71	8.7	12.2	9.6
	$1P'_1$	270.6	246	272	285	56	1.09	18.4	15
	$1P_1$	262	263	284	294	6.28	44	44.6	64
1^3D_1	1^3P_2	249.4	229	265	255	1.74	0.7	2.2	1.8
	$1P'_1$	270.6	242	279	273	5.12	12	3.3	4.4
	$1P_1$	262	259	291	281	22.79	29	39.2	28
	1^3P_0	317.5	299	325	315	52.36	65	79.9	55
1^3F_4	1^3D_3	204	194	···	222	63.93	69	···	81
$1F'_3$	1^3D_3	215.7	207	···	227	3.59	4.76	···	5.4
	$1D'_2$	211.8	205	···	231	1.32	32	···	82
	$1D_2$	220.5	212	···	236	73.2	0.04	···	0.04
$1F_3$	1^3D_3	203	191	···	218	4.0	4.91	···	3.7
	$1D'_2$	199.2	189	···	222	56.24	0.22	···	0.5
	$1D_2$	208	197	···	226	0.58	29	···	78
1^3F_2	1^3D_3	212.8	202	···	221	0.32	0.12	···	0.4
	$1D'_2$	208.9	200	···	224	5.0	5.72	···	5.3
	$1D_2$	217.6	208	···	229	6.57	6.36	···	6.5
	1^3D_1	217.6	212	···	221	61.39	78	···	75

TABLE XVI. Partial widths and branching ratios of OZI-allowed strong decay for *G*-wave B_c states. The width results are in units of MeV.

State	Channels	This work	Br (%)	State	Channels	This work	Br (%)
$1\mathrm{G}'_4$	BD^*	45.5	58.5	$1^3\mathrm{G}_3$	BD	63.7	46.6
	B^*D^*	18.4	23.6		B^*D	42.5	31.1
	B^*D	13.8	17.9		BD^*	20.3	14.9
					B^*D^*	9	6.59
					B_sD_s	0.916	0.671
	Total	77.8	100		Total	137	100
$1\mathrm{G}_4$	B^*D	88.3	87	$1^3\mathrm{G}_5$	B^*D^*	28.6	64.1
	B^*D^*	11.6	11.2		BD	9.04	20.3

(Table continued)

TABLE XVI. *(Continued)*

State	Channels	This work	Br (%)	State	Channels	This work	Br (%)
	BD^*	1.64	1.62		B^*D	6.18	13.8
	$B_s^*D_s$	0.126	0.122		BD^*	0.821	1.84
	Total	102	100		Total	44.6	100
$2G_4'$	B^*D^*	41	61.5	2^3G_3	B^*D^*	74.1	57.5
	B^*D	12.3	18.5		BD	23.6	18.9
	BD^*	7.73	11.6		B^*D	13.1	10.5
	$B_s^*D_s^*$	1.49	2.23		$B_2^*(5747)D$	10.6	8.28
	$B_1(5721)D$	1.31	1.97		$B_1(5721)D$	7.62	5.69
	$B_s^*D_s$	1.17	1.75		BD^*	3.3	2.58
	$B_sD_s^*$	0.88	1.32		$B_s^*D_s^*$	1.2	0.937
	$B_2^*(5747)D$	0.693	1.04		$B_sD_s^*$	0.384	0.3
	Total	66.8	100		Total	134	100
$2G_4$	$B_2^*(5747)D$	56.8	36.5	2^3G_5	B^*D^*	22.4	41.1
	B^*D^*	40.1	25.8		BD^*	15.4	28.3
	B^*D	28.5	18.3		B^*D	7.56	14.1
	BD^*	27.8	17.9		BD	3.12	5.82
	$B_s^*D_s^*$	1.32	0.848		$B_s^*D_s^*$	2.55	4.68
	$B_1(5721)D$	0.561	0.36		$B_1(5721)D$	0.699	1.28
	$B_sD_s^*$	0.551	0.354		$B_2^*(5747)D$	0.237	0.442
	Total	156	100		Total	54.4	100

TABLE XVII. Partial widths of the E1 dominant radiative transitions for the $2D$- and $2F$-wave B_c states.

Initial state	Final state	Γ_{E1} (keV)	Initial state	Final state	Γ_{E1} (keV)
2^3D_1	$1P_1'$	0.526	2^3D_3	1^3P_2	7.46
	$1P_1$	2.47		2^3P_2	48.7
	$2P_1'$	5.12		1^3F_4	7.28
	$2P_1$	15.5			
	1^3P_0	13.5			
	1^3P_2	0.1			
	2^3P_0	37.2			
	2^3P_2	1.29			
	1^3F_2	5.43			
$2D_2'$	$1P_1'$	1.7	$2D_2$	$2P_1'$	50.2
	$1P_1$	7.36		$2P_1$	2.6
	$2P_1'$	8.4		2^3P_2	6.36
	$2P_1$	39.54		$1F_3$	0.05
	1^3P_2	0.6			
	2^3P_2	5.51			
	$1F_3'$	0.11			
2^3F_2	$2D_2'$	3.83	2^3F_4	1^3D_3	4.96
	$2D_2$	4.54		2^3D_3	51.2
	1^3D_1	6.3		1^3G_5	5.38
	2^3D_1	48.9			
	1^3G_3	4.61			
$2F_3'$	$2D_2'$	1.05	$2F_3$	$1D_2$	0.06
	$1D_2$	5.73		$2D_2$	0.29
	$1G_4$	7.29		2^3D_3	3.12
	$1G_4'$	0.1			
	$1D_2'$	0.11			

TABLE XVIII. Partial widths of the E1 dominant radiative transitions for the G-wave B_c states.

Initial state	Final state	Γ_{E1} (keV)	Initial state	Final state	Γ_{E1} (keV)
1^3G_3	$1F_3'$	2.25	2^3G_3	$1F_3'$	0.145
	1^3F_2	62.4		$2F_3'$	1.88
	$1F_3$	3.68		1^3F_2	4.55
	1^3F_4	0.1		$1F_3$	0.21
				1^3F_4	0.006
				2^3F_2	52.86
				$2F_3$	3.05
				2^3F_4	0.083
				1^3H_4	3.8
1^3G_5	1^3F_4	64.7	2^3G_5	1^3F_4	3.92
				2^3F_4	52.59
				$1H_5'$	0.042
				1^3H_4	0.0008
				$1H_5$	0.08
$1G_4'$	$1F_3'$	1.43	$1G_4$	$1F_3'$	48.83
	$1F_3$	78.66		$1F_3$	0.83
	1^3F_4	2.2		1^3F_4	2.23
$2G_4'$	$1F_3'$	0.08	$2G_4$	$1F_3'$	3.78
	$2F_3'$	1.1		$2F_3'$	40.97
	$1F_3$	4.48		$1F_3$	0.059
	1^3F_4	0.123		1^3F_4	0.145
	$2F_3$	64.9		$2F_3$	0.639
	2^3F_4	1.82		2^3F_4	1.84
	$1H_5'$	0.057		$1H_5'$	2.36
	1^3H_4	0.07		1^3H_4	0.056
	$1H_5$	6.23		$1H_5$	0.081

V. CONCLUSION

The mass spectra of B_c mesons are studied in this paper using Cornell potential with the screening effect. The parameters of the potential model by fitting B_c, $b\bar{b}$, $c\bar{c}$, B, B_s, D and D_s mesons are given, we also predict the masses of the excited states of B_c mesons. According to our research, the mass of the higher exited states in this work is lower than most other potential models, which is caused by the screening effect. For the radiative decays of B_c mesons, we obtain the decay widths of the E1 dipole for their $S-G$ waves. The decay widths are all small, within 1 MeV. We also study the decay width of the S-wave B_c meson state to G-wave B_c meson state using the ${}^3P_0$ model and give the branching ratios. Analysis the strong two-body decay of the B_c mesons, the important decay channel is B^*D^* for many B_c mesons. In addition, $B_s^*D_s$ and $B_sD_s^*$ make small contribution. Some main results are emphasized as follows.

For the S-wave states, the low-lying S-wave states $B_c^*(2S)$ and $B_c(2S)$ only decay via radiative transitions. Other S-wave states all decay via radiative transitions and strong decay and B^*D^* makes large contribution for almost all higher S-wave states. For the P-wave states, it is found that the decay modes of the $1P$ only has $E1$ decay processes with small total width less than 10^{-1} MeV and BD channel is the most important decay mode of $B_c(2^3P_2)$ state. For the D-wave and F-wave states, some of them present some radiative decay channels with large widths which are allowed to observation. This may help for establishing the D-wave and F- wave B_c states in the future experiments. All the G-wave B_c states have strong decays and radiative transitions, but the contribution of radiative transitions is very small.

We expect that our research can provide some helpful information for searching for B_c mesons in future experiments.

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