Kaon and strangeonium spectrum in Regge phenomenology

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(Received 22 January 2023; revised 26 April 2023; accepted 10 July 2023; published 27 July 2023)

In the present work, the mass spectra of the light mesons, the kaons ($u\bar{s}$), and strangeonium ($s\bar{s}$) is systematically studied within the framework of Regge phenomenology. Several relations between Regge slope, intercept, and meson masses are extracted with the assumption of linear Regge trajectories. Using these relations, the ground state masses (1^1S_0 and 1^3S_1) of the pure $s\bar{s}$ states are evaluated. Further, the Regge slopes are extracted for kaons and strangeonium to obtain the orbitally excited state masses in the (J, M^2) plane. Similarly, the values of Regge parameters are calculated in the (n, M^2) plane for each Regge trajectory and we obtain the radially excited state masses of mesons lying on that Regge trajectory. We compared our obtained spectrum with the experimental observations where available and with the predictions of other theoretical approaches. Here, we predict the possible quantum numbers of several recently observed experimental states, which still require further verification, and also evaluate the higher orbital and radial excited states that may be detected in the near future. We expect our predicted results could provide valuable information for future experimental searches for missing excited kaons and strangeonium mesons.

DOI: 10.1103/PhysRevD.108.014034

I. INTRODUCTION

As a result of significant experimental progress in recent years, many new light hadronic states have been observed, and studying the properties of these states has been of great interest. The quark model holds that mesons are made up of quark and antiquark ($q\bar{q}$), while baryons are composed of three quarks (qqq). In addition to these, many exotic states have also been detected experimentally, comprising more than three quarks. Experimental evidence of new hidden charm tetraquark states, the P_c pentaquark states [1,2], and the hexaquark candidates [3] has been observed in the past two decades. In a recent study, the heptaquark states composed of seven quarks, having two charmed mesons and one nucleon, is investigated [4]. In the present work, we concentrate on the light strange and strangeonium states.

Several strange mesons have been discovered and observed over the course of many years by various experimental facilities, such as *BABAR* [5], Belle [6], LHCb [7–9], BESIII [10,11], etc. Recently, e^+e^- collision experiments in BESIII and *BABAR* provided data over 2.0–2.2 GeV with excellent precision, which helped to study the excitations of light mesons [12–15]. There are ideas and plans for future measurements of strange mesons at other experimental facilities, including LHCb, Belle II, and BESIII Collaborations, employing either τ or D- meson decays, and also the upcoming experimental facility PANDA seeking these strange mesons [16]. In addition, J-PARC will soon construct a new kaon beam line [17]. In the most recent Particle Data Group (PDG) review [18], 14 kaon mesons with their corresponding spin and parity are firmly known, but 11 states still need to be confirmed, with two of them, $K(1630)$ and $K(3100)$, still awaiting the determination of their spin-parity quantum numbers. In the present work, we obtained the mass spectra of kaons and tried to assign the possible spin-parity to the observed states.

Other than strange mesons, an interesting quarkonium state named strangeonium ($s\bar{s}$) states predicted in the quark model, which are bound states of the s quark and anti-s quark, lie between the light up, down quarks and heavy charm, bottom quarks. The study of strangeonium states is related to non-$q\bar{q}$ states (glueballs, hybrids, and tetraquarks, etc.) with the same quantum numbers as conventional $q\bar{q}$ systems. For experimental confirmation of non-$q\bar{q}$ states, one must have a thorough understanding of the conventional $q\bar{q}$ states. There are very few strangeonium resonances listed in the PDG [18] that have been experimentally well confirmed and are widely accepted as pure $s\bar{s}$ states. Except for certain low-lying $1P$ and $1D$ states, many excited strangeonium states have yet to be detected. To study these states, the BESIII experiment provides a strong platform by not only confirming the previously observed $s\bar{s}$ states, but also discovering some new resonances by the decays of J/ψ and $\psi(2S)$ [19–24]. For instance, in 2016, several states around 2.0–2.4 GeV were detected at BESIII; the $f_2(2010)$ resonance mentioned in PDG [18] was confirmed in this process. Another state $X(2500)$ observed with mass $M = 2470^{+15+101}_{-19-23}$ MeV still needs more confirmation [24].

In 2018, the BESIII Collaboration detected several isoscalar mesonic states, out of which the $f_0(2330)$ state with $J^{PC} = 0^{++}$ might be a good candidate of strangeonium state [23]. Experimentally, this resonance still needs more confirmation. The LASS Collaboration observed a narrow resonance $f_4(2210)$ with $J^{PC} = 4^{++}$ having mass 2209^{+17}_{-15} MeV [25]. Later on, the results obtained by the MARK-III Collaboration [26] and WA67 (CERN SPS) [27] matches with the observation of [25]. This resonance is still not confirmed and needs further verification. Recently, in 2019, evidence of new resonances $X(2060)$ having mass $M = (2062.8 \pm 13.1 \pm 7.2)$ MeV and $X(2000)$ with mass $M = (2002.1 \pm 27.5 \pm 15.0)$ MeV were observed in $J/\psi \to \phi\eta\eta'$ at BESIII [22]. These might be the candidates of $s\bar{s}$ states and need more confirmation. In a recent experimental study, the resonance $\phi(2170)$, listed in PDG [18], was also confirmed by BESIII Collaboration [28,29]. Also, the most precise resonance parameters for the state $h_1(1415)$ were also determined at BESIII [30].

Other than the experimental study of the kaon and strangeonium families, it is very interesting to study these mesons theoretically as well. A vast amount of literature is available on the spectroscopy of light mesons, using various theoretical approaches, including relativistic quark model [31,32], nonrelativistic linear potential quark model [33], QCD-motivated relativistic quark model [34], and the Regge trajectory approach [35–37]. In Refs. [38,39], a detailed analysis of light pseudoscalar and vector mesons is carried out and $q\bar{q}$ spectrum is studied in a generalized constituent quark model. In the recent article, the authors revisited the kaon spectrum in which the quark-antiquark interaction is based on the nonperturbative phenomena of dynamical chiral symmetry breaking and color confinement as well as the perturbative one-gluon exchange force [40]. The authors of Ref. [41] give an overview of the strangeonium states and their experimental status, and the experimental techniques are described to explore strangeonium with the BESIII detector. The mass spectrum of the kaon family is analyzed by using the modified Godfrey-Isgur model with a color screening effect in Ref. [42].

Since various theoretical approaches give different predictions for these mesons, more comparison of computations with the experimental measurements is required to identify the excited kaon and strangeonium mesons. The main aim of the present work is to systematically study the light strange mesons and the pure $s\bar{s}$ states and to discuss the possible spin-parity of the experimentally observed excited resonances, as well as to predict the J^P values of the resonances that are yet to be confirmed experimentally. With the assumption of the existence of the quasilinear Regge trajectories, relations between Regge slope, intercept, and the meson masses have been extracted. With the help of these derived relations, the ground state masses of strangeonium are computed. Further, the Regge parameters of kaon and strangeonium mesons are extracted to obtain the orbitally and radially excited state masses in both the (J, M^2) and (n, M^2) planes.

The paper is structured as follows. In Sec. II, a detailed description of the theoretical model is presented and the complete calculation of mass spectra of kaon and strangeonium is given. Regge trajectories are drawn in both (J, M^2) and (n, M^2) planes. Section III is mostly devoted to the discussion of our obtained results and comparing our masses with those produced by other theoretical approaches. Finally, we concluded our work in Sec. IV.

II. THEORETICAL FRAMEWORK

One of the most defining aspects of Regge phenomenology is the Regge trajectories, which relate the mass and the spin of hadrons. Several theories have been put forth to explain the Regge trajectories. In the 1970s, Nambu [43,44] proposed one of the simplest explanations for linear Regge trajectories by assuming the uniform interactions between quark and antiquark pair which results in the formation of a strong flux tube. Light quarks rotating with the speed of light at radius R at the end of the tube and the mass originating in this flux tube is estimated as [45]

$$M = 2\int_0^R \frac{\sigma}{\sqrt{1-\nu^2(r)}} dr = \pi\sigma R, \tag{1}$$

where σ represents the string tension, i.e., the mass density per unit length. $\nu(r)$ is the velocity as a function of the distance from the center of the string. Similarly, the angular momentum of this flux tube is estimated as

$$J = 2\int_0^R \frac{\sigma r \nu(r)}{\sqrt{1-\nu^2(r)}} dr = \frac{\pi\sigma R^2}{2} + c', \tag{2}$$

From the above relations, one can also write

$$J = \frac{M^2}{2\pi\sigma} + c'', \tag{3}$$

where c' and c'' are the constants of integration. Hence we can say that, J and M^2 are linearly related to each other. Assuming the linear Regge trajectories for light mesons, one can write the most general form of the linear Regge trajectories as [46–48]

$$J = \alpha(M) = a(0) + \alpha' M^2, \tag{4}$$

where $a(0)$ and α' represent the intercept and slope of the trajectory, respectively. For a meson multiplet, the Regge parameters for different quark constituents can be related by the following relations:

$$a_{i\bar{i}}(0) + a_{j\bar{j}}(0) = 2a_{i\bar{j}}(0), \tag{5}$$

$$\frac{1}{\alpha'_{i\bar{i}}} + \frac{1}{\alpha'_{j\bar{j}}} = \frac{2}{\alpha'_{i\bar{j}}}, \tag{6}$$

where i and j represent quark flavors. Equation (5), the additivity of intercepts, was derived from the dual resonance model [49] and was found to be satisfied in two-dimensional QCD [50] and the quark bremsstrahlung model [51]. Equation (6), the additivity of inverse slopes, was derived in a model based on the topological expansion and the quark-antiquark string picture of hadrons [52]; it is also satisfied in the formal chiral limit and the heavy quark limit for both mesons and baryons [53].

Using Eqs. (4) and (5), after solving them, we get

$$\alpha'_{i\bar{i}}M^2_{i\bar{i}} + \alpha'_{j\bar{j}}M^2_{j\bar{j}} = 2\alpha'_{i\bar{j}}M^2_{i\bar{j}}. \tag{7}$$

Combining Eqs. (6) and (7), two pairs of solutions are obtained in terms of slope ratios and mesons masses, which are expressed as [46]

$$\frac{\alpha'_{j\bar{j}}}{\alpha'_{i\bar{i}}} = \frac{1}{2M^2_{j\bar{j}}} \times \left[\left(4M^2_{i\bar{j}} - M^2_{i\bar{i}} - M^2_{j\bar{j}}\right) \pm \sqrt{(4M^2_{i\bar{j}} - M^2_{i\bar{i}} - M^2_{j\bar{j}})^2 - 4M^2_{i\bar{i}}M^2_{j\bar{j}}}\right] \tag{8}$$

and

$$\frac{\alpha'_{i\bar{j}}}{\alpha'_{i\bar{i}}} = \frac{1}{4M^2_{i\bar{j}}} \times \left[\left(4M^2_{i\bar{j}} + M^2_{i\bar{i}} - M^2_{j\bar{j}}\right) \pm \sqrt{(4M^2_{i\bar{j}} - M^2_{i\bar{i}} - M^2_{j\bar{j}})^2 - 4M^2_{i\bar{i}}M^2_{j\bar{j}}}\right]. \tag{9}$$

Now, using Eq. (8), we can extract the high-power mass equalities for mesons, which are expressed as [46]

$$\frac{\alpha'_{j\bar{j}}}{\alpha'_{i\bar{i}}} = \frac{\alpha'_{k\bar{k}}}{\alpha'_{i\bar{i}}} \times \frac{\alpha'_{j\bar{j}}}{\alpha'_{k\bar{k}}}, \tag{10}$$

where k can be any quark flavor. We have

$$\frac{\left[\left(4M^2_{i\bar{j}} - M^2_{i\bar{i}} - M^2_{j\bar{j}}\right) + \sqrt{(4M^2_{i\bar{j}} - M^2_{i\bar{i}} - M^2_{j\bar{j}})^2 - 4M^2_{i\bar{i}}M^2_{j\bar{j}}}\right]}{2M^2_{j\bar{j}}} = \frac{\left[\left(4M^2_{i\bar{k}} - M^2_{i\bar{i}} - M^2_{k\bar{k}}\right) + \sqrt{(4M^2_{i\bar{k}} - M^2_{i\bar{i}} - M^2_{k\bar{k}})^2 - 4M^2_{i\bar{i}}M^2_{k\bar{k}}}\right]/2M^2_{k\bar{k}}}{\left[\left(4M^2_{j\bar{k}} - M^2_{j\bar{j}} - M^2_{k\bar{k}}\right) + \sqrt{(4M^2_{j\bar{k}} - M^2_{j\bar{j}} - M^2_{k\bar{k}})^2 - 4M^2_{j\bar{j}}M^2_{k\bar{k}}}\right]/2M^2_{k\bar{k}}}. \tag{11}$$

In terms of different flavors of meson masses, we have derived the general relation expressed above, which can be used to evaluate the mass of any meson state if all other masses are known.

A. Excited state masses of kaons and strangeonium mesons in the (J,M^2) plane

In this section, first we obtained the ground state (1^1S_0 and 1^3S_1) masses of strangeonium meson. Since, experimentally, it is very difficult to measure the masses of the pure strangeonium state due to the usual mixing of pure $n\bar{n}$ and $s\bar{s}$ states, in the present work we do not consider the states with mixing of flavors. Further, the orbitally excited state masses of strangeonium and kaons are evaluated in the (J, M^2) plane using the above extracted relations. To determine the ground state masses, we use relation (11). Since strangeonium mesons are composed of one strange and one antistrange quark, we put $i = n$ (u or d), $j = s$, and $k = c$ in Eq. (11) and get the quadratic mass expression in terms of well-established light and heavy mesons, which is expressed as

$$\frac{\left[\left(4M^2_{n\bar{s}} - M^2_{n\bar{n}} - M^2_{s\bar{s}}\right) + \sqrt{(4M^2_{n\bar{s}} - M^2_{n\bar{n}} - M^2_{s\bar{s}})^2 - 4M^2_{n\bar{n}}M^2_{s\bar{s}}}\right]}{2M^2_{s\bar{s}}} = \frac{\left[\left(4M^2_{n\bar{c}} - M^2_{n\bar{n}} - M^2_{c\bar{c}}\right) + \sqrt{(4M^2_{n\bar{c}} - M^2_{n\bar{n}} - M^2_{c\bar{c}})^2 - 4M^2_{n\bar{n}}M^2_{c\bar{c}}}\right]}{\left[\left(4M^2_{s\bar{c}} - M^2_{s\bar{s}} - M^2_{c\bar{c}}\right) + \sqrt{(4M^2_{s\bar{c}} - M^2_{s\bar{s}} - M^2_{c\bar{c}})^2 - 4M^2_{s\bar{s}}M^2_{c\bar{c}}}\right]}. \tag{12}$$

By inserting the experimentally observed mass values of $n\bar{n}$, $n\bar{s}$, $n\bar{c}$, $s\bar{c}$, and $c\bar{c}$ mesons for $J^P = 0^-$ from PDG [18] in the above relation, we get $M_{s\bar{s}} = 695.8$ MeV. Similarly, we can obtain the ground state mass for $J^P = 1^-$ as 1005.6 MeV.

Now for the determination of orbitally excited state masses of kaons ($n\bar{s}$) and strangeonium ($s\bar{s}$), first we extract the values of Regge slopes (α') using the relations (8) and (9) for these light flavored mesons. For instance, according to the quark composition of kaons, we insert $i = n$ and $j = s$ into Eq. (9), and we get an expression in terms of slope ratios and light flavored mesons,

$$\frac{\alpha'_{n\bar{s}}}{\alpha'_{n\bar{n}}} = \frac{1}{4M^2_{n\bar{s}}} \times \left[\left(4M^2_{n\bar{s}} + M^2_{n\bar{n}} - M^2_{s\bar{s}}\right) + \sqrt{(4M^2_{n\bar{s}} - M^2_{n\bar{n}} - M^2_{s\bar{s}})^2 - 4M^2_{n\bar{n}}M^2_{s\bar{s}}}\right]. \quad (13)$$

Since we have taken the experimental masses as inputs, we have also incorporated the experimental errors while calculating the Regge parameters and evaluating the excited state masses. Hence, we insert the values $M_{n\bar{n}} = 134.98 \pm 0.0005$ MeV and $M_{n\bar{s}} = 497.61 \pm 0.013$ MeV from PDG [18] and $M_{s\bar{s}}$ obtained above for $J^P = 0^-$ into Eq. (13). Now, with the help of $\alpha'_{n\bar{n}} = 2/(M_{\pi_2(1670)} - M_\pi)$, we can get the value of $\alpha'_{n\bar{s}}$ as 0.7101 ± 0.001 GeV^{-2} for 1^1S_0 trajectory. Here we expressed our obtained Regge slopes as $\alpha' \pm \delta\alpha'$, where $\delta\alpha'$ is the experimental error in the slope. Similarly, we can calculate the Regge slope $\alpha'_{n\bar{s}} = 0.8203 \pm 0.02$ GeV^{-2} for 1^3S_1 trajectory.

In the same manner for strangeonium ($s\bar{s}$), which is composed of one strange quark and one antistrange quark, we put $i = n$ and $j = s$ in Eq. (8) and get

$$\frac{\alpha'_{s\bar{s}}}{\alpha'_{n\bar{n}}} = \frac{1}{2M^2_{s\bar{s}}} \times \left[\left(4M^2_{n\bar{s}} - M^2_{n\bar{n}} - M^2_{s\bar{s}}\right) \pm \sqrt{(4M^2_{n\bar{s}} - M^2_{n\bar{n}} - M^2_{s\bar{s}})^2 - 4M^2_{n\bar{n}}M^2_{s\bar{s}}}\right]. \quad (14)$$

Again inserting the values of $M_{n\bar{n}}$, $M_{n\bar{s}}$, and $M_{s\bar{s}}$ in the above relation and extracting the values of Regge slopes for 0^- and 1^- trajectories as 0.6750 ± 0.001 and 0.7721 ± 0.032 GeV^{-2}, respectively, in the similar manner. From Eq. (4), an another relation can be obtained in terms of meson masses and Regge slope, which is expressed as

$$M_{J+1} = \sqrt{M_J^2 + \frac{1}{\alpha'}}. \quad (15)$$

Hence, with the help of Eq. (15) and extracted values of $\alpha'_{n\bar{s}}$ and $\alpha'_{s\bar{s}}$, we can obtain the excited state masses of kaon and strangeonium mesons lying on 1^1S_0 and 1^3S_1 trajectories for unnatural ($1^1P_1, 1^1D_2, 1^1F_3\ldots$) and natural ($1^3P_2, 1^3D_3, 1^3F_4\ldots$) parity states. Now, other than natural and unnatural parities, we have also calculated the other remaining states in the same manner. First, using Eq. (12) we have obtained the masses for 1^3P_0 and 1^3P_1 strangeonium states as we evaluated for the ground states earlier by inserting the masses of other light and heavy mesons from PDG [18] for 1^3P_0 and 1^3P_1 states. For some mesonic states, due to the unavailability of experimental masses, we have taken the input masses from Ref. [34]. Further, the Regge slopes for 1^3P_0 and 1^3P_1 trajectories have been extracted using the same procedure to calculate the higher excited states $1^3D_1, 1^3F_2, 1^3G_4\ldots$ and $1^3D_2, 1^3F_3, 1^3G_5\ldots$, respectively, lying on those trajectories. We have include the experimental error in the calculated excited state masses. We expressed our obtained mass as $M \pm \delta M$, where M and δM denote the calculated mass and the experimental error in the mass, respectively. The estimated results calculated in the (J, M^2) plane are represented in Tables I and II for kaons and strangeonium, respectively, along with the comparison of experimental values where available and the predictions of other theoretical approaches.

B. Excited state masses of kaons and strangeonium mesons in (n, M^2) plane

After obtaining the orbitally excited state masses of kaon and strangeonium mesons, in this section the Regge parameters have been extracted and radial excitations are obtained in the (n, M^2) plane. The general equation for linear Regge trajectories in the (n, M^2) plane can be expressed as

$$n = \beta_0 + \beta M^2, \quad (16)$$

where n represents the radial principal quantum number, $1, 2, 3, \ldots$, while β_0 and β are the intercept and slope of the trajectories. The meson multiplets lying on the single Regge trajectory have the same Regge slope (β) and Regge intercept (β_0). Using relation (16), the values of β and β_0 for these light mesons can be extracted for S, P, D,... states to evaluate the excited state masses lying on Regge trajectories. For instance, for strange mesons (kaons), with the help of the slope equation, we can have $\beta_{(S)} = 1/(M^2_{K(2S)} - M^2_{K(1S)})$. Here the mass values of first radial excitations ($n = 2$) are taken as inputs from PDG [18], the experimentally observed masses wherever available. Whereas, due to the unavailability of experimental masses in some states, we have taken the theoretical predictions of [34] for the calculation. Here also, the error analysis is incorporated for the calculation of Regge parameters. Hence, by putting the values of $M_{K(1S)}$ and $M_{K(2S)}$ for $J^P = 0^-$, we can get $\beta_{(S)} = 0.51285 \pm 0.003$ GeV^{-2} for S states with spin $s = 0$. Now, from relation (16), we can write

$$1 = \beta_{0(S)} + \beta_{(S)} M^2_{K(1S)},$$
$$2 = \beta_{0(S)} + \beta_{(S)} M^2_{K(2S)}. \quad (17)$$

TABLE I. Excited state masses of kaons in the (J, M^2) plane (in MeV). The numbers in boldface are taken as inputs.

$N^{2S+1}L_J$	This work	Mesons	PDG [18]	[34]	[42]	[40]	[31]	[38]
1^1S_0	$\mathbf{497.61 \pm 0.013}$	$K^{\pm}$	497.61 ± 0.013	482	497.7	481	462	496
1^3S_1	$\mathbf{891.67 \pm 0.26}$	$K^*(892)$	891.67 ± 0.26	897	896	900	903	910
1^1P_1	1286.81 ± 0.77	$K_1(1270)$	1253 ± 7	1294	1364	1370	1352	1372
1^3P_0	1362.00 ± 0.00			1362	1257	1305	1234	1394
1^3P_1	1403.00 ± 7.00			1412	1377	1455	1366	
1^3P_2	1419.20 ± 10.47	$K_2^*(1430)$	1425.6 ± 1.5	1424	1431	1454	1428	1450
1^1D_2	1750.46 ± 0.80	$K_2(1770)$	1773 ± 8	1709	1778	1760	1791	1747
1^3D_1	1700.77 ± 30.12	$K^*(1680)$	1718 ± 18	1699	1766	1787	1776	1698
1^3D_2	1756.84 ± 61.44	$K_2(1820)$	1819 ± 2	1824	1789	1854	1804	1741
1^3D_3	1798.11 ± 11.69	$K_3^*(1780)$	1776 ± 7	1789	1781	1810	1794	1766
1^1F_3	2114.79 ± 0.81			2009	2075	2047	2131	
1^3F_2	1982.46 ± 36.54			1964	2093	2095	2151	1968
1^3F_3	2050.50 ± 74.29			2080	2084	2132	2143	
1^3F_4	2110.04 ± 12.20	$K_4^*(2045)$	2054 ± 9	2096	2058	2080	2108	
1^1G_4	2424.99 ± 0.82			2255	2309	2270	2422	
1^3G_3	2228.84 ± 39.81			2207	2336	2316	2458	
1^3G_4	2307.08 ± 80.81			2285	2317	2337	2433	
1^3G_5	2381.46 ± 12.48	$K_5^*(2380)$	2382 ± 24	2356	2286	2291	2388	
1^1H_5	2699.79 ± 0.82					2442		
1^3H_4	2450.57 ± 41.81					2477		
1^3H_5	2537.85 ± 84.80					2489		
1^3H_6	2624.96 ± 12.66							

TABLE II. Excited state masses of strangeonium in the (J, M^2) plane (in MeV).

$N^{2S+1}L_J$	This work	Mesons	PDG [18]	[34]	[33]	[32]	[54]	[38]
1^1S_0	695.80 ± 0.00			743	797	675	690	956
1^3S_1	1005.63 ± 0.00	ϕ	1019.46 ± 0.016	1038	1017	1009	1020	1020
1^1P_1	1402.01 ± 0.78	$h_1(1415)$	1416 ± 8	1485	1462	1473	1460	1511
1^3P_0	1392.00 ± 0.00	$f_0(1370)$	$1350 \pm 9^{+12}_{-2}$ [23]	1420	1373	1355	1180	1340
1^3P_1	1514.90 ± 0.00			1464	1492	1480	1430	1508
1^3P_2	1518.70 ± 17.67	$f_2'(1525)$	1517.4 ± 2.5	1529	1513	1539	1480	1556
1^1D_2	1856.64 ± 0.83	$\eta_2(1870)$	1842 ± 8	1909	1825	1893	1830	1853
1^3D_1	1731.50 ± 30.24			1845	1809	1883	1750	
1^3D_2	1848.07 ± 95.98			1908	1840	1904	1810	
1^3D_3	1897.79 ± 19.99	$\phi_3(1850)$	1854 ± 7	1950	1822	1897	1830	1875
1^1F_3	2220.04 ± 0.85			2209	2111	2223	2130	
1^3F_2	2014.59 ± 36.75			2143	2146	2243	2090	
1^3F_3	2129.74 ± 117.78			2215	2128	2234	2120	
1^3F_4	2212.87 ± 21.01	$f_4(2210)$	2209^{+17}_{-15} [25]	2286	2078	2202	2130	
1^1G_4	2531.81 ± 0.86			2469		2507		
1^3G_3	2262.52 ± 40.08			2403				
1^3G_4	2378.29 ± 129.18			2481				
1^3G_5	2488.37 ± 21.57			2559				
1^1H_5	2809.19 ± 0.87			2706				
1^3H_4	2485.85 ± 42.12			2634				
1^3H_5	2603.21 ± 136.28			2720				
1^3H_6	2736.26 ± 21.93			2809				

Simplifying the above equations, we get $\beta_{0(S)} = 0.87301 \pm 0.0007$. With the help of $\beta_{(S)}$ and $\beta_{0(S)}$, we can evaluate the masses of the excited K-meson states for $n = 3, 4, 5\ldots$. Similarly, we can express these relations for P and D waves as

$$1 = \beta_{0(P)} + \beta_{(P)} M^2_{K(1P)},$$
$$2 = \beta_{0(P)} + \beta_{(P)} M^2_{K(2P)},$$
$$1 = \beta_{0(D)} + \beta_{(D)} M^2_{K(1D)},$$
$$2 = \beta_{0(D)} + \beta_{(D)} M^2_{K(2D)}. \quad (18)$$

With the help of above obtained relations, we have extracted the values of Regge slopes and intercepts for kaon and strangeonium mesons for each Regge trajectory with spin $s = 0$ and 1, and obtain the radially excited state masses lying on that trajectory. The predicted mass spectra for kaon and strangeonium are shown in Tables III and IV, respectively, along with the predictions of other theoretical approaches.

C. Regge trajectories in the (J, M^2) and (n, M^2) planes

In this section, based on the calculated results of kaons and strangeonium mesons, the plots of total angular momentum J and principal quantum number n against the square of resonance mass M^2 are constructed. Additionally, in all of our M^2 plots, we have also incorporated the error, which is given by $\Delta M^2 = \pm \Gamma M$, where Γ represents the width of the resonance. Hence, we have taken the width of each state as an estimate of the error of the resonance mass and this mass uncertainty is referred to as the half-width rule [37,55,56]. The squared mass of each meson is then represented as $M^2 \pm \Gamma M$. The numerical values of the resonance widths are the experimental data, which are taken from the latest PDG [18]. Tables V and VI represent the estimated mass uncertainty $\Delta M^2 = \pm \Gamma M$ of the states belonging to kaons and strangeonium mesons, respectively.

The Regge trajectories in the (J, M^2) plane are drawn for kaon and strangeonium with unnatural and natural parity states as shown in Figs. 1–4. The solid straight lines represent our calculated results and the cross symbol represents the experimental masses. Also, the error bars represent the uncertainty. Similarly, the trajectories in the (n, M^2) plane are depicted in Figs. (5)–(8).

III. RESULTS AND DISCUSSION

The main purpose of the present work is to study the strange mesons (kaons) and the states that can be considered as pure $s\bar{s}$. Here we do not consider the states having strong flavor mixing between $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$. The mass spectra of these light flavored mesons have been obtained successfully using Regge phenomenology. The numerical results are shown in Tables I–IV. We have also

TABLE III. Excited state masses of kaons in the (n, M^2) plane (in MeV). The numbers in boldface are taken as inputs.

$N^{2S+1}L_J$	This work	[40]	[42]	[31]	[38]
1^1S_0	497.61 ± 0.013	481	497.7	462	496
2^1S_0	$\mathbf{1482.40 \pm 3.60}$ [18]	1512	1457	1454	1472
3^1S_0	2036.52 ± 5.57	2018	1924	2065	1899
4^1S_0	2469.27 ± 6.74	2318	2248		
5^1S_0	2836.76 ± 7.75	2488			
6^1S_0	3161.82 ± 8.63	2467			
1^3S_1	891.67 ± 0.26	900	896	903	910
2^3S_1	$\mathbf{1675.00 \pm 0.00}$ [34]	1676	1548	1579	1620
3^3S_1	2194.57 ± 0.24	2112	1983	1950	
4^3S_1	2612.79 ± 0.27	2372	2287		
5^3S_1	2972.75 ± 0.31	2516			
6^3S_1	3293.59 ± 0.34	2576			
1^1P_1	1286.81 ± 0.77	1370	1364	1352	1372
2^1P_1	$\mathbf{1757.00 \pm 0.00}$ [34]	1925	1840	1897	1841
3^1P_1	2125.61 ± 1.66	2260	2177	2164	
4^1P_1	2439.14 ± 1.84	2458	2422		
5^1P_1	2716.72 ± 2.01	2556			
1^3P_0	1362.00 ± 0.00	1305	1257	1234	1213
2^3P_0	$\mathbf{1791.00 \pm 0.00}$ [34]	1894	1829	1890	1768
3^3P_0	2135.49 ± 0.00	2242	2176	2160	
4^3P_0	2431.66 ± 0.00	2447	2424		
5^3P_0	2695.48 ± 0.00	2552			
1^3P_1	1403.00 ± 7.00	1455	1377	1366	1394
2^3P_1	$\mathbf{1893.00 \pm 0.00}$ [34]	1971	1861	1928	1850
3^3P_1	2280.11 ± 15.44	2286	2192	2200	
4^3P_1	2610.39 ± 16.95	2471	2434		
5^3P_1	2903.34 ± 18.40	2561			
1^3P_2	1419.20 ± 10.47	1454	1431	1428	1450
2^3P_2	$\mathbf{1994.00 \pm 60.00}$ [18]	1975	1870	1938	
3^3P_2	2436.79 ± 158.22	2290	2198	2206	
4^3P_2	2810.66 ± 178.30	2474	2438		
5^3P_2	3140.34 ± 197.00	2563			
1^1D_2	1750.46 ± 0.80	1760	1778	1791	1747
2^1D_2	$\mathbf{2066.00 \pm 0.00}$ [34]	2160	2121	2238	
3^1D_2	2339.35 ± 3.17	2402	2380		
4^1D_2	2583.95 ± 3.35	2535	2570		
5^1D_2	2807.31 ± 3.53	2588			
1^3D_1	1700.77 ± 30.12	1787	1766	1776	1698
2^3D_1	$\mathbf{2063.00 \pm 0.00}$ [34]	2173	2127	2251	
3^3D_1	2370.51 ± 102.48	2408	2385		
4^3D_1	2642.47 ± 109.20	2537	2573		
5^3D_1	2888.95 ± 123.27	2585			
1^3D_2	1756.84 ± 61.44	1854	1789	1804	1741
2^3D_2	$\mathbf{2163.00 \pm 0.00}$ [34]	2214	2131	2254	
3^3D_2	2504.13 ± 194.07	2432	2388		
4^3D_2	2804.06 ± 207.83	2547	2575		
5^3D_2	3074.87 ± 222.09	2588			
1^3D_3	1798.11 ± 11.69	1810	1781	1794	1766
2^3D_3	$\mathbf{2182.00 \pm 0.00}$ [34]	2191	2131	2237	
3^3D_3	2507.80 ± 39.67	2421	2382		
4^3D_3	2795.88 ± 42.28	2543	2571		
5^3D_3	3056.94 ± 45.01	2587			

TABLE IV. Masses of excited states of the strangeonium in the (n, M^2) plane (in MeV). The numbers in boldface are taken as inputs.

$N^{2S+1}L_J$	This work	PDG [18]	[32]	[33]	[54]	[31]	[38]
1^1S_0	695.80 ± 0.00		657	797	690	960	956
2^1S_0	**1475.00 ± 4.00** [18]	1475 ± 4	1578	1619	1440	1630	1795
3^1S_0	1966.51 ± 6.70		2125	2144	1970		
4^1S_0	2357.68 ± 8.25		2568	2580	2260		
5^1S_0	2692.61 ± 9.40		2949				
6^1S_0	2990.26 ± 10.43		3328				
1^3S_1	1005.63 ± 0.00	1019.46 ± 0.016	1009	1017	1020	1020	1020
2^3S_1	**1680.00 ± 20.00** [18]		1688	1699	1740	1690	1726
3^3S_1	2152.56 ± 39.98	2162 ± 7	2204	2198	2250		
4^3S_1	2538.62 ± 46.53		2627	2623	2540		
5^3S_1	2873.20 ± 52.42		2996				
6^3S_1	3172.83 ± 57.74		3327				
1^1P_1	1402.01 ± 0.78	1416 ± 8	1473	1462	1460	1470	1511
2^1P_1	**2024.00 ± 0.00** [34]		2008	1991	2040	2010	1973
3^1P_1	2495.51 ± 1.46		2449	2435	2490		
4^1P_1	2891.11 ± 1.62		2832				
5^1P_1	3238.75 ± 1.78		3174				
1^3P_0	1392.00 ± 0.00		1355	1373	1180	1360	1340
2^3P_0	**1909.00 ± 0.00** [34]		1986	1971	1800	1990	1894
3^3P_0	2411.69 ± 0.00	2411 ± 10 ± 7 [25]	2444	2434	2280		
4^3P_0	2784.88 ± 0.00		2834				
5^3P_0	3113.66 ± 0.00		3179				
1^3P_1	1514.90 ± 0.00		1480	1492	1430	1480	1508
2^3P_1	**2016.00 ± 0.00** [34]		2027	2027	2020	2030	
3^3P_1	2415.28 ± 0.00		2468	2470	2480		
4^3P_1	2757.34 ± 0.00		2850				
5^3P_1	3061.41 ± 0.00		3191				
1^3P_2	1518.70 ± 17.67	1517.4 ± 2.5	1539	1513	1480	1530	1556
2^3P_2	**2011.00 ± 60.00** [18]		2046	2030	2080	2040	1999
3^3P_2	2404.54 ± 184.30		2480	2466	2540		
4^3P_2	2742.16 ± 204.13		2859				
5^3P_2	3042.55 ± 223.12		3198				
1^1D_2	1856.64 ± 0.83	1842 ± 8	1893	1825	1830	1890	1853
2^1D_2	**2321.00 ± 0.00** [34]		2336	2282	2340		
3^1D_2	2706.84 ± 2.42		2723	2685			
4^1D_2	3044.16 ± 2.60		3070				
5^1D_2	3347.66 ± 2.79		3387				
1^3D_1	1731.50 ± 30.24		1883	1809	1750	1880	
2^3D_1	**2258.00 ± 0.00** [34]		2342	2272	2260		
3^3D_1	2683.10 ± 75.03		2732	2681			
4^3D_1	3049.51 ± 81.69		3079				
5^3D_1	3376.39 ± 88.39		3395				
1^3D_2	1848.07 ± 95.98		1904	1840	1810	1910	
2^3D_2	**2323.00 ± 0.00** [34]		2348	2297	2330		
3^3D_2	2716.12 ± 275.84		2734	2701			
4^3D_2	3059.13 ± 297.30		3080				
5^3D_2	3367.38 ± 319.27		3396				
1^3D_3	1897.79 ± 19.99	1854 ± 7	1897	1822	1830	1900	1875
2^3D_3	**2338.00 ± 0.00** [34]		2337	2285	2360		
3^3D_3	2707.56 ± 62.94		2725	2691			
4^3D_3	3032.41 ± 67.42		3073				
5^3D_3	3325.69 ± 72.06		3390				

TABLE V. The uncertainty in the squared masses of kaons, which is given by $\Delta M^2 = \pm\Gamma M$. The numerical values of widths (Γ) are the experimental values taken from PDG [18].

$N^{2S+1}L_J$	Resonances	Experimental mass [18] (MeV)	Mass M (ours) (MeV)	Width (Γ) [18] (MeV)	$\Delta M^2 = \pm\Gamma M$ (GeV^2)
1^3S_1	$K(892)$	891.67	891.67	51.4	0.0458
2^1S_0	$K(1460)$	1482.40	1482.40	335.60	0.4975
1^1P_1	$K_1(1270)$	1253.00	1286.81	90	0.1158
1^3P_2	$K_2^*(1430)$	1425.60	1419.20	100	0.1419
2^3P_2	$K_2^*(1980)$	1994.00	1994.00	348	0.6939
1^1D_2	$K_2(1770)$	1773.00	1750.46	186	0.3256
1^3D_3	$K_3^*(1780)$	1776.00	1798.11	161	0.2895
1^3F_4	$K_4^*(2045)$	2054.00	2110.04	199	0.4199
1^3G_5	$K_5^*(2380)$	2382.00	2381.46	178	0.4239

TABLE VI. The uncertainty in the squared masses of strangeonium, which is given by $\Delta M^2 = \pm\Gamma M$. The numerical values of widths (Γ) are the experimental values taken from PDG [18].

$N^{2S+1}L_J$	Resonances	Experimental mass (MeV)	Mass M (ours) (MeV)	Width (Γ) [18] (MeV)	$\Delta M^2 = \pm\Gamma M$ (GeV^2)
1^3S_1	$\phi(1020)$	1019.46	1005.63	4.25	0.0043
2^1S_0	$\eta(1475)$	1475.00	1475.00	90	0.1328
2^3S_1	$\phi(1680)$	1680.00	1680.00	150	0.2520
3^3S_1	$\phi(2170)$	2162.00	2152.56	103	0.2217
1^1P_1	$h_1(1415)$	1416.00	1402.01	78	0.1094
1^3P_2	$f_2'(1525)$	1517.40	1518.70	86	0.1306
1^1D_2	$\eta_2(1870)$	1842.00	1856.64	225	0.4177
1^3D_3	$\phi_3(1850)$	1854.00	1897.79	87	0.1651
1^3F_4	$f_4(2210)$	2209.00	2212.87	60	0.1328

given a global picture of the predicted kaon and strangeonium mass spectra along with the comparison of experimentally observed states for more clear picture, as shown in Figs. 9 and 10. The colored straight lines represent the predicted masses obtained from our model, and the cross sign represents the experimentally detected masses mentioned in PDG [18]. From Figs. 9 and 10, we can say that our estimated masses are very close to the experimental masses. The detailed discussion of the calculated theoretical predictions for kaons and strangeonium is given below.

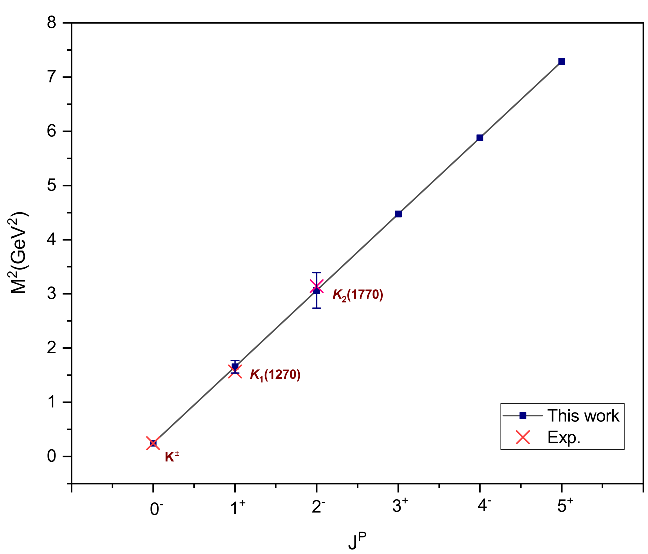

FIG. 1. Regge trajectory in the (J, M^2) plane for kaon with unnatural parity states. Error bars correspond to take $\Delta M^2 = \pm\Gamma M$.

A. Kaon spectrum

With the amount of experimental data available for strange mesons, we compared our obtained results presented in Tables I and III with well-established experimental masses and also with the predictions obtained from other theoretical approaches. The orbitally excited state masses calculated for kaons in the (J, M^2) plane are shown in Table I. The low-lying $1P$ state $K_1(1270)$ having spin-parity 1^+ listed in PDG [18], with well-established mass 1253 ± 7 MeV, is close to our predicted mass 1286.81 ± 0.77 MeV for 1^1P_1 state with a mass difference of 33.8 MeV. The studies [34,40] also assigned $K_1(1270)$ resonance to be a 1^1P_1 state. Also, our predicted mass shows a good agreement with the results obtained in the relativistic quark model [34], having a mass difference

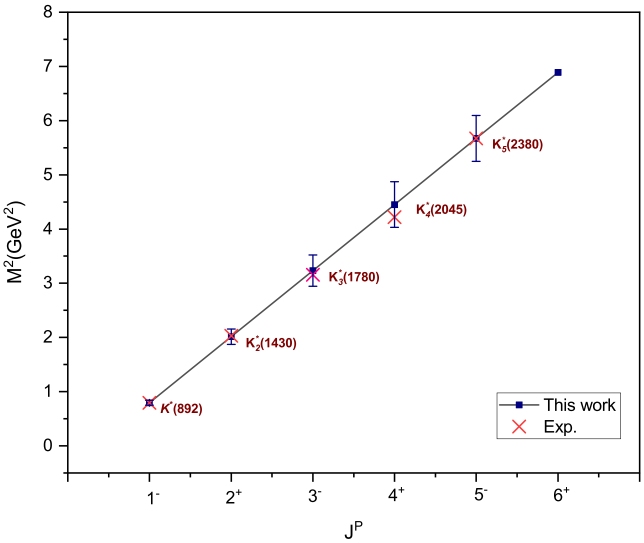

FIG. 2. Regge trajectory in the (J, M^2) plane for kaon with natural parity states. Error bars correspond to take $\Delta M^2 = \pm\Gamma M$.

of only 8 MeV, and slightly lower than the predictions of [31,40,42].

Another $1P$ state, $K_2^*(1430)$ having $J^P = 2^+$ mentioned in PDG [18], with experimental mass 1425.6 ± 1.5 MeV, which is assigned to be 1^3P_2 by various theoretical studies [34,38,40], is observed to be very close to our calculated mass 1419.20 ± 10.47 MeV for the 1^3P_2 state, having a mass difference of only 6.4 MeV. We compared our calculated mass value with those obtained by other theoretical models. Very less mass difference in the range of 5–12 MeV is seen with the predicted masses of Refs. [31,34,42] and a slightly higher mass difference of around 50 MeV is shown with the results obtained in [38,54].

The strange meson $K(1680)$ reported in PDG [18], having measured mass 1718 ± 18 MeV with $J^P = 1^-$, can either belong to the 1^3D_1 or 2^3S_1 state. In the present work, the predicted mass for the 1^3D_1 state as 1700.77 ± 30.12 MeV agrees well with the experimental mass of $K(1680)$; hence we assign this resonance to the 1^3D_1 state. The two experimentally well-established K-meson resonances $K_2(1770)$ and $K_2^*(1820)$ are listed in PDG with quantum numbers $J^P = 2^-$. Our mass value prediction for the 1^1D_2 state as 1750.46 ± 0.80 MeV agrees very well with the experimental mass 1773 ± 8 MeV, with a mass difference of around 23 MeV. The predicted mass for the 1^3D_2 state is a bit smaller than the experimental mass of $K_2^*(1820)$. Another candidate $K_3^*(1780)$ reported in PDG, having measured mass 1776 ± 7 with spin-parity 3^-, belongs to the $1D$ state, and the model prediction for the 1^3D_3 state as 1798.11 ± 11.69 MeV is very close to the experimental mass, having a mass difference of 22 MeV. Also, our mass value predictions for $1D$ states shows a general agreement with the results obtained in other theoretical approaches [31,34,38,40,42].

The $K_4^*(2045)$ state collected in the PDG [18], with the confirmed spin-parity 4^+, has mass 2054 ± 9 MeV. Our obtained mass for the 1^3F_4 state is slightly higher than its experimentally observed mass, having a mass difference of 56 MeV. There is an agreement between our prediction by Regge phenomenology and the masses obtained by [31,34,40], while [42] predicts this state to be 52 MeV lower than our mass. The only state having spin-parity 5^+ listed in PDG [18] is $K_5^*(2380)$, with a mass of 2383 ± 24 MeV, which was discovered in 1986 but has

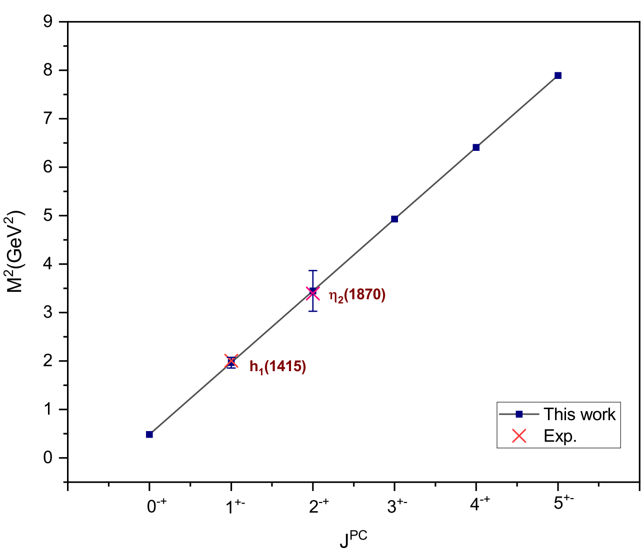

FIG. 3. Regge trajectory in the (J, M^2) plane for strangeonium with unnatural parity states. Error bars correspond to take $\Delta M^2 = \pm\Gamma M$.

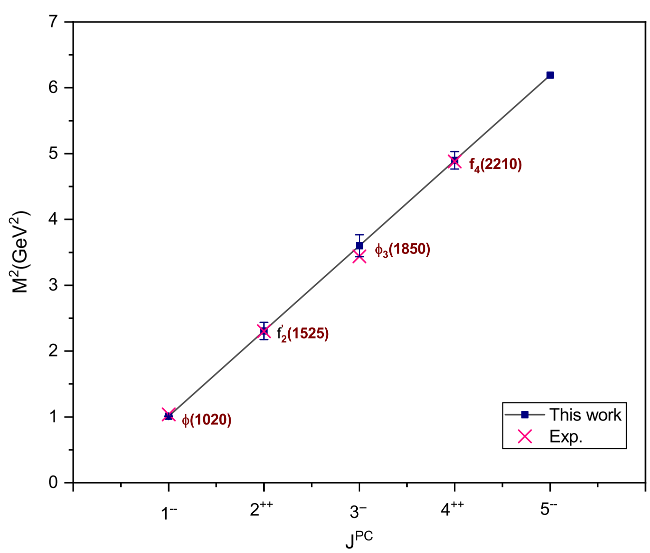

FIG. 4. Regge trajectory in the (J, M^2) plane for strangeonium with natural parity states. Error bars correspond to take $\Delta M^2 = \pm\Gamma M$.

not been validated by other experimental collaborations and this resonance needs more confirmation. Our calculated mass 2381.46 ± 12.48 for the 1^3G_5 state is almost the same as the experimental mass of the $K_5^*(2380)$ state, having a mass difference of less than 1 MeV. Hence, in this work, we predict this resonance belongs to the 1^3G_5 state. The results obtained in theoretical studies [31,34] are well in agreement with our predicted mass, having a mass difference of a few MeV, whereas Refs. [40,42] show a mass difference of 90–95 MeV with our estimated mass. The remaining excited states, whose experimental evidences are not yet seen, show a general agreement with the results obtained in other theoretical approaches [31,34,42].

Further, the radially excited state masses estimated in the present work for kaons are represented in Table III. The obtained mass spectra for strange mesons is compared with various theoretical approaches, and a general agreement can be seen with the results of Refs. [31,38,40,42].

Figures 5 and 6 depict the Regge plots for radial excitations. The trajectories for S, P, and D states are not exactly parallel and equidistant. The uncertainty in the resonance masses is shown by the error bars and quite large errors have been seen in the resonance masses of kaons.

B. Strangeonium spectrum

Since there are only a few experimentally well-established resonances observed till now that are acquired as pure $s\bar{s}$ states. The calculated ground state and the orbitally excited state masses in the (J, M^2) plane are summarized in Table II. The ground state (1^1S_0) mass 695.8 MeV predicted by our model is compared with various results obtained in different theories. It ranges from 650–960 MeV. Some theories predict the η' state mentioned in PDG [18] as the ground

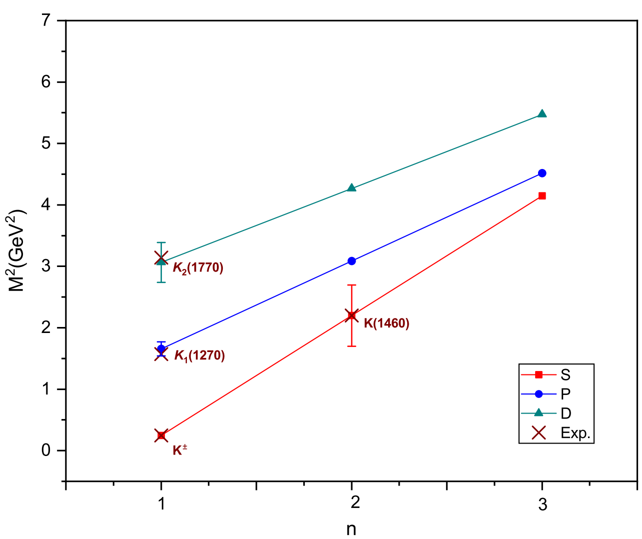

FIG. 5. Regge trajectories in the (n, M^2) plane for kaons with unnatural parity states. Error bars correspond to take $\Delta M^2 = \pm\Gamma M$.

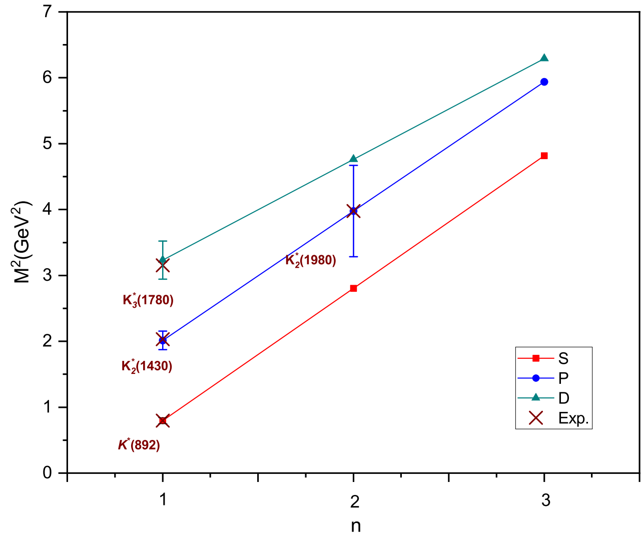

FIG. 6. Regge trajectories in the (n, M^2) plane for kaons with natural parity states. Error bars correspond to take $\Delta M^2 = \pm\Gamma M$.

state mass of strangeonium [31,38]. But some of the recent studies denied to accept this state as a strangeonium, as it is a mixed flavor state [32–34,54]. Our obtained mass is consistent with the prediction of [32–34,54], having a mass difference of a few MeV. The lowest S-wave vector state ϕ was first detected in a bubble chamber in 1962. It is well established and listed in PDG with $J^P = 1^-$, having mass * 1019.46 ± 0.016 MeV. The estimated mass for the 1^3S_1 state as 1005.63 MeV is well produced by our employed theory and seems to be very close to the experimental mass of ϕ, having a mass difference of 13.8 MeV only. Other than this, our calculated mass is also in good agreement with the mass obtained for the 1^3S_1 state by various theoretical models [32–34,54].

The $h_1(1415)$ resonance is listed by the PDG with $J^P = 1^+$ and mass 1416 ± 8 MeV. The recent BESIII measurements have improved the precision of the observed mass and width of $h_1(1415)$. Various theoretical studies predicted this state to be a good candidate of the 1^1P_1 $s\bar{s}$ state [32–34]. Our estimated mass 1402.01 ± 0.78 MeV for the 1^1P_1 state is in good agreement with the experimental mass of $h_1(1415)$, having a mass difference of 14 MeV. While it shows slight disagreement with the predictions of various theoretical models, our estimated mass seems to be lower than the results obtained in [32–34,38].

Another state, $f_2'(1525)$, which is widely accepted as the 1^3P_2 $s\bar{s}$ state listed in the PDG [18], with spin-parity 2^+ having mass 1517.4 ± 2.5 MeV, is very close to the mass predicted by our model as 1518.70 ± 17.67 MeV with a mass difference of only 1 MeV. Hence, the J^P of the experimentally observed state $f_2'(1525)$ is confirmed to be 2^+ in this work. Also, our estimated mass is consistent and very close to the outcomes of Refs. [32–34].

The experimental status of the 1^3P_0 $s\bar{s}$ state is a long-standing puzzle. The 0^{++} meson state $f_0(1370)$ mentioned in PDG [18] has a mass in the range of ≈1200–1500 MeV. Recently, the BESIII Collaboration [23] established the $f_0(1370)$ state with the measured mass $1350 \pm 9^{+12}_{-2}$ MeV, which is close to our predicted mass 1392.0 MeV with a mass difference of 42 MeV. Hence, $f_0(1370)$ can be a good candidate for the 1^3P_0 $s\bar{s}$ state.

The resonance $\eta_2(1870)$ listed in PDG [18] with $J^P = 2^-$, having averaged mass 1842 ± 8 MeV, is observed to be near to our estimated mass 1856.64 ± 0.83 MeV for the 1^1D_2 state with a mass difference of nearly 15 MeV. The relativistic quark model theory also suggests $\eta_2(1870)$ resonance as the $s\bar{s}$ state belongs to the 1^1D_2 state. Here, we assigned this state as the $1D$ $s\bar{s}$ state with $J^P = 2^-$ by our model. Also, our obtained mass value prediction shows a general agreement with the masses generated by various theoretical approaches. The results in Refs. [33,38,54] are close to the mass obtained by our model with a mass difference of few MeV, whereas the studies [32,34] show a slight large mass difference. One more state, $\phi_3(1850)$, which is accepted as $s\bar{s}$ resonance, is assigned to be 1^3D_3 state in recent theoretical studies [33,34]. This state was first identified in the $K\bar{K}$ invariant mass spectrum at CERN [57] and further experiments at CERN [58] and SLAC LASS [59] determined its spin-parity to be 3^-. Our estimated mass for the 1^3D_3 state as 1897.79 ± 19.99 MeV is slightly higher than the experimentally observed mass of the $\phi_3(1850)$ state as 1854 ± 7 MeV mentioned in PDG [18] with a mass difference of 43 MeV. The theoretical mass values predicted for this state in Refs. [32,38] are very close to the mass obtained by our model, having a mass difference of a few MeV, while a slight large mass difference can be seen with the results obtained in studies [33,34,54].

The LASS Collaboration detected a narrow resonance $f_4(2210)$ with J^{PC} value 4^{++} having a measured mass 2209^{+17}_{-15} MeV [25]. Later on, the mass and decay width of this resonance shows agreement with the results obtained by the MARKIII Collaboration [26] and WA67 (CERN SPS) [27]. Our predicted mass for 1^3F_4 as 2212.87 ± 21.01 MeV is very close to the experimentally detected mass [25]. Hence, the resonance $f_4(2210)$ might be a good candidate for the 1^3F_4 state.

Similarly, the radially excited state masses obtained in the present work for strangeonium are shown in Table IV. We compared our estimated $s\bar{s}$ states with available experimentally observed masses and the mass spectra obtained by different theoretical models. Again, a general trend of agreement is seen in the case of $s\bar{s}$ states. The $\phi(2170)$ resonance, which is mentioned in PDG [18] with spin-parity 1^-, is a possible radial $s\bar{s}$ state. Our calculated mass for the 3^3S_1 state as 2152.56 ± 39.98 MeV is very close to the experimental mass 2162 ± 7 MeV, having a mass difference of nearly 10 MeV. Hence we assigned this state to be a good candidate of the 3^3S_1 state. In the relativistic quark model [34], the authors also suggest that this state as a radial state belongs to 3^3S_1. Also, one more state, $f_0(2330)$ listed in PDG with $J^{PC} = 0^{++}$, needs more confirmation and the mass of this state is still not established. Our predicted mass for the 3^3P_0 state as 2411.69 MeV matches exactly the same with the mass observed at BESIII, which is $2411 \pm 10 \pm 7$ MeV by an amplitude analysis of the K_SK_S system produced in radiative J/ψ decays [23]. Also, our obtained mass is very close to the recent observation of [60], which predicts the

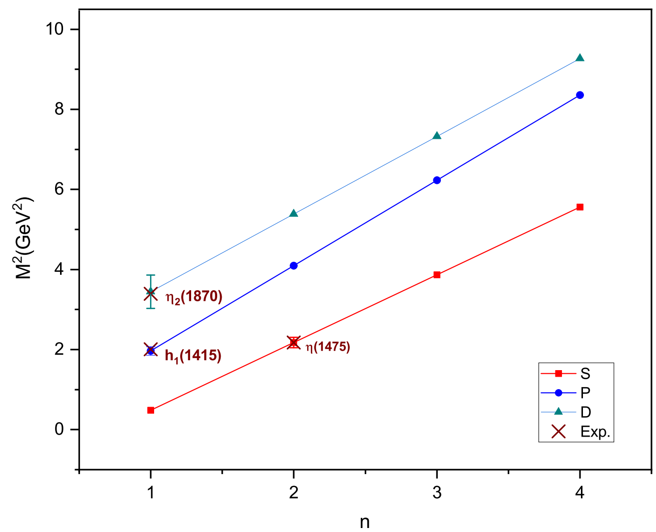

FIG. 7. Regge trajectories in the (n, M^2) plane for strangeonium with unnatural parity states. Error bars correspond to take $\Delta M^2 = \pm\Gamma M$.

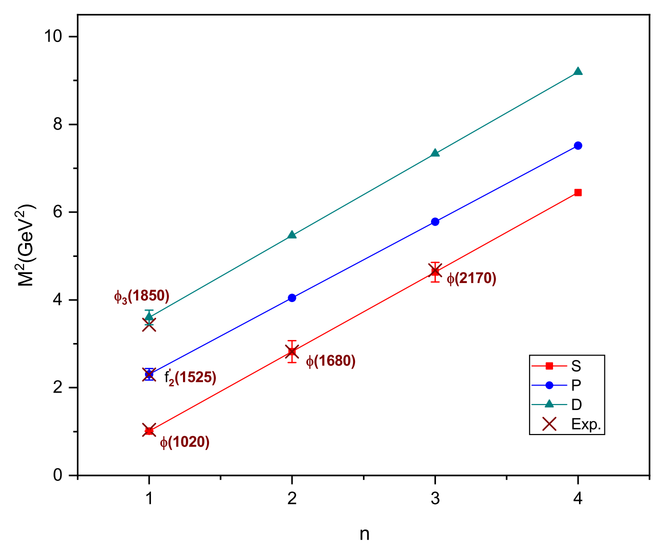

FIG. 8. Regge trajectories in the (n, M^2) plane for strangeonium with natural parity states. Error bars correspond to take $\Delta M^2 = \pm\Gamma M$.

mass of $f_0(2330)$ as 2419 ± 64 MeV. Hence the resonance $f_0(2330)$ listed by PDG may belong to the 3^3P_0 $s\bar{s}$ state. Also, the predicted mass spectra show good agreement with the outcomes of Refs. [32,54] with mass difference of few MeV, while some discrepancies can be shown with the results of Refs. [31,38].

The Regge trajectories plotted for the strangeonium meson in the (n, M^2) plane are shown in Figs. 7 and 8. A slight deviation is observed in the trajectories drawn for S, P, and D states from being exactly parallel and equally distant. The error bars in the line show uncertainty in the masses.

IV. CONCLUSION

Our model has been successfully employed to evaluate the spectrum of kaons and strangeonium. Unlike strangeonium, kaons have a large number of experimentally established states. In the present work, we confirmed the spin-parity of $K_1(1270)$, $K_2(1770)$, $K_2^*(1430)$, $K^*(1680)$, $K_2(1820)$, $K_3^*(1780)$, and $K_4^*(2045)$, which also gives a validation to the evaluated results from our employed model. The $K_5^*(2380)$ state needs more confirmation and we predict the quantum numbers of this resonance, which might be useful for future experimental progress.

Since only a few experimental data are available for pure strangeonium states, the J^P values of ϕ, $h_1(1415)$, $\eta_2(1870)$, $f_2'(1525)$, and $\phi_3(1850)$ states are confirmed in this work. Some recently observed resonances by various experimental groups still need further verification and have yet to be confirmed. In the present work, we try to predict the possible quantum numbers of such states that might be the candidates of strangeonium states, namely, $f_0(1370)$, $\phi(2170)$, $f_0(2330)$, and $f_4(2210)$ might be candidates of $s\bar{s}$ states, which will provide a valuable contribution in future experiments.

The reliability of the model depends upon the mass inputs we have taken for calculation. When the experimental masses are taken as inputs, the estimated results are very close to the experimentally observed masses. But when we take the theoretical masses as inputs, our calculation shows small discrepancies with the experimental masses. The nonparallelism in the plots of n and M^2 is clearly

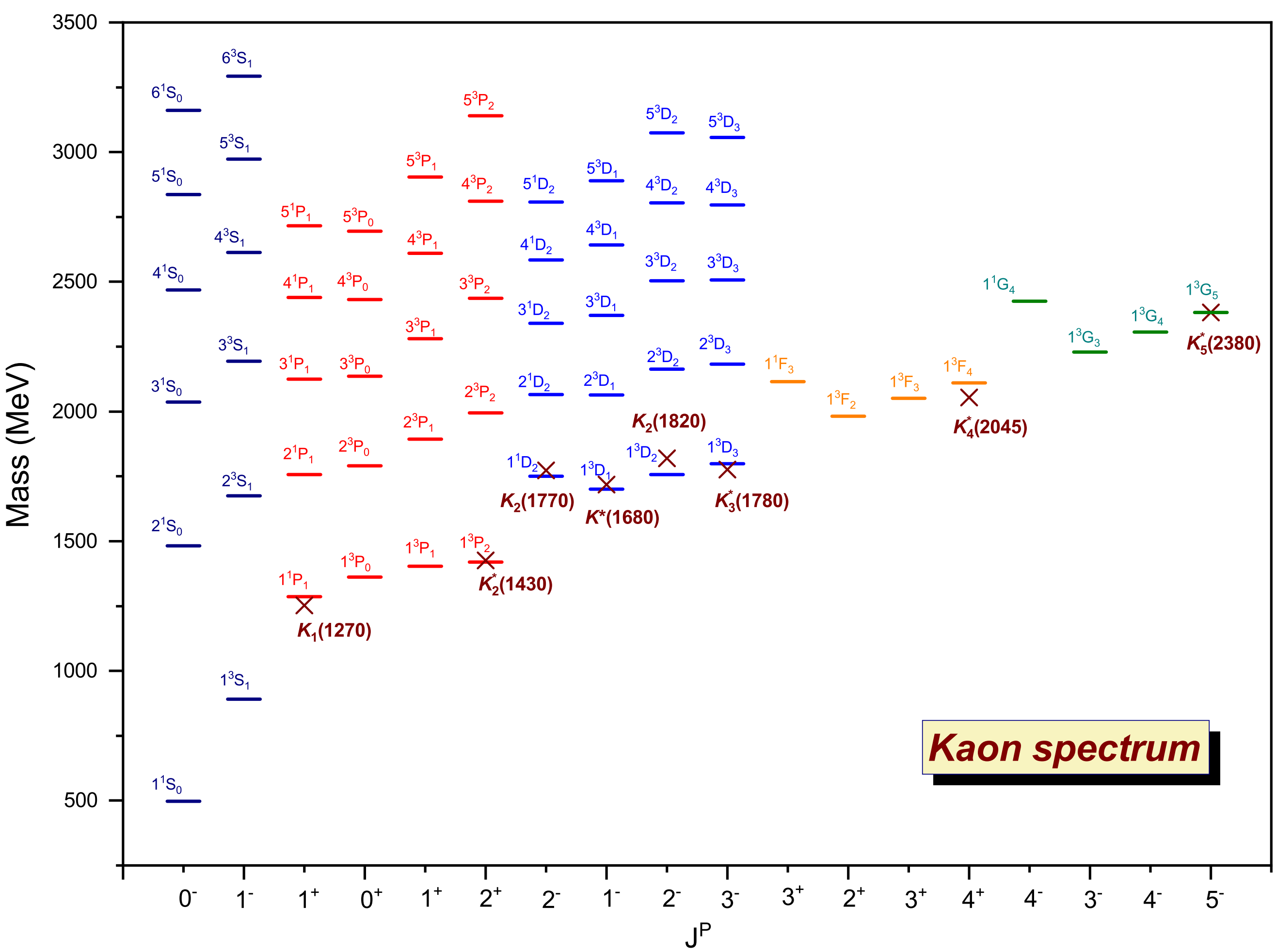

FIG. 9. The mass spectra of kaons predicted by our model with comparison of experimentally established states mentioned in PDG [18]. The colored straight lines represents our predicted masses and the cross sign represents the experimental masses.

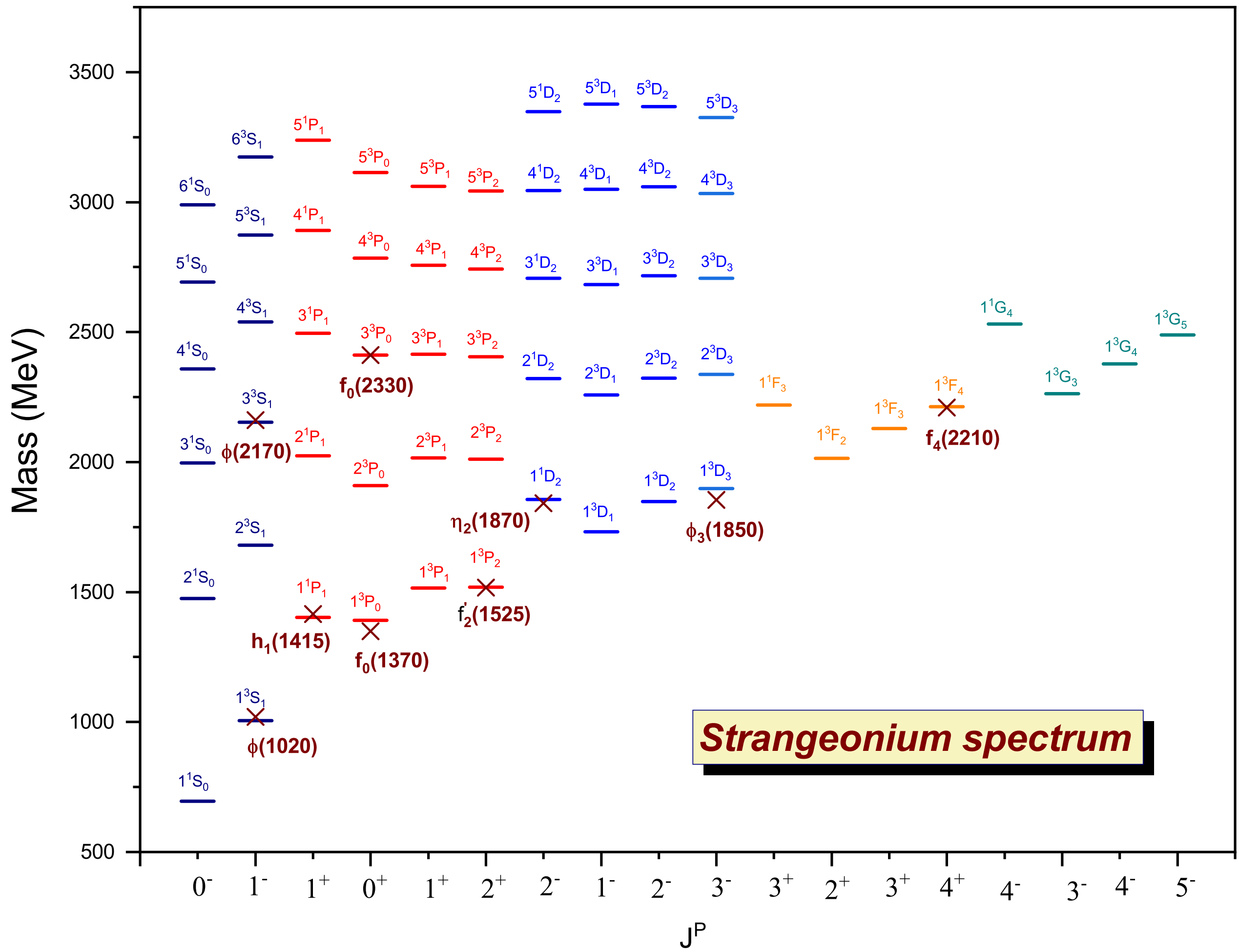

FIG. 10. The mass spectra of strangeonium predicted by our model with comparison of experimentally established states mentioned in PDG [18]. The colored straight lines represents our predicted masses and the cross sign represents the experimental masses.

observed. We have incorporated the half-width rule as an uncertainty in the resonance masses. Quite large error bars are observed in the Regge lines of kaons, which are more deviated as compared to the trajectories of strangeonium mesons, where deviation is less and uncertainty in the masses is small.

Figures 9 and 10 clearly depict that our obtained results are very close to the experimental observations where available. Also, the evaluated mass spectra show a general agreement with the predictions of other theoretical approaches. Hence, this study with a large number of mass predictions might be helpful and will provide important clues to the experimental facilities such as BESIII, LHCb, *BABAR*, J-PARC, etc. in future searches and the missing excited states.

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