Semileptonic decays of spin-entangled baryon-antibaryon pairs

Varvara Batozskaya,[1,2,*] Andrzej Kupsc,[3,2] Nora Salone,[2] and Jakub Wiechnik[2,4]

[1]*Institute of High Energy Physics, Beijing 100049, People's Republic of China*
[2]*National Centre for Nuclear Research, Pasteura 7, 02-093 Warsaw, Poland*
[3]*Department of Physics and Astronomy, Uppsala University, Box 516, SE-75120 Uppsala, Sweden*
[4]*Faculty of Physics, University of Warsaw, Pasteura 5, 02–093 Warsaw, Poland*

(Received 20 February 2023; accepted 28 June 2023; published 18 July 2023; corrected 26 July 2023)

A modular representation for the semileptonic decays of baryons originating from spin-polarized and correlated baryon-antibaryon pairs is derived. The complete spin information of the decaying baryon is propagated to the daughter baryon via a real-valued matrix. It allows us to obtain joint differential distributions in sequential processes involving the semileptonic decay in a straightforward way. The formalism is suitable for extraction of the semileptonic form factors in experiments where strange baryon-antibaryon pairs are produced in electron-positron annihilation or in charmonia decays. We give examples such as the complete angular distributions in the $e^+e^- \to \Lambda\bar{\Lambda}$ process, where $\Lambda \to pe^-\bar{\nu}_e$ and $\bar{\Lambda} \to \bar{p}\pi^+$. The formalism can also be used to describe the distributions in semileptonic decays of charm and bottom baryons. Using the same principles, the modules to describe electromagnetic and neutral current weak baryon decay processes involving a charged lepton-antilepton pair can be obtained. As an example, we provide the decay matrix for the Dalitz transition between two spin-1/2 baryons.

DOI: 10.1103/PhysRevD.108.016011

I. INTRODUCTION

Baryon semileptonic (SL) decays are an important tool to study transitions between ground state baryons. Comparing to the nonleptonic baryon decays where at least three hadronic currents are involved, the SL transition involves only a two-point hadronic vertex and the external W-boson field coupled to the leptonic current. The properties of the hadronic vertices are described by a set of scalar functions—form factors, that depend on the invariant mass squared of the emitted virtual W boson. In particular, the semileptonic processes allow us to probe the kinematic regions of the form factors that are dominated by the static properties of the baryons. The recent progress in the lattice quantum chromodynamics gives a hope to determine the properties of the form factors from the first principles with the accuracy sufficient for a comparison with precise experimental data [1]. Once the hadronic effects are well-understood, the SL decays will provide a complementary method to determine Cabbibo-Kobayashi-Maskawa matrix elements [2] and to search for beyond the Standard Model effects such as violation of lepton flavor and charge-conjugation-parity symmetries [3]. In this article, we provide a modular description of the semileptonic decays that can be used to extract properties of the form factors in the experiments using spin-entangled baryon-antibaryon pairs.

The helicity amplitude method [4–6] that is commonly used in the analyses of semileptonic decays allows us to express the angular distributions in an efficient and compact way. The complete process is described as a sequence of two-body decays, where each of them is analyzed in the rest frame of the subsequent decaying particle. For a semileptonic decay $B_1 \to B_2 + \ell^-\bar{\nu}_\ell$, the first decay step $B_1 \to B_2 W^-_{\text{off-shell}}$ is analyzed in the B_1 rest frame, whereas $W^-_{\text{off-shell}} \to \ell^-\bar{\nu}_\ell$ is analyzed in the $W^-_{\text{off-shell}}$ rest frame. The resulting expressions for the differential distributions are compact and can be written in a quasifactorized form. The formalism also describes joint angular distributions in the semileptonic decays of a spin-polarized baryons.

A novel approach to study strange baryon decays is to use hyperon-antihyperon pairs from J/ψ resonances produced in electron-positron annihilations [7]. The complete angular distribution in such processes can be conveniently represented using a product of real-valued matrices that describe the initial spin-entangled baryon-antibaryon state and chains of two-body weak decays. These matrices can be rearranged to describe many decay scenarios in the $e^+e^- \to \Lambda\bar{\Lambda}$, $e^+e^- \to \Xi\bar{\Xi}$ and similar processes [7–10]. Several high-profile analyses using multidimensional

*varvara.batozskaya@ncbj.gov.pl

maximum-likelihood fits to angular distributions were performed by the electron-positron collider experiment BESIII [11,12] using this modular formalism. These multidimensional analyses have demonstrated increased precision of the decay parameters measurements and enabled to observe effects that were averaged out in previous studies, such as a polarization of the hyperon-antihyperon pair from charmonia decays.

The same spin-entangled hyperon-antihyperon system can be used to study semileptonic decays such as $\Lambda \to pe^-\bar{\nu}_e$ or $\Xi^- \to \Lambda e^-\bar{\nu}_e$. The processes are relatively rare with the branching fractions (BFs) $8.32(14)\times 10^{-4}$ and $5.63(31)\times 10^{-4}$, respectively [13]. In the reactions $e^+e^- \to J/\psi \to \Lambda\bar{\Lambda}$ and $e^+e^- \to J/\psi \to \Xi^-\bar{\Xi}^+$ the hyperon semileptonic decay is tagged via a common decay of the antihyperon; $\bar{\Lambda} \to \bar{p}\pi^+$ and $\bar{\Xi}^+ \to \bar{\Lambda}\pi^+$, respectively. The tagging processes involve only charged particles in the final state, therefore their momenta can be precisely determined. This allows one to reconstruct the momentum of the antineutrino in the semileptonic process and to determine the four-momentum squared of the lepton pair that is needed to study the dynamics of the process. The polarization of the hyperons is given by the angular distributions in their decays, but usually the polarization of the leptons is not measured. Such a double-tag (DT) technique is often used to determine absolute branching fractions in electron-positron collider experiments [14]. With a large number of collected events in experiments such as BESIII [15] studies of decay distributions in the semileptonic hyperon decays are possible. A formalism that uses spin correlations and polarization of the produced baryon-antibaryon system is needed to determine the decay parameters with the best precision. The purpose of this report is to extend the approach from Refs. [8,10] to include decay matrices representing the three-body semileptonic processes. Our starting point is the helicity formalism for semileptonic decays from Ref. [6]. We construct a real-valued decay matrix relating the initial and final baryons' spin states, represented by the Pauli matrices. The obtained decay matrix is used to construct the full joint decay distributions of the spin-entangled baryon-antibaryon pair in a modular way.

The paper is organized as follows: In Sec. II and Sec. IV we review the formalism to describe baryon-antibaryon production process and semileptonic decays, respectively. In Sec. V the main result is derived—the spin-density matrix of the daughter baryon in the semileptonic decay. Section VI presents modular formulas to describe the angular distributions of the semileptonic hyperon decays. Finally, in Sec. VII we collect some numerical results.

II. PRODUCTION PROCESS

In general a state of two spin-$1/2$ particles e.g., a baryon-antibaryon pair $B_1\bar{B}_1$ can be written as [8]

$$\rho_{B_1\bar{B}_1} = \sum_{\mu,\bar{\nu}=0}^{3} C_{\mu\bar{\nu}}\sigma_\mu^{B_1} \otimes \sigma_{\bar{\nu}}^{\bar{B}_1}, \tag{1}$$

where a set of four Pauli matrices $\sigma_\mu^{B_1}(\sigma_{\bar{\nu}}^{\bar{B}_1})$ acting in the rest frame of a baryon $B_1(\bar{B}_1)$ is used and $C_{\mu\bar{\nu}}$ is a 4×4 real matrix representing polarizations and spin correlations of the baryons. Here we consider mainly baryon-antibaryon systems created in the $e^+e^- \to B_1\bar{B}_1$ process. However, the formalism can be applied for the pairs from decays of (pseudo)scalar or tensor particles such as $\psi(2S), \eta_c, \chi_{c0}, \chi_{c2} \to B_1\bar{B}_1$ or in a fact to any pair of spin-$1/2$ particles (for example baryon-baryon, muon-antimuon, and others). The spin matrices $\sigma_\mu^{B_1}$ and $\sigma_{\bar{\nu}}^{\bar{B}_1}$ are given in the coordinate systems with the axes denoted $\hat{\mathbf{x}}_1, \hat{\mathbf{y}}_1, \hat{\mathbf{z}}_1$ and $\hat{\mathbf{x}}_3, \hat{\mathbf{y}}_3, \hat{\mathbf{z}}_3$ as shown in Fig. 1. The directions of the two right-handed coordinate systems are related as $(\hat{\mathbf{x}}_3, \hat{\mathbf{y}}_3, \hat{\mathbf{z}}_3) = (\hat{\mathbf{x}}_1, -\hat{\mathbf{y}}_1, -\hat{\mathbf{z}}_1)$. The spin-correlation matrix $C_{\mu\bar{\nu}}$ for the reaction $e^+e^- \to B_1\bar{B}_1$ depends in the lowest order on two parameters, $\alpha_\psi \in [-1, 1]$ and $\Delta\Phi \in [-\pi, \pi)$. The elements of the $C_{\mu\bar{\nu}}$ matrix are functions of the baryon B_1 production angle θ_1 in the electron-positron center-of-momentum system. The matrix for the single-photon annihilation of unpolarized electrons and positrons is [8]

$$C_{\mu\bar{\nu}} \propto \begin{pmatrix} 1+\alpha_\psi\cos^2\theta_1 & 0 & \beta_\psi\sin\theta_1\cos\theta_1 & 0 \\ 0 & \sin^2\theta_1 & 0 & \gamma_\psi\sin\theta_1\cos\theta_1 \\ -\beta_\psi\sin\theta_1\cos\theta_1 & 0 & \alpha_\psi\sin^2\theta_1 & 0 \\ 0 & -\gamma_\psi\sin\theta_1\cos\theta_1 & 0 & -\alpha_\psi-\cos^2\theta_1 \end{pmatrix}, \tag{2}$$

where the parameters β_ψ and γ_ψ are expressed via α_ψ and $\Delta\Phi$ as $\beta_\psi = \sqrt{1-\alpha_\psi^2}\sin(\Delta\Phi)$ and $\gamma_\psi = \sqrt{1-\alpha_\psi^2}\cos(\Delta\Phi)$. We will also use a more general formula from Ref. [10] that describes the annihilation processes with polarized electron beams.

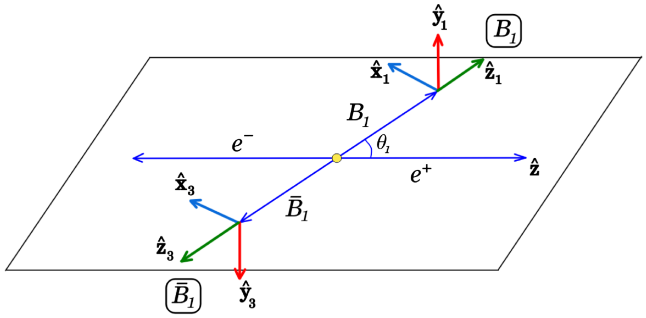

FIG. 1. Definition of the three coordinate systems used to describe the spin-entangled $B_1\bar{B}_1$ state. The overall c.m. frame with $\hat{\mathbf{z}}$ axis (e.g., for $e^+e^- \to B_1\bar{B}_1$ it is defined along the positron momentum). The axes in baryon B_1 and antibaryon $\bar{B}_1$ rest frames (helicity frames) are denoted $(\hat{\mathbf{x}}_1, \hat{\mathbf{y}}_1, \hat{\mathbf{z}}_1)$ and $(\hat{\mathbf{x}}_3, \hat{\mathbf{y}}_3, \hat{\mathbf{z}}_3)$, respectively.

III. INVARIANT FORM FACTORS

Let us consider a semileptonic decay of a $1/2^+$ hyperon B_1 into a $1/2^+$ baryon B_2 and an off-shell W^- boson decaying to the lepton pair $l^-\bar{\nu}_l$ with the momenta and masses denoted as $B_1(p_1,M_1)\to B_2(p_2,M_2)+l^-(p_l,m_l)+\bar{\nu}_l(p_\nu,0)$. The matrix elements due to the vector J^V_μ and axial-vector J^A_μ currents in notation from Ref. [6] are

$$\begin{aligned}\langle B_2|J^V_\mu + J^A_\mu|B_1\rangle = \bar{u}(p_2)\Big[&\gamma_\mu(F^V_1(q^2)+F^A_1(q^2)\gamma_5)\\ &+\frac{i\sigma_{\mu\nu}q^\nu}{M_1}(F^V_2(q^2)+F^A_2(q^2)\gamma_5)\\ &+\frac{q_\mu}{M_1}(F^V_3(q^2)+F^A_3(q^2)\gamma_5)\Big]u(p_1),\end{aligned} \tag{3}$$

where $q_\mu \coloneqq (p_1-p_2)_\mu = (p_l+p_\nu)_\mu$ is the four-momentum transfer. The four-momentum squared q^2 ranges from m_l^2 to $(M_1-M_2)^2$. The form factors $F^{V,A}_{1,2,3}(q^2)$ are complex functions of q^2 that describe hadronic effects in the transition. Neglecting possible CP-odd weak phases, the corresponding form factors are the same for the $(l^-,\bar{\nu}_l)$ and (l^+,ν_l) transitions. To fully determine the hadronic part of a semileptonic decay, the six involved form factors should be extracted as a function of q^2. The form factors are usually parametrized by the axial-vector to vector g_{av} coupling, the weak-magnetism g_w coupling, and the pseudoscalar g_{av3} coupling. They are obtained by normalizing to $F^V_1(0)$,

$$g_{av} = \frac{F^A_1(0)}{F^V_1(0)}, \qquad g_w = \frac{F^V_2(0)}{F^V_1(0)}, \qquad g_{av3} = \frac{F^A_3(0)}{F^V_1(0)}. \tag{4}$$

For experiments with a limited number of events, the q^2-dependence of the form factors is assumed using a model. The standard approach is to include one or more poles of the mesons that have the correct quantum numbers to mix with the W boson and have the masses close to the q^2 range in the decay. Traditionally, one pole is explicitly included together with an effective contribution from other poles [16] such as in the Becirevic-Kaidalov (BK) [17] parametrization,

$$F_i(q^2) = \frac{F_i(0)}{1-\frac{q^2}{M^2}}\frac{1}{1-\alpha_{\rm BK}\frac{q^2}{M^2}}, \tag{5}$$

where the dominant pole mass M is outside the kinematic region and the parameter $\alpha_{\rm BK}$ represent an effective contribution from the meson poles with higher messes. Here the case $\alpha_{\rm BK}=0$ represents the dominant pole contribution. This parametrization gives real-valued form factors. If more data is available, one or more extra parameters can be added to describe the q^2 distribution. In the hyperon decays the range of $q^2 \le (M_1-M_2)^2$ is limited and in the first order can completely neglect the q^2 dependence using the values of the couplings at the $q^2=0$ point. A better approximation is to include an effective-range parameter r_i that represent linear dependence on q^2,

$$F_i(q^2) = F_i(0)[1+r_iq^2+\cdots]. \tag{6}$$

For example, using (5) the effective-range parameter is $r_i = (1+\alpha_{\rm BK})/M^2$. The main takeaway message from the above discussion is that, for practical purposes, the q^2 dependence of an SL form factor can be represented by one or two parameters. In experiments, these parameters can be determined from the observed distributions. The optimal method for such parametric estimation is the maximum likelihood method using multidimensional unbinned data. We will first construct modular formulas for the angular distributions and then in Sec. VII discuss the attainable statistical uncertainties for the SL form factors parameters as the function of the number of observed events.

IV. HELICITY AMPLITUDES

We will describe the $B_1 \to B_2 + W^-_{\text{off-shell}}$ process using three-coordinate systems attached to the three involved particles. In the baryon B_1 rest frame $\mathbb{R}_1$, with the $(\hat{\mathbf{x}}_1, \hat{\mathbf{y}}_1, \hat{\mathbf{z}}_1)$ Cartesian coordinate system shown in Fig. 1, the B_1-spin projection on the quantization axis $\hat{\mathbf{z}}_1$ is $\kappa = \pm 1/2$. The daughter baryon B_2 is emitted in the direction given by the spherical coordinates θ_2, ϕ_2 in $\mathbb{R}_1$ and the B_2-helicity is $\lambda_2 = \pm 1/2$. The off-shell W^- boson is emitted in the direction $\theta_W = \pi - \theta_2$, $\phi_W = \pi + \phi_2$ in the $\mathbb{R}_1$ frame. It has helicity $\underline{\lambda}_W = \{t, -1, 0, +1\}$ where the time component, $\underline{\lambda}_W = t$, corresponds to $J_W = 0$ and the remaining three components to $J_W = 1$. Therefore, $\underline{\lambda}_W$ uniquely defines both spin J_W and helicity λ_W as $J_W(\underline{\lambda}_W) = \{0,1,1,1\}$ and $\lambda_W(\underline{\lambda}_W) = \{0,-1,0,1\}$, respectively. The four-momentum vector of the off-shell W^- is $q_\mu = (q_0, p\sin\theta_W\cos\phi_W, p\sin\theta_W\sin\phi_W, p\cos\theta_W)$ in the $\mathbb{R}_1$

system. The energy q_0 of the off-shell W^- boson and the magnitude of the three-momentum p are the following functions of the q^2 invariant

$$q_0(q^2) = \frac{1}{2M_1}(M_1^2 - M_2^2 + q^2) \tag{7}$$

and

$$p(q^2) = |\mathbf{p}_2| = \frac{1}{2M_1}\sqrt{Q_+Q_-}, \tag{8}$$

where

$$Q_\pm = (M_1 \pm M_2)^2 - q^2. \tag{9}$$

The spin direction and subsequent decays of the baryon B_2 and boson $W^-_{\text{off-shell}}$ are described in two helicity systems denoted $\mathbb{R}_2$ and $\mathbb{R}_W$, respectively. The helicity frame $\mathbb{R}_2$ is obtained by performing three active rotations: (a) around the $\hat{\mathbf{z}}_1$–axis by $-\phi_2$; (b) a rotation around the new $\hat{\mathbf{y}}$–axis by $-\theta_2$; (c) a rotation around the $\hat{\mathbf{z}}_2$-axis by $+\chi_2$, see Fig. 2 [18]. The first two rotations are sufficient to align $\mathbf{p}_2$ with the z-axis and such two-rotations prescription is used e.g., in Ref. [8]. Here we allow for an additional rotation that can be e.g., used to bring the momenta $\mathbf{p}_2$, $\mathbf{p}_l$, and $\mathbf{p}_\nu$ to one plane. Initially, we consider the angle χ_2 of this rotation as an arbitrary parameter. The combined (a)–(c) three-dimensional rotation is given by the product of three axial rotations $\mathcal{R}(\chi_2, -\theta_2, -\phi_2) = R_z(\chi_2)R_y(-\theta_2)R_z(-\phi_2)$. Subsequently, one then boosts to the B_2 rest frame. The $\mathbb{R}_W$ frame is defined using the same procedure with the rotation matrix $\mathcal{R}(\chi_W, -\theta_W, -\phi_W)$ and the subsequent boost to the

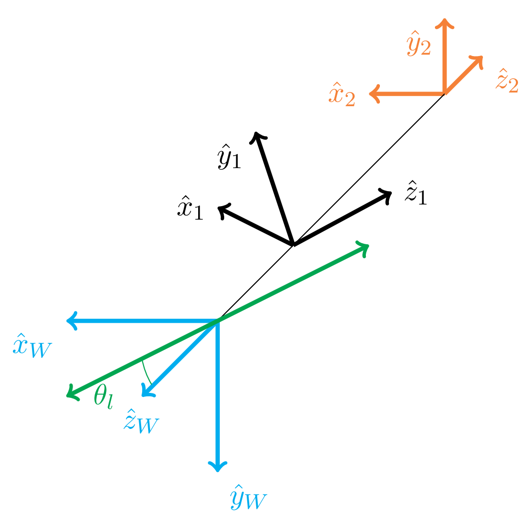

FIG. 2. Definition of the three coordinate systems used to describe the semileptonic decay $B_1 \to B_2 + W^-_{\text{off-shell}}$. The axes in the B_1, B_2 and $W^-_{\text{off-shell}}$ rest frames (helicity frames: $\mathbb{R}_1$, $\mathbb{R}_2$ and $\mathbb{R}_W$) are denoted $(\hat{\mathbf{x}}_1, \hat{\mathbf{y}}_1, \hat{\mathbf{z}}_1)$, $(\hat{\mathbf{x}}_2, \hat{\mathbf{y}}_2, \hat{\mathbf{z}}_2)$ and $(\hat{\mathbf{x}}_W, \hat{\mathbf{y}}_W, \hat{\mathbf{z}}_W)$, respectively.

$W^-_{\text{off-shell}}$ rest frame. Since the $W^-_{\text{off-shell}}$ direction is opposite to B_2 in $\mathbb{R}_1$, one has $\phi_W = \pi + \phi_2$ and $\theta_W = \pi - \theta_2$. In order to assure that the coordinate systems in $\mathbb{R}_2$ and $\mathbb{R}_W$ are related as $(\hat{\mathbf{x}}_2, \hat{\mathbf{y}}_2, \hat{\mathbf{z}}_2) = (\hat{\mathbf{x}}_W, -\hat{\mathbf{y}}_W, -\hat{\mathbf{z}}_W)$ we set $\chi_W = -\chi_2$.

The matching transition amplitude between B_1 and the two daughter particles expressed using the defined above helicity frames is [8,18]

$$\begin{aligned}
&\langle \Omega_2, \lambda_2, \lambda_W | S | J = 1/2, \kappa \rangle \\
&\quad = \sqrt{\frac{2J+1}{8\pi^2}} \langle \lambda_2, \lambda_W | S | J = 1/2, \kappa \rangle \mathcal{D}^{1/2*}_{\kappa, \lambda_2 - \lambda_W}(\Omega_2) \\
&\quad = \frac{1}{2\pi} H_{\lambda_2, \lambda_W}(q^2) \mathcal{D}^{1/2*}_{\kappa, \lambda_2 - \lambda_W}(\Omega_2),
\end{aligned} \tag{10}$$

where $\mathcal{D}^J_{m_1,m_2}(\Omega_2) \coloneqq \mathcal{D}^J_{m_1,m_2}(\phi_2, \theta_2, -\chi_2)$ is the Wigner rotation matrix, where the convention $\mathcal{D}^J_{m_1,m_2}(\phi,\theta,\chi) = e^{-im_1\phi - im_2\chi}\mathcal{D}^J_{m_1,m_2}(0,\theta,0) = e^{-im_1\phi - im_2\chi} d^J_{m_1,m_2}(\theta)$ is used (see Appendix A). The order and the signs of the angles $\Omega_2 = \{\phi_2, \theta_2, -\chi_2\}$ in the Wigner functions are opposite to the used in the rotations to define the helicity reference frames. In addition, the normalization factor is different since we allow for three independent rotation angles. The helicity amplitudes $H_{\lambda_2, \lambda_W}(q^2)$ are functions of q^2 and depend on the helicities of the daughter particles. The vector and axial-vector helicity amplitudes $H_{\lambda_2, \lambda_W} = H^V_{\lambda_2, \lambda_W} + H^A_{\lambda_2, \lambda_W}$ are related to the invariant form factors in the following way:

$$\begin{aligned}
H^V_{\frac{1}{2}t} &= \frac{\sqrt{Q_+}}{\sqrt{q^2}}\left[(M_1 - M_2)F_1^V + \frac{q^2}{M_1}F_3^V\right], \\
H^V_{\frac{1}{2}1} &= \sqrt{2Q_-}\left[-F_1^V - \frac{M_1 + M_2}{M_1}F_2^V\right], \\
H^V_{\frac{1}{2}0} &= \frac{\sqrt{Q_-}}{\sqrt{q^2}}\left[(M_1 + M_2)F_1^V + \frac{q^2}{M_1}F_2^V\right], \\
H^A_{\frac{1}{2}t} &= \frac{\sqrt{Q_-}}{\sqrt{q^2}}\left[-(M_1 + M_2)F_1^A + \frac{q^2}{M_1}F_3^A\right], \\
H^A_{\frac{1}{2}1} &= \sqrt{2Q_+}\left[F_1^A - \frac{M_1 - M_2}{M_1}F_2^A\right], \\
H^A_{\frac{1}{2}0} &= \frac{\sqrt{Q_+}}{\sqrt{q^2}}\left[-(M_1 - M_2)F_1^A + \frac{q^2}{M_1}F_2^A\right],
\end{aligned} \tag{11}$$

where the remaining helicity amplitudes are obtained by applying the parity operator,

$$H^V_{-\lambda_2, -\lambda_W} = H^V_{\lambda_2, \lambda_W}, \qquad H^A_{-\lambda_2, -\lambda_W} = -H^A_{\lambda_2, \lambda_W}. \tag{12}$$

The decay $W^- \to l^- \bar{\nu}_l$ is described in $\mathbb{R}_W$ where the emission angles of the l^- lepton are θ_l and ϕ_l. The value of the lepton momentum in this frame is

$$|\mathbf{p}_l| = \frac{q^2 - m_l^2}{2q}. \tag{13}$$

The decay amplitude reads

$$\langle \Omega_l, \lambda_l, \lambda_\nu | S_l | J_W, q^2, \lambda_W \rangle = \sqrt{\frac{2J_W + 1}{4\pi}} (-1)^{J_W} h^l_{\lambda_l \lambda_\nu}(q^2) \mathcal{D}^{J_W *}_{\lambda_W, \lambda_l - \lambda_\nu}(\Omega_l), \tag{14}$$

where $\Omega_l = \{\phi_l, \theta_l, 0\}$. The helicity amplitudes $h^l_{\lambda_l \lambda_\nu}$ for the elementary transition to the final lepton pair can be calculated directly by evaluating the Feynman diagrams. The neutrino helicities are $\lambda_\nu = 1/2$ and $\lambda_\nu = -1/2$ for $(l^-, \bar{\nu}_l)$ and (l^+, ν_l), respectively. The moduli squared of $h^l_{\lambda_l \lambda_\nu}$ are [6]:

$$\text{nonflip}(\underline{\lambda}_W = \mp 1) \colon |h^l_{\lambda_l = \mp\frac{1}{2}, \lambda_\nu = \pm\frac{1}{2}}|^2 = 8\delta(\lambda_l + \lambda_\nu)(q^2 - m_l^2), \tag{15}$$

$$\text{flip}(\underline{\lambda}_W = 0, t) \colon |h^l_{\lambda_l = \pm\frac{1}{2}, \lambda_\nu = \pm\frac{1}{2}}|^2 = 8\delta(\lambda_l - \lambda_\nu) \frac{m_l^2}{2q^2}(q^2 - m_l^2), \tag{16}$$

where here and in the following the upper and lower signs refer to the configurations $(l^-, \bar{\nu}_l)$ and (l^+, ν_l), respectively.

The representations in Eqs. (10) and (14) imply that the complete amplitude for the $B_1(\kappa) \to B_2(\lambda_2)$ transition reads,

$$\sum_{\underline{\lambda}_W} \langle \Omega_l, \lambda_l, \lambda_\nu | S_l | q^2, \underline{\lambda}_W \rangle \langle \Omega_2, \lambda_2, \underline{\lambda}_W | S | 1/2, \kappa \rangle, \tag{17}$$

where the $\underline{\lambda}_W$ sum runs over the four W-boson helicity components $\{t, -1, 0, +1\}$. An explicit representation of the amplitude with the angular part separated is

$$\begin{aligned} &\sum_{\underline{\lambda}_W} (-1)^{J_W} h^l_{\lambda_l \lambda_\nu} \mathcal{D}^{J_W *}_{\lambda_W, \lambda_l - \lambda_\nu}(\Omega_l) H_{\lambda_2, \underline{\lambda}_W} \mathcal{D}^{1/2 *}_{\kappa, \lambda_2 - \lambda_W}(\Omega_2) \\ &= \sum_{\underline{\lambda}_W} (-1)^{J_W} h^l_{\lambda_l \lambda_\nu} d^{J_W}_{\lambda_W, \lambda_l - \lambda_\nu}(\theta_l) H_{\lambda_2, \underline{\lambda}_W} d^{1/2}_{\kappa, \lambda_2 - \lambda_W}(\theta_2) \\ &\quad \times \exp[i\kappa\phi_2 + i\lambda_2\chi_2 - i\lambda_W(\chi_2 - \phi_l)], \end{aligned} \tag{18}$$

where the final expression combines all azimuthal-angle rotations in one term. One can consider two options for selecting χ_2 to define the transversal orientation of the $\mathbb{R}_2$ and $\mathbb{R}_W$ helicity frames. The first option is to set $\chi_2 = 0$ as in Ref. [8] where the corresponding azimuthal angle of the charged lepton in the $\mathbb{R}_W$ system is ϕ_l^0. An alternative is to select $\chi_2^{\text{3-b}}$ so that $\hat{x}_2$ is in the decay plane of the semileptonic decay. In this case the momenta of the leptons are in this plane which corresponds to $\phi_l^{\text{3-b}} = 0$ and the $\chi_2^{\text{3-b}} = \phi_l^0$ relation holds.

The amplitude can be rearranged by inserting a complete spin basis for the baryon B_2 to represent transition between $B_1(\kappa)$ and $B_2(\lambda_2)$,

$$\sum_{\lambda' = -1/2}^{1/2} \sum_{\underline{\lambda}_W} \langle \Omega_l, \lambda_l, \lambda_\nu | S_l | q^2, \underline{\lambda}_W \rangle \langle \lambda_2, \underline{\lambda}_W | \lambda' \rangle \langle \Omega_2, \lambda' | S | 1/2, \kappa \rangle \tag{19}$$

$$= \frac{1}{2\pi} \sum_{\lambda' = -1/2}^{1/2} \mathcal{D}^{1/2 *}_{\kappa, \lambda'}(\Omega_2) \left\{ \sum_{\underline{\lambda}_W} \langle \Omega_l, \lambda_l, \lambda_\nu | S_l | q^2, \underline{\lambda}_W \rangle \times \langle \lambda_2, \underline{\lambda}_W | \lambda' \rangle H_{\lambda_2, \underline{\lambda}_W}(q^2) \right\} \tag{20}$$

$$= \frac{1}{2\pi} \sum_{\lambda' = -1/2}^{1/2} \mathcal{D}^{1/2 *}_{\kappa, \lambda'}(\Omega_2) \mathcal{H}_{\lambda', \lambda_2}(\Omega_l, q^2, \lambda_l, \lambda_\nu). \tag{21}$$

Therefore the angular dependence on Ω_2 can be separated in the amplitude of the complete process. Since usually experiments do not measure polarization of the leptons, it is useful to consider a tensor that describes the $W^\pm$-boson decay with the lepton helicities summed over,

$$L_{\underline{\lambda}_W, \underline{\lambda}'_W}(q^2, \Omega_l) \coloneqq \sum_{\lambda_l = -1/2}^{1/2} \langle \Omega_l, \lambda_l, \lambda_\nu | S_l |, q^2, \underline{\lambda}'_W \rangle^* \langle \Omega_l, \lambda_l, \lambda_\nu | S_l | q^2, \underline{\lambda}_W \rangle \tag{22}$$

$$= \frac{3}{4\pi} \sum_{\lambda_l = -1/2}^{1/2} |h^l_{\lambda_l \lambda_\nu}(q^2)|^2 (-1)^{J_W + J'_W} \mathcal{D}^{J_W *}_{\lambda_W, \lambda_l - \lambda_\nu}(\Omega_l) \mathcal{D}^{J'_W}_{\lambda'_W, \lambda_l - \lambda_\nu}(\Omega_l) \tag{23}$$

$$= \frac{3}{4\pi} e^{i(\lambda_W - \lambda'_W)\phi_l} \sum_{\lambda_l = -1/2}^{1/2} |h^l_{\lambda_l \lambda_\nu}(q^2)|^2 (-1)^{J_W + J'_W} d^{J_W}_{\lambda_W, \lambda_l - \lambda_\nu}(\theta_l) d^{J'_W}_{\lambda'_W, \lambda_l - \lambda_\nu}(\theta_l). \tag{24}$$

The interference contribution from $\underline{\lambda}_W = t$ and $\underline{\lambda}_W = 0$ gives an extra minus sign. We write the tensor as

$$L_{\underline{\lambda}_W,\underline{\lambda}'_W}(q^2,\Omega_l) = \frac{6}{\pi}(q^2 - m_l^2)\left[\ell^{\rm nf}_{\underline{\lambda}_W,\underline{\lambda}'_W}(\Omega_l) + \varepsilon\ell^{\rm f}_{\underline{\lambda}_W,\underline{\lambda}'_W}(\Omega_l)\right], \tag{25}$$

where $\varepsilon = m_l^2/(2q^2)$. The Hermitian matrix for the nonflip transition reads

$$\ell^{\rm nf}_{\underline{\lambda}_W,\underline{\lambda}'_W}(\Omega_l) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{(1\pm\cos\theta_l)^2}{4} & \mp\frac{e^{-i\phi_l}\sin\theta_l(1\pm\cos\theta_l)}{2\sqrt{2}} & \frac{1}{4}e^{-2i\phi_l}\sin^2\theta_l \\ 0 & \mp\frac{e^{i\phi_l}\sin\theta_l(1\pm\cos\theta_l)}{2\sqrt{2}} & \frac{1}{2}\sin^2\theta_l & \mp\frac{e^{-i\phi_l}\sin\theta_l(1\mp\cos\theta_l)}{2\sqrt{2}} \\ 0 & \frac{1}{4}e^{2i\phi_l}\sin^2\theta_l & \mp\frac{e^{i\phi_l}\sin\theta_l(1\mp\cos\theta_l)}{2\sqrt{2}} & \frac{(1\mp\cos\theta_l)^2}{4} \end{pmatrix}, \tag{26}$$

while for the flip transition

$$\ell^{\rm f}_{\underline{\lambda}_W,\underline{\lambda}'_W}(\Omega_l) = \begin{pmatrix} 1 & -\frac{e^{i\phi_l}\sin\theta_l}{\sqrt{2}} & -\cos\theta_l & \frac{e^{-i\phi_l}\sin\theta_l}{\sqrt{2}} \\ -\frac{e^{-i\phi_l}\sin\theta_l}{\sqrt{2}} & \frac{\sin^2\theta_l}{2} & \frac{e^{-i\phi_l}\sin\theta_l\cos\theta_l}{\sqrt{2}} & -\frac{1}{2}e^{-2i\phi_l}\sin^2\theta_l \\ -\cos\theta_l & \frac{e^{i\phi_l}\sin\theta_l\cos\theta_l}{\sqrt{2}} & \cos^2\theta_l & -\frac{e^{-i\phi_l}\sin\theta_l\cos\theta_l}{\sqrt{2}} \\ \frac{e^{i\phi_l}\sin\theta_l}{\sqrt{2}} & -\frac{1}{2}e^{2i\phi_l}\sin^2\theta_l & -\frac{e^{i\phi_l}\sin\theta_l\cos\theta_l}{\sqrt{2}} & \frac{1}{2}\sin^2\theta_l \end{pmatrix}. \tag{27}$$

V. DECAY MATRIX

Here, we derive a matrix that relates the spin of the baryon B_2 to the spin of the baryon B_1 in $B_1 \to B_2\ell\nu_\ell$ where the state of the lepton pair with the summed spin projections is given by the $L_{\underline{\lambda}_W,\underline{\lambda}'_W}(q^2,\Omega_l)$ tensor in Eq. (23). The transition can be represented by a tensor $T^{\kappa\kappa',\lambda_2\lambda'_2}$ that describes how the initial spin-density matrix $\rho_1^{\kappa\kappa'}$ of the baryon B_1 transforms to the density matrix $\rho_2^{\lambda_2\lambda'_2}$ of the baryon B_2,

$$\rho_2^{\lambda_2\lambda'_2} = T^{\kappa\kappa',\lambda_2\lambda'_2}\rho_1^{\kappa\kappa'}. \tag{28}$$

Using Eq. (10) the transition tensor is given as

$$T^{\kappa\kappa',\lambda_2\lambda'_2} = \frac{1}{4\pi^2}\sum_{\underline{\lambda}_W,\underline{\lambda}'_W} H_{\lambda_2\underline{\lambda}_W}H^*_{\lambda'_2\underline{\lambda}'_W}\mathcal{D}^{1/2*}_{\kappa,\lambda_2-\lambda_W}(\Omega_2) \times \mathcal{D}^{1/2}_{\kappa',\lambda'_2-\lambda'_W}(\Omega_2)L_{\lambda_W,\lambda'_W}(q^2,\Omega_l) \tag{29}$$

$$\equiv \frac{1}{4\pi^2}\sum_{\underline{\lambda}_W,\underline{\lambda}'_W} T^{\kappa\kappa',\lambda_2\lambda'_2}_{\underline{\lambda}_W,\underline{\lambda}'_W}(q^2,\Omega_2)L_{\underline{\lambda}_W,\underline{\lambda}'_W}(q^2,\Omega_l). \tag{30}$$

The explicit expression for the phases of the hadronic tensor due to the azimuthal rotations is

$$T^{\kappa\kappa',\lambda_2\lambda_2'}_{\underline{\lambda}_W,\underline{\lambda}_W'}(q^2,\Omega_2) = H_{\lambda_2\underline{\lambda}_W}H^*_{\lambda_2'\underline{\lambda}_W'}d^{1/2}_{\kappa,\lambda_2-\lambda_W}(\theta_2)d^{1/2}_{\kappa',\lambda_2'-\lambda'_W}(\theta_2) \times \exp[i\kappa\phi_2 + i\lambda_2\chi_2 - i\lambda_W\chi_2] \times \exp[-i\kappa'\phi_2 - i\lambda_2'\chi_2 + i\lambda'_W\chi_2], \tag{31}$$

where we use the generic case with $\Omega_2 = \{\phi_2,\theta_2,\chi_2\}$ and $\Omega_l = \{\phi_l,\theta_l,0\}$. The overall phases of the contraction of the above hadronic tensor and the leptonic tensor in Eq. (24) for the two choices of the orientations of the coordinate systems $\mathbb{R}_2$ and $\mathbb{R}_W$ are

$$(\chi_2 = 0) \to \exp[i(\kappa-\kappa')\phi_2 + i(\lambda_W - \lambda'_W)\phi_l^0], \tag{32}$$

$$(\phi_l^{\text{3-b}} = 0) \to \exp\left[i(\kappa-\kappa')\phi_2 + i(\lambda_2 - \lambda'_2)\chi_2^{\text{3-b}}\right] \tag{33}$$

$$= \exp[i(\kappa-\kappa')\phi_2 + i(\lambda_2 - \lambda'_2)\phi_l^0]. \tag{34}$$

The two representations are not equivalent but can be written in terms of the tensors evaluated for $\Omega_2^0 \coloneqq \{\phi_2,\theta_2,0\}$ and $\Omega_l^0 = \{0,\theta_l,0\}$ as

$$T^{\kappa\kappa',\lambda_2\lambda'_2}(\chi_2 = 0) = \frac{1}{4\pi^2}\sum_{\underline{\lambda}_W,\underline{\lambda}'_W}\exp[i(\lambda_W - \lambda'_W)\phi_l^0] \times T^{\kappa\kappa',\lambda_2\lambda'_2}_{\underline{\lambda}_W,\underline{\lambda}'_W}(\Omega_2^0)L_{\underline{\lambda}_W,\underline{\lambda}'_W}(\Omega_l^0), \tag{35}$$

$$T^{\kappa\kappa',\lambda_2\lambda_2'}(\phi_l^{3\text{-b}}=0)=\frac{1}{4\pi^2}\exp[i(\lambda_2-\lambda_2')\phi_l^0]\sum_{\underline{\lambda}_W,\underline{\lambda}_W'}T^{\kappa\kappa',\lambda_2\lambda_2'}_{\underline{\lambda}_W,\underline{\lambda}_W'}\times(\Omega_2^0)L_{\underline{\lambda}_W,\underline{\lambda}_W'}(\Omega_l^0). \tag{36}$$

Instead of the helicities, the transition can be written as in Ref. [8] using spin base vectors $\sigma_\mu^{B_1}$ and $\sigma_\nu^{B_2}$ in the mother and daughter reference systems $\mathbb{R}_1$ and $\mathbb{R}_2$, respectively. The 4×4 matrix $\mathcal{B}_{\mu\nu}$ describes how the decay process transforms the base Pauli matrices,

$$\sigma_\mu^{B_1}\to\frac{3(q^2-m_l^2)}{4\pi^3}\sum_{\nu=0}^{3}\mathcal{B}_{\mu\nu}\sigma_\nu^{B_2}. \tag{37}$$

The real coefficients $\mathcal{B}_{\mu\nu}$ can be obtained by inserting Pauli σ_μ matrices for the mother and the daughter baryons in the expression for the tensor $T^{\kappa\kappa',\lambda_2\lambda_2'}$,

$$\mathcal{B}_{\mu\nu}\coloneqq\frac{2\pi^3}{3(q^2-m_l^2)}\sum_{\lambda_2,\lambda_2'=-1/2}^{1/2}\sum_{\kappa,\kappa'=-1/2}^{1/2}T^{\kappa\kappa',\lambda_2\lambda_2'}\sigma_\mu^{\kappa,\kappa'}\sigma_\nu^{\lambda_2',\lambda_2}. \tag{38}$$

However, as we show in Appendix B the coefficients can be represented as

$$\mathcal{B}_{\mu\nu}=\sum_{\kappa=0}^{3}\mathcal{R}^{(4)}_{\mu\kappa}(\Omega_2)b_{\kappa\nu}(q^2,\Omega_l), \tag{39}$$

where $\mathcal{R}^{(4)}_{\mu\kappa}(\Omega_2)$ is the 4×4 spacelike rotation matrix obtained as the direct sum of identity and 3D rotation $\mathcal{R}(\Omega_2)$; $\mathcal{R}^{(4)}(\Omega_2)=\mathrm{diag}(1,\mathcal{R}(\Omega_2))$. The argument $\Omega_2=\{\phi_2,\theta_2,-\chi_2\}$ assures that the rotation is the inverse of the rotation $\mathcal{R}(\{\chi_2,-\theta_2,-\phi_2\})$ that was used to define the helicity frame $\mathbb{R}_2$. The coefficients $b_{\mu\nu}$ correspond to the $B_1\to B_2$ transition where the orientations of the axes of the reference systems are aligned $\Omega_2=\{0,0,0\}$. They can be obtained by inserting Pauli σ_μ matrices for the mother and the daughter baryons in the expression for the tensor $T^{\kappa\kappa',\lambda_2\lambda_2'}$ with Ω_2 set to $\{0,0,0\}$ which implies replacement $\mathcal{D}^{1/2}_{m_1,m_2}(\{0,0,0\})=\delta(m_1-m_2)$,

$$b_{\mu\nu}\coloneqq\frac{\pi}{6(q^2-m_l^2)}\sum_{\underline{\lambda}_W,\underline{\lambda}_W'}\sum_{\lambda_2,\lambda_2'=-1/2}^{1/2}H_{\lambda_2\underline{\lambda}_W}H^*_{\lambda_2'\underline{\lambda}_W'}\underbrace{\sigma_\mu^{\lambda_2-\lambda_W,\lambda_2'-\lambda_W'}\sigma_\nu^{\lambda_2',\lambda_2}L_{\underline{\lambda}_W,\underline{\lambda}_W'}(q^2,\Omega_l)}_{\mathcal{T}_{\mu\nu}^{\underline{\lambda}_W,\underline{\lambda}_W',\lambda_2,\lambda_2'}}, \tag{40}$$

$$\begin{aligned}b_{\mu\nu}&=\sum_{\underline{\lambda}_W,\underline{\lambda}_W'}\sum_{\lambda_2,\lambda_2'=-1/2}^{1/2}H_{\lambda_2\underline{\lambda}_W}H^*_{\lambda_2'\underline{\lambda}_W'}\mathcal{T}_{\mu\nu}^{\underline{\lambda}_W,\underline{\lambda}_W',\lambda_2,\lambda_2'}\\&=\sum_{\underline{\lambda}_W}\sum_{\lambda_2=-1/2}^{1/2}\left\{|H_{\lambda_2\underline{\lambda}_W}|^2\mathcal{T}_{\mu\nu}^{\underline{\lambda}_W,\underline{\lambda}_W,\lambda_2,\lambda_2}+2\sum_{\underline{\lambda}_W'<\underline{\lambda}_W}\sum_{\lambda_2'<\lambda_2}\left[\Re(H_{\lambda_2\underline{\lambda}_W}H^*_{\lambda_2'\underline{\lambda}_W'})\Re\mathcal{T}_{\mu\nu}^{\underline{\lambda}_W,\underline{\lambda}_W',\lambda_2,\lambda_2'}+\Im(H_{\lambda_2\underline{\lambda}_W}H^*_{\lambda_2'\underline{\lambda}_W'})\Im\mathcal{T}_{\mu\nu}^{\underline{\lambda}_W,\underline{\lambda}_W',\lambda_2,\lambda_2'}\right]\right\}.\end{aligned} \tag{41}$$

The last form involves only real-valued tensors $\mathcal{T}_{\mu\nu}^{\underline{\lambda}_W,\underline{\lambda}_W,\lambda_2,\lambda_2}$, $\Re\mathcal{T}_{\mu\nu}^{\underline{\lambda}_W,\underline{\lambda}_W',\lambda_2,\lambda_2'}$ and $\Im\mathcal{T}_{\mu\nu}^{\underline{\lambda}_W,\underline{\lambda}_W',\lambda_2,\lambda_2'}$. The hadronic part is encoded in the real-valued functions of q^2: $|H_{\lambda_2\underline{\lambda}_W}|^2$, $\Re(H_{\lambda_2\underline{\lambda}_W}H^*_{\lambda_2'\underline{\lambda}_W'})$ and $\Im(H_{\lambda_2\underline{\lambda}_W}H^*_{\lambda_2'\underline{\lambda}_W'})$, where $\underline{\lambda}_W'<\underline{\lambda}_W$ and $\lambda_2'<\lambda_2$. Moreover, the form factors $H_{-\frac{1}{2}1}=H_{\frac{1}{2}-1}=0$ reducing number of the functions.

We will represent the $b_{\mu\nu}$ matrix as the sum of the nonflip and flip contributions $b_{\mu\nu}=b_{\mu\nu}^{\mathrm{nf}}+\varepsilon b_{\mu\nu}^{\mathrm{f}}$. The cross section term is written as $b_{00}=b_{00}^{\mathrm{nf}}+\varepsilon b_{00}^{\mathrm{f}}$, where

$$b_{00}^{\mathrm{nf}}=\frac{1}{4}(1\mp\cos\theta_l)^2|H_{\frac{1}{2}1}|^2+\frac{1}{4}(1\pm\cos\theta_l)^2|H_{-\frac{1}{2}-1}|^2+\frac{1}{2}\sin^2\theta_l(|H_{-\frac{1}{2}0}|^2+|H_{\frac{1}{2}0}|^2), \tag{42}$$

$$b_{00}^{\mathrm{f}}=|H_{\frac{1}{2}t}|^2+|H_{-\frac{1}{2}t}|^2+\frac{1}{2}\sin^2\theta_l(|H_{\frac{1}{2}1}|^2+|H_{-\frac{1}{2}-1}|^2)+\cos^2\theta_l(|H_{\frac{1}{2}0}|^2+|H_{-\frac{1}{2}0}|^2)-2\cos\theta_l\Re(H^*_{\frac{1}{2}0}H_{\frac{1}{2}t}+H^*_{-\frac{1}{2}0}H_{-\frac{1}{2}t}), \tag{43}$$

define the angular distributions for the decay of unpolarized baryon B_1 when the spins of all final particles are summed over. The differential decay rate is obtained by multiplying by the kinematic and spinor normalization factors that depend on q^2,

$$\mathrm{d}\Gamma=\frac{G_F^2}{(2\pi)^5}|V_{us}|^2\frac{|\mathbf{p}_l||\mathbf{p}_2|}{16M_1^2}(q^2-m_l^2)b_{00}\mathrm{d}q\mathrm{d}\Omega_2\mathrm{d}\Omega_l \tag{44}$$

$$=G_F^2|V_{us}|^2V_{Ph}(q^2)(q^2-m_l^2)b_{00}\mathrm{d}q\mathrm{d}\Omega_2\mathrm{d}\Omega_l, \tag{45}$$

where $V_{Ph}(q^2) = (2\pi)^{-5}(4M_1)^{-2}|\mathbf{p}_l||\mathbf{p}_2|$ is the three-body phase-space density factor [13]. The momenta $|\mathbf{p}_2|$ and $|\mathbf{p}_l|$ of the baryon B_2 and the lepton are given in Eqs. (8) and (13), respectively.

The first row of the b_{0i} matrix, where $i = 1, 2, 3(x, y, z)$, gives the polarization vector $\mathbf{P} = (P_x, P_y, P_z)$ of the baryon B_2 in the reference frame $\mathbb{R}_2$ corresponding to the decay of unpolarized baryon B_1. These elements are

$$\begin{aligned} b_{01} &= -\Re(\mathcal{I}_{01})\cos\phi_l + \Im(\mathcal{I}_{01})\sin\phi_l = P_x b_{00}, \\ b_{02} &= \Re(\mathcal{I}_{01})\sin\phi_l + \Im(\mathcal{I}_{01})\cos\phi_l = P_y b_{00}, \\ b_{03} &= b_{03}^{\mathrm{nf}} + \varepsilon b_{03}^{\mathrm{f}} = P_z b_{00}, \end{aligned} \tag{46}$$

where $\mathcal{I}_{\mu\nu}$ are complex. We use notation $\mathcal{I}_{\mu\nu} = \mathcal{I}_{\mu\nu}^{\mathrm{nf}} + \varepsilon\mathcal{I}_{\mu\nu}^{\mathrm{f}}$ and

$$\begin{aligned} \mathcal{I}_{01}^{\mathrm{nf}} &= \pm\frac{1}{\sqrt{2}}\sin\theta_l\Big[(1\pm\cos\theta_l)H^*_{-\frac{1}{2}-1}H_{\frac{1}{2}0} + (1\mp\cos\theta_l)H^*_{-\frac{1}{2}0}H_{\frac{1}{2}1}\Big], \\ \mathcal{I}_{01}^{\mathrm{f}} &= \sqrt{2}\sin\theta_l\Big[(H^*_{-\frac{1}{2}-1}H_{\frac{1}{2}t} - H^*_{-\frac{1}{2}t}H_{\frac{1}{2}1}) + \cos\theta_l(H^*_{-\frac{1}{2}0}H_{\frac{1}{2}1} - H^*_{-\frac{1}{2}-1}H_{\frac{1}{2}0})\Big], \\ b_{03}^{\mathrm{nf}} &= \frac{1}{4}(1\mp\cos\theta_l)^2|H_{\frac{1}{2}1}|^2 - \frac{1}{4}(1\pm\cos\theta_l)^2|H_{-\frac{1}{2}-1}|^2 - \frac{1}{2}\sin^2\theta_l(|H_{-\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2), \\ b_{03}^{\mathrm{f}} &= |H_{\frac{1}{2}t}|^2 - |H_{-\frac{1}{2}t}|^2 + \frac{1}{2}\sin^2\theta_l(|H_{\frac{1}{2}1}|^2 - |H_{-\frac{1}{2}-1}|^2) - \cos^2\theta_l(|H_{-\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2) - 2\cos\theta_l\Re(H^*_{\frac{1}{2}0}H_{\frac{1}{2}t} - H^*_{-\frac{1}{2}0}H_{-\frac{1}{2}t}). \end{aligned} \tag{47}$$

The first column b_{i0} of the matrix corresponds to the decay of the spin-polarized baryon B_1. The element $b_{30} = b_{30}^{\mathrm{nf}} + \varepsilon b_{30}^{\mathrm{f}}$ is

$$\begin{aligned} b_{30}^{\mathrm{nf}} &= \frac{1}{4}(1\pm\cos\theta_l)^2|H_{-\frac{1}{2}-1}|^2 - \frac{1}{4}(1\mp\cos\theta_l)^2|H_{\frac{1}{2}1}|^2 - \frac{1}{2}\sin^2\theta_l(|H_{-\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2), \\ b_{30}^{\mathrm{f}} &= |H_{\frac{1}{2}t}|^2 - |H_{-\frac{1}{2}t}|^2 - \frac{1}{2}\sin^2\theta_l(|H_{\frac{1}{2}1}|^2 - |H_{-\frac{1}{2}-1}|^2) - \cos^2\theta_l(|H_{-\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2) - 2\cos\theta_l\Re(H^*_{\frac{1}{2}0}H_{\frac{1}{2}t} - H^*_{-\frac{1}{2}0}H_{-\frac{1}{2}t}). \end{aligned} \tag{48}$$

The elements b_{10} and b_{20} are

$$\begin{aligned} b_{10} &= -\cos\phi_l\Re(\mathcal{I}_{10}) + \sin\phi_l\Im(\mathcal{I}_{10}), \\ b_{20} &= \sin\phi_l\Re(\mathcal{I}_{10}) + \cos\phi_l\Im(\mathcal{I}_{10}), \end{aligned} \tag{49}$$

where

$$\begin{aligned} \mathcal{I}_{10}^{\mathrm{nf}} &= \pm\frac{1}{\sqrt{2}}\sin\theta_l\Big[(1\pm\cos\theta_l)H^*_{-\frac{1}{2}-1}H_{-\frac{1}{2}0} + (1\mp\cos\theta_l)H^*_{\frac{1}{2}0}H_{\frac{1}{2}1}\Big], \\ \mathcal{I}_{10}^{\mathrm{f}} &= \sqrt{2}\sin\theta_l\Big[(H^*_{-\frac{1}{2}-1}H_{-\frac{1}{2}t} - H^*_{\frac{1}{2}t}H_{\frac{1}{2}1}) + \cos\theta_l(H^*_{\frac{1}{2}0}H_{\frac{1}{2}1} - H^*_{-\frac{1}{2}-1}H_{-\frac{1}{2}0})\Big]. \end{aligned} \tag{50}$$

The decay-plane representation which requires three rotation angles for baryon B_2 gives simple formulas for the remaining terms of the decay matrix. The terms of the nonflip contributions for the aligned (with $\phi_l = 0$) decay matrix $b_{\mu\nu}^{\mathrm{nf}}$ are

$$b_{\mu\nu}^{\mathrm{nf}} = \begin{pmatrix} b_{00}^{\mathrm{nf}} & -\Re(\mathcal{I}_{01}^{\mathrm{nf}}) & \Im(\mathcal{I}_{10}^{\mathrm{nf}}) & b_{03}^{\mathrm{nf}} \\ -\Re(\mathcal{I}_{10}^{\mathrm{nf}}) & \Re(\mathcal{E}_{00}^{\mathrm{nf}} + \mathcal{E}_{11}^{\mathrm{nf}}) & -\Im(\mathcal{E}_{00}^{\mathrm{nf}} + \mathcal{E}_{11}^{\mathrm{nf}}) & \Re(\mathcal{I}_{13}^{\mathrm{nf}}) \\ \Im(\mathcal{I}_{10}^{\mathrm{nf}}) & \Im(\mathcal{E}_{00}^{\mathrm{nf}} - \mathcal{E}_{11}^{\mathrm{nf}}) & \Re(\mathcal{E}_{00}^{\mathrm{nf}} - \mathcal{E}_{11}^{\mathrm{nf}}) & -\Im(\mathcal{I}_{13}^{\mathrm{nf}}) \\ b_{30}^{\mathrm{nf}} & -\Re(\mathcal{I}_{31}^{\mathrm{nf}}) & \Im(\mathcal{I}_{31}^{\mathrm{nf}}) & b_{33}^{\mathrm{nf}} \end{pmatrix}, \tag{51}$$

where

$$b_{33}^{\mathrm{nf}} = +\frac{1}{2}\sin^2\theta_l(|H_{-\frac{1}{2}0}|^2 + |H_{\frac{1}{2}0}|^2) - \frac{1}{4}(1\mp\cos\theta_l)^2|H_{\frac{1}{2}1}|^2 - \frac{1}{4}(1\pm\cos\theta_l)^2|H_{-\frac{1}{2}-1}|^2 \tag{52}$$

and

$$\begin{aligned}
\mathcal{I}^{\rm nf}_{13} &= \pm\frac{1}{\sqrt{2}}\sin\theta_l\Big\{(1\pm\cos\theta_l)H^*_{-\frac{1}{2}-1}H_{-\frac{1}{2}0}-(1\mp\cos\theta_l)H^*_{\frac{1}{2}0}H_{\frac{1}{2}1}\Big\},\\
\mathcal{I}^{\rm nf}_{31} &= \pm\frac{1}{\sqrt{2}}\sin\theta_l\Big\{(1\pm\cos\theta_l)H^*_{-\frac{1}{2}-1}H_{\frac{1}{2}0}-(1\mp\cos\theta_l)H^*_{-\frac{1}{2}0}H_{\frac{1}{2}1}\Big\},\\
\mathcal{E}^{\rm nf}_{00} &= \sin^2\theta_l H^*_{-\frac{1}{2}0}H_{\frac{1}{2}0},\\
\mathcal{E}^{\rm nf}_{11} &= \frac{1}{2}\sin^2\theta_l H^*_{-\frac{1}{2}-1}H_{\frac{1}{2}1}.
\end{aligned} \tag{53}$$

The terms of the flip contributions for the aligned decay matrix $b^{\rm f}_{\mu\nu}$ are

$$b^{\rm f}_{\mu\nu} = \begin{pmatrix} b^{\rm f}_{00} & -\Re(\mathcal{I}^{\rm f}_{01}) & \Im(\mathcal{I}^{\rm f}_{10}) & b^{\rm f}_{03}\\ -\Re(\mathcal{I}^{\rm f}_{10}) & \Re(\mathcal{E}^{\rm f}_{00}-\mathcal{E}^{\rm f}_{11}) & -\Im(\mathcal{E}^{\rm f}_{00}-\mathcal{E}^{\rm f}_{11}) & \Re(\mathcal{I}^{\rm f}_{13})\\ \Im(\mathcal{I}^{\rm f}_{10}) & \Im(\mathcal{E}^{\rm f}_{00}+\mathcal{E}^{\rm f}_{11}) & \Re(\mathcal{E}^{\rm f}_{00}+\mathcal{E}^{\rm f}_{11}) & -\Im(\mathcal{I}^{\rm f}_{13})\\ b^{\rm f}_{30} & -\Re(\mathcal{I}^{\rm f}_{31}) & \Im(\mathcal{I}^{\rm f}_{31}) & b^{\rm f}_{33}\end{pmatrix}, \tag{54}$$

where

$$b^{\rm f}_{33} = |H_{-\frac{1}{2}t}|^2+|H_{\frac{1}{2}t}|^2+\cos^2\theta_l(|H_{-\frac{1}{2}0}|^2+|H_{\frac{1}{2}0}|^2)-\frac{1}{2}\sin^2\theta_l(|H_{\frac{1}{2}1}|^2+|H_{-\frac{1}{2}-1}|^2)-2\cos\theta_l\Re(H^*_{\frac{1}{2}0}H_{\frac{1}{2}t}+H^*_{-\frac{1}{2}0}H_{-\frac{1}{2}t}) \tag{55}$$

and

$$\begin{aligned}
\mathcal{I}^{\rm f}_{13} &= \sqrt{2}\sin\theta_l\Big\{H^*_{-\frac{1}{2}-1}H_{-\frac{1}{2}t}+H^*_{\frac{1}{2}1}H_{\frac{1}{2}t}-\cos\theta_l(H^*_{\frac{1}{2}0}H_{\frac{1}{2}1}+H^*_{-\frac{1}{2}-1}H_{-\frac{1}{2}0})\Big\},\\
\mathcal{I}^{\rm f}_{31} &= \sqrt{2}\sin\theta_l\Big\{H^*_{-\frac{1}{2}-1}H_{\frac{1}{2}t}+H^*_{-\frac{1}{2}t}H_{\frac{1}{2}1}-\cos\theta_l(H^*_{-\frac{1}{2}0}H_{\frac{1}{2}1}+H^*_{-\frac{1}{2}-1}H_{\frac{1}{2}0})\Big\},\\
\mathcal{E}^{\rm f}_{00} &= 2\Big\{H^*_{-\frac{1}{2}t}H_{\frac{1}{2}t}+\cos^2\theta_l H^*_{-\frac{1}{2}0}H_{\frac{1}{2}0}-\cos\theta_l(H^*_{-\frac{1}{2}0}H_{\frac{1}{2}t}+H^*_{-\frac{1}{2}t}H_{\frac{1}{2}0})\Big\},\\
\mathcal{E}^{\rm f}_{11} &= \sin^2\theta_l H^*_{-\frac{1}{2}-1}H_{\frac{1}{2}1}.
\end{aligned} \tag{56}$$

If the form factors have no complex phases, meaning the $\mathcal{I}_{\mu\nu}$ terms are real functions, the decay matrix reads as

$$b_{\mu\nu} = \begin{pmatrix} b_{00} & -\mathcal{I}_{01} & 0 & \mathcal{I}_{03}\\ -\mathcal{I}_{10} & b_{11} & 0 & \mathcal{I}_{13}\\ 0 & 0 & b_{22} & 0\\ -\mathcal{I}_{30} & \mathcal{I}_{31} & 0 & b_{33}\end{pmatrix}. \tag{57}$$

The terms of the $b_{\mu\nu}$ matrix in general form for an arbitrary ϕ_l value are given in Appendix C. They should be used if two rotation angle representation as in Ref. [8] was applied.

VI. JOINT ANGULAR DISTRIBUTIONS

Here we provide examples how to construct modular expressions for the angular distributions of semileptonic decays of baryons. First, using our formalism, we rewrite the results from Ref. [6] for the single baryon B_2 decay. The simplest case is the decay of a spin-polarized baryon $B_1 \to B_2 l^- \bar{\nu}_l$. If the polarization of the final particles is not measured the fully differential angular distribution $d\Gamma \propto \mathcal{W} = V_{Ph}(q^2)(q^2-m_l^2)\mathrm{Tr}\rho_{B_2}$, where

$$\begin{aligned}
\mathrm{Tr}\rho_{B_2} &\propto \sum_{\mu=0}^{3} C_{\mu 0}\mathcal{B}^{B_1B_2}_{\mu 0}\\
&= \sum_{\mu=0}^{3} C_{\mu 0}\sum_{\kappa=0}^{3}\mathcal{R}^{(4)}_{\mu\kappa}(\Omega_2)b^{B_1B_2}_{\kappa 0}(q^2,\Omega_l),
\end{aligned} \tag{58}$$

with the baryon B_1 spin state in its rest frame described by the polarization vector $C_{\mu 0} = (1, P_x, P_y, P_z)$. The elements of the decay matrix $b^{B_1B_2}_{\mu 0}(q^2,\Omega_l) \coloneqq b_{\mu 0}(q^2,\Omega_l;\boldsymbol{\omega}_{B_1B_2})$ are given in Eq. (49). For example, if the initial polarization has only P_z component the joint angular distribution for the decay process $B_1 \to B_2 l^- \bar{\nu}_l$ is

$$\begin{aligned}
\mathcal{W}(\boldsymbol{\xi};\boldsymbol{\omega}) = V_{Ph}(q^2)(q^2-m_l^2)[&b_{00}(q^2,\Omega_l;\boldsymbol{\omega}_{B_1B_2})\\
&+P_z b_{30}(q^2,\Omega_l;\boldsymbol{\omega}_{B_1B_2})\cos\theta_2],
\end{aligned} \tag{59}$$

where the vector $\boldsymbol{\xi} \coloneqq (\theta_2, \phi_2, q^2, \Omega_l)$ represents a complete set of the kinematic variables describing an event configuration and the parameter vector $\boldsymbol{\omega}_{B_1 B_2}$ represents the polarization P_z, the semileptonic couplings in Eq. (4) and the range parameters in Eq. (6). If the baryon B_2 decays weakly as $B_2 \to B_4 \pi$ the complete angular distribution is $\mathcal{W} = V_{Ph}(q^2)(q^2 - m_l^2)\mathrm{Tr}\rho_{B_4}$ with

$$\mathrm{Tr}\rho_{B_4} \propto \sum_{\mu,\nu=0}^{3} C_{\mu 0} \mathcal{B}_{\mu\nu}^{B_1 B_2} a_{\nu 0}^{B_2 B_4} = \sum_{\mu=0}^{3} C_{\mu 0} \sum_{\kappa,\nu=0}^{3} \mathcal{R}_{\mu\kappa}^{(4)}(\Omega_2) b_{\kappa\nu}^{B_1 B_2}(q^2, \Omega_l) a_{\nu 0}(\theta_4, \phi_4; \alpha_{B_2}). \tag{60}$$

The decay matrix $a_{\nu 0}(\theta_4, \phi_4; \alpha_{B_2})$ [8] describes the nonleptonic decay $B_2 \to B_4 \pi$ and using the representation from Appendix D is given as

$$\begin{bmatrix} a_{00} \\ a_{10} \\ a_{20} \\ a_{30} \end{bmatrix} = \mathcal{R}^{(4)}(\{0, \theta_4, \phi_4\}) \begin{bmatrix} 1 \\ 0 \\ 0 \\ \alpha_{B_2} \end{bmatrix} = \begin{bmatrix} 1 \\ \alpha_{B_2} \sin\theta_4 \cos\phi_4 \\ \alpha_{B_2} \sin\theta_4 \sin\phi_4 \\ \alpha_{B_2} \cos\theta_4 \end{bmatrix}, \tag{61}$$

where θ_4 and ϕ_4 are the helicity angles of B_4 in the $\mathbb{R}_2$ frame and α_{B_2} is the decay asymmetry parameter. The corresponding angular distribution for charge-conjugated decay mode is obtained by the replacements $H_{\lambda_2 \lambda_W}^{B_1} \to H_{\lambda_2 \lambda_W}^{\bar{B}_1}$, $g_{av/w}^{B_1} \to g_{av/w}^{\bar{B}_1}$ and swapping between $(l^-, \bar{\nu}_l)$ and (l^+, ν_l). Neglecting hadronic CP-violating effects, one has $H_{\lambda_2, \lambda_W}^{V(\bar{B}_1)} = H_{\lambda_2, \lambda_W}^{V(B_1)}$ and $H_{\lambda_2, \lambda_W}^{A(\bar{B}_1)} = -H_{\lambda_2, \lambda_W}^{A(B_1)}$ Eq. (12) meaning that $g_w^{\bar{B}_1} = g_w^{B_1}$ and $g_{av}^{\bar{B}_1} = -g_{av}^{B_1}$ [19,20].

Now we consider a decay of a spin-entangled baryon-antibaryon system $B_1 \bar{B}_1$, where the initial state is given by the spin-correlation matrix $C_{\mu\bar{\nu}}^{B_1 \bar{B}_1}$ defined in Eq. (1) with $B_1 \to B_2 l^- \bar{\nu}_l$. The semileptonic decay is tagged by a common decay of the antibaryon $\bar{B}_1$. For hyperon decay studies, a nonleptonic decay $\bar{B}_1 \to \bar{B}_3 \bar{\pi}$ is used. One obvious advantage of the studies using baryon-antibaryon pairs is that the charge-conjugated decays, corresponding to the $\bar{B}_1 \to \bar{B}_2 l^+ \nu_l$ and $B_1 \to B_3 \pi$ scenario, can be studied simultaneously. A common practice is to implicitly combine events corresponding to the charge-conjugated channels in the analyses to determine the decay properties in the CP-symmetry limit. In such analyses, the quantities that are even (odd) with respect to the parity operation have the same (opposite sign) values when combining the two cases. At the same time, the CP-symmetry can be tested by comparing values of the separately determined parameters for the baryon and antibaryon decays. Using as a building block the semileptonic decay matrix one constructs the angular distribution for the case when polarization of baryons B_2 and $\bar{B}_3$ is not measured,

$$\mathrm{Tr}\rho_{B_2 \bar{B}_3} \propto \sum_{\mu,\bar{\nu}=0}^{3} C_{\mu\bar{\nu}}^{B_1 \bar{B}_1} \mathcal{B}_{\mu 0}^{B_1 B_2} a_{\bar{\nu} 0}^{\bar{B}_1 \bar{B}_3}. \tag{62}$$

The matrix $\mathcal{B}_{\mu 0}^{B_1 B_2} \coloneqq \mathcal{B}_{\mu 0}(\theta_2, \phi_2, q^2, \Omega_l; \boldsymbol{\omega}_{B_1 B_2})$ describes the semileptonic decay and $a_{\bar{\nu} 0}^{\bar{B}_1 \bar{B}_3} \coloneqq a_{\bar{\nu} 0}(\theta_3, \phi_3; \bar{\alpha}_{B_1})$ [8] describes the nonleptonic decay $\bar{B}_1 \to \bar{B}_3 \pi$, where θ_3 and ϕ_3 are the helicity angles of $\bar{B}_3$ in the $\bar{B}_1$ rest frame and $\bar{\alpha}_{B_1}$ is the decay asymmetry parameter. The joint angular distribution for the process is $\mathcal{W}(\boldsymbol{\xi}; \boldsymbol{\omega}) = V_{Ph}(q^2)(q^2 - m_l^2)\mathrm{Tr}\rho_{B_2 \bar{B}_3}$, where

$$\begin{aligned} \mathrm{Tr}\rho_{B_2 \bar{B}_3} = {} & C_{00}^{B_1 \bar{B}_1}(\theta_1) b_{00}(\boldsymbol{\xi}') + \sum_{i,j=1}^{3} C_{ij}^{B_1 \bar{B}_1}(\theta_1) \mathcal{B}_{i0}(\boldsymbol{\xi}') \\ & \times a_{j0}(\theta_3, \phi_3; \bar{\alpha}_{B_1}) + \sum_{i=1}^{3} C_{i0}^{B_1 \bar{B}_1}(\theta_1) \mathcal{B}_{i0}(\boldsymbol{\xi}') \\ & + b_{00}(\boldsymbol{\xi}') \sum_{j=1}^{3} C_{0j}^{B_1 \bar{B}_1}(\theta_1) a_{j0}(\theta_3, \phi_3; \bar{\alpha}_{B_1}) \end{aligned} \tag{63}$$

with $C_{\mu\bar{\nu}}^{B_1 \bar{B}_1}$ given in Eq. (2) for the annihilation of the unpolarized electron-positron beams. The vectors of the kinematic variables are $\boldsymbol{\xi} = (\theta_1, \theta_2, \phi_2, q^2, \Omega_l, \theta_3, \phi_3)$ while $\boldsymbol{\xi}' = (\theta_2, \phi_2, q^2, \Omega_l)$. The full vector of parameters is denoted as $\boldsymbol{\omega} \coloneqq (\alpha_\psi, \Delta\Phi, g_{av}^{B_1}, g_w^{B_1}, \bar{\alpha}_{B_1})$.

VII. SENSITIVITIES FOR SL FORM FACTORS PARAMETERS

Here we present estimates for the statistical uncertainties of the parameters describing form factors of selected semileptonic hyperon decays. The derived angular distributions are used to construct the normalized multidimensional probability density function for an event configuration. They are functions of q^2 and the helicity angles, and depend on the form factor parameters such as g_{av} and g_w (4). The parameters can be determined in an experiment using maximum likelihood (ML) method, which guarantees consistency and efficiency properties. We provide uncertainties of the parameters in the large number of events limit and assuming the detection efficiency does not depend on the kinematic variables as described in Refs. [9,10]. Since the ML estimators are asymptotically normal, the product of their standard deviations, σ, and $\sqrt{N}$, where N is the number of the observed events, does not depend on N. The uncertainties are obtained by calculating elements of the Fisher information

TABLE I. Properties of selected semileptonic decays of the ground-state-octet hyperons. The column labeled $M_1 - M_2$ gives the upper range of the $\sqrt{q^2}$ variable.

Decay	Transition	$\mathcal{B}(\times 10^{-4})$	g_{av}	g_w	$M_1 - M_2$ [MeV]	Comment
$\Lambda \to p e^- \bar{\nu}_e$	V_{us}	8.32(14)	0.718(15)	1.066	177	[2,13]
$\Sigma^+ \to \Lambda e^+ \nu_e$ [a]	V_{ud}	0.20(05)	0.01(10)	2.4(17)	74	[13]
$\Xi^- \to \Lambda e^- \bar{\nu}_e$	V_{us}	5.63(31)	0.25(5)	0.085	206	[2,22]
$\Xi^- \to \Sigma^0 e^- \bar{\nu}_e$	V_{us}	0.87(17)	1.25(15)	2.609	129	[2,22]
$\Xi^0 \to \Sigma^+ e^- \bar{\nu}_e$	V_{us}	2.52(8)	1.22(5)	2.0(9)	125	[13]

[a] Since for Σ^+ $F_1 = 0$, the coupling constants g_{av} and g_w are defined as F_1^V/F_1^A and F_2^V/F_1^A, respectively.

matrix that is inverted to obtain the covariance matrix for the parameters.

We consider the semileptonic decays of hyperons listed in Table I. We neglect form factors F_3^V and F_2^A which vainsh in the limit of of the $SU(3)$ flavor symmetry [21]. Equation (11) allows one to estimate the relative contribution of different form factors to the angular distributions. Based on the g_{av} and g_w values from Table I the q^2 dependence of the six helicity amplitudes for the Λ semileptonic decays is shown in Fig. 3(a). To allow a better comparison the amplitudes are multiplied by $\sqrt{q^2}$. Close to the lower boundary, $q^2 = m_e^2$, the longitudinal and scalar helicity amplitudes dominate, with $H^{V(A)}_{\frac{1}{2}0} \approx H^{V(A)}_{\frac{1}{2}t}$. Close to the upper boundary at the zero recoil point, $q^2 = (M_1 - M_2)^2$, the contributions $H^V_{\frac{1}{2}t}$ and $H^A_{\frac{1}{2}1} = -\sqrt{2} H^A_{\frac{1}{2}0}$ are dominant with $H^V_{\frac{1}{2}t} = -H^A_{\frac{1}{2}0}/g_{av}$. We do not consider the decay $\Sigma^- \to n e^- \bar{\nu}_e$ since the final state includes two neutral particles, neutron and neutrino, making it impossible to fully reconstruct the events. In addition, no measurements exist for the production parameters in the $e^+e^- \to \Sigma^- \bar{\Sigma}^+$ process.

The first case is the decay $\Lambda \to p e^- \bar{\nu}_e$ studied in the exclusive process $e^+e^- \to J/\psi \to \Lambda\bar{\Lambda}$, where $\bar{\Lambda} \to \bar{p}\pi^+$ is used for tagging. The angular distribution is given by Eq. (62) where the parameters of the production process $e^+e^- \to J/\psi \to \Lambda\bar{\Lambda}$ needed to define the spin-correlation–polarization matrix $C_{\mu\nu}$ are given in Table II. The properties of $\bar{\Lambda} \to \bar{p}\pi^+$ decay and the charge conjugated process that is used to tag the SL decay are given in Table III. We assume the production parameters and the decay parameters of the nonleptonic decays used for the tagging to be well known and fixed. Since the coupling g_{av3} is multiplied by m_e in the transition amplitude [23], we set it to zero because it cannot be determined from experiment with a reasonable uncertainty. In addition the parameters $r_{v,w}$ and r_{av} defined in Eq. (6) are fixed to the values deduced from the ansatz for the $s \to u$ transition of Refs. [2,24] and listed in Table IV. The statistical uncertainties $\sigma(g_{av})$ and $\sigma(g_w)$ for the coupling constants g_{av} and g_w, respectively, are given in the first row of Table IV. The main feature is that the uncertainty for the g_{av} coupling is nearly one order of magnitude less than for g_w since the latter is suppressed by the $q^2/M_1^2 < (M_1 - M_2)^2/M_1^2 \approx 0.025$ factor (11). The second row corresponds to an independent method to study $\Lambda \to p e^- \bar{\nu}_e$ using the $e^+e^- \to J/\psi \to \Xi^- \bar{\Xi}^+$ process with

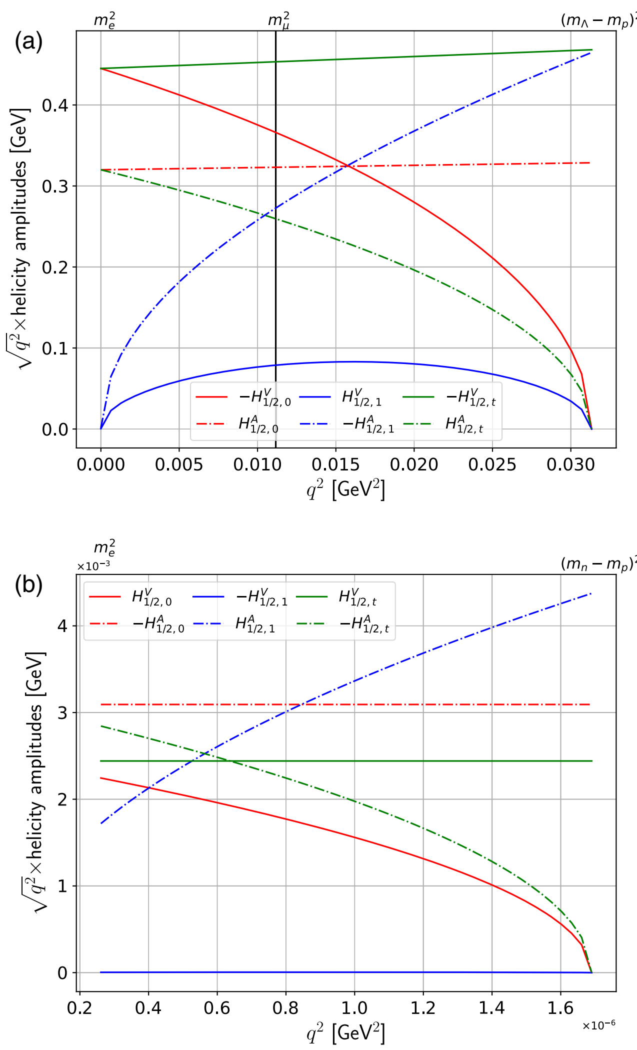

FIG. 3. The q^2 dependence of the six helicity amplitudes for (a) $\Lambda \to p l^- \bar{\nu}_l$ and (b) $n \to p e^- \bar{\nu}_e$ decays. For the Λ decay the kinematic range for the μ mode is to the right of the vertical line $q^2 = m_\mu^2$.

TABLE II. Properties of the $e^+e^- \to J/\psi \to B_1\bar{B}_1$ decays to the pairs of ground-state octet hyperons.

	$\mathcal{B}(\times 10^{-4})$	α_ψ	$\Delta\Phi$ [rad]	Comment
$\Lambda\bar{\Lambda}$	19.43(33)	0.475(4)	0.752(8)	[11,25]
$\Sigma^+\bar{\Sigma}^-$	15.0(24)	−0.508(7)	−0.270(15)	[26,27]
$\Xi^-\bar{\Xi}^+$	9.7(8)	0.586(16)	1.213(49)	[12,13]
$\Xi^0\bar{\Xi}^0$	11.65(4)	0.514(16)	1.168(26)	[28,29]

TABLE III. Properties of the main decays of the ground-state octet hyperons that can be used to tag the SL decays. The decay asymmetry $\bar{\alpha}_D$ for the charge conjugated decay modes in the CP-symmetry conservation limit is $\bar{\alpha}_D = -\alpha_D$.

D	$\mathcal{B}(\%)$	α_D	Comment
$\Lambda \to p\pi^-$	064	0.755(3)	[12,30]
$\Sigma^+ \to p\pi^0$	052	−0.994(4)	[27]
$\Xi^- \to \Lambda\pi^-$	100	−0.379(4)	[12,13]
$\Xi^0 \to \Lambda\pi^0$	96	−0.375(3)	[13,29]

the $\Xi^- \to [\Lambda \to pe^-\bar{\nu}_e]\pi^-$ sequence and $\bar{\Xi}^+ \to [\bar{\Lambda} \to \bar{p}\pi^+]\pi^-$ for the tagging. The modular expression for the angular distribution of such process reads

$$\mathrm{Tr}\rho_{p\bar{p}} \propto \sum_{\mu,\bar{\nu}=0}^{3} C_{\mu\bar{\nu}}^{\Xi\bar{\Xi}} \sum_{\mu'=0}^{3} a_{\mu\mu'}^{\Xi\Lambda} \mathcal{B}_{\mu'0}^{\Lambda p} \sum_{\bar{\nu}'=0}^{3} a_{\bar{\nu}\bar{\nu}'}^{\bar{\Xi}\bar{\Lambda}} a_{\bar{\nu}'0}^{\bar{\Lambda}\bar{p}}. \quad (64)$$

The polarization of the Λ originating from the nonleptonic weak decay $\Xi^- \to \Lambda\pi^-$, is ∼40%, to be compared to the root-mean-squared value of the Λ polarization in $e^+e^- \to J/\psi \to \Lambda\bar{\Lambda}$ of 11% [10]. However, the uncertainties of the weak couplings are the same for both methods. To further investigate dependence on the initial polarization of Λ we set $\Delta\Phi = 0$ to have the zero polarization, while to obtain maximally polarized Λ we include the longitudinal polarization of the electron beam and use the production matrix $C_{\mu\bar{\nu}}$ from Ref. [10]. The impact of the spin correlations for the uncertainties can be studied by comparing the results using the angular distributions (62) or (64) with full production matrices $C_{\mu\bar{\nu}}$ to the ones where all elements except $C_{\mu 0}$ are set to zero. This arrangement assures that the spin-correlation terms are excluded. In all these tests the uncertainties of $\sigma(g_{av})$ and $\sigma(g_w)$ remain unchanged, meaning that the polarization and the spin correlations of the mother hyperon in the decay play almost no role for the measurements of properties of the semileptonic decays to baryons whose polarization is not measured.

The entries from the third row and below in Table IV correspond to the decays where the polarization of the daughter baryon is measured and the angular distributions include the complete $\mathcal{B}_{\mu\mu'}$ matrices. For example the angular distribution for $\Xi^- \to \Lambda e^-\bar{\nu}_e$ measurement in $e^+e^- \to J/\psi \to \Xi^-\bar{\Xi}^+$ is

$$\mathrm{Tr}\rho_{p\bar{p}} \propto \sum_{\mu,\bar{\nu}=0}^{3} C_{\mu\bar{\nu}}^{\Xi\bar{\Xi}} \sum_{\mu'=0}^{3} \mathcal{B}_{\mu\mu'}^{\Xi\Lambda} a_{\mu'0}^{\Lambda p} \sum_{\bar{\nu}'=0}^{3} a_{\bar{\nu}\bar{\nu}'}^{\bar{\Xi}\bar{\Lambda}} a_{\bar{\nu}'0}^{\bar{\Lambda}\bar{p}}. \quad (65)$$

Since the uncertainties depend on the values of the weak couplings it is difficult to compare the results for different decays in Table IV. By repeating the studies with variation of $\Delta\Phi$ and the electron-beam polarization some impact is seen for the uncertainties, specially for the g_w parameter in $\Sigma^+ \to \Lambda e^+\nu_e$. In addition we study the uncertainties for single spin-polarized baryon decays with the angular distributions given by Eq. (58). The baryon B_1 polarization vector is set to $C_{\mu 0} = (1, 0, P_y, 0)$. The results for $\sigma(g_{av})$ and $\sigma(g_w)$ are shown in Fig. 4. The uncertainty for large P_y decreases typically by 20% comparing to the unpolarized case.

Our formalism applies also to the $n \to pe^-\bar{\nu}_e$ decay and it should be equivalent to the approach from Ref. [31] for the single neutron decay. However, we can also describe decay correlations for a spin entangled neutron–neutron pair. As an example we take nn spin singlet state given by the spin-correlation matrix $C_{\mu\nu} = \mathrm{diag}(1, -1, 1, 1)$. The coupling constants g_{av} and g_w are 1.2754(13) and 1.853, respectively [13,31]. The q^2 dependence of the form factors is neglected due to the tiny range, $m_e < \sqrt{q^2} < M_n - M_p$, of the variable. The corresponding helicity amplitudes for the neutron beta decay are shown in Fig. 3(b). The resulting uncertainty of the g_{av} measurement in the double beta

TABLE IV. Statistical uncertainties for the g_{av} and g_w couplings for some semileptonic decays reconstructed using the double-tag method [Eq. (62)].

Decay	$\sigma(g_{av})\sqrt{N}$	$\sigma(g_w)\sqrt{N}$	$r_{v,w}$ [GeV^{-2}]	r_a [GeV^{-2}]
$\Lambda \to pe^-\bar{\nu}_e$	1.8	12	1.94	1.28
$\Xi^- \to [\Lambda \to pe^-\bar{\nu}_e]\pi^-$	1.8	12		
$\Xi^- \to [\Lambda \to p\pi^-]e^-\bar{\nu}_e$	0.6	9		
$\Xi^- \to [\Sigma^0 \to [\Lambda \to p\pi^-]\gamma]e^-\bar{\nu}_e$	5.0	29		
$\Xi^0 \to [\Sigma^+ \to p\pi^0]e^-\bar{\nu}_e$	4.0	28		
$\Sigma^+ \to [\Lambda \to p\pi^-]e^+\nu_e$	0.5	19	2.83	1.71

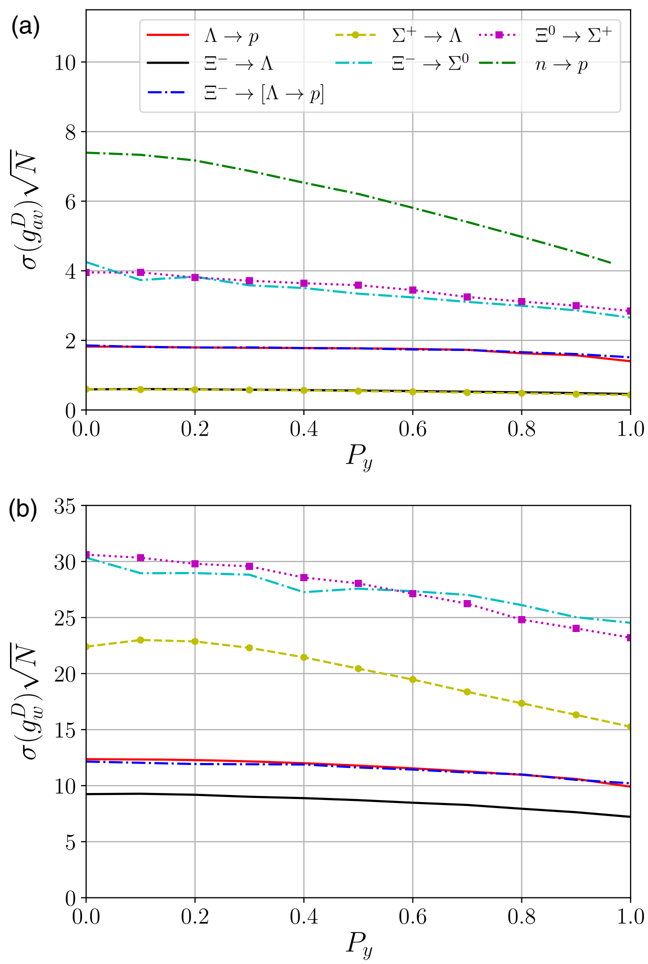

FIG. 4. Statistical uncertainties (a) $\sigma(g_{av})\sqrt{N}$ and (b) $\sigma(g_w)\sqrt{N}$ for semileptonic decays decays as a function of the initial baryon polarization. Note that there is no estimate of $\sigma(g_w)$ for $n \to pe^-\nu_e$ in panel (b) as explained in the text.

decay of the singlet pair is $\sigma(g_{av})\sqrt{N} = 4.3$. It should be compared to the uncertainties in the measurements with single neutrons that are shown in Fig. 4(a) as a function of the neutron polarization. For unpolarized neutron $\sigma(g_{av})\sqrt{N} = 7.4$ and it decreases to 4.1 when the polarization is equal one. The flip contribution to the helicity amplitudes (54) of about 8% was neglected in the estimates. The g_w coupling cannot be determined since its contribution to the helicity amplitude $H^V_{\frac{1}{2}0}$ is suppressed by a factor q^2/M_n^2. Moreover, the second amplitude that includes g_w, $H^V_{\frac{1}{2}1}$, is suppressed by $\sqrt{q^2}$ and as seen in Fig. 3(b) it is consistent with zero.

VIII. CONCLUSIONS

We have constructed a modular description of the differential distributions for baryon semileptonic decays where the baryons are originating from entangled baryon-antibaryon pairs produced in the electron-positron annihilations or in charmonia decays. The formalism allows to extract the weak form factors using complete information available in such experiments. The lepton mass effects as well as polarization effects of the decaying parent hyperon are included in the formalism. The presented modular expressions are applicable to various sequential processes like $B_1 \to B_2(\to B_3 + \pi) + l + \nu_l$ that involve a semileptonic decay. Two conventions for defining transversal directions of the helicity frames were considered. The daughter baryon spin-density matrix in a semileptonic decay takes the simplest form when expressed using the angles in the decay plane. The two representations are equivalent, provided that one uses the matching set of rotations to define the helicity angles.

We have not included radiative corrections in our estimates but they have to be considered in the experimental analyses. Over the years, the radiative corrections to hadronic β-decays have been extensively studied [32] and the specific applications to the hyperon semileptonic decays are discussed in Ref. [33]. The state-of-the-art in experimental analyses is to use PHOTOS program [34] that is based on leading-logarithmic (collinear) approximation. The procedure is applied to all final particles but the electron(positron) tracks are most affected.

The BESIII experiment has collected 10^{10} J/ψ [15] meaning that for semileptonic decays data samples of less than 10^4 events are available. Therefore, a rough estimate of the achievable uncertainties with this data set is given by dividing the values in Table IV by 100. The J/ψ decays into a hyperon-antihyperon pair can provide a clean setting with low systematic uncertainties for the CP-symmetry conservation tests in semileptonic decays since the decays of the charge conjugated modes can be done simultaneously.

A similar modular approach with decay matrices might be useful for studies of radiative and Dalitz decays. As a cross-check and illustration in Appendix D we provide formulas for the Dalitz transition $B_1 \to B_2 l^+ l^-$ between baryons with spin-$1/2$ as well as decay matrix for a weak radiative decay with real photon $B_1 \to B_2\gamma$.

For the studies of semileptonic decays of heavy baryons induced by the quark transitions $c \to s + l^+ + \nu_l$ or $b \to c + l^- + \bar{\nu}_l$ the previously available formalism [6] is likely sufficient if only beams of polarized baryons are used. This might change in near future with BESIII and Belle II experiments where entangled charmed baryon-antibaryon pairs will be available. One difference would be a measurement of the polarization for the tagging reactions which probably has to use three-body hadronic weak decays. However, even for the case of single baryon decays our approach provides an easy and flexible way to implement different decay sequences in the event generators that propagate spin information of the decaying baryons.

ACKNOWLEDGMENTS

We thank our colleagues Stefan Leupold and Patrik Adlarson for useful discussion and for motivation of this study. This work was supported in part by National Natural Science Foundation of China (NSFC) under Contract No. 11935018, the CAS President's International Fellowship Initiative (PIFI) Grant No. 2021PM0014, Polish National Science Centre through the Grant No. 2019/35/O/ST2/02907 and Swedish Research Council through the Grant No. 2021-04567. In addition, a support from the European Union's Horizon 2020 research and innovation programme under Grant Agreement STRONG-2020—No. 824093 and the Munich Institute for Astro-, Particle and BioPhysics (MIAPbP) which is funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy—EXC-2094-390783311 is acknowledged.

APPENDIX A: CONVENTIONS FOR WIGNER FUNCTIONS AND PAULI MATRICES

Conventions for Pauli matrices: rows $m_1 = 1/2, -1/2$ are numbered from top to bottom and columns $m_2 = 1/2, -1/2$ are numbered from left to right,

$$\sigma_0^{m_1,m_2} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \sigma_1^{m_1,m_2} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2^{m_1,m_2} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3^{m_1,m_2} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A1}$$

The corresponding Wigner functions $\mathcal{D}^J_{m_1,m_2}(0,\theta,0) = d^J_{m_1,m_2}(\theta)$ are for $J = 1/2$

$$\mathcal{D}^{1/2}_{m_1,m_2}(0,\theta,0) = \begin{pmatrix} \cos\theta/2 & -\sin\theta/2 \\ \sin\theta/2 & \cos\theta/2 \end{pmatrix}, \tag{A2}$$

with the columns and rows expressed using the same convention. For $J = 1$ the functions are

$$\mathcal{D}^{1}_{m_1,m_2}(0,\theta,0) = \begin{pmatrix} \frac{1}{2}(1+\cos\theta) & \frac{1}{\sqrt{2}}\sin\theta & \frac{1}{2}(1-\cos\theta) \\ -\frac{1}{\sqrt{2}}\sin\theta & \cos\theta & \frac{1}{\sqrt{2}}\sin\theta \\ \frac{1}{2}(1-\cos\theta) & -\frac{1}{\sqrt{2}}\sin\theta & \frac{1}{2}(1+\cos\theta) \end{pmatrix}, \tag{A3}$$

where the rows (m_1) and columns (m_2) are labeled in the order $(-1, 0, 1)$ from left to right and top to bottom, respectively. This convention matches the complete Wigner $\mathcal{D}$ functions given as $\mathcal{D}^J_{m_1,m_2}(\phi,\theta,\chi) = \exp(-im_1\phi)\mathcal{D}^J_{m_1,m_2}(0,\theta,0)\times \exp(-im_2\chi)$. The Pauli matrices are related to the 3D rotation matrices $R_{jk}(\Omega)$ for $\Omega \equiv \{\phi,\theta,\chi\}$ in the following way:

$$\begin{aligned} R_{jk}(\Omega) &= \frac{1}{2}\sum_{\kappa,\kappa'}\sum_{\zeta,\zeta'}(\sigma_j)^{\kappa,\kappa'}(\sigma_k)^{\zeta',\zeta}\mathcal{D}^{1/2*}_{\kappa,\zeta}(\Omega)\mathcal{D}^{1/2}_{\kappa',\zeta'}(\Omega) \\ &= \begin{pmatrix} \cos\theta\cos\chi\cos\phi - \sin\chi\sin\phi & -\cos\theta\sin\chi\cos\phi - \cos\chi\sin\phi & \sin\theta\cos\phi \\ \cos\theta\cos\chi\sin\phi + \sin\chi\cos\phi & \cos\chi\cos\phi - \cos\theta\sin\chi\sin\phi & \sin\theta\sin\phi \\ -\sin\theta\cos\chi & \sin\theta\sin\chi & \cos\theta \end{pmatrix}, \end{aligned} \tag{A4}$$

where the columns (k) and rows (j) are labeled $k, j = 1, 2, 3$ (x, y, z) from left to right and from top to bottom, respectively.

APPENDIX B: DERIVATION OF THE DECAY MATRIX DECOMPOSITION

Starting from the amplitude representation in Eq. (21) we derive expression Eq. (39). Multiplying the amplitude in Eq. (21) by its conjugate to obtain spin-density matrix and by inserting basis Pauli matrices for the mother and the daughter baryon,

$$
\begin{aligned}
\mathcal{B}^D_{\mu\nu} &= \frac{1}{8\pi^2}\sum_{\lambda,\lambda'}\sum_{\kappa,\kappa'}\sum_{\zeta,\zeta'}\mathcal{H}_{\zeta,\lambda}\mathcal{H}^*_{\zeta',\lambda'}(\sigma_\mu)^{\kappa,\kappa'}(\sigma_\nu)^{\lambda',\lambda}\mathcal{D}^{1/2*}_{\kappa,\zeta}(\Omega)\mathcal{D}^{1/2}_{\kappa',\zeta'}(\Omega)\\
&= \frac{1}{8\pi^2}\sum_{\zeta,\zeta'}\underbrace{\left[\sum_{\lambda,\lambda'}(\sigma_\nu)^{\lambda',\lambda}\mathcal{H}_{\zeta,\lambda}\mathcal{H}^*_{\zeta',\lambda'}\right]}_{I_\nu^{\zeta,\zeta'}}\underbrace{\left[\sum_{\kappa,\kappa'}(\sigma_\mu)^{\kappa,\kappa'}\mathcal{D}^{1/2*}_{\kappa,\zeta}(\Omega)\mathcal{D}^{1/2}_{\kappa',\zeta'}(\Omega)\right]}_{R_\mu^{\zeta,\zeta'}(\Omega)}\\
&= \frac{1}{8\pi^2}\sum_{\zeta,\zeta'}R_\mu^{\zeta,\zeta'}(\Omega)I_\nu^{\zeta,\zeta'},
\end{aligned}
\tag{B1}
$$

where the running indices in all sums $\kappa, \kappa', \lambda', \lambda$ and ζ, ζ' are $-1/2$ and $+1/2$. Despite $\mathcal{B}^D_{\mu\nu}$ being a real-value matrix, the matrices $I_\nu^{\zeta,\zeta'}$ and $R_\mu^{\zeta,\zeta'}(\Omega)$ are not real valued. We would like to rewrite Eq. (B1) as a product of a 4D rotation matrix and a 4×4 matrix $b_{\rho\nu}$ in the form given in Eq. (39),

$$
\mathcal{B}^D_{\mu\nu} = \frac{1}{8\pi^2}\sum_{\rho=0}^{3}\mathcal{R}^{(4)}_{\mu\rho}(\Omega)b_{\rho\nu}.
$$

In order to derive form of $\mathcal{R}^{(4)}_{\mu\rho}(\Omega)$ we set the matrix $b_{\mu\nu}$ to the identity 4×4 matrix. This can be achieved by setting $\mathcal{H}_{\zeta,\lambda} = \delta_{\zeta\lambda}$ since

$$
\sum_{\zeta,\zeta'}\sum_{\lambda,\lambda'}(\sigma_\nu)^{\lambda',\lambda}\delta_{\zeta,\lambda}\delta_{\zeta',\lambda'}(\sigma_\mu)^{\zeta,\zeta'} = 2\delta_{\nu\mu}.
\tag{B2}
$$

Such replacement in Eq. (B1) gives,

$$
\begin{aligned}
\mathcal{R}^{(4)}_{\mu\nu}(\Omega) &= \frac{1}{2}\sum_{\lambda,\lambda'}\sum_{\kappa,\kappa'}\sum_{\zeta,\zeta'}\delta_{\zeta,\lambda}\delta_{\zeta',\lambda'}(\sigma_\mu)^{\kappa,\kappa'}(\sigma_\nu)^{\lambda',\lambda}\mathcal{D}^{1/2*}_{\kappa,\zeta}(\Omega)\mathcal{D}^{1/2}_{\kappa',\zeta'}(\Omega)\\
&= \frac{1}{2}\sum_{\kappa,\kappa'}\sum_{\zeta,\zeta'}(\sigma_\mu)^{\kappa,\kappa'}(\sigma_\nu)^{\zeta',\zeta}\mathcal{D}^{1/2*}_{\kappa,\zeta}(\Omega)\mathcal{D}^{1/2}_{\kappa',\zeta'}(\Omega).
\end{aligned}
\tag{B3}
$$

By evaluating the above expression one gets the explicit form for $\mathcal{R}^{(4)}_{\mu\nu}(\Omega)$,

$$
\begin{pmatrix}
1 & 0 & 0 & 0\\
0 & \cos\theta\cos\chi\cos\phi - \sin\chi\sin\phi & -\cos\theta\sin\chi\cos\phi - \cos\chi\sin\phi & \sin\theta\cos\phi\\
0 & \cos\theta\cos\chi\sin\phi + \sin\chi\cos\phi & \cos\chi\cos\phi - \cos\theta\sin\chi\sin\phi & \sin\theta\sin\phi\\
0 & -\sin\theta\cos\chi & \sin\theta\sin\chi & \cos\theta
\end{pmatrix},
\tag{B4}
$$

which is the 4D rotation where the spatial part $\mathcal{R}_{jk}(\Omega)$ corresponds to the product of the following three axial rotations,

$$
\begin{aligned}
\mathcal{R}_{jk}(\Omega) &= R_z(\phi)R_y(\theta)R_z(\chi)\\
&= \begin{pmatrix}\cos\phi & -\sin\phi & 0\\ \sin\phi & \cos\phi & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{pmatrix}\begin{pmatrix}\cos\chi & -\sin\chi & 0\\ \sin\chi & \cos\chi & 0\\ 0 & 0 & 1\end{pmatrix}.
\end{aligned}
\tag{B5}
$$

The expression for $b_{\mu\nu}$ can be deduced by setting $\mathcal{R}^{(4)}_{\mu\rho}(\Omega)$ to the 4×4 identity matrix i.e., by setting $\Omega = \{0, 0, 0\}$,

$$b_{\mu\nu} = \mathcal{B}^D_{\mu\nu}(\Omega \equiv 0) = \sum_{\zeta,\zeta'}\left[\sum_{\lambda,\lambda'}(\sigma_\nu)^{\lambda',\lambda}\mathcal{H}_{\zeta,\lambda}\mathcal{H}^*_{\zeta',\lambda'}\right] \times \left[\sum_{\kappa,\kappa'}(\sigma_\mu)^{\kappa,\kappa'}\delta_{\kappa,\zeta}\delta_{\kappa',\zeta'}\right] = \sum_{\zeta,\zeta'}\sum_{\lambda,\lambda'}(\sigma_\nu)^{\lambda',\lambda}\mathcal{H}_{\zeta,\lambda}\mathcal{H}^*_{\zeta',\lambda'}(\sigma_\mu)^{\zeta,\zeta'}. \tag{B6}$$

The elements of the real-valued matrix $b_{\rho\nu}$ expressed in terms of amplitudes $\mathcal{H}$ are

$$b_{\rho 0} = \begin{pmatrix} |\mathcal{H}_{--}|^2 + |\mathcal{H}_{+-}|^2 + |\mathcal{H}_{-+}|^2 + |\mathcal{H}_{++}|^2 \\ 2\Re(\mathcal{H}_{++}\mathcal{H}^*_{-+} + \mathcal{H}_{--}\mathcal{H}^*_{+-}) \\ 2\Im(\mathcal{H}_{++}\mathcal{H}^*_{-+} - \mathcal{H}_{--}\mathcal{H}^*_{+-}) \\ -|\mathcal{H}_{--}|^2 + |\mathcal{H}_{++}|^2 + |\mathcal{H}_{+-}|^2 - |\mathcal{H}_{-+}|^2 \end{pmatrix}, \tag{B7}$$

$$b_{\rho 1} = \begin{pmatrix} 2\Re(\mathcal{H}_{++}\mathcal{H}^*_{+-} + \mathcal{H}_{--}\mathcal{H}^*_{-+}) \\ 2\Re(\mathcal{H}_{++}\mathcal{H}^*_{--} + \mathcal{H}_{-+}\mathcal{H}^*_{+-}) \\ 2\Im(\mathcal{H}_{++}\mathcal{H}^*_{--} - \mathcal{H}_{-+}\mathcal{H}^*_{+-}) \\ 2\Re(\mathcal{H}_{++}\mathcal{H}^*_{+-} - \mathcal{H}_{--}\mathcal{H}^*_{-+}) \end{pmatrix}, \tag{B8}$$

$$b_{\rho 2} = \begin{pmatrix} -2\Im(\mathcal{H}_{++}\mathcal{H}^*_{+-} - \mathcal{H}_{--}\mathcal{H}^*_{-+}) \\ -2\Im(\mathcal{H}_{++}\mathcal{H}^*_{--} + \mathcal{H}_{-+}\mathcal{H}^*_{+-}) \\ 2\Re(\mathcal{H}_{++}\mathcal{H}^*_{--} - \mathcal{H}_{-+}\mathcal{H}^*_{+-}) \\ -2\Im(\mathcal{H}_{++}\mathcal{H}^*_{+-} + \mathcal{H}_{--}\mathcal{H}^*_{-+}) \end{pmatrix}, \tag{B9}$$

$$b_{\rho 3} = \begin{pmatrix} -|\mathcal{H}_{--}|^2 - |\mathcal{H}_{+-}|^2 + |\mathcal{H}_{-+}|^2 + |\mathcal{H}_{++}|^2 \\ 2\Re(\mathcal{H}_{++}\mathcal{H}^*_{-+} - \mathcal{H}_{--}\mathcal{H}^*_{+-}) \\ 2\Im(\mathcal{H}_{++}\mathcal{H}^*_{-+} + \mathcal{H}_{--}\mathcal{H}^*_{+-}) \\ |\mathcal{H}_{--}|^2 + |\mathcal{H}_{++}|^2 - |\mathcal{H}_{+-}|^2 - |\mathcal{H}_{-+}|^2 \end{pmatrix}. \tag{B10}$$

The matrix elements $b_{\mu\nu}$ are interrelated since they are expressed by the four complex amplitudes $\mathcal{H}_{\lambda,\lambda'}$. Therefore, neglecting the unobservable overall phase there are up to six independent real-valued functions in addition to the unpolarized cross section term b_{00}. The b-matrix can be considered as a generalization of Lee-Yang baryon polarization formula [35] which has maximum two independent parameters (see example in Appendix D 2). The terms b_{i0}/b_{00} are discussed in [36] in the context of hadronic decays and are called aligned polarimeter fields $\alpha_{x,y,z}$. In Appendix D we give the b matrices for few example processes.

APPENDIX C: COMPLETE DECAY MATRIX FOR SL DECAYS

The terms of the nonflip contributions for the unaligned (with arbitrary ϕ_l) decay matrix $b^{\mathrm{nf}}_{\mu\nu}$ are (the term b^{nf}_{33} does not depend on the angle and it is not repeated),

$$\begin{aligned}
b^{\mathrm{nf}}_{11} &= \Re(\mathcal{E}^{\mathrm{nf}}_{00}) + \{\Re(\mathcal{E}^{\mathrm{nf}}_{11})\cos 2\phi_l - \Im(\mathcal{E}^{\mathrm{nf}}_{11})\sin 2\phi_l\},\\
b^{\mathrm{nf}}_{12} &= -\Im(\mathcal{E}^{\mathrm{nf}}_{00}) - \{\Re(\mathcal{E}^{\mathrm{nf}}_{11})\sin 2\phi_l + \Im(\mathcal{E}^{\mathrm{nf}}_{11})\cos 2\phi_l\},\\
b^{\mathrm{nf}}_{13} &= \Re(\mathcal{I}^{\mathrm{nf}}_{13})\cos\phi_l - \Im(\mathcal{I}^{\mathrm{nf}}_{13})\sin\phi_l,\\
b^{\mathrm{nf}}_{21} &= \Im(\mathcal{E}^{\mathrm{nf}}_{00}) - \{\Re(\mathcal{E}^{\mathrm{nf}}_{11})\sin 2\phi_l + \Im(\mathcal{E}^{\mathrm{nf}}_{11})\cos 2\phi_l\},\\
b^{\mathrm{nf}}_{22} &= \Re(\mathcal{E}^{\mathrm{nf}}_{00}) - \{\Re(\mathcal{E}^{\mathrm{nf}}_{11})\cos 2\phi_l - \Im(\mathcal{E}^{\mathrm{nf}}_{11})\sin 2\phi_l\},\\
b^{\mathrm{nf}}_{23} &= -(\Re(\mathcal{I}^{\mathrm{nf}}_{13})\sin\phi_l + \Im(\mathcal{I}^{\mathrm{nf}}_{13})\cos\phi_l),\\
b^{\mathrm{nf}}_{31} &= -(\Re(\mathcal{I}^{\mathrm{nf}}_{31})\cos\phi_l - \Im(\mathcal{I}^{\mathrm{nf}}_{31})\sin\phi_l),\\
b^{\mathrm{nf}}_{32} &= \Re(\mathcal{I}^{\mathrm{nf}}_{31})\sin\phi_l + \Im(\mathcal{I}^{\mathrm{nf}}_{31})\cos\phi_l.
\end{aligned} \tag{C1}$$

The remaining terms of the flip contributions for the decay matrix $b^{\mathrm{f}}_{\mu\nu}$ are

$$\begin{aligned}
b^{\mathrm{f}}_{11} &= \Re(\mathcal{E}^{\mathrm{f}}_{00}) - \{\Re(\mathcal{E}^{\mathrm{f}}_{11})\cos 2\phi_l - \Im(\mathcal{E}^{\mathrm{f}}_{11})\sin 2\phi_l\},\\
b^{\mathrm{f}}_{12} &= -\Im(\mathcal{E}^{\mathrm{f}}_{00}) + \{\Re(\mathcal{E}^{\mathrm{f}}_{11})\sin 2\phi_l + \Im(\mathcal{E}^{\mathrm{f}}_{11})\cos 2\phi_l\},\\
b^{\mathrm{f}}_{13} &= \Re(\mathcal{I}^{\mathrm{f}}_{13})\cos\phi_l - \Im(\mathcal{I}^{\mathrm{f}}_{13})\sin\phi_l,\\
b^{\mathrm{f}}_{21} &= \Im(\mathcal{E}^{\mathrm{f}}_{00}) + \{\Re(\mathcal{E}^{\mathrm{f}}_{11})\sin 2\phi_l + \Im(\mathcal{E}^{\mathrm{f}}_{11})\cos 2\phi_l\},\\
b^{\mathrm{f}}_{22} &= \Re(\mathcal{E}^{\mathrm{f}}_{00}) + \{\Re(\mathcal{E}^{\mathrm{f}}_{11})\cos 2\phi_l - \Im(\mathcal{E}^{\mathrm{f}}_{11})\sin 2\phi_l\},\\
b^{\mathrm{f}}_{23} &= -(\Re(\mathcal{I}^{\mathrm{f}}_{13})\sin\phi_l + \Im(\mathcal{I}^{\mathrm{f}}_{13})\cos\phi_l),\\
b^{\mathrm{f}}_{31} &= -(\Re(\mathcal{I}^{\mathrm{f}}_{31})\cos\phi_l - \Im(\mathcal{I}^{\mathrm{f}}_{31})\sin\phi_l),\\
b^{\mathrm{f}}_{32} &= \Re(\mathcal{I}^{\mathrm{f}}_{31})\sin\phi_l + \Im(\mathcal{I}^{\mathrm{f}}_{31})\cos\phi_l.
\end{aligned} \tag{C2}$$

APPENDIX D: EXAMPLES OF ALIGNED DECAY MATRICES

1. $B_1 \to B_2\gamma$

The amplitude Eq. (10) for the weak decay $B_1 \to B_2\gamma$ simplifies by replacing $\underline{\lambda}_W \to \lambda_\gamma$ where $\lambda_\gamma = \{-1, 1\}$. For the hadronic tensor only terms $H_{\frac{1}{2}1}$ and $H_{-\frac{1}{2}-1}$ are nonzero. The transition tensor for decay with real photon in helicity representation reads,

$$T^{\kappa\kappa',\lambda_2\lambda_2'} = \frac{1}{4\pi}\sum_{\lambda_\gamma} H_{\lambda_2\lambda_\gamma}H^*_{\lambda_2'\lambda_\gamma}\mathcal{D}^{1/2*}_{\kappa,\lambda_2-\lambda_\gamma}(\Omega_2)\mathcal{D}^{1/2}_{\kappa',\lambda_2'-\lambda_\gamma}(\Omega_2). \tag{D1}$$

The decay matrix $b^\gamma_{\mu\nu}$ is the following:

$$b^\gamma_{\mu\nu} \coloneqq \sum_{\lambda_\gamma}\sum_{\lambda_2,\lambda_2'=-1/2}^{1/2} H_{\lambda_2\lambda_\gamma}H^*_{\lambda_2'\lambda_\gamma}\sigma_\mu^{\lambda_2-\lambda_\gamma,\lambda_2'-\lambda_\gamma}\sigma_\nu^{\lambda_2',\lambda_2} = |H_{-1/2,-1}|^2\sigma_\mu^{1/2,1/2}\sigma_\nu^{-1/2,-1/2} + |H_{1/2,+1}|^2\sigma_\mu^{-1/2,-1/2}\sigma_\nu^{1/2,1/2} \tag{D2}$$

or

$$b^{\gamma}_{\mu\nu} \propto \begin{pmatrix} 1 & 0 & 0 & \alpha_\gamma \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\alpha_\gamma & 0 & 0 & -1 \end{pmatrix}, \tag{D3}$$

where

$$\alpha_\gamma = |H_{1/2,+1}|^2 - |H_{-1/2,-1}|^2; \\ |H_{1/2,+1}|^2 + |H_{-1/2,-1}|^2 = 1. \tag{D4}$$

2. $B_1 \to B_2\pi$

For weak nonleptonic decay $D(B_1 \to B_2\pi)$ we present the results from Ref. [8] as a product of rotation matrix and the aligned decay matrix,

$$T^{\kappa\kappa',\lambda_2\lambda_2'} = \frac{1}{4\pi} H_{\lambda_2,0}H^*_{\lambda_2',0}\mathcal{D}^{1/2*}_{\kappa,\lambda_2}(\Omega_2)\mathcal{D}^{1/2}_{\kappa',\lambda_2'}(\Omega_2). \tag{D5}$$

The decay matrix $b^D_{\mu\nu}$ is rewritten as

$$\begin{aligned} b^D_{\mu\nu} &:= \sum_{\lambda_2,\lambda_2'=-1/2}^{1/2} H_{\lambda_2,0}H^*_{\lambda_2',0}\sigma_\mu^{\lambda_2,\lambda_2'}\sigma_\nu^{\lambda_2',\lambda_2} \\ &= |H_{-1/2,0}|^2\sigma_\mu^{-1/2,-1/2}\sigma_\nu^{-1/2,-1/2} \\ &\quad + |H_{1/2,0}|^2\sigma_\mu^{1/2,1/2}\sigma_\nu^{1/2,1/2} \\ &\quad + H_{1/2,0}H^*_{-1/2,0}\sigma_\mu^{1/2,-1/2}\sigma_\nu^{-1/2,1/2} \\ &\quad + H_{-1/2,0}H^*_{1/2,0}\sigma_\mu^{-1/2,1/2}\sigma_\nu^{1/2,-1/2} \end{aligned} \tag{D6}$$

or

$$b^D_{\mu\nu} \propto \begin{pmatrix} 1 & 0 & 0 & \alpha_D \\ 0 & \gamma_D & -\beta_D & 0 \\ 0 & \beta_D & \gamma_D & 0 \\ \alpha_D & 0 & 0 & 1 \end{pmatrix}, \tag{D7}$$

where

$$\alpha_D = |H_{1/2,0}|^2 - |H_{-1/2,0}|^2; \qquad |H_{1/2,0}|^2 + |H_{-1/2,0}|^2 = 1, \tag{D8}$$

$$\beta_D = 2\Im(H_{1/2,0}H^*_{-1/2,0}); \qquad \gamma_D = 2\Re(H_{1/2,0}H^*_{-1/2,0}). \tag{D9}$$

3. $B_1 \to B_2\gamma^* \to B_2 l^+ l^-$

The decay matrices for the $B_1 \to B_2\gamma^* \to B_2 l^+ l^-$ electromagnetic decay can be obtained by simplifying the hadronic tensor by setting to zero all form factors except for $H^V_{\frac{1}{2}1} = H^V_{-\frac{1}{2}-1}$ and $H^V_{\frac{1}{2}0} = H^V_{-\frac{1}{2}0}$ that are nonzero in this parity-conserving process. The decay $\gamma^* \to l^- l^+$ is described in the $\mathbb{R}_\gamma$ frame where the emission angles of the l^- lepton are θ_l and ϕ_l. The value of the lepton momentum in this frame is

$$|\mathbf{p}_l| = \frac{\sqrt{q^2 - 4m_l^2}}{2}. \tag{D10}$$

The leptonic tensor for the γ^* decay $\lambda_\gamma = \{-1, 0, 1\}$ with the lepton helicities summed over is

$$L_{\lambda_\gamma,\lambda_\gamma'}(q^2,\Omega_l) := \sum_{\lambda_+=-1/2}^{1/2}\sum_{\lambda_-=-1/2}^{1/2} \langle\Omega_-,\lambda_-,\lambda_+|S_l|,q^2,\lambda_\gamma'\rangle^*\langle\Omega_-,\lambda_-,\lambda_+|S_l|q^2,\lambda_\gamma\rangle \tag{D11}$$

$$= \sum_{\lambda_+=-1/2}^{1/2}\sum_{\lambda_-=-1/2}^{1/2} |h^l_{\lambda_+\lambda_-}(q^2)|^2\mathcal{D}^{1*}_{\lambda_\gamma,\lambda_--\lambda_+}(\Omega_l)\mathcal{D}^1_{\lambda_\gamma',\lambda_--\lambda_+}(\Omega_l) \tag{D12}$$

$$= e^{i(\lambda_\gamma-\lambda_\gamma')\phi_l}\sum_{\lambda_+=-1/2}^{1/2}\sum_{\lambda_-=-1/2}^{1/2} |h^l_{\lambda_+\lambda_-}(q^2)|^2 d^1_{\lambda_\gamma,\lambda_--\lambda_+}(\theta_l)d^1_{\lambda_\gamma',\lambda_--\lambda_+}(\theta_l). \tag{D13}$$

The moduli squared of $h^l_{\lambda_-\lambda_+}$ corresponding to the vertex $\bar{u}(p_z,\lambda_-)\gamma^\mu v(-p_z,\lambda_+)\epsilon_\mu$ calculated using the charged-lepton spinor representation from Appendix in Ref. [37] are

$$\text{nonflip}(\lambda_\gamma = \mp 1)\colon\ |h^l_{\lambda_-=\mp\frac{1}{2},\lambda_+=\pm\frac{1}{2}}|^2 = 2q^2, \tag{D14}$$

$$\text{flip}(\lambda_\gamma = 0)\colon\ |h^l_{\lambda_-=\pm\frac{1}{2},\lambda_+=\pm\frac{1}{2}}|^2 = 4m_l^2. \tag{D15}$$

The resulting leptonic tensor reads

$$L_{\lambda_\gamma,\lambda'_\gamma}(q^2,\Omega_l) = (q^2-4m_l^2)\begin{pmatrix} \cos^2\theta_l & -\sqrt{2}e^{-i\phi_l}\sin\theta_l\cos\theta_l & e^{-2i\phi_l}\sin^2\theta_l \\ -\sqrt{2}e^{i\phi_l}\sin\theta_l\cos\theta_l & -\cos 2\theta_l & \sqrt{2}e^{-i\phi_l}\sin\theta_l\cos\theta_l \\ e^{2i\phi_l}\sin^2\theta_l & \sqrt{2}e^{i\phi_l}\sin\theta_l\cos\theta_l & \cos^2\theta_l \end{pmatrix} + (q^2+4m_l^2)\mathrm{diag}(1,1,1). \tag{D16}$$

The differential decay rate of the unpolarized baryon B_1 in the electromagnetic conversion process where the spins of all final particles are summed is

$$\mathrm{d}\Gamma \propto \frac{\alpha_{\mathrm{em}}^2}{q^2}V_{Ph}(q^2)\left(1-\frac{4m_l^2}{q^2}\right)b_{00}^{\mathrm{em}}\mathrm{d}q\mathrm{d}\Omega_2\mathrm{d}\Omega_l, \tag{D17}$$

where $V_{Ph}(q^2)$ is the three-body phase space density factor given by the product of the momenta $|\mathbf{p}_2|$ and $|\mathbf{p}_l|$ of the baryon B_2 and the lepton, given in Eqs. (8) and (D10), respectively. The unrotated decay matrix can be obtained adapting (40),

$$b_{\mu\nu}^{\mathrm{em}} := \frac{1}{2(q^2-4m_l^2)}\sum_{\lambda_\gamma,\lambda'_\gamma=-1,0}^{1}\sum_{\lambda_2,\lambda'_2=-1/2}^{1/2} H_{\lambda_2\lambda_\gamma}H^*_{\lambda'_2\lambda'_\gamma}\mathcal{T}_{\mu\nu}^{\lambda_\gamma,\lambda'_\gamma,\lambda_2,\lambda'_2}. \tag{D18}$$

Its elements are

$$b_{\mu\nu}^{\mathrm{em}} = \begin{pmatrix} b_{00}^{\mathrm{em}} & b_{01}^{\mathrm{em}} & b_{02}^{\mathrm{em}} & 0 \\ b_{01}^{\mathrm{em}} & b_{11}^{\mathrm{em}} & b_{12}^{\mathrm{em}} & b_{13}^{\mathrm{em}} \\ b_{02}^{\mathrm{em}} & b_{12}^{\mathrm{em}} & b_{22}^{\mathrm{em}} & b_{23}^{\mathrm{em}} \\ 0 & -b_{13}^{\mathrm{em}} & -b_{23}^{\mathrm{em}} & b_{33}^{\mathrm{em}} \end{pmatrix}, \tag{D19}$$

where

$$\begin{aligned}
b_{00}^{\mathrm{em}} &= \left[\cos^2\theta_l + \frac{q^2+4m_l^2}{q^2-4m_l^2}\right]|H^V_{\frac{1}{2}1}|^2 + 2\left[\sin^2\theta_l + \frac{4m_l^2}{q^2-4m_l^2}\right]|H^V_{\frac{1}{2}0}|^2,\\
b_{33}^{\mathrm{em}} &= -\left[\cos^2\theta_l + \frac{q^2+4m_l^2}{q^2-4m_l^2}\right]|H^V_{\frac{1}{2}1}|^2 + 2\left[\sin^2\theta_l + \frac{4m_l^2}{q^2-4m_l^2}\right]|H^V_{\frac{1}{2}0}|^2,\\
b_{01}^{\mathrm{em}} &= -\sqrt{2}\sin 2\theta_l\sin\phi_l\Im(H^V_{\frac{1}{2}1}H^{V*}_{\frac{1}{2}0}),\\
b_{02}^{\mathrm{em}} &= -\sqrt{2}\sin 2\theta_l\cos\phi_l\Im(H^V_{\frac{1}{2}1}H^{V*}_{\frac{1}{2}0}),\\
b_{11}^{\mathrm{em}} &= \cos 2\phi_l\sin^2\theta_l|H^V_{\frac{1}{2}1}|^2 + 2\left[\sin^2\theta_l + \frac{4m_l^2}{q^2-4m_l^2}\right]|H^V_{\frac{1}{2}0}|^2,\\
b_{22}^{\mathrm{em}} &= -\cos 2\phi_l\sin^2\theta_l|H^V_{\frac{1}{2}1}|^2 + 2\left[\sin^2\theta_l + \frac{4m_l^2}{q^2-4m_l^2}\right]|H^V_{\frac{1}{2}0}|^2,\\
b_{12}^{\mathrm{em}} &= -\sin 2\phi_l\sin^2\theta_l|H^V_{\frac{1}{2}1}|^2,\\
b_{23}^{\mathrm{em}} &= -\sqrt{2}\sin 2\theta_l\sin\phi_l\Re(H^V_{\frac{1}{2}1}H^{V*}_{\frac{1}{2}0}),\\
b_{13}^{\mathrm{em}} &= \sqrt{2}\sin 2\theta_l\cos\phi_l\Re(H^V_{\frac{1}{2}1}H^{V*}_{\frac{1}{2}0}).
\end{aligned} \tag{D20}$$

Decay plane-aligned parameters reduce to the following form:

$$b_{\mu\nu}^{\mathrm{em}} = \begin{pmatrix} b_{00}^{\mathrm{em}} & 0 & b_{02}^{\mathrm{em}} & 0 \\ 0 & b_{11}^{\mathrm{em}} & 0 & b_{13}^{\mathrm{em}} \\ b_{02}^{\mathrm{em}} & 0 & b_{22}^{\mathrm{em}} & 0 \\ 0 & -b_{13}^{\mathrm{em}} & 0 & b_{33}^{\mathrm{em}} \end{pmatrix}, \tag{D21}$$

where in the real form factors limit additionally the term b_{02}^{em} vanishes. Thus, no polarization is induced, but the initial polarization and spin correlations of the baryon B_1 are transferred to the daughter baryon.

4. $B_1 \to B_2[V^* \to P_1P_2]$

Here we consider a decay of spin-1/2 baryon to a spin-1/2 baryon and a pair of pseudoscalar mesons P_1 and P_2 via an intermediate vector meson V e.g., $B_1 \to B_2\rho^0 \to \pi^+\pi^-$. The decay matrices are obtained as in Appendix D 3 by replacing the dilepton with the pseudoscalars, and the virtual photon with a massive vector meson decaying strongly. Since the initial baryon decays weakly into the intermediate state $B_1 \to B_2 V^*$, all vector and axial vector form factors should be used. The decay $V^*(q) \to P_1(m_1, \mathbf{p}_\pi) P_2(m_2, -\mathbf{p}_\pi)$ is described in the $\mathbb{R}_V$ frame where the emission angles of the P_1 pseudoscalar are θ_π and ϕ_π. The value of the momentum $\mathbf{p}_\pi$ is

$$|\mathbf{p}_\pi| = \sqrt{\frac{q^4 + m_2^4 + m_1^4 - 2q^2 m_1^2 - 2q^2 m_2^2 - 2m_1^2 m_2^2}{4q^2}}. \tag{D22}$$

The tensor for the $V^* \to P_1 P_2$ decay for the helicities $\lambda_V, \lambda'_V = \{-1, 0, 1\}$ is

$$\mathfrak{H}_{\lambda_V \lambda'_V}(\Omega_\pi) \coloneqq e^{i(\lambda_V - \lambda'_V)\phi_\pi} |h^V|^2 d^1_{\lambda_V,0}(\theta_\pi) d^1_{\lambda'_V,0}(\theta_\pi), \tag{D23}$$

where the h^V is a constant and it can be absorbed as a normalization factor. The resulting tensor reads

$$\mathfrak{H}_{\lambda_V \lambda'_V}(\Omega_\pi) = \begin{pmatrix} \frac{\sin^2\theta_\pi}{2} & \frac{e^{-i\phi_\pi}\sin\theta_\pi\cos\theta_\pi}{\sqrt{2}} & -\frac{e^{-2i\phi_\pi}}{2}\sin^2\theta_\pi \\ \frac{e^{i\phi_\pi}\sin\theta_\pi\cos\theta_\pi}{\sqrt{2}} & \cos^2\theta_\pi & -\frac{e^{-i\phi_\pi}\sin\theta_\pi\cos\theta_\pi}{\sqrt{2}} \\ -\frac{e^{2i\phi_\pi}}{2}\sin^2\theta_\pi & -\frac{e^{i\phi_\pi}\sin\theta_\pi\cos\theta_\pi}{\sqrt{2}} & \frac{\sin^2\theta_\pi}{2} \end{pmatrix}. \tag{D24}$$

The unrotated decay matrix can be obtained by replacing the leptonic tensor with the tensor $\mathfrak{H}_{\lambda_V,\lambda'_V}$ in (40),

$$b^V_{\mu\nu} \coloneqq \sum_{\lambda_V,\lambda'_V=-1}^{1} H_{\lambda_2\lambda_V} H^*_{\lambda'_2\lambda'_V} \mathcal{T}^{\lambda_V,\lambda'_V,\lambda_2,\lambda'_2}_{\mu\nu}. \tag{D25}$$

Its elements are

$$\begin{aligned}
b^V_{00} &= \left(|H_{\frac{1}{2}0}|^2 + |H_{-\frac{1}{2}0}|^2\right)\cos^2\theta_\pi + \frac{1}{2}\left(|H_{\frac{1}{2}1}|^2 + |H_{-\frac{1}{2}-1}|^2\right)\sin^2\theta_\pi, \\
b^V_{01} &= \Re(\mathcal{A})\cos\phi_\pi + \Im(\mathcal{A})\sin\phi_\pi, \\
b^V_{02} &= \Im(\mathcal{A})\cos\phi_\pi - \Re(\mathcal{A})\sin\phi_\pi, \\
b^V_{03} &= \left(|H_{\frac{1}{2}0}|^2 - |H_{-\frac{1}{2}0}|^2\right)\cos^2\theta_\pi + \frac{1}{2}\left(|H_{\frac{1}{2}1}|^2 - |H_{-\frac{1}{2}-1}|^2\right)\sin^2\theta_\pi, \\
b^V_{10} &= \Re(\mathcal{B})\cos\phi_\pi + \Im(\mathcal{B})\sin\phi_\pi, \\
b^V_{20} &= \Im(\mathcal{B})\cos\phi_\pi - \Re(\mathcal{B})\sin\phi_\pi, \\
b^V_{11} &= \Re(\mathcal{C}) - \Re(\mathcal{D})\cos 2\phi_\pi - \Im(\mathcal{D})\sin 2\phi_\pi, \\
b^V_{12} &= \Im(\mathcal{C}) - \Im(\mathcal{D})\cos 2\phi_\pi + \Re(\mathcal{D})\sin 2\phi_\pi, \\
b^V_{21} &= -\Im(\mathcal{C}) - \Im(\mathcal{D})\cos 2\phi_\pi + \Re(\mathcal{D})\sin 2\phi_\pi, \\
b^V_{22} &= \Re(\mathcal{C}) + \Re(\mathcal{D})\cos 2\phi_\pi + \Im(\mathcal{D})\sin 2\phi_\pi, \\
b^V_{13} &= -\Re(\mathcal{E})\cos\phi_\pi - \Im(\mathcal{E})\sin\phi_\pi, \\
b^V_{23} &= -\Im(\mathcal{E})\cos\phi_\pi + \Re(\mathcal{E})\sin\phi_\pi, \\
b^V_{30} &= \left(|H_{\frac{1}{2}0}|^2 - |H_{-\frac{1}{2}0}|^2\right)\cos^2\theta_\pi - \frac{1}{2}\left(|H_{\frac{1}{2}1}|^2 - |H_{-\frac{1}{2}-1}|^2\right)\sin^2\theta_\pi, \\
b^V_{31} &= \Re(\mathcal{F})\cos\phi_\pi + \Im(\mathcal{F})\sin\phi_\pi, \\
b^V_{32} &= \Im(\mathcal{F})\cos\phi_\pi - \Re(\mathcal{F})\sin\phi_\pi, \\
b^V_{33} &= \left(|H_{\frac{1}{2}0}|^2 + |H_{-\frac{1}{2}0}|^2\right)\cos^2\theta_\pi - \frac{1}{2}\left(|H_{\frac{1}{2}1}|^2 + |H_{-\frac{1}{2}-1}|^2\right)\sin^2\theta_\pi,
\end{aligned} \tag{D26}$$

with

$$\mathcal{A} = \sqrt{2}\cos\theta_\pi \sin\theta_\pi \left(H^*_{\frac{1}{2}0} H_{-\frac{1}{2}-1} - H^*_{\frac{1}{2}1} H_{-\frac{1}{2}0}\right), \tag{D27}$$

$$\mathcal{B} = \sqrt{2}\cos\theta_\pi \sin\theta_\pi \Big(H^*_{-\frac{1}{2}0} H_{-\frac{1}{2}-1} - H^*_{\frac{1}{2}1} H_{\frac{1}{2}0}\Big), \quad \text{(D28)}$$

$$\mathcal{C} = 2H^*_{\frac{1}{2}0} H_{-\frac{1}{2}0} \cos^2\theta_\pi, \quad \text{(D29)}$$

$$\mathcal{D} = H^*_{\frac{1}{2}1} H_{-\frac{1}{2}-1} \sin^2\theta_\pi, \quad \text{(D30)}$$

$$\mathcal{E} = \sqrt{2}\cos\theta_\pi \sin\theta_\pi \Big(H^*_{-\frac{1}{2}0} H_{-\frac{1}{2}-1} + H^*_{\frac{1}{2}1} H_{\frac{1}{2}0}\Big), \quad \text{(D31)}$$

$$\mathcal{F} = \sqrt{2}\cos\theta_\pi \sin\theta_\pi \Big(H^*_{\frac{1}{2}0} H_{-\frac{1}{2}-1} + H^*_{\frac{1}{2}1} H_{-\frac{1}{2}0}\Big). \quad \text{(D32)}$$

Decay plane aligned parameters reduce to the following form

$$b^V_{\mu\nu} = \begin{pmatrix} b^V_{00} & \Re(\mathcal{A}) & \Im(\mathcal{A}) & b^V_{03} \\ \Re(\mathcal{B}) & \Re(\mathcal{C}-\mathcal{D}) & \Im(\mathcal{C}-\mathcal{D}) & -\Re(\mathcal{E}) \\ \Im(\mathcal{B}) & -\Im(\mathcal{C}+D) & \Re(\mathcal{C}+\mathcal{D}) & -\Im(\mathcal{E}) \\ b^V_{30} & \Re(\mathcal{F}) & \Im(\mathcal{F}) & b^V_{33} \end{pmatrix}. \quad \text{(D33)}$$

The differential decay rate of the process with unpolarized baryon B_1 and the spins of B_2 summed over is

$$\mathrm{d}\Gamma \propto V_{Ph}(q^2) b^V_{00} \mathrm{d}q \mathrm{d}\Omega_2 \mathrm{d}\Omega_\pi, \quad \text{(D34)}$$

where $V_{Ph}(q^2)$ is the three-body phase space density factor given by the product of the momenta $|\mathbf{p}_2|$ and $|\mathbf{p}_\pi|$.

[1] W. Detmold, C. Lehner, and S. Meinel, Phys. Rev. D **92**, 034503 (2015).
[2] N. Cabibbo, E. C. Swallow, and R. Winston, Annu. Rev. Nucl. Part. Sci. **53**, 39 (2003).
[3] E. Goudzovski *et al.*, Rep. Prog. Phys. **86**, 016201 (2023).
[4] J. G. Korner and G. A. Schuler, Z. Phys. C **46**, 93 (1990).
[5] J. G. Korner and M. Kramer, Phys. Lett. B **275**, 495 (1992).
[6] A. Kadeer, J. G. Korner, and U. Moosbrugger, Eur. Phys. J. C **59**, 27 (2009).
[7] G. Fäldt and A. Kupsc, Phys. Lett. B **772**, 16 (2017).
[8] E. Perotti, G. Faldt, A. Kupsc, S. Leupold, and J. J. Song, Phys. Rev. D **99**, 056008 (2019).
[9] P. Adlarson and A. Kupsc, Phys. Rev. D **100**, 114005 (2019).
[10] N. Salone, P. Adlarson, V. Batozskaya, A. Kupsc, S. Leupold, and J. Tandean, Phys. Rev. D **105**, 116022 (2022).
[11] M. Ablikim *et al.* (BESIII Collaboration), Nat. Phys. **15**, 631 (2019).
[12] M. Ablikim *et al.* (BESIII Collaboration), Nature (London) **606**, 64 (2022).
[13] R. L. Workman *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).
[14] J. Adler *et al.* (MARK-III Collaboration), Phys. Rev. Lett. **62**, 1821 (1989).
[15] M. Ablikim *et al.* (BESIII Collaboration), Chin. Phys. C **46**, 074001 (2022).
[16] Y. S. Amhis *et al.* (HFLAV Collaboration), Eur. Phys. J. C **81**, 226 (2021).
[17] D. Becirevic and A. B. Kaidalov, Phys. Lett. B **478**, 417 (2000).
[18] M. Jacob and G. C. Wick, Ann. Phys. (N.Y.) **7**, 404 (1959); **281**, 774 (2000).
[19] S. Weinberg, Phys. Rev. **112**, 1375 (1958).
[20] P. H. Frampton and W.-K. Tung, Phys. Rev. D **3**, 1114 (1971).
[21] R. E. Marshak and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (New York Wiley Interscience, New York, 1969).
[22] M. Bourquin *et al.* (Bristol-Geneva-Heidelberg-Orsay-Rutherford-Strasbourg Collaboration), Z. Phys. C **21**, 1 (1983).
[23] M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 354 (1958).
[24] J. M. Gaillard and G. Sauvage, Annu. Rev. Nucl. Part. Sci. **34**, 351 (1984).
[25] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **95**, 052003 (2017).
[26] M. Ablikim *et al.* (BES Collaboration), Phys. Rev. D **78**, 092005 (2008).
[27] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **125**, 052004 (2020).
[28] M. Ablikim *et al.* (BESIII Collaboration), Phys. Lett. B **770**, 217 (2017).
[29] M. Ablikim *et al.* (BESIII Collaboration), arXiv:2305.09218.
[30] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **129**, 131801 (2022).
[31] S. Groote, J. G. Körner, and B. Melić, Eur. Phys. J. C **79**, 948 (2019).
[32] A. Sirlin and A. Ferroglia, Rev. Mod. Phys. **85**, 263 (2013).
[33] A. Garcia and P. Kielanowski, Lect. Notes Phys. **222**, 1 (1985).
[34] E. Barberio and Z. Was, Comput. Phys. Commun. **79**, 291 (1994).
[35] T. D. Lee and C.-N. Yang, Phys. Rev. **108**, 1645 (1957).
[36] LHCb Collaboration, arXiv:2301.07010.
[37] C. F. Perdrisat, V. Punjabi, and M. Vanderhaeghen, Prog. Part. Nucl. Phys. **59**, 694 (2007).

Correction: Typographical errors in Eqs. (51), (52), and in (C1) in Appendix C have been fixed.