Baryogenesis from Higgs inflation

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If the inflaton field is coupled to the hypercharge Chern-Simons density $F\tilde{F}$, an explosive production of helical gauge fields when inflation ends can trigger baryogenesis at the electroweak phase transition. Besides, Higgs inflation identifies the inflaton with the Higgs field $\mathcal{H}$, thus relating cosmological observables to properties of electroweak physics. In this paper we merge both approaches; the helical gauge fields are produced at the end of Higgs inflation from the coupling $|\mathcal{H}|^2 F\tilde{F}$. In the metric formulation of gravity we found a window in the parameter space for electroweak baryogenesis consistent with all experimental observations. Conversely, for the Palatini formalism the non-Gaussianity bounds strongly constrain the helicity produced at the end of inflation, forbidding an efficient baryogenesis.

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I. INTRODUCTION

The Standard Model (SM) of electroweak and strong interactions of particle physics is a well established theory that, until today, has successfully passed all experimental tests at high-energy colliders (LEP, Tevatron, LHC,...), as well as the low-energy ones. Still there are a number of phenomena that cannot be easily coped by the SM, in particular a dynamical explanation of the baryon asymmetry of the Universe (BAU), or baryogenesis [1], and the existence of cosmological inflation [2–4] in the early stages of the Universe, both features usually requiring the introduction of beyond the SM (BSM) physics. Still the reluctancy of experimental data to confirm deviations with respect to the SM predictions has motivated people to reanalyze those phenomena with SM tools as much as possible.

Two main obstacles for the baryogenesis mechanism to work within the SM are (i) The required out of equilibrium condition in the electroweak phase transition is forbidden by the present value of the Higgs mass. In fact it has been shown that the electroweak phase transition is not even first order, but a continuous crossover [5]. (ii) The CP violation in the CKM matrix is too weak to generate the BAU [6–9], so an extra source of CP violation is required. Both problems were solved assuming that inflation is driven by a scalar field, the inflaton, with a dimension-5 operator coupling it to the (CP-odd) hypercharge Chern-Simons density $F\tilde{F}$ and generating, at the end of inflation, an explosive production of helical hypermagnetic fields [10–12] relaxing its helicity into the baryon asymmetry at the electroweak crossover [13–19].

Cosmological inflation is supposed to be driven by a BSM scalar field with an appropriate potential. The Higgs field $\mathcal{H}$ with minimal coupling to gravity is excluded as an inflaton candidate, as the quartic coupling is too large to cope with the measured amplitude of density perturbations. It was however observed that if the Higgs field is non-minimally coupled to gravity $\xi_h|\mathcal{H}|^2 R$, with a large coupling ξ_h, it can generate cosmological inflation, dubbed as Higgs inflation (HI), consistently with the value of the density perturbations [20]. Still HI faces two fundamental problems which possibly require some UV completion of the SM: (i) Assuming that the Higgs quartic coupling λ_h be $\mathcal{O}(m_h^2/(2v^2))$, the tree-level value of the SM, the amplitude of cosmological perturbations [21] require that $\xi_h = \mathcal{O}(10^4)$, which can be at odd with the validity of the effective field theory and violate unitarity constraints. (ii) When radiative corrections are considered in the SM effective potential, the value of the quartic coupling becomes a function of the Higgs background, $\lambda_h(h)$, which becomes negative, mainly by the contribution from the top quark, at a value of $h_I \sim 10^{11}$ GeV [22], much below the values for which inflation takes place, i.e. $h \sim 10^{-2} M_{\rm Pl}$. Problem (i) has recently been addressed in Refs. [23,24] where it was proven that, while in the electroweak vacuum tree-level unitarity is violated at the scale $M_{\rm Pl}/\xi_h$, in the inflationary large field background the unitarity limit is at $M_{\rm Pl}/\sqrt{\xi_h}$. Problem (ii) usually requires an ultraviolet (UV) completion of the model [25] which can modify the relation between the low-energy and high-energy SM parameters, and in particular the value of the coupling λ_h at the inflationary scale.[1] Moreover critical HI (CHI) theories [27] aim to solve both problems if UV physics modify the running of the SM couplings in such a way that $\lambda_h \ll 1$ at inflationary scales, so that the Higgs mass is near its critical value, while staying positive all the way towards the electroweak scale.[2] For a recent approach in this direction see e.g. Ref. [29]. In models of CHI it turns out that the amplitude of density perturbations requires values $\xi_h \lesssim 10$ thus greatly alleviating the unitarity problem.

In this paper we will unify both previous approaches and consider cosmological inflation triggered by the Higgs field, i.e. Higgs inflation, while we will assume that the Higgs is coupled to the hypercharge Chern-Simons density with a *CP*-odd dimension-6 operator, $|\mathcal{H}|^2 F\tilde{F}$.[3] Helical hypermagnetic fields will be generated at the end of inflation, relaxing the helicity into baryon asymmetry at the electroweak crossover. We will then assume ordinary HI with arbitrary value of the quartic coupling, and we will be agnostic about the origin on its value and the mechanism stabilizing the Higgs potential. In this way the value of the coupling λ_h during inflation will be considered as a free parameter, which should depend on the particular UV completion of the theory.

The contents of this paper are as follows. In Sec. II we review the results on cosmological observables in Higgs inflation. The equations of motion for gauge fields is presented in Sec. III, where we prove the (almost) constancy of the parameter ξ which is responsible for the energy transfer from the inflaton to the gauge field, and whose value is the critical quantity for the explosive production of helical gauge field. The cases of no Schwinger effect (motivated by a solution to the flavor problem by means of a Froggatt-Nielsen mechanism with the flavon coupled to the inflaton) and with Schwinger effect (using a well-motivated analytical approximation) are considered in detail. Also the consistency condition for the nonbackreaction of the gauge field on the inflaton sector, and the bounds from the non-Gaussianity are studied in detail. The baryogenesis capabilities of the model are analyzed in Sec. IV, both in the cases of Higgs inflation and critical Higgs inflation. We show here that there is an available window where all constraints can be satisfied. As all previous results are done (by default) in the metric formulation of gravity, we have considered in Sec. V the Palatini formulation of gravity, where it is known that the inflationary predictions are different than those in the metric formulation. We have also proven that the baryogenesis predictions are different, and in fact baryogenesis by helical gauge fields is forbidden in the Palatini formulation of gravity. Finally, we have drawn our conclusions and outlook in Sec. VI, while in the Appendix we present the technical details of the Froggatt-Nielsen mechanism where the flavon field is coupled to the inflaton field.

[1] The case of low-scale Higgs inflation has been considered in Ref. [26].
[2] For a detailed analysis of renormalized Higgs inflation, see Ref. [28].
[3] For related works on magnetogenesis see Refs. [30,31].

II. HIGGS INFLATION

The model of Higgs inflation is based on an action where the Higgs field has a nonminimal coupling to gravity. In particular the action in the Jordan frame is

$$S_J = \int d^4x \left[\sqrt{-g} \left(-\frac{M_{\rm pl}^2}{2} R - \xi_h |\mathcal{H}|^2 R + (D_\mu \mathcal{H})^\dagger D^\mu \mathcal{H} - \frac{1}{4} Y_{\mu\nu} Y^{\mu\nu} - U(\mathcal{H}) \right) - \frac{|\mathcal{H}|^2}{2 f_h^2} Y_{\mu\nu} \tilde{Y}^{\mu\nu} \right], \qquad (2.1)$$

where $\mathcal{H} = \exp(i\chi^a \sigma^a/2)(0, h/\sqrt{2})^T$ is the Higgs doublet with the three Goldstone bosons χ^a and the physical Higgs h, and U the Higgs potential in the Jordan frame. The parameter f_h (with mass dimension) provides the inverse coupling of the Higgs to the Chern-Simons term, a *CP*-odd dimension-6 operator which will be responsible for the baryogenesis mechanism. $Y^{\mu\nu}$ is the field-strength of the hypercharge gauge field Y^μ, and $\tilde{Y}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} Y_{\rho\sigma}$ is its dual tensor. D_μ is the electroweak covariant derivative given by

$$D_\mu = \partial_\mu - \frac{ig'}{2} Y_\mu - \frac{ig}{2} \sigma^a W_\mu^a, \qquad (2.2)$$

where g', g are the $U(1)_Y$ and $SU(2)_L$ couplings and σ^a the Pauli matrices. A possible UV completion giving rise to this dimension-6 *CP*-odd operator was provided in Ref. [17]. The action also contains a general nonminimal coupling ξ_h of the Higgs field to the Ricci scalar.

During the inflationary stage the background value of the Higgs field h is large and the electroweak (EW) symmetry is broken. We can then consider the trivial background solution $W_\mu^\pm = 0$, $Z_\mu = \cos\theta_W W_\mu^3 - \sin\theta_W Y_\mu = 0$. Given that the photon field is given by $A_\mu = \sin\theta_W W_\mu^3 + \cos\theta_W Y_\mu$, equation $Z_\mu = 0$ implies $Y_\mu = \cos\theta_W A_\mu$ and $W_\mu^3 = \sin\theta_W A_\mu$. We can thus keep in our equations just the photon gauge field A_μ and, in the higher-dimensional coupling in Eq. (2.1), the Chern-Simons term, replace Y_μ by $\cos\theta_W A_\mu$. Moreover, in the above background the covariant derivative is written as $D_\mu = \partial_\mu - ieQA_\mu$ where $e = g\sin\theta_W$ and Q is the electric charge of the field the covariant derivative is acting on. For the Higgs sector, the Goldstone bosons χ^a are eaten up in the unitary gauge $\omega(\alpha^a) = \exp(i\alpha^a\sigma^a/2)$, with $\alpha^a = -\chi^a$, while the Higgs field h is chargeless, $Q = 0$, hence its covariant derivative reduces to ∂_μ, leading to the action

$$S_J = \int d^4x \left[\sqrt{-g} \left(-\frac{M_{\rm pl}^2 + \xi_h h^2}{2} R + \frac{1}{2} \partial_\mu h \partial^\mu h - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - U(h) \right) - \frac{\cos^2\theta_W}{4} \frac{h^2}{f_h^2} F_{\mu\nu} \tilde{F}^{\mu\nu} \right]. \qquad (2.3)$$

To alleviate the notation, and for the rest of this paper, we will use units where $M_{\rm pl} \equiv 1$. For the large values of h involved in inflation, we can use $U(h) \simeq \lambda_h h^4/4$, where λ_h is taken as a positive parameter. As we have explained in the introduction, we are not considering any particular UV completion stabilizing the Higgs potential and, instead, we will consider λ_h as a free parameter of the theory.

To go to the Einstein frame, we perform a Weyl redefinition of the metric with $g_{\mu\nu} \to \Theta g_{\mu\nu}$ with

$$\Theta(h) = \frac{1}{1+\xi_h h^2} \tag{2.4}$$

chosen such that we recover the Einstein-Hilbert action explicitly. The potential becomes

$$V(h) \simeq \frac{\lambda}{4\xi_h^2}\left(1 - \frac{1}{\xi_h h^2}\right)^2, \tag{2.5}$$

where the approximation is valid in the regime $\xi_h h^2 \gg 1$, where the Einstein frame departs from the Jordan frame.

The inflaton field χ, with canonical kinetic term, is related to h, by the following change of variables

$$\frac{d\chi}{dh} = \sqrt{\Theta(1+6\xi_h^2 h^2 \Theta)}, \tag{2.6}$$

such that, in the Einstein frame,

$$S_E = \int d^4x\sqrt{-g}\left[-\frac{R}{2} + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V[h(\chi)]\right] - \int d^4x \frac{\cos^2\theta_W}{4}\frac{h(\chi)^2}{f_h^2}F_{\mu\nu}\tilde{F}^{\mu\nu}, \tag{2.7}$$

where, in the limit $\xi_h h^2 \gg 1$, we obtain

$$h(\chi) \simeq \frac{1}{2\sqrt{\xi_h(1+6\xi_h)}}\exp\left(\sqrt{\frac{\xi_h}{1+6\xi_h}}\chi\right) \tag{2.8}$$

and the potential in terms of the canonically normalized field χ is then

$$V(\chi) \simeq \frac{\lambda}{4\xi_h^2}\left[1 - 4(1+6\xi_h)\exp\left(-\sqrt{\frac{4\xi_h}{1+6\xi_h}}\chi\right)\right]^2. \tag{2.9}$$

Assuming $\xi_h \gg 1$, which will be proven in the next section, we can write

$$h(\chi) \simeq \frac{e^{\frac{\chi}{\sqrt{6}}}}{\sqrt{24\xi_h}} \tag{2.10}$$

and

$$V(\chi) \simeq \frac{\lambda}{4\xi_h^2}\left[1 - 24\xi_h e^{-\sqrt{\frac{2}{3}}\chi}\right]^2. \tag{2.11}$$

Variation of the action (2.7) with respect to χ and the gauge field $A_\mu = (A_0, \boldsymbol{A})$ leads to the gauge equations of motion in the radiation gauge, $A_0 = 0$ and $\boldsymbol{\nabla}\cdot\boldsymbol{A} = 0$,

$$\ddot{\chi} + 3H\dot{\chi} + V'(\chi) = K(\chi)\frac{\langle \boldsymbol{E}\cdot\boldsymbol{B}\rangle}{a^4 f_\chi}, \tag{2.12a}$$

$$\left(\frac{\partial^2}{\partial\tau^2} - \nabla^2 - K(\chi)\frac{a\dot{\chi}}{f_\chi}\boldsymbol{\nabla}\times\right)\boldsymbol{A} = \boldsymbol{J}, \tag{2.12b}$$

where we are using derivatives with respect to the cosmic time t for the inflaton, as $\dot{\chi} = d\chi/dt$, and with respect to the conformal time τ for the gauge field, where $d/d\tau = a d/dt$, and

$$K(\chi) \equiv \frac{e^{\sqrt{\frac{2}{3}}\chi}}{6^{3/2}\xi_h^2}, \qquad f_\chi \equiv \frac{2f_h^2}{\cos^2\theta_W}. \tag{2.13}$$

Moreover, we have used that $F_{\mu\nu}\tilde{F}^{\mu\nu} = -4\boldsymbol{E}\cdot\boldsymbol{B}$ and $J^\mu = (\rho_c, \boldsymbol{J}) = ieQ\bar{\psi}\gamma^\mu\psi$, where ψ are the light fermions. We assume that initially the Universe does not contain charged particles, and that these ones are produced only later in particle-antiparticle pairs. Therefore, we set the charge density to zero, $\rho_c = 0$. The current $\boldsymbol{J}$ is given by the Ohm's law $\boldsymbol{J} = \sigma\boldsymbol{E} = -\sigma\boldsymbol{A}'$, where σ is the generalized conductivity, to be defined later (see Sec. III B 2). We assume a homogeneous inflaton with only zero-mode, $\chi(t, \boldsymbol{x}) = \chi(t)$. All gauge field quantities are $U(1)$ ordinary electromagnetic fields.

Assuming now slow roll inflation, we will neglect the right-hand side of Eq. (2.12a), i.e. we will assume $K(\chi)\langle\mathbf{E}\cdot\mathbf{B}\rangle \ll a^4 f_\chi V'(\chi)$, a consistency condition that will be checked *a posteriori*, after solving the system in Eq. (2.12). Using the usual slow-roll techniques one can easily find the value of the inflaton field at the end of inflation, χ_E, as well as its value N_* e-folds before, χ_*, as

$$\chi_E = \sqrt{\frac{3}{2}}\log(24\xi_h\beta), \qquad \beta \equiv 1 + \frac{2}{\sqrt{3}},$$
$$\chi_* = \sqrt{\frac{3}{2}}\left(\log(24\xi_h\beta) - \frac{4N_*}{3} - \beta - W_*\right), \tag{2.14}$$

where W_* is the Lambert function evaluated at

$$W_* \equiv \mathcal{W}_{-1}\left[-\beta\exp\left(-\beta - \frac{4N_*}{3}\right)\right]. \tag{2.15}$$

The slow roll parameters and the cosmic observables can be written as

$$\epsilon_* = \frac{4}{3(1+W_*)^2}, \qquad \eta_* = \frac{4(2+W_*)}{3(1+W_*)^2}, \tag{2.16}$$

so that they are independent on the value of ξ_h and λ_h. In particular, for $N_* = 60(50)$ one has

$$\epsilon_* \simeq 0.00019(0.00026), \qquad \eta_* \simeq -0.0155(-0.0184),$$
$$n_s \simeq 0.968(0.962), \qquad r_* \simeq 0.003(0.004), \tag{2.17}$$

inside the experimental range measured by Planck [21].

Finally the constraint on the amplitude of scalar fluctuations translates, for $N_* = 60(50)$, into the values for the parameter ξ_h

$$\xi_h \simeq 50(42) \times 10^3 \sqrt{\lambda_h}, \tag{2.18}$$

which validates our previous approximation $\xi_h \gg 1$ for $\lambda_h \gtrsim 10^{-8}$.

III. GAUGE FIELDS PRODUCTION

We now quantize the gauge field $\boldsymbol{A}$ in momentum space

$$\boldsymbol{A}(\tau, \boldsymbol{x}) = \sum_{\lambda=\pm} \int \frac{d^3k}{(2\pi)^3} [\boldsymbol{\epsilon}_\lambda(\boldsymbol{k}) a_\lambda(\boldsymbol{k}) A_\lambda(\tau, \boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + \text{H.c.}], \tag{3.1}$$

where λ is the photon polarization and $a_\lambda(\boldsymbol{k})$ ($a_\lambda^\dagger(\boldsymbol{k})$) are annihilation (creation) operators that fulfill the canonical commutation relations $[a_\lambda(\boldsymbol{k}), a_{\lambda'}^\dagger(\boldsymbol{k}')] = (2\pi)^3 \delta_{\lambda\lambda'} \delta^{(3)}(\boldsymbol{k} - \boldsymbol{k}')$. The polarization vectors $\boldsymbol{\epsilon}_\lambda(\boldsymbol{k})$ satisfy the conditions

$$\boldsymbol{k} \cdot \boldsymbol{\epsilon}_\lambda(\boldsymbol{k}) = 0, \qquad \boldsymbol{k} \times \boldsymbol{\epsilon}_\lambda(\boldsymbol{k}) = -i\lambda k \boldsymbol{\epsilon}_\lambda(\boldsymbol{k}),$$
$$\boldsymbol{\epsilon}_{\lambda'}^*(\boldsymbol{k}) \cdot \boldsymbol{\epsilon}_\lambda(\boldsymbol{k}) = \delta_{\lambda\lambda'}, \qquad \boldsymbol{\epsilon}_\lambda^*(\boldsymbol{k}) = \boldsymbol{\epsilon}_\lambda(-\boldsymbol{k}), \tag{3.2}$$

where $k \equiv |\boldsymbol{k}|$. Therefore, the equation of motion for the gauge modes (2.12b) yields

$$A_\lambda'' + \sigma A_\lambda' + k(k + 2\lambda\xi a H) A_\lambda = 0, \tag{3.3}$$

where we defined the instability parameter as

$$\xi = -K(\chi) \frac{\dot{\chi}}{2H f_\chi}. \tag{3.4}$$

From the solution of Eq. (3.3), the electric and magnetic energy density, as well as the helicity and helicity time derivative are given by

$$\rho_E \equiv \frac{1}{a^4} \int^{k_c} dk \frac{k^2}{4\pi^2} (|A_+'|^2 + |A_-'|^2), \tag{3.5a}$$

$$\rho_B \equiv \frac{1}{a^4} \int^{k_c} dk \frac{k^4}{4\pi^2} (|A_+|^2 + |A_-|^2), \tag{3.5b}$$

$$\mathcal{H} \equiv \lim_{V\to\infty} \frac{1}{V} \int_V d^3x \frac{\langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle}{a^3} = \frac{1}{a^3} \int^{k_c} dk \frac{k^3}{2\pi^2} (|A_+|^2 - |A_-|^2), \tag{3.5c}$$

$$\mathcal{G} \equiv \frac{1}{2a} \frac{d\mathcal{H}}{d\tau} = -\lim_{V\to\infty} \frac{1}{V} \int_V d^3x \frac{\langle \boldsymbol{E} \cdot \boldsymbol{B} \rangle}{a^4}, \tag{3.5d}$$

where $A_\pm$ are the solutions of (3.3). The upper integration limit

$$k_c = \left| \frac{a\dot{h}}{2f_h} \right| + \sqrt{\left(\frac{a\dot{h}}{2f_h} \right)^2 + \frac{a^2}{2} \left[\dot{\hat{\sigma}} + \hat{\sigma} \left(\frac{\hat{\sigma}}{2} + H \right) \right]},$$
$$\hat{\sigma} = \sigma/a \tag{3.6}$$

comes because subhorizon modes have an oscillatory behavior and should be regarded as quantum fluctuations. Therefore, such modes do not contribute to the above classical observables and are excluded from the integration (see Ref. [19] for more details).

A. Almost constancy of ξ

The methods usually employed in the literature to analytically compute the electromagnetic energy density and helicity (at least in the absence of the Schwinger effect) require a constant ξ (see Refs. [10–12]. We then aim to demonstrate in this section that this parameter barely changes during inflation while its main dependence lies in the couplings f_h and λ_h.

First, let us draw our attention on the fact that HI is a good candidate for an (almost) constant ξ as the leading interaction term between the Higgs and the gauge field is a dimension six operator. Let us consider the following interaction term in the action between the inflaton ϕ and the Chern-Simons density

$$S_E \supset \int d^4x F(\phi) Y_{\mu\nu} \tilde{Y}^{\mu\nu}. \tag{3.7}$$

As the instability parameter is defined by

$$\xi = 2F'(\phi) \frac{V'(\phi)}{V(\phi)}. \tag{3.8}$$

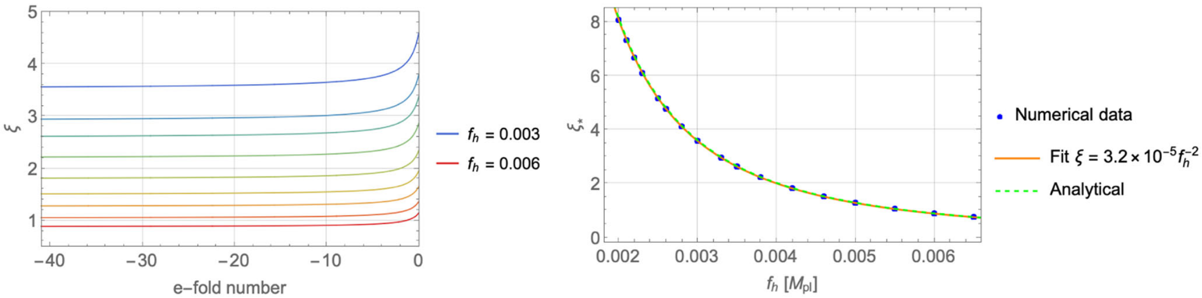

FIG. 1. Left: Plot of the parameters ξ for various values of the coupling f_h. Right: Instability parameter at CMB value ξ_* for various values of f_h from numerical simulations (blue dots) and their numerical fit (orange). Both perfectly overlap with the analytical relation (3.10) in dashed green.

a constant value of ξ is guaranteed by the condition $F'(\phi) \propto V(\phi)/V'(\phi)$.[4] In HI this condition is naturally satisfied as, $\mathcal{H}$ being an $SU(2)$ doublet, the lowest term in a power expansion is $F(h) \propto h^2$.[5] In this special case we find an (almost) constant value for the parameter ξ during Higgs inflation. In fact, in the slow roll approximation, the instability parameter is given by

$$\xi = \frac{K(\chi)}{f_\chi}\sqrt{\frac{\epsilon}{2}} = \frac{K(\chi)}{f_\chi}\frac{8\sqrt{6}\xi_h}{\exp\left[\sqrt{\frac{2}{3}}\chi\right] - 24\xi_h}, \quad (3.9)$$

where, in the second equality, we have used the definition of the potential (2.11). Now, using the definition of $K(\chi)$ and f_χ, we obtain that ξ is approximately constant, provided that $\chi \gtrsim \sqrt{\frac{3}{2}}M_{\rm pl}\log(24\xi_h) \simeq \chi_E$, and given by

$$\xi \simeq \frac{4}{3f_\chi\xi_h} \simeq 3.2\times10^{-5}\sqrt{\frac{0.1}{\lambda}}f_h^{-2}. \quad (3.10)$$

We verified this computation numerically[6] by solving the full system (2.12) without making the slow roll approximation and found the behavior displayed on the left panel of Fig. 1, where we plot the parameter ξ as a function of the number of e-folds during inflation N for various values of the parameter f_h and $\lambda_h = 0.1$. We see in the figure that ξ stays constant during most of the inflationary period and only increases at the end of inflation. The fact that ξ stays almost constant during the inflationary era provides confidence to analytically solve the equation of motion (EoM) (3.3), while its small variation provides a window for generating baryogenesis, as we will see later on in this paper.

To know how much ξ does vary during the N_* e-fold in inflation, we compute, using Eqs. (2.14),

$$\xi(\chi_E) \equiv \xi_E = \frac{4}{3f_\chi\xi_h}\frac{\beta}{\beta-1}, \quad (3.11a)$$

$$\xi(\chi_*) \equiv \xi_* = \frac{4}{3f_\chi\xi_h}\frac{\beta}{\beta - e^{\frac{4N_*}{3}+\beta+W_*}}. \quad (3.11b)$$

Hence

$$\frac{\xi_E}{\xi_*} = \frac{\beta - e^{\frac{4N_*}{3}+\beta+W_*}}{\beta-1} \simeq 1.84, \quad (3.12)$$

this ratio being insensitive to the value of N_* up the second digit. Notice that it does not contain the self-coupling λ_h, nor f_h.

In conclusion, we see that the instability parameter ξ is flat, regardless on when the simulation begins or on the chosen value of f_h. Only for the very end of the simulation, ξ deviates from its constant value. In fact, if we plot how this constant value changes with the parameter ξ_h, we found a perfect agreement between the numerical calculation and the analytical one (3.10), as we can see in the right panel of Fig. 1, where we plot ξ_* as a function of f_h for the different estimates.

B. Solution of the gauge equations of motion

In the presence of strong gauge fields, light fermions charged under the gauge group are produced by the backreaction of gauge fields that source the fermion equations of motion. The corresponding currents can then, in turn, backreact on the produced gauge fields, a phenomenon called the *Schwinger effect*. The backreaction of fermion currents on the produced gauge fields acts as a damping force in the explosive production of helical gauge fields. There is nevertheless a condition for a fermion f to contribute to the magnetic conductivity which is, for the fermion Yukawa coupling,

[4]Note that a dimension-5 operator $\phi F\tilde{F}$ in a model where the inflaton ϕ is an axionlike particle does not provide a constant ξ, but rather an exponential behavior given by (3.9) with $K(\chi) = 1$, where ξ changes the most at the end of inflation.

[5]A linear term in the function F would explicitly break gauge invariance in the symmetric phase.

[6]See Ref. [19] for the method.

$$Y_f \lesssim 0.45 \left(\frac{\rho_E}{H^4}\right)^{1/4} \sqrt{|Q_f|}, \tag{3.13}$$

and we have computed all couplings at the characteristic scale $\mu \simeq (\langle \boldsymbol{E} \rangle^2 + \langle \boldsymbol{B} \rangle^2)^{1/4}$ where the Schwinger effect takes place. If the three generations of fermions satisfy the above condition then the conductivity for the magnetic field is given by

$$\sigma \simeq \frac{e^3}{\pi^2} \frac{a}{H} \sqrt{2\rho_B} \coth\left(\pi \sqrt{\frac{\rho_B}{\rho_E}}\right), \tag{3.14}$$

where $e = gg'/\sqrt{g^2 + g'^2}$, $e^3 \simeq 0.36$.

The case of a constant ξ is suitable for the following scenarios as they both have been studied with this assumption:

(i) Absence of the Schwinger effect i.e. $\sigma \simeq 0$.
(ii) Presence of the Schwinger effect in the so-called equilibrium estimate [32].

In this section we shall review both cases and compute the baryogenesis parameter space accordingly.

1. No Schwinger effect

In this section we study the case when the conductivity σ vanishes in the equation of motion (3.3). One possibility that can guarantee this result would be a dynamical mechanism such that all fermion Yukawa couplings at the inflation scale are $\mathcal{O}(1)$, such that the criterion (3.13) is not met anymore, and after inflation they relax to the physical values which correspond to fermion masses and mixing angles. A possible mechanism described in the Appendix appears if flavor is explained by a Froggatt-Nielsen mechanism [33], where the flavon field is coupled to the inflaton and gets a very large vacuum expectation value (VEV) of $\sim h$ during inflation, while the flavon VEV relaxes to its low-energy value when $h \simeq v$.

In this setup, we can rewrite (3.3) as

$$A''_\lambda + k\left(k - \lambda \frac{2\xi}{\tau}\right) A_\lambda = 0, \tag{3.15}$$

where we use the scale factor definition $a = -(H\tau)^{-1}$ as we are in de Sitter space. We solve this equation of motion asymptotically in the slow-roll regime. At early time, when $|k\tau| \gg 2\xi$, the modes are in their Bunch-Davies vacuum. When $|k\tau| \sim 2\xi$, one of the modes develops both parametric and tachyonic instabilities leading to exponential growth while the other stays in the vacuum. During the last e-folds of inflation, i.e. $|k\tau| \ll 2\xi$, the growing mode (with polarization λ) has the solution [11,34]

$$A_\lambda \simeq \frac{1}{\sqrt{2k}} \left(\frac{k}{2\xi a_E H_E}\right)^{\frac{1}{4}} \exp\left\{\pi\xi - 2\sqrt{\frac{2\xi k}{a_E H_E}}\right\}, \tag{3.16}$$

where a_E and H_E are, respectively, the scale factor and the Hubble parameter at the end of inflation. Here, as we assume a slow-roll regime, we consider H_E constant and we take the convention $a_E = 1$.

Using (3.16), the physical quantities in Eq. (3.5) become

$$\rho_E \simeq \frac{63}{2^{16}} \frac{H_E^4}{\pi^2 \xi^3} e^{2\pi\xi}, \qquad \rho_B \simeq \frac{315}{2^{18}} \frac{H_E^4}{\pi^2 \xi^5} e^{2\pi\xi},$$
$$\mathcal{H} \simeq \frac{45}{2^{15}} \frac{H_E^3}{\pi^2 \xi^4} e^{2\pi\xi}, \qquad \mathcal{G} \simeq \frac{135}{2^{16}} \frac{H_E^4}{\pi^2 \xi^4} e^{2\pi\xi}. \tag{3.17}$$

In this setup the Hubble can be computed from $3M_{\rm pl}^2 H_E^2 \simeq V(\chi_E)$ where the potential is given by Eq. (2.11).

These results are only valid when the absence of backreaction on the inflaton EoM (2.12a) is guaranteed, as we will see in Sec. III C. This model-dependent condition puts a lower bound on the parameter f_h or, equivalently, a higher bound on ξ.

2. With Schwinger effect

In cases where the Schwinger effect is at work, we can use the equilibrium Schwinger estimate [32] and redefine $\xi \to \xi_{\rm eff}$ with $\sigma \neq 0$ such that

$$A''_\lambda + k(k - 2\lambda \xi_{\rm eff} a H) A_\lambda = 0, \tag{3.18}$$

with $\xi_{\rm eff}$ given by the solution of [32]

$$\frac{63}{2^{17}\pi^2} \frac{e^{2\pi\xi_{\rm eff}}}{\xi_{\rm eff}^3} = \left(\frac{\pi^2}{e^3}\right)^2 (\xi - \xi_{\rm eff})^2 \tanh^2\left(\sqrt{\frac{5}{4}} \frac{\pi}{\xi_{\rm eff}}\right). \tag{3.19}$$

We show its behavior on Fig. 2 where we plot the effective parameter $\xi_{\rm eff}$ as a function of ξ. In this approximation the prediction for the gauge quantities in Eqs. (3.17) as those in the backreactionless scenario with the replacement $\xi \to \xi_{\rm eff}$. The consistency condition and

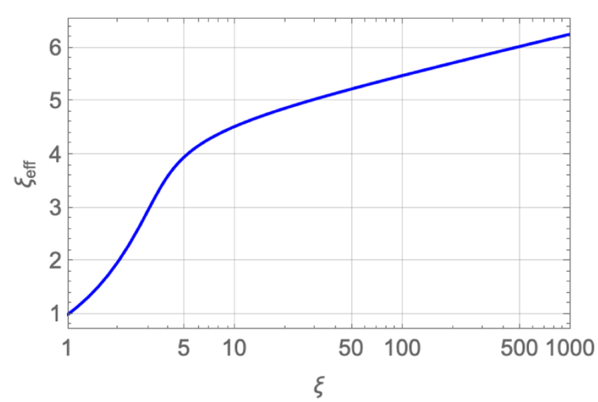

FIG. 2. In the Schwinger equilibrium estimate, the instability parameter ξ is replaced with an effective one that mimic the fermion backreaction on the gauge fields. We display their relation in the above plot.

the non-Gaussianity bounds that we will present, respectively, in Secs. III C and III D, should apply to the parameter ξ in the backreactionless case, and to the effective parameter $\xi_{\rm eff}$ in the case of the Schwinger effect with equilibrium solution.

C. Backreactionless consistency condition

In the absence of backreaction of the gauge field on the inflaton EoM, the inflationary equation (2.12a) with slow roll conditions reduces to $3H\dot{\chi} \simeq -V'(\chi)$. Thus, in order to consistently neglect the backreaction on the inflaton, we must simply enforce that, in the inflaton EoM (2.12a), the right-hand side term is negligible as compared to the kinetic term, i.e.

$$3H\dot{\chi} \gg K(\chi)\frac{\mathcal{G}}{f_\chi}. \tag{3.20}$$

Using the result (3.17) for $\mathcal{G}$ and the definition of ξ (3.4), this condition becomes

$$\frac{45}{2^{13}}\frac{e^{2\pi\xi}}{\xi^3} \ll \mathcal{P}_\zeta^{-1} \tag{3.21}$$

where the spectrum of primordial perturbations, for around 60 e-folds before the end of inflation (i.e. for $\chi = \chi_*$) is $\mathcal{P}_\zeta^{1/2} = H^2/(2\pi|\dot{\chi}|) \simeq 4.7\times 10^{-5}$ [21]. This leads to the upper bound $\xi_* \lesssim 4.74$, for which we can neglect the backreaction of the gauge fields on the inflaton EoM for the value of the inflaton field $\chi = \chi_*$. As we will see in the next section this condition is superseded by the condition of non-Gaussianity effects.

We must however ensure that condition (3.20) is valid throughout the end of inflation. Using the slow roll conditions, and the fact that, for our model, $V'(\chi_E) > V'(\chi_*)$, we found a stronger bound than the former one as Eq. (3.20) can be written as

$$\xi\mathcal{G} \ll \frac{V'^2}{6H^2}, \tag{3.22}$$

which leads to $\xi_E \lesssim 6.45$ (i.e. $\xi_* \lesssim 3.48$), at the end of inflation.

Once the nonbackreaction condition on the inflaton equation is satisfied, the no backreaction condition on the Friedmann equation

$$\frac{\langle \mathbf{E}^2+\mathbf{B}^2\rangle}{2a^4} = \frac{63}{2^{16}}\frac{H^4}{\pi^2\xi^3}e^{2\pi\xi}\left(1+\frac{5}{4\xi}\right) \ll V \simeq 3H^2 \tag{3.23}$$

holds automatically. In particular the latter condition leads to $\xi_E \lesssim 6.55$ (i.e. $\xi_* \lesssim 3.54$).

D. Non-Gaussianity bounds for HI

As pointed out in Refs. [35,36], even if the nonbackreaction conditions are satisfied, the coupling $h^2F\tilde{F}$ can generate cosmological fluctuations in the HI model. The perturbations on the inflaton are obtained by replacing $\chi(t,\vec{x}) = \bar{\chi}(t) + \delta\chi(t,\vec{x})$, where $\bar{\chi}(t)$ is the inflationary background and $\delta\chi(t,\vec{x})$ the fluctuation. The equation for the fluctuation is given by

$$\left[\frac{\partial^2}{\partial t^2} + 3H\frac{\partial}{\partial t} - \frac{\nabla^2}{a^2} + V''(\bar{\chi}) - \bar{K}'\frac{\mathcal{G}}{f_\chi}\right]\delta\chi = K(\chi)\frac{\delta\mathcal{G}}{f_\chi} \tag{3.24}$$

where $\bar{K} \equiv K(\bar{\chi})$ and $\delta\mathcal{G} = (\mathbf{E}\cdot\mathbf{B} - \langle\mathbf{E}\cdot\mathbf{B}\rangle)/a^4$.

The function $\bar{K}$ satisfies the condition $\bar{K}' = \sqrt{2/3}\bar{K}$, while for our potential, during the inflationary period, it turns out that $V''(\bar{\chi}) \simeq -\sqrt{2/3}V'(\bar{\chi})$. Then, the last two terms of the left-hand side of Eq. (3.24) are

$$V'' - \frac{\bar{K}'}{f_\chi}\mathcal{G} \simeq -\sqrt{\frac{2}{3}}\left(V' + \frac{\bar{K}}{f_\chi}\mathcal{G}\right) \simeq -\sqrt{\frac{2}{3}}V' \simeq V'' \tag{3.25}$$

where we have made use of the nonbackreaction condition (3.20). In this way the last term in the left-hand side of Eq. (3.24) can be safely neglected.

The resulting fluctuation equation has been explicitly solved in Ref. [37], provided the backreactionless consistency condition of Sec. III C is satisfied, as well as the correlation functions for the curvature perturbations on uniform density hypersurfaces $\zeta(t,\vec{x}) = -H\delta\chi(t,\vec{x})/\dot{\bar{\chi}}$. A good fit for the equilateral configuration of the three-point function yields the fit, valid for values $2 \lesssim \xi \lesssim 3$ [37],

$$f_{\rm NL}^{\rm equil} \simeq \frac{1.6\times 10^{-16}}{\xi^{8.1}}e^{6\pi\xi}. \tag{3.26}$$

The current Planck bound on $f_{\rm NL}^{\rm equil}$ [38], $f_{\rm NL}^{\rm equil} = -26 \pm 47$ yields, at CMB scales, the upper bound $\xi_* \lesssim 2.55$, at 95% C.L. A much stronger condition than that leading to the absence of backreaction. Given that in our model the almost constancy of ξ leads to the relation (3.12) the non-Gaussianity bound translates in our model into the bound $\xi_E \lesssim 4.71$. As already stated, all the calculations done in the absence of the Schwinger effect apply, in the presence of the Schwinger effect in the equilibrium approximation, to corresponding bounds on the effective parameter, i.e. $\xi_{\rm eff*} < 2.55$.

IV. BARYOGENESIS

We will follow in this section the formalism and technical details from Ref. [18] for the baryogenesis mechanism. In particular the value of the baryon-to-entropy ratio generated by the decay of the helicity at the electroweak phase transition is given by

$$\eta_B \simeq 4\times 10^{-12} f_{\theta_W} \frac{\mathcal{H}}{H_E^3}\left(\frac{H_E}{10^{13}\ \text{GeV}}\right)^{\frac{3}{2}}\left(\frac{T_{\text{rh}}}{T_{\text{rh}}^{\text{ins}}}\right) \simeq 9\times 10^{-11}, \tag{4.1}$$

where the last equality is the observational value [39]. T_{rh} is the reheating temperature after the period of inflation, while $T_{\text{rh}}^{\text{ins}}$ is the instant reheating temperature assuming that all the inflaton energy is converted into radiation, so that it is given by

$$V(\chi_E) \simeq \frac{\pi^2}{30} g_* (T_{\text{rh}}^{\text{ins}})^4, \tag{4.2}$$

where $g_* = 106.75$ is the SM number of degrees of freedom. When reheating is produced perturbatively by coherent oscillations of the inflaton around its potential minimum, then

$$T_{\text{rh}} = \left(\frac{90}{\pi^2 g_*}\right)^{\frac{1}{4}} \sqrt{\Gamma_h M_{\text{pl}}} \tag{4.3}$$

in which case $T_{\text{rh}}^{\text{ins}}$ is attained for values $\Gamma_h \simeq H_E$. For the present case of HI it was proved in Refs. [40–42] that for $\xi_h \gtrsim 100$ the total energy of the Higgs-inflaton condensate is transferred to Higgs particles and gauge bosons (leading to an instantaneous nonperturbative preheating), which efficiently decay into SM fermions, filling the Universe with a thermal plasma. Following Refs. [16,17] we define the parameter f_{θ_W}, which encodes all the details of the EW phase transition and its uncertainties,

$$f_{\theta_W} = -\sin(2\theta_W)\frac{d\theta_W}{d\ln T}\bigg|_{T=135\ \text{GeV}}, \quad 5.6\times 10^{-4} \lesssim f_{\theta_W} \lesssim 0.32. \tag{4.4}$$

We will then consider instant reheating [40–42], $T_{\text{rh}} \simeq T_{\text{rh}}^{\text{ins}}$, hence the ratio $T_{\text{rh}}/T_{\text{rh}}^{\text{ins}}$ drops in Eq. (4.1). However, in addition to their dependence on the gauge sector observables, the quantities used in this section vary according to the quartic coupling λ_h as $\xi \propto \lambda_h^{-1/2}$, see Eq. (3.10). Besides, the Hubble ratio at the end of inflation $H_E \simeq \sqrt{V(\chi_E)/3}$ also depend on λ_h as V does.

A. Constraints

There are however, a number of constraints that must be fulfilled before any claim on the BAU can be made, see Ref. [18]. To ensure that the required magnetohydrodynamical conditions are fulfilled for the (hyper)magnetic fields to survive until the electroweak crossover, we must demand that the magnetic Reynolds number at reheating $\mathcal{R}_m^{\text{rh}}$ is bigger than one. As we are in the viscous regime, we can compute

$$\mathcal{R}_m^{\text{rh}} \approx 5.9\times 10^{-6} \frac{\rho_B \ell_B^2}{H_E^2}\left(\frac{H_E}{10^{13}\ \text{GeV}}\right)\left(\frac{T_{\text{rh}}}{T_{\text{rh}}^{\text{ins}}}\right)^{\frac{2}{3}}, \tag{4.5}$$

where ℓ_B is the physical correlation length of the magnetic field given by

$$\ell_B = \frac{2\pi}{\rho_B a^3}\int^{k_c} dk \frac{k^3}{4\pi^2}(|A_+|^2 + |A_-|^2) \simeq \frac{8}{7}\frac{\pi\xi}{H_E}, \tag{4.6}$$

where in the second step we use the solution (3.16).

Then, the chiral plasma instability (CPI) temperature must be low enough to ensure that the CPI time scale is long enough to allow all right-handed fermionic states to come into chemical equilibrium with the left-handed ones via Yukawa coupling interactions (so that sphalerons can erase their corresponding asymmetries in particle number densities) before CPI can happen. The estimated temperature at which CPI takes place is

$$T_{\text{CPI}}/\text{GeV} \approx 4\times 10^{-7} \frac{\mathcal{H}^2}{H_E^6}\left(\frac{H_E}{10^{13}\ \text{GeV}}\right)^3\left(\frac{T_{\text{rh}}}{T_{\text{rh}}^{\text{ins}}}\right)^2. \tag{4.7}$$

The constraint $T_{\text{CPI}} \lesssim 10^5$ GeV (the temperature at which e_R comes into chemical equilibrium) guarantees that the CPI cannot occur before the smallest Yukawa coupling reaches equilibrium and all particle-number density asymmetries are erased, preventing thus the cancellation of the helicity generated at the reheating temperature.

Finally, with the values of energy densities and helicity at our hand we checked that the generation of baryon isocurvature perturbation provides no constraint.

B. Higgs inflation

As we have previously explained we will be agnostic about the mechanism stabilizing the Higgs potential and then just will consider λ_h as a free parameter. The corresponding plot, for values $10^{-3} \lesssim \lambda_h \lesssim 1$, is shown in Fig. 3, for the backreactionless case (left panel) and the Schwinger equilibrium solution (right panel), which shows that condition (4.1) provides a wide window for baryogenesis (in blue). Then we display in orange the region where $\mathcal{R}_m^{\text{rh}} > 1$, see Eq. (4.5), and in green the region where $T_{\text{CPI}} \lesssim 10^5$ GeV, see Eq. (4.7). In both plots the red region is excluded because of the CMB non-Gaussianity bound.

We can see that in this scenario, the BAU is attained for values

$$3.6 \lesssim \xi_E \lesssim 4.1. \tag{4.8}$$

This range is the same for both the backreactionless and the Schwinger equilibrium case by construction of the latter. However, because of the replacement $\xi \to \xi_{\text{eff}}$, the relation between ξ and the couplings λ and f_h is different in both

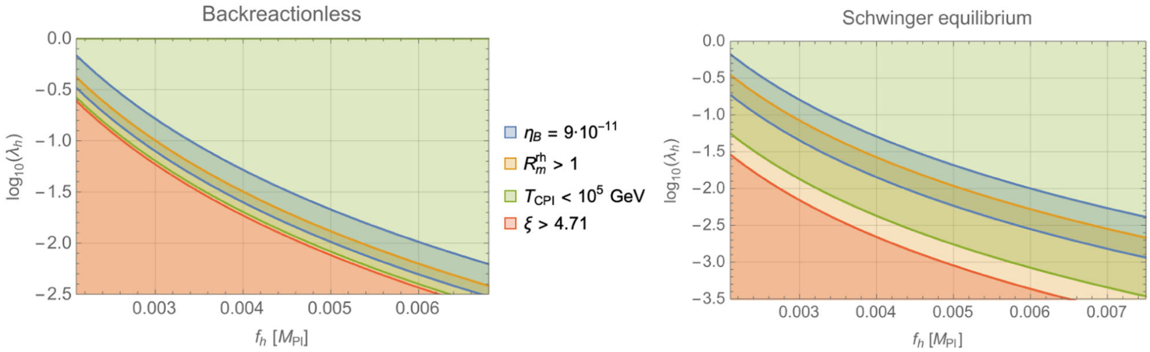

FIG. 3. The baryogenesis parameter space for the backreactionless (left panel) and Schwinger equilibrium (right panel) cases. The red region is excluded because of CMB non-Gaussianity. We seek the overlapping region between the first three one. The condition on CPI temperature is no constraint since it overlaps the entire region for η_B. Hence the tradeoff must be made between η_B and the magnetic Reynolds number.

cases (this is why we showed two panels on Fig. 3). These bounds correspond to the values

$$1.4\times10^4 \lesssim \frac{\rho_E}{H_E^4} \lesssim 1.7\times10^5, \qquad 1.4\times10^3 \lesssim \frac{\rho_B}{H_E^4} \lesssim 1.3\times10^4,$$
$$5.6\times10^3 \lesssim \frac{\mathcal{H}}{H_E^3} \lesssim 6.2\times10^4, \qquad 8.4\times10^3 \lesssim \frac{\mathcal{G}}{H_E^4} \lesssim 9.3\times10^4. \tag{4.9}$$

C. Critical Higgs inflation

Depending on the values of the Higgs and top-quark masses, λ_h could remain positive till the Planck scale, and such that $\lambda_h \ll 1$ and $\beta_{\lambda_h} \ll 1$ (exhibiting a *critical behavior*) without any need of new physics. In particular this should happen if the top-quark mass is $m_t \simeq 171.3$ GeV [22,43,44], which however exceeds its current value from direct measurements, $m_t = 172.76 \pm 0.30$ [45] by $\sim 3\sigma$. Those models initially proposed in Refs. [27,46–51] were dubbed critical Higgs inflation and in principle would not need any UV completion for the Higgs potential stabilization.

Nevertheless, in view of the actual experimental values of the Higgs and top-quark masses, people have been proposing UV completions changing the size of the quartic β-function, and such that λ_h, and β_{λ_h}, can attain a critical behavior for the values of the Higgs for which HI takes place, and stay positive all the way down to the electroweak scale [29].

In all cases, for critical values of λ_h, CHI has the advantage that the required value of the coupling to the Ricci scalar ξ_h, as given by Eq. (2.18), is considerably reduced with respect to ordinary HI. In particular $\xi_h \lesssim \mathcal{O}(10)$ for $\lambda_h \lesssim 4\times10^{-8}$. For these reasons, we found it interesting to show a wider parameter window of Fig. 3 that covers smaller values of the self-coupling parameter λ_h. We show, in Fig. 4, the overlapping region of Fig. 3 for $\lambda_h \ll 1$ where all conditions are met to successfully produce the BAU. As in this case, the Higgs self coupling can be arbitrary small, we used the exact solutions (2.8) and (2.9) instead of their approximations (2.10) and (2.11), with only minor differences.

V. PALATINI FORMULATION

In this paper we have used the metric formulation of gravity, where the connection giving rise to the Ricci scalar is identified with the Levi-Civita connection $\Gamma^\rho{}_{\mu\nu}$, and thus related to the metric $g_{\mu\nu}$. There is an alternative formulation,

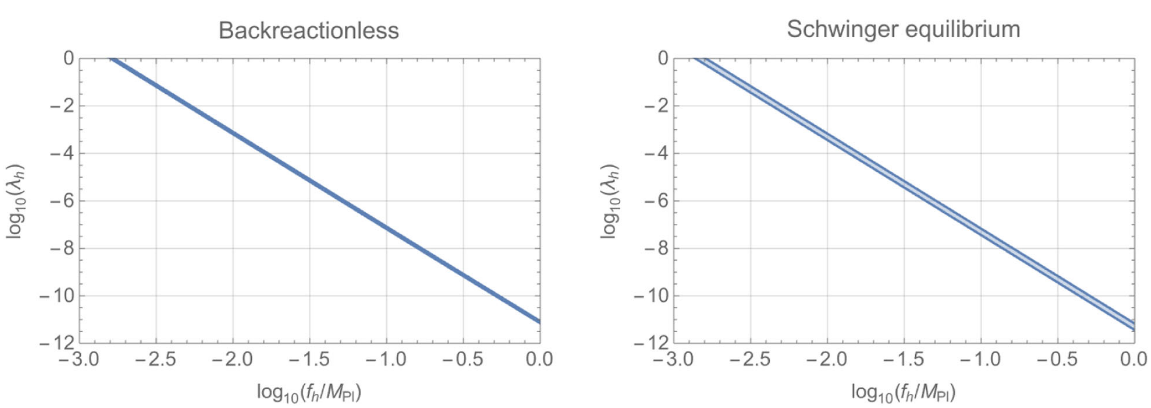

FIG. 4. Region where the BAU can successfully be achieved, for a wider range of the parameters.

the Palatini formulation of gravity, where the connection is arbitrary and torsion free, i.e. $\Gamma^{\rho}{}_{\mu\nu} = \Gamma^{\rho}{}_{\nu\mu}$. One of the main features of the Palatini formalism is that the inflationary predictions are different than those in the metric one [52].

In the Palatini HI (for a review, see e.g. Ref. [41]), the connexion from which the Ricci tensor is calculated does not depend on the metric, and the Weyl rescaling (2.4) leaves R invariant. Hence, in the Einstein frame, the Palatini action is written as

$$S_E = \int d^4x\sqrt{-g}\left[-\frac{R}{2} + \frac{\Theta^2}{2}\partial_\mu h \partial^\mu h - \frac{1}{4}Y_{\mu\nu}Y^{\mu\nu} - V(h)\right] - \int d^4x F(h) F_{\mu\nu}\tilde{F}^{\mu\nu}, \tag{5.1}$$

where Θ is given by (2.4), and the canonical inflaton χ is obtained by

$$\frac{d\chi}{dh} = \Theta = \frac{1}{\sqrt{1+\xi_h h^2}}. \tag{5.2}$$

This considerably simplifies the equations in terms of χ as we can now write exact analytical relations such as

$$h(\chi) = \frac{\sinh(\sqrt{\xi_h}\chi)}{\sqrt{\xi_h}}, \tag{5.3a}$$

$$V(\chi) = \frac{\lambda}{4\xi_h^2}\tanh^4(\sqrt{\xi_h}\chi). \tag{5.3b}$$

The slow-roll analysis from HI in the metric formulation is then modified as we now have

$$\sinh(2\sqrt{\xi_h}\chi_E) = 4\sqrt{2\xi_h}, \tag{5.4a}$$

$$\cosh(2\sqrt{\xi_h}\chi_*) \simeq 16\xi_h N_*. \tag{5.4b}$$

Using Eq. (5.4b), the amplitude of scalar fluctuations at N_*

$$A_s = \frac{\lambda_h}{768\pi^2}\frac{\sinh^4(\sqrt{\xi_h}\chi_*)\tanh^2(\sqrt{\xi_h}\chi_*)}{\xi_h^3} \tag{5.5}$$

leads to

$$\xi_h \simeq 1.4\times 10^9 \frac{\lambda}{0.1}\left(\frac{N_*}{60}\right)^2. \tag{5.6}$$

Finally, considering the coupling to the Chern-Simons density $F(\chi)F\tilde{F}$ given by the quadratic function

$$F(\chi) = \frac{\cos^2\theta_W}{4}\frac{h^2(\chi)}{f_h^2} = \frac{\cos^2\theta_W}{4}\frac{\sinh^2(\sqrt{\xi_h}\chi)}{\xi_h f_h^2} \tag{5.7}$$

the parameter ξ is given, using Eq. (3.8), by

$$\xi = \frac{4\cos^2\theta_W}{f_h^2}, \tag{5.8}$$

which is constant throughout all the inflationary period.

Notice the important difference between HI in the metric and Palatini formalisms. While in the former the parameter ξ is almost constant, just providing a small growth $\xi_E \sim 1.84\xi_*$ at the end of inflation, in the latter the parameter ξ is exactly a constant throughout all the inflationary period. Therefore while in the metric formulation the non-Gaussianity bound on ξ at the CMB $\xi_* < 2.55$ translates into the bound $\xi_E < 4.71$ at the end of inflation, when the helical magnetic fields are generated, relaxing its helicity into the baryon asymmetry at the electroweak phase transition, in the Palatini formulation the non-Gaussianity bound at the end of inflation is $\xi_E < 2.55$. Given the baryogenesis window (4.8) we have found, this result means that, while Palatini HI can be a viable candidate to produce cosmological inflation, however the magnetic fields produced at the end of Palatini HI have not enough strength to generate the baryon asymmetry of the Universe.

VI. CONCLUSION

Baryogenesis and cosmological inflation are two main issues which usually require the existence of BSM physics. (i) The baryogenesis mechanism is too weak in the SM for the present values of the Higgs boson, as the electroweak phase transition is too weak (a crossover) and the amount of CP-violation induced by the CKM phase too small due to the presence of light-quark masses. Thus, most baryogenesis mechanisms rely on BSM extensions for which the electroweak phase transition is strong first order and have an extra source of CP-violation. Still there is a tension between electric dipole moment (EDM) bounds and the required amount of BAU. (ii) On the other hand, cosmological inflation requires the presence of an extra BSM field χ, the inflaton, with an appropriately flat potential. In view of the lack of experimental evidence for BSM physics at low energy, there have been attempts to solve the above problems with as much as possible SM physics.

(i) Concerning the baryogenesis mechanism, in the presence of the inflaton coupling to the Chern-Simons hypercharge density $\chi F\tilde{F}$, generating CP-violation, helical gauge fields can be produced at the end of inflation and the helicity relaxes to baryon asymmetry at the electroweak crossover generating the observed BAU. In this way the physics at the electroweak breaking scale is that provided by the SM of electroweak and strong interactions.

(ii) Concerning the problem of cosmological inflation, it was proven that the Higgs field $\mathcal{H}$ can generate enough inflation, consistent with cosmological observations by the Planck Collaboration, provided that it is nonminimally coupled to gravity. In this case one could achieve cosmological inflation with the SM degrees of freedom. Still this approach has some caveats. One of them being that, in the SM, for the current values of the Higgs and top-quark masses, the Higgs self coupling is driven to negative values at scales $\sim 10^{11}$ GeV, much lower than the inflationary (Planckian) scales, so one needs some UV completion to change the RGE evolution of the SM couplings, or perhaps some criticality value of the SM quartic coupling at the inflationary scales.

In this paper we have merged both above approaches. In particular we have considered Higgs inflation, where the Higgs is nonminimally coupled to gravity, and added a dimension-6CP-violating operator coupling the Higgs to the hypercharge Chern-Simons density, $|\mathcal{H}|^2 F\tilde{F}$. We have proven there is an explosive production of helical hypermagnetic fields which can produce baryogenesis when the helicity relaxes into the BAU at the electroweak crossover. The parameter ξ responsible for the energy transfer from the inflaton to the gauge fields is almost a constant, due to the particular shape of the inflationary potential and the coupling of the Higgs to the Chern-Simons density, and we can thus fully rely on analytic approximations to consider the gauge field solutions. We have also proven that the helicity produced at the end of inflation satisfies the required magnetohydrodynamical conditions to survive to the electroweak phase transition, and produce the observed BAU, for a window of ξ at the CMB scales given by $1.96 < \xi_* < 2.23$ (corresponding at the end of inflation to $3.6 < \xi_E < 4.1$), thus satisfying the bound $\xi_* < 2.55$ on non-Gaussianity.

In the above analysis we have worked in the metric formulation of gravity and considered two especially simple cases: (a) In the absence of Schwinger effect; and (b) In the presence of Schwinger effect. We have implemented case (a) by assuming that the SM flavor problem is implemented by means of a Froggatt-Nielsen mechanism, in the case where the flavon field is coupled to the inflaton. As a consequence of this coupling, during inflation one can easily impose the condition that all fermions be heavy (say as heavy as the top quark) in such a way that the Schwinger conductivity, which is exponentially suppressed by the fermion mass squared, is negligible and the Schwinger effect turns out to also be negligible. After inflation the flavon field relaxes to its usual minimum which can describe all fermion masses and mixing angles at the electroweak scale. The details of the mechanism are described in the Appendix. As for case (b), in the presence of the Schwinger effect, we have taken advantage of the (almost) constancy of the parameter ξ to use the simple Schwinger equilibrium approximation, which simply amounts to a redefinition of the ξ parameter. In all cases we have extended our calculation to the case of critical Higgs inflation and found that for values of the quartic Higgs self-coupling $\lesssim 10^{-10}$ the coupling $1/f_h$ of the Higgs to the Chern-Simons density $\frac{h^2}{f_h^2} F\tilde{F}$ can be $\lesssim M_{\rm Pl}^{-1}$, in the weakly coupled region.

We also have considered the Palatini formulation of gravity. In this case the equations for the change from the Jordan to the Einstein frame are analytic, as well as the inflationary potential and the relation between the inflaton χ and the Higgs field h. As a consequence of the shape of the inflationary potential it turns out that in this model the parameter ξ is exactly a constant, i.e. $\xi_* = \xi_E$. In this formalism helical gauge fields can be produced, however the bounds on non-Gaussianity impose that its production is not so explosive as required to trigger electroweak baryogenesis, which is then forbidden in this model. It was already known that the two formalisms of gravity, the metric and the Palatini formulations, lead to different inflationary predictions. In this paper we have also proven that they behave differently concerning the baryogenesis capabilities of the helical gauge fields produced at the end of inflation.

There are a number of physics problems that are left open in the present work, and deserve future analysis, some of them being related to the classical problems of Higgs inflation. One of them is related to the stabilization of the Higgs potential, and the possibility of getting critical values of the Higgs mass at the inflationary scales. This problem is particularly relevant in the case where the SM flavor problem is solved by a Froggatt-Nielsen mechanism where the flavon field is coupled to the inflaton, in the way we have described in this paper. This analysis clearly requires a more detailed analysis of the renormalization group running in the presence of the Froggatt-Nielsen mechanism, working at the inflationary scales. Another obvious problem, which was outside the scope of the present paper, is the analysis of the Schwinger effect, in Higgs inflation, by numerical methods as those used in Refs. [19,53–55].

Finally, some comments on the effective, CP-violating, operator in Eq. (2.3) are in order here. From Eqs. (2.10) and (2.14) we find that, for $\lambda_h \simeq 0.1$, it turns out that $h_E \simeq 10^{-2}$, while the value of the scale f_h which provides the BAU, from Fig. 4, is such that the expansion parameter $h_E/f_h \gtrsim \mathcal{O}(1)$. For smaller values of λ_h characteristic of CHI we obtain similar results. For instance for $\lambda_h = 10^{-8}$, $h_E \simeq 0.6$ and the value of f_h which provides the correct value of the BAU, is such that $h_E/f_h \gtrsim O(1)$. As these values are at the limit of validity of the effective theory, the UV completion should be such that higher dimensional operators do not spoil the validity of the results. An example of such UV completion was provided in Ref. [17].

ACKNOWLEDGMENTS

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APPENDIX: FROGGATT-NIELSEN MECHANISM IN DE SITTER SPACE

The Froggatt-Nielsen (FN) mechanism [33] is one of the simplest and most elegant solutions to the problem of flavor for the SM fermions. The hierarchy of masses and mixing angles for quarks and leptons can be explained by a global, generation dependent, $U(1)$ symmetry under which the fermions are charged. This symmetry is spontaneously broken by the radial part of scalar field $S \equiv \sigma e^{i\theta}$, the "flavon field", which is charged under the $U(1)$ (with charge conventionally normalized to -1) and which has a VEV, $\langle\sigma\rangle = v_\sigma$. The breaking is communicated to the fermion sector at different orders in the parameter $\lambda(\langle\sigma\rangle) = \langle\sigma\rangle/M_\star$, where M_* is the scale of flavor dynamics, which depend on the charges of the SM fermions q_i, u_i^c, d_i^c, ℓ_i, e_i^c involved in Yukawa couplings.

If we denote the $U(1)$ charge of the fermion f by $[f]$, the Yukawa coupling matrices are given by

$$Y_u^{ij} \sim \lambda^{[q_i]+[u_j^c]}, \quad Y_d^{ij} \sim \lambda^{[q_i]+[d_j^c]}, \quad Y_\ell^{ij} \sim \lambda^{[\ell_i]+[e_j^c]}. \quad \text{(A1)}$$

When the field σ is at its minimum, and provided that $\lambda(v_\sigma) \simeq 0.2$, of the order of the Cabibbo angle, one can choose the $U(1)$ charges such that the SM fermion mass spectrum and mixing angles are correctly described. A simple example is provided by (see e.g. Ref. [56] for a pedagogical introduction) $[q_{3,2,1}] = [u_{3,2,1}^c] = (0,2,4)$, $d_{3,2,1}^c = (2,2,3)$, $[\ell_{3,2,1}] = (2,2,3)$, $[e_{3,2,1}^c] = (0,2,4)$. However, the details of the model are not important for our argument here.

We will introduce a coupling between the flavon and the inflaton (Higgs fields) as $|S|^2|H|^2$, and assume that the flavon field has a potential given, in the Jordan frame, by

$$U(\sigma) = \lambda_1(|S|^2 - v_\sigma^2 - \lambda_2|H|^2)^2 \quad \text{(A2)}$$

which corresponds, in the Einstein frame, to the potential

$$V(\sigma) = \frac{\lambda_1(\sigma^2 - v_\sigma^2 - \frac{1}{2}\lambda_2 h^2)^2}{(1+\xi_h h^2/M_p^2)^2}, \quad \text{(A3)}$$

where $v_\sigma \gg v$, so that at electroweak scales $(h \sim v)$ the vacuum expectation value $\langle\sigma\rangle \simeq v_\sigma$, which spontaneously breaks the flavor symmetry.[7]

[7]After the global $U(1)$ symmetry breaking a (massless) Goldstone boson will remain in the spectrum. To avoid phenomenological problems it is usually assumed that there is a small explicit soft breaking of the $U(1)$ symmetry giving a mass to the (pseudo) Goldstone boson. These model details are also orthogonal to our argument here.

At the electroweak phase transition, when the field σ is at its minimum v_σ, and provided that the flavor scale be $M_* \simeq 5v_\sigma$, it is possible to solve the flavor problem for fermion masses. Moreover, there is an extra quartic coupling for the Higgs field from the potential (A2) which is negligible, compared to the SM one, provided that $\lambda_1\lambda_2^2 \ll \lambda_h$, where λ_h is the SM Higgs quartic coupling evaluated at the electroweak scale. This condition can be widely satisfied e.g. for typical values of the couplings

$$\lambda_1 = \lambda_2 = 0.1. \quad \text{(A4)}$$

However during the de Sitter phase, things can be pretty much different. We will study the possibility that at the end of inflation $\lambda(\langle\sigma\rangle) \simeq 1$. In fact, at the end of inflation $h_E \simeq 10^{-2}M_p$ and one can safely neglect v_σ^2 as compared to $\frac{1}{2}\lambda_2 h_E^2$, so that $\langle\sigma\rangle \simeq \sqrt{\lambda_2/2}\,h_E$, which dictates the flavor scale M_* by imposing the condition $\lambda(\langle\sigma\rangle) \simeq 1$ as

$$M_* \simeq \sqrt{\lambda_2/2}\,h_E, \quad \text{(A5)}$$

which yields, e.g. for the values of the couplings in (A4), $v_\sigma \simeq 10^{15}$ GeV.

Moreover, the condition for the de Sitter fluctuations to be suppressed, so that the field σ stays anchored to its minimum $V(\langle\sigma\rangle) = 0$, during inflation $V''(\langle\sigma\rangle) > \frac{9}{4}H_E^2$ [57], translates into the condition

$$\frac{8\lambda_1\langle\sigma\rangle^2}{(1+\xi_h h_E^2/M_p^2)^2} > \frac{9}{4}H_E^2 \quad \text{(A6)}$$

which, using the value of h_E above and $H_E \simeq 2\times 10^{13}$ GeV, yields the condition $\sqrt{\lambda_1\lambda_2} \gtrsim 10^{-3}$, which is satisfied for the choice in Eq. (A4).

What are the implications of the above scenario for the conductivity in the Schwinger effect? As we have seen the conductivity from a Dirac fermion f, of electric charge Q_f and Yukawa coupling Y_f, is exponentially suppressed as $\sim e^{-A_f}$, where

$$A_f = \frac{\pi Y_f^2 h^2}{2|eQ_f||E|} \quad \text{(A7)}$$

and for $A_f \gg 1$ it does not contribute to the Schwinger effect. Now, considering, at the end of HI, $Y_f \sim 1$ and $h_E \simeq 10^{-2}M_{\rm pl}$, the condition for the fermion f to not create any conductivity, $A_f \gg 1$, self-consistently translates into an upper bound on the generated electric field $|E|$ in the absence of Schwinger effect, as

$$\frac{|E|}{H_E^2} \ll \frac{10^7}{|Q_f|}. \tag{A8}$$

The strongest bound is then provided by the leptons, for which $|Q_\ell| = 1$ so that a (conservative) safe bound for all charged SM fermions to not contribute to the Schwinger effect is $E \lesssim 10^6 H_E^2$. If we use the analytic expression for zero conductivity, $\rho_E = 63/(2^{16}\pi^2\xi^3)e^{2\pi\xi}H_E^4$, we get the corresponding upper bound $\xi \lesssim 6.7$, which translates into the lower bound on the parameter f_h, as $f_h \gtrsim 0.0022 M_{\rm pl}$.

The previous model is just an example of UV completion of the FN mechanism which allows for the absence of Schwinger effect at the end of inflation. In fact any UV completion providing $\langle\sigma\rangle \gtrsim M_*$ at the end of inflation and relaxing to $\langle\sigma\rangle \simeq 0.2 M_*$ would do a similar job.

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