Automatic Nelson-Barr solutions to the strong *CP* puzzle

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We discuss a simple model, based on the gauge group $\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \otimes \mathrm{U}(1)_R$, where the Nelson-Barr solution to the strong *CP* problem is implemented. This model automatically provides a high quality solution to the strong *CP* puzzle. Weak *CP* violation in the lepton sector arises in the same fashion as in the quark sector. We derive explicit expressions for the flavor changing couplings of the electroweak and Higgs bosons. These expressions are more general than the particular model considered. Constraints from finite naturalness are briefly discussed. We also briefly discuss related models based on the gauge group B-L.

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I. INTRODUCTION

In the standard model (SM) the *CP* symmetry is explicitly broken by the QCD vacuum angle and by the interactions of the left-handed quarks with the *W*-gauge bosons. The QCD vacuum angle, θ_{QCD}, appears in the *CP*-violating term

$$\mathcal{L}_{\mathrm{QCD}} \supset \frac{\alpha_s \theta_{\mathrm{QCD}}}{16\pi} G^a_{\mu\nu} \epsilon^{\mu\nu\rho\sigma} G^a_{\rho\sigma}, \tag{1}$$

where $a = 1\ldots8$, α_s is the strong coupling, and $G^a_{\mu\nu}$ is the field strength tensor for gluons. The basis invariant parameter that enters in the neutron electric dipole moment is given by

$$\bar{\theta}_{\mathrm{QCD}} = \theta_{\mathrm{QCD}} + \arg\{\mathrm{Det}(\mathcal{M}_u \mathcal{M}_d)\}, \tag{2}$$

where the last term is the contribution from *CP*-violating phases in the quark mass matrices. The vacuum angle $\bar{\theta}_{\mathrm{QCD}}$ must be very small, $\bar{\theta}_{\mathrm{QCD}} < 10^{-10}$ [1], to satisfy the experimental bounds on the neutron electric dipole moment. The second term in Eq. (2) is related to the large *CP*-violating phase in the Cabibbo-Kobayaski-Maskawa (CKM) matrix, and an awkward cancellation between it and θ_{QCD} is required for $\bar{\theta}_{\mathrm{QCD}}$ to be very small. This is the strong *CP* problem.

There are two well-studied solutions to the strong *CP*-problem: the Peccei-Quinn (PQ) mechanism [2,3] and the Nelson-Barr (NB) mechanism [4–6]. In the case of the PQ mechanism one postulates the existence of an anomalous global $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry that is spontaneous broken and gives rise to the existence of the axion [7–12], a possible dark matter candidate [13–15]. When the Peccei-Quinn symmetry is an automatic consequence of a spontaneously broken gauge symmetry, it is difficult to get a small enough value of $\bar{\theta}_{\mathrm{QCD}}$ since operators of very high dimension must also preserve the symmetry [16]. The degree to which higher dimension operators preserve the smallness of $\bar{\theta}_{\mathrm{QCD}}$ is called the quality of the solution.

In the case of the Nelson-Barr mechanism *CP* is spontaneously broken and new quarks are added in such way that the $\arg\{\mathrm{Det}(\mathcal{M}_u \mathcal{M}_d)\}$ vanishes at tree level.[1] Nonetheless, a large *CP*-violating CKM phase is permitted. See Bento, Branco, and Parada in Ref. [19] for a discussion of the CKM matrix. The quality of the solution is easier to ensure in NB models because forbidding dimension five operators can be sufficient.

Gauge extensions of the SM that implement the NB mechanism have been constructed. For previous studies see Refs. [20–23]. The spontaneous breaking of *CP* gives rise to domain walls, which will dominate the energy density of the universe unless inflation [24–26] (or whatever solves the horizon problem) occurs after the spontaneous breaking of *CP* [27,28]. As the authors of Ref. [23] pointed out, a new gauge symmetry can help increase the quality of the

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[1]For a related mechanism that predates Nelson-Barr see Ref. [17]. An implementation of this idea is given in Ref. [18].

NB mechanism by forbidding the lowest higher dimensional CP-violating operators. Assuming that CP invariance is restored at high temperatures, this relaxes the upper bound on the CP breaking scale, which otherwise would be in tension with the simplest models of inflation. Note that there are cases where CP is not restored at high temperatures [29].

In this article, we discuss simple gauge theories based on the gauge group $\mathcal{G}_{\rm SM} \otimes {\rm U}(1)_R$ (see, for example, [30]), where $\mathcal{G}_{\rm SM}$ is the SM gauge group. In these models the NB mechanism is an automatic consequence of the gauge theory and matter content. The gauge symmetry increases the quality of the NB mechanism, forbidding dimension-five operators that can spoil it. This relieves tension between solving the strong CP problem and cosmology. In this model, CP violation in the lepton sector arises in the same fashion as in the quark sector. We also briefly discuss models based on $\mathcal{G}_{\rm SM} \otimes {\rm U}(1)_{B-L}$.

This article is organized as follows: In Sec. II, we discuss the gauge theories for spontaneous CP violation that we study. In Sec. III, we discuss the implementation of the Nelson-Barr mechanism and also the quality of the solution to the strong CP problem. In Sec. IV, we review how to obtain a realistic CKM and Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrices. Flavor-violating interactions are discussed in Sec. V, while the bounds from finite naturalness are discussed in Sec. VI. We summarize our main findings in Sec. VII.

II. THEORETICAL FRAMEWORK

A gauge theory for spontaneous CP violation (SCPV) that implements automatically the Nelson-Barr mechanism can be constructed using the gauge symmetry group:

$$\mathcal{G}_R = {\rm SU}(3)_C \otimes {\rm SU}(2)_L \otimes {\rm U}(1)_Y \otimes {\rm U}(1)_R. \quad (3)$$

The SM fermion content (plus three right-handed neutrinos) with the right-handed fields carrying a charge r,

$$\begin{aligned} &q_L \sim (3,2,1/6,0), && \ell_L \sim (1,2,-1/2,0),\\ &u_R \sim (3,1,2/3,r), && e_R \sim (1,1,-1,-r),\\ &d_R \sim (3,1,-1/3,-r), && \nu_R \sim (1,1,0,r), \end{aligned} \quad (4)$$

is anomaly free under $\mathcal{G}_R$. The SM Higgs, $H \sim (1,2,1/2,r)$, is also charged under ${\rm U}(1)_R$ to give mass to the fermions through the usual Yukawa interactions,

$$\begin{aligned} -\mathcal{L} \supset\ & Y_u \bar{q}_L \tilde{H} u_R + Y_d \bar{q}_L H d_R \\ & + Y_\nu \bar{\ell}_L \tilde{H} \nu_R + Y_e \bar{\ell}_L H e_R + {\rm H.c.}, \end{aligned} \quad (5)$$

where $\tilde{H} = i\sigma_2 H^*$. To implement the NB mechanism, let us consider extra vectorlike fermions that under $\mathcal{G}_R$ transform as[2]

$$\begin{aligned} &U_R \sim (3,1,2/3,-R), && U_L \sim (3,1,2/3,L),\\ &D_R \sim (3,1,-1/3,R), && D_L \sim (3,1,-1/3,-L),\\ &E_R \sim (1,1,-1,R), && E_L \sim (1,1,-1,-L),\\ &N_R \sim (1,1,0,-R), && N_L \sim (1,1,0,L). \end{aligned} \quad (6)$$

This set of vectorlike under the SM[3] fermions is anomaly-free for any value of the charges L and R. The new fermions get mass through the spontaneous breaking of ${\rm U}(1)_R$ by the vacuum expectation value (VEV) of the scalar $S \sim (1,1,0,L+R)$, via the following interactions:

$$\begin{aligned} -\mathcal{L} \supset\ & (\lambda_U \bar{U}_L U_R + \lambda_N \bar{N}_L N_R) S \\ & + (\lambda_D \bar{D}_L D_R + \lambda_E \bar{E}_L E_R) S^* + {\rm H.c.}, \end{aligned} \quad (7)$$

where we have assumed $L \neq -R$. For generic ${\rm U}(1)_{\rm R}$ charges, all the fermions (including the neutrinos) have Dirac masses, and baryon and lepton numbers are conserved at the renormalizable level.

At this point the vectorlike fermions are stable. To implement the NB mechanism [4,5] and allow the vectorlike fermions to decay, new scalar fields are needed. These scalars trigger SCPV through a complex VEV. There are two complex scalar fields $X_a \sim (1,1,0,L-r)$ with $a = 1,\ 2$. The ${\rm U}(1)_R$ charge of X_a fields allows the following interaction terms in the Lagrangian:

$$\begin{aligned} -\mathcal{L} \supset\ & \bar{U}_L u_R^i \left(\sum_{a=1}^{2} \lambda_{u,a}^i X_a \right) + \bar{D}_L d_R^i \left(\sum_{a=1}^{2} \lambda_{d,a}^i X_a^* \right) \\ & + \bar{N}_L \nu_R^i \left(\sum_{a=1}^{2} \lambda_{\nu,a}^i X_a \right) + \bar{E}_L e_R^i \left(\sum_{a=1}^{2} \lambda_{e,a}^i X_a^* \right) \\ & + {\rm H.c.} \end{aligned} \quad (8)$$

Note that $r \neq R$; otherwise, the Higgs doublet can couple to the right-handed new fermions, spoiling the Nelson-Barr mechanism.

Because the Higgs boson is charged under ${\rm U}(1)_{\rm R}$ in this model, there is a tree-level mixing between the electroweak neutral gauge boson, Z, and the Abelian generator of ${\rm U}(1)_{\rm R}$, Z_R, in the broken phase:

$$\mathcal{L} \supset \frac{1}{2} \begin{pmatrix} Z^\mu & Z_R^\mu \end{pmatrix} \begin{pmatrix} M_Z^2 & -\frac{2 g_R r M_Z^2}{\sqrt{g_1^2 + g_2^2}} \\ -\frac{2 g_R r M_Z^2}{\sqrt{g_1^2 + g_2^2}} & M_{Z_R}^2 \end{pmatrix} \begin{pmatrix} Z_\mu \\ Z_{R\mu} \end{pmatrix}. \quad (9)$$

[2] We call this new Abelian gauge symmetry ${\rm U}(1)_R$ because that is what it is acting on the SM fermions. For the new vectorlike fermions the left-handed fields also have the ${\rm U}(1)_R$ charge.

[3] Henceforth we will just use *vectorlike* to refer to these fermions.

In the following we neglect the mixing angle between Z and Z_R, given by $2g_R r M_Z^2/(\sqrt{g_1^2+g_2^2} M_{Z_R}^2)$, since $M_{Z_R} > M_Z$ and $g_R \ll 1$ to ensure an acceptably small correction to the Z mass and small enough flavor changing neutral currents.

Although this paper focuses on $\mathrm{U}(1)_R$, it is worth noting that one could change the new gauge group from $\mathrm{U}(1)_R$ to $\mathrm{U}(1)_X$, as long as the SM fermions are anomaly-free under the new gauge symmetry, with L and R now interpreted as X charges. For example, X could be $B-L$. For the $\mathrm{U}(1)_{B-L}$ case, the X_i's cannot simultaneously couple to quarks and leptons because they have different $B-L$ charges. This will lead to charged stable relics[4] except for the case when $L = 1/3$. In this case, $X_i \sim (1, 1, 0, 2/3)$ couple to both the charged leptons and the down-type quarks, while the vectorlike up-type quarks are connected to the SM through the mass term $\bar{U}_L u_R$. The lepton number is violated by the interaction $N_L^T C N_L X_i^*$. The vectorlike neutrino, however, is stable and therefore could be a candidate for cold dark matter [31].

III. SCPV AND THE STRONG *CP* PROBLEM

Working in the basis where the VEV of S is made real via a $\mathrm{U}(1)_R$ transformation, one can generate a vacuum expectation value with a nonzero phase for at least one of the X_i fields using the scalar potential,

$$V \supset -\mu_X^2 (X_1^\dagger X_2) + \lambda_X (X_1^\dagger X_2)^2 + \mathrm{H.c.} \qquad (10)$$

Defining the VEVs as $\langle X_a \rangle = e^{i\theta_a} v_{X_a}/\sqrt{2}$, one finds

$$V \supset -\mu_X^2 v_{X_1} v_{X_2} \cos\theta_X + \lambda_X \frac{v_{X_1}^2 v_{X_2}^2}{2} \cos 2\theta_X, \qquad (11)$$

with $\theta_X = \theta_2 - \theta_1$. Here, μ_X is an "effective mass" containing all possible contributions to the term $X_1^\dagger X_2$ (e.g., $X_1^\dagger X_1$). The minimization condition, $\partial V/\partial \theta_X = 0$, gives us

$$\cos\theta_X = \frac{\mu_X^2}{2\lambda_X v_{X_1} v_{X_2}}. \qquad (12)$$

Even in the limit of large VEVs, $\cos\theta_X$ can still be order one as long as $\mu_X \sim v_X$. Only the combination of phases θ_X is determined by the scalar potential. To determine the remaining free phase, particular values of the charges are required that allow for additional terms.[5] For example, $2L = 3r + R$ allows $S^* X_i X_j X_k$ terms. Note that adding these terms will modify Eq. (12). These terms explicitly break the $\mathrm{U}(1)_{X_1 - X_2}$ global symmetry that otherwise would lead to a Nambu-Goldstone boson.

In the broken phase, the fermion masses are given by

$$-\mathcal{L} \supset \bar{f}_L'^A \mathcal{M}_f^{AB} f_R'^B + \mathrm{H.c.}, \qquad (13)$$

with the mass matrices at tree level given by

$$\mathcal{M}_f^{AB} = \frac{1}{\sqrt{2}} \begin{pmatrix} Y_f^{ij} v_H & 0^{i4} \\ \sum_{a=1}^2 \lambda_{f,a}^j v_{X_a} e^{\pm i\theta_a} & \lambda_F v_S \end{pmatrix}, \qquad (14)$$

where $\langle S \rangle = v_S/\sqrt{2}$. In our convention, capital letters $A, B, \ldots = 1, 2, 3, 4$, roman letters $i, j, \ldots = 1, 2, 3$ run over the indices of the light quarks, and 4 will refer to the new vectorlike fermions. In the above matrix when A goes over 1,2,3, it is represented by i, and when B goes over 1,2,3, it is represented by j. Above, the primes are used for the weak eigenstates. In Eq. (14), $\pm$ takes $+$ $(-)$ for U and N (E and D) fermions.

We have used the $\mathrm{U}(1)_R$ gauge symmetry to set the phase of $\langle S \rangle$ to be zero. However, if we had not done that, this phase cancels out in $\arg\{\mathrm{Det}(\mathcal{M}_\mathrm{u} \mathcal{M}_\mathrm{d})\}$ because S gives mass to U's while S^* gives mass to D's.

Both a CKM phase in the quark sector and a PMNS phase in the lepton sector are generated from the same phase θ_X.

In this model $\theta_{\mathrm{QCD}} = 0$ by CP invariance and the argument of the determinant of the mass matrices can be seen to be zero by expanding these determinants in minors about the last column. Hence in this model the renormalizable couplings give $\bar{\theta}_{\mathrm{QCD}} = 0$ at tree level.

A. Nonrenormalizable operators and loops

Nonrenormalizable operators and radiative corrections can give rise to CP-violating corrections to the mass matrices (see Refs. [19,23]), $\delta\mathcal{M}_q$, giving rise to the following correction to $\bar{\theta}_{\mathrm{QCD}}$:

$$\Delta\bar{\theta}_{\mathrm{QCD}} \simeq \mathrm{Im}\{\mathrm{Tr}\{\mathcal{M}_q^{-1} \delta\mathcal{M}_q\}\}. \qquad (15)$$

Assuming an order one CP-violating phase, the one-loop diagrams in Fig. 1 give

$$\Delta\bar{\theta}_{\mathrm{QCD}} \sim \frac{\lambda}{16\pi^2} \left(\frac{\tilde{M}_U}{m_X}\right)^2 \ln\left(\frac{m_X^2}{m_t^2}\right), \qquad (16)$$

where λ is the $H^\dagger H X_i^* X_j$ quartic coupling and $\tilde{M}_U$ is the mass of the up-type new vectorlike quark. One way for this to be consistent with the experimental bounds is to have $m_X \sim \tilde{M}_U$ and λ very small (which does not require fine-tuning). Another way is to have λ of order one and $\tilde{M}_U \ll m_X$. For example, if $m_X \sim 10^{11}$ GeV and the mass

[4]Stable relics are not necessarily a problem if inflation occurs at a scale lower than their mass.

[5]We thank Lisa Randall for pointing this out to us.

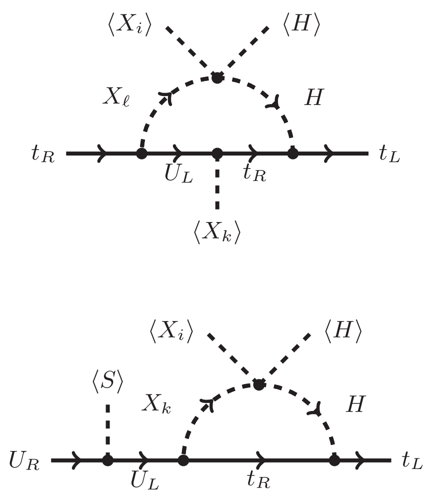

FIG. 1. Some one-loop diagrams contributing to the fermion mass matrices that give a nonzero $\bar{\theta}_{\rm QCD}$.

of the new vectorlike fermions is $M_U \sim 5$ TeV, λ is not constrained by the neutron electric dipole moment.

Contributions to the vectorlike fermion mass terms from higher dimension operators can spoil the NB solution to the strong CP puzzle. For example,

$$\frac{1}{\Lambda_{\rm EFT}^2}\bar{U}_L U_R S X_1 X_2^* + \frac{1}{\Lambda_{\rm EFT}^2}\bar{q}_L \tilde{H} u_R X_1 X_2^* + \cdots. \tag{17}$$

Note that if the vectorlike fermion has a bare mass term, then there would be dimension-five operators that spoil the NB mechanism, which might be problematic given cosmological constraints on completely stable domain walls arising from SCP [28]. Such a mass term is forbidden if $L \neq R$. According to Eq. (A5), the higher dimensional operators explicitly displayed in Eq. (17) shift the $\bar{\theta}_{\rm QCD}$ as follows:

$$\Delta\bar{\theta}_{\rm QCD} \sim \frac{1}{\lambda_F}\frac{v_X^2}{\Lambda_{\rm EFT}^2} + \frac{1}{Y_u}\frac{v_X^2}{\Lambda_{\rm EFT}^2}, \tag{18}$$

where we assumed order one CP-violating phases. This leads to the following condition:

$$v_X \lesssim 10^{14}\ {\rm GeV}\left(\frac{\Lambda_{\rm EFT}}{M_{\rm Pl}}\right)\Big/\sqrt{\frac{1}{\lambda_F}+\frac{1}{Y_u}}. \tag{19}$$

Note that the upper bound above cannot surpass $v_X \lesssim 10^{11}$ GeV since $Y_u \sim m_u/v_H$. This is high enough for inflation to have occurred after the spontaneous breaking of CP, but too low for the gravitational waves from inflation [32–35] to be observable in the B modes of the CMB [36–39].

IV. THE CKM AND PMNS MATRICES

We start by discussing how the CKM and PMNS phases arise in Nelson-Barr models. The 4×4 fermion mass matrix in Eq. (14) is generic to those models when there is only one generation of new vectorlike fermions. Let us redefine the fermions as follows:

$$f'_L = O_{\rm FL} f''_L \quad \text{and} \quad f'_R = O_{\rm FR} f''_R, \tag{20}$$

where $\mathcal{O}_{{\rm FL},R}$ is an orthogonal matrix,

$$O^{AB}_{{\rm FL},R} = \begin{pmatrix} & & & 0\\ & o^{ij}_{{\rm FL},R} & & 0\\ & & & 0\\ 0 & 0 & 0 & 1\end{pmatrix}, \tag{21}$$

that diagonalizes the light quark masses 3×3 block,[6]

$$(o^T_{\rm FL} Y_f o_{\rm FR})\frac{v_H}{\sqrt{2}} = {\rm diag}(m_1, m_2, m_3). \tag{22}$$

Thus,

$$\begin{aligned} -\mathcal{L} &= \bar{f}''_L O^T_{\rm FL}\mathcal{M}_f O_{\rm FR} f''_R + {\rm H.c.}\\ &= \bar{f}''_L\begin{pmatrix} m_1 & 0 & 0 & 0\\ 0 & m_2 & 0 & 0\\ 0 & 0 & m_3 & 0\\ \mu_1 & \mu_2 & \mu_3 & M_F\end{pmatrix} f''_R + {\rm H.c.}, \end{aligned} \tag{23}$$

where

$$\mu_j = \sum_k \sum_{a=1}^{2} \lambda^k_{f,a} v_{X_a} e^{\pm i\theta_a} o^{kj}_{\rm FR}. \tag{24}$$

Further transformations on the left-handed and right-handed fields are required to fully diagonalize the fermion mass matrix.

Diagonalizing the Hermitian matrix $\mathcal{M}_f\mathcal{M}_f^\dagger$ determines the CKM matrix (and the PMNS matrix). We will denote the matrix that diagonalizes $\mathcal{M}_f\mathcal{M}_f^\dagger$ by $\tilde{V}_{\rm FL}$. Starting from the matrix in Eq. (23), we find that the Hermitian matrix $\mathcal{M}_f\mathcal{M}_f^\dagger$ in the double primed basis is given by

$$O^T_{\rm FL}\mathcal{M}_f\mathcal{M}_f^\dagger O_{\rm FL} = \begin{pmatrix} m_1^2 & 0 & 0 & \mu_1^* m_1\\ 0 & m_2^2 & 0 & \mu_2^* m_2\\ 0 & 0 & m_3^2 & \mu_3^* m_3\\ \mu_1 m_1 & \mu_2 m_2 & \mu_3 m_3 & \tilde{M}_F^2\end{pmatrix}, \tag{25}$$

where $\tilde{M}_F^2 = M_F^2 + \sum_i |\mu_i|^2$. The matrix $V_{\rm FL}$ is given approximately by

[6] We note that m_i are not the mass eigenstates of the light quarks, as the total 4×4 matrix is not diagonalized yet. However, as we will show later, they are expected to be of the same order.

$$V_{\rm FL} = \begin{pmatrix} 1 & 0 & 0 & \frac{m_1\mu_1^*}{\tilde{M}_F^2} \\ 0 & 1 & 0 & \frac{m_2\mu_2^*}{\tilde{M}_F^2} \\ 0 & 0 & 1 & \frac{m_3\mu_3^*}{\tilde{M}_F^2} \\ -\frac{m_1\mu_1}{\tilde{M}_F^2} & -\frac{m_2\mu_2}{\tilde{M}_F^2} & -\frac{m_3\mu_3}{\tilde{M}_F^2} & 1 \end{pmatrix}. \tag{26}$$

The nonzero off-diagonal elements are correct up to $\mathcal{O}(m^2\mu^2/\tilde{M}_F^4)$ and $V_{\rm FL}^\dagger V_{\rm FL} = \mathbb{I} + \mathcal{O}(m^2\mu^2/\tilde{M}_F^4)$. The matrix in Eq. (26) approximately diagonalizes Eq. (25), leading to the following matrix:

$$V_{\rm FL}^\dagger O_{\rm FL}^T \mathcal{M}_f \mathcal{M}_f^\dagger O_{\rm FL} V_{\rm FL} = \begin{pmatrix} m_1^2(1-\frac{|\mu_1|^2}{\tilde{M}_F^2}) & -m_1m_2\frac{\mu_1^*\mu_2}{\tilde{M}_F^2} & -m_1m_3\frac{\mu_1^*\mu_3}{\tilde{M}_F^2} & \mathcal{O}(\frac{m^3\mu}{\tilde{M}_F^2}) \\ -m_2m_1\frac{\mu_2^*\mu_1}{\tilde{M}_F^2} & m_2^2(1-\frac{|\mu_2|^2}{\tilde{M}_F^2}) & -m_2m_3\frac{\mu_2^*\mu_3}{\tilde{M}_F^2} & \mathcal{O}(\frac{m^3\mu}{\tilde{M}_F^2}) \\ -m_3m_1\frac{\mu_3^*\mu_1}{\tilde{M}_F^2} & -m_3m_2\frac{\mu_3^*\mu_2}{\tilde{M}_F^2} & m_3^2(1-\frac{|\mu_3|^2}{\tilde{M}_F^2}) & \mathcal{O}(\frac{m^3\mu}{\tilde{M}_F^2}) \\ \mathcal{O}(\frac{m^3\mu}{\tilde{M}_F^2}) & \mathcal{O}(\frac{m^3\mu}{\tilde{M}_F^2}) & \mathcal{O}(\frac{m^3\mu}{\tilde{M}_F^2}) & \tilde{M}_F^2 + 2\sum_i m_i^2\frac{|\mu_i|^2}{\tilde{M}_F^2} + \mathcal{O}(\frac{m^4}{\tilde{M}_F^4}) \end{pmatrix}. \tag{27}$$

The off-diagonal elements in the fourth row and column are very suppressed. Therefore, the CKM matrix comes mostly from diagonalizing the 3×3 upper-left block. Furthermore, if $\mu/\tilde{M}_F \ll 1$, then the upper-left 3×3 block is approximately diagonal and the CKM phase is small, as already noted in Refs. [40,41].

Since $\mu/\tilde{M}_F$ is not small, the 3×3 upper-left block of the above matrix still needs to be diagonalized. As it is Hermitian, it will be diagonalized by a unitary (complex) matrix,

$$U_{\rm FL}^{AB} = \begin{pmatrix} & & & 0 \\ & {\rm U}_{\rm FL}^{ij} & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{28}$$

The unitary matrix that diagonalizes $\mathcal{M}_f\mathcal{M}_f^\dagger$ is then

$$\tilde{V}_{\rm FL} = O_{\rm FL} V_{\rm FL} U_{\rm FL}, \tag{29}$$

and therefore the CKM matrix is given by the following matrix:

$$V_{\rm CKM}^{ij} = \sum_k (\tilde{V}_{\rm UL}^\dagger)^{ik}(\tilde{V}_{\rm DL})^{kj}$$
$$= \sum_{k,\ell} ({\rm U}_{\rm UL}^\dagger)^{ik}(o_{\rm UL}^T o_{\rm DL})^{k\ell}{\rm U}_{\rm DL}^{\ell j} + \mathcal{O}\left(\frac{m^2\mu^2}{\tilde{M}_U^2\tilde{M}_D^2}\right), \tag{30}$$

which agrees with the previous literature [42].

In these models, CP violation in the lepton sector arises in the same fashion as in the quark sector and a CP-violating phase in the PMNS matrix is expected. However, since it is unconstrained by experiment, in the lepton sector μ could be much smaller than $\tilde{M}_F$.

V. FLAVOR VIOLATION

In this theory we have a new flavor-violating interaction due to mixing between the SM fermions with the new heavy fermions.

A. Z boson couplings

Flavor changing neutral currents only enter through the left-handed fermions and are suppressed by $\mathcal{O}(m^2\mu^2/\tilde{M}_F^4)$,

$$\mathcal{L} \supset -\frac{2eT_3^{f_L}}{\sin 2\theta_W}Z^\mu \times \bar{f}_L^i\gamma_\mu\left(\sum_{\ell,k}({\rm U}_{\rm FL}^\dagger)^{ik}\frac{m_k m_\ell \mu_k^*\mu_\ell}{\tilde{M}_F^4}({\rm U}_{\rm FL})^{\ell j}\right)f_L^j, \tag{31}$$

where $T_3^{f_L} = \pm 1/2$ is the weak isospin of the left-handed fermion f_L. Even though Eq. (31) is of $\mathcal{O}(m^2\mu^2/\tilde{M}_F^4)$, which is of the same order as the error in the elements of the matrix in Eq. (26), the interaction above arises from the product of two entries that are of $\mathcal{O}(m\mu/\tilde{M}_F^2)$. These are much more suppressed that the flavor changing neutral currents in Ref. [43]. Equation (31) agrees with the results of Ref. [44].

The coupling of the heavy vectorlike fermions to the SM fermions and the Z,

$$\mathcal{L} \supset \frac{2eT_3^{f_L}}{\sin 2\theta_W}Z^\mu \bar{f}_L^i\gamma_\mu\left(\sum_k({\rm U}_{\rm FL}^\dagger)^{ik}\frac{m_k\mu_k^*}{\tilde{M}_F^2}\right)f_L^4 + {\rm H.c.} \tag{32}$$

Since in this theory the new fermions are heavy, bounds from flavor-violating processes can be satisfied. For example, let us consider the stringent constraint on the μ to e conversion in nuclei. Given the Feynman rule for the Z boson mediating flavor-violating interactions with electrons and muons from Eq. (31), the limit on the conversion of muons to electrons in gold [45] requires [46,47]

$$\left(\frac{m_\ell}{\tilde{M}_E}\right) < 5\times 10^{-4}\left(\frac{\mathcal{B}(\mu\to e)[\mathrm{Au}]}{7\times 10^{-13}}\right)^{1/4}\left(\frac{\tilde{M}_E/\mu_\ell}{1}\right). \quad (33)$$

For $m_\ell \sim m_\tau$, the above constraint is satisfied if $\tilde{M}_E > 4$ TeV. Projected bounds on this process [48] are about 4 orders of magnitude stronger and will increase the bound on $\tilde{M}_E$ by an order of magnitude. A more comprehensive analysis may be warranted.

B. *W* boson couplings

The couplings of the heavy vectorlike fermions with the W bosons and the SM fermions are given by

$$\mathcal{L}\supset \frac{g_2}{\sqrt{2}}W^+_\mu \sum_{k,p,\ell}\Bigg(\bar{u}^i_L(\mathrm{U}^\dagger_{\mathrm{UL}})^{i\ell}(\mathrm{O}^T_{\mathrm{UL}})^{\ell p}(\mathrm{o}_{\mathrm{D}L})^{pk}\frac{m_{\mathrm{D}k}\mu^*_{\mathrm{D}k}}{\tilde{M}^2_D}\gamma^\mu d^4_L + \bar{u}^4_L\frac{m_{\mathrm{U}k}\mu_{\mathrm{U}k}}{\tilde{M}^2_U}(\mathrm{o}^T_{\mathrm{U}L})^{kp}(\mathrm{o}_{\mathrm{D}L})^{p\ell}(\mathrm{U}_{\mathrm{D}L})^{\ell i}\gamma^\mu d^i_L\Bigg) + \mathrm{H.c.} \quad (34)$$

We specify the involved fermion in the label of m and μ to note which mass matrix they come from.

The SM fermion couplings with the W boson are given by the CKM matrix, explicitly given in Eq. (30).

Similar formulas hold for the leptons.

C. SM-Higgs couplings

The couplings of the light fermions to the SM Higgs are given by the following Yukawa interaction:

$$-\mathcal{L}\supset \left(\frac{h}{v_H}\right)\bar{f}'^i_L\mathcal{M}^{ij}_f f'^j_R + \mathrm{H.c.}, \quad (35)$$

where the primes indicate that the fermions are weak eigenstates. Using the following identity:

$$\sum_{\ell,k}(\tilde{V}^\dagger_{\mathrm{F}L})^{i\ell}\mathcal{M}^{\ell k}_f(\tilde{V}_{\mathrm{F}R})^{kj} = (\tilde{V}^\dagger_{\mathrm{F}L}\mathcal{M}_f\tilde{V}_{\mathrm{F}R})^{ij} - \sum_A(\tilde{V}^\dagger_{\mathrm{F}L})^{i4}\mathcal{M}^{4A}_f(\tilde{V}_{\mathrm{F}R})^{Aj}, \quad (36)$$

where $\tilde{V}_{\mathrm{F}R}$ diagonalizes $\mathcal{M}^\dagger_f\mathcal{M}_f$, we can rewrite the above Lagrangian in the mass eigenstate basis as follows:

$$-\mathcal{L}\supset\frac{h}{v_H}\bar{f}^i_L\left(\tilde{m}_i\delta^{ij} - \sum_A(\tilde{V}^\dagger_{\mathrm{F}L})^{i4}\mathcal{M}^{4A}_f\tilde{V}^{Aj}_{\mathrm{F}R}\right)f^j_R + \mathrm{H.c.} \quad (37)$$

where $\tilde{m}_i$ are the physical light fermion masses. The first part, which corresponds to the diagonal matrix $\tilde{m}_i\delta^{ij}$, is the standard model coupling to the Higgs boson. The second term will lead to flavor changing interactions that are CP-violating. We can use the identity $(\mathcal{M}_f\tilde{V}_{\mathrm{F}R})^{4j} = (\tilde{V}_{\mathrm{F}L}\mathcal{M}_{\mathrm{diag}})^{4j}$ to rewrite the second term above as

$$-\mathcal{L}\supset -\frac{h}{v_H}\bar{f}^i_L(\tilde{V}^\dagger_{\mathrm{F}L})^{i4}(\tilde{V}_{\mathrm{F}L}\mathcal{M}_{\mathrm{diag}})^{4j}f^j_R + \mathrm{H.c.}, \quad (38)$$

where $\mathcal{M}_{\mathrm{diag}} = \mathrm{diag}(\tilde{m}_1, \tilde{m}_2, \tilde{m}_3, \tilde{M}_F)$ is the diagonal matrix with the physical masses of the fermions. Expressing $\tilde{V}_{\mathrm{F}L}$ as in Eq. (29), exploiting the fact that $(U_{\mathrm{F}L})^{A4} = (U_{\mathrm{F}L})^{4A} = \delta^{A4}$ and $(O_{\mathrm{F}L})^{A4} = (O_{\mathrm{F}L})^{4A} = \delta^{A4}$, and finding that $(V_{\mathrm{F}L})^{4i} = -m_i\mu_i/\tilde{M}^2_F$, we can write the non-$CP$ conserving light fermion couplings to the standard model Higgs boson as

$$-\mathcal{L}\supset -h\left(\frac{\tilde{m}_j}{v_H}\right)\bar{f}^i_L\sum_{\ell,k}(\mathrm{U}^\dagger_{\mathrm{F}L})^{i\ell}\frac{m_\ell\mu^*_\ell m_k\mu_k}{\tilde{M}^4_F}(\mathrm{U}_{\mathrm{F}L})^{kj}f^j_R + \mathrm{H.c.} \quad (39)$$

Similarly, we can write the coupling between the new fermions, the standard model fermions, and the Higgs boson in the following way:

$$-\mathcal{L}\supset \bar{f}^i_L\sum_k(\mathrm{U}^\dagger_{\mathrm{F}L})^{ik}\frac{\mu^*_k}{\tilde{M}_F}\left(\frac{m_k}{v_H}\right)hf^4_R + \mathrm{H.c.}, \quad (40)$$

which will be relevant for the decay rates of the heavy vectorlike fermions. Note that for $\mu\sim\tilde{M}_F$ the amplitude for the interaction that allows f_4 to decay to a Higgs and a SM fermion is not suppressed by $\tilde{M}_F$.

Even without a detailed discussion of the couplings of the X and S scalars and the Z_R gauge boson to the fermions, upper bounds can be derived on the lifetimes of the new vectorlike fermions. These upper bounds are dominated by decays through the Higgs boson via the couplings in Eq. (40),

$$\tau_F < \left[\frac{\tilde{M}_F}{8\pi}\left(\frac{\mu_F}{\tilde{M}_F}\right)^2\left(\frac{m_f}{v_H}\right)^2\right]^{-1}, \quad (41)$$

where m_f is the mass of the heaviest fermion of its kind (electrons, neutrinos, up-type or down-type quarks).

The measured CKM phase implies $\mu_Q\sim\tilde{M}_Q$, which fixes the upper bound on the lifetime of the new quarks to be around 10^{-21} s (10^{-26} s) for tera-electron-volt (TeV) down (up) vectorlike quark masses. On the other hand, the PMNS phase in the lepton sector is unknown, which renders more freedom to the hierarchy between μ and $\tilde{M}_F$. Particularly interesting is the case of the vectorlike neutrino, N, as the upper bound on its lifetime is strongly suppressed by the SM neutrino masses:

$$\tau_N < 0.1\ \mathrm{s}\left(\frac{1\ \mathrm{TeV}}{\tilde{M}_N}\right)\left(\frac{\tilde{M}_N/\mu_N}{1}\right)^2\left(\frac{0.1\ \mathrm{eV}}{m_\nu}\right)^2. \quad (42)$$

If the PMNS phase turns out to be of the same magnitude or bigger than the CKM phase, then $\mu_{\rm N} \sim \tilde{M}_N$ and a TeV scale vectorlike neutrino would be expected to decay before big bang nucleosynthesis.

VI. FINITE NATURALNESS

In a renormalizable theory after all divergencies are canceled, one can check if there are needed cancellations between different contributions to a physical quantity. To avoid awkward fine-tuning one can impose bounds on the masses and couplings of the new fields. This simple criteria is often called *finite naturalness*; see Ref. [49] for a detailed discussion.

The Nelson-Barr mechanism is an attractive solution to the strong CP problem. Without it, a very precise cancellation between the strong CP phase $\theta_{\rm QCD}$ and $\arg\{\det(\mathcal{M}_u\mathcal{M}_d)\}$ is needed to satisfy $\bar{\theta}_{\rm QCD} < 10^{-10}$. However, generically in Nelson-Barr models, new colored fermions, vectorlike under the standard model, are required to implement the aforementioned mechanism. Moreover, because of the stable domain wall generated from the spontaneous breaking of a spacetime symmetry as CP, v_X must be large. Given the simultaneous presence of several scales in the theory, a similar cancellation problem to the one that Nelson-Barr models succeed to solve may arise in the effective scalar potential. Therefore, it is reasonable to explore for which coupling values the theory can remain free of such cancellations.

The new vectorlike fermions, as long as they carry standard model quantum numbers, will contribute at three loops to the Higgs mass, as shown in the upper panel of Fig. 2. Particularly, for the new quarks [49],

$$\delta m_H^2 \sim \frac{\alpha_s^2}{(4\pi)^4} M_F^2 \ln\left(\frac{M_F^2}{m_t^2}\right). \qquad (43)$$

This implies that

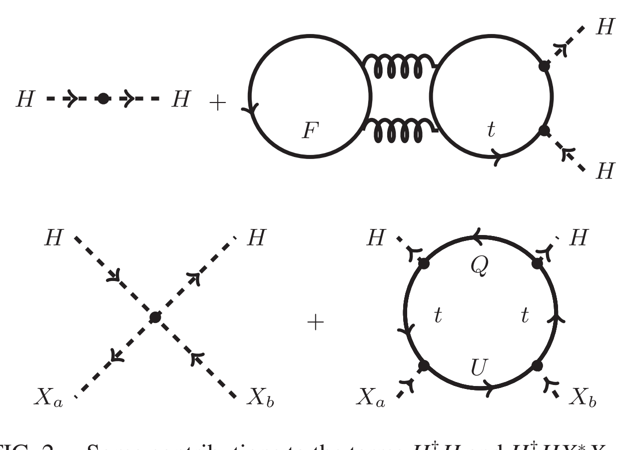

FIG. 2. Some contributions to the terms $H^\dagger H$ and $H^\dagger H X_a^* X_b$ in the effective scalar potential.

$$M_F \lesssim 500\ \text{TeV}\left(\frac{0.118}{\alpha_S}\right)\left(\frac{\delta m_H^2/m_H^2}{100}\right)^{1/2}. \qquad (44)$$

Since inflating the domain walls away demands the VEV of X to be large, one can assume that all couplings between the X_i and the SM Higgs are suppressed, avoiding thus cancellations arising at tree level in the scalar potential. For example, the coupling of the $H^\dagger H X_i X_j^*$ quartic interaction at tree level,

$$\lambda < 10^{-14}\left(\frac{10^{10}\ \text{GeV}}{v_X}\right)^2\left(\frac{\delta m_H^2/m_H^2}{100}\right). \qquad (45)$$

This is consistent with having a large θ_X and an acceptably small contribution to $\bar{\theta}_{\rm QCD}$ from loops [see Eq. (16)].

On the other hand, such crossed terms can be generated by loop effects. For instance, the diagram displayed in the bottom right panel of Fig. 2 contributes to the SM-Higgs mass in the following way:

$$\delta m_H^2 \sim \frac{\lambda_u^2}{(4\pi)^2} v_X^2 \ln\left(\frac{M_U^2}{m_t^2}\right). \qquad (46)$$

Using the fact that $\lambda_u v_X \sim \tilde{M}_{\rm U}$, Eq. (46) implies

$$M_{\rm U} \lesssim 6\ \text{TeV}\left(\frac{\delta m_H^2/m_H^2}{100}\right)^{1/2}. \qquad (47)$$

Equation (47) suggests that in this model vectorlike fermions may be accessible to the LHC or future colliders. To achieve masses at the TeV scale with SCPV at a much larger scale (which is required by the domain walls) implies very small couplings.

There are other finite naturalness constraints that we will not discuss since there may be other physics that makes tunings in the Higgs effective potential acceptable.

VII. SUMMARY

In this article we have discussed simple gauge theories where the Nelson-Barr mechanism is realized. Using a gauge symmetry that is chiral under the new vectorlike fermions improves the quality of the solution to the strong CP puzzle.

We studied models that correspond to $\mathrm{U}(1)_R$ when acting on the SM fermions. These models have CP violation in both the quark and thelepton sectors.

We derived explicit expressions for flavor-violating interactions that apply in Nelson-Barr models which hold quite generally. These are more suppressed than one may naively expect, occurring at order $m^2/\tilde{M}_F^2$, where m ($\tilde{M}_F$) is the magnitude of the standard model (new vector-like) fermion masses. This additional suppression ensures that they can be consistent with experimental constraints.

If finite naturalness is taken seriously, it suggests that the new particle content is near the TeV scale. Another way that new particle content can be at the TeV scale is to have *CP* symmetry not restored at high temperature.

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APPENDIX: CORRECTIONS TO $\bar{\theta}_{\mathrm{QCD}}$

Nonrenormalizable operators and loops can contribute to $\bar{\theta}_{\mathrm{QCD}}$ by adding corrections to the tree-level mass matrix of the quarks. Then, the total mass matrix of the quarks, $\tilde{\mathcal{M}}_q$, can be decomposed into

$$\tilde{\mathcal{M}}_q = \mathcal{M}_q + \delta\mathcal{M}_q, \qquad \text{(A1)}$$

where $\mathcal{M}_q$ is the tree-level mass matrix of the quarks [see Eq. (14)] and $\delta\mathcal{M}_q$ parametrizes any subleading contribution.

The determinant of a matrix close to the identity can be expanded as follows:

$$\begin{aligned}\mathrm{Det}(\tilde{\mathcal{M}}_q) &= \mathrm{Det}(\mathcal{M}_q)\mathrm{Det}(\mathbb{I} + \mathcal{M}_q^{-1}\delta\mathcal{M}_q)\\ &= \mathrm{Det}(\mathcal{M}_q)(1 + \mathrm{Tr}\{M_q^{-1}\delta\mathcal{M}_q\}), \qquad \text{(A2)}\end{aligned}$$

where we have employed the identity $\mathrm{Det}(e^A) = e^{\mathrm{Tr}\{A\}}$. Since the argument of a complex number is given by $\arg\{z\} = -i(\ln z - \ln|z|)$, and using that

$$\ln \mathrm{Det}(\tilde{\mathcal{M}}_q) = \ln \mathrm{Det}(\mathcal{M}_q) + \mathrm{Tr}\{\mathcal{M}_q^{-1}\delta\mathcal{M}_q\}, \qquad \text{(A3)}$$

the following expression is derived:

$$\begin{aligned}\arg\{\mathrm{Det}(\tilde{\mathcal{M}}_q)\} &= \arg\{\mathrm{Det}(\mathcal{M}_q)\} - i\mathrm{Tr}\{\mathcal{M}_q^{-1}\delta\mathcal{M}_q\}\\ &\quad + i\ln|1 + \mathrm{Tr}\{\mathcal{M}_q^{-1}\delta\mathcal{M}_q\}|. \qquad \text{(A4)}\end{aligned}$$

Using that $|z| = \sqrt{z^* z}$ and expanding the logarithm of the square root up to first order in $\delta\mathcal{M}_q$, the subleading corrections to the strong *CP* phase can be written as

$$\Delta\bar{\theta}_{\mathrm{QCD}} = \mathrm{Im}\{\mathrm{Tr}\{\mathcal{M}_q^{-1}\delta\mathcal{M}_q\}\} + \mathcal{O}(\delta M_q^2). \qquad \text{(A5)}$$

[1] C. Abel *et al.*, Phys. Rev. Lett. **124**, 081803 (2020).
[2] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977).
[3] R. D. Peccei and H. R. Quinn, Phys. Rev. D **16**, 1791 (1977).
[4] A. E. Nelson, Phys. Lett. **136B**, 387 (1984).
[5] S. M. Barr, Phys. Rev. Lett. **53**, 329 (1984).
[6] A. E. Nelson, Phys. Lett. **143B**, 165 (1984).
[7] F. Wilczek, Phys. Rev. Lett. **40**, 279 (1978).
[8] S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978).
[9] J. E. Kim, Phys. Rev. Lett. **43**, 103 (1979).
[10] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B166**, 493 (1980).
[11] M. Dine, W. Fischler, and M. Srednicki, Phys. Lett. **104B**, 199 (1981).
[12] A. R. Zhitnitsky, Sov. J. Nucl. Phys. **31**, 260 (1980).
[13] J. Preskill, M. B. Wise, and F. Wilczek, Phys. Lett. **120B**, 127 (1983).
[14] L. F. Abbott and P. Sikivie, Phys. Lett. **120B**, 133 (1983).
[15] M. Dine and W. Fischler, Phys. Lett. **120B**, 137 (1983).
[16] H. M. Georgi, L. J. Hall, and M. B. Wise, Nucl. Phys. **B192**, 409 (1981).
[17] R. N. Mohapatra and G. Senjanovic, Phys. Lett. **79B**, 283 (1978).
[18] K. S. Babu and R. N. Mohapatra, Phys. Rev. D **41**, 1286 (1990).
[19] L. Bento, G. C. Branco, and P. A. Parada, Phys. Lett. B **267**, 95 (1991).
[20] P. H. Frampton and D. Ng, Phys. Rev. D **43**, 3034 (1991).
[21] J. Schwichtenberg, P. Tremper, and R. Ziegler, Eur. Phys. J. C **78**, 910 (2018).
[22] A. Valenti and L. Vecchi, J. High Energy Phys. 07 (2021) 152.
[23] P. Asadi, S. Homiller, Q. Lu, and M. Reece, arXiv: 2212.03882.
[24] A. H. Guth, Phys. Rev. D **23**, 347 (1981).
[25] A. D. Linde, Phys. Lett. **108B**, 389 (1982).
[26] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).
[27] A. Vilenkin, Phys. Rep. **121**, 263 (1985).
[28] J. McNamara and M. Reece, arXiv:2212.00039.
[29] G. R. Dvali, A. Melfo, and G. Senjanovic, Phys. Rev. D **54**, 7857 (1996).
[30] T. Appelquist, B. A. Dobrescu, and A. R. Hopper, Phys. Rev. D **68**, 035012 (2003).
[31] P. Fileviez Perez and C. Murgui, Phys. Lett. B **777**, 381 (2018).
[32] V. A. Rubakov, M. V. Sazhin, and A. V. Veryaskin, Phys. Lett. **115B**, 189 (1982).
[33] R. Fabbri and M. d. Pollock, Phys. Lett. **125B**, 445 (1983).
[34] L. F. Abbott and M. B. Wise, Nucl. Phys. **B244**, 541 (1984).
[35] A. A. Starobinsky, Sov. Astron. Lett. **11**, 133 (1985).

[36] M. Kamionkowski, A. Kosowsky, and A. Stebbins, Phys. Rev. Lett. **78**, 2058 (1997).
[37] M. Kamionkowski, A. Kosowsky, and A. Stebbins, Phys. Rev. D **55**, 7368 (1997).
[38] U. Seljak and M. Zaldarriaga, Phys. Rev. Lett. **78**, 2054 (1997).
[39] M. Zaldarriaga and U. Seljak, Phys. Rev. D **55**, 1830 (1997).
[40] L. Vecchi, J. High Energy Phys. 04 (2017) 149.
[41] A. Valenti and L. Vecchi, J. High Energy Phys. 07 (2021) 203.
[42] G. C. Branco, P. A. Parada, and M. N. Rebelo, arXiv: hep-ph/0307119.
[43] K. Ishiwata, Z. Ligeti, and M. B. Wise, J. High Energy Phys. 10 (2015) 027.
[44] G. Perez and A. Shalit, J. High Energy Phys. 02 (2021) 118.
[45] W. H. Bertl *et al.* (SINDRUM II Collaboration), Eur. Phys. J. C **47**, 337 (2006).
[46] A. de Gouvea and P. Vogel, Prog. Part. Nucl. Phys. **71**, 75 (2013).
[47] J. Bernabeu, E. Nardi, and D. Tommasini, Nucl. Phys. **B409**, 69 (1993).
[48] P. Dornan, EPJ Web Conf. **118**, 01010 (2016).
[49] M. Farina, D. Pappadopulo, and A. Strumia, J. High Energy Phys. 08 (2013) 022.