

Novel approach to investigate η decays via $\eta' \rightarrow \pi\pi\eta$

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To avoid the impact from the background events directly from e^+e^- annihilations or J/ψ decays, we propose a novel approach to investigate η decays, in particular for its rare or forbidden decays, by using $\eta' \rightarrow \pi\pi\eta$ produced in J/ψ decays at the τ -charm factories. Based on the Monte Carlo studies of a few typical decays, $\eta \rightarrow \pi\pi$, $\gamma l^+ l^-$ ($l = e, \mu$), $l^+ l^-$, as well as $l^+ l^- \pi^0$, the sensitivities could be obviously improved by taking advantage of the extra constraint of η' . Using 1×10^{12} J/ψ events accumulated at the super τ -charm facility, the precision on the investigation of η decays could be improved significantly and the observation of the rare decay $\eta \rightarrow e^+e^-$ is even accessible.

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I. INTRODUCTION

Since its strong, electromagnetic, and weak decays are forbidden in the first order, η meson plays an important role as a test of low-energy quantum chromodynamics calculations in the framework of chiral perturbation theory (ChPT). In addition, η is an eigenstate of the charge conjugation (C) and parity (P) operators, and thus it provides an important experimental probe for investigations of the degree of conservation of these symmetries in strong and electromagnetic interactions. In addition to the promising numbers of η directly produced from hadron- or photoproduction processes, huge samples of the η can be collected in the radiative decays of the vector meson from the e^+e^- annihilations ($\phi \rightarrow \gamma\eta$ at KLOE-2 [1] and $J/\psi \rightarrow \gamma\eta$ at BESIII [2]). In recent years, with the world's largest J/ψ samples collected with the BESIII detector, a series of interesting results on η decays was achieved with the decays of $J/\psi \rightarrow \gamma\eta$ (see the reviews [3–6] for details).

However, it was found that the large background contributions from J/ψ decays make it hard to improve the

sensitivity for the investigation on the η rare or forbidden decays. Taking $\eta \rightarrow \pi^0\pi^0$ as an example, the dominant background events come from $J/\psi \rightarrow \gamma\pi^0\pi^0$ due to the direct pions production. In particular, the production of the intermediate state $f_0(600)$ makes the background events irreducible [7]. To avoid the background impacts directly from J/ψ decays, we introduce a novel approach to investigate the η decays via the $\eta' \rightarrow \pi\pi\eta$ process. According to the Particle Data Group (PDG) [8], the product of branching fraction of $J/\psi \rightarrow \gamma\eta'$ and $\eta' \rightarrow \pi^+\pi^-\eta$ is $(2.23 \pm 0.04) \times 10^{-3}$, which is about 2 times larger than that of $J/\psi \rightarrow \gamma\eta$. After taking into account the tracking efficiency of two charged pions, the selected η samples from this approach are larger than, and at least compatible with, the directly obtained sample from $J/\psi \rightarrow \gamma\eta$. On the other hand, since the η' is quite narrow, one more constraint on the η' peak makes it easier to suppress the background events directly from J/ψ decays.

Most recently, a project of the super τ -charm facility (STCF) [9] was proposed for exploring the τ -charm physics and searching for the physics beyond the Standard Model (SM). The STCF is an electron-positron collider, operating at energies from 2 to 7 GeV, together with a state-of-the-art particle detector. The designed luminosity, $0.5 \times 10^{35} \text{ cm}^{-2} \text{ s}^{-1}$ or higher is about 100 times larger than that of the BEPCII [10], which enables one to collect unprecedented high statistics data samples in one year. As advocated by the BEPCII/BESIII, not only will this facility play a leading role in the investigation of τ -charm physics, but they will offer an unprecedented

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opportunity to explore the light meson decays benefited from the high production rates of light mesons in the charmonium decays.

According to the latest conceptual design report [9], 3.4×10^{12} J/ψ events can be produced in one year at STCF. To have a conservation estimation on the investigation of η decays, the sensitivities are estimated based on 1×10^{12} J/ψ events, which corresponds to 5.2×10^9 $J/\psi \rightarrow \gamma\eta'$ decays. Therefore, a simulated sample of 5.2×10^9 $J/\psi \rightarrow \gamma\eta'$ with η' inclusive decays are simulated based on the basic STCF fast simulation package [11]. All the branching fractions of η' decays are taken from the PDG [8]. This sample will be denoted as “pseudodata” throughout the text and used to estimate the potential background contributions. Then exclusive Monte Carlo (MC) studies of a few typical decays of η meson are performed in this article to elucidate the feasibility for investigating η decays with $\eta' \rightarrow \pi^+\pi^-\eta$. It is worth mentioning that the detector geometry and performance and the reconstruction software are still under further optimization, such as the spatial resolution for tracks and clusters, the energy resolution for clusters, the efficiency for tracking, and particle identification.

II. $\eta \rightarrow \pi\pi$

The P and CP violating decays $\eta \rightarrow \pi\pi$ are usually regarded as the golden channels to search for the unconventional source of CP violation [12]. The SM and its extended sector predicted the branching fraction of $\eta \rightarrow \pi\pi$ at the level of $\sim 10^{-15}$ [13]. While the experimental upper limits are highly limited due to the irreducible background production at both hadronic collisions and e^+e^- annihilations. That is why a possible new test in the decay into four pions is performed by many experiments, even though the detection efficiency is lower than that of $\eta \rightarrow \pi^0\pi^0$. The present upper limit for the branching fraction of $\eta \rightarrow \pi^0\pi^0$, 3.5×10^{-4} [14], is 2 orders of magnitude larger than that of $\eta \rightarrow 4\pi^0$. While the upper limit for the branching fraction of $\eta \rightarrow \pi^+\pi^-$ is 4.4×10^{-6} [15] from the KLOE-2 experiment.

With a sample of 2.2×10^8 J/ψ events, BESIII performed the search for $\eta \rightarrow \pi\pi$ via the $J/\psi \rightarrow \gamma\eta \rightarrow \gamma\pi\pi$ process [7]. The dominant background contributions are from $J/\psi \rightarrow \pi^+\pi^-\pi^0$, e^+e^- , and $\mu^+\mu^-$ for $\eta \rightarrow \pi^+\pi^-$, and $J/\psi \rightarrow \gamma\pi^0\pi^0$ with the direct pions production for $\eta \rightarrow \pi^0\pi^0$. In particular, the production of the intermediate state $f_0(600)$ makes the background events irreducible, as illustrated in Fig. 1. The high background level makes the sensitivity of searching this rare decay quite low via $J/\psi \rightarrow \gamma\eta$, which set the upper limits as 3.9×10^{-4} and 6.9×10^{-4} for $\eta \rightarrow \pi^+\pi^-$ and $\eta \rightarrow \pi^0\pi^0$, respectively.

To check the sensitivity of searching the rare decay of $\eta \rightarrow \pi\pi$ via $J/\psi \rightarrow \gamma\eta'(\pi^+\pi^-\eta)$, MC studies are performed with the pseudodata sample. The main background events are found to be $\eta' \rightarrow \pi^+\pi^-\pi^+\pi^-$ for the charged channel

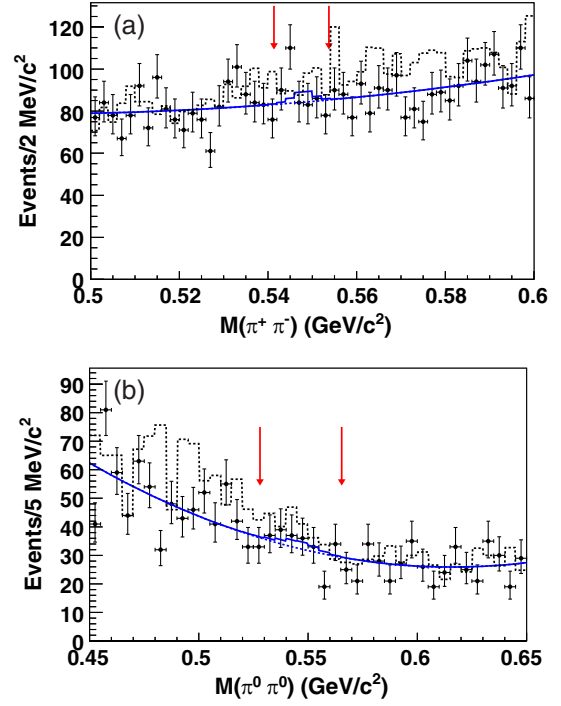


FIG. 1. Adapted from Ref. [7], which is from $J/\psi \rightarrow \gamma\pi\pi$ based on a sample of 2.2×10^8 J/ψ events at BESIII. The (a) $\pi^+\pi^-$ and (b) $\pi^0\pi^0$ invariant mass distributions of the final candidate events in the η signal region. The dots with error bars are data, the solid lines are the fit results, and the dashed histograms are the sum of all the simulated normalized backgrounds. The arrows show mass regions that contain around 95% of the signal according to MC simulations.

and $\eta' \rightarrow \pi^+\pi^-\pi^0\pi^0$ for the neutral channel, respectively, which can be well described by the combination of the ChPT and vector meson dominance model. In addition, there are also small amounts of backgrounds from $\eta \rightarrow \gamma\pi^+\pi^-$ with η from $\eta' \rightarrow \pi\pi\eta$, which contribute as peaks in the mass spectra of $\pi^+\pi^-\pi^+\pi^-$ and also $\pi^+\pi^-$ for the charged channel, but both are below the η' and η signal regions. To eliminate $\eta \rightarrow \gamma\pi^+\pi^-$ backgrounds and other continuum background contributions under the η' peak, the same approach as in Ref. [16] is adopted. The $M(\pi^+\pi^-)$ or $M(\pi^0\pi^0)$ can be divided into a number of bins around the η signal region and a fit to $M(\pi^+\pi^-\pi^+\pi^-)$ or $M(\pi^+\pi^-\pi^0\pi^0)$ for each bin is performed to extract the strength of $\eta' \rightarrow 4\pi$ and other background contributions. Then the background-subtracted $\pi^+\pi^-$ and $\pi^0\pi^0$ mass spectra are obtained and shown in Fig. 2, together with the possible $\eta \rightarrow \pi\pi$ signal with an arbitrary normalization. Please note that one $\eta' \rightarrow \pi^+\pi^-\pi^+\pi^-$ event contributes more than one entry in $M(\pi^+\pi^-)$.

We then made a test by determining the production upper limit of $\eta \rightarrow \pi\pi$ using the Bayesian approach. A series of maximum likelihood fits is performed to the mass spectrum of $\pi\pi$ with an expected signal. In the fit, the line shape of

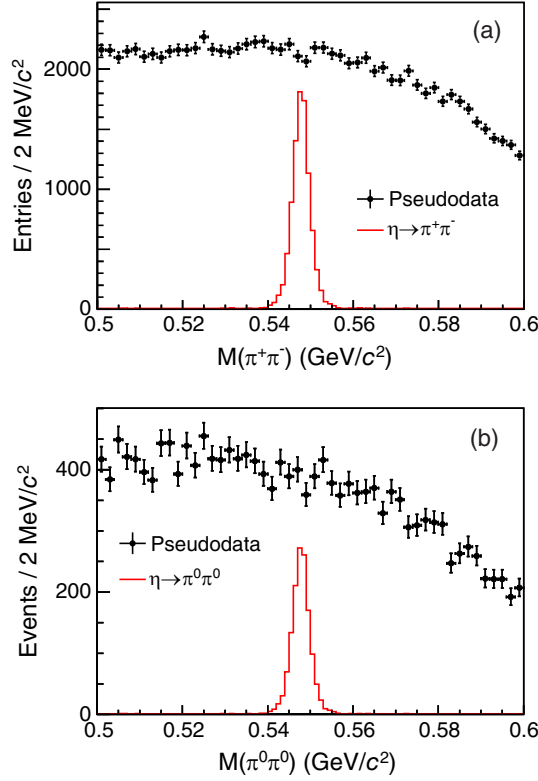


FIG. 2. The (a) $\pi^+\pi^-$ and (b) $\pi^0\pi^0$ invariant mass distributions for $\eta' \rightarrow \pi^+\pi^-\eta(\pi^+\pi^-)$ and $\eta' \rightarrow \pi^+\pi^-\eta(\pi^0\pi^0)$ candidates in the η signal region, respectively. The dots with error bars are from pseudodata after subtracting the non- $\eta' \rightarrow 4\pi$ background contributions and the histograms are the simulated $\eta \rightarrow \pi\pi$ signal with a random scale.

the η signal is determined by MC simulation, and the background is represented with a second-order Chebyshev polynomial. The likelihood distributions of the fit are taken as the probability density function directly. The upper limit on the number of signal events at the 90% confidence level corresponds to the number of events at 90% of the integral of the probability density function. Considering the estimated detection efficiency, the upper limits on the branching fractions of $\eta \rightarrow \pi^+\pi^-$ and $\eta \rightarrow \pi^0\pi^0$ are determined to be 7.5×10^{-8} and 6.9×10^{-7} , respectively, which will be the most stringent upper limits, and the one for $\eta \rightarrow \pi^0\pi^0$ is 3 orders of magnitude better than the present upper limit [8].

A full systematic uncertainty evaluation requires both experimental data and full MC simulation, therefore, we only have a qualitative discussion below. The possible systematic uncertainty sources for the upper limits include the number of J/ψ events, the intermediate branching fractions, and the event selection. The number of J/ψ events can be determined precisely with its hadronic decays, as described in Ref. [17]. The uncertainties associated with the intermediate process will be taken from PDG. The uncertainties associated with event selection are mainly from the difference between MC

simulation and experimental data in tracking, particle identification, and photon reconstruction, which can be studied with clean and high statistics control samples and are still under optimization. The total systematic uncertainty at STCF is expected to be at the level of several percent or even less, which only has a minor impact on the sensitivities of η rare decays.

III. $\eta \rightarrow \gamma e^+ e^-$ AND $\eta \rightarrow \gamma \mu^+ \mu^-$

The $\eta \rightarrow \gamma l^+ l^-$ ($l = e, \mu$) decays are the simplest radiative dilepton decays, also called Dalitz decays, where the lepton pair is formed by internal conversion of an intermediate virtual photon. The deviation of the spectrum $M(l^+ l^-)$, from the quantum electrodynamics (QED) prediction, allows one to investigate the electromagnetic structure of the η in terms of a timelike transition form factor, which plays an important role in the evaluation of the hadronic light-by-light contribution to the muon anomalous magnetic moment.

The latest slope of the form factor measurements for η meson are $\Lambda^{-2} = 1.97 \pm 0.11$ and $\Lambda^{-2} = 1.934 \pm 0.067 \pm 0.050$ (GeV/c^2) $^{-2}$, respectively, from the A2 Collaboration using $\eta \rightarrow \gamma e^+ e^-$ [18] and the NA60 Collaboration using $\eta \rightarrow \gamma \mu^+ \mu^-$ [19], while the branching fractions of them have not been updated for more than one decade.

In the study of the $\eta \rightarrow \gamma l^+ l^-$ decays with $J/\psi \rightarrow \gamma \eta$ by the BESIII experiment [20], it was found that these decays suffer from the background events directly from $e^+ e^-$ annihilations and J/ψ decays that have charged pions in the final states. In particular, for the $\eta \rightarrow \gamma \mu^+ \mu^-$ decay, the impact of the backgrounds should be large because of its low branching fraction and the misidentification of muons and pions. However, the MC study indicates that both of these two decay modes could be easily distinguished from events obtained through the $\eta' \rightarrow \pi^+ \pi^- \eta$ decay.

Using the pseudodata sample of 1×10^{12} J/ψ events, we selected 1747071 ± 1321 $\eta \rightarrow \gamma e^+ e^-$ events and 200193 ± 447 $\eta \rightarrow \gamma \mu^+ \mu^-$ events, respectively. It was found that the background contribution is at a level of 10^{-3} , which indicates that the selected sample of η Dalitz decays from $\eta' \rightarrow \pi^+ \pi^- \eta$ could provide a clean laboratory to measure the transition form factor. After normalization with the QED contribution, the transition form factors, defined as $F(M_{l^+ l^-}^2; 0)$, as a function of $M(l^+ l^-)$ are displayed in Fig. 3. With the single pole model [21], $F(M_{l^+ l^-}^2; 0) \equiv (1 - M_{l^+ l^-}^2/\Lambda^2)^{-1}$, the slopes of the transition form factor, defined as $dF(M_{l^+ l^-}^2; 0)/dM^2(l^+ l^-)|_{M^2(l^+ l^-)=0} = \Lambda^{-2}$, are measured to be 1.653 ± 0.038 (GeV/c^2) $^{-2}$ for $\eta \rightarrow \gamma \mu^+ \mu^-$ and 1.644 ± 0.012 (GeV/c^2) $^{-2}$ for $\eta \rightarrow \gamma e^+ e^-$, where the errors are statistical only. From the above study, it is clear that the precision of branching fractions and the transition form factor measurement will be improved significantly.

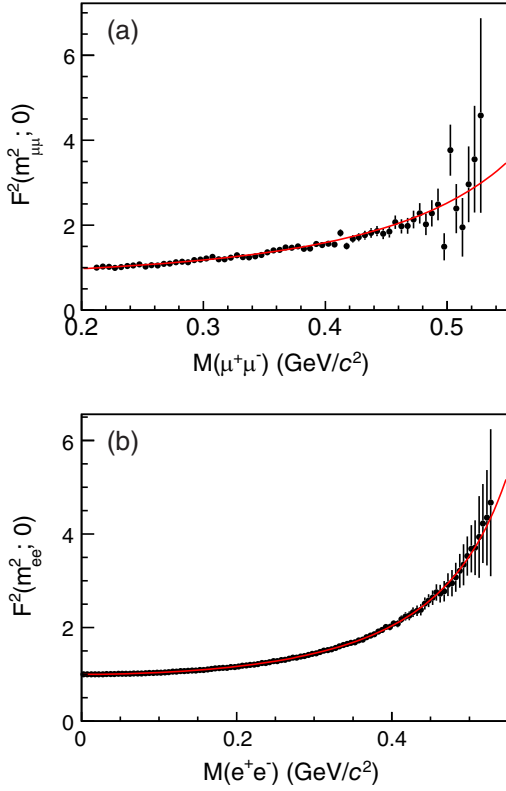


FIG. 3. The distribution of $F^2(M_{l^+l^-}^2; 0)$ over the (a) $M(\mu^+\mu^-)$ and (b) $M(e^+e^-)$. The dots with error bars are the ratio of the background-subtracted pseudodata at STCF to the signal MC, which is simulated using $F^2(M_{l^+l^-}^2; 0) \equiv 1$. The solid lines are normalized fit results.

In addition, the clean sample of $\eta \rightarrow \gamma\mu^+\mu^-$ allows one to search for the electromagnetic bound states of a $\mu^+\mu^-$ pair, called muonium [22,23], which, experimentally, has never been observed yet due to its low production rate. The observation of the muonium will be essential for understanding the various potential anomalies involving muons [24] and the possible contributions from the physics beyond the SM [25].

IV. $\eta \rightarrow e^+e^-$ AND $\eta \rightarrow \mu^+\mu^-$

$\eta \rightarrow l^+l^-$ is a fourth-order electromagnetic transition and the branching fraction is expected to be tiny, especially for $\eta \rightarrow e^+e^-$, which is suppressed compared to $\eta \rightarrow \mu^+\mu^-$ as a consequence of the helicity factor of the electrons. The unitarity limit gives the branching fraction at a level of 10^{-9} [26], which makes $\eta \rightarrow e^+e^-$ an attractive prospect for a leptoquark search. New theories [27,28] beyond the SM, such as composite, grand unified, and technicolor models, require the existence of new particles. An especially popular type is known as leptoquark, which couples directly to quarks and leptons. In addition, the interest in the decays was revived due to the observed excess rate of the $\pi^0 \rightarrow e^+e^-$ decay [29] with respect to the SM

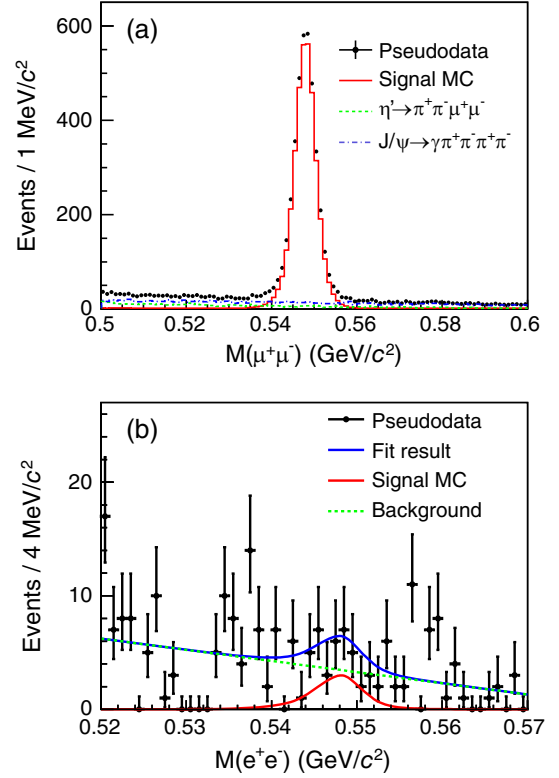


FIG. 4. The (a) $\mu^+\mu^-$ and (b) e^+e^- invariant mass distribution for $\eta' \rightarrow \pi^+\pi^-\eta(\mu^+\mu^-)$ and $\eta' \rightarrow \pi^+\pi^-\eta(e^+e^-)$ candidates in the η signal region, respectively. The dots with error bars are for the pseudodata, the dashed lines are backgrounds, the solid lines are the signal and also the fit result for the $\eta \rightarrow e^+e^-$ channel.

predictions [30]. This triggered theoretical speculations that the excess might be caused by a neutral vector meson responsible for annihilation of a neutral scalar dark matter particle [31]. The consequence could be large (even an order of magnitude) enhancement of the $\eta \rightarrow e^+e^-$ decay rate. Therefore, a telling clue to the existence of these new effects would be the enhancement of $\mathcal{B}(\eta \rightarrow e^+e^-)$ much above the unitarity limit, which implies that the rare decay of $\eta \rightarrow e^+e^-$ can be an important probe for the new physics beyond the SM.

Since the high production cross section of $e^+e^- \rightarrow l^+l^-$ and the large branching fraction of $J/\psi \rightarrow l^+l^-$, it is hard to investigate $\eta \rightarrow l^+l^-$ processes using the radiative decay of $J/\psi \rightarrow \gamma\eta$. However, theoretically, the $\eta' \rightarrow \pi^+\pi^-l^+l^-$ decay proceeds via a virtual photon intermediate state, $\eta \rightarrow \pi^+\pi^-\gamma^* \rightarrow \pi^+\pi^-l^+l^-$. A peak with a long tail just above $2m_e$ is expected to be seen in the $M(l^+l^-)$ and a dominant ρ contribution in $M(\pi^+\pi^-)$. These two prominent features could make these decays well separated from the decays of $\eta' \rightarrow \pi^+\pi^-\eta$ with $\eta \rightarrow l^+l^-$, which is illustrated in Fig. 4.

Based on 1.3×10^9 J/ψ events, BESIII first observed the $\eta' \rightarrow \pi^+\pi^-\mu^+\mu^-$ signal and found a few dozen events peaked around the η meson mass in the dimuon mass

spectrum [32]. These events come from the $\eta' \rightarrow \pi^+\pi^-\eta$, followed by the rare decay $\eta \rightarrow \mu^+\mu^-$, which could give a compatible branching fraction with the present world average value $\mathcal{B}(\eta \rightarrow \mu^+\mu^-) = (5.8 \pm 0.8) \times 10^{-6}$ [8]. With the current available $10 \times 10^9 J/\psi$ events at the BESIII experiments, which is about 8 times larger than that used in Ref. [32], the precision of the evaluated branching fraction of $\eta \rightarrow \mu^+\mu^-$ can be extracted with a relative uncertainty on the order of 10%.

To estimate the background contribution, we performed a MC study by generating $J/\psi \rightarrow \gamma\eta', \eta' \rightarrow \pi^+\pi^-\mu^+\mu^-$, and $J/\psi \rightarrow \gamma\pi^+\pi^-\pi^+\pi^-$ samples based on the STCF fast simulation package, which are also shown in Fig. 4(a). Based on the pseudodata sample, the signal yields of $\eta \rightarrow \mu^+\mu^-$ are estimated to be 3847 ± 62 and the corresponding branching fraction is calculated to be $(5.88 \pm 0.09) \times 10^{-6}$; the precision is improved by 1 order of magnitude.

With the same pseudodata sample, the possible $\eta' \rightarrow \pi^+\pi^-\eta$ with $\eta \rightarrow e^+e^-$ candidates are also selected. The obtained e^+e^- mass spectra is shown as the blacks dots in Fig. 4(b). An unbinned maximum likelihood fit is then performed to the $M(e^+e^-)$ distribution, where the signal is described by the MC simulated shape, and the background contribution is described by a first-order Chebyshev polynomial function. The branching fraction is expected to reach a level of 10^{-9} with $1 \times 10^{12} J/\psi$ events at STCF, which is just close to the theoretical calculation [33]. Therefore, an observation of $\eta \rightarrow e^+e^-$ decay with a branching fraction exceeding the theoretical prediction might be a signature of physics beyond the SM.

V. $\eta \rightarrow \pi^0 e^+ e^-$ AND $\eta \rightarrow \pi^0 \mu^+ \mu^-$

The investigation of the charge conjugation invariance in the electromagnetic interactions can be done by studying the $\eta \rightarrow \pi^0 l^+ l^-$ decay. Within the framework of the SM, the matrix element for this process should involve the two virtual photon exchange [34] as illustrated in Fig. 5, with the transition according to the reaction of $\eta \rightarrow \pi^0 + \gamma^* + \gamma^* \rightarrow \pi^0 + l^+ + l^-$. The theoretical predictions on the decay rate of this C-conserving process ranges from 10^{-11} to 10^{-8} [35–37] depending on the undertaken

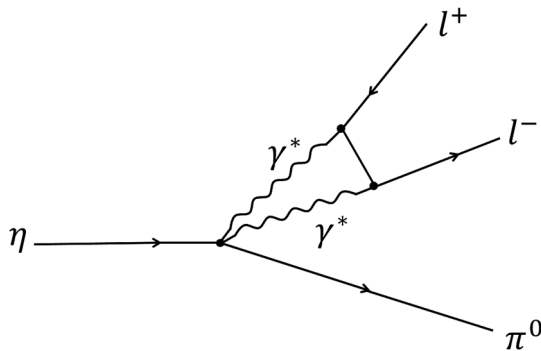


FIG. 5. $\eta \rightarrow \pi^0 \gamma^* \gamma^* \rightarrow \pi^0 l^+ l^-$ occurring via the C-conserving second-order electromagnetic process.

assumptions. Since the first-order electromagnetic η decays are forbidden and $\eta \rightarrow \pi^0 \gamma$ also violates the conservation of angular momentum, in principle, the decay $\eta \rightarrow \pi^0 l^+ l^-$ proceeding with a virtual photon is forbidden.

At present, the experimental upper limit for the branching fraction $\mathcal{B}(\eta \rightarrow \pi^0 e^+ e^-)$ was determined to be 8×10^{-6} [8], which is still at least 3 orders of magnitude and remains to be experimentally investigated until the prediction based on the SM is reached. Whereas the experimental upper limit for $\eta \rightarrow \pi^0 \mu^+ \mu^-$, 5×10^{-6} [8] has not been updated for more than 40 years. The observation of any higher branching fraction than one calculated in the framework of the SM could provide the evidence of violation of the charge conjugation symmetry.

To testing the feasibility of the exploring $\eta \rightarrow \pi^0 l^+ l^-$ via $J/\psi \rightarrow \gamma\eta', \eta' \rightarrow \pi^+\pi^-\eta$, an extensive study is performed with the pseudodata sample and the dedicated signal MC samples. The results indicate that the dominant background contribution is from $\eta \rightarrow \gamma e^+ e^-$, which presents as a sharp peak in the mass spectrum of $\pi^0 e^+ e^-$ in the η mass region, but a smooth shape in the distribution of $\gamma\gamma$ invariant mass. Therefore, we can easily extract the possible $\eta \rightarrow \pi^0 e^+ e^-$ signal by fitting to the mass spectrum of $\gamma\gamma$ with the requirement of $M(e^+ e^- \gamma\gamma)$ in η signal region. Figure 6(a)

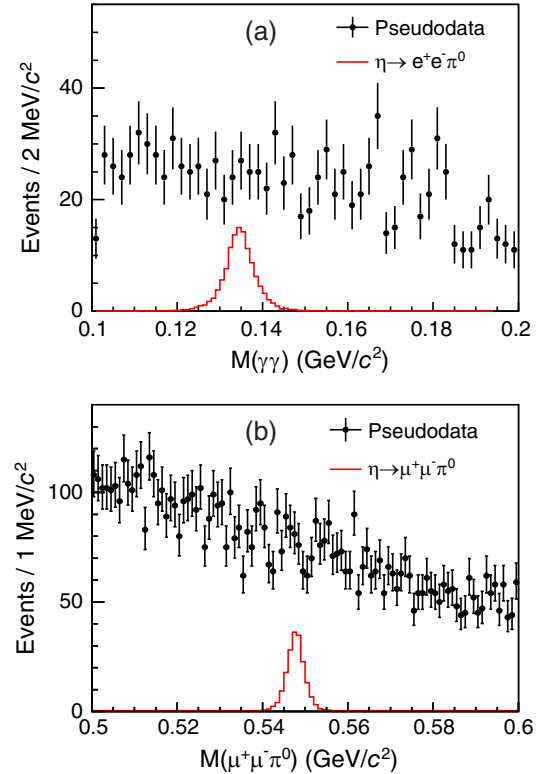


FIG. 6. (a) The $\gamma\gamma$ mass spectrum with $M(e^+ e^- \gamma\gamma)$ within the η signal region. (b) The $\mu^+\mu^-\pi^0$ mass spectrum for the $\eta \rightarrow \mu^+\mu^-\pi^0$ channel. The dots with error bars are backgrounds estimated from the pseudodata at STCF and the histograms are the possible $\eta \rightarrow \pi^0 l^+ l^-$ signal with a random scale.

shows the obtained $\gamma\gamma$ mass spectrum from the pseudodata sample and the possible $\eta \rightarrow \pi^0 e^+ e^-$ signal with a random scale. With 1×10^{12} J/ψ events at STCF, the upper limit is expected around 2×10^{-7} , which is improved by 1 order of magnitude compared with the PDG value [8].

For the $\eta \rightarrow \pi^0 \mu^+ \mu^-$ channel, the main background contribution from $\eta \rightarrow \pi^+ \pi^- \pi^0$, which is flat in the mass spectrum of $\mu^+ \mu^- \pi^0$ around the η signal region. Figure 6(b) shows the background contributions estimated from the pseudodata sample and the possible $\eta \rightarrow \pi^0 \mu^+ \mu^-$ signal with an arbitrary normalization. By fitting to $M(\mu^+ \mu^- \pi^0)$, we can determine the possible $\eta \rightarrow \pi^0 \mu^+ \mu^-$ signal yields. Together with the estimated efficiency, the upper limit on the branching fraction for $\eta \rightarrow \pi^0 \mu^+ \mu^-$ is expected to reach 8.5×10^{-8} with 1×10^{12} J/ψ events at STCF, which is improved by 2 orders of magnitude compared with the PDG value [8] and quite close to the theoretical prediction.

VI. SUMMARY

Despite the impressive progress on the investigation of η mesons that has been achieved in recent years, the data on the decay modes of the η are still scarcer and much less accurate than those for the pions and kaons. The reason is that the η mesons were produced with low intensity, which inspired new facilities proposed to be dedicated to explore the η/η' decays [38,39]. Moreover, the STCF is unique since the charmonium decay (J/ψ) provide very clean laboratory of studying light meson decays as advocated by the BESIII experiment [9].

For the investigation on the η decays, since its production rate of $J/\psi \rightarrow \gamma\eta$ is 5 times less than that of η' in J/ψ radiative decays and the irreducible background contributions directly from both J/ψ decays and e^+e^- annihilations, it is hard to improve the sensitivity for exploring the η rare decays. However, $\eta' \rightarrow \pi^+ \pi^- \eta$ is one of dominant

decays with a branching fraction of $(42.5 \pm 0.5)\%$ [8] and the η mesons could be well tagged; these features make the decay of $\eta' \rightarrow \pi\pi\eta$ particularly attractive for the study of η decays, which inspired us to present a proposal for exploring the η decays by tagging η with $\eta' \rightarrow \pi^+ \pi^- \eta$ at the STCF [9].

STCF was proposed to perform an extensive study of τ -charm physics [9] and the designed luminosity is about 100 times larger than that of BEPCII. Therefore, the unprecedented charmonium decays, e.g., J/ψ and $\psi(2S)$, are expected to be accumulated in one year. We then present several feasibility studies on η decays with the fast simulation package developed for STCF. The examples are not intended to deliver an applicable message for this novel approach; instead, they are provided to illustrate the STCF capabilities to fulfill this physics program. The MC study indicates that STCF opens the possibility to investigate the η decays with an excellent sensitivity and may make feasible observation of η rare decays. Actually, the above study also advocates that the available 10×10^9 J/ψ events [17] at BESIII can already yield a series of measurements, such as $\eta \rightarrow 2\pi$ and $\eta \rightarrow l^+ l^- \pi^0$, with accuracy competitive with the current world averages.

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