Three-particle distribution in the B meson and charm-quark loops in FCNC B decays

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We discuss a nonfactorizable (NF) contribution of a charm loop to the FCNC B-decay amplitude given through the three-particle Bethe-Salpeter amplitude (3BS) of the B-meson. This 3BS contains one heavy-quark field and two light fields (a light quark and a gluon). Our discussion is aimed at clarifying properties of the B-meson 3BS necessary to describe properly charm-loop contributions to the amplitudes of FCNC B-decays. We demonstrate that the dominant contribution of nonfactorizable charm to FCNC B-decay amplitude is given in the heavy-quark limit by a convolution of some hard kernel and the B-meson 3BS in a "double-collinear" light cone (LC) configuration: one of the light degrees of freedom $\phi(x)$, $x^2 = 0$, lies on the $(+)$-direction of the LC, whereas another light degree of freedom $\phi'(x')$, $x'^2 = 0$ lies along the $(-)$-direction. We show the emergence of new constraints on the distribution amplitudes which parametrize the 3BS in this double-collinear configuration.

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I. INTRODUCTION

Charming loops in rare flavor-changing neutral current (FCNC) decays of the B-meson have impact on the B-decay observables [1] and provide an unpleasant noise for the studies of possible new physics effects (see, e.g., [2–9]).

A number of theoretical analyses of nonfactorizable (NF) charming loops in FCNC B-decays have been published. In [10], an effective gluon-photon local operator describing the charm-quark loop was calculated as an expansion in inverse charm-quark mass m_c and applied to inclusive $B \to X_s\gamma$ decays (see also [11,12]). In [13] NF corrections in $B \to K^*\gamma$ using local operator product expansion (OPE) have been studied. NF corrections induced by the local photon-gluon operator were calculated in [14,15] in terms of the light cone (LC) 3-particle antiquark-quark-gluon Bethe-Salpeter amplitude (3BS) of K^*-meson [16–18] with two field operators having equal coordinates, $\langle 0|\bar{s}(0)G_{\mu\nu}(0)u(x)|K^*(p)\rangle$, $x^2 = 0$. However, local OPE for the charm-quark loop in FCNC B decays leads to a power series in $\Lambda_{\rm QCD}m_b/m_c^2$; numerically this parameter is close to one. To sum up $O(\Lambda_{\rm QCD}m_b/m_c^2)^n$ corrections, Ref. [19] obtained a nonlocal photon-gluon operator describing the charm-quark loop and evaluated its effect making use of 3BS of the B-meson in a collinear LC configuration $\langle 0|\bar{s}(x)G_{\mu\nu}(ux)b(0)|B(p)\rangle$, $x^2 = 0$ [20,21]. This approximation was later used for the analysis of other FCNC B-decays [22].

The collinear LC configuration was known to provide the dominant 3BS contribution to meson tree-level form factors [23,24]; in particular, to form factors of semileptonic (SL) B-decay induced by $b \to u$ weak charged current (CC). It was tempting to use the collinear 3BS for the description of FCNC B-decays. However, the 3BS contribution to the CC B-decay and to the FCNC B-decay have a qualitative difference. To demonstrate this difference, let us consider the B-decay in the B-meson rest frame. In CC B-decays, the b-quark emits a fast light u-quark which is later hit by a soft gluon and thus keeps moving in the same space direction. In the case of charming loops in FCNC B-decays, a fast light s-quark and a pair of fast c-quarks emitted by the b-quark move in the opposite space directions. In [25–27] it was demonstrated that the FCNC B-decay amplitude is dominated not by the collinear configuration of 3BS, but rather by a double-collinear configuration $\langle 0|\bar{q}(x)G_{\mu\nu}(x')b(0)|B(p)\rangle$, $x^2 = 0$, $x'^2 = 0$, but $(x - x')^2 \neq 0$. The first application of a double-collinear 3BS to FCNC B-decays was presented in [28].

The noncollinear configuration of the B-meson 3BS leads to the appearance of new Lorentz structures of the type $(x_\mu p_\nu - x_\mu p_\nu)/(xp)$ and $(x'_\mu p_\nu - x'_\mu p_\nu)/(x'p)$. These Lorentz structures contain kinematical singularities at $1/(xp)$ and $1/(x'p)$ which should not be singularities of the physical 3BS. This requirement and the continuity of the 3BS at the point $x^2 = 0$, $x'^2 = 0$, $xp = 0$, and $x'p = 0$ yield constraints on the corresponding distribution amplitudes (DAs). Moreover, these constraints turn out to be valid also for those DAs which appear in the collinear 3BS. To avoid complications related to the spinorial structure of the 3BS in QCD, Sec. II demonstrates the derivation of these constraints in the case of field theory with scalar "quarks". A generalization to QCD is straightforward.

In Sec. III we analyze Feynman diagrams of the type corresponding to charming loops in FCNC B-decays (i.e., those diagrams in which the heavy field hits the middle point of the line along which light degrees of freedom propagate; hereafter referred to as FCNC-type diagrams) and show that to the leading order in the heavy-quark (HQ) expansion, the amplitude is given by the convolution of the hard kernel composed of propagators of the light degrees of freedom and the B-meson 3BS in the double-collinear configuration. We also derive the exact expression for the B-decay amplitudes of FCNC-type using the α-representation and show that the exact amplitudes and the double-collinear approximation for these amplitudes coincide in the heavy-quark limit.

II. PROPERTIES OF THE 3BS WAVE FUNCTION OF THE B-MESON

Leaving aside technical details related to the spins of the constituent fields and of the bound state, and considering heavy B-meson as bound state of scalar fields, one of which, $\phi_b(x)$, is heavy, the 3BS of the B-meson may be parametrized as follows ([27] and references therein):

$$\langle 0|\phi(x)\phi_b(0)\phi'(x')|B(p)\rangle = \int D(\omega,\omega')e^{-ipx\omega - ipx'\omega'} \times \Psi(\omega,\omega',x^2,x'^2,(x-x')^2), \tag{2.1}$$

where

$$D(\omega,\omega') \equiv d\omega d\omega' \theta(\omega)\theta(\omega')\theta(1-\omega-\omega'). \tag{2.2}$$

For $\Psi(\omega,\omega',x^2,x'^2,(x-x')^2)$ one can write down Taylor expansion in its variables x^2, x'^2, $(x-x')^2$ as

$$\Psi(\omega,\omega',x^2,x'^2,(x-x')^2) = \Psi_0(\omega,\omega') + x^2\Psi_{12}(\omega,\omega') + x'^2\Psi_{23}(\omega,\omega') + (x-x')^2\Psi_{13}(\omega,\omega') + \ldots, \tag{2.3}$$

where $\ldots$ stands for higher powers of $x^2, x'^2, (x-x')^2$. The distribution amplitudes Ψ_i have support in the region $0 < \omega$, $0 < \omega'$, $\omega + \omega' < 1$, and peak at small values $(\omega, \omega') = O(\Lambda_{\mathrm{QCD}}/M_B)$, reflecting the fact that the heavy b-quark carries almost the full momentum of the heavy B-meson, whereas the light degrees of freedom carry its small fraction $O(\Lambda_{\mathrm{QCD}}/M_B)$. (In practical model calculations, the DAs may be nonzero in a narrower region, e.g., $0 < \omega$, $0 < \omega$, $\omega + \omega' < 2\omega_0 < 1$.)

Interesting constraints on the DAs emerge in those cases when the 3BS has a nontrivial Lorentz structure. We shall consider a more complicated 3BS, when one of the light scalar fields is replaced by the gauge field, $\phi(x) \to G_{\mu\nu}(x)$,[1]

$$\langle 0|G_{\mu\nu}(x)\phi_b(0)\phi'(x')|B(p)\rangle. \tag{2.4}$$

Our discussion will be technically rather simple but will allow a direct generalization to QCD, in which case more Lorentz structures emerge.

A. Collinear 3BS of B-meson

In the literature, much attention has been given to the collinear 3BS, where the coordinates of the light degrees of freedom are proportional to each other [24],

$$\begin{aligned}&\langle 0|G_{\mu\nu}(ux)\phi_b(0)\phi'(x)|B(p))\rangle \\ &= \int D(\omega,\omega')e^{-i(u\omega+\omega')px} \\ &\quad\times\left(\frac{x_\mu p_\nu - x_\nu p_\mu}{xp}\right)[X_A(\omega,\omega') + O(x^2)], \qquad 0 < u < 1.\end{aligned} \tag{2.5}$$

For $x^2 = 0$ and $xp \to 0$, all components of the 4-vector x_μ vanish in the rest frame of the B-meson ($p_+ = p_-$, $p_\perp = 0$),

$$\begin{aligned}&xp = x_+p_- + x_-p_+ - x_\perp p_\perp = 0 \Rightarrow x_+ = -x_-,\\ &x^2 = 2x_+x_- - x_\perp^2 = -2x_+^2 - x_\perp^2 = 0 \Rightarrow x_+ = -x_- = 0,\\ &x_\perp = 0.\end{aligned}$$

So, if (2.5) is defined only for $x^2 = 0$, one can introduce the new variable τ, $x_\mu = \tau n_\mu$, $n^2 = 0$ [n_μ lies, e.g., along the $(+)$ direction of the light cone]. Then the Lorentz structure $(x_\mu p_\nu - x_\nu p_\mu)/xp$ has no singularity at $\tau \to 0$ and therefore the amplitude (2.5) is finite at $xp \to 0$ [21].

However, if one considers the collinear 3BS (2.5) also at $x^2 \neq 0$, then the singularity at $xp \to 0$ in the Lorentz structure emerges. The 3BS (2.5) as the function of two arguments x^2 and xp should be continuous and finite at

[1] In QCD, the analogous 3BS is $\langle 0|\bar{q}(x')G_{\mu\nu}(x)b(0)|B(p)\rangle$. This amplitude is not gauge-invariant as it contains field operators at different locations. Gauge-invariant 3BS is constructed by introducing the Wilson lines as follows: $\langle 0|\bar{q}(x')U(x',x) G_{\mu\nu}(x)U(x,0))b(0)|B(p)\rangle$ with $U(x,y) = P\exp(i\int_y^x A_\mu(z)dz_\mu)$.

$x^2 = 0$ and $xp = 0$, so one has to require that the kinematical singularity of the Lorentz structure should not be the singularity of the 3BS (2.5). Expanding the exponential in (2.5) and requiring the absence of a singularity at $xp \to 0$ in the rhs of Eq. (2.5) leads to a well-known constraint [see Eq. (5.5) of [21]],

$$\int D(\omega,\omega')X_A(\omega,\omega') = 0. \tag{2.6}$$

B. Generalization to a noncollinear kinematics

For a proper description of the nonfactorizable charm in FCNC B-decay, 3BS of the B-meson in a noncollinear configuration is necessary (see [25–28] and the demonstration in the next section). We therefore consider the generalization of the amplitude (2.5) and allow the gluon field $G_{\mu\nu}(x)$ and the light-quark field $\phi'(x')$ to have arbitrary different noncollinear coordinates. Then a more general decomposition emerges,[2]

$$\langle 0|G_{\mu\nu}(x)\phi_b(0)\phi'(x')|B(p)\rangle = \int D(\omega,\omega')e^{-i\omega xp - i\omega' x'p} \times \left\{\left(\frac{x_\mu p_\nu - x_\nu p_\mu}{xp}\right)X_A^{(1)}(\omega,\omega') + \left(\frac{x'_\mu p_\nu - x'_\nu p_\mu}{x'p}\right)X_A^{(2)}(\omega,\omega') + \dots\right\}. \tag{2.7}$$

The amplitude (2.7) contains factors $1/xp$ and $1/x'p$ in the Lorentz structures. If (2.7) is defined for $x^2 = 0$ and $x'^2 = 0$ only, one can introduce two new variables τ and τ' such that $x_\mu = \tau n_\mu$, $x'_\mu = \tau' n'_\mu$, $n^2 = 0$ and $n'^2 = 0$, but $n'n \neq 0$ [e.g., n_μ lies along the (+)-direction of the light cone, and n'_μ lies along the (−)-direction, see also [28]]. Then the Lorentz structure $(x_\mu p_\nu - x_\nu p_\mu)/xp$ has no singularity at $\tau \to 0$, the Lorentz structure $(x'_\mu p_\nu - x'_\nu p_\mu)/x'p$ has no singularity at $\tau' \to 0$, and therefore the amplitude (2.7) has no singularity at $xp \to 0$ and $x'p \to 0$.

However, the noncollinear 3BS (2.7) should be a regular continuous function of its arguments at the point $x^2 = 0$, $x'^2 = 0$, $xp = 0$, and $x'p = 0$ independently of the way one approaches this point. If one first takes the limit $xp \to 0$ and $x'p \to 0$ keeping $x^2 \neq 0$ and $x'^2 \neq 0$, then the singularities at $xp \to 0$ and $x'p \to 0$ in the Lorentz structures emerge. To compensate these singularities, the DAs should satisfy the following constraints [obtained by expanding the exponential in (2.7)]

$$\int_0^{1-\omega'} d\omega X_A^{(1)}(\omega,\omega') = 0 \quad \forall\omega' \quad \text{and}$$
$$\int_0^{1-\omega} d\omega' X_A^{(2)}(\omega,\omega') = 0 \quad \forall\omega. \tag{2.8}$$

For X_A parametrizing the 3BS in the collinear limit, $x = ux'$, one finds the relation

$$X_A(\omega,\omega') = X_A^{(1)}(\omega,\omega') + X_A^{(2)}(\omega,\omega'). \tag{2.9}$$

Obviously, the condition (2.6) follows from (2.8). However, Eq. (2.8) shows that each of the parts of $X_A(\omega,\omega')$ should satisfy more restrictive constraints.

For further use we introduce the primitives:

$$\bar{X}_A^{(1)}(\omega,\omega') = \int_0^{\omega} d\underline{\omega} X_A^{(1)}(\underline{\omega},\omega'),$$
$$X_A^{(1)}(\omega,\omega') = \partial_\omega \bar{X}_A^{(1)}(\omega,\omega'), \tag{2.10}$$

$$\bar{X}_A^{(2)}(\omega,\omega') = \int_0^{\omega'} d\underline{\omega}' X_A^{(2)}(\omega,\underline{\omega}'),$$
$$X_A^{(2)}(\omega,\omega') = \partial_{\omega'} \bar{X}_A^{(2)}(\omega,\omega'). \tag{2.11}$$

By virtue of (2.8), we obtain the following constraints on the primitives:

$$\bar{X}_A^{(1)}(\omega=0,\omega') = \bar{X}_A^{(1)}(\omega=1-\omega',\omega') = 0 \quad \forall\omega'; \tag{2.12}$$

$$\bar{X}_A^{(2)}(\omega,\omega'=0) = \bar{X}_A^{(2)}(\omega,\omega'=1-\omega) = 0 \quad \forall\omega. \tag{2.13}$$

These relations mean that the primitives should vanish on the boundary of the DA's support region.

Performing parts integration in ω or ω', and taking into account that the surface terms vanish due to (2.12) and (2.13), we may rewrite (2.7) in the form that does not contain factors $1/xp$ and $1/x'p$,[3]

[2]We do not display here further structures like $(x_\mu x'_\nu - x'_\mu x_\nu)/((xp)(x'p))$. The way one should properly treat the corresponding DAs will become obvious.

[3]Note, the functions of the type X_A extensively used in the literature [19,21,22] do not satisfy (2.8). In this case, the surface terms do not vanish and the 3BS given by Eqs. (2.7) and (2.14) are not equivalent to each other.

$$\begin{aligned}&\langle 0|G_{\mu\nu}(x)\phi_b(0)\phi'(x')|B(p)\rangle\\&= i\int D(\omega,\omega')e^{-i\omega xp-i\omega' x'p}\\&\quad\times\{(x_\mu p_\nu - x_\nu p_\mu)\bar{X}_A^{(1)}(\omega,\omega')\\&\quad+(x'_\mu p_\nu - x'_\nu p_\mu)\bar{X}_A^{(2)}(\omega,\omega')+\ldots\}.\end{aligned} \tag{2.14}$$

Let us summarize the material of this section. We required that the noncollinear 3BS is a continuous function of its arguments at $x^2=0$, $x'^2=0$, $xp=0$, and $x'p=0$ and has a finite value at this point. Then, the kinematical singularities in the Lorentz structures at $xp\to 0$ and $x'p\to 0$ should be compensated by certain properties of the DAs: namely, for the DA of the X_A-type, its primitives should vanish on the boundary of X_A-support region. This property also guarantees that two forms of the 3BS, Eqs. (2.7) and (2.14), are equivalent and may be safely applied to the calculation of the amplitudes of B-meson decays.

III. B-DECAY IN THE KINEMATICS OF CHARMING LOOPS IN FCNC DECAYS

In this section we focus on 3BS contributions involving charming loops in FCNC B-decays and show that to the leading order in the HQ expansion the amplitude is given by the convolution of the hard kernel and the B-meson 3BS in the noncollinear kinematical configuration.

A. The dominant contribution to the FCNC B-decay amplitude involving charming loops

As noticed in [27] (see the Appendix for details) the kinematics of FCNC B-decay amplitudes involving charming loops is equivalent to the 3BS correction to the B-decay form factor with the difference that the heavy field, $\phi_b(0)$, is in the middle of the quark line. Placing the heavy-quark field at zero, $\phi_b(0)$, we have [Fig. 1]

$$\begin{aligned}A(p|q,q') = &\int\frac{dx dx' dk dk'}{(\mu^2-k^2)(m^2-k'^2)}e^{iqx+ikx+iq'x'-ik'x'}\\&\times\langle 0|\phi(x)\phi_b(0)\phi'(x')|B(p)\rangle,\end{aligned} \tag{3.1}$$

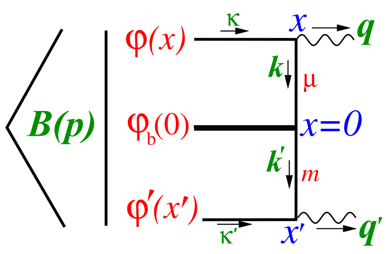

FIG. 1. The generic 3BS amplitude of the form factor topology. If the field ϕ_b is heavy, while other fields ϕ and ϕ' are light, the diagram corresponds to the topology of charming loops in FCNC B-decay amplitude.

where ϕ and ϕ' are light degrees of freedom (in practical calculations, the gluon and the light quark). We are going to derive the leading-order behavior of this amplitude.

1. The x-vertex

Let us discuss the x-vertex and introduce $\kappa = k+q$, the momentum carried by the constituent field $\phi(x)$,

$$\int\frac{dx d\kappa}{\mu^2-(\kappa-q)^2}e^{i\kappa x}\langle 0|\phi(x)\ldots|B(p)\rangle. \tag{3.2}$$

We consider the case $q^2=q'^2=0$ and work in the rest frame of the B-meson and take the axes such that momentum q is along $(+)$-axis, whereas momentum q' is along the $(-)$-axis. [One can formulate this in a covariant form by introducing vectors n_μ and $\bar{n}_\mu$ [21,28].]

Due to the properties of the B-meson 3BS, the vector κ is soft, i.e., all its components are $\kappa_\mu\sim O(\Lambda_{\rm QCD})$. The component q_+ is large, $q_+\sim M_B$, and the propagator is highly virtual, $\mu^2-2\kappa_-(\kappa_+-q_+)+\kappa_\perp^2\sim\Lambda_{\rm QCD}M_B$.

Let us expand the field operator $\phi(x)$ near $x=0$ (this would correspond to considering a tower of local operators of the increasing dimension). The expansion in powers of x_- and $x_\perp$ leads to a well-behaved Taylor series as

$$\begin{aligned}&x_- e^{i\kappa_+x_-}\frac{1}{\mu^2-2\kappa_-(\kappa_+-q_+)+\kappa_\perp^2}\\&\to\frac{1}{\mu^2-2\kappa_-(\kappa_+-q_+)+\kappa_\perp^2}\partial_{\kappa_+}e^{i\kappa_+x_-}\\&\to e^{i\kappa_+x_-}\partial_{\kappa_+}\left(\frac{1}{\mu^2-2\kappa_-(\kappa_+-q_+)+\kappa_\perp^2}\right)\\&\to e^{i\kappa_+x_-}\frac{\kappa_-}{(\mu^2-2\kappa_-(\kappa_+-q_+)+\kappa_\perp^2)^2}.\end{aligned} \tag{3.3}$$

Since $\kappa_-=O(\Lambda_{\rm QCD})$ and the virtuality of the propagator is $O(\Lambda_{\rm QCD}M_B)$, any term $(x_-)^n$ is suppressed by a factor $(1/M_B)^n$ compared to the term $(x_-)^0$. The same property holds for $(x_\perp)^n$.

However, for powers of the variable x_+ the situation is different,

$$\begin{aligned}&x_+ e^{i\kappa_-x_+}\frac{1}{\mu^2-2\kappa_-(\kappa_+-q_+)+\kappa_\perp^2}\\&\to\frac{1}{\mu^2-2\kappa_-(\kappa_+-q_+)+\kappa_\perp^2}\partial_{\kappa_-}e^{i\kappa_-x_+}\\&\to\partial_{\kappa_-}\left(\frac{1}{\mu^2-2\kappa_-(\kappa_+-q_+)+\kappa_\perp^2}\right)e^{i\kappa_-x_+}\\&\to e^{i\kappa_-x_+}\frac{q_+}{(\mu^2-2\kappa_-(\kappa_+-q_+)+\kappa_\perp^2)^2}.\end{aligned} \tag{3.4}$$

Since $q_+\sim M_B$, all powers of x_+^n in the expansion of $\phi(x_+)$ near $x_+=0$ have the same order of magnitude. The Taylor expansion of $\phi(x_+)$ near $x_+ = 0$ leads to no hierarchy in the corresponding expansion of the B-decay amplitude, and we need to keep the full x_+ dependence of the operator $\phi(x_+)$ on the light cone ($x^2 = 0$). This result is valid for two phenomenologically interesting cases: (i) the charm-quark loop contribution to the FCNC amplitude, $\mu^2 \sim m_c^2 = O(\Lambda_{\mathrm{QCD}} m_b)$; and (ii) the light-quark loop contribution to the FCNC amplitude, μ being the light-quark mass.

So, the leading term of the expansion of the B-decay amplitude related to the x-vertex corresponds to the expansion near $x_- = 0$, $x_\perp = 0$ and has the form

$$\int dx_+ dx_- dx_\perp d\kappa_+ d\kappa_- d\kappa_\perp \frac{1}{\mu^2 - 2(\kappa_+ - q_+)k_- + k_\perp^2} \times e^{i\kappa_+ x_- + i\kappa_- x_+ - i k_\perp x_\perp} \langle 0|\ldots\phi(x_+)\ldots|B(p)\rangle. \quad (3.5)$$

The x_- and $x_\perp$ integrals here may be taken and lead to $\delta(\kappa_\perp)\delta(\kappa_+)$. Integrating these δ-functions, we obtain for the part of the amplitude related to the x-vertex (we denote $\tau = x_+$, and recall that q has only the $(+)$-component),

$$\int d\tau d\kappa_- \frac{1}{\mu^2 + 2q_+\kappa_-} e^{i\kappa_- \tau} \langle 0|\phi(\tau)\ldots|B(p)\rangle. \quad (3.6)$$

2. The x'-vertex

We can perform a similar analysis of the x'-vertex. The crucial difference is that now $q_- \sim M_B$ is the only nonzero component of the vector q'. The propagator has the form $m^2 - 2\kappa'_-(\kappa'_- - q_-) + \kappa'^2_\perp \sim \Lambda_{\mathrm{QCD}} M_B$. Obviously, we can perform Taylor expansion of $\phi'(x')$ near $x'_+ = 0$ and $x'_\perp = 0$ but have to keep its full dependence on the variable x_-. Taking into account this property and denoting $\tau' = x'_-$, the dominant contribution of the x'-vertex reads ($q_- \sim M_B$ is the only nonzero component of the vector q')

$$\int d\tau' d\kappa'_+ \frac{1}{m^2 + 2q'_-\kappa'_+} e^{i\kappa'_+ \tau'} \langle 0|\ldots\phi'(\tau')\ldots|B(p)\rangle. \quad (3.7)$$

3. The amplitude of the FCNC B-decay

Making use of the leading contributions of the x- and x'-vertices we obtain the leading contribution to the B-decay amplitude in the form (see also [28]),

$$A(p|q,q') = \int d\tau d\kappa_- \frac{1}{\mu^2 + 2q_+\kappa_-} e^{i\kappa_- \tau} \times \int d\tau' d\kappa'_+ \frac{1}{m^2 + 2q'_-\kappa'_+} \times e^{i\kappa'_+ \tau'} \langle 0|\phi(\tau)\phi_b(0)\phi'(\tau')|B(p)\rangle. \quad (3.8)$$

This relation is the factorization theorem that represents the dominant contribution to the FCNC amplitude as the convolution of the hard kernel composed from the propagators of the light degrees of freedom and the 3BS in the kinematical configuration which may be called "double collinear"; the upper and the lower parts of the diagram are aligned along the different light cone directions.

4. On the collinear light cone 3DA in FCNC B-decay

We would like to emphasize that the 3BS in a collinear light cone configuration

$$\langle 0|\phi(x)\phi_b(0)\phi'(\lambda x)|B(p)\rangle, \quad x^2 = 0, \quad 0 < \lambda < 1, \quad (3.9)$$

is not related to the dominant contribution to the amplitude of FCNC B-decay. So, we do not find justification of the statement of [19] that the dominant contribution to the FCNC amplitude may be calculated via the collinear light cone 3DA of the B-meson (3.9).

5. Multiparticle BS contributions to amplitudes of B-decays

Our analysis may be generalized to other contributions to B-decay amplitudes of the type shown in Fig. 2. The corresponding diagram involves a multiparticle BS of the B-meson,

$$\langle 0|\phi(x)\phi_1(x_1)\ldots\phi_n(x_n)\phi_b(0)\phi'_{n'}(x'_{n'})\ldots \times \phi'_1(x'_1)\phi'(\tau')|B(p)\rangle, \quad (3.10)$$

where all fields except for ϕ_b are light. Combining the analysis presented above and the discussion of Sec. 3 of [27], one can easily prove by induction that the dominant contribution to the B-decay amplitude comes from the double-collinear light cone configuration $[a_\mu = (a_+, a_-, a_\perp)]$,

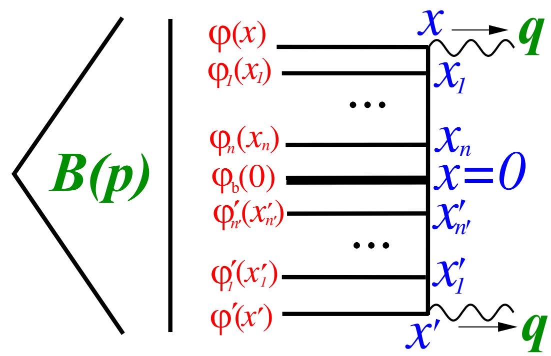

FIG. 2. An example of a multiparticle BS contribution to the B-decay amplitude. The field ϕ_b is heavy, while all other fields, $\phi, \phi', \ldots,$ are light. The dominant contribution comes from the double collinear LC configuration: $x_1 = u_1 x, \ldots, x_n = u_n x, 0 < u_n < \ldots < u_1 < 1, x^2 = 0$; $x'_1 = u'_1 x', \ldots, x'_{n'} = u_{n'} x', 0 < u'_{n'} < \ldots < u'_1 < 1, x'^2 = 0$, whereas $xx' \neq 0$, i.e., the set of collinear field coordinates in the upper part of the diagram is not aligned with the set of collinear field coordinates in the lower part of the diagram.

$$x = (\tau, 0, 0), \qquad x_1 = (\tau u_1, 0, 0), \quad \ldots,$$
$$x_n = (\tau u_n, 0, 0), \qquad 0 < u_n < \ldots < u_1 < 1,$$
$$x' = (0, \tau', 0), \qquad x'_1 = (0, \tau' u'_1, 0), \quad \ldots,$$
$$x'_{n'} = (0, \tau' u'_{n'}, 0), \qquad 0 < u'_{n'} < \ldots < u'_1 < 1. \tag{3.11}$$

The coordinates $x, x_1, \ldots, x_n$ are ordered and lie on the (+)-axis of the LC, and the coordinates $x', x'_1, \ldots, x'_{n'}$ are ordered and lie on the (−)-axis of the LC. That is why we refer to this configuration as to the double collinear light cone configuration.

6. Collinear vs double-collinear 3DAs

Here we would like to emphasize the differences between the collinear and the double-collinear 3DAs. Let us get back to Eq. (2.7). Again introduce LC variables in the B-meson rest frame and consider two vectors $n_\mu = (\sqrt{2}, 0, 0)$ and $n'_\mu = (0, \sqrt{2}, 0)$ such that $n^2 = 0$, $n'^2 = 0$, $nn' = 2$, $v_\mu = \frac{1}{2}(n_\mu + n'_\mu)$, $nv = n'v = 1$:

(i) In a collinear LC configuration, $x^2 = 0$, $x'^2 = 0$, $x'_\mu = u x_\mu$, we can choose x_μ and x'_μ along n_μ: $x_\mu = \tau n_\mu$, $x'_\mu = \tau' n'_\mu$. Then two Lorentz structures in (2.7) reduce to one Lorentz structure [we use relation $v = (n + n')/2$],

$$\langle 0|G_{\mu\nu}(n\tau)\phi_b(0)\phi(n\tau')|B(p)\rangle = \int D(\omega,\omega')e^{-iM_B(\omega\tau+\omega'\tau')} \times \frac{1}{2}(n_\mu n'_\nu - n_\nu n'_\mu)\left(X_A^{(1)} + X_A^{(2)}\right), \tag{3.12}$$

where $X_A \equiv X_A^{(1)} + X_A^{(2)}$ is the collinear LC 3DA.

(ii) We now turn to the double-collinear LC configuration. Since the 3BS (2.7) is antisymmetric in $\mu \leftrightarrow \nu$, both Lorentz structures in (2.7) are again reduced to the same Lorentz structure as in (3.12) with however a different 3DA,

$$\langle 0|G_{\mu\nu}(n\tau)\phi_b(0)\phi(n'\tau')|B(p)\rangle = \int D(\omega,\omega')e^{-iM_B(\omega\tau+\omega'\tau'} \times \frac{1}{2}(n_\mu n'_\nu - n_\nu n'_\mu)\left(X_A^{(1)} - X_A^{(2)}\right). \tag{3.13}$$

So, we conclude; although the 3BS of Eq. (2.7) both in the collinear and the double-collinear configurations contain one and the same Lorentz structure, the corresponding 3DAs in the collinear and the double-collinear configurations are different and in general are independent of each other, see also [28].

B. Gauge field as one of the light fields: Double-collinear approximation

Let us now turn to the situation with more Lorentz structures. We have to take into account that the kinematics is double-collinear and the new Lorentz structures and new DAs emerge compared to collinear kinematics considered in [21]. Still, some features established for the collinear 3DA survive also in this more complicated case. For instance, let us consider the Lorentz structures containing xp and $x'p$ in the denominator

$$\frac{x_\mu p_\nu - x_\nu p_\mu}{xp} \quad \text{and} \quad \frac{x'_\mu p_\nu - x'_\nu p_\mu}{x'p}. \tag{3.14}$$

If we keep the "large" components [i.e., x_μ along the (+)-axis and x'_μ along the (−)-axis], then powers of τ and τ' in the numerator and the denominator cancel. In the end, the τ- and τ'-integrations may be easily performed.

To be more specific, we consider the case when the light field $\phi(x)$ is replaced by the vector gauge field $G_{\mu\nu}(x)$ and discuss one structure in the B-meson 3BS

$$\langle 0|G_{\mu\nu}(x)\phi_b(0)\phi(x')|B(p)\rangle = \int D(\omega,\omega')e^{-i\omega xp - i\omega' x' p}\left(\frac{x_\mu p_\nu - x_\nu p_\mu}{xp}\right)X_A(\omega,\omega'). \tag{3.15}$$

The B-decay amplitude induced by this 3BS is antisymmetric in two indices μ and ν and thus contains one form factor,

$$A_{\mu\nu}(p|q,q') = (q_\mu q'_\nu - q_\nu q'_\mu)F(q^2, q'^2) = \int \frac{dx dx' d\kappa d\kappa'}{(\mu^2 - (\kappa - q)^2)(m^2 - (\kappa' - q')^2)} e^{i\kappa x + i\kappa' x'} \langle 0|G_{\mu\nu}(x)\phi_b(0)\phi'(x')|B(p)\rangle. \tag{3.16}$$

Taking into account the derivation of the previous subsection, the leading contribution may be written as

$$A_{\mu\nu}(p|q,q') = \int d\tau d\kappa_- \frac{1}{\mu^2 + 2q_+\kappa_-} e^{i\kappa_-\tau} \int d\tau' d\kappa'_+ \frac{1}{m^2 + 2q'_-\kappa'_+} e^{i\kappa'_+\tau'} \frac{x_\mu p_\nu - x_\nu p_\mu}{xp} \times \int D(\omega,\omega')X_A(\omega,\omega')e^{-i\omega p_-\tau - i\omega' p_+\tau'}. \tag{3.17}$$

Obviously, the main contribution to the amplitude comes from the following regions:

(i) In the upper part of the diagram, x_μ has nonzero $(+)$-component, q_μ has nonzero $(+)$-component, κ_μ has nonzero $(-)$-component, and $xp = \tau p_+$.

(ii) In the lower part of the diagram, the situation is opposite: x'_μ has nonzero $(-)$-component, q'_μ has nonzero $(-)$-component, κ'_μ has nonzero $(+)$-component, and $x'p = \tau' p_-$.

Essentially, we have two independent one-dimensional configurations. These configurations talk to each other via the ω and ω' dependence of the 3DA $X_A(\omega, \omega')$.

In the double-collinear configuration, the 4-vector x has only one nonzero component, x_+, and the combination $x_\mu p_\nu - x_\nu p_\mu$ is linear in x_+. Therefore, the combination

$$\frac{x_\mu p_\nu - x_\nu p_\mu}{xp} \tag{3.18}$$

is nonsingular for $x_+ \to 0$. [The same property holds e.g., for the structure $(x_\mu \gamma_\nu - x_\nu \gamma_\mu)/xp$ for spinor fields.]

Taking into account the nonzero components, for calculating the form factor $F(q^2, q'^2)$ we may set $\mu = (+)$ and $\nu = (-)$, or $\mu = (-)$ and $\nu = (+)$. In the double-collinear configuration, we find

$$\begin{aligned} &\frac{x_\mu p_\nu - x_\nu p_\mu}{xp} \to 1 \quad \text{for } \mu = (+), \nu = (-), \quad \text{and} \\ &\frac{x_\mu p_\nu - x_\nu p_\mu}{xp} \to -1 \quad \text{for } \mu = (-), \nu = (+). \end{aligned} \tag{3.19}$$

In the end, we obtain the same result as for the scalar form factor of the previous section,

$$\begin{aligned} q' q F(q^2, q'^2) = &\int d\tau d\kappa_- \frac{1}{\mu^2 + 2q_+ \kappa_-} e^{i\kappa_- \tau} \\ &\times \int d\tau' d\kappa'_+ \frac{1}{m^2 + 2q'_- \kappa'_+} e^{i\kappa'_+ \tau'} \\ &\times \int D(\omega, \omega') e^{-i\omega p_- \tau - i\omega' p_+ \tau'} X_A(\omega, \omega'). \end{aligned} \tag{3.20}$$

C. Gauge field as one of the light fields: The α-representation

Another possibility to handle the $1/xp$ factor without making approximations and without performing parts integration is to make use of the α-representation

$$\frac{1}{ixp} = \int_0^\infty e^{-iapx - \epsilon a} da. \tag{3.21}$$

This representation leads to a shift of the expressions containing ω in the denominator of the quark propagator and enables us to take explicitly the α-integration in the amplitude (3.16). So, we do not need to make the parts integration in ω. In this way we could avoid making approximations and considering the collinear kinematics.

Taking into account that the quadratic form in the denominator is the quark propagator, we reconstruct the imaginary part as follows:

$$\begin{aligned} &\int_0^\infty \frac{da}{(a - a_+)(a - a_-) - i0} \\ &= \frac{1}{a_+ - a-} \log\left(-\frac{a_-}{a_+}\right) + i\pi \frac{1}{a_+ - a-}, \\ &a_- < 0, \qquad a_+ > 0 \quad \text{for } 0 < \omega < 1. \end{aligned} \tag{3.22}$$

Let us discuss the contribution to the form factor from the "upper part of the diagram" including the propagator of the quark with mass μ; the "lower" part of the diagram is treated precisely the same way. Important is that we do not need to make any approximations and the calculation is fully covariant. By virtue of the α-representation, the contribution of the 3DA structure (3.15) to the form factor (3.8) takes the form [we explicitly write only the upper part of the diagram containing the vertex x that emits the momentum q, the quark propagator $1/(\mu^2 - k^2 - i0)$ and the relevant part of the B-meson 3BS],

$$\begin{aligned} A_{\mu\nu}(p|q, q') = &\int dx dk e^{iqx + ikx - i\omega px} \frac{x_\mu p_\nu - x_\nu p_\mu}{ixp} \\ &\times \frac{1}{\mu^2 - k^2 - i0} d\omega X_A(\omega, \omega') \{\ldots\}. \end{aligned} \tag{3.23}$$

We substitute $1/px$ as the α-integral and represent the x_β as the q_β-derivative of $\exp(iqx)$. This q_β derivative may be taken out of the integral bringing us to the following expression:

$$A_{\mu\nu}(p|q, q') = \left(p_\nu \frac{\partial}{\partial q_\mu} - p_\mu \frac{\partial}{\partial q_\nu}\right) \int_0^\infty da\, dx\, dk\, e^{iqx + ikx - i\omega px - iapx - \epsilon a} \frac{1}{\mu^2 - k^2 - i0} d\omega X_A(\omega, \omega' \{\ldots\}. \tag{3.24}$$

Taking x- and k-integrations yields

$$A_{\mu\nu}(p|q,q') = \left(p_\nu \frac{\partial}{\partial q_\mu} - p_\mu \frac{\partial}{\partial q_\nu}\right) \int_0^\infty da e^{-\epsilon a} \frac{1}{\mu^2 - (q - p(a+\omega))^2 - i0} d\omega X_A(\omega,\omega')\{\ldots\}. \tag{3.25}$$

The q-derivative may be written as

$$\begin{aligned} \frac{\partial}{\partial q_\beta}\left(\frac{1}{\mu^2 - (q - p(a+\omega))^2 - i0}\right) &= -2\frac{\partial}{\partial \mu^2}\left(\frac{1}{\mu^2 - (q - p(a+\omega))^2 - i0\cdots}\right)(q_\beta - p_\beta(a+\omega)) \\ &\to -2q_\beta \frac{\partial}{\partial \mu^2}\left(\frac{1}{\mu^2 - (q - p(a+\omega))^2 - i0\cdots}\right), \end{aligned} \tag{3.26}$$

since the term proportional to p_β does not contribute to the antisymmetric amplitude $A_{\mu\nu}$.

The next steps in our handling of Eq. (3.25) are as follows:

The a-integral converges so we may take the limit $\epsilon \to 0$ and set $\exp(-\epsilon a) \to 1$.

Isolating the factor M_B^2 in the denominator, the a-integral in Eq. (3.25) is equal to (3.22) with

$$\begin{aligned} a_+ &= \frac{-1 + \sqrt{1 + 4\bar\mu^2} + 2\omega}{2} > 0, \\ a_- &= \frac{-1 - \sqrt{1 + 4\bar\mu^2} + 2\omega}{2} < 0, \quad \text{for } 0 < \omega < 1, \end{aligned} \tag{3.27}$$

where $\bar\mu^2 \equiv \mu^2/M_B^2$ and $a_+ - a_- = \sqrt{1 + 4\bar\mu^2}$. For further use we denote the real part of (3.22) as $K(\omega,\bar\mu^2)$,

$$K(\omega,\bar\mu^2) = \frac{1}{a_+ - a-}\log\left(-\frac{a_-}{a_+}\right), \tag{3.28}$$

with $a_\pm$ given by (3.27). Notice a useful relation

$$\partial_\omega K(\omega,\bar\mu^2) = \frac{1}{\bar\mu^2 + \omega(1-\omega)}. \tag{3.29}$$

Adding the contribution of the "lower" part of the diagram yields

$$A_{\mu\nu}(p|q,q') \sim (q'_\nu q_\nu - q_\nu q'_\mu)\frac{\partial}{\partial \bar\mu^2} F_s(\bar\mu^2, \bar m^2), \tag{3.30}$$

with (recall that we consider $q^2 = q'^2 = 0$)

$$\begin{aligned} F_s(\bar\mu^2,\bar m^2) = &\int D(\omega,\omega')X_A(\omega,\omega')K(\omega,\bar\mu^2) \\ &\times \frac{1}{\bar m^2 + \omega'(1+\omega')} + i\pi \frac{1}{\sqrt{1+4\bar\mu^2}} \\ &\times \int D(\omega,\omega')X_A(\omega,\omega')\frac{1}{\bar m^2 + \omega'(1+\omega')}. \end{aligned} \tag{3.31}$$

Here $\bar m^2 \equiv m^2/M_B^2$.

To compare the amplitude in the double-collinear approximation, Eq. (3.20), and the exact amplitude, Eq. (3.30), in the HQ limit ($M_B \to \infty$, m and μ fixed, $\omega \sim \Lambda_{\mathrm{QCD}}/M_B$), we make use of the following expansions:

$$-\partial_{\bar\mu^2} K(\omega,\bar\mu^2) = \frac{1}{\omega} - (1 + 2\log(\omega) - 2i\pi) - 3\omega + O(\omega^2), \tag{3.32}$$

$$\frac{1}{\bar\mu^2 + \omega(1-\omega)} = \frac{1}{\omega} + 1 + \omega + O(\omega^2). \tag{3.33}$$

Taking into account that any 3DA is peaked in region $(\omega,\omega') \sim \Lambda_{\mathrm{QCD}}/m_b$, we observe the following important features of the amplitude in the HQ limit:

(a) The amplitude in the "double-collinear" approximation, Eqs. (3.20) and (3.33) and the exact amplitude, Eqs. (3.30) and (3.32), have one and the same leading behavior in the HQ limit.

(b) The imaginary part of the exact amplitude, see Eqs. (3.30) and (3.31), is parametrically suppressed compared to its real part. The amplitude in the "double-collinear" approximation (3.20) does not have any imaginary part.

(c) The "strong" imaginary part of the exact $B \to \gamma\gamma$ amplitude, Eq. (3.31), gained due to soft gluon interactions, is unphysical as there are no appropriate hadron intermediate states which might lead to the appearance of such imaginary part at $q^2 = 0$ and $q'^2 = 0$. Note that the imaginary part vanishes identically if X_A satisfies the constraint (2.8); the constraint on X_A given by Eq. (2.6) is not sufficient to guarantee the absence of the unphysical imaginary part.

IV. CONCLUSIONS

We studied a generic B-decay amplitude of the FCNC-type—i.e., an amplitude given by diagrams in which the heavy field hits the middle point of the line along which light degrees of freedom propagate—and obtained the following results:

(i) As already demonstrated in the literature [25–27], the leading contribution to the amplitude of a B-decay of FCNC-type, $B \to j(q)j'(q')$, is given by the convolution formula of the hard kernel composed of propagators of the light degrees of freedom and the 3BS of the B-meson

$$\langle 0|\phi(x)\phi_b(0)\phi'(x')|B(p)\rangle \tag{4.1}$$

in the following configuration:

$$x^2 = 0; \qquad x'^2 = 0,\ xx' \neq 0. \tag{4.2}$$

We have now formulated this result as a factorization theorem by a direct analysis of Feynman diagrams. Considering the FCNC-type B-decay into two real photons ($q^2 = q'^2 = 0$) in the rest frame of the B-meson and choosing the momenta q and q' along the $(+)$ and $(-)$ axes of the light cone, respectively, we have shown that the dominant contribution to the amplitude in the heavy-quark limit comes from the "double-collinear" configuration

$$\langle 0|\phi(x_+)\phi_b(0)\phi'(x'_-)|B(p)\rangle. \tag{4.3}$$

Equation (3.8) represents the factorization formula for the amplitudes of FCNC-type; corrections to the contribution given by Eq. (3.8) are suppressed by powers of the heavy-quark mass.

(ii) The B-meson 3BS in a collinear LC configuration

$$\langle 0|\phi(x)\phi_b(0)\phi'(\lambda x)|B(p)\rangle, \qquad x^2 = 0, \qquad 0 < \lambda < 1, \tag{4.4}$$

does not appear in the convolution formula (3.8) for the leading 3BS contribution to the FCNC B-decay amplitude. This means that the calculation of the FCNC B-decay amplitude as the convolution of the hard kernel and the collinear 3BS as is done e.g., in [19] does not provide a well-defined starting point; corrections to this approximation are not suppressed by any large parameter.

This makes an essential difference between the 3DA contributions to charming loops in FCNC B-decays and to SL B-decays; the 3DA contribution to the SL form factor factorizes into the convolution of a hard kernel and the collinear LC 3BS [24], corrections to this configuration being suppressed by $1/m_b$.

(iii) The generic 3BS involves more Lorentz structures and 3DAs than the collinear 3BS. In particular, it involves structures of the type

$$\langle 0|G_{\mu\nu}(x)\phi_b(0)\phi'(x')|B(p)\rangle = \int D(\omega,\omega')e^{-ipx\omega - ipx'\omega'}\left\{\left(\frac{p_\mu x_\nu - p_\nu x_\mu}{xp}\right)X_A^{(1)}(\omega,\omega') + \left(\frac{p_\mu x'_\nu - p_\nu x'_\mu}{x'p}\right)X_A^{(2)}(\omega,\omega')\right\} \tag{4.5}$$

with $D(\omega,\omega') = d\omega d\omega'\theta(\omega)\theta(\omega)\theta(1-\omega-\omega')$. These Lorentz structures contain kinematical singularities in $1/xp$ or $1/x'p$. Such singularities are unphysical. Requiring that the 3DA is a continuous and finite function of its variables at $x^2 = 0$, $x'^2 = 0$, $xp = 0$ and $x'p = 0$, we conclude that the kinematical singularities should vanish due to the properties of the 3DAs $X_A^{(1)}$ and $X_A^{(2)}$, leading to the following constraints:

$$\int_0^{1-\omega} d\omega X_A^{(1)}(\omega,\omega') = 0 \quad \forall\omega'; \qquad \int_0^{1-\omega'} d\omega' X_A^{(2)}(\omega,\omega') = 0 \quad \forall\omega. \tag{4.6}$$

In other words, the primitives $\bar{X}_A^{(1)}(\omega,\omega')$ and $\bar{X}_A^{(2)}(\omega,\omega')$ [Eqs. (2.10) and (2.11)] should vanish on the boundaries of the X_A-suppport area,

$$\begin{aligned}
&\bar{X}_A^{(1)}(\omega = 0,\omega') = \bar{X}_A^{(1)}(1-\omega',\omega') = 0 \quad \forall\omega', \\
&\bar{X}_A^{(2)}(\omega,\omega' = 0) = \bar{X}_A^{(2)}(\omega,1-\omega) = 0 \quad \forall\omega.
\end{aligned} \tag{4.7}$$

Constraints of this type emerge for all 3DAs which parametrize the Lorents structures x_μ/xp, $x'_\mu/x'p$, $x_\mu x_\nu/(xp)^2$, $x_\mu x'_\nu/(xp)(x'p)$, etc., in the 3BS of the B-meson. These constraints should be taken into account when building models for the 3DAs.

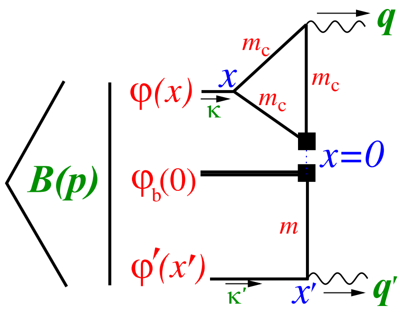

FIG. 3. Diagram describing $A_{\rm FCNC}$, a nonfactorizable charming-loop corrections to the amplitude of FCNC B-decay.

(iv) The B-meson 3BS of the X_A-type of Eq. (4.5) in the collinear LC configuration $x_\mu = \tau n_\mu$, $x'_\mu = \tau' n_\mu$, $n^2 = 0$, Eq. (3.12), and in the double-collinear LC configuration, $x_\mu = \tau n_\mu$, $x'_\mu = \tau' n'_\mu$, $n^2 = n'^2 = 0$, $n'n = 2$, Eq. (3.13), may be parametrized via one and the same Lorentz structure $n_\mu n'_\nu - n_\nu n'_\mu$. However, the corresponding 3DAs are different and in general independent of each other; the collinear LC 3DA is equal to $X_A^{(1)} + X_A^{(2)}$, whereas the double-collinear LC 3DA is equal to $X_A^{(1)} - X_A^{(2)}$.

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APPENDIX: NONFACTORIZABLE CHARMING LOOP IN FCNC *B*-DECAY AMPLITUDE

Nonfactorizable charm-loop contribution to the amplitude of FCNC decay is given by the diagram of Fig. 3. The corresponding analytic expression reads [27]

$$A_{\rm FCNC}(p|q,q') = \frac{G_F}{\sqrt{2}}\int d\kappa' dx' e^{i\kappa' x'}\frac{1}{m^2-(\kappa'-q')^2-i0}d\kappa dx e^{i\kappa x}\Gamma_{cc}(\kappa,q)\langle 0|\phi(x)\phi_b(0)\phi'(x')|B(p)\rangle, \tag{A1}$$

where the expression for the charm-quark loop in the case of scalar "quarks" has the form

$$\Gamma_{cc}(\kappa,q) = \frac{1}{8\pi^2}\int_0^1 du\int_0^{1-u} dv\frac{1}{m_c^2-2uv\kappa q-\kappa^2 u(1-u)-q^2 v(1-v)-i0}. \tag{A2}$$

The amplitude (A1) corresponds to the generic amplitude describing 3BS correction to the form factor (3.1) with the replacement of the usual quark propagator by an "effective" propagator describing the charm-quark triangle $\Gamma_{cc}(\kappa,q)$

$$\frac{1}{\mu^2-(\kappa-q)^2-i0}\to\Gamma_{cc}(\kappa,q). \tag{A3}$$

It is important that $\Gamma_{cc}(\kappa,q)$, similar to the usual propagator, is a quadratic function of its momentum variables. Therefore the consideration presented in the text for the amplitude (3.1) can be directly applied to the amplitude (A1).

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