

Chiral Lagrangians for spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ doubly charmed baryons

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The relativistic chiral Lagrangians for both spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ doubly charmed baryons are constructed up to the order $\mathcal{O}(p^4)$. From $\mathcal{O}(p^2)$ to $\mathcal{O}(p^4)$, there are 19, 74, and 452 independent terms in the two-flavor case and 25, 112, and 864 independent terms in the three-flavor case. The chiral Lagrangians in the heavy diquark limit are also obtained. From $\mathcal{O}(p^2)$ to $\mathcal{O}(p^4)$, there are 7, 23, and 118 independent terms in the two-flavor case and 8, 31, and 189 independent terms in the three-flavor case. We present the low-energy constant relations between the relativistic case and the case in the heavy diquark limit up to the order $\mathcal{O}(p^3)$. With the heavy diquark-antiquark symmetry, the low-energy constant relations between the doubly charmed baryon case and the heavy-light meson case are also obtained up to the order $\mathcal{O}(p^3)$.

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I. INTRODUCTION

Heavy quark baryons play an important role in hadron physics and the study of their properties may deepen our understanding of QCD. Their spectra are tightly related to the nonperturbative effects of QCD [1]. Until now, all types of singly heavy baryons have been observed [2] while only one doubly heavy baryon Ξ_{cc}^{++} is reported by two experiments, SELEX [3] and LHCb [4]. Although the SELEX Collaboration claimed the observation of Ξ_{cc}^{+} with mass 3519 ± 1 MeV [5], other measurements from FOCUS [6], BABAR [7], Belle [8], and LHCb [9] did not confirm this result. For the mass of the Ξ_{cc} baryon, the LHCb Collaboration obtains a value about 100 MeV higher than the SELEX Collaboration. The puzzle for the mass inconsistency needs more studies to solve.

The observation of Ξ_{cc}^{++} in the LHCb experiment inspired lots of discussions on the properties about doubly charmed baryons. Various theoretical approaches were adopted in the discussions, such as chiral perturbation theory (ChPT), QCD sum rule, lattice QCD, and heavy quark effective theory. Some properties of spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ doubly charmed baryons have been explored extensively, like the mass spectra [10–12,12–22], electromagnetic properties [23–32], weak and strong decay

properties [33,34,34,35,35–52], and so on. Here, we would like to discuss the chiral Lagrangians in studying the properties of doubly charmed baryons in ChPT.

As an effective theory of QCD, ChPT provides an alternative approach to deal with strong interactions over long distances [53–55]. It is a powerful tool in studying low-energy processes involving pions. In ChPT, the eight low-lying pseudoscalar mesons are treated as the Goldstone bosons generated by spontaneous breaking of chiral symmetry of QCD. These mesons instead of the complicated quarks and gluons mediate the interactions between hadrons. Only the pseudoscalar mesons were initially incorporated in the framework. Later, various ChPTs including baryons were developed [56–60].

In ChPT, the chirally invariant Lagrangian is the most basic premise for practical applications. For a theoretical study, in principle, more higher-order chiral Lagrangians are involved, and therefore more precise results would be obtained. However, with the increase of chiral order, the number of independent interaction terms and the corresponding unknown low-energy constants (LECs) would become considerable. These LECs cannot be determined by ChPT itself. Even worse, they are also hard to be obtained in other ways. This makes it quite difficult to work with high-order chiral Lagrangians. In order to solve this problem to some extent, available approximate symmetries, e.g. heavy quark symmetry and heavy diquark-antiquark symmetry [61,62], could be introduced to constraint the number of LECs or to give some approximate LEC relations. The present study will involve such symmetries.

Of the six flavor quarks in QCD, three light quarks are below the scale of the chiral symmetry breaking, $\Lambda_\chi \sim 1$ GeV. The charm quark is above Λ_χ and its mass

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M_c is much larger than those of light quarks (M_q) as well as the scale $\Lambda_{\text{QCD}} \sim 200$ MeV. Then, the ratios M_q/M_c and Λ_{QCD}/M_c are both considered small enough. In the extreme limit $M_c \rightarrow \infty$, both M_q/M_c and Λ_{QCD}/M_c could be ignored and the heavy quark symmetry of QCD appears [63,64]. In this limit, the dynamics of the considered hadron may not be affected by the change of the heavy quark flavor (spin), which corresponds to the heavy quark flavor (spin) symmetry. For the doubly charmed baryons we are considering, one may assume that the two charm quarks form a compact heavy diquark with spin = 1. We will use *heavy diquark limit* to denote the case that the mass of the diquark does not affect the dynamics of the doubly charmed baryon [62].

With the chiral Lagrangians, many properties of doubly charmed baryons have been studied in the literature, e.g. their masses [18,22,65], magnetic moments [23,25,31], and electromagnetic form factors [24,26]. At present, most studies involve the low-order Lagrangians. Parts of high-order chiral Lagrangians are given in a few studies on some specific problems [22–26,29]. Recently, the chiral Lagrangian for spin- $\frac{1}{2}$ doubly charmed baryons in the three-flavor case has been constructed up to the order $\mathcal{O}(p^4)$ [66]. In the present work, for considerations to complete the necessary ingredient of one-loop level investigations, to check convergence of chiral expansion better, to motivate future LEC studies, and so on, we construct the relativistic chiral Lagrangian involving both spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ doubly charmed baryons up to the order $\mathcal{O}(p^4)$. In principle, when one determines the values of LECs from experimental data, all the operators should be independent. Otherwise, the overfitting problem would appear. The Lagrangians given in the present study will be minimal and the terms will be independent. Besides the relativistic case, we will also consider Lagrangians in the heavy diquark case. In that case, the spin-1/2 and spin-3/2 baryons can be put into a superfield and some LEC relations can be obtained.

This paper is organized as follows. Section II introduces the building blocks in constructing the chiral Lagrangians for doubly charmed baryons. In Sec. III, the properties of the building blocks, the needed linear relations, and the procedure to get the relativistic and nonrelativistic forms of Lagrangians are given. In Sec. IV, we discuss how to find constraints on LECs with the heavy quark symmetry from different Lagrangians. Section V shows our results and some discussions, and Sec. VI gives a short summary.

II. DEFINITIONS AND BUILDING BLOCKS

This section presents the definitions related with pseudoscalar Goldstone boson fields, external sources, and doubly charmed baryons in both relativistic and non-relativistic forms. One may find more relevant details in Refs. [22,23,54,55,67–74].

A. Goldstone boson fields and external sources

Considering the lightest N_f -flavor quarks ($N_f = 2$ or 3), one may write the related QCD Lagrangian with external sources as

$$\mathcal{L} = \mathcal{L}_{\text{QCD}}^0 + \bar{q}(\not{p} + \not{a} \gamma_5 - s + i p \gamma_5)q, \quad (1)$$

where $\mathcal{L}_{\text{QCD}}^0$ is the original QCD Lagrangian and q denotes the light quark fields. Here, s , p , v^μ , and a^μ are scalar, pseudoscalar, vector, and axial-vector external sources, respectively. In this work, the tensor source and the θ term are not considered. The external source a^μ is always traceless, but v^μ is not. To consider the electromagnetic interactions, v^μ is taken to be traceable in the relativistic case. In the heavy diquark limit, it is traceable (traceless) in the $SU(2)$ [$SU(3)$] case because the heavy diquark is just a spectator and the electromagnetic interaction is mainly determined by the light quark.

When the light quarks are taken to be massless, QCD exhibits a global $SU(N_f)_L \times SU(N_f)_R$ chiral symmetry, but it spontaneously breaks into $SU(N_f)_V$ because of the non-vanishing quark condensate $\langle \bar{q}q \rangle$. The generated $(N_f^2 - 1)$ Goldstone bosons are considered to be the lowest pseudoscalar pion mesons. These mesonic fields are collected in the nonlinear representation $u = \exp(i\pi^j \tau^j / 2F_0)$ or $u = \exp(i\pi^j \lambda^j / 2F_0)$ where F_0 is the pion decay constant in the chiral limit, τ^j ($j = 1, 2, 3$) are the Pauli matrices, and λ^j ($j = 1, 2, \dots, 8$) are the Gell-Mann matrices. As the basic ingredient of ChPT, u has the chiral transformation behavior

$$u \rightarrow g_L u h^\dagger = h u g_R^\dagger, \quad (2)$$

where g_L and g_R are the group elements of $SU(N_f)_L$ and $SU(N_f)_R$, respectively, and h is a compensator field related to the pion fields.

The necessary building blocks in constructing the chiral Lagrangians are combinations of the mesonic fields and the external sources. The conventional choice of their explicit forms reads

$$\begin{aligned} u^\mu &= i\{u^\dagger(\partial^\mu - i r^\mu)u - u(\partial^\mu - i l^\mu)u^\dagger\}, \\ h^{\mu\nu} &= \nabla^\mu u^\nu + \nabla^\nu u^\mu, \\ f_+^{\mu\nu} &= u F_L^{\mu\nu} u^\dagger + u^\dagger F_R^{\mu\nu} u, \\ f_-^{\mu\nu} &= u F_L^{\mu\nu} u^\dagger - u^\dagger F_R^{\mu\nu} u = -\nabla^\mu u^\nu + \nabla^\nu u^\mu, \\ \chi_\pm &= u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \end{aligned} \quad (3)$$

where $r^\mu = v^\mu + a^\mu$, $l^\mu = v^\mu - a^\mu$, $F_R^{\mu\nu} = \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu]$, $F_L^{\mu\nu} = \partial^\mu l^\nu - \partial^\nu l^\mu - i[l^\mu, l^\nu]$, and $\chi = 2B_0(s + i p)$ with B_0 being a constant related to the quark condensate. The covariant derivative of a building block X is defined by

$$\nabla^\mu X = \partial^\mu X + [\Gamma^\mu, X], \quad (4)$$

$$\Gamma^\mu = \frac{1}{2} \{u^\dagger (\partial^\mu - i r^\mu) u + u (\partial^\mu - i l^\mu) u^\dagger\}. \quad (5)$$

The chiral dimension of ∇^μ is one in ChPT. All the defined building blocks have the same behavior under the chiral rotation R ,

$$X \xrightarrow{R} X' = h X h^\dagger. \quad (6)$$

B. Doubly charmed baryons

The spin of an S -wave doubly charmed baryon containing two charmed quarks and one light quark may be $1/2$ or $3/2$. Then the doubly charmed baryons belong to two triplets in the three-flavor case,

$$B = \begin{pmatrix} \Xi_{cc}^{++} \\ \Xi_{cc}^+ \\ \Omega_{cc}^+ \end{pmatrix}, \quad T^\mu = \begin{pmatrix} \Xi_{cc}^{*++} \\ \Xi_{cc}^{*+} \\ \Omega_{cc}^{*+} \end{pmatrix}, \quad (7)$$

where B and T^μ denote the $\text{spin-}\frac{1}{2}$ and $\text{spin-}\frac{3}{2}$ fields, respectively. Only the $\Xi_{cc}^{(*)}$ states are needed if one considers the two-flavor case. The covariant derivative of a baryon field and its Dirac conjugate are

$$D^\mu \tilde{B} = (\partial^\mu + \Gamma^\mu) \tilde{B}, \quad D^\mu \bar{\tilde{B}} = \bar{\tilde{B}} (\overleftarrow{\partial}^\mu + \Gamma^{\mu\dagger}), \quad (8)$$

where $\tilde{B} = B$ or T^ν . If more than one covariant derivatives act on $\tilde{B}$, a totally symmetrical derivative is introduced

$$D^{\mu\nu\cdots\rho} \equiv \frac{1}{n!} (\underbrace{D^\mu D^\nu \cdots D^\rho}_n + \text{full permutation of } D\text{'s}). \quad (9)$$

Any antisymmetrical derivative is related to the higher-order contributions [72]. The chiral rotations of the baryon fields (and their derivatives) are

$$\tilde{B} \xrightarrow{R} \tilde{B}' = h \tilde{B}, \quad \bar{\tilde{B}} \xrightarrow{R} \bar{\tilde{B}}' = \bar{\tilde{B}} h^\dagger. \quad (10)$$

In the heavy diquark limit, a doubly charmed baryon with velocity v^μ [75] can be simply separated into a heavy diquark component and a light degree of freedom component. The diquark behaves just like a static color source for the light component and the suppression for the spin interaction between the diquark and the light component results in the degenerate B and T^μ . In this situation, the heavy diquark symmetry exists and one may put the $\text{spin-}\frac{1}{2}$ and $\text{spin-}\frac{3}{2}$ baryon fields into a superfield ψ^μ [62],

$$\psi^\mu = T^\mu + \sqrt{\frac{1}{3}} (\gamma^\mu + v^\mu) \gamma^5 B, \quad (11)$$

$$\bar{\psi}^\mu = \bar{T}^\mu - \sqrt{\frac{1}{3}} \bar{B} \gamma^5 (\gamma^\mu + v^\mu). \quad (12)$$

Here, B and T^μ as nonrelativistic fields only contain the annihilation operators. It is obvious that the chiral rotation behaviors of ψ^μ and $\bar{\psi}^\mu$ are the same as $\tilde{B}$ and $\bar{\tilde{B}}$, respectively. In order to modify the energy measure, the superfield is scaled by $e^{-iMv \cdot x}$ with M being the doubly charmed baryon mass. Hence, in the heavy diquark limit, the covariant derivative on matter field becomes

$$D^\mu \psi^\nu(x) = -iM v^\mu \psi^\nu(x). \quad (13)$$

III. CONSTRUCTION OF CHIRAL LAGRANGIANS FOR DOUBLY CHARMED BARYONS

The basic procedure to construct the chirally invariant Lagrangians for doubly charmed baryons is as follows. First, the structures of the chiral Lagrangians are introduced. Second, the necessary properties of the building blocks are given. One constructs all possible terms based on such properties. Third, all linear relations to reduce the number of Lagrangian terms are listed. Finally, the linearly dependent terms would be eliminated and the independent ones are retained.

A. Structures of Lagrangians

The relativistic chiral Lagrangian $\mathcal{L}$ for doubly charmed baryons contains terms involving only $\text{spin-}\frac{1}{2}$ baryons, those involving only $\text{spin-}\frac{3}{2}$ baryons, and those involving both $\text{spin-}\frac{1}{2}$ and $\text{spin-}\frac{3}{2}$ baryons. We use $\mathcal{L}_{BB}$, $\mathcal{L}_{TT}$, and $\mathcal{L}_{BT}$ to represent these three sectors of interaction terms, respectively. That is,

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{BB} + \mathcal{L}_{TT} + \mathcal{L}_{BT} \\ &= \sum_n E_n \bar{B} \cdots B + \sum_m E_m \bar{T} \cdots T \\ &\quad + \sum_p E_p (\bar{B} \cdots T + \bar{T} \cdots B), \end{aligned} \quad (14)$$

where E_n , E_m , and E_p are LECs and the symbol “...” represents the allowed combinations of building blocks. If the trace of a combination X in the flavor space is necessary, it will be denoted as $\langle X \rangle$. To keep the hermiticity of $\mathcal{L}$, there would exist some appropriate coefficients $\pm i$ in “...”.

In the heavy diquark limit, the Lagrangian has the structure

$$\mathcal{L}_H = \sum_n F_n \bar{\psi}^\mu \cdots \psi_\mu \quad (15)$$

where F_n are LECs.

B. Properties of building blocks

The chiral Lagrangian should be invariant under the parity transformation, charge conjugation, and Hermitian conjugation. Table I shows the transformation behaviors of the building blocks about mesons and external sources. Table II presents those of the Clifford algebra, the velocity of doubly charmed baryons, Levi-Civita tensor, and the covariant derivatives acting on baryons. With these transformation properties, all the constructed terms would have the structures shown in Eqs. (14) and (15). In Table II, we only show the transformation signs. Some explanations are as follows. Under the parity transformation, one has

$$B \xrightarrow{P} \gamma_0 B, \quad T_\mu \xrightarrow{P} -\gamma_0 T_\mu. \quad (16)$$

For convenience in the Lagrangian construction, we omit the minus sign in the second transformation and compensate it in the transformations of the five Clifford algebra elements and the antisymmetric tensor $\epsilon^{\mu\nu\lambda\rho}$. That is why we distinguish the transformations between $\mathcal{L}_{BT}$ and $\mathcal{L}_{BB(TT)}$ in Table II. Moreover, we use a convention that the covariant derivatives in $\mathcal{L}_{BB}$ and $\mathcal{L}_{TT}$ of Eq. (14) act on the right baryon field $\tilde{B}$, but those in $\mathcal{L}_{BT}$ act only on the spin-3/2 field T_μ or $\tilde{T}_\mu$. Therefore, the properties of covariant derivatives in $\mathcal{L}_{BB(TT)}$ and $\mathcal{L}_{BT}$ may also be different. However, the

TABLE I. Chiral dimension (Dim), parity transformation (P), charge conjugation (C), and Hermiticity (H.c.) of the building blocks.

	Dim	P	C	H.c.
u^μ	1	$-u^\mu$	$(u^\mu)^T$	u^μ
$h^{\mu\nu}$	2	$-h^{\mu\nu}$	$(h^{\mu\nu})^T$	$h^{\mu\nu}$
$\chi_\pm$	2	$\pm\chi_\pm$	$(\chi_\pm)^T$	$\pm\chi_\pm$
$f_\pm^{\mu\nu}$	2	$\pm f_\pm^{\mu\nu}$	$\mp(f_\pm^{\mu\nu})^T$	$f_\pm^{\mu\nu}$

TABLE II. Chiral dimension (Dim), parity transformation (P), charge conjugation (C), and Hermiticity (H.c.) of the Clifford algebra, the velocity of doubly charmed baryons, Levi-Civita tensor, and the covariant derivatives acting on baryons. The subscripts BB , TT , and BT represent $\mathcal{L}_{BB}$, $\mathcal{L}_{TT}$, and $\mathcal{L}_{BT}$, respectively.

	Dim	$P_{BB(TT)}$	$C_{BB(TT)}$	H.c. $_{BB(TT)}$	P_{BT}	C_{BT}	H.c. $_{BT}$
1	0	+	+	+	-	+	+
γ_5	1	-	+	-	+	+	-
γ^μ	0	+	-	+	-	-	+
$\gamma_5\gamma^\mu$	0	-	+	+	+	+	+
$\sigma^{\mu\nu}$	0	+	-	+	-	-	+
v^μ	0	+	-	+	+	-	+
$\epsilon^{\mu\nu\lambda\rho}$	0	-	+	+	+	+	+
$D^\mu \tilde{B}$	0	+	-	-	+	+	+

convention difference has no impact on the construction of $\mathcal{L}_{BT}$. The related properties for $\mathcal{L}_H$ in Eq. (15) are the same as those for $\mathcal{L}_{TT}$.

With these tables, a complete set of interaction terms can be obtained. However, these constructed terms are not always linearly independent. The next subsection will list all possible linear relations with which the redundant terms can be eliminated.

C. Linear relations

The linear relations in the construction of effective Lagrangians have been discussed widely. We here just give a short description of them. More details can be found in Refs. [66,68–70,72,76].

1. Partial integration

A derivative acting on any Lagrangian term does not affect the physics. Therefore, the covariant derivative acting on one baryon field can be changed to that acting on the other one, which induces a high-order difference, i.e.

$$0 \doteq \tilde{B} \tilde{D}^\nu X B + \tilde{B} X D^\nu B, \quad (17)$$

$$0 \doteq \tilde{T}^\mu \tilde{D}^\nu X T_\mu + \tilde{T}^\mu X D^\nu T_\mu, \quad (18)$$

$$0 \doteq \tilde{B} \tilde{D}^\nu X T^\mu + \tilde{B} X D^\nu T^\mu, \quad (19)$$

$$0 \doteq \tilde{T}^\mu \tilde{D}^\nu X B + \tilde{T}^\mu X D^\nu B, \quad (20)$$

where X denotes any possible combination of the building blocks and “ $\doteq$ ” means that the higher-order terms are ignored. We will use the first two relations to move the positions of covariant derivatives. The third and fourth relations will not give any new constraints in the present work.

2. Schouten identity

The Schouten identity involves the four-dimension Levi-Civita tensor,

$$\epsilon^{\mu\nu\lambda\rho} A^\sigma - \epsilon^{\sigma\nu\lambda\rho} A^\mu - \epsilon^{\mu\sigma\lambda\rho} A^\nu - \epsilon^{\mu\nu\sigma\rho} A^\lambda - \epsilon^{\mu\nu\lambda\sigma} A^\rho = 0, \quad (21)$$

where A can be any building block containing at least one Lorentz index. This relation exists because the five indices in the equation are antisymmetric, but the dimension of the spacetime is four.

3. Equations of motions

The equations of motions (EoMs) for the light pseudo-scalar mesons and the doubly charmed baryons read

$$\nabla_\mu u^\mu \doteq \frac{i}{2} \left(\chi_- - \frac{1}{N_f} \langle \chi_- \rangle \right), \quad (22)$$

$$(i\not{D} - m_T)T^\mu \doteq 0, \quad (23)$$

$$(i\not{D} - m_B)B \doteq 0. \quad (24)$$

For the spin-3/2 fields, two subsidiary conditions are needed to eliminate the redundant degrees of freedom,

$$D_\mu T^\mu \doteq 0, \quad (25)$$

$$\gamma_\mu T^\mu \doteq 0. \quad (26)$$

In the heavy diquark limit, one has

$$v_\mu \psi^\mu = 0, \quad (27)$$

$$\not{v}\psi^\mu = \psi^\mu. \quad (28)$$

4. Covariant derivatives and Bianchi identity

The commutation of two covariant derivatives gives

$$[\nabla^\mu, \nabla^\nu]X = [\Gamma^{\mu\nu}, X], \quad (29)$$

$$\Gamma^{\mu\nu} = \frac{1}{4}[u^\mu, u^\nu] - \frac{i}{2}f_+^{\mu\nu}. \quad (30)$$

The Bianchi identity involving $\Gamma^{\mu\nu}$ reads

$$\nabla^\mu \Gamma^{\nu\lambda} + \nabla^\nu \Gamma^{\lambda\mu} + \nabla^\lambda \Gamma^{\mu\nu} = 0. \quad (31)$$

An alternative form with building blocks is more useful,

$$\begin{aligned} \nabla^\mu f_+^{\nu\lambda} + \nabla^\nu f_+^{\lambda\mu} + \nabla^\lambda f_+^{\mu\nu} + \frac{i}{2}[u^\mu, f_-^{\nu\lambda}] \\ + \frac{i}{2}[u^\nu, f_-^{\lambda\mu}] + \frac{i}{2}[u^\lambda, f_-^{\mu\nu}] = 0. \end{aligned} \quad (32)$$

5. Cayley-Hamilton relations

All the building blocks in Sec. II A are $N_f \times N_f$ matrices in the flavor space. For any 2×2 matrices A and B , there exists a relation

$$AB + BA - A\langle B \rangle - B\langle A \rangle - \langle AB \rangle + \langle A \rangle\langle B \rangle = 0. \quad (33)$$

For any 3×3 matrices A , B , and C , the relation is

$$\begin{aligned} 0 = & ABC + ACB + BAC + BCA + CAB + CBA \\ & - AB\langle C \rangle - AC\langle B \rangle - BA\langle C \rangle - BC\langle A \rangle - CA\langle B \rangle \\ & - CB\langle A \rangle - A\langle BC \rangle - B\langle AC \rangle - C\langle AB \rangle - \langle ABC \rangle \\ & - \langle ACB \rangle + A\langle B \rangle\langle C \rangle + B\langle A \rangle\langle C \rangle + C\langle A \rangle\langle B \rangle \\ & + \langle A \rangle\langle BC \rangle + \langle B \rangle\langle AC \rangle + \langle C \rangle\langle AB \rangle - \langle A \rangle\langle B \rangle\langle C \rangle. \end{aligned} \quad (34)$$

6. Contact terms

For contact terms, the LR basis is more convenient for the Lagrangian construction. From the building blocks in Eq. (3), one gets

$$F_L^{\mu\nu} = \frac{1}{2}u^\dagger(f_+^{\mu\nu} + f_-^{\mu\nu})u, \quad (35)$$

$$F_R^{\mu\nu} = \frac{1}{2}u(f_+^{\mu\nu} - f_-^{\mu\nu})u^\dagger, \quad (36)$$

$$\chi = \frac{1}{2}u(\chi_+ + \chi_-)u, \quad (37)$$

$$\chi^\dagger = \frac{1}{2}u^\dagger(\chi_+ - \chi_-)u^\dagger. \quad (38)$$

Their transform properties can be found in Table I of Ref. [68].

IV. RELATIONS BETWEEN LECs IN DIFFERENT CASES

In the heavy diquark limit, the LECs in the relativistic Lagrangian are no longer independent. Some relations among these LECs appear. With these relations, the number of independent relativistic LECs could be reduced. A method to obtain these relations is to compare the relativistic Lagrangian with the Lagrangian in the heavy diquark limit. To remove the dimensions in the results and to simplify the forms of the relations, we define a new set of terms and LECs in the relativistic Lagrangians as follows,

$$\tilde{O}_n = O_n/M^r, \quad \tilde{E}_n = E_n M^r, \quad (39)$$

where r is the number of covariant derivatives acting on the doubly charmed baryons. Now, all the $\tilde{E}_n$'s have the same dimension at a given order.

There are three types of Lorentz structures in the constructed terms of Eq. (15). With the definition of the superfield ψ^μ in Eq. (11), one gets

$$\bar{\psi}^\mu \psi_\mu \rightarrow \bar{T}^\mu T_\mu - \bar{B}B, \quad (40)$$

$$\bar{\psi}^\mu \gamma_5 \gamma^\lambda \psi_\mu \rightarrow \bar{T}^\mu \gamma_5 \gamma^\lambda T_\mu + \frac{1}{3}\bar{B}\gamma_5 \gamma^\lambda B + \frac{2}{\sqrt{3}}\bar{T}^\lambda B + \frac{2}{\sqrt{3}}\bar{B}T^\lambda, \quad (41)$$

$$\begin{aligned} \bar{\psi}^\mu \sigma^{\lambda\rho} \psi_\mu \rightarrow \bar{T}^\mu \sigma^{\lambda\rho} T_\mu + \frac{1}{3}\bar{B}\sigma^{\lambda\rho} B - \frac{2i}{\sqrt{3}}\bar{B}\gamma_5 \gamma^\rho T^\lambda \\ + \frac{2i}{\sqrt{3}}\bar{B}\gamma_5 \gamma^\lambda T^\rho + \frac{2i}{\sqrt{3}}\bar{T}^\lambda \gamma_5 \gamma^\rho B - \frac{2i}{\sqrt{3}}\bar{T}^\rho \gamma_5 \gamma^\lambda B. \end{aligned} \quad (42)$$

The right-hand side expansions imply the possible LEC relations in the relativistic Lagrangians. Substituting the

above structures into Eq. (15) and comparing the relativistic and nonrelativistic terms, one can express the relations between $\tilde{E}_m$ and F_n as

$$F_n = \sum_m \tilde{E}_m A_{mn}. \quad (43)$$

The number of F_n (N_F) is less than the number of $\tilde{E}_m$ (N_E). Hence, these relations give $N_E - N_F$ constraint conditions among $\tilde{E}_m$ in the heavy diquark limit. With the help of linear algebra, one can pick up the independent relativistic LECs and eliminate the dependent ones. In this paper, we only consider the relations up to the order $\mathcal{O}(p^3)$, which is restricted by computational conditions.

If one treats the heavy diquark as a compact object, the properties of the doubly charmed baryons can be related to those of the singly heavy mesons, which is the result of the heavy diquark-antiquark symmetry [77]. This symmetry then tells us that the doubly charmed baryons and heavy-light mesons with quark content $\bar{Q}q$ share the same LECs in ChPT [62]. In Ref. [78], we have obtained the chiral Lagrangian for heavy-light mesons with quark content $Q\bar{q}$ and the LEC relations in the heavy quark limit in that case. Noticing the charge-conjugation transformations for building blocks and superfields, one can understand that the LECs in $Q\bar{q}$ and $\bar{Q}q$ cases are the same. Now, the LEC relations between the doubly charmed baryon case and the heavy-light meson case could also be obtained by linear algebra.

V. RESULTS AND DISCUSSIONS

Now, we are ready to present the final results for the independent chiral-invariant terms. The details to get such terms are almost the same as those given in Refs. [74,76,79] and we do not talk about them any more. Here, the Lagrangians are expressed as

$$\mathcal{L}^{(m)} = \sum_n E_n^{(m)} O_n^{(m)} = \sum_n \tilde{E}_n^{(m)} \tilde{O}_n^{(m)}, \quad N_f = 3, \quad (44)$$

$$\mathcal{L}^{(m)} = \sum_n e_n^{(m)} o_n^{(m)} = \sum_n \tilde{e}_n^{(m)} \tilde{o}_n^{(m)}, \quad N_f = 2, \quad (45)$$

$$\mathcal{L}_H^{(m)} = \sum_n F_n^{(m)} P_n^{(m)} N_f = 3, \quad (46)$$

$$\mathcal{L}_H^{(m)} = \sum_n f_n^{(m)} p_n^{(m)} N_f = 2, \quad (47)$$

where m denotes the chiral dimension.

A. $\mathcal{O}(p^1)$ order

The relativistic result at the order $\mathcal{O}(p^1)$ is [23]

$$\begin{aligned} \mathcal{L}^{(1)} = & \bar{B}(i\not{D} - m_1)B + \bar{T}^\mu [g_{\mu\nu}(i\not{D} - m_2) \\ & + iA(\gamma_\mu \not{D}_\nu + \gamma_\nu \not{D}_\mu) + \frac{i}{2}(3A^2 + 2A + 1)\gamma_\mu \not{D}_\nu \\ & + m_2(3A^2 + 3A + 1)\gamma_\mu \gamma_\nu]T^\nu + \frac{E_1}{2}\bar{B}u_\mu\gamma_5\gamma^\mu B \\ & + \frac{E_2}{2}\bar{T}^\mu u_\lambda\gamma_5\gamma^\lambda T_\mu + \frac{E_3}{2}(\bar{B}u^\mu T_\mu + \text{H.c.}), \end{aligned} \quad (48)$$

where $A \neq 1/2$ is an unphysical parameter and it is usually taken to be $A = -1$. Generally speaking, the Lagrangian for spin-3/2 fields is invariant under the “point” or “contact” transformation and the relations between different choices of A can be found [76]. Since the value of A does not affect the structure of chiral Lagrangians, we do not discuss more about it here. The Lagrangian at the leading order in the heavy diquark limit reads [62]

$$\mathcal{L}_{HQ}^{(1)} = \bar{\psi}_\nu i v^\mu D_\mu \psi^\nu - \frac{1}{2} g \bar{\psi}_\nu u_\lambda \gamma_5 \gamma^\lambda \psi^\nu. \quad (49)$$

For the relations between these LECs, one gets [62]

$$E_1 = \frac{1}{3}E_2 = \frac{1}{2\sqrt{3}}E_3 = -\frac{1}{3}g. \quad (50)$$

B. $\mathcal{O}(p^2)$ order

The interaction terms of $\mathcal{O}(p^2)$ relativistic Lagrangian are collected in Table III. The first column lists the forms of independent terms. The second and sixth columns label the number for each term in the $SU(2)$ and $SU(3)$ cases, respectively. The terms without a number in the $SU(2)$ case mean that they are not independent. If one takes no account of the electromagnetic effect, the terms containing $\langle f_+^{\mu\nu} \rangle$ could be ignored. The third and the seventh columns show the relations between the relativistic LECs and the LECs in the heavy diquark limit. To assign the independent terms in the relativistic Lagrangian, we put a symbol “I” in the forth and eighth columns. These two columns also present the relations of the dependent LECs to the independent ones. The fifth and ninth columns give the LEC relations in the heavy diquark-antiquark symmetry, where $\tilde{c}$ and $\tilde{C}$ defined in Ref. [78] are LECs for heavy-light mesons. We show the $\mathcal{O}(p^2)$ Lagrangian terms for the superfield ψ^μ in Table IV.

From Table III, the numbers of $\mathcal{O}(p^2)$ terms in $\mathcal{L}_{BB}$, $\mathcal{L}_{TT}$, and $\mathcal{L}_{BT}$ in the $SU(2)$ case are 7, 8, and 4, respectively. Those in the $SU(3)$ case are 9, 11, and 5, respectively. From Table IV, the heavy diquark symmetry reduces significantly the total number of independent terms from 19 (25) to be 7 (8) in the $SU(2)$ [$SU(3)$] case.

TABLE III. Independent terms in the relativistic Lagrangian at the order $\mathcal{O}(p^2)$. The columns 2, 3, 4, and 5 (6, 7, 8, and 9) are for the two-flavor (three-flavor) case. Columns 2 and 6 label the number for each term. The terms without a number are not independent. Columns 3 and 7 list the relations between LECs in the relativistic case and those in the heavy diquark limit. Columns 4 and 8 show the LEC relations among different terms in the relativistic case by using the heavy diquark symmetry. “I” means that the term is chosen as an independent term in the heavy diquark limit. Columns 5 and 9 give the LEC relations with the help of the heavy diquark-antiquark symmetry, where $\tilde{c}$ and $\tilde{C}$ are LECs in the heavy-light meson case. Such LECs are defined in Ref. [78]. “0” means that the LECs in the heavy diquark limit vanish.

O_n/o_n	$SU(2)$	$\tilde{e}_n^{(2)}$	$\tilde{e}_n^{(2)}$	$\tilde{e}_n^{(2)}$	$SU(3)$	$\tilde{E}_n^{(2)}$	$\tilde{E}_n^{(2)}$	$\tilde{E}_n^{(3)}$
$\bar{B}\langle u^\mu u_\mu \rangle B$	1	$-f_1^{(2)}$	I	$\frac{1}{4}\tilde{c}_1^{(2)}$	1	$-F_1^{(2)}$	I	$\frac{1}{2}\tilde{C}_3^{(2)}$
$\bar{B}\langle u^\mu u^\nu \rangle D_{\mu\nu} B$	2	$f_2^{(2)}$	I	$\frac{1}{4}\tilde{c}_2^{(2)}$	2	$F_2^{(2)}$	I	$\frac{1}{2}\tilde{C}_4^{(2)}$
$\bar{B}u^\mu u_\mu B$					3	$-F_3^{(2)}$	I	$\frac{1}{2}\tilde{C}_1^{(2)}$
$\bar{B}u^\mu u^\nu D_{\mu\nu} B$					4	$F_4^{(2)}$	I	$\frac{1}{2}\tilde{C}_2^{(2)}$
$i\bar{B}u^\mu u^\nu \sigma_{\mu\nu} B$	3	$\frac{1}{3}f_3^{(2)}$	I	$-\frac{1}{6}\tilde{c}_5^{(2)}$	5	$\frac{1}{3}F_5^{(2)}$	I	$-\frac{1}{6}\tilde{C}_7^{(2)}$
$\bar{B}\langle f_+^{\mu\nu} \rangle \sigma_{\mu\nu} B$	4	$\frac{1}{3}f_4^{(2)}$	I	$\frac{1}{12}\tilde{c}_{10}^{(2)}$	6	0	0	0
$\bar{B}f_+^{\mu\nu} \sigma_{\mu\nu} B$	5	$\frac{1}{3}f_5^{(2)}$	I	$\frac{1}{12}\tilde{c}_9^{(2)}$	7	$\frac{1}{3}F_6^{(2)}$	I	$\frac{1}{12}\tilde{C}_{14}^{(2)}$
$\bar{B}\langle \chi_+ \rangle B$	6	$-f_6^{(2)}$	I	$\frac{1}{2}\tilde{c}_4^{(2)}$	8	$-F_7^{(2)}$	I	$\frac{1}{2}\tilde{C}_6^{(2)}$
$\bar{B}\chi_+ B$	7	$-f_7^{(2)}$	I	$\frac{1}{2}\tilde{c}_3^{(2)}$	9	$-F_8^{(2)}$	I	$\frac{1}{2}\tilde{C}_5^{(2)}$
$\bar{T}^\mu \langle u_\mu u^\nu \rangle T_\nu$	8	$f_3^{(2)}$	$3\tilde{e}_3^{(2)}$	$-\frac{1}{2}\tilde{c}_5^{(2)}$	10	0	0	0
$\bar{T}^\mu \langle u^\nu u_\nu \rangle T_\mu$	9	$f_1^{(2)}$	$-\tilde{e}_1^{(2)}$	$-\frac{1}{4}\tilde{c}_1^{(2)}$	11	$F_1^{(2)}$	$-\tilde{E}_1^{(2)}$	$-\frac{1}{2}\tilde{C}_3^{(2)}$
$\bar{T}^\mu \langle u^\nu u^\lambda \rangle D_{\nu\lambda} T_\mu$	10	$-f_2^{(2)}$	$-\tilde{e}_2^{(2)}$	$-\frac{1}{4}\tilde{c}_2^{(2)}$	12	$-F_2^{(2)}$	$-\tilde{E}_2^{(2)}$	$-\frac{1}{2}\tilde{C}_4^{(2)}$
$\bar{T}^\mu u_\mu u^\nu T_\nu$	11	$-2f_3^{(2)}$	$-6\tilde{e}_3^{(2)}$	$\tilde{c}_5^{(2)}$	13	$-F_5^{(2)}$	$-3\tilde{E}_5^{(2)}$	$\frac{1}{2}\tilde{C}_7^{(2)}$
$\bar{T}^\mu u^\nu u_\mu T_\nu$					14	$F_5^{(2)}$	$3\tilde{E}_5^{(2)}$	$-\frac{1}{2}\tilde{C}_7^{(2)}$
$\bar{T}^\mu u^\nu u_\nu T_\mu$					15	$F_3^{(2)}$	$-\tilde{E}_3^{(2)}$	$-\frac{1}{2}\tilde{C}_1^{(2)}$
$\bar{T}^\mu u^\nu u^\lambda D_{\nu\lambda} T_\mu$					16	$-F_4^{(2)}$	$-\tilde{E}_4^{(2)}$	$-\frac{1}{2}\tilde{C}_2^{(2)}$
$i\bar{T}^\mu \langle f_{+\mu}{}^\nu \rangle T_\nu$	12	$2f_4^{(2)}$	$6\tilde{e}_4^{(2)}$	$\frac{1}{2}\tilde{c}_{10}^{(2)}$	17	0	0	0
$i\bar{T}^\mu f_{+\mu}{}^\nu T_\nu$	13	$2f_5^{(2)}$	$6\tilde{e}_5^{(2)}$	$\frac{1}{2}\tilde{c}_9^{(2)}$	18	$2F_6^{(2)}$	$6\tilde{E}_6^{(2)}$	$\frac{1}{2}\tilde{C}_{14}^{(2)}$
$\bar{T}^\mu \langle \chi_+ \rangle T_\mu$	14	$f_6^{(2)}$	$-\tilde{e}_6^{(2)}$	$-\frac{1}{2}\tilde{c}_4^{(2)}$	19	$F_7^{(2)}$	$-\tilde{E}_7^{(2)}$	$-\frac{1}{2}\tilde{C}_6^{(2)}$
$\bar{T}^\mu \chi_+ T_\mu$	15	$f_7^{(2)}$	$-\tilde{e}_7^{(2)}$	$-\frac{1}{2}\tilde{c}_3^{(2)}$	20	$F_8^{(2)}$	$-\tilde{E}_8^{(2)}$	$-\frac{1}{2}\tilde{C}_5^{(2)}$
$\bar{B}\langle u^\mu u^\nu \rangle \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$	16	$\frac{2}{\sqrt{3}}f_3^{(2)}$	$2\sqrt{3}\tilde{e}_3^{(2)}$	$-\frac{1}{\sqrt{3}}\tilde{c}_5^{(2)}$	21	0	0	0
$\bar{B}u^\mu u^\nu \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$	17	$-\frac{4}{\sqrt{3}}f_3^{(2)}$	$-4\sqrt{3}\tilde{e}_3^{(2)}$	$\frac{2}{\sqrt{3}}\tilde{c}_5^{(2)}$	22	$-\frac{2}{\sqrt{3}}F_5^{(2)}$	$-2\sqrt{3}\tilde{E}_5^{(2)}$	$\frac{1}{\sqrt{3}}\tilde{C}_7^{(2)}$
$\bar{B}u^\mu u^\nu \gamma_5 \gamma_\nu T_\mu + \text{H.c.}$					23	$\frac{2}{\sqrt{3}}F_5^{(2)}$	$2\sqrt{3}\tilde{E}_5^{(2)}$	$-\frac{1}{\sqrt{3}}\tilde{C}_7^{(2)}$
$i\bar{B}\langle f_+^{\mu\nu} \rangle \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$	18	$\frac{4}{\sqrt{3}}f_4^{(2)}$	$4\sqrt{3}\tilde{e}_4^{(2)}$	$\frac{1}{\sqrt{3}}\tilde{c}_{10}^{(2)}$	24	0	0	0
$i\bar{B}f_+^{\mu\nu} \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$	19	$\frac{4}{\sqrt{3}}f_5^{(2)}$	$4\sqrt{3}\tilde{e}_5^{(2)}$	$\frac{1}{\sqrt{3}}\tilde{c}_9^{(2)}$	25	$\frac{4}{\sqrt{3}}F_6^{(2)}$	$4\sqrt{3}\tilde{E}_6^{(2)}$	$\frac{1}{\sqrt{3}}\tilde{C}_{14}^{(2)}$

C. $\mathcal{O}(p^3)$ and $\mathcal{O}(p^4)$ orders

The numbers of interaction terms are large at the $\mathcal{O}(p^3)$ and $\mathcal{O}(p^4)$ orders. We list the third- and fourth-order results in Appendixes A and B, respectively. Table VI (Table VII) in Appendix A is for the $\mathcal{O}(p^3)$ relativistic (nonrelativistic) terms. Since the LEC relations may be long, part of them are given outside the Table VI. The $\mathcal{O}(p^4)$ relativistic (nonrelativistic) Lagrangian terms are shown in Table VIII (Table IX) in Appendix B. In Table VIII, we only label the number for each term and mark the independent terms in the heavy diquark limit.

From Table VI, the numbers of $\mathcal{O}(p^3)$ terms in $\mathcal{L}_{BB}$, $\mathcal{L}_{TT}$, and $\mathcal{L}_{BT}$ in the $SU(2)$ case are 23, 26, and 25, respectively. Those in the $SU(3)$ case are 33, 38, and 41, respectively. From Table VII, the heavy diquark symmetry reduces significantly the total number of independent terms from 74 (112) to be 23 (31) in the $SU(2)$ [$SU(3)$] case.

From Table VIII, the numbers of $\mathcal{O}(p^4)$ terms in $\mathcal{L}_{BB}$, $\mathcal{L}_{TT}$, and $\mathcal{L}_{BT}$ in the $SU(2)$ case are 118, 154, and 180, respectively. Those in the $SU(3)$ case are 216, 304, and 344, respectively. From Table IX, the heavy diquark symmetry reduces significantly the total number of independent terms from 452 (864) to be 118 (189) in the $SU(2)$ [$SU(3)$] case.

TABLE IV. Independent Lagrangian terms in the heavy diquark limit at the order $\mathcal{O}(p^2)$. Column 2 (3) labels the number for each term in the two-flavor (three-flavor) case. The terms without a number are not independent.

P_n/p_n	$SU(2)$	$SU(3)$
$\bar{\psi}^\mu \langle u^\nu u_\nu \rangle \psi_\mu$	1	1
$\bar{\psi}^\mu \langle u^\nu u^\lambda \rangle v_\nu v_\lambda \psi_\mu$	2	2
$\bar{\psi}^\mu u^\nu u_\nu \psi_\mu$		3
$\bar{\psi}^\mu u^\nu u^\lambda v_\nu v_\lambda \psi_\mu$		4
$i\bar{\psi}^\mu u^\nu u^\lambda \sigma_{\nu\lambda} \psi_\mu$	3	5
$\bar{\psi}^\mu \langle f_+^{\nu\lambda} \rangle \sigma_{\nu\lambda} \psi_\mu$	4	
$\bar{\psi}^\mu f_+^{\nu\lambda} \sigma_{\nu\lambda} \psi_\mu$	5	6
$\bar{\psi}^\mu \langle \chi_+ \rangle \psi_\mu$	6	7
$\bar{\psi}^\mu \chi_+ \psi_\mu$	7	8

D. Discussions

In the literature, one may find part of the constructed chiral Lagrangians for the doubly charmed baryons. Here, we make some comparisons. Our $\mathcal{L}_{BB}$ in the two-flavor case has the same number of independent terms as that in Ref. [72], but the choice of independent structures is different. In the three-flavor case, the terms in our $\mathcal{L}_{BB}$ are consistent with those in Ref. [66] at the first two orders, but we have one less term at the third chiral order. In fact, the 32nd term $O_{32}^{(3)} = \bar{\psi}[D^\lambda, f_+^{\mu\nu}]\gamma_5\gamma_\mu D_{\nu\lambda}\psi + \text{H.c.}$ over there can be eliminated with Eq. (2.41) of Ref. [72]. At the fourth order, there exist some redundant terms similarly, such as $O_{195}^{(4)} = \bar{\psi}[D^{\mu\nu}, f_+^{\lambda\rho}]\sigma_{\lambda\rho} D_{\mu\nu}\psi + \text{H.c.}$, $O_{196}^{(4)} = \bar{\psi}[D^{\mu\nu}, f_+^{\lambda\rho}]\sigma_{\mu\lambda} D_{\nu\rho}\psi + \text{H.c.}$, and so on. Moreover, linear relations, such as Cayley-Hamilton relations, are not completely used. We have checked some of these new relations by hand explicitly and have removed all redundant terms by computer. Hence, we have much fewer terms at the $\mathcal{O}(p^4)$ order. In the heavy diquark limit, our results are consistent with the $\mathcal{O}(p^2)$ terms obtained in Ref. [62].

One may wonder whether the $\mathcal{O}(p^4)$ -order Lagrangian is useful because the number of LECs is very large. At present, there are studies of masses [18,22] and magnetic moments [23,24] for the doubly heavy baryons with chiral Lagrangians up to the $\mathcal{O}(p^4)$ order. The complete Lagrangian we obtain can be used to confirm the terms constructed in the literature. A calculated observable contains different order LECs and one needs to determine the involved LECs by fitting available data. Usually, several LECs or LEC combinations are involved in the study of a special problem. One may, in principle, use various theoretical methods (quark model, large N_c , lattice QCD, etc.) to determine or constrain such LEC values. It is true that available data to determine the $\mathcal{O}(p^4)$ LECs in the existing studies about doubly heavy baryons are still scarce. However, once the LECs are determined, the convergence of chiral expansion, an important issue one should check in

understanding the power counting problem in ChPT, can be certainly better understood in the investigations up to the $\mathcal{O}(p^4)$ order than that to the $\mathcal{O}(p^3)$ order. Moreover, the complete Lagrangian can serve for future studies of any other quantities.

In fact, the complete Lagrangian may motivate future studies of LEC determinations or LEC relations. Since the numbers of LECs are significantly increased for high-order terms, theoretical methods to study LECs in a systematic way are welcome. In Ref. [80], an analytical method was adopted to calculate the LECs for mesonic chiral Lagrangians. A possible method in the present case would also be developed, which needs complete Lagrangians with independent terms. In Refs. [81,82], we preliminarily studied the LEC relations with the help of the chiral quark model up to the third chiral order in the light baryon cases, which is based on operator correspondences between hadron-level structures and quark-level structures. In principle, such a study can be extended to any chiral order, but it also needs complete Lagrangians with independent terms.

Since the LEC determination is very important in the application of ChPT, here we discuss a little more about the chiral quark model method which is feasible in constraining LECs before enough data are available. In Ref. [81], we found LEC relations between the $SU(2)$ πN Lagrangians and the $SU(3)$ meson-baryon Lagrangians, by employing the idea that the hadron-level interactions can be equivalently described in the chiral quark model and by noticing that quarks are placed in the fundamental representations of both $SU(2)$ and $SU(3)$. Since the LECs in the πN Lagrangians can be extracted from a large number of experimental data, one may get some constraints on LECs in the $SU(3)$ ϕB Lagrangians with ϕ (B) being the pseudoscalar meson (octet baryon). Similar to the Lagrangian construction at the hadron level, we have constructed high-order Lagrangians at the quark level. Such Lagrangians may be applied to all the qqq , Qqq , and QQq baryon cases where q (Q) represents a light (heavy) quark. In Ref. [82], when studying the LEC relations for the spin-3/2 Δ baryon case, we found that several multiplication factors to be determined are actually needed. At that time, we only determined values of two such factors from a phenomenological perspective. It is possible to determine such factors with the heavy quark symmetry by using the chiral Lagrangians with QQq and Qqq [83] baryons. Since only one light quark exists in the doubly charmed baryon and the pion interaction is mainly induced by the light quark, it is obvious that the LECs may have some relations with the coupling parameters in the chiral quark model. In the heavy diquark limit, the heavy quarks decouple with the light quark and the spin-1/2 and spin-3/2 doubly charmed baryons can be described simultaneously by the superfield ψ^μ . The number of independent Lagrangian terms for ψ^μ should be the same as that in the chiral quark model description. One may confirm this by

TABLE V. Number of independent terms at each chiral order.

Chiral order	Relativistic Lagrangian				Nonrelativistic Lagrangian			
	$\mathcal{O}(p^1)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$	$\mathcal{O}(p^4)$	$\mathcal{O}(p^1)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$	$\mathcal{O}(p^4)$
$SU(2)$	3	19	74	452	1	7	23	118
$SU(3)$	3	25	112	864	1	8	31	189

consulting Ref. [81] for the relevant Lagrangian up to the third chiral order. Since we mainly focus on the search for a minimal set of Lagrangian terms in the present paper, we leave the discussions to get LEC relations and to determine the multiplication factors for spin-3/2 baryons in a separate work.

It is usually interesting to know how many independent structures there are for the Lagrangian at a certain order in ChPT. The constructed Lagrangian with explicit terms can surely give an answer. Besides the method to construct Lagrangians, it is also possible to find an answer with other approaches. In the mesonic case [84], the Hilbert series techniques have been used to analyze numbers of independent terms at different orders, where the role of the concrete Lagrangians is to validate the method. When a similar study for the doubly heavy baryon case is performed, the Lagrangian constructed here would also be helpful to validate the analysis.

If one replaces the cc diquark by the bb diquark in the doubly charmed baryons, the chiral Lagrangians for the doubly bottom baryons are obtained. For the bcq baryons, the situation is slightly different. In the case that the spin of the bc diquark is 1, the chiral Lagrangians have the same structures discussed above. Since the spin of the bc diquark can also be 0, an additional flavor triplet exists for the spin-1/2 bcq baryons. The Lagrangian structures for the spin-1/2 bcq baryons in the scalar and axial-vector bc diquark cases are the same.

VI. SUMMARY

In this paper, we constructed the relativistic chiral Lagrangians for both spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ doubly charmed baryons up to the order $\mathcal{O}(p^4)$, in both two- and three-flavor cases. The chiral Lagrangians in the heavy diquark limit are also obtained up to the order $\mathcal{O}(p^4)$. Table V collects the number of independent terms at each order. It seems that the numbers in the heavy diquark limit are about one-third (one-fourth) of those in the relativistic case at the order $\mathcal{O}(p^3)$ [$\mathcal{O}(p^4)$]. Obviously, the heavy diquark symmetry is helpful for us to reduce the number of unknown parameters in employing ChPT. We present the LEC relations between the relativistic case and the case in the heavy diquark limit up to the order $\mathcal{O}(p^3)$. In addition, the heavy diquark-antiquark symmetry is also considered. With this symmetry, we obtain the LEC relations between the doubly charmed baryon case and the heavy-light meson case.

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APPENDIX A: $\mathcal{O}(p^3)$ -ORDER RESULTS

This appendix gives Tables VI and VII.

$$\begin{aligned}
\tilde{e}_{26}^{(3)} &= f_1^{(3)} + f_4^{(3)}, & \tilde{e}_{27}^{(3)} &= f_2^{(3)} - f_4^{(3)}, & \tilde{e}_{28}^{(3)} &= -f_3^{(3)} - f_4^{(3)}, & \tilde{e}_{30}^{(3)} &= f_{13}^{(3)} + f_6^{(3)}, & \tilde{e}_{31}^{(3)} &= -2f_{14}^{(3)} + 2f_6^{(3)}, \\
\tilde{e}_{37}^{(3)} &= 2f_{11}^{(3)} - f_8^{(3)}, & \tilde{e}_{39}^{(3)} &= 2f_{12}^{(3)} - f_8^{(3)}, & \tilde{e}_{42}^{(3)} &= f_7^{(3)} - f_8^{(3)}, & \tilde{e}_{52}^{(3)} &= -\frac{2}{\sqrt{3}}f_4^{(3)} - \frac{2}{\sqrt{3}}f_8^{(3)}, & \tilde{e}_{53}^{(3)} &= -\frac{2}{\sqrt{3}}f_3^{(3)} + \frac{2}{\sqrt{3}}f_8^{(3)}, \\
\tilde{e}_{55}^{(3)} &= -\frac{1}{\sqrt{3}}f_{13}^{(3)} + \frac{1}{\sqrt{3}}f_6^{(3)}, & \tilde{e}_{58}^{(3)} &= \frac{2}{\sqrt{3}}f_{13}^{(3)} - \frac{2}{\sqrt{3}}f_6^{(3)} - \frac{4}{\sqrt{3}}f_{14}^{(3)}.
\end{aligned} \tag{A1}$$

$$\begin{aligned}
\tilde{e}_{26}^{(3)} &= 3\tilde{e}_1^{(3)} - 3\tilde{e}_4^{(3)}, & \tilde{e}_{27}^{(3)} &= 3\tilde{e}_2^{(3)} + 3\tilde{e}_4^{(3)}, & \tilde{e}_{28}^{(3)} &= 3\tilde{e}_3^{(3)} + 3\tilde{e}_4^{(3)}, & \tilde{e}_{30}^{(3)} &= 3\tilde{e}_6^{(3)} + 3\tilde{e}_8^{(3)}, & \tilde{e}_{31}^{(3)} &= -6\tilde{e}_7^{(3)} + 6\tilde{e}_8^{(3)}, \\
\tilde{e}_{37}^{(3)} &= -2\tilde{e}_{15}^{(3)} + 3\tilde{e}_{14}^{(3)}, & \tilde{e}_{39}^{(3)} &= -2\tilde{e}_{16}^{(3)} + 3\tilde{e}_{14}^{(3)}, & \tilde{e}_{42}^{(3)} &= 3\tilde{e}_{13}^{(3)} + 3\tilde{e}_{14}^{(3)}, & \tilde{e}_{52}^{(3)} &= 2\sqrt{3}\tilde{e}_{14}^{(3)} + 2\sqrt{3}\tilde{e}_4^{(3)}, \\
\tilde{e}_{53}^{(3)} &= -2\sqrt{3}\tilde{e}_{14}^{(3)} + 2\sqrt{3}\tilde{e}_3^{(3)}, & \tilde{e}_{55}^{(3)} &= -\sqrt{3}\tilde{e}_6^{(3)} + \sqrt{3}\tilde{e}_8^{(3)}, & \tilde{e}_{58}^{(3)} &= 2\sqrt{3}\tilde{e}_6^{(3)} - 2\sqrt{3}\tilde{e}_8^{(3)} - 4\sqrt{3}\tilde{e}_7^{(3)}.
\end{aligned} \tag{A2}$$

TABLE VI. Independent terms in the relativistic Lagrangian at the order $\mathcal{O}(p^3)$. Columns 2, 3, 4, and 5 (6, 7, 8, and 9) are for the two-flavor (three-flavor) case. Columns 2 and 6 label the number for each term. The terms without a number are not independent in the $SU(2)$ case. Columns 3 and 7 list the relations between LECs in the relativistic case and those in the heavy diquark limit. Columns 4 and 8 show the LEC relations among different terms in the relativistic case by using the heavy diquark symmetry. “I” means that the term is chosen as an independent term in the heavy diquark limit. Columns 5 and 9 give the LEC relations with the help of the heavy diquark-antiquark symmetry, where $\tilde{c}$ and $\tilde{C}$ are LECs in the heavy-light meson case. Such LECs are defined in Ref. [78]. “0” means that the LECs in the heavy diquark limit vanish. The symbol “*” indicates that the relation is long and we show relevant results in Eqs. (A1)–(A6) in this appendix.

O_n/o_n	$SU(2)$	$\tilde{e}_n^{(3)}$	$\tilde{e}_n^{(3)}$	$\tilde{e}_n^{(3)}$	$SU(3)$	$\tilde{E}_n^{(3)}$	$\tilde{E}_n^{(3)}$	$\tilde{E}_n^{(3)}$
$\bar{B}\langle u^\mu u_\mu \rangle u^\nu \gamma_5 \gamma_\nu B$	1	$\frac{1}{3}f_1^{(3)}$	I	*	1	$\frac{1}{3}F_1^{(3)}$	I	*
$\bar{B}\langle u^\mu u^\nu \rangle u_\mu \gamma_5 \gamma_\nu B$	2	$\frac{1}{3}f_2^{(3)}$	I	$-\frac{1}{12}\tilde{c}_{10}^{(3)}$	2	$\frac{1}{3}F_2^{(3)}$	I	*
$\bar{B}\langle u^\mu u^\nu \rangle u^\lambda \gamma_5 \gamma_\mu D_{\nu\lambda} B$	3	$-\frac{1}{3}f_3^{(3)}$	I	*	3	$-\frac{1}{3}F_3^{(3)}$	I	*
$\bar{B}\langle u^\mu u^\nu \rangle u^\lambda \gamma_5 \gamma_\lambda D_{\mu\nu} B$	4	$-\frac{1}{3}f_4^{(3)}$	I	*	4	$-\frac{1}{3}F_4^{(3)}$	I	*
$\bar{B}\langle u^\mu u_\mu u^\nu \rangle \gamma_5 \gamma_\nu B$					5	$\frac{1}{3}F_5^{(3)}$	I	*
$\bar{B}\langle u^\mu u^\nu u^\lambda \rangle \gamma_5 \gamma_\mu D_{\nu\lambda} B$					6	$-\frac{1}{3}F_6^{(3)}$	I	*
$\bar{B}u^\mu u_\mu u^\nu \gamma_5 \gamma_\nu B + \text{H.c.}$					7	$\frac{1}{3}F_7^{(3)}$	I	*
$\bar{B}u^\mu u^\nu u^\lambda \gamma_5 \gamma_\mu D_{\nu\lambda} B + \text{H.c.}$					8	$-\frac{1}{3}F_8^{(3)}$	I	*
$\epsilon^{\mu\nu\lambda\rho} \bar{B}\langle u_\mu u_\nu u_\lambda \rangle D_\rho B$	5	$f_5^{(3)}$	I	$-\tilde{c}_1^{(3)}$	9	$F_9^{(3)}$	I	$-\frac{1}{2}\tilde{C}_2^{(3)}$
$\epsilon^{\mu\nu\lambda\rho} \bar{B}u_\mu u_\nu u_\lambda D_\rho B$					10	$F_{10}^{(3)}$	I	$-\frac{1}{2}\tilde{C}_1^{(3)}$
$i\bar{B}\langle u^\mu f_{-\nu\lambda} \rangle \sigma_{\mu\nu} D_\lambda B$	6	$\frac{1}{3}f_{13}^{(3)}$	I	$-\frac{1}{6}\tilde{c}_{15}^{(3)}$	11	$\frac{1}{3}F_{19}^{(3)}$	I	$-\frac{1}{6}\tilde{C}_{30}^{(3)}$
$i\bar{B}\langle u^\mu f_{-\nu\lambda} \rangle \sigma_{\nu\lambda} D_\mu B$	7	$\frac{1}{3}f_{14}^{(3)}$	I	$\frac{1}{12}\tilde{c}_{16}^{(3)}$	12	$\frac{1}{3}F_{20}^{(3)}$	I	$\frac{1}{12}\tilde{C}_{31}^{(3)}$
$i\bar{B}\langle u^\mu h^{\nu\lambda} \rangle \sigma_{\mu\nu} D_\lambda B$	8	$\frac{1}{3}f_6^{(3)}$	I	$\frac{1}{6}\tilde{c}_{19}^{(3)}$	13	$\frac{1}{3}F_{11}^{(3)}$	I	$\frac{1}{6}\tilde{C}_{32}^{(3)}$
$\bar{B}u^\mu f_{-\mu}{}^\nu D_\nu B + \text{H.c.}$	9	$f_{15}^{(3)}$	I	$-\frac{1}{2}\tilde{c}_2^{(3)}$	14	$F_{21}^{(3)}$	I	$-\frac{1}{2}\tilde{C}_3^{(3)}$
$i\bar{B}u^\mu f_{-\nu\lambda} \sigma_{\mu\nu} D_\lambda B + \text{H.c.}$					15	$\frac{1}{3}F_{22}^{(3)}$	I	$-\frac{1}{6}\tilde{C}_{21}^{(3)}$
$i\bar{B}u^\mu f_{-\nu\lambda} \sigma_{\nu\lambda} D_\mu B + \text{H.c.}$					16	$\frac{1}{3}F_{23}^{(3)}$	I	$\frac{1}{12}\tilde{C}_{22}^{(3)}$
$\bar{B}u^\mu h_{\mu}{}^\nu D_\nu B + \text{H.c.}$	10	$f_9^{(3)}$	I	$-\frac{1}{2}\tilde{c}_3^{(3)}$	17	$F_{14}^{(3)}$	I	$-\frac{1}{2}\tilde{C}_4^{(3)}$
$\bar{B}u^\mu h^{\nu\lambda} D_{\mu\nu\lambda} B + \text{H.c.}$	11	$-f_{10}^{(3)}$	I	$-\frac{1}{2}\tilde{c}_4^{(3)}$	18	$-F_{15}^{(3)}$	I	$-\frac{1}{2}\tilde{C}_5^{(3)}$
$i\bar{B}u^\mu h^{\nu\lambda} \sigma_{\mu\nu} D_\lambda B + \text{H.c.}$					19	$\frac{1}{3}F_{16}^{(3)}$	I	$\frac{1}{6}\tilde{C}_{25}^{(3)}$
$\bar{B}\nabla^\mu f_{-\mu}{}^\nu \gamma_5 \gamma_\nu B$	12	$\frac{1}{3}f_{18}^{(3)}$	I	$\frac{1}{6}\tilde{c}_{24}^{(3)}$	20	$\frac{1}{3}F_{25}^{(3)}$	I	$\frac{1}{6}\tilde{C}_{33}^{(3)}$
$i\bar{B}f_{+\mu}{}^{\mu\nu} u_\mu \gamma_5 \gamma_\nu B + \text{H.c.}$	13	$\frac{1}{3}f_7^{(3)}$	I	$\frac{1}{6}\tilde{c}_{30}^{(3)}$	21	$\frac{1}{3}F_{12}^{(3)}$	I	$\frac{1}{6}\tilde{C}_{39}^{(3)}$
$i\bar{B}f_{+\mu}{}^{\mu\nu} u^\lambda \gamma_5 \gamma_\mu D_{\nu\lambda} B + \text{H.c.}$	14	$-\frac{1}{3}f_8^{(3)}$	I	$\frac{1}{6}\tilde{c}_{41}^{(3)}$	22	$-\frac{1}{3}F_{13}^{(3)}$	I	$\frac{1}{3}\tilde{C}_{37}^{(3)}$
$i\epsilon^{\mu\nu\lambda\rho} \bar{B}\langle f_{+\mu\nu} \rangle u_\lambda D_\rho B$	15	$-f_{11}^{(3)}$	I	$-\frac{1}{2}\tilde{c}_6^{(3)}$	23	0	0	0
$i\epsilon^{\mu\nu\lambda\rho} \bar{B}\langle f_{+\mu\nu} u_\lambda \rangle D_\rho B$	16	$-f_{12}^{(3)}$	I	*	24	$-F_{17}^{(3)}$	I	$-\frac{1}{2}\tilde{C}_7^{(3)}$
$i\epsilon^{\mu\nu\lambda\rho} \bar{B}f_{+\mu\nu} u_\lambda D_\rho B + \text{H.c.}$					25	$-F_{18}^{(3)}$	I	$-\frac{1}{2}\tilde{C}_6^{(3)}$
$i\bar{B}\langle \nabla^\mu f_{+\mu}{}^\nu \rangle D_\nu B$	17	$-f_{16}^{(3)}$	I	$-\frac{1}{2}\tilde{c}_8^{(3)}$	26	0	0	0
$i\bar{B}\nabla^\mu f_{+\mu}{}^\nu D_\nu B$	18	$-f_{17}^{(3)}$	I	$-\frac{1}{2}\tilde{c}_7^{(3)}$	27	$-F_{24}^{(3)}$	I	$-\frac{1}{2}\tilde{C}_8^{(3)}$
$\bar{B}\langle u^\mu \chi_+ \rangle \gamma_5 \gamma_\mu B$	19	$\frac{1}{3}f_{19}^{(3)}$	I	$-\frac{1}{6}\tilde{c}_{38}^{(3)}$	28	$\frac{1}{3}F_{26}^{(3)}$	I	$-\frac{1}{6}\tilde{C}_{45}^{(3)}$
$\bar{B}\langle \chi_+ \rangle u^\mu \gamma_5 \gamma_\mu B$	20	$\frac{1}{3}f_{20}^{(3)}$	I	*	29	$\frac{1}{3}F_{27}^{(3)}$	I	$-\frac{1}{6}\tilde{C}_{46}^{(3)}$
$\bar{B}u^\mu \chi_+ \gamma_5 \gamma_\mu B + \text{H.c.}$					30	$\frac{1}{3}F_{28}^{(3)}$	I	$-\frac{1}{6}\tilde{C}_{44}^{(3)}$
$i\bar{B}u^\mu \chi_- D_\mu B + \text{H.c.}$	21	$-f_{21}^{(3)}$	I	$-\frac{1}{2}\tilde{c}_9^{(3)}$	31	$-F_{29}^{(3)}$	I	$-\frac{1}{2}\tilde{C}_9^{(3)}$
$i\bar{B}\langle \nabla^\mu \chi_- \rangle \gamma_5 \gamma_\mu B$	22	$\frac{1}{3}f_{22}^{(3)}$	I	$-\frac{1}{6}\tilde{c}_{41}^{(3)}$	32	$\frac{1}{3}F_{30}^{(3)}$	I	$-\frac{1}{6}\tilde{C}_{49}^{(3)}$
$i\bar{B}\nabla^\mu \chi_- \gamma_5 \gamma_\mu B$	23	$\frac{1}{3}f_{23}^{(3)}$	I	$-\frac{1}{6}\tilde{c}_{40}^{(3)}$	33	$\frac{1}{3}F_{31}^{(3)}$	I	$-\frac{1}{6}\tilde{C}_{48}^{(3)}$
$\bar{T}^\mu \langle u_\mu u^\nu \rangle u^\lambda \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$	24	$f_4^{(3)}$	$-3\tilde{e}_4^{(3)}$	*	34	*	*	*
$\bar{T}^\mu \langle u_\mu u^\nu \rangle u^\lambda \gamma_5 \gamma_\lambda T_\nu$	25	$-2f_4^{(3)}$	$6\tilde{e}_4^{(3)}$	*	35	*	*	*
$\bar{T}^\mu \langle u^\nu u_\nu \rangle u^\lambda \gamma_5 \gamma_\lambda T_\mu$	26	*	*	*	36	*	*	*
$\bar{T}^\mu \langle u^\nu u^\lambda \rangle u_\nu \gamma_5 \gamma_\lambda T_\mu$	27	*	*	*	37	*	*	*
$\bar{T}^\mu \langle u^\nu u^\lambda \rangle u^\rho \gamma_5 \gamma_\nu D_{\lambda\rho} T_\mu$	28	*	*	*	38	*	*	*

(Table continued)

TABLE VI. (Continued)

O_n/o_n	$SU(2)$	$\tilde{e}_n^{(3)}$	$\tilde{e}_n^{(3)}$	$\tilde{e}_n^{(3)}$	$SU(3)$	$\tilde{E}_n^{(3)}$	$\tilde{E}_n^{(3)}$	$\tilde{E}_n^{(3)}$
$\bar{T}^\mu \langle u_\mu u^\nu u^\lambda \rangle \gamma_5 \gamma_\nu T_\lambda$	29	$-6f_5^{(3)}$	$-6\tilde{e}_5^{(3)}$	$6\tilde{c}_1^{(3)}$	39	*	*	*
$\bar{T}^\mu \langle u_\mu u^\nu u^\lambda \rangle \gamma_5 \gamma_\lambda T_\nu$					40	*	*	*
$\bar{T}^\mu \langle u^\nu u_\nu u^\lambda \rangle \gamma_5 \gamma_\lambda T_\mu$					41	*	*	*
$\bar{T}^\mu \langle u^\nu u^\lambda u^\rho \rangle \gamma_5 \gamma_\nu D_{\lambda\rho} T_\mu$					42	*	*	*
$\bar{T}^\mu u_\mu u^\nu u^\lambda \gamma_5 \gamma_\nu T_\lambda$					43	$-2F_{10}^{(3)}$	$-2\tilde{E}_{10}^{(3)}$	$\tilde{C}_1^{(3)}$
$\bar{T}^\mu u_\mu u^\nu u^\lambda \gamma_5 \gamma_\lambda T_\nu + \text{H.c.}$					44	$-F_8^{(3)}$	$3\tilde{E}_8^{(3)}$	*
$\bar{T}^\mu u^\nu u_\mu u^\lambda \gamma_5 \gamma_\lambda T_\nu + \text{H.c.}$					45	*	*	*
$\bar{T}^\mu u^\nu u_\nu u^\lambda \gamma_5 \gamma_\lambda T_\mu + \text{H.c.}$					46	*	*	*
$\bar{T}^\mu \langle u_\mu f_{-}^{\nu\lambda} \rangle D_\nu T_\lambda + \text{H.c.}$	30	*	*	*	47	*	*	*
$\bar{T}^\mu \langle u^\nu f_{-}^{\lambda} \rangle D_\nu T_\lambda$	31	*	*	*	48	*	*	*
$\bar{T}^\mu u_\mu f_{-}^{\nu\lambda} D_\nu T_\lambda + \text{H.c.}$	32	0	0	0	49	*	*	*
$\bar{T}^\mu u^\nu f_{-}^{\lambda} D_\nu T_\lambda + \text{H.c.}$					50	*	*	*
$\bar{T}^\mu u^\nu f_{-}^{\lambda} D_\lambda T_\nu + \text{H.c.}$					51	*	*	*
$\bar{T}^\mu u^\nu f_{-}^{\lambda} D_\lambda T_\mu + \text{H.c.}$	33	$-f_{15}^{(3)}$	$-\tilde{e}_9^{(3)}$	$\frac{1}{2}\tilde{c}_2^{(3)}$	52	$-F_{21}^{(3)}$	$-\tilde{E}_{14}^{(3)}$	$\frac{1}{2}\tilde{C}_3^{(3)}$
$\bar{T}^\mu u_\mu h^{\nu\lambda} D_\nu T_\lambda + \text{H.c.}$	34	0	0	0	53	0	0	0
$\bar{T}^\mu u^\nu h_\nu^\lambda D_\lambda T_\mu + \text{H.c.}$	35	$-f_9^{(3)}$	$-\tilde{e}_{10}^{(3)}$	$\frac{1}{2}\tilde{c}_3^{(3)}$	54	$-F_{14}^{(3)}$	$-\tilde{E}_{17}^{(3)}$	$\frac{1}{2}\tilde{C}_4^{(3)}$
$\bar{T}^\mu u^\nu h^{\lambda\rho} D_{\nu\lambda\rho} T_\mu + \text{H.c.}$	36	$f_{10}^{(3)}$	$-\tilde{e}_{11}^{(3)}$	$\frac{1}{2}\tilde{c}_4^{(3)}$	55	$F_{15}^{(3)}$	$-\tilde{E}_{18}^{(3)}$	$\frac{1}{2}\tilde{C}_5^{(3)}$
$i\bar{T}^\mu \langle f_{+}^{\nu} \rangle u^\lambda \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$	37	*	*	*	56	0	0	0
$i\bar{T}^\mu \langle f_{+}^{\nu} \rangle u^\lambda \gamma_5 \gamma_\lambda T_\nu$	38	$-2f_{11}^{(3)}$	$2\tilde{e}_{15}^{(3)}$	$-\tilde{c}_6^{(3)}$	57	0	0	0
$i\bar{T}^\mu \langle f_{+}^{\nu} u^\lambda \rangle \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$	39	*	*	*	58	$2F_{17}^{(3)}$	$-2\tilde{E}_{24}^{(3)}$	$\tilde{C}_7^{(3)}$
$i\bar{T}^\mu \langle f_{+}^{\nu} u^\lambda \rangle \gamma_5 \gamma_\lambda T_\nu$	40	$-2f_{12}^{(3)}$	$2\tilde{e}_{16}^{(3)}$	*	59	$-2F_{17}^{(3)}$	$2\tilde{E}_{24}^{(3)}$	$-\tilde{C}_7^{(3)}$
$i\bar{T}^\mu f_{+}^{\nu} u^\lambda \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$	41	$2f_8^{(3)}$	$-6\tilde{e}_{14}^{(3)}$	$-\tilde{c}_{41}^{(3)}$	60	*	*	*
$i\bar{T}^\mu f_{+}^{\nu} u^\lambda \gamma_5 \gamma_\lambda T_\nu + \text{H.c.}$					61	$-2F_{18}^{(3)}$	$2\tilde{E}_{25}^{(3)}$	$-\tilde{C}_6^{(3)}$
$i\bar{T}^\mu f_{+}^{\nu\lambda} u_\mu \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$					62	*	*	*
$i\bar{T}^\mu f_{+}^{\nu\lambda} u_\nu \gamma_5 \gamma_\lambda T_\mu + \text{H.c.}$	42	*	*	*	63	*	*	*
$i\bar{T}^\mu \langle \nabla^\nu f_{+}^{\lambda} \rangle D_\lambda T_\mu$	43	$f_{16}^{(3)}$	$-\tilde{e}_{17}^{(3)}$	$\frac{1}{2}\tilde{c}_8^{(3)}$	64	0	0	0
$i\bar{T}^\mu \nabla^\nu f_{+}^{\lambda} D_\lambda T_\mu$	44	$f_{17}^{(3)}$	$-\tilde{e}_{18}^{(3)}$	$\frac{1}{2}\tilde{c}_7^{(3)}$	65	$F_{24}^{(3)}$	$-\tilde{E}_{27}^{(3)}$	$\frac{1}{2}\tilde{C}_8^{(3)}$
$\bar{T}^\mu \langle u^\nu \chi_{+} \rangle \gamma_5 \gamma_\nu T_\mu$	45	$f_{19}^{(3)}$	$3\tilde{e}_{19}^{(3)}$	$-\frac{1}{2}\tilde{c}_{38}^{(3)}$	66	$F_{26}^{(3)}$	$3\tilde{E}_{26}^{(3)}$	$-\frac{1}{2}\tilde{C}_{45}^{(3)}$
$\bar{T}^\mu \langle \chi_{+} \rangle u^\nu \gamma_5 \gamma_\nu T_\mu$	46	$f_{20}^{(3)}$	$3\tilde{e}_{20}^{(3)}$	*	67	$F_{27}^{(3)}$	$3\tilde{E}_{29}^{(3)}$	$-\frac{1}{2}\tilde{C}_{46}^{(3)}$
$\bar{T}^\mu u^\nu \chi_{+} \gamma_5 \gamma_\nu T_\mu + \text{H.c.}$					68	$F_{28}^{(3)}$	$3\tilde{E}_{30}^{(3)}$	$-\frac{1}{2}\tilde{C}_{44}^{(3)}$
$i\bar{T}^\mu u^\nu \chi_{-} D_\nu T_\mu + \text{H.c.}$	47	$f_{21}^{(3)}$	$-\tilde{e}_{21}^{(3)}$	$\frac{1}{2}\tilde{c}_9^{(3)}$	69	$F_{29}^{(3)}$	$-\tilde{E}_{31}^{(3)}$	$\frac{1}{2}\tilde{C}_9^{(3)}$
$i\bar{T}^\mu \langle \nabla^\nu \chi_{-} \rangle \gamma_5 \gamma_\nu T_\mu$	48	$f_{22}^{(3)}$	$3\tilde{e}_{22}^{(3)}$	$-\frac{1}{2}\tilde{c}_{41}^{(3)}$	70	$F_{30}^{(3)}$	$3\tilde{E}_{32}^{(3)}$	$-\frac{1}{2}\tilde{C}_{49}^{(3)}$
$i\bar{T}^\mu \nabla^\nu \chi_{-} \gamma_5 \gamma_\nu T_\mu$	49	$f_{23}^{(3)}$	$3\tilde{e}_{23}^{(3)}$	$-\frac{1}{2}\tilde{c}_{40}^{(3)}$	71	$F_{31}^{(3)}$	$3\tilde{E}_{33}^{(3)}$	$-\frac{1}{2}\tilde{C}_{48}^{(3)}$
$\bar{B} \langle u^\mu u_\mu \rangle u^\nu T_\nu + \text{H.c.}$	50	$\frac{2}{\sqrt{3}}f_1^{(3)}$	$2\sqrt{3}\tilde{e}_1^{(3)}$	*	72	*	*	*
$\bar{B} \langle u^\mu u^\nu \rangle u_\mu T_\nu + \text{H.c.}$	51	$\frac{2}{\sqrt{3}}f_2^{(3)}$	$2\sqrt{3}\tilde{e}_2^{(3)}$	$-\frac{1}{2\sqrt{3}}\tilde{c}_{10}^{(3)}$	73	*	*	*
$\bar{B} \langle u^\mu u^\nu \rangle u^\lambda D_{\mu\nu} T_\lambda + \text{H.c.}$	52	*	*	*	74	*	*	*
$\bar{B} \langle u^\mu u^\nu \rangle u^\lambda D_{\mu\lambda} T_\nu + \text{H.c.}$	53	*	*	*	75	*	*	*
$i\bar{B} \langle u^\mu u^\nu \rangle u^\lambda \sigma_{\mu\lambda} T_\nu + \text{H.c.}$	54	0	0	0	76	0	0	0
$\bar{B} \langle u^\mu u_\mu u^\nu \rangle T_\nu + \text{H.c.}$					77	*	*	*
$\bar{B} \langle u^\mu u^\nu u^\lambda \rangle D_{\mu\nu} T_\lambda + \text{H.c.}$					78	*	*	*
$\bar{B} u^\mu u_\mu u^\nu T_\nu + \text{H.c.}$					79	0	0	0
$\bar{B} u^\mu u^\nu u_\mu T_\nu + \text{H.c.}$					80	$-\frac{2}{\sqrt{3}}F_7^{(3)}$	$-2\sqrt{3}\tilde{E}_7^{(3)}$	*
$\bar{B} u^\mu u^\nu u^\lambda D_{\mu\nu} T_\lambda + \text{H.c.}$					81	0	0	0
$\bar{B} u^\mu u^\nu u^\lambda D_{\mu\lambda} T_\nu + \text{H.c.}$					82	*	*	*
$i\bar{B} u^\mu u^\nu u^\lambda \sigma_{\mu\nu} T_\lambda + \text{H.c.}$					83	0	0	0
$i\bar{B} u^\mu u^\nu u^\lambda \sigma_{\mu\lambda} T_\nu + \text{H.c.}$					84	0	0	0

(Table continued)

TABLE VI. (Continued)

O_n/o_n	$SU(2)$	$\tilde{e}_n^{(3)}$	$\tilde{e}_n^{(3)}$	$\tilde{e}_n^{(3)}$	$SU(3)$	$\tilde{E}_n^{(3)}$	$\tilde{E}_n^{(3)}$	$\tilde{E}_n^{(3)}$
$\epsilon^{\mu\nu\lambda\rho}\bar{B}\langle u_\mu f_{-\nu\lambda}\rangle T_\rho + \text{H.c.}$	55	*	*	*	85	*	*	*
$\epsilon^{\mu\nu\lambda\rho}\bar{B}u_\mu f_{-\nu\lambda}T_\rho + \text{H.c.}$	56	0	0	0	86	*	*	*
$\epsilon^{\mu\nu\lambda\rho}\bar{B}f_{-\mu\nu}u_\lambda T_\rho + \text{H.c.}$					87	$-\frac{2}{\sqrt{3}}F_{23}^{(3)}$	$-2\sqrt{3}\tilde{E}_{16}^{(3)}$	$-\frac{1}{2\sqrt{3}}\tilde{C}_{22}^{(3)}$
$\bar{B}\langle u^\mu f_{-\nu\lambda}\rangle\gamma_5\gamma_\mu D_\nu T_\lambda + \text{H.c.}$	57	$\frac{4}{\sqrt{3}}f_6^{(3)}$	$4\sqrt{3}\tilde{e}_8^{(3)}$	$\frac{2}{\sqrt{3}}\tilde{c}_{19}^{(3)}$	88	$\frac{4}{\sqrt{3}}F_{11}^{(3)}$	$4\sqrt{3}\tilde{E}_{13}^{(3)}$	$\frac{2}{\sqrt{3}}\tilde{C}_{32}^{(3)}$
$\bar{B}\langle u^\mu f_{-\nu\lambda}\rangle\gamma_5\gamma_\nu D_\mu T_\lambda + \text{H.c.}$	58	*	*	*	89	*	*	*
$\bar{B}\langle u^\mu h^{\nu\lambda}\rangle\gamma_5\gamma_\mu D_\nu T_\lambda + \text{H.c.}$	59	$-\frac{4}{\sqrt{3}}f_6^{(3)}$	$-4\sqrt{3}\tilde{e}_8^{(3)}$	$-\frac{2}{\sqrt{3}}\tilde{c}_{19}^{(3)}$	90	$-\frac{4}{\sqrt{3}}F_{11}^{(3)}$	$-4\sqrt{3}\tilde{E}_{13}^{(3)}$	$-\frac{2}{\sqrt{3}}\tilde{C}_{32}^{(3)}$
$\bar{B}u^\mu f_{-\nu\lambda}\gamma_5\gamma_\mu D_\nu T_\lambda + \text{H.c.}$	60	0	0	0	91	*	*	*
$\bar{B}u^\mu f_{-\nu\lambda}\gamma_5\gamma_\nu D_\mu T_\lambda + \text{H.c.}$	61	0	0	0	92	*	*	*
$\bar{B}u^\mu h^{\nu\lambda}\gamma_5\gamma_\mu D_\nu T_\lambda + \text{H.c.}$	62	0	0	0	93	$-\frac{4}{\sqrt{3}}F_{16}^{(3)}$	$-4\sqrt{3}\tilde{E}_{19}^{(3)}$	$-\frac{2}{\sqrt{3}}\tilde{C}_{25}^{(3)}$
$\bar{B}u^\mu h^{\nu\lambda}\gamma_5\gamma_\nu D_\mu T_\lambda + \text{H.c.}$					94	*	*	*
$\bar{B}u^\mu h^{\nu\lambda}\gamma_5\gamma_\nu D_\lambda T_\mu + \text{H.c.}$					95	*	*	*
$\bar{B}f_{-\mu\nu}u^\lambda\gamma_5\gamma_\mu D_\nu T_\lambda + \text{H.c.}$					96	*	*	*
$\bar{B}\nabla^\mu f_{-\mu}{}^\nu T_\nu + \text{H.c.}$	63	$\frac{2}{\sqrt{3}}f_{18}^{(3)}$	$2\sqrt{3}\tilde{e}_{12}^{(3)}$	$\frac{1}{\sqrt{3}}\tilde{c}_{24}^{(3)}$	97	$\frac{2}{\sqrt{3}}F_{25}^{(3)}$	$2\sqrt{3}\tilde{E}_{20}^{(3)}$	$\frac{1}{\sqrt{3}}\tilde{C}_{33}^{(3)}$
$i\bar{B}\langle f_+{}^{\mu\nu}\rangle u_\mu T_\nu + \text{H.c.}$	64	$-\frac{2}{\sqrt{3}}f_7^{(3)}$	$-2\sqrt{3}\tilde{e}_{13}^{(3)}$	$-\frac{1}{\sqrt{3}}\tilde{c}_{30}^{(3)}$	98	0	0	0
$i\bar{B}\langle f_+{}^{\mu\nu}\rangle u^\lambda D_{\mu\lambda} T_\nu + \text{H.c.}$	65	0	0	0	99	0	0	0
$\bar{B}\langle f_+{}^{\mu\nu}\rangle u^\lambda\sigma_{\mu\nu}T_\lambda + \text{H.c.}$	66	0	0	0	100	0	0	0
$i\bar{B}\langle f_+{}^{\mu\nu}u_\mu\rangle T_\nu + \text{H.c.}$	67	$-\frac{2}{\sqrt{3}}f_7^{(3)}$	$-2\sqrt{3}\tilde{e}_{13}^{(3)}$	$-\frac{1}{\sqrt{3}}\tilde{c}_{30}^{(3)}$	101	0	0	0
$i\bar{B}\langle f_+{}^{\mu\nu}u^\lambda\rangle D_{\mu\lambda} T_\nu + \text{H.c.}$	68	0	0	0	102	0	0	0
$\bar{B}\langle f_+{}^{\mu\nu}u^\lambda\rangle\sigma_{\mu\nu}T_\lambda + \text{H.c.}$	69	0	0	0	103	0	0	0
$i\bar{B}f_+{}^{\mu\nu}u_\mu T_\nu + \text{H.c.}$	70	$\frac{4}{\sqrt{3}}f_7^{(3)}$	$4\sqrt{3}\tilde{e}_{13}^{(3)}$	$\frac{2}{\sqrt{3}}\tilde{c}_{30}^{(3)}$	104	$\frac{2}{\sqrt{3}}F_{12}^{(3)}$	$2\sqrt{3}\tilde{E}_{21}^{(3)}$	$\frac{1}{\sqrt{3}}\tilde{C}_{39}^{(3)}$
$i\bar{B}f_+{}^{\mu\nu}u^\lambda D_{\mu\lambda} T_\nu + \text{H.c.}$					105	0	0	0
$\bar{B}f_+{}^{\mu\nu}u^\lambda\sigma_{\mu\nu}T_\lambda + \text{H.c.}$	71	0	0	0	106	0	0	0
$i\bar{B}u^\mu f_{+\mu}{}^\nu T_\nu + \text{H.c.}$					107	$-\frac{2}{\sqrt{3}}F_{12}^{(3)}$	$-2\sqrt{3}\tilde{E}_{21}^{(3)}$	$-\frac{1}{\sqrt{3}}\tilde{C}_{39}^{(3)}$
$\bar{B}u^\mu f_{+\mu}{}^\nu\sigma_{\mu\nu}T_\lambda + \text{H.c.}$					108	0	0	0
$\bar{B}\langle u^\mu\chi_+\rangle T_\mu + \text{H.c.}$	72	$\frac{2}{\sqrt{3}}f_{19}^{(3)}$	$2\sqrt{3}\tilde{e}_{19}^{(3)}$	$-\frac{1}{\sqrt{3}}\tilde{c}_{38}^{(3)}$	109	$\frac{2}{\sqrt{3}}F_{26}^{(3)}$	$2\sqrt{3}\tilde{E}_{28}^{(3)}$	$-\frac{1}{\sqrt{3}}\tilde{C}_{45}^{(3)}$
$\bar{B}\langle\chi_+\rangle u^\mu T_\mu + \text{H.c.}$	73	$\frac{2}{\sqrt{3}}f_{20}^{(3)}$	$2\sqrt{3}\tilde{e}_{20}^{(3)}$	*	110	$\frac{2}{\sqrt{3}}F_{27}^{(3)}$	$2\sqrt{3}\tilde{E}_{29}^{(3)}$	$-\frac{1}{\sqrt{3}}\tilde{C}_{46}^{(3)}$
$\bar{B}u^\mu\chi_+T_\mu + \text{H.c.}$	74	0	0	0	111	$\frac{2}{\sqrt{3}}F_{28}^{(3)}$	$2\sqrt{3}\tilde{E}_{30}^{(3)}$	$-\frac{1}{\sqrt{3}}\tilde{C}_{44}^{(3)}$
$\bar{B}\chi_+u^\mu T_\mu + \text{H.c.}$					112	$\frac{2}{\sqrt{3}}F_{28}^{(3)}$	$2\sqrt{3}\tilde{E}_{30}^{(3)}$	$-\frac{1}{\sqrt{3}}\tilde{C}_{44}^{(3)}$

$$\begin{aligned}
\tilde{e}_1^{(3)} &= -\frac{1}{12}\tilde{c}_{13}^{(3)} + \frac{1}{24}\tilde{c}_{10}^{(3)}, & \tilde{e}_3^{(3)} &= -\frac{1}{12}\tilde{c}_{10}^{(3)} - \frac{1}{6}\tilde{c}_{11}^{(3)} - \frac{1}{6}\tilde{c}_{14}^{(3)}, & \tilde{e}_4^{(3)} &= \frac{1}{3}\tilde{c}_{11}^{(3)} - \frac{1}{3}\tilde{c}_{14}^{(3)} + \frac{1}{6}\tilde{c}_{10}^{(3)}, & \tilde{e}_{16}^{(3)} &= \frac{1}{2}\tilde{c}_5^{(3)} - \frac{1}{2}\tilde{c}_6^{(3)}, \\
\tilde{e}_{20}^{(3)} &= \frac{1}{6}\tilde{c}_{37}^{(3)} - \frac{1}{6}\tilde{c}_{38}^{(3)}, & \tilde{e}_{24}^{(3)} &= -\frac{1}{2}\tilde{c}_{10}^{(3)} - \tilde{c}_{11}^{(3)} + \tilde{c}_{14}^{(3)}, & \tilde{e}_{25}^{(3)} &= 2\tilde{c}_{11}^{(3)} - 2\tilde{c}_{14}^{(3)} + \tilde{c}_{10}^{(3)}, & \tilde{e}_{26}^{(3)} &= -\frac{1}{4}\tilde{c}_{13}^{(3)} - \frac{3}{8}\tilde{c}_{10}^{(3)} - \tilde{c}_{11}^{(3)} + \tilde{c}_{14}^{(3)}, \\
\tilde{e}_{27}^{(3)} &= \frac{1}{4}\tilde{c}_{10}^{(3)} + \tilde{c}_{11}^{(3)} - \tilde{c}_{14}^{(3)}, & \tilde{e}_{28}^{(3)} &= \frac{1}{2}\tilde{c}_{11}^{(3)} + \frac{1}{4}\tilde{c}_{10}^{(3)} - \frac{3}{2}\tilde{c}_{14}^{(3)}, & \tilde{e}_{30}^{(3)} &= -\frac{1}{2}\tilde{c}_{15}^{(3)} + \frac{1}{2}\tilde{c}_{19}^{(3)}, & \tilde{e}_{31}^{(3)} &= -\frac{1}{2}\tilde{c}_{16}^{(3)} + \tilde{c}_{19}^{(3)}, \\
\tilde{e}_{37}^{(3)} &= \frac{1}{2}\tilde{c}_{41}^{(3)} + \tilde{c}_6^{(3)}, & \tilde{e}_{39}^{(3)} &= \frac{1}{2}\tilde{c}_{41}^{(3)} - \tilde{c}_5^{(3)} + \tilde{c}_6^{(3)}, & \tilde{e}_{40}^{(3)} &= \tilde{c}_5^{(3)} - \tilde{c}_6^{(3)}, & \tilde{e}_{42}^{(3)} &= \frac{1}{2}\tilde{c}_{30}^{(3)} + \frac{1}{2}\tilde{c}_{41}^{(3)}, & \tilde{e}_{46}^{(3)} &= \frac{1}{2}\tilde{c}_{37}^{(3)} - \frac{1}{2}\tilde{c}_{38}^{(3)}, \\
\tilde{e}_{50}^{(3)} &= -\frac{1}{2\sqrt{3}}\tilde{c}_{13}^{(3)} + \frac{1}{4\sqrt{3}}\tilde{c}_{10}^{(3)}, & \tilde{e}_{52}^{(3)} &= \frac{1}{\sqrt{3}}\tilde{c}_{10}^{(3)} + \frac{1}{\sqrt{3}}\tilde{c}_{41}^{(3)} + \frac{2}{\sqrt{3}}\tilde{c}_{11}^{(3)} - \frac{2}{\sqrt{3}}\tilde{c}_{14}^{(3)}, \\
\tilde{e}_{53}^{(3)} &= -\frac{1}{2\sqrt{3}}\tilde{c}_{10}^{(3)} - \frac{1}{\sqrt{3}}\tilde{c}_{11}^{(3)} - \frac{1}{\sqrt{3}}\tilde{c}_{14}^{(3)} - \frac{1}{\sqrt{3}}\tilde{c}_{41}^{(3)}, & \tilde{e}_{55}^{(3)} &= \frac{1}{2\sqrt{3}}\tilde{c}_{15}^{(3)} + \frac{1}{2\sqrt{3}}\tilde{c}_{19}^{(3)}, & \tilde{e}_{58}^{(3)} &= -\frac{1}{\sqrt{3}}\tilde{c}_{15}^{(3)} - \frac{1}{\sqrt{3}}\tilde{c}_{16}^{(3)} - \frac{1}{\sqrt{3}}\tilde{c}_{19}^{(3)}, \\
\tilde{e}_{73}^{(3)} &= \frac{1}{\sqrt{3}}\tilde{c}_{37}^{(3)} - \frac{1}{\sqrt{3}}\tilde{c}_{38}^{(3)}.
\end{aligned} \tag{A3}$$

TABLE VII. Independent Lagrangian terms in the heavy diquark limit at the order $\mathcal{O}(p^3)$. Columns 2 and 5 (3 and 6) label the number for each term in the two-flavor (three-flavor) case. The terms without a number are not independent.

P_n/p_n	$SU(2)$	$SU(3)$	P_n/p_n	$SU(2)$	$SU(3)$
$\bar{\psi}^\mu \langle u^\nu u_\nu \rangle u^\lambda \gamma_5 \gamma_\lambda \psi_\mu$	1	1	$\epsilon^{\mu\nu\lambda\rho} \bar{\psi}^\sigma \langle f_{+\mu\nu} u_\lambda \rangle v_\rho \psi_\sigma$	12	17
$\bar{\psi}^\mu \langle u^\nu u^\lambda \rangle u_\nu \gamma_5 \gamma_\lambda \psi_\mu$	2	2	$\epsilon^{\mu\nu\lambda\rho} \bar{\psi}^\sigma f_{+\mu\nu} u_\lambda v_\rho \psi_\sigma + \text{H.c.}$		18
$\bar{\psi}^\mu \langle u^\nu u^\lambda \rangle u^\rho \gamma_5 \gamma_\nu v_\lambda v_\rho \psi_\mu$	3	3	$\bar{\psi}^\mu \langle u^\nu f_{-\lambda\rho} \rangle \sigma_{\nu\lambda} v_\rho \psi_\mu$	13	19
$\bar{\psi}^\mu \langle u^\nu u^\lambda \rangle u^\rho \gamma_5 \gamma_\rho v_\nu v_\lambda \psi_\mu$	4	4	$\bar{\psi}^\mu \langle u^\nu f_{-\lambda\rho} \rangle \sigma_{\lambda\rho} v_\nu \psi_\mu$	14	20
$\bar{\psi}^\mu \langle u^\nu u_\nu u^\lambda \rangle \gamma_5 \gamma_\lambda \psi_\mu$		5	$i \bar{\psi}^\mu u^\nu f_{-\nu}^\lambda v_\lambda \psi_\mu + \text{H.c.}$	15	21
$\bar{\psi}^\mu \langle u^\nu u^\lambda u^\rho \rangle \gamma_5 \gamma_\nu v_\lambda v_\rho \psi_\mu$		6	$\bar{\psi}^\mu \langle \nabla^\nu f_{+\nu}^\lambda \rangle v_\lambda \psi_\mu$	16	
$\bar{\psi}^\mu u^\nu u_\nu u^\lambda \gamma_5 \gamma_\lambda \psi_\mu + \text{H.c.}$		7	$\bar{\psi}^\mu u^\nu f_{-\lambda\rho} \sigma_{\nu\lambda} v_\rho \psi_\mu + \text{H.c.}$		22
$\bar{\psi}^\mu u^\nu u^\lambda u^\rho \gamma_5 \gamma_\nu v_\lambda v_\rho \psi_\mu + \text{H.c.}$		8	$\bar{\psi}^\mu u^\nu f_{-\lambda\rho} \sigma_{\lambda\rho} v_\nu \psi_\mu + \text{H.c.}$		23
$i \epsilon^{\mu\nu\lambda\rho} \bar{\psi}^\sigma \langle u_\mu u_\nu u_\lambda \rangle v_\rho \psi_\sigma$	5	9	$\bar{\psi}^\mu \nabla^\nu f_{+\nu}^\lambda v_\lambda \psi_\mu$	17	24
$i \epsilon^{\mu\nu\lambda\rho} \bar{\psi}^\sigma u_\mu u_\nu u_\lambda v_\rho \psi_\sigma$		10	$\bar{\psi}^\mu \nabla^\nu f_{-\nu}^\lambda \gamma_5 \gamma_\lambda \psi_\mu$	18	25
$\bar{\psi}^\mu \langle u^\nu h^{\lambda\rho} \rangle \sigma_{\nu\lambda} v_\rho \psi_\mu$	6	11	$\bar{\psi}^\mu \langle u^\nu \chi_+ \rangle \gamma_5 \gamma_\nu \psi_\mu$	19	26
$i \bar{\psi}^\mu f_{+}^{\nu\lambda} u_\nu \gamma_5 \gamma_\lambda \psi_\mu + \text{H.c.}$	7	12	$\bar{\psi}^\mu \langle \chi_+ \rangle u^\nu \gamma_5 \gamma_\nu \psi_\mu$	20	27
$i \bar{\psi}^\mu f_{+}^{\nu\lambda} u^\rho \gamma_5 \gamma_\nu v_\lambda v_\rho \psi_\mu + \text{H.c.}$	8	13	$\bar{\psi}^\mu u^\nu \chi_+ \gamma_5 \gamma_\nu \psi_\mu + \text{H.c.}$		28
$i \bar{\psi}^\mu u^\nu h_{\nu}^{\lambda} v_\lambda \psi_\mu + \text{H.c.}$	9	14	$\bar{\psi}^\mu u^\nu \chi_- v_\nu \psi_\mu + \text{H.c.}$	21	29
$i \bar{\psi}^\mu u^\nu h^{\lambda\rho} v_\nu v_\lambda v_\rho \psi_\mu + \text{H.c.}$	10	15	$i \bar{\psi}^\mu \langle \nabla^\nu \chi_- \rangle \gamma_5 \gamma_\nu \psi_\mu$	22	30
$\epsilon^{\mu\nu\lambda\rho} \bar{\psi}^\sigma \langle f_{+\mu\nu} \rangle u_\lambda v_\rho \psi_\sigma$	11		$i \bar{\psi}^\mu \nabla^\nu \chi_- \gamma_5 \gamma_\nu \psi_\mu$	23	31
$\bar{\psi}^\mu u^\nu h^{\lambda\rho} \sigma_{\nu\lambda} v_\rho \psi_\mu + \text{H.c.}$		16			

$$\begin{aligned}
\tilde{E}_{34}^{(3)} &= F_{10}^{(3)} + F_4^{(3)} + F_8^{(3)}, & \tilde{E}_{35}^{(3)} &= -2F_4^{(3)} + F_{10}^{(3)}, & \tilde{E}_{36}^{(3)} &= F_1^{(3)} + F_4^{(3)}, & \tilde{E}_{37}^{(3)} &= F_2^{(3)} - F_4^{(3)} - F_8^{(3)}, \\
\tilde{E}_{38}^{(3)} &= -F_3^{(3)} - F_4^{(3)} - F_8^{(3)}, & \tilde{E}_{39}^{(3)} &= -3F_9^{(3)} + F_{10}^{(3)} + \frac{2}{3}F_8^{(3)}, & \tilde{E}_{40}^{(3)} &= 3F_9^{(3)} + F_{10}^{(3)} + \frac{2}{3}F_8^{(3)}, & \tilde{E}_{41}^{(3)} &= F_5^{(3)} - \frac{2}{3}F_8^{(3)}, \\
\tilde{E}_{42}^{(3)} &= -F_6^{(3)} - \frac{2}{3}F_8^{(3)}, & \tilde{E}_{45}^{(3)} &= -2F_{10}^{(3)} - F_8^{(3)}, & \tilde{E}_{46}^{(3)} &= F_7^{(3)} + F_8^{(3)}, & \tilde{E}_{47}^{(3)} &= F_{11}^{(3)} + F_{19}^{(3)}, & \tilde{E}_{48}^{(3)} &= 2F_{11}^{(3)} - 2F_{20}^{(3)}, \\
\tilde{E}_{49}^{(3)} &= F_{16}^{(3)} + F_{22}^{(3)}, & \tilde{E}_{50}^{(3)} &= 2F_{16}^{(3)} - 2F_{23}^{(3)}, & \tilde{E}_{51}^{(3)} &= F_{16}^{(3)} + F_{22}^{(3)}, & \tilde{E}_{60}^{(3)} &= 2F_{18}^{(3)} + F_{13}^{(3)}, & \tilde{E}_{62}^{(3)} &= 2F_{18}^{(3)} - F_{13}^{(3)}, \\
\tilde{E}_{63}^{(3)} &= F_{12}^{(3)} - F_{13}^{(3)}, & \tilde{E}_{72}^{(3)} &= \frac{1}{\sqrt{3}}F_7^{(3)} + \frac{2}{\sqrt{3}}F_1^{(3)}, & \tilde{E}_{73}^{(3)} &= \frac{2}{\sqrt{3}}F_2^{(3)} + \frac{2}{\sqrt{3}}F_7^{(3)}, & \tilde{E}_{74}^{(3)} &= -\frac{1}{2\sqrt{3}}F_{13}^{(3)} - \frac{1}{\sqrt{3}}F_8^{(3)} - \frac{2}{\sqrt{3}}F_4^{(3)}, \\
\tilde{E}_{75}^{(3)} &= -\frac{1}{\sqrt{3}}F_{13}^{(3)} - \frac{2}{\sqrt{3}}F_3^{(3)} - \frac{2}{\sqrt{3}}F_8^{(3)}, & \tilde{E}_{77}^{(3)} &= \frac{2}{\sqrt{3}}F_5^{(3)} + \frac{2}{\sqrt{3}}F_7^{(3)}, & \tilde{E}_{78}^{(3)} &= -\frac{1}{\sqrt{3}}F_{13}^{(3)} - \frac{2}{\sqrt{3}}F_6^{(3)} - \frac{2}{\sqrt{3}}F_8^{(3)}, \\
\tilde{E}_{82}^{(3)} &= \frac{2}{\sqrt{3}}F_8^{(3)} + \sqrt{3}F_{13}^{(3)}, & \tilde{E}_{85}^{(3)} &= \frac{1}{\sqrt{3}}F_{11}^{(3)} - \frac{1}{\sqrt{3}}F_{19}^{(3)}, & \tilde{E}_{86}^{(3)} &= -\frac{1}{2\sqrt{3}}F_{16}^{(3)} + \frac{1}{\sqrt{3}}F_{23}^{(3)} - \frac{\sqrt{3}}{2}F_{22}^{(3)}, \\
\tilde{E}_{89}^{(3)} &= -\frac{2}{\sqrt{3}}F_{11}^{(3)} + \frac{2}{\sqrt{3}}F_{19}^{(3)} - \frac{4}{\sqrt{3}}F_{20}^{(3)}, & \tilde{E}_{91}^{(3)} &= \frac{1}{\sqrt{3}}F_{16}^{(3)} - \frac{1}{\sqrt{3}}F_{22}^{(3)} + \frac{2}{\sqrt{3}}F_{23}^{(3)}, & \tilde{E}_{92}^{(3)} &= \frac{2}{\sqrt{3}}F_{16}^{(3)} + \frac{2}{\sqrt{3}}F_{22}^{(3)} - \frac{4}{\sqrt{3}}F_{23}^{(3)}, \\
\tilde{E}_{94}^{(3)} &= \frac{1}{\sqrt{3}}F_{16}^{(3)} - \frac{1}{\sqrt{3}}F_{22}^{(3)} + \frac{2}{\sqrt{3}}F_{23}^{(3)}, & \tilde{E}_{95}^{(3)} &= \frac{1}{\sqrt{3}}F_{22}^{(3)} - \frac{2}{\sqrt{3}}F_{23}^{(3)} + \sqrt{3}F_{16}^{(3)}, & \tilde{E}_{96}^{(3)} &= -\frac{2}{\sqrt{3}}F_{16}^{(3)} + \frac{2}{\sqrt{3}}F_{22}^{(3)} - \frac{4}{\sqrt{3}}F_{23}^{(3)}. \quad (\text{A4})
\end{aligned}$$

$$\begin{aligned}
\tilde{E}_{34}^{(3)} &= -3\tilde{E}_4^{(3)} - 3\tilde{E}_8^{(3)} + \tilde{E}_{10}^{(3)}, & \tilde{E}_{35}^{(3)} &= 6\tilde{E}_4^{(3)} + \tilde{E}_{10}^{(3)}, & \tilde{E}_{36}^{(3)} &= 3\tilde{E}_1^{(3)} - 3\tilde{E}_4^{(3)}, & \tilde{E}_{37}^{(3)} &= 3\tilde{E}_2^{(3)} + 3\tilde{E}_4^{(3)} + 3\tilde{E}_8^{(3)}, \\
\tilde{E}_{38}^{(3)} &= 3\tilde{E}_3^{(3)} + 3\tilde{E}_4^{(3)} + 3\tilde{E}_8^{(3)}, & \tilde{E}_{39}^{(3)} &= -2\tilde{E}_8^{(3)} - 3\tilde{E}_9^{(3)} + \tilde{E}_{10}^{(3)}, & \tilde{E}_{40}^{(3)} &= -2\tilde{E}_8^{(3)} + 3\tilde{E}_9^{(3)} + \tilde{E}_{10}^{(3)}, \\
\tilde{E}_{41}^{(3)} &= 2\tilde{E}_8^{(3)} + 3\tilde{E}_5^{(3)}, & \tilde{E}_{42}^{(3)} &= 2\tilde{E}_8^{(3)} + 3\tilde{E}_6^{(3)}, & \tilde{E}_{45}^{(3)} &= -2\tilde{E}_{10}^{(3)} + 3\tilde{E}_8^{(3)}, & \tilde{E}_{46}^{(3)} &= 3\tilde{E}_7^{(3)} - 3\tilde{E}_8^{(3)}, & \tilde{E}_{47}^{(3)} &= 3\tilde{E}_{11}^{(3)} + 3\tilde{E}_{13}^{(3)}, \\
\tilde{E}_{48}^{(3)} &= -6\tilde{E}_{12}^{(3)} + 6\tilde{E}_{13}^{(3)}, & \tilde{E}_{49}^{(3)} &= 3\tilde{E}_{15}^{(3)} + 3\tilde{E}_{19}^{(3)}, & \tilde{E}_{50}^{(3)} &= -6\tilde{E}_{16}^{(3)} + 6\tilde{E}_{19}^{(3)}, & \tilde{E}_{51}^{(3)} &= 3\tilde{E}_{15}^{(3)} + 3\tilde{E}_{19}^{(3)},
\end{aligned}$$

$$\begin{aligned}
\tilde{E}_{60}^{(3)} &= -2\tilde{E}_{25}^{(3)} - 3\tilde{E}_{22}^{(3)}, & \tilde{E}_{62}^{(3)} &= -2\tilde{E}_{25}^{(3)} + 3\tilde{E}_{22}^{(3)}, & \tilde{E}_{63}^{(3)} &= 3\tilde{E}_{21}^{(3)} + 3\tilde{E}_{22}^{(3)}, & \tilde{E}_{72}^{(3)} &= 2\sqrt{3}\tilde{E}_1^{(3)} + \sqrt{3}\tilde{E}_7^{(3)}, \\
\tilde{E}_{73}^{(3)} &= 2\sqrt{3}\tilde{E}_2^{(3)} + 2\sqrt{3}\tilde{E}_7^{(3)}, & \tilde{E}_{74}^{(3)} &= 2\sqrt{3}\tilde{E}_4^{(3)} + \frac{\sqrt{3}}{2}\tilde{E}_{22}^{(3)} + \sqrt{3}\tilde{E}_8^{(3)}, & \tilde{E}_{75}^{(3)} &= 2\sqrt{3}\tilde{E}_3^{(3)} + 2\sqrt{3}\tilde{E}_8^{(3)} + \sqrt{3}\tilde{E}_{22}^{(3)}, \\
\tilde{E}_{77}^{(3)} &= 2\sqrt{3}\tilde{E}_5^{(3)} + 2\sqrt{3}\tilde{E}_7^{(3)}, & \tilde{E}_{78}^{(3)} &= 2\sqrt{3}\tilde{E}_6^{(3)} + 2\sqrt{3}\tilde{E}_8^{(3)} + \sqrt{3}\tilde{E}_{22}^{(3)}, & \tilde{E}_{82}^{(3)} &= -2\sqrt{3}\tilde{E}_8^{(3)} - 3\sqrt{3}\tilde{E}_{22}^{(3)}, \\
\tilde{E}_{85}^{(3)} &= -\sqrt{3}\tilde{E}_{11}^{(3)} + \sqrt{3}\tilde{E}_{13}^{(3)}, & \tilde{E}_{86}^{(3)} &= -\frac{3\sqrt{3}}{2}\tilde{E}_{15}^{(3)} - \frac{\sqrt{3}}{2}\tilde{E}_{19}^{(3)} + \sqrt{3}\tilde{E}_{16}^{(3)}, & \tilde{E}_{89}^{(3)} &= 2\sqrt{3}\tilde{E}_{11}^{(3)} - 2\sqrt{3}\tilde{E}_{13}^{(3)} - 4\sqrt{3}\tilde{E}_{12}^{(3)}, \\
\tilde{E}_{91}^{(3)} &= 2\sqrt{3}\tilde{E}_{16}^{(3)} - \sqrt{3}\tilde{E}_{15}^{(3)} + \sqrt{3}\tilde{E}_{19}^{(3)}, & \tilde{E}_{92}^{(3)} &= 2\sqrt{3}\tilde{E}_{15}^{(3)} + 2\sqrt{3}\tilde{E}_{19}^{(3)} - 4\sqrt{3}\tilde{E}_{16}^{(3)}, & \tilde{E}_{94}^{(3)} &= 2\sqrt{3}\tilde{E}_{16}^{(3)} - \sqrt{3}\tilde{E}_{15}^{(3)} + \sqrt{3}\tilde{E}_{19}^{(3)}, \\
\tilde{E}_{95}^{(3)} &= -2\sqrt{3}\tilde{E}_{16}^{(3)} + 3\sqrt{3}\tilde{E}_{19}^{(3)} + \sqrt{3}\tilde{E}_{15}^{(3)}, & \tilde{E}_{96}^{(3)} &= 2\sqrt{3}\tilde{E}_{15}^{(3)} - 2\sqrt{3}\tilde{E}_{19}^{(3)} - 4\sqrt{3}\tilde{E}_{16}^{(3)}.
\end{aligned} \tag{A5}$$

$$\begin{aligned}
\tilde{E}_1^{(3)} &= \frac{1}{12}\tilde{C}_{19}^{(3)} - \frac{1}{4}\tilde{C}_{17}^{(3)}, & \tilde{E}_2^{(3)} &= \frac{1}{12}\tilde{C}_1^{(3)} + \frac{13}{24}\tilde{C}_{19}^{(3)} + \frac{1}{6}\tilde{C}_{10}^{(3)} + \frac{1}{6}\tilde{C}_{13}^{(3)} - \frac{5}{8}\tilde{C}_{17}^{(3)}, & \tilde{E}_3^{(3)} &= \frac{1}{2}\tilde{C}_{17}^{(3)} + \frac{1}{2}\tilde{C}_{18}^{(3)} - \frac{1}{6}\tilde{C}_{20}^{(3)}, \\
\tilde{E}_4^{(3)} &= -\frac{1}{12}\tilde{C}_1^{(3)} - \frac{1}{6}\tilde{C}_{10}^{(3)} - \frac{1}{6}\tilde{C}_{11}^{(3)} - \frac{1}{6}\tilde{C}_{15}^{(3)} + \frac{1}{6}\tilde{C}_{20}^{(3)} - \frac{3}{2}\tilde{C}_{17}^{(3)} - \frac{3}{2}\tilde{C}_{18}^{(3)}, & \tilde{E}_5^{(3)} &= -\frac{1}{6}\tilde{C}_{17}^{(3)} + \frac{1}{6}\tilde{C}_{19}^{(3)}, \\
\tilde{E}_6^{(3)} &= \frac{1}{3}\tilde{C}_{17}^{(3)} + \frac{1}{3}\tilde{C}_{18}^{(3)}, & \tilde{E}_7^{(3)} &= \frac{1}{12}\tilde{C}_1^{(3)} + \frac{1}{6}\tilde{C}_{10}^{(3)} - \frac{1}{6}\tilde{C}_{13}^{(3)} + \frac{1}{6}\tilde{C}_{17}^{(3)} - \frac{1}{6}\tilde{C}_{19}^{(3)}, \\
\tilde{E}_8^{(3)} &= \frac{1}{12}\tilde{C}_1^{(3)} - \frac{1}{3}\tilde{C}_{17}^{(3)} - \frac{1}{3}\tilde{C}_{18}^{(3)} + \frac{1}{6}\tilde{C}_{10}^{(3)} + \frac{1}{6}\tilde{C}_{11}^{(3)} - \frac{1}{6}\tilde{C}_{15}^{(3)}, & \tilde{E}_{34}^{(3)} &= \frac{11}{2}\tilde{C}_{17}^{(3)} + \frac{11}{2}\tilde{C}_{18}^{(3)} - \frac{1}{2}\tilde{C}_1^{(3)} - \frac{1}{2}\tilde{C}_{20}^{(3)} + \tilde{C}_{15}^{(3)}, \\
\tilde{E}_{35}^{(3)} &= -9\tilde{C}_{17}^{(3)} - 9\tilde{C}_{18}^{(3)} - \tilde{C}_{10}^{(3)} - \tilde{C}_{11}^{(3)} - \tilde{C}_1^{(3)} - \tilde{C}_{15}^{(3)} + \tilde{C}_{20}^{(3)}, \\
\tilde{E}_{36}^{(3)} &= \frac{1}{2}\tilde{C}_{10}^{(3)} + \frac{1}{2}\tilde{C}_{11}^{(3)} + \frac{1}{2}\tilde{C}_{15}^{(3)} - \frac{1}{2}\tilde{C}_{20}^{(3)} + \frac{1}{4}\tilde{C}_1^{(3)} + \frac{1}{4}\tilde{C}_{19}^{(3)} + \frac{15}{4}\tilde{C}_{17}^{(3)} + \frac{9}{2}\tilde{C}_{18}^{(3)}, \\
\tilde{E}_{37}^{(3)} &= -\frac{11}{2}\tilde{C}_{18}^{(3)} + \frac{1}{2}\tilde{C}_{10}^{(3)} + \frac{1}{2}\tilde{C}_{13}^{(3)} + \frac{1}{2}\tilde{C}_{20}^{(3)} + \frac{13}{8}\tilde{C}_{19}^{(3)} + \frac{1}{4}\tilde{C}_1^{(3)} - \frac{59}{8}\tilde{C}_{17}^{(3)} - \tilde{C}_{15}^{(3)}, & \tilde{E}_{38}^{(3)} &= -4\tilde{C}_{17}^{(3)} - 4\tilde{C}_{18}^{(3)} - \tilde{C}_{15}^{(3)}, \\
\tilde{E}_{39}^{(3)} &= -\frac{1}{3}\tilde{C}_{10}^{(3)} - \frac{1}{3}\tilde{C}_{11}^{(3)} + \frac{1}{3}\tilde{C}_{15}^{(3)} - \frac{2}{3}\tilde{C}_1^{(3)} + \frac{2}{3}\tilde{C}_{17}^{(3)} + \frac{2}{3}\tilde{C}_{18}^{(3)} + \frac{3}{2}\tilde{C}_2^{(3)}, \\
\tilde{E}_{40}^{(3)} &= -\frac{1}{3}\tilde{C}_{10}^{(3)} - \frac{1}{3}\tilde{C}_{11}^{(3)} + \frac{1}{3}\tilde{C}_{15}^{(3)} - \frac{2}{3}\tilde{C}_1^{(3)} + \frac{2}{3}\tilde{C}_{17}^{(3)} + \frac{2}{3}\tilde{C}_{18}^{(3)} - \frac{3}{2}\tilde{C}_2^{(3)}, \\
\tilde{E}_{41}^{(3)} &= \frac{1}{2}\tilde{C}_{19}^{(3)} + \frac{1}{3}\tilde{C}_{10}^{(3)} + \frac{1}{3}\tilde{C}_{11}^{(3)} - \frac{1}{3}\tilde{C}_{15}^{(3)} + \frac{1}{6}\tilde{C}_1^{(3)} - \frac{2}{3}\tilde{C}_{18}^{(3)} - \frac{7}{6}\tilde{C}_{17}^{(3)}, \\
\tilde{E}_{42}^{(3)} &= \frac{1}{3}\tilde{C}_{10}^{(3)} + \frac{1}{3}\tilde{C}_{11}^{(3)} - \frac{1}{3}\tilde{C}_{15}^{(3)} + \frac{1}{3}\tilde{C}_{17}^{(3)} + \frac{1}{3}\tilde{C}_{18}^{(3)} + \frac{1}{6}\tilde{C}_1^{(3)}, & \tilde{E}_{44}^{(3)} &= \frac{1}{2}\tilde{C}_{10}^{(3)} + \frac{1}{2}\tilde{C}_{11}^{(3)} - \frac{1}{2}\tilde{C}_{15}^{(3)} + \frac{1}{4}\tilde{C}_1^{(3)} - \tilde{C}_{17}^{(3)} - \tilde{C}_{18}^{(3)}, \\
\tilde{E}_{45}^{(3)} &= \frac{1}{2}\tilde{C}_{10}^{(3)} + \frac{1}{2}\tilde{C}_{11}^{(3)} - \frac{1}{2}\tilde{C}_{15}^{(3)} + \frac{5}{4}\tilde{C}_1^{(3)} - \tilde{C}_{17}^{(3)} - \tilde{C}_{18}^{(3)}, & \tilde{E}_{46}^{(3)} &= -\frac{1}{2}\tilde{C}_{11}^{(3)} - \frac{1}{2}\tilde{C}_{13}^{(3)} + \frac{1}{2}\tilde{C}_{15}^{(3)} - \frac{1}{2}\tilde{C}_{19}^{(3)} + \frac{3}{2}\tilde{C}_{17}^{(3)} + \tilde{C}_{18}^{(3)}, \\
\tilde{E}_{47}^{(3)} &= -\frac{1}{2}\tilde{C}_{30}^{(3)} + \frac{1}{2}\tilde{C}_{32}^{(3)}, & \tilde{E}_{48}^{(3)} &= -\frac{1}{2}\tilde{C}_{31}^{(3)} + \tilde{C}_{32}^{(3)}, & \tilde{E}_{49}^{(3)} &= -\frac{1}{2}\tilde{C}_{21}^{(3)} + \frac{1}{2}\tilde{C}_{25}^{(3)}, & \tilde{E}_{50}^{(3)} &= -\frac{1}{2}\tilde{C}_{22}^{(3)} + \tilde{C}_{25}^{(3)}, \\
\tilde{E}_{51}^{(3)} &= -\frac{1}{2}\tilde{C}_{21}^{(3)} + \frac{1}{2}\tilde{C}_{25}^{(3)}, & \tilde{E}_{60}^{(3)} &= -\tilde{C}_{37}^{(3)} + \tilde{C}_6^{(3)}, & \tilde{E}_{62}^{(3)} &= \tilde{C}_{37}^{(3)} + \tilde{C}_6^{(3)}, & \tilde{E}_{63}^{(3)} &= \frac{1}{2}\tilde{C}_{39}^{(3)} + \tilde{C}_{37}^{(3)}, \\
\tilde{E}_{72}^{(3)} &= \frac{1}{2\sqrt{3}}\tilde{C}_{10}^{(3)} - \frac{1}{2\sqrt{3}}\tilde{C}_{13}^{(3)} + \frac{1}{4\sqrt{3}}\tilde{C}_1^{(3)} - \frac{1}{\sqrt{3}}\tilde{C}_{17}^{(3)}, & \tilde{E}_{73}^{(3)} &= -\frac{11}{4\sqrt{3}}\tilde{C}_{17}^{(3)} + \frac{1}{\sqrt{3}}\tilde{C}_1^{(3)} + \frac{2}{\sqrt{3}}\tilde{C}_{10}^{(3)} + \frac{3\sqrt{3}}{4}\tilde{C}_{19}^{(3)}, \\
\tilde{E}_{74}^{(3)} &= -\frac{10}{\sqrt{3}}\tilde{C}_{17}^{(3)} - \frac{10}{\sqrt{3}}\tilde{C}_{18}^{(3)} - \frac{1}{2\sqrt{3}}\tilde{C}_{10}^{(3)} - \frac{1}{2\sqrt{3}}\tilde{C}_{11}^{(3)} + \frac{1}{2\sqrt{3}}\tilde{C}_{37}^{(3)} - \frac{1}{4\sqrt{3}}\tilde{C}_1^{(3)} + \frac{1}{\sqrt{3}}\tilde{C}_{20}^{(3)} - \frac{\sqrt{3}}{2}\tilde{C}_{15}^{(3)}, \\
\tilde{E}_{75}^{(3)} &= \frac{1}{2\sqrt{3}}\tilde{C}_1^{(3)} + \frac{1}{\sqrt{3}}\tilde{C}_{10}^{(3)} + \frac{1}{\sqrt{3}}\tilde{C}_{11}^{(3)} - \frac{1}{\sqrt{3}}\tilde{C}_{15}^{(3)} + \frac{1}{\sqrt{3}}\tilde{C}_{17}^{(3)} + \frac{1}{\sqrt{3}}\tilde{C}_{18}^{(3)} - \frac{1}{\sqrt{3}}\tilde{C}_{20}^{(3)} + \frac{1}{\sqrt{3}}\tilde{C}_{37}^{(3)}, \\
\tilde{E}_{77}^{(3)} &= \frac{1}{2\sqrt{3}}\tilde{C}_1^{(3)} + \frac{1}{\sqrt{3}}\tilde{C}_{10}^{(3)} - \frac{1}{\sqrt{3}}\tilde{C}_{13}^{(3)}, & \tilde{E}_{78}^{(3)} &= \frac{1}{2\sqrt{3}}\tilde{C}_1^{(3)} + \frac{1}{\sqrt{3}}\tilde{C}_{10}^{(3)} + \frac{1}{\sqrt{3}}\tilde{C}_{11}^{(3)} - \frac{1}{\sqrt{3}}\tilde{C}_{15}^{(3)} + \frac{1}{\sqrt{3}}\tilde{C}_{37}^{(3)},
\end{aligned}$$

$$\begin{aligned}
\tilde{E}_{80}^{(3)} &= -\frac{1}{2\sqrt{3}}\tilde{C}_1^{(3)} - \frac{1}{\sqrt{3}}\tilde{C}_{10}^{(3)} + \frac{1}{\sqrt{3}}\tilde{C}_{13}^{(3)} - \frac{1}{\sqrt{3}}\tilde{C}_{17}^{(3)} + \frac{1}{\sqrt{3}}\tilde{C}_{19}^{(3)}, \\
\tilde{E}_{82}^{(3)} &= -\frac{1}{2\sqrt{3}}\tilde{C}_1^{(3)} - \frac{1}{\sqrt{3}}\tilde{C}_{10}^{(3)} - \frac{1}{\sqrt{3}}\tilde{C}_{11}^{(3)} + \frac{1}{\sqrt{3}}\tilde{C}_{15}^{(3)} + \frac{2}{\sqrt{3}}\tilde{C}_{17}^{(3)} + \frac{2}{\sqrt{3}}\tilde{C}_{18}^{(3)} - \sqrt{3}\tilde{C}_{37}^{(3)}, & \tilde{E}_{85}^{(3)} &= \frac{1}{2\sqrt{3}}\tilde{C}_{30}^{(3)} + \frac{1}{2\sqrt{3}}\tilde{C}_{32}^{(3)}, \\
\tilde{E}_{86}^{(3)} &= \frac{1}{4\sqrt{3}}\tilde{C}_{22}^{(3)} - \frac{1}{4\sqrt{3}}\tilde{C}_{25}^{(3)} + \frac{\sqrt{3}}{4}\tilde{C}_{21}^{(3)}, & \tilde{E}_{89}^{(3)} &= -\frac{1}{\sqrt{3}}\tilde{C}_{30}^{(3)} - \frac{1}{\sqrt{3}}\tilde{C}_{31}^{(3)} - \frac{1}{\sqrt{3}}\tilde{C}_{32}^{(3)}, \\
\tilde{E}_{91}^{(3)} &= \frac{1}{2\sqrt{3}}\tilde{C}_{21}^{(3)} + \frac{1}{2\sqrt{3}}\tilde{C}_{22}^{(3)} + \frac{1}{2\sqrt{3}}\tilde{C}_{25}^{(3)}, & \tilde{E}_{92}^{(3)} &= -\frac{1}{\sqrt{3}}\tilde{C}_{21}^{(3)} - \frac{1}{\sqrt{3}}\tilde{C}_{22}^{(3)} + \frac{1}{\sqrt{3}}\tilde{C}_{25}^{(3)}, \\
\tilde{E}_{94}^{(3)} &= \frac{1}{2\sqrt{3}}\tilde{C}_{21}^{(3)} + \frac{1}{2\sqrt{3}}\tilde{C}_{22}^{(3)} + \frac{1}{2\sqrt{3}}\tilde{C}_{25}^{(3)}, & \tilde{E}_{95}^{(3)} &= -\frac{1}{2\sqrt{3}}\tilde{C}_{21}^{(3)} - \frac{1}{2\sqrt{3}}\tilde{C}_{22}^{(3)} + \frac{\sqrt{3}}{2}\tilde{C}_{25}^{(3)}, \\
\tilde{E}_{96}^{(3)} &= -\frac{1}{\sqrt{3}}\tilde{C}_{21}^{(3)} - \frac{1}{\sqrt{3}}\tilde{C}_{22}^{(3)} - \frac{1}{\sqrt{3}}\tilde{C}_{25}^{(3)}.
\end{aligned} \tag{A6}$$

APPENDIX B: $\mathcal{O}(p^4)$ -ORDER RESULTS

This appendix gives Tables VIII and IX.

TABLE VIII. Independent terms in the relativistic Lagrangian at the order $\mathcal{O}(p^4)$. Columns 2 and 7 (4 and 9) are for the two-flavor (three-flavor) case. Columns 2, 4, 7, and 9 label the number for each term. The terms without a number are not independent. “T” means that the term is chosen as an independent term in the heavy diquark limit.

O_n/o_n	$SU(2)$	$\tilde{e}_n^{(4)}$	$SU(3)$	$\tilde{E}_n^{(4)}$	O_n	$SU(2)$	$\tilde{e}_n^{(4)}$	$SU(3)$	$\tilde{E}_n^{(4)}$
$\bar{B}\langle u^\mu u_\mu \rangle \langle u^\nu u_\nu \rangle B$	1	I	1	I	$\bar{T}^\mu \langle f_+^{\nu\lambda} \rangle \langle f_{+\nu\lambda} \rangle T_\mu$				436
$\bar{B}\langle u^\mu u^\nu \rangle \langle u_\mu u_\nu \rangle B$	2	I	2	I	$\bar{T}^\mu \langle f_{+\mu}^{\nu\lambda} \rangle \langle f_{+\nu\lambda} \rangle D_{\nu\lambda} T_\rho$				437
$\bar{B}\langle u^\mu u_\mu \rangle \langle u^\nu u^\lambda \rangle D_{\nu\lambda} B$	3	I	3	I	$\bar{T}^\mu \langle f_+^{\nu\lambda} \rangle \langle f_{+\nu}^{\rho\lambda} \rangle D_{\lambda\rho} T_\mu$				438
$\bar{B}\langle u^\mu u^\nu \rangle \langle u_\mu u^\lambda \rangle D_{\nu\lambda} B$	4	I	4	I	$i\bar{T}^\mu \langle f_{+\mu}^{\nu\lambda} \rangle \langle f_{+\nu}^{\rho\lambda} \rangle \sigma_{\nu\lambda} T_\rho$				439
$\bar{B}\langle u^\mu u^\nu \rangle \langle u^\lambda u^\rho \rangle D_{\mu\nu\lambda\rho} B$	5	I	5	I	$\bar{T}^\mu \langle f_{+\mu}^{\nu\lambda} \rangle f_{+\nu}^{\lambda\rho} T_\lambda + \text{H.c.}$	213			440
$\bar{B}\langle u^\mu u_\mu \rangle u^\nu u_\nu B$			6	I	$\bar{T}^\mu \langle f_+^{\nu\lambda} \rangle f_{+\nu\lambda} T_\mu$	214			441
$\bar{B}\langle u^\mu u^\nu \rangle u_\mu u_\nu B$			7	I	$\bar{T}^\mu \langle f_{+\mu}^{\nu\lambda} \rangle f_{+\nu}^{\lambda\rho} D_{\nu\lambda} T_\rho + \text{H.c.}$	215			442
$\bar{B}\langle u^\mu u_\mu \rangle u^\nu u^\lambda D_{\nu\lambda} B$			8	I	$\bar{T}^\mu \langle f_+^{\nu\lambda} \rangle f_{+\nu}^{\rho\lambda} D_{\lambda\rho} T_\mu$	216			443
$\bar{B}\langle u^\mu u^\nu \rangle u_\mu u^\lambda D_{\nu\lambda} B + \text{H.c.}$			9	I	$i\bar{T}^\mu \langle f_{+\mu}^{\nu\lambda} \rangle f_{+\nu}^{\lambda\rho} \sigma_{\nu\lambda} T_\rho + \text{H.c.}$	217			444
$\bar{B}\langle u^\mu u^\nu \rangle u^\lambda u^\rho D_{\mu\nu\lambda\rho} B$			10	I	$\bar{T}^\mu \langle f_{+\mu}^{\nu\lambda} \rangle f_{+\nu}^{\lambda\rho} T_\lambda$	218			445
$\bar{B}\langle u^\mu u^\nu \rangle u^\lambda u^\rho D_{\mu\nu\lambda\rho} B$			11	I	$\bar{T}^\mu \langle f_+^{\nu\lambda} \rangle f_{+\nu\lambda} T_\mu$	219			446
$i\bar{B}\langle u^\mu u_\mu \rangle u^\nu u^\lambda \sigma_{\nu\lambda} B$	6	I	12	I	$\bar{T}^\mu \langle f_{+\mu}^{\nu\lambda} \rangle f_{+\nu}^{\lambda\rho} D_{\nu\lambda} T_\rho$	220			447
$i\bar{B}\langle u^\mu u^\nu \rangle u_\mu u^\lambda \sigma_{\nu\lambda} B + \text{H.c.}$	7	I	13	I	$\bar{T}^\mu \langle f_+^{\nu\lambda} \rangle f_{+\nu}^{\rho\lambda} D_{\lambda\rho} T_\mu$	221			448
$i\bar{B}\langle u^\mu u^\nu \rangle u^\lambda u^\rho \sigma_{\mu\lambda} D_{\nu\rho} B + \text{H.c.}$	8	I	14	I	$i\bar{T}^\mu \langle f_{+\mu}^{\nu\lambda} \rangle f_{+\nu}^{\lambda\rho} \sigma_{\nu\lambda} T_\rho$	222			449
$i\bar{B}\langle u^\mu u^\nu \rangle u^\lambda u^\rho \sigma_{\lambda\rho} D_{\mu\nu} B$	9	I	15	I	$\bar{T}^\mu f_{+\mu}^{\nu\lambda} f_{+\nu}^{\lambda\rho} T_\lambda$	223			450
$\bar{B}\langle u^\mu u_\mu u^\nu \rangle u_\nu B$			16	I	$\bar{T}^\mu f_{+\mu}^{\nu\lambda} f_{+\nu\mu} T_\lambda$				451
$\bar{B}\langle u^\mu u_\mu u^\nu \rangle u^\lambda D_{\nu\lambda} B$			17	I	$\bar{T}^\mu f_{+\mu}^{\nu\lambda} f_{+\nu\lambda} T_\mu$				452
$\bar{B}\langle u^\mu u^\nu u^\lambda \rangle u_\mu D_{\nu\lambda} B$			18	I	$\bar{T}^\mu f_{+\mu}^{\nu\lambda} f_{+\nu}^{\lambda\rho} D_{\nu\lambda} T_\rho$				453
$\bar{B}\langle u^\mu u^\nu u^\lambda \rangle u^\rho D_{\mu\nu\lambda\rho} B$			19	I	$\bar{T}^\mu f_{+\mu}^{\nu\lambda} f_{+\nu}^{\rho\lambda} D_{\lambda\rho} T_\mu$				454
$i\bar{B}\langle u^\mu u^\nu u^\lambda \rangle u_\mu \sigma_{\nu\lambda} B$			20	I	$i\bar{T}^\mu f_{+\mu}^{\nu\lambda} f_{+\nu}^{\lambda\rho} \sigma_{\nu\lambda} T_\rho$				455
$i\bar{B}\langle u^\mu u^\nu u^\lambda \rangle u^\rho \sigma_{\mu\nu} D_{\lambda\rho} B$			21	I	$\bar{T}^\mu \langle u_\mu u^\nu \rangle \chi_+ T_\nu$	224			456
$\bar{B}\langle u^\mu u_\mu u^\nu u_\nu \rangle B$			22	I	$\bar{T}^\mu \langle u^\nu u_\nu \rangle \chi_+ T_\mu$	225			457
$\bar{B}\langle u^\mu u_\mu u^\nu u^\lambda \rangle D_{\nu\lambda} B$			23	I	$\bar{T}^\mu \langle u^\nu u^\lambda \rangle \chi_+ D_{\nu\lambda} T_\mu$	226			458
$i\bar{B}\langle u^\mu u_\mu u^\nu u^\lambda \rangle \sigma_{\nu\lambda} B$			24	I	$\bar{T}^\mu \langle u_\mu u^\nu \rangle \chi_+ T_\nu$	227			459
$i\bar{B}\langle u^\mu u^\nu u^\lambda u^\rho \rangle \sigma_{\mu\nu} D_{\lambda\rho} B$			25	I	$\bar{T}^\mu \langle u^\nu u_\mu \rangle \chi_+ T_\nu$	228			460
$\bar{B}u^\mu u_\mu u^\nu u_\nu B$			26	I	$\bar{T}^\mu \langle u^\nu u_\nu \rangle \chi_+ T_\mu$	229			461

(Table continued)

TABLE VIII. (Continued)

O_n/o_n	$SU(2)$	$\tilde{e}_n^{(4)}$	$SU(3)$	$\tilde{E}_n^{(4)}$	O_n	$SU(2)$	$\tilde{e}_n^{(4)}$	$SU(3)$	$\tilde{E}_n^{(4)}$
$\bar{B}u^\mu u_\mu u^\nu u^\lambda D_{\nu\lambda}B + \text{H.c.}$			27	I	$\bar{T}^\mu \langle u^\nu u^\lambda \chi_+ \rangle D_{\nu\lambda} T_\mu$	230		462	
$i\bar{B}u^\mu u_\mu u^\nu u^\lambda \sigma_{\nu\lambda}B + \text{H.c.}$			28	I	$\bar{T}^\mu \langle u_\mu \chi_+ \rangle u^\nu T_\nu + \text{H.c.}$	231		463	
$i\bar{B}u^\mu u^\nu u_\mu u^\lambda \sigma_{\nu\lambda}B + \text{H.c.}$			29	I	$\bar{T}^\mu \langle u^\nu \chi_+ \rangle u_\nu T_\mu$	232		464	
$i\bar{B}u^\mu u^\nu u^\lambda u^\rho \sigma_{\mu\nu} D_{\lambda\rho}B + \text{H.c.}$			30	I	$\bar{T}^\mu \langle u^\nu \chi_+ \rangle u^\lambda D_{\nu\lambda} T_\mu$	233		465	
$i\bar{B}u^\mu u^\nu u^\lambda u^\rho \sigma_{\mu\lambda} D_{\nu\rho}B + \text{H.c.}$			31	I	$\bar{T}^\mu \langle \chi_+ \rangle \langle u_\mu u^\nu \rangle T_\nu$			466	
$\bar{B} \langle u^\mu u^\nu f_{-\mu}{}^\lambda \rangle \gamma_5 \gamma_\nu D_\lambda B + \text{H.c.}$	10	I	32	I	$\bar{T}^\mu \langle \chi_+ \rangle \langle u^\nu u_\nu \rangle T_\mu$			467	
$\bar{B} \langle u^\mu u^\nu f_{-\mu}{}^\lambda \rangle \gamma_5 \gamma_\lambda D_\nu B + \text{H.c.}$	11	I	33	I	$\bar{T}^\mu \langle \chi_+ \rangle \langle u^\nu u^\lambda \rangle D_{\nu\lambda} T_\mu$			468	
$\bar{B} \langle u^\mu u^\nu h_\mu{}^\lambda \rangle \gamma_5 \gamma_\nu D_\lambda B + \text{H.c.}$	12	I	34	I	$\bar{T}^\mu \langle \chi_+ \rangle u_\mu u^\nu T_\nu$	234		469	
$\bar{B} \langle u^\mu u^\nu h_\mu{}^\lambda \rangle \gamma_5 \gamma_\lambda D_\nu B + \text{H.c.}$	13	I	35	I	$\bar{T}^\mu \langle \chi_+ \rangle u^\nu u_\mu T_\nu$			470	
$\bar{B} \langle u^\mu u^\nu h^{\lambda\rho} \rangle \gamma_5 \gamma_\mu D_{\nu\lambda\rho}B + \text{H.c.}$	14	I	36	I	$\bar{T}^\mu \langle \chi_+ \rangle u^\nu u_\nu T_\mu$			471	
$\bar{B}u^\mu u_\mu f_{-\nu}{}^\lambda \gamma_5 \gamma_\nu D_\lambda B + \text{H.c.}$			37	I	$\bar{T}^\mu \langle \chi_+ \rangle u^\nu u^\lambda D_{\nu\lambda} T_\mu$			472	
$\bar{B}u^\mu u^\nu f_{-\mu}{}^\lambda \gamma_5 \gamma_\nu D_\lambda B + \text{H.c.}$			38	I	$\bar{T}^\mu u_\mu u^\nu \chi_+ T_\nu + \text{H.c.}$			473	
$\bar{B}u^\mu u^\nu f_{-\mu}{}^\lambda \gamma_5 \gamma_\lambda D_\nu B + \text{H.c.}$			39	I	$\bar{T}^\mu u^\nu u_\mu \chi_+ T_\nu + \text{H.c.}$			474	
$\bar{B}u^\mu u^\nu f_{-\nu}{}^\lambda \gamma_5 \gamma_\mu D_\lambda B + \text{H.c.}$			40	I	$\bar{T}^\mu u^\nu u_\nu \chi_+ T_\mu + \text{H.c.}$			475	
$\bar{B}u^\mu u^\nu f_{-\nu}{}^\lambda \gamma_5 \gamma_\lambda D_\mu B + \text{H.c.}$			41	I	$\bar{T}^\mu u^\nu u^\lambda \chi_+ D_{\nu\lambda} T_\mu + \text{H.c.}$			476	
$\bar{B}u^\mu u^\nu f_{-\lambda}{}^\rho \gamma_5 \gamma_\rho D_{\mu\nu}B + \text{H.c.}$			42	I	$\bar{T}^\mu u_\mu \chi_+ u^\nu T_\nu$			477	
$\bar{B}u^\mu u_\mu h^{\nu\lambda} \gamma_5 \gamma_\nu D_\lambda B + \text{H.c.}$			43	I	$\bar{T}^\mu u^\nu \nabla^\lambda \chi_+ \gamma_5 \gamma_\nu D_\lambda T_\mu + \text{H.c.}$	235		478	
$\bar{B}u^\mu u^\nu h_\mu{}^\lambda \gamma_5 \gamma_\nu D_\lambda B + \text{H.c.}$			44	I	$\bar{T}^\mu u^\nu \nabla^\lambda \chi_+ \gamma_5 \gamma_\lambda D_\nu T_\mu + \text{H.c.}$	236		479	
$\bar{B}u^\mu u^\nu h_\mu{}^\lambda \gamma_5 \gamma_\lambda D_\nu B + \text{H.c.}$			45	I	$\bar{T}^\mu f_{-\nu}{}^\lambda \chi_+ \gamma_5 \gamma_\nu D_\lambda T_\mu + \text{H.c.}$	237		480	
$\bar{B}u^\mu u^\nu h_\nu{}^\lambda \gamma_5 \gamma_\mu D_\lambda B + \text{H.c.}$			46	I	$\bar{T}^\mu \langle \nabla^\nu \nabla_\nu \chi_+ \rangle T_\mu$	238		481	
$\bar{B}u^\mu u^\nu h_\nu{}^\lambda \gamma_5 \gamma_\lambda D_\mu B + \text{H.c.}$			47	I	$\bar{T}^\mu \nabla^\nu \nabla_\nu \chi_+ T_\mu$	239		482	
$\bar{B}u^\mu u^\nu h^{\lambda\rho} \gamma_5 \gamma_\mu D_{\nu\lambda\rho}B + \text{H.c.}$			48	I	$i\bar{T}^\mu \langle f_{+\mu}{}^\nu \rangle \langle \chi_+ \rangle T_\nu$	240		483	
$\bar{B}u^\mu u^\nu h^{\lambda\rho} \gamma_5 \gamma_\nu D_{\mu\lambda\rho}B + \text{H.c.}$			49	I	$i\bar{T}^\mu \langle f_{+\mu}{}^\nu \rangle \chi_+ T_\nu$	241		484	
$\bar{B}u^\mu u^\nu h^{\lambda\rho} \gamma_5 \gamma_\lambda D_{\mu\nu\rho}B + \text{H.c.}$			50	I	$i\bar{T}^\mu \langle f_{+\mu}{}^\nu \chi_+ \rangle T_\nu$	242		485	
$\bar{B}u^\mu f_{-\mu}{}^\nu u^\lambda \gamma_5 \gamma_\nu D_\lambda B + \text{H.c.}$			51	I	$i\bar{T}^\mu \langle \chi_+ \rangle f_{+\mu}{}^\nu T_\nu$	243		486	
$\bar{B}u^\mu f_{-\mu}{}^\nu u^\lambda \gamma_5 \gamma_\lambda D_\nu B + \text{H.c.}$			52	I	$i\bar{T}^\mu f_{+\mu}{}^\nu \chi_+ T_\nu + \text{H.c.}$			487	
$\bar{B}u^\mu h_\mu{}^\nu u^\lambda \gamma_5 \gamma_\nu D_\lambda B + \text{H.c.}$			53	I	$\bar{T}^\mu \langle \chi_+ \rangle \langle \chi_+ \rangle T_\mu$	244		488	
$\epsilon^{\mu\nu\lambda\rho} \bar{B} \langle u_\mu u^\sigma \rangle f_{-\nu\lambda} D_{\rho\sigma}B$	15	I	54	I	$\bar{T}^\mu \langle \chi_+ \rangle \chi_+ T_\mu$	245		489	
$\epsilon^{\mu\nu\lambda\rho} \bar{B} \langle u_\mu u^\sigma f_{-\nu\lambda} \rangle D_{\rho\sigma}B + \text{H.c.}$			55	I	$\bar{T}^\mu \langle \chi_+^2 \rangle T_\mu$			490	
$\epsilon^{\mu\nu\lambda\rho} \bar{B} \langle u_\mu f_{-\nu\lambda} \rangle u_\rho B$	16	I	56	I	$\bar{T}^\mu \chi_+^2 T_\mu$			491	
$\epsilon^{\mu\nu\lambda\rho} \bar{B} \langle u_\mu f_{-\nu\lambda} \rangle u^\sigma D_{\rho\sigma}B$	17	I	57	I	$i\bar{T}^\mu \langle u^\nu u^\lambda \chi_- \rangle \gamma_5 \gamma_\nu D_\lambda T_\mu + \text{H.c.}$	246		492	
$\epsilon^{\mu\nu\lambda\rho} \bar{B} \langle u_\mu f_{-\nu}{}^\sigma \rangle u_\lambda D_{\rho\sigma}B$	18	I	58	I	$i\bar{T}^\mu \langle \chi_- \rangle u^\nu u^\lambda \gamma_5 \gamma_\nu D_\lambda T_\mu + \text{H.c.}$	247		493	
$\epsilon^{\mu\nu\lambda\rho} \bar{B} \langle u_\mu h_\nu{}^\sigma \rangle u_\lambda D_{\rho\sigma}B$	19	I	59	I	$i\bar{T}^\mu u^\nu u^\lambda \chi_- \gamma_5 \gamma_\nu D_\lambda T_\mu + \text{H.c.}$			494	
$\epsilon^{\mu\nu\lambda\rho} \bar{B} u_\mu u_\nu f_{-\lambda\rho}B + \text{H.c.}$			60	I	$i\bar{T}^\mu u^\nu u^\lambda \chi_- \gamma_5 \gamma_\lambda D_\nu T_\mu + \text{H.c.}$			495	
$\epsilon^{\mu\nu\lambda\rho} \bar{B} u_\mu u_\nu f_{-\lambda}{}^\sigma D_{\rho\sigma}B + \text{H.c.}$			61	I	$i\bar{T}^\mu u^\nu \chi_- u^\lambda \gamma_5 \gamma_\nu D_\lambda T_\mu + \text{H.c.}$			496	
$\epsilon^{\mu\nu\lambda\rho} \bar{B} u_\mu u^\sigma f_{-\nu\lambda} D_{\rho\sigma}B + \text{H.c.}$			62	I	$i\bar{T}^\mu \langle u_\mu \nabla^\nu \chi_- \rangle T_\nu + \text{H.c.}$	248		497	
$\epsilon^{\mu\nu\lambda\rho} \bar{B} u_\mu u_\nu h_\lambda{}^\sigma D_{\rho\sigma}B + \text{H.c.}$			63	I	$i\bar{T}^\mu \langle u^\nu \nabla_\nu \chi_- \rangle T_\mu$	249		498	
$\bar{B} \langle u^\mu \nabla^\nu f_{-\mu\nu} \rangle B$	20	I	64	I	$i\bar{T}^\mu \langle u^\nu \nabla^\lambda \chi_- \rangle D_{\nu\lambda} T_\mu$	250		499	
$\bar{B} \langle u^\mu \nabla^\nu f_{-\mu}{}^\lambda \rangle D_{\nu\lambda}B$	21	I	65	I	$i\bar{T}^\mu \langle \chi_- \rangle h_\mu{}^\nu T_\nu$	251		500	
$\bar{B} \langle u^\mu \nabla^\nu f_{-\nu}{}^\lambda \rangle D_{\mu\lambda}B$	22	I	66	I	$i\bar{T}^\mu \langle \chi_- \rangle h^{\nu\lambda} D_{\nu\lambda} T_\mu$	252		501	
$\bar{B} \langle u^\mu \nabla_\mu h^{\nu\lambda} \rangle D_{\nu\lambda}B$	23	I	67	I	$i\bar{T}^\mu \langle \nabla^\nu \chi_- \rangle u_\nu T_\mu$	253		502	
$\bar{B} \langle u^\mu \nabla^\nu h^{\lambda\rho} \rangle D_{\mu\nu\lambda\rho}B$	24	I	68	I	$i\bar{T}^\mu u_\mu \nabla^\nu \chi_- T_\nu + \text{H.c.}$	254		503	
$\bar{B} \langle h^{\mu\nu} h_{\mu\nu} \rangle B$	25	I	69	I	$i\bar{T}^\mu u^\nu \nabla_\mu \chi_- T_\nu + \text{H.c.}$			504	
$\bar{B}u^\mu \nabla^\nu f_{-\mu\nu}B + \text{H.c.}$			70	I	$i\bar{T}^\mu u^\nu \nabla_\nu \chi_- T_\mu + \text{H.c.}$			505	
$\bar{B}u^\mu \nabla^\nu f_{-\mu}{}^\lambda D_{\nu\lambda}B + \text{H.c.}$			71	I	$i\bar{T}^\mu u^\nu \nabla^\lambda \chi_- D_{\nu\lambda} T_\mu + \text{H.c.}$			506	
$\bar{B}u^\mu \nabla^\nu f_{-\nu}{}^\lambda D_{\mu\lambda}B + \text{H.c.}$			72	I	$\bar{T}^\mu \langle f_{+\nu}{}^\lambda \rangle \langle \chi_- \rangle \gamma_5 \gamma_\nu D_\lambda T_\mu$	255		507	
$i\bar{B}u^\mu \nabla_\mu f_{-\nu}{}^\lambda \sigma_{\nu\lambda}B + \text{H.c.}$	26	I	73	I	$\bar{T}^\mu \langle f_{+\nu}{}^\lambda \rangle \chi_- \gamma_5 \gamma_\nu D_\lambda T_\mu$	256		508	
$i\bar{B}u^\mu \nabla^\nu f_{-\nu}{}^\lambda \sigma_{\mu\lambda}B + \text{H.c.}$	27	I	74	I	$\bar{T}^\mu \langle f_{+\nu}{}^\lambda \chi_- \rangle \gamma_5 \gamma_\nu D_\lambda T_\mu$	257		509	

(Table continued)

TABLE VIII. (Continued)

O_n/o_n	$SU(2)$	$\tilde{e}_n^{(4)}$	$SU(3)$	$\tilde{E}_n^{(4)}$	O_n	$SU(2)$	$\tilde{e}_n^{(4)}$	$SU(3)$	$\tilde{E}_n^{(4)}$
$i\bar{B}u^\mu\nabla^\nu f_{-\lambda\rho}\sigma_{\mu\lambda}D_{\nu\rho}B + \text{H.c.}$	28	I	75	I	$\bar{T}^\mu\langle\chi_-\rangle f_{+\nu\lambda}\gamma_5\gamma_\nu D_\lambda T_\mu$	258		510	
$i\bar{B}u^\mu\nabla^\nu f_{-\lambda\rho}\sigma_{\nu\lambda}D_{\mu\rho}B + \text{H.c.}$	29	I	76	I	$\bar{T}^\mu f_{+\nu\lambda}\gamma_5\gamma_\nu D_\lambda T_\mu + \text{H.c.}$			511	
$\bar{B}u^\mu\nabla_\mu h^{\nu\lambda}D_{\nu\lambda}B + \text{H.c.}$			77	I	$\bar{T}^\mu\langle\chi_-\rangle\langle\chi_-\rangle T_\mu$	259		512	
$\bar{B}u^\mu\nabla^\nu h^{\lambda\rho}D_{\mu\nu\lambda\rho}B + \text{H.c.}$			78	I	$\bar{T}^\mu\langle\chi_-\rangle\chi_- T_\mu$	260		513	
$i\bar{B}u^\mu\nabla^\nu h^{\lambda\rho}\sigma_{\mu\nu}D_{\lambda\rho}B + \text{H.c.}$	30	I	79	I	$\bar{T}^\mu\chi_-^2 T_\mu$			514	
$\bar{B}f_{-}^{\mu\nu}f_{-\mu\nu}B$			80	I	$\bar{T}^\mu\langle F_{L\mu}{}^\nu F_{L\nu}{}^\lambda\rangle T_\lambda + P$	261		515	
$\bar{B}f_{-}^{\mu\nu}f_{-\mu}{}^\lambda D_{\nu\lambda}B$			81	I	$\bar{T}^\mu\langle F_{L\mu}{}^{\nu\lambda} F_{L\nu\lambda}\rangle T_\mu + P$	262		516	
$i\bar{B}f_{-}^{\mu\nu}f_{-\mu}{}^\lambda\sigma_{\nu\lambda}B$	31	I	82	I	$\bar{T}^\mu\langle F_{L\mu}{}^\nu F_{L\nu}{}^{\lambda\rho}\rangle D_{\nu\lambda}T_\rho + P$	263		517	
$i\bar{B}f_{-}^{\mu\nu}f_{-\mu}{}^{\lambda\rho}\sigma_{\mu\lambda}D_{\nu\rho}B$	32	I	83	I	$\bar{T}^\mu\langle F_{L\mu}{}^{\nu\lambda} F_{L\nu}{}^{\rho\lambda}\rangle D_{\lambda\rho}T_\mu + P$	264		518	
$i\bar{B}f_{-}^{\mu\nu}h_{\mu}{}^\lambda\sigma_{\nu\lambda}B + \text{H.c.}$	33	I	84	I	$i\bar{T}^\mu\langle F_{L\mu}{}^\nu F_{L\nu}{}^{\lambda\rho}\rangle\sigma_{\nu\lambda}T_\rho + P$	265		519	
$\bar{B}h^{\mu\nu}h_{\mu\nu}B$			85	I	$\bar{T}^\mu\langle F_{L\mu}{}^\nu\rangle\langle F_{L\nu}{}^\lambda\rangle T_\lambda + P$	266			
$i\bar{B}h^{\mu\nu}h_{\mu}{}^\lambda\sigma_{\nu\lambda}B$	34	I	86	I	$\bar{T}^\mu\langle F_{L\mu}{}^{\nu\lambda}\rangle\langle F_{L\nu\lambda}\rangle T_\mu + P$	267			
$\bar{B}\langle f_{+}^{\mu\nu}\rangle\langle u_{\mu}u^{\lambda}\rangle\sigma_{\nu\lambda}B$	35	I	87		$\bar{T}^\mu\langle F_{L\mu}{}^\nu\rangle\langle F_{L\nu}{}^{\lambda\rho}\rangle D_{\nu\lambda}T_\rho + P$	268			
$\bar{B}\langle f_{+}^{\mu\nu}\rangle\langle u^{\lambda}u_{\lambda}\rangle\sigma_{\mu\nu}B$	36	I	88		$\bar{T}^\mu\langle F_{L\mu}{}^{\nu\lambda}\rangle\langle F_{L\nu}{}^{\rho\lambda}\rangle D_{\lambda\rho}T_\mu + P$	269			
$\bar{B}\langle f_{+}^{\mu\nu}\rangle\langle u^{\lambda}u^{\rho}\rangle\sigma_{\mu\nu}D_{\lambda\rho}B$	37	I	89		$i\bar{T}^\mu\langle F_{L\mu}{}^\nu\rangle\langle F_{L\nu}{}^{\lambda\rho}\rangle\sigma_{\nu\lambda}T_\rho + P$	270			
$\bar{B}\langle f_{+}^{\mu\nu}\rangle\langle u^{\lambda}u^{\rho}\rangle\sigma_{\mu\lambda}D_{\nu\rho}B$	38	I	90		$\bar{T}^\mu\langle\chi\chi^\dagger\rangle T_\mu$	271		520	
$i\bar{B}\langle f_{+}^{\mu\nu}\rangle u_{\mu}u_{\nu}B$	39	I	91		$\bar{T}^\mu\det\chi T_\mu + \text{H.c.}$	272			
$i\bar{B}\langle f_{+}^{\mu\nu}\rangle u_{\mu}u^{\lambda}D_{\nu\lambda}B + \text{H.c.}$	40	I	92		$\bar{B}\langle u^\mu u_{\mu}\rangle\langle u^\nu u^{\lambda}\rangle\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$	273		521	
$\bar{B}\langle f_{+}^{\mu\nu}\rangle u_{\mu}u^{\lambda}\sigma_{\nu\lambda}B + \text{H.c.}$			93		$\bar{B}\langle u^\mu u^{\nu}\rangle\langle u_{\mu}u^{\lambda}\rangle\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$	274		522	
$\bar{B}\langle f_{+}^{\mu\nu}\rangle u^{\lambda}u_{\lambda}\sigma_{\mu\nu}B$			94		$\bar{B}\langle u^\mu u^{\nu}\rangle\langle u^{\lambda}u^{\rho}\rangle\gamma_5\gamma_\mu D_{\nu\lambda}T_\rho + \text{H.c.}$	275		523	
$\bar{B}\langle f_{+}^{\mu\nu}\rangle u^{\lambda}u^{\rho}\sigma_{\mu\nu}D_{\lambda\rho}B$			95		$\bar{B}\langle u^\mu u^{\nu}\rangle\langle u^{\lambda}u^{\rho}\rangle\gamma_5\gamma_\mu D_{\lambda\rho}T_\nu + \text{H.c.}$	276		524	
$\bar{B}\langle f_{+}^{\mu\nu}\rangle u^{\lambda}u^{\rho}\sigma_{\mu\lambda}D_{\nu\rho}B + \text{H.c.}$			96		$\bar{B}\langle u^\mu u_{\mu}\rangle u^{\nu}u^{\lambda}\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$	277		525	
$\bar{B}\langle f_{+}^{\mu\nu}u_{\mu}\rangle u^{\lambda}\sigma_{\nu\lambda}B$	41	I	97	I	$\bar{B}\langle u^\mu u_{\mu}\rangle u^{\nu}u^{\lambda}\gamma_5\gamma_\lambda T_\nu + \text{H.c.}$			526	
$\bar{B}\langle f_{+}^{\mu\nu}u_{\mu}\rangle u_{\mu}\sigma_{\nu\lambda}B$	42	I	98	I	$\bar{B}\langle u^\mu u^{\nu}\rangle u_{\mu}u^{\lambda}\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$	278		527	
$\bar{B}\langle f_{+}^{\mu\nu}u_{\mu}\rangle u_{\lambda}\sigma_{\mu\nu}B$	43	I	99	I	$\bar{B}\langle u^\mu u^{\nu}\rangle u_{\mu}u^{\lambda}\gamma_5\gamma_\lambda T_\nu + \text{H.c.}$	279		528	
$\bar{B}\langle f_{+}^{\mu\nu}u_{\mu}\rangle u^{\rho}\sigma_{\mu\nu}D_{\lambda\rho}B$	44	I	100	I	$\bar{B}\langle u^\mu u^{\nu}\rangle u^{\lambda}u_{\mu}\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$			529	
$\bar{B}\langle f_{+}^{\mu\nu}u_{\mu}\rangle u^{\rho}\sigma_{\mu\lambda}D_{\nu\rho}B$	45	I	101	I	$\bar{B}\langle u^\mu u^{\nu}\rangle u^{\lambda}u_{\mu}\gamma_5\gamma_\lambda T_\nu + \text{H.c.}$			530	
$\bar{B}\langle f_{+}^{\mu\nu}u_{\mu}\rangle u^{\rho}\sigma_{\mu\rho}D_{\nu\lambda}B$	46	I	102	I	$\bar{B}\langle u^\mu u^{\nu}\rangle u^{\lambda}u_{\lambda}\gamma_5\gamma_\mu T_\nu + \text{H.c.}$			531	
$i\bar{B}\langle f_{+}^{\mu\nu}u_{\mu}u_{\nu}\rangle B$	47	I	103	I	$\bar{B}\langle u^\mu u^{\nu}\rangle u^{\lambda}u^{\rho}\gamma_5\gamma_\mu D_{\nu\lambda}T_\rho + \text{H.c.}$	280		532	
$i\bar{B}\langle f_{+}^{\mu\nu}u_{\mu}u^{\lambda}\rangle D_{\nu\lambda}B + \text{H.c.}$	48	I	104	I	$\bar{B}\langle u^\mu u^{\nu}\rangle u^{\lambda}u^{\rho}\gamma_5\gamma_\mu D_{\nu\rho}T_\lambda + \text{H.c.}$			533	
$\bar{B}\langle f_{+}^{\mu\nu}u_{\mu}u^{\lambda}\rangle\sigma_{\nu\lambda}B + \text{H.c.}$			105	I	$\bar{B}\langle u^\mu u^{\nu}\rangle u^{\lambda}u^{\rho}\gamma_5\gamma_\mu D_{\lambda\rho}T_\nu + \text{H.c.}$			534	
$\bar{B}\langle f_{+}^{\mu\nu}u_{\mu}\rangle\sigma_{\mu\nu}B$			106	I	$\bar{B}\langle u^\mu u^{\nu}\rangle u^{\lambda}u^{\rho}\gamma_5\gamma_\lambda D_{\mu\nu}T_\rho + \text{H.c.}$	281		535	
$\bar{B}\langle f_{+}^{\mu\nu}u_{\mu}\rangle\sigma_{\mu\nu}D_{\lambda\rho}B$			107	I	$\bar{B}\langle u^\mu u^{\nu}\rangle u^{\lambda}u^{\rho}\gamma_5\gamma_\lambda D_{\mu\rho}T_\nu + \text{H.c.}$	282		536	
$\bar{B}\langle f_{+}^{\mu\nu}u_{\mu}\rangle\sigma_{\mu\lambda}D_{\nu\rho}B + \text{H.c.}$			108	I	$\bar{B}\langle u^\mu u^{\nu}\rangle u^{\lambda}u^{\rho}\gamma_5\gamma_\rho D_{\mu\nu}T_\lambda + \text{H.c.}$			537	
$\bar{B}\langle u^\mu u_{\mu}\rangle f_{+}^{\nu\lambda}\sigma_{\nu\lambda}B$	49	I	109	I	$\bar{B}\langle u^\mu u^{\nu}\rangle u^{\lambda}u^{\rho}\gamma_5\gamma_\rho D_{\mu\lambda}T_\nu + \text{H.c.}$			538	
$\bar{B}\langle u^\mu u^{\nu}\rangle f_{+}^{\mu\lambda}\sigma_{\nu\lambda}B$	50	I	110	I	$\bar{B}\langle u^\mu u_{\mu}u^{\nu}\rangle u^{\lambda}\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$			539	
$\bar{B}\langle u^\mu u^{\nu}\rangle f_{+}^{\lambda\rho}\sigma_{\mu\lambda}D_{\nu\rho}B$	51	I	111	I	$\bar{B}\langle u^\mu u_{\mu}u^{\nu}\rangle u^{\lambda}\gamma_5\gamma_\lambda T_\nu + \text{H.c.}$			540	
$\bar{B}\langle u^\mu u^{\nu}\rangle f_{+}^{\lambda\rho}\sigma_{\lambda\rho}D_{\mu\nu}B$	52	I	112	I	$\bar{B}\langle u^\mu u^{\nu}u^{\lambda}\rangle u_{\mu}\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$			541	
$i\bar{B}f_{+}^{\mu\nu}u_{\mu}u_{\nu}B + \text{H.c.}$			113	I	$\bar{B}\langle u^\mu u^{\nu}u^{\lambda}\rangle u_{\mu}\gamma_5\gamma_\lambda T_\nu + \text{H.c.}$			542	
$i\bar{B}f_{+}^{\mu\nu}u_{\mu}u^{\lambda}D_{\nu\lambda}B + \text{H.c.}$			114	I	$\bar{B}\langle u^\mu u^{\nu}u^{\lambda}\rangle u^{\rho}\gamma_5\gamma_\mu D_{\nu\lambda}T_\rho + \text{H.c.}$			543	
$i\bar{B}f_{+}^{\mu\nu}u_{\mu}u^{\lambda}D_{\nu\lambda}B + \text{H.c.}$			115	I	$\bar{B}\langle u^\mu u^{\nu}u^{\lambda}\rangle u^{\rho}\gamma_5\gamma_\mu D_{\nu\rho}T_\lambda + \text{H.c.}$			544	
$\bar{B}f_{+}^{\mu\nu}u_{\mu}u^{\lambda}\sigma_{\nu\lambda}B + \text{H.c.}$			116	I	$\bar{B}\langle u^\mu u^{\nu}u^{\lambda}\rangle u^{\rho}\gamma_5\gamma_\mu D_{\lambda\rho}T_\nu + \text{H.c.}$			545	
$\bar{B}f_{+}^{\mu\nu}u^{\lambda}u_{\mu}\sigma_{\nu\lambda}B + \text{H.c.}$			117	I	$\bar{B}\langle u^\mu u^{\nu}u^{\lambda}\rangle u^{\rho}\gamma_5\gamma_\rho D_{\mu\nu}T_\lambda + \text{H.c.}$			546	
$\bar{B}f_{+}^{\mu\nu}u^{\lambda}u_{\lambda}\sigma_{\mu\nu}B + \text{H.c.}$			118	I	$\bar{B}\langle u^\mu u_{\mu}u^{\nu}u^{\lambda}\rangle\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$			547	
$\bar{B}f_{+}^{\mu\nu}u^{\lambda}u^{\rho}\sigma_{\mu\nu}D_{\lambda\rho}B + \text{H.c.}$			119	I	$\bar{B}\langle u^\mu u_{\mu}u^{\nu}u^{\lambda}\rangle\gamma_5\gamma_\lambda T_\nu + \text{H.c.}$			548	
$\bar{B}f_{+}^{\mu\nu}u^{\lambda}u^{\rho}\sigma_{\mu\lambda}D_{\nu\rho}B + \text{H.c.}$			120	I	$\bar{B}\langle u^\mu u^{\nu}u^{\lambda}u^{\rho}\rangle\gamma_5\gamma_\mu D_{\nu\lambda}T_\rho + \text{H.c.}$			549	
$\bar{B}f_{+}^{\mu\nu}u^{\lambda}u^{\rho}\sigma_{\mu\rho}D_{\nu\lambda}B + \text{H.c.}$			121	I	$\bar{B}\langle u^\mu u^{\nu}u^{\lambda}u^{\rho}\rangle\gamma_5\gamma_\mu D_{\nu\rho}T_\lambda + \text{H.c.}$			550	
$i\bar{B}u^\mu f_{+}^{\nu}u_{\nu}B$			122	I	$\bar{B}u^\mu u_{\mu}u^{\nu}u^{\lambda}\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$			551	

(Table continued)

TABLE VIII. (Continued)

O_n/o_n	$SU(2)$	$\tilde{e}_n^{(4)}$	$SU(3)$	$\tilde{E}_n^{(4)}$	O_n	$SU(2)$	$\tilde{e}_n^{(4)}$	$SU(3)$	$\tilde{E}_n^{(4)}$
$i\bar{B}u^\mu f_{+\mu} u^\lambda D_{\nu\lambda} B + \text{H.c.}$			123	I	$\bar{B}u^\mu u_\mu u^\nu u^\lambda \gamma_5 \gamma_\lambda T_\nu + \text{H.c.}$			552	
$i\bar{B}\langle f_{+\mu} \rangle f_{-\mu}^\lambda \gamma_5 \gamma_\nu D_\lambda B$	53	I	124		$\bar{B}u^\mu u^\nu u_\mu u^\lambda \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$			553	
$i\bar{B}\langle f_{+\mu} \rangle f_{-\mu}^\lambda \gamma_5 \gamma_\lambda D_\nu B$	54	I	125		$\bar{B}u^\mu u^\nu u_\mu u^\lambda \gamma_5 \gamma_\lambda T_\nu + \text{H.c.}$			554	
$i\bar{B}\langle f_{+\mu} \rangle h_\mu^\lambda \gamma_5 \gamma_\nu D_\lambda B$	55	I	126		$\bar{B}u^\mu u^\nu u^\lambda u_\mu \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$			555	
$i\bar{B}\langle f_{+\mu} \rangle h_\mu^\lambda \gamma_5 \gamma_\lambda D_\nu B$	56	I	127		$\bar{B}u^\mu u^\nu u^\lambda u^\rho \gamma_5 \gamma_\mu D_{\nu\lambda} T_\rho + \text{H.c.}$			556	
$i\bar{B}\langle f_{+\mu} \rangle h^{\lambda\rho} \gamma_5 \gamma_\mu D_{\nu\lambda\rho} B$	57	I	128		$\bar{B}u^\mu u^\nu u^\lambda u^\rho \gamma_5 \gamma_\mu D_{\nu\rho} T_\lambda + \text{H.c.}$			557	
$i\bar{B}\langle f_{+\mu} \rangle f_{-\mu}^\lambda \gamma_5 \gamma_\nu D_\lambda B$	58	I	129	I	$\bar{B}u^\mu u^\nu u^\lambda u^\rho \gamma_5 \gamma_\nu D_{\mu\lambda} T_\rho + \text{H.c.}$			558	
$i\bar{B}\langle f_{+\mu} \rangle f_{-\mu}^\lambda \gamma_5 \gamma_\lambda D_\nu B$	59	I	130	I	$\bar{B}u^\mu u^\nu u^\lambda u^\rho \gamma_5 \gamma_\nu D_{\lambda\rho} T_\mu + \text{H.c.}$			559	
$i\bar{B}\langle f_{+\mu} \rangle h_\mu^\lambda \gamma_5 \gamma_\nu D_\lambda B$	60	I	131	I	$\bar{B}u^\mu u^\nu u^\lambda u^\rho \gamma_5 \gamma_\lambda D_{\mu\rho} T_\nu + \text{H.c.}$			560	
$i\bar{B}\langle f_{+\mu} \rangle h_\mu^\lambda \gamma_5 \gamma_\lambda D_\nu B$	61	I	132	I	$\bar{B}\langle u^\mu u_\mu \rangle f_{-\mu}^\lambda D_\nu T_\lambda + \text{H.c.}$	283		561	
$i\bar{B}\langle f_{+\mu} \rangle h^{\lambda\rho} \gamma_5 \gamma_\mu D_{\nu\lambda\rho} B$	62	I	133	I	$\bar{B}\langle u^\mu u^\nu \rangle f_{-\mu}^\lambda D_\nu T_\lambda + \text{H.c.}$	284		562	
$i\bar{B}\langle \nabla^\mu f_{+\mu} \rangle u^\lambda \gamma_5 \gamma_\nu D_\lambda B$	63	I	134		$\bar{B}\langle u^\mu u^\nu \rangle f_{-\mu}^\lambda D_\lambda T_\nu + \text{H.c.}$	285		563	
$i\bar{B}\langle \nabla^\mu f_{+\mu} \rangle u^\lambda \gamma_5 \gamma_\lambda D_\nu B$	64	I	135		$\bar{B}\langle u^\mu u^\nu \rangle f_{-\mu}^\lambda D_{\mu\nu} T_\rho + \text{H.c.}$	286		564	
$i\bar{B}\langle \nabla^\mu f_{+\mu} \rangle u_\mu \gamma_5 \gamma_\nu D_\lambda B$	65	I	136		$i\bar{B}\langle u^\mu u^\nu \rangle f_{-\mu}^\lambda \sigma_{\mu\lambda} D_\nu T_\rho + \text{H.c.}$	287		565	
$i\bar{B}\langle \nabla^\mu f_{+\mu} \rangle u^\lambda \gamma_5 \gamma_\nu D_\lambda B$	66	I	137	I	$i\bar{B}\langle u^\mu u^\nu \rangle f_{-\mu}^\lambda \sigma_{\mu\lambda} D_\rho T_\nu + \text{H.c.}$	288		566	
$i\bar{B}\langle \nabla^\mu f_{+\mu} \rangle u^\lambda \gamma_5 \gamma_\lambda D_\nu B$	67	I	138	I	$\bar{B}\langle u^\mu u_\mu \rangle h^{\lambda\rho} D_\nu T_\lambda + \text{H.c.}$	289		567	
$i\bar{B}\langle \nabla^\mu f_{+\mu} \rangle u^\lambda u_\mu \gamma_5 \gamma_\nu D_\lambda B$	68	I	139	I	$\bar{B}\langle u^\mu u^\nu \rangle h_\mu^\lambda D_\nu T_\lambda + \text{H.c.}$	290		568	
$i\bar{B}f_{+\mu} f_{-\mu}^\lambda \gamma_5 \gamma_\nu D_\lambda B + \text{H.c.}$			140	I	$\bar{B}\langle u^\mu u^\nu \rangle h_\mu^\lambda D_\lambda T_\nu + \text{H.c.}$	291		569	
$i\bar{B}f_{+\mu} f_{-\mu}^\lambda \gamma_5 \gamma_\lambda D_\nu B + \text{H.c.}$			141	I	$\bar{B}\langle u^\mu u^\nu \rangle h^{\lambda\rho} D_{\mu\nu\lambda} T_\rho + \text{H.c.}$	292		570	
$i\bar{B}f_{+\mu} h_\mu^\lambda \gamma_5 \gamma_\nu D_\lambda B + \text{H.c.}$			142	I	$\bar{B}\langle u^\mu u^\nu \rangle h^{\lambda\rho} D_{\mu\lambda\rho} T_\nu + \text{H.c.}$	293		571	
$i\bar{B}f_{+\mu} h_\mu^\lambda \gamma_5 \gamma_\lambda D_\nu B + \text{H.c.}$			143	I	$i\bar{B}\langle u^\mu u^\nu \rangle h^{\lambda\rho} \sigma_{\mu\lambda} D_\nu T_\rho + \text{H.c.}$	294		572	
$i\bar{B}f_{+\mu} h^{\lambda\rho} \gamma_5 \gamma_\mu D_{\nu\lambda\rho} B + \text{H.c.}$			144	I	$i\bar{B}\langle u^\mu u^\nu \rangle h^{\lambda\rho} \sigma_{\mu\lambda} D_\rho T_\nu + \text{H.c.}$	295		573	
$i\bar{B}\nabla^\mu f_{+\mu} u^\lambda \gamma_5 \gamma_\nu D_\lambda B + \text{H.c.}$			145	I	$\bar{B}\langle u^\mu u_\mu f_{-\mu}^\lambda \rangle D_\nu T_\lambda + \text{H.c.}$			574	
$i\bar{B}\nabla^\mu f_{+\mu} u^\lambda \gamma_5 \gamma_\lambda D_\nu B + \text{H.c.}$			146	I	$\bar{B}\langle u^\mu u^\nu f_{-\mu}^\lambda \rangle D_\nu T_\lambda + \text{H.c.}$	296		575	
$i\bar{B}\nabla^\mu f_{+\mu} u^\lambda u_\mu \gamma_5 \gamma_\nu D_\lambda B + \text{H.c.}$			147	I	$\bar{B}\langle u^\mu u^\nu f_{-\mu}^\lambda \rangle D_\lambda T_\nu + \text{H.c.}$	297		576	
$i\epsilon^{\mu\nu\lambda\rho} \bar{B}f_{+\mu\nu} f_{-\lambda\rho} B + \text{H.c.}$	69	I	148	I	$\bar{B}\langle u^\mu u^\nu f_{-\nu}^\lambda \rangle D_\mu T_\lambda + \text{H.c.}$			577	
$i\epsilon^{\mu\nu\lambda\rho} \bar{B}f_{+\mu\nu} f_{-\lambda}^\sigma D_{\rho\sigma} B + \text{H.c.}$	70	I	149	I	$\bar{B}\langle u^\mu u^\nu f_{-\nu}^\lambda \rangle D_\lambda T_\mu + \text{H.c.}$			578	
$i\epsilon^{\mu\nu\lambda\rho} \bar{B}f_{+\mu\nu} h_\lambda^\sigma D_{\rho\sigma} B + \text{H.c.}$	71	I	150	I	$\bar{B}\langle u^\mu u^\nu f_{-\nu}^\lambda \rangle D_{\mu\lambda} T_\rho + \text{H.c.}$			579	
$\bar{B}\langle \nabla^\mu \nabla_\mu f_{+\mu} \rangle u^\lambda \gamma_5 \gamma_\nu D_\lambda B$			151		$i\bar{B}\langle u^\mu u^\nu f_{-\nu}^\lambda \rangle \sigma_{\mu\nu} D_\lambda T_\rho + \text{H.c.}$	298		580	
$\bar{B}\nabla^\mu \nabla_\mu f_{+\mu} u^\lambda \gamma_5 \gamma_\lambda D_\nu B$	72	I	152	I	$i\bar{B}\langle u^\mu u^\nu f_{-\nu}^\lambda \rangle \sigma_{\mu\lambda} D_\nu T_\rho + \text{H.c.}$	299		581	
$\bar{B}\langle f_{+\mu} \rangle \langle f_{+\mu\nu} \rangle B$			153		$i\bar{B}\langle u^\mu u^\nu f_{-\nu}^\lambda \rangle \sigma_{\mu\lambda} D_\rho T_\nu + \text{H.c.}$			582	
$\bar{B}\langle f_{+\mu} \rangle \langle f_{+\mu}^\lambda \rangle D_{\nu\lambda} B$			154		$i\bar{B}\langle u^\mu u^\nu f_{-\nu}^\lambda \rangle \sigma_{\nu\lambda} D_\mu T_\rho + \text{H.c.}$			583	
$\bar{B}\langle f_{+\mu} \rangle f_{+\mu\nu} B$	73	I	155		$\bar{B}\langle u^\mu u_\mu h^{\nu\lambda} \rangle D_\nu T_\lambda + \text{H.c.}$			584	
$\bar{B}\langle f_{+\mu} \rangle f_{+\mu}^\lambda D_{\nu\lambda} B$	74	I	156		$\bar{B}\langle u^\mu u^\nu h_\mu^\lambda \rangle D_\nu T_\lambda + \text{H.c.}$	300		585	
$\bar{B}\langle f_{+\mu} \rangle f_{+\mu\nu} B$	75	I	157	I	$\bar{B}\langle u^\mu u^\nu h_\mu^\lambda \rangle D_\lambda T_\nu + \text{H.c.}$	301		586	
$\bar{B}\langle f_{+\mu} \rangle f_{+\mu}^\lambda D_{\nu\lambda} B$	76	I	158	I	$\bar{B}\langle u^\mu u^\nu h^{\lambda\rho} \rangle D_{\mu\lambda\rho} T_\nu + \text{H.c.}$	302		587	
$\bar{B}f_{+\mu} f_{+\mu\nu} B$			159	I	$i\bar{B}\langle u^\mu u^\nu h^{\lambda\rho} \rangle \sigma_{\mu\nu} D_\lambda T_\rho + \text{H.c.}$	303		588	
$\bar{B}f_{+\mu} f_{+\mu}^\lambda D_{\nu\lambda} B$			160	I	$i\bar{B}\langle u^\mu u^\nu h^{\lambda\rho} \rangle \sigma_{\mu\lambda} D_\nu T_\rho + \text{H.c.}$			589	
$i\bar{B}f_{+\mu} f_{+\mu}^\lambda \sigma_{\nu\lambda} B$	77	I	161	I	$i\bar{B}\langle u^\mu u^\nu h^{\lambda\rho} \rangle \sigma_{\mu\lambda} D_\rho T_\nu + \text{H.c.}$			590	
$i\bar{B}f_{+\mu} f_{+\mu}^\lambda \sigma_{\mu\lambda} D_{\nu\rho} B$	78	I	162	I	$\bar{B}\langle u^\mu f_{-\mu}^\nu \rangle u^\lambda D_\nu T_\lambda + \text{H.c.}$	304		591	
$\bar{B}\langle u^\mu u_\mu \rangle \chi_+ B$	79	I	163	I	$\bar{B}\langle u^\mu f_{-\mu}^\nu \rangle u^\lambda D_\lambda T_\nu + \text{H.c.}$	305		592	
$\bar{B}\langle u^\mu u^\nu \rangle \chi_+ D_{\mu\nu} B$	80	I	164	I	$\bar{B}\langle u^\mu f_{-\mu}^\nu \rangle u_\mu D_\nu T_\lambda + \text{H.c.}$	306		593	
$\bar{B}\langle u^\mu u_\mu \chi_+ \rangle B$	81	I	165	I	$\bar{B}\langle u^\mu f_{-\mu}^\nu \rangle u_\nu D_\mu T_\lambda + \text{H.c.}$	307		594	
$\bar{B}\langle u^\mu u^\nu \chi_+ \rangle D_{\mu\nu} B$	82	I	166	I	$\bar{B}\langle u^\mu f_{-\mu}^\nu \rangle u_\nu D_\lambda T_\mu + \text{H.c.}$	308		595	
$i\bar{B}\langle u^\mu u^\nu \chi_+ \rangle \sigma_{\mu\nu} B$	83	I	167	I	$\bar{B}\langle u^\mu f_{-\mu}^\nu \rangle u^\rho D_{\mu\nu\rho} T_\lambda + \text{H.c.}$	309		596	
$\bar{B}\langle u^\mu \chi_+ \rangle u_\mu B$	84	I	168	I	$i\bar{B}\langle u^\mu f_{-\mu}^\nu \rangle u^\rho \sigma_{\mu\nu} D_\lambda T_\rho + \text{H.c.}$	310		597	
$\bar{B}\langle u^\mu \chi_+ \rangle u^\nu D_{\mu\nu} B$	85	I	169	I	$i\bar{B}\langle u^\mu f_{-\mu}^\nu \rangle u^\rho \sigma_{\mu\nu} D_\rho T_\lambda + \text{H.c.}$	311		598	
$\bar{B}\langle \chi_+ \rangle \langle u^\mu u_\mu \rangle B$			170	I	$i\bar{B}\langle u^\mu f_{-\mu}^\nu \rangle u^\rho \sigma_{\mu\rho} D_\nu T_\lambda + \text{H.c.}$	312		599	

(Table continued)

TABLE VIII. (Continued)

O_n/o_n	$SU(2)$	$\tilde{e}_n^{(4)}$	$SU(3)$	$\tilde{E}_n^{(4)}$	O_n	$SU(2)$	$\tilde{e}_n^{(4)}$	$SU(3)$	$\tilde{E}_n^{(4)}$
$\bar{B}\langle\chi_+\rangle\langle u^\mu u^\nu\rangle D_{\mu\nu}B$			171	I	$i\bar{B}\langle u^\mu f_{-\nu}^{\lambda}\rangle u^\rho\sigma_{\nu\lambda}D_\mu T_\rho + \text{H.c.}$	313		600	
$\bar{B}\langle\chi_+\rangle u^\mu u_\mu B$			172	I	$\bar{B}\langle u^\mu h^{\nu\lambda}\rangle u_\mu D_\nu T_\lambda + \text{H.c.}$	314		601	
$\bar{B}\langle\chi_+\rangle u^\mu u^\nu D_{\mu\nu}B$			173	I	$i\bar{B}\langle u^\mu h^{\nu\lambda}\rangle u^\rho\sigma_{\mu\nu}D_\lambda T_\rho + \text{H.c.}$	315		602	
$i\bar{B}\langle\chi_+\rangle u^\mu u^\nu\sigma_{\mu\nu}B$	86	I	174	I	$i\bar{B}\langle u^\mu h^{\nu\lambda}\rangle u^\rho\sigma_{\mu\nu}D_\rho T_\lambda + \text{H.c.}$	316		603	
$\bar{B}u^\mu u_\mu\chi_+B + \text{H.c.}$			175	I	$\bar{B}u^\mu u_\mu f_{-\nu}^{\lambda}D_\nu T_\lambda + \text{H.c.}$			604	
$\bar{B}u^\mu u^\nu\chi_+D_{\mu\nu}B + \text{H.c.}$			176	I	$\bar{B}u^\mu u^\nu f_{-\mu}^{\lambda}D_\nu T_\lambda + \text{H.c.}$			605	
$i\bar{B}u^\mu u^\nu\chi_+\sigma_{\mu\nu}B + \text{H.c.}$			177	I	$\bar{B}u^\mu u^\nu f_{-\mu}^{\lambda}D_\lambda T_\nu + \text{H.c.}$			606	
$i\bar{B}u^\mu\chi_+u^\nu\sigma_{\mu\nu}B$			178	I	$\bar{B}u^\mu u^\nu f_{-\nu}^{\lambda}D_\mu T_\lambda + \text{H.c.}$			607	
$\bar{B}u^\mu\nabla^\nu\chi_+\gamma_5\gamma_\mu D_\nu B + \text{H.c.}$	87	I	179	I	$\bar{B}u^\mu u^\nu f_{-\nu}^{\lambda}D_\lambda T_\mu + \text{H.c.}$			608	
$\bar{B}u^\mu\nabla^\nu\chi_+\gamma_5\gamma_\nu D_\mu B + \text{H.c.}$	88	I	180	I	$\bar{B}u^\mu u^\nu f_{-\mu}^{\lambda\rho}D_{\mu\nu\lambda}T_\rho + \text{H.c.}$			609	
$\bar{B}f_{-}^{\mu\nu}\chi_+\gamma_5\gamma_\mu D_\nu B + \text{H.c.}$	89	I	181	I	$i\bar{B}u^\mu u^\nu f_{-\mu}^{\lambda\rho}\sigma_{\mu\nu}D_\lambda T_\rho + \text{H.c.}$			610	
$\bar{B}\langle\nabla^\mu\nabla_\mu\chi_+\rangle B$	90	I	182	I	$i\bar{B}u^\mu u^\nu f_{-\mu}^{\lambda\rho}\sigma_{\mu\lambda}D_\nu T_\rho + \text{H.c.}$			611	
$\bar{B}\nabla^\mu\nabla_\mu\chi_+B$	91	I	183	I	$i\bar{B}u^\mu u^\nu f_{-\mu}^{\lambda\rho}\sigma_{\mu\lambda}D_\rho T_\nu + \text{H.c.}$			612	
$\bar{B}\langle f_{+}^{\mu\nu}\rangle\langle\chi_+\rangle\sigma_{\mu\nu}B$	92	I	184		$i\bar{B}u^\mu u^\nu f_{-\mu}^{\lambda\rho}\sigma_{\nu\lambda}D_\mu T_\rho + \text{H.c.}$			613	
$\bar{B}\langle f_{+}^{\mu\nu}\rangle\chi_+\sigma_{\mu\nu}B$	93	I	185		$\bar{B}u^\mu u_\mu h^{\nu\lambda}D_\nu T_\lambda + \text{H.c.}$			614	
$\bar{B}\langle f_{+}^{\mu\nu}\chi_+\rangle\sigma_{\mu\nu}B$	94	I	186	I	$\bar{B}u^\mu u^\nu h_\mu^{\lambda}D_\nu T_\lambda + \text{H.c.}$			615	
$\bar{B}\langle\chi_+\rangle f_{+}^{\mu\nu}\sigma_{\mu\nu}B$	95	I	187	I	$\bar{B}u^\mu u^\nu h_\mu^{\lambda}D_\lambda T_\nu + \text{H.c.}$			616	
$\bar{B}f_{+}^{\mu\nu}\chi_+\sigma_{\mu\nu}B + \text{H.c.}$			188	I	$\bar{B}u^\mu u^\nu h_\nu^{\lambda}D_\mu T_\lambda + \text{H.c.}$			617	
$\bar{B}\langle\chi_+\rangle\langle\chi_+\rangle B$	96	I	189	I	$\bar{B}u^\mu u^\nu h_\nu^{\lambda}D_\lambda T_\mu + \text{H.c.}$			618	
$\bar{B}\langle\chi_+\rangle\chi_+B$	97	I	190	I	$\bar{B}u^\mu u^\nu h^{\lambda\rho}D_{\mu\nu\lambda}T_\rho + \text{H.c.}$			619	
$\bar{B}\langle\chi_+^2\rangle B$			191	I	$\bar{B}u^\mu u^\nu h^{\lambda\rho}D_{\mu\lambda\rho}T_\nu + \text{H.c.}$			620	
$\bar{B}\chi_+^2 B$			192	I	$\bar{B}u^\mu u^\nu h^{\lambda\rho}D_{\nu\lambda\rho}T_\mu + \text{H.c.}$			621	
$i\bar{B}\langle u^\mu u^\nu\chi_-\rangle\gamma_5\gamma_\mu D_\nu B + \text{H.c.}$	98	I	193	I	$i\bar{B}u^\mu u^\nu h^{\lambda\rho}\sigma_{\mu\nu}D_\lambda T_\rho + \text{H.c.}$			622	
$i\bar{B}\langle\chi_-\rangle u^\mu u^\nu\gamma_5\gamma_\mu D_\nu B + \text{H.c.}$	99	I	194	I	$i\bar{B}u^\mu u^\nu h^{\lambda\rho}\sigma_{\mu\lambda}D_\nu T_\rho + \text{H.c.}$			623	
$i\bar{B}u^\mu u^\nu\chi_-\gamma_5\gamma_\mu D_\nu B + \text{H.c.}$			195	I	$i\bar{B}u^\mu u^\nu h^{\lambda\rho}\sigma_{\mu\lambda}D_\rho T_\nu + \text{H.c.}$			624	
$i\bar{B}u^\mu u^\nu\chi_-\gamma_5\gamma_\nu D_\mu B + \text{H.c.}$			196	I	$i\bar{B}u^\mu u^\nu h^{\lambda\rho}\sigma_{\nu\lambda}D_\mu T_\rho + \text{H.c.}$			625	
$i\bar{B}u^\mu\chi_+u^\nu\gamma_5\gamma_\mu D_\nu B + \text{H.c.}$			197	I	$\bar{B}u^\mu f_{-\mu}^{\nu}u^\lambda D_\nu T_\lambda + \text{H.c.}$			626	
$i\bar{B}\langle u^\mu\nabla_\mu\chi_-\rangle B$	100	I	198	I	$\bar{B}u^\mu f_{-\mu}^{\nu}u^\lambda D_\lambda T_\nu + \text{H.c.}$			627	
$i\bar{B}\langle u^\mu\nabla^\nu\chi_-\rangle D_{\mu\nu}B$	101	I	199	I	$\bar{B}u^\mu f_{-\nu}^{\lambda}u_\mu D_\nu T_\lambda + \text{H.c.}$			628	
$i\bar{B}\langle\chi_-\rangle h^{\mu\nu}D_{\mu\nu}B$	102	I	200	I	$\bar{B}u^\mu f_{-\nu}^{\lambda}u_\nu D_\mu T_\lambda + \text{H.c.}$			629	
$i\bar{B}\langle\nabla^\mu\chi_-\rangle u_\mu B$	103	I	201	I	$\bar{B}u^\mu f_{-\nu}^{\lambda}u_\nu D_\lambda T_\mu + \text{H.c.}$			630	
$i\bar{B}u^\mu\nabla_\mu\chi_-B + \text{H.c.}$			202	I	$\bar{B}u^\mu f_{-\nu}^{\lambda}u^\rho D_{\mu\nu\rho}T_\lambda + \text{H.c.}$			631	
$i\bar{B}u^\mu\nabla^\nu\chi_-D_{\mu\nu}B + \text{H.c.}$			203	I	$i\bar{B}u^\mu f_{-\nu}^{\lambda}u^\rho\sigma_{\mu\nu}D_\lambda T_\rho + \text{H.c.}$			632	
$\bar{B}u^\mu\nabla^\nu\chi_-\sigma_{\mu\nu}B + \text{H.c.}$	104	I	204	I	$i\bar{B}u^\mu f_{-\nu}^{\lambda}u^\rho\sigma_{\mu\nu}D_\rho T_\lambda + \text{H.c.}$			633	
$\bar{B}f_{-}^{\mu\nu}\chi_-\sigma_{\mu\nu}B + \text{H.c.}$	105	I	205	I	$i\bar{B}u^\mu f_{-\nu}^{\lambda}u^\rho\sigma_{\mu\rho}D_\nu T_\lambda + \text{H.c.}$			634	
$\bar{B}\langle f_{+}^{\mu\nu}\rangle\langle\chi_-\rangle\gamma_5\gamma_\mu D_\nu B$	106	I	206		$i\bar{B}u^\mu f_{-\nu}^{\lambda}u^\rho\sigma_{\nu\lambda}D_\mu T_\rho + \text{H.c.}$			635	
$\bar{B}\langle f_{+}^{\mu\nu}\rangle\chi_-\gamma_5\gamma_\mu D_\nu B$	107	I	207		$\bar{B}u^\mu h_\mu^{\nu}u^\lambda D_\nu T_\lambda + \text{H.c.}$			636	
$\bar{B}\langle f_{+}^{\mu\nu}\chi_-\rangle\gamma_5\gamma_\mu D_\nu B$	108	I	208	I	$\bar{B}u^\mu h_\mu^{\nu}u^\lambda D_\lambda T_\nu + \text{H.c.}$			637	
$\bar{B}\langle\chi_-\rangle f_{+}^{\mu\nu}\gamma_5\gamma_\mu D_\nu B$	109	I	209	I	$\bar{B}u^\mu h^{\nu\lambda}u_\mu D_\nu T_\lambda + \text{H.c.}$			638	
$\bar{B}f_{+}^{\mu\nu}\chi_-\gamma_5\gamma_\mu D_\nu B + \text{H.c.}$			210	I	$\bar{B}u^\mu h^{\nu\lambda}u^\rho D_{\mu\nu\lambda}T_\rho + \text{H.c.}$			639	
$\bar{B}\langle\chi_-\rangle\langle\chi_-\rangle B$	110	I	211	I	$\bar{B}u^\mu h^{\nu\lambda}u^\rho D_{\mu\rho\nu}T_\lambda + \text{H.c.}$			640	
$\bar{B}\langle\chi_-\rangle\chi_-B$	111	I	212	I	$i\bar{B}u^\mu h^{\nu\lambda}u^\rho\sigma_{\mu\nu}D_\rho T_\lambda + \text{H.c.}$			641	
$\bar{B}\chi_-^2 B$			213	I	$i\bar{B}u^\mu h^{\nu\lambda}u^\rho\sigma_{\mu\rho}D_\nu T_\lambda + \text{H.c.}$			642	
$\bar{B}\langle D^\mu D_\mu F_L^{\nu\lambda}\rangle\sigma_{\nu\lambda}B + P$	112	I			$\bar{B}f_{-}^{\mu\nu}u_\mu u^\lambda D_\nu T_\lambda + \text{H.c.}$			643	
$\bar{B}\langle F_L^{\mu\nu}F_{L\mu\nu}\rangle B + P$	113	I	214	I	$\bar{B}f_{-}^{\mu\nu}u_\mu u^\lambda D_\lambda T_\nu + \text{H.c.}$			644	
$\bar{B}\langle F_L^{\mu\nu}F_{L\mu}^{\lambda}\rangle D_{\nu\lambda}B + P$	114	I	215	I	$i\bar{B}f_{-}^{\mu\nu}u^\lambda u^\rho\sigma_{\mu\nu}D_\lambda T_\rho + \text{H.c.}$			645	
$\bar{B}\langle F_L^{\mu\nu}\rangle\langle F_{L\mu\nu}\rangle B + P$	115	I			$\bar{B}\langle u^\mu\nabla_\mu f_{-\nu}^{\lambda}\rangle\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$	317		646	
$\bar{B}\langle F_L^{\mu\nu}\rangle\langle F_{L\mu}^{\lambda}\rangle D_{\nu\lambda}B + P$	116	I			$\bar{B}\langle u^\mu\nabla^\nu f_{-\mu}^{\lambda}\rangle\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$	318		647	

(Table continued)

TABLE VIII. (Continued)

O_n/o_n	$SU(2)$	$\tilde{e}_n^{(4)}$	$SU(3)$	$\tilde{E}_n^{(4)}$	O_n	$SU(2)$	$\tilde{e}_n^{(4)}$	$SU(3)$	$\tilde{E}_n^{(4)}$
$\bar{B}\langle\chi\chi^\dagger\rangle B$	117	I	216	I	$\bar{B}\langle u^\mu\nabla^\nu f_{-\nu}^\lambda\rangle\gamma_5\gamma_\mu T_\lambda + \text{H.c.}$	319		648	
$\bar{B}\det\chi B + \text{H.c.}$	118	I			$\bar{B}\langle u^\mu\nabla^\nu f_{-\nu}^\lambda\rangle\gamma_5\gamma_\lambda T_\mu + \text{H.c.}$	320		649	
$\bar{T}^\mu\langle u_\mu u^\nu\rangle\langle u_\nu u^\lambda\rangle T_\lambda$	119		217		$\bar{B}\langle u^\mu\nabla^\nu f_{-\nu}^\lambda\rangle\gamma_5\gamma_\mu D_{\nu\lambda}T_\rho + \text{H.c.}$	321		650	
$\bar{T}^\mu\langle u_\mu u^\nu\rangle\langle u^\lambda u_\lambda\rangle T_\nu$	120		218		$\bar{B}\langle u^\mu\nabla^\nu f_{-\nu}^\lambda\rangle\gamma_5\gamma_\nu D_{\mu\lambda}T_\rho + \text{H.c.}$	322		651	
$\bar{T}^\mu\langle u^\nu u_\nu\rangle\langle u^\lambda u_\lambda\rangle T_\mu$	121		219		$\bar{B}\langle u^\mu\nabla^\nu f_{-\nu}^\lambda\rangle\gamma_5\gamma_\lambda D_{\mu\nu}T_\rho + \text{H.c.}$	323		652	
$\bar{T}^\mu\langle u^\nu u^\lambda\rangle\langle u_\nu u_\lambda\rangle T_\mu$	122		220		$\bar{B}\langle u^\mu\nabla_\mu h^{\nu\lambda}\rangle\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$	324		653	
$\bar{T}^\mu\langle u_\mu u^\nu\rangle\langle u^\lambda u^\rho\rangle D_{\nu\lambda}T_\rho$	123		221		$\bar{B}\langle u^\mu\nabla^\nu h^{\lambda\rho}\rangle\gamma_5\gamma_\mu D_{\nu\lambda}T_\rho + \text{H.c.}$	325		654	
$\bar{T}^\mu\langle u_\mu u^\nu\rangle\langle u^\lambda u^\rho\rangle D_{\lambda\rho}T_\nu$	124		222		$\bar{B}\langle f_{-}^{\mu\nu}h_{\mu}^\lambda\rangle\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$	326		655	
$\bar{T}^\mu\langle u^\nu u_\nu\rangle\langle u^\lambda u^\rho\rangle D_{\lambda\rho}T_\mu$	125		223		$\bar{B}\langle h^{\mu\nu}h_{\mu}^\lambda\rangle\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$	327		656	
$\bar{T}^\mu\langle u^\nu u^\lambda\rangle\langle u_\nu u^\rho\rangle D_{\lambda\rho}T_\mu$	126		224		$\bar{B}u^\mu\nabla_{\mu}f_{-}^{\nu\lambda}\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$	328		657	
$\bar{T}^\mu\langle u^\nu u^\lambda\rangle\langle u^\rho u^\sigma\rangle D_{\nu\lambda\rho\sigma}T_\mu$	127		225		$\bar{B}u^\mu\nabla^\nu f_{-}^{\lambda\rho}\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$	329		658	
$i\bar{T}^\mu\langle u_\mu u^\nu\rangle\langle u^\lambda u^\rho\rangle\sigma_{\nu\lambda}T_\rho$	128		226		$\bar{B}u^\mu\nabla^\nu f_{-}^{\lambda\rho}\gamma_5\gamma_\mu T_\lambda + \text{H.c.}$	330		659	
$\bar{T}^\mu\langle u_\mu u^\nu\rangle u_\nu u^\lambda T_\lambda + \text{H.c.}$	129		227		$\bar{B}u^\mu\nabla^\nu f_{-}^{\lambda\rho}\gamma_5\gamma_\lambda T_\mu + \text{H.c.}$	331		660	
$\bar{T}^\mu\langle u_\mu u^\nu\rangle u^\lambda u_\nu T_\lambda + \text{H.c.}$			228		$\bar{B}u^\mu\nabla^\nu f_{-}^{\lambda\rho}\gamma_5\gamma_\mu D_{\nu\lambda}T_\rho + \text{H.c.}$	332		661	
$\bar{T}^\mu\langle u_\mu u^\nu\rangle u^\lambda u_\lambda T_\nu$			229		$\bar{B}u^\mu\nabla^\nu f_{-}^{\lambda\rho}\gamma_5\gamma_\nu D_{\mu\lambda}T_\rho + \text{H.c.}$	333		662	
$\bar{T}^\mu\langle u^\nu u_\nu\rangle u_\mu u^\lambda T_\lambda$	130		230		$\bar{B}u^\mu\nabla^\nu f_{-}^{\lambda\rho}\gamma_5\gamma_\lambda D_{\mu\nu}T_\rho + \text{H.c.}$	334		663	
$\bar{T}^\mu\langle u^\nu u_\nu\rangle u^\lambda u_\mu T_\lambda$			231		$\bar{B}u^\mu\nabla^\nu f_{-}^{\lambda\rho}\gamma_5\gamma_\lambda D_{\nu\rho}T_\mu + \text{H.c.}$	335		664	
$\bar{T}^\mu\langle u^\nu u_\nu\rangle u^\lambda u_\lambda T_\mu$			232		$\bar{B}u^\mu\nabla_\mu h^{\lambda\rho}\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$	336		665	
$\bar{T}^\mu\langle u^\nu u^\lambda\rangle u_\nu u_\lambda T_\mu$			233		$\bar{B}u^\mu\nabla^\nu h^{\lambda\rho}\gamma_5\gamma_\mu D_{\nu\lambda}T_\rho + \text{H.c.}$	337		666	
$\bar{T}^\mu\langle u_\mu u^\nu\rangle u^\lambda u^\rho D_{\nu\lambda}T_\rho + \text{H.c.}$	131		234		$\bar{B}u^\mu\nabla^\nu h^{\lambda\rho}\gamma_5\gamma_\nu D_{\mu\lambda}T_\rho + \text{H.c.}$			667	
$\bar{T}^\mu\langle u_\mu u^\nu\rangle u^\lambda u^\rho D_{\nu\rho}T_\lambda + \text{H.c.}$			235		$\bar{B}u^\mu\nabla^\nu h^{\lambda\rho}\gamma_5\gamma_\nu D_{\lambda\rho}T_\mu + \text{H.c.}$			668	
$\bar{T}^\mu\langle u_\mu u^\nu\rangle u^\lambda u^\rho D_{\lambda\rho}T_\nu$			236		$\bar{B}f_{-}^{\mu\nu}f_{-}^{\lambda\rho}\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$	338		669	
$\bar{T}^\mu\langle u^\nu u_\nu\rangle u^\lambda u^\rho D_{\lambda\rho}T_\mu$			237		$\bar{B}f_{-}^{\mu\nu}f_{-}^{\lambda\rho}\gamma_5\gamma_\lambda T_\nu + \text{H.c.}$			670	
$\bar{T}^\mu\langle u^\nu u^\lambda\rangle u_\mu u^\rho D_{\nu\lambda}T_\rho$	132		238		$\bar{B}f_{-}^{\mu\nu}f_{-}^{\lambda\rho}\gamma_5\gamma_\mu D_{\nu\lambda}T_\rho + \text{H.c.}$			671	
$\bar{T}^\mu\langle u^\nu u^\lambda\rangle u_\nu u^\rho D_{\lambda\rho}T_\mu + \text{H.c.}$			239		$\bar{B}f_{-}^{\mu\nu}h_{\mu}^\lambda\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$	339		672	
$\bar{T}^\mu\langle u^\nu u^\lambda\rangle u^\rho u_\mu D_{\nu\lambda}T_\rho$			240		$\bar{B}f_{-}^{\mu\nu}h_{\mu}^\lambda\gamma_5\gamma_\lambda T_\nu + \text{H.c.}$			673	
$\bar{T}^\mu\langle u^\nu u^\lambda\rangle u^\rho u_\rho D_{\nu\lambda}T_\mu$			241		$\bar{B}f_{-}^{\mu\nu}h^{\lambda\rho}\gamma_5\gamma_\mu D_{\nu\lambda}T_\rho + \text{H.c.}$			674	
$\bar{T}^\mu\langle u^\nu u^\lambda\rangle u^\rho u^\sigma D_{\nu\lambda\rho\sigma}T_\mu$			242		$\bar{B}f_{-}^{\mu\nu}h^{\lambda\rho}\gamma_5\gamma_\mu D_{\lambda\rho}T_\nu + \text{H.c.}$			675	
$i\bar{T}^\mu\langle u_\mu u^\nu\rangle u^\lambda u^\rho\sigma_{\nu\lambda}T_\rho + \text{H.c.}$	133		243		$\bar{B}\nabla^\mu f_{-}^{\nu\lambda}u^\lambda\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$			676	
$i\bar{T}^\mu\langle u_\mu u^\nu\rangle u^\lambda u^\rho\sigma_{\nu\rho}T_\lambda + \text{H.c.}$			244		$\bar{B}\nabla^\mu f_{-}^{\nu\lambda}u^\lambda\gamma_5\gamma_\lambda T_\nu + \text{H.c.}$			677	
$\bar{T}^\mu\langle u_\mu u^\nu u_\nu\rangle u^\lambda T_\lambda + \text{H.c.}$			245		$\bar{B}\nabla^\mu f_{-}^{\nu\lambda}u_\nu\gamma_5\gamma_\mu T_\lambda + \text{H.c.}$			678	
$\bar{T}^\mu\langle u_\mu u^\nu u^\lambda\rangle u_\nu T_\lambda$			246		$\bar{B}h^{\mu\nu}f_{-}^{\lambda\rho}\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$			679	
$\bar{T}^\mu\langle u_\mu u^\nu u^\lambda\rangle u_\lambda T_\nu$			247		$\bar{B}h^{\mu\nu}h_{\mu}^\lambda\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$	340		680	
$\bar{T}^\mu\langle u^\nu u_\nu u^\lambda\rangle u_\lambda T_\mu$			248		$\bar{B}h^{\mu\nu}h_{\mu}^\lambda\gamma_5\gamma_\lambda T_\nu + \text{H.c.}$			681	
$\bar{T}^\mu\langle u_\mu u^\nu u^\lambda\rangle u^\rho D_{\nu\lambda}T_\rho + \text{H.c.}$			249		$\bar{B}\nabla^\mu h^{\nu\lambda}u_\mu\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$			682	
$\bar{T}^\mu\langle u_\mu u^\nu u^\lambda\rangle u^\rho D_{\nu\rho}T_\lambda$			250		$i\bar{B}\langle f_{+}^{\mu\nu}\rangle\langle u_\mu u^\lambda\rangle\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$	341		683	
$\bar{T}^\mu\langle u_\mu u^\nu u^\lambda\rangle u^\rho D_{\lambda\rho}T_\nu$			251		$i\bar{B}\langle f_{+}^{\mu\nu}\rangle\langle u_\mu u^\lambda\rangle\gamma_5\gamma_\lambda T_\nu + \text{H.c.}$	342		684	
$\bar{T}^\mu\langle u^\nu u_\nu u^\lambda\rangle u^\rho D_{\lambda\rho}T_\mu$			252		$i\bar{B}\langle f_{+}^{\mu\nu}\rangle\langle u^\lambda u_\lambda\rangle\gamma_5\gamma_\mu T_\nu + \text{H.c.}$	343		685	
$\bar{T}^\mu\langle u^\nu u^\lambda u^\rho\rangle u_\nu D_{\lambda\rho}T_\mu$			253		$i\bar{B}\langle f_{+}^{\mu\nu}\rangle\langle u^\lambda u^\rho\rangle\gamma_5\gamma_\mu D_{\nu\lambda}T_\rho + \text{H.c.}$	344		686	
$\bar{T}^\mu\langle u^\nu u^\lambda u^\rho\rangle u^\sigma D_{\nu\lambda\rho\sigma}T_\mu$			254		$i\bar{B}\langle f_{+}^{\mu\nu}\rangle\langle u^\lambda u^\rho\rangle\gamma_5\gamma_\mu D_{\lambda\rho}T_\nu + \text{H.c.}$	345		687	
$\bar{T}^\mu\langle u_\mu u^\nu u_\nu u^\lambda\rangle T_\lambda$			255		$i\bar{B}\langle f_{+}^{\mu\nu}\rangle\langle u^\lambda u^\rho\rangle\gamma_5\gamma_\lambda D_{\mu\rho}T_\nu + \text{H.c.}$	346		688	
$\bar{T}^\mu\langle u_\mu u^\nu u^\lambda u_\nu\rangle T_\lambda$			256		$i\bar{B}\langle f_{+}^{\mu\nu}\rangle u_\mu u^\lambda\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$	347		689	
$\bar{T}^\mu\langle u^\nu u_\nu u^\lambda u_\lambda\rangle T_\mu$			257		$i\bar{B}\langle f_{+}^{\mu\nu}\rangle u_\mu u^\lambda\gamma_5\gamma_\lambda T_\nu + \text{H.c.}$	348		690	
$\bar{T}^\mu\langle u_\mu u^\nu u^\lambda u^\rho\rangle D_{\nu\lambda}T_\rho$			258		$i\bar{B}\langle f_{+}^{\mu\nu}\rangle u^\lambda u_\mu\gamma_5\gamma_\nu T_\lambda + \text{H.c.}$			691	
$\bar{T}^\mu\langle u_\mu u^\nu u^\lambda u^\rho\rangle D_{\nu\rho}T_\lambda$			259		$i\bar{B}\langle f_{+}^{\mu\nu}\rangle u^\lambda u_\mu\gamma_5\gamma_\lambda T_\nu + \text{H.c.}$			692	
$\bar{T}^\mu\langle u^\nu u_\nu u^\lambda u^\rho\rangle D_{\lambda\rho}T_\mu$			260		$i\bar{B}\langle f_{+}^{\mu\nu}\rangle u^\lambda u_\lambda\gamma_5\gamma_\mu T_\nu + \text{H.c.}$			693	
$i\bar{T}^\mu\langle u_\mu u^\nu u^\lambda u^\rho\rangle\sigma_{\nu\lambda}T_\rho$			261		$i\bar{B}\langle f_{+}^{\mu\nu}\rangle u^\lambda u^\rho\gamma_5\gamma_\mu D_{\nu\lambda}T_\rho + \text{H.c.}$	349		694	
$\bar{T}^\mu u_\mu u^\nu u_\nu u^\lambda T_\lambda$			262		$i\bar{B}\langle f_{+}^{\mu\nu}\rangle u^\lambda u^\rho\gamma_5\gamma_\mu D_{\nu\rho}T_\lambda + \text{H.c.}$			695	

(Table continued)

TABLE VIII. (Continued)

O_n/o_n	$SU(2)$	$\tilde{e}_n^{(4)}$	$SU(3)$	$\tilde{E}_n^{(4)}$	O_n	$SU(2)$	$\tilde{e}_n^{(4)}$	$SU(3)$	$\tilde{E}_n^{(4)}$
$\bar{T}^\mu u_\mu u^\nu u^\lambda u_\nu T_\lambda + \text{H.c.}$			263		$i\bar{B}\langle f_+^{\mu\nu} \rangle u^\lambda u^\rho \gamma_5 \gamma_\mu D_{\lambda\rho} T_\nu + \text{H.c.}$				696
$\bar{T}^\mu u^\nu u_\mu u^\lambda u_\lambda T_\nu + \text{H.c.}$			264		$i\bar{B}\langle f_+^{\mu\nu} \rangle u^\lambda u^\rho \gamma_5 \gamma_\lambda D_{\mu\rho} T_\nu + \text{H.c.}$	350			697
$\bar{T}^\mu u^\nu u_\nu u^\lambda u_\lambda T_\mu$			265		$i\bar{B}\langle f_+^{\mu\nu} \rangle u^\lambda u^\rho \gamma_5 \gamma_\rho D_{\mu\lambda} T_\nu + \text{H.c.}$				698
$\bar{T}^\mu u_\mu u^\nu u^\lambda u^\rho D_{\nu\lambda} T_\rho$			266		$i\bar{B}\langle f_+^{\mu\nu} u_\mu \rangle u^\lambda \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$	351			699
$\bar{T}^\mu u_\mu u^\nu u^\lambda u^\rho D_{\nu\rho} T_\lambda + \text{H.c.}$			267		$i\bar{B}\langle f_+^{\mu\nu} u_\mu \rangle u^\lambda \gamma_5 \gamma_\lambda T_\nu + \text{H.c.}$	352			700
$\bar{T}^\mu u^\nu u_\mu u^\lambda u^\rho D_{\lambda\rho} T_\nu + \text{H.c.}$			268		$i\bar{B}\langle f_+^{\mu\nu} u_\mu \rangle u^\lambda \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$	353			701
$\bar{T}^\mu u^\nu u_\nu u^\lambda u^\rho D_{\lambda\rho} T_\mu + \text{H.c.}$			269		$i\bar{B}\langle f_+^{\mu\nu} u_\mu \rangle u^\lambda \gamma_5 \gamma_\lambda T_\nu + \text{H.c.}$	354			702
$i\bar{T}^\mu u_\mu u^\nu u^\lambda u^\rho \sigma_{\nu\lambda} T_\rho$			270		$i\bar{B}\langle f_+^{\mu\nu} u_\lambda \rangle u_\mu \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$	355			703
$i\bar{T}^\mu u_\mu u^\nu u^\lambda u^\rho \sigma_{\nu\rho} T_\lambda + \text{H.c.}$			271		$i\bar{B}\langle f_+^{\mu\nu} u_\mu \rangle u^\rho \gamma_5 \gamma_\mu D_{\nu\lambda} T_\rho + \text{H.c.}$	356			704
$\bar{T}^\mu \langle u_\mu u^\nu \rangle f_-^{\lambda\rho} \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$	134		272		$i\bar{B}\langle f_+^{\mu\nu} u^\lambda \rangle u^\rho \gamma_5 \gamma_\mu D_{\nu\rho} T_\lambda + \text{H.c.}$	357			705
$\bar{T}^\mu \langle u_\mu u^\nu \rangle f_-^{\lambda\rho} \gamma_5 \gamma_\lambda D_\nu T_\rho + \text{H.c.}$	135		273		$i\bar{B}\langle f_+^{\mu\nu} u^\lambda \rangle u^\rho \gamma_5 \gamma_\mu D_{\lambda\rho} T_\nu + \text{H.c.}$	358			706
$\bar{T}^\mu \langle u_\mu u^\lambda \rangle f_-^{\rho\lambda} \gamma_5 \gamma_\nu D_\lambda T_\rho$	136		274		$i\bar{B}\langle f_+^{\mu\nu} u^\lambda \rangle u^\rho \gamma_5 \gamma_\lambda D_{\mu\rho} T_\nu + \text{H.c.}$	359			707
$\bar{T}^\mu \langle u_\mu u^\nu \rangle h^{\lambda\rho} \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$	137		275		$i\bar{B}\langle f_+^{\mu\nu} u^\lambda \rangle u^\rho \gamma_5 \gamma_\rho D_{\mu\lambda} T_\nu + \text{H.c.}$				708
$\bar{T}^\mu \langle u_\mu u^\nu \rangle h^{\lambda\rho} \gamma_5 \gamma_\lambda D_\nu T_\rho + \text{H.c.}$	138		276		$i\bar{B}\langle f_+^{\mu\nu} u_\mu u^\lambda \rangle \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$	360			709
$\bar{T}^\mu \langle u_\mu u^\nu f_-^{\lambda\rho} \rangle \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$	139		277		$i\bar{B}\langle f_+^{\mu\nu} u_\mu u^\lambda \rangle \gamma_5 \gamma_\lambda T_\nu + \text{H.c.}$	361			710
$\bar{T}^\mu \langle u_\mu u^\nu f_-^{\lambda\rho} \rangle \gamma_5 \gamma_\lambda D_\nu T_\rho + \text{H.c.}$	140		278		$i\bar{B}\langle f_+^{\mu\nu} u^\lambda u_\mu \rangle \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$				711
$\bar{T}^\mu \langle u^\nu u_\mu f_-^{\lambda\rho} \rangle \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$			279		$i\bar{B}\langle f_+^{\mu\nu} u^\lambda u_\mu \rangle \gamma_5 \gamma_\lambda T_\nu + \text{H.c.}$				712
$\bar{T}^\mu \langle u^\nu u_\mu f_-^{\lambda\rho} \rangle \gamma_5 \gamma_\lambda D_\nu T_\rho + \text{H.c.}$			280		$i\bar{B}\langle f_+^{\mu\nu} u^\lambda u_\lambda \rangle \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$				713
$\bar{T}^\mu \langle u^\nu u^\lambda f_-^{\rho\lambda} \rangle \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$			281		$i\bar{B}\langle f_+^{\mu\nu} u^\lambda u^\rho \rangle \gamma_5 \gamma_\mu D_{\nu\lambda} T_\rho + \text{H.c.}$	362			714
$\bar{T}^\mu \langle u_\mu u^\nu h^{\lambda\rho} \rangle \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$	141		282		$i\bar{B}\langle f_+^{\mu\nu} u^\lambda u^\rho \rangle \gamma_5 \gamma_\mu D_{\nu\rho} T_\lambda + \text{H.c.}$				715
$\bar{T}^\mu \langle u_\mu u^\nu h^{\lambda\rho} \rangle \gamma_5 \gamma_\lambda D_\nu T_\rho + \text{H.c.}$	142		283		$i\bar{B}\langle f_+^{\mu\nu} u^\lambda u^\rho \rangle \gamma_5 \gamma_\mu D_{\lambda\rho} T_\nu + \text{H.c.}$				716
$\bar{T}^\mu \langle u^\nu u_\mu h^{\lambda\rho} \rangle \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$			284		$i\bar{B}\langle f_+^{\mu\nu} u^\lambda u^\rho \rangle \gamma_5 \gamma_\lambda D_{\mu\rho} T_\nu + \text{H.c.}$	363			717
$\bar{T}^\mu \langle u^\nu u^\lambda h_\nu^\rho \rangle \gamma_5 \gamma_\lambda D_\rho T_\mu + \text{H.c.}$	143		285		$i\bar{B}\langle f_+^{\mu\nu} u^\lambda u^\rho \rangle \gamma_5 \gamma_\rho D_{\mu\lambda} T_\nu + \text{H.c.}$				718
$\bar{T}^\mu \langle u_\mu f_-^{\nu\lambda} \rangle u^\rho \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$	144		286		$i\bar{B}\langle u^\mu u_\mu \rangle f_+^{\nu\lambda} \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$	364			719
$\bar{T}^\mu \langle u_\mu f_-^{\nu\lambda} \rangle u^\rho \gamma_5 \gamma_\nu D_\rho T_\lambda + \text{H.c.}$	145		287		$i\bar{B}\langle u^\mu u^\nu \rangle f_+^{\lambda\mu} \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$	365			720
$\bar{T}^\mu \langle u_\mu f_-^{\nu\lambda} \rangle u^\rho \gamma_5 \gamma_\rho D_\nu T_\lambda + \text{H.c.}$	146		288		$i\bar{B}\langle u^\mu u^\nu \rangle f_+^{\lambda\mu} \gamma_5 \gamma_\lambda T_\nu + \text{H.c.}$	366			721
$\bar{T}^\mu \langle u^\nu f_-^{\rho\lambda} \rangle u^\rho \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$	147		289		$i\bar{B}\langle u^\mu u^\nu \rangle f_+^{\lambda\rho} \gamma_5 \gamma_\mu D_{\nu\lambda} T_\rho + \text{H.c.}$	367			722
$\bar{T}^\mu \langle u^\nu f_-^{\rho\lambda} \rangle u^\rho \gamma_5 \gamma_\nu D_\rho T_\lambda + \text{H.c.}$	148		290		$i\bar{B}\langle u^\mu u^\nu \rangle f_+^{\lambda\rho} \gamma_5 \gamma_\lambda D_{\mu\nu} T_\rho + \text{H.c.}$	368			723
$\bar{T}^\mu \langle u^\nu f_-^{\rho\lambda} \rangle u^\rho \gamma_5 \gamma_\lambda D_\nu T_\rho + \text{H.c.}$	149		291		$i\bar{B}\langle u^\mu u^\nu \rangle f_+^{\lambda\rho} \gamma_5 \gamma_\lambda D_{\mu\rho} T_\nu + \text{H.c.}$	369			724
$\bar{T}^\mu \langle u^\nu f_-^{\rho\lambda} \rangle u^\rho \gamma_5 \gamma_\rho D_\nu T_\lambda + \text{H.c.}$	150		292		$i\bar{B}f_+^{\mu\nu} u_\mu u^\lambda \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$				725
$\bar{T}^\mu \langle u_\mu h^{\nu\lambda} \rangle u^\rho \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$	151		293		$i\bar{B}f_+^{\mu\nu} u_\mu u^\lambda \gamma_5 \gamma_\lambda T_\nu + \text{H.c.}$				726
$\bar{T}^\mu u_\mu u^\nu f_-^{\lambda\rho} \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$			294		$i\bar{B}f_+^{\mu\nu} u^\lambda u_\mu \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$				727
$\bar{T}^\mu u_\mu u^\nu f_-^{\lambda\rho} \gamma_5 \gamma_\lambda D_\nu T_\rho + \text{H.c.}$			295		$i\bar{B}f_+^{\mu\nu} u^\lambda u_\mu \gamma_5 \gamma_\lambda T_\nu + \text{H.c.}$				728
$\bar{T}^\mu u_\mu u^\nu f_-^{\lambda\rho} \gamma_5 \gamma_\lambda D_\rho T_\nu + \text{H.c.}$			296		$i\bar{B}f_+^{\mu\nu} u^\lambda u_\lambda \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$				729
$\bar{T}^\mu u^\nu u_\mu f_-^{\lambda\rho} \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$			297		$i\bar{B}f_+^{\mu\nu} u^\lambda u^\rho \gamma_5 \gamma_\mu D_{\nu\lambda} T_\rho + \text{H.c.}$				730
$\bar{T}^\mu u^\nu u_\mu f_-^{\lambda\rho} \gamma_5 \gamma_\lambda D_\nu T_\rho + \text{H.c.}$			298		$i\bar{B}f_+^{\mu\nu} u^\lambda u^\rho \gamma_5 \gamma_\mu D_{\nu\rho} T_\lambda + \text{H.c.}$				731
$\bar{T}^\mu u^\nu u_\mu f_-^{\lambda\rho} \gamma_5 \gamma_\lambda D_\rho T_\nu + \text{H.c.}$			299		$i\bar{B}f_+^{\mu\nu} u^\lambda u^\rho \gamma_5 \gamma_\mu D_{\lambda\rho} T_\nu + \text{H.c.}$				732
$\bar{T}^\mu u^\nu u_\nu f_-^{\lambda\rho} \gamma_5 \gamma_\lambda D_\rho T_\mu + \text{H.c.}$			300		$i\bar{B}f_+^{\mu\nu} u^\lambda u^\rho \gamma_5 \gamma_\lambda D_{\mu\rho} T_\nu + \text{H.c.}$				733
$\bar{T}^\mu u^\nu u^\lambda f_-^{\rho\lambda} \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$			301		$i\bar{B}f_+^{\mu\nu} u^\lambda u^\rho \gamma_5 \gamma_\rho D_{\mu\lambda} T_\nu + \text{H.c.}$				734
$\bar{T}^\mu u^\nu u^\lambda f_-^{\rho\lambda} \gamma_5 \gamma_\nu D_\rho T_\lambda + \text{H.c.}$			302		$i\bar{B}u^\mu f_+^{\nu\lambda} u^\nu \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$				735
$\bar{T}^\mu u^\nu u^\lambda f_-^{\rho\lambda} \gamma_5 \gamma_\lambda D_\nu T_\rho + \text{H.c.}$			303		$i\bar{B}u^\mu f_+^{\nu\lambda} u^\nu \gamma_5 \gamma_\lambda T_\nu + \text{H.c.}$				736
$\bar{T}^\mu u^\nu u^\lambda f_-^{\rho\lambda} \gamma_5 \gamma_\lambda D_\rho T_\nu + \text{H.c.}$			304		$i\bar{B}u^\mu f_+^{\nu\lambda} u_\mu \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$				737
$\bar{T}^\mu u^\nu u^\lambda f_-^{\rho\lambda} \gamma_5 \gamma_\rho D_\nu T_\lambda + \text{H.c.}$			305		$i\bar{B}u^\mu f_+^{\nu\lambda} u_\nu \gamma_5 \gamma_\mu T_\lambda + \text{H.c.}$				738
$\bar{T}^\mu u^\nu u^\lambda f_-^{\rho\lambda} \gamma_5 \gamma_\rho D_\lambda T_\nu + \text{H.c.}$			306		$i\bar{B}u^\mu f_+^{\nu\lambda} u_\nu \gamma_5 \gamma_\lambda T_\mu + \text{H.c.}$				739
$\bar{T}^\mu u^\nu u^\lambda f_-^{\rho\lambda} \gamma_5 \gamma_\lambda D_\rho T_\mu + \text{H.c.}$			307		$i\bar{B}u^\mu f_+^{\nu\lambda} u^\rho \gamma_5 \gamma_\mu D_{\nu\rho} T_\lambda + \text{H.c.}$				740
$\bar{T}^\mu u_\mu u^\nu h^{\lambda\rho} \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$			308		$i\bar{B}u^\mu f_+^{\nu\lambda} u^\rho \gamma_5 \gamma_\nu D_{\mu\lambda} T_\rho + \text{H.c.}$				741
$\bar{T}^\mu u_\mu u^\nu h^{\lambda\rho} \gamma_5 \gamma_\lambda D_\nu T_\rho + \text{H.c.}$			309		$i\bar{B}u^\mu f_+^{\nu\lambda} u^\rho \gamma_5 \gamma_\nu D_{\mu\rho} T_\lambda + \text{H.c.}$				742
$\bar{T}^\mu u_\mu u^\nu h^{\lambda\rho} \gamma_5 \gamma_\lambda D_\rho T_\nu + \text{H.c.}$			310		$i\bar{B}u^\mu f_+^{\nu\lambda} u^\rho \gamma_5 \gamma_\nu D_{\lambda\rho} T_\mu + \text{H.c.}$				743

(Table continued)

TABLE VIII. (Continued)

O_n/o_n	$SU(2)$	$\tilde{e}_n^{(4)}$	$SU(3)$	$\tilde{E}_n^{(4)}$	O_n	$SU(2)$	$\tilde{e}_n^{(4)}$	$SU(3)$	$\tilde{E}_n^{(4)}$
$\bar{T}^\mu u^\nu u_\mu h^{\lambda\rho} \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$			311		$i\bar{B}u^\mu u^\nu f_{+\mu}{}^\lambda \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$			744	
$\bar{T}^\mu u^\nu u_\mu h^{\lambda\rho} \gamma_5 \gamma_\lambda D_\nu T_\rho + \text{H.c.}$			312		$i\bar{B}u^\mu u^\nu f_{+\mu}{}^\lambda \gamma_5 \gamma_\lambda T_\nu + \text{H.c.}$			745	
$\bar{T}^\mu u^\nu u_\mu h^{\lambda\rho} \gamma_5 \gamma_\lambda D_\rho T_\nu + \text{H.c.}$			313		$i\bar{B}u^\mu u^\nu f_{+\mu}{}^\lambda \gamma_5 \gamma_\lambda D_{\mu\rho} T_\nu + \text{H.c.}$			746	
$\bar{T}^\mu u^\nu u_\nu h^{\lambda\rho} \gamma_5 \gamma_\lambda D_\rho T_\mu + \text{H.c.}$			314		$i\bar{B}\langle f_{+\mu}{}^{\mu\nu} \rangle f_{-\mu}{}^\lambda D_\nu T_\lambda + \text{H.c.}$	370		747	
$\bar{T}^\mu u^\nu u^\lambda h_{\mu}{}^\rho \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$			315		$i\bar{B}\langle f_{+\mu}{}^{\mu\nu} \rangle f_{-\mu}{}^\lambda D_\lambda T_\nu + \text{H.c.}$	371		748	
$\bar{T}^\mu u^\nu u^\lambda h_{\mu}{}^\rho \gamma_5 \gamma_\nu D_\rho T_\lambda + \text{H.c.}$			316		$\bar{B}\langle f_{+\mu}{}^{\mu\nu} \rangle f_{-\mu}{}^\lambda \sigma_{\mu\nu} D_\lambda T_\rho + \text{H.c.}$	372		749	
$\bar{T}^\mu u^\nu u^\lambda h_{\mu}{}^\rho \gamma_5 \gamma_\lambda D_\nu T_\rho + \text{H.c.}$			317		$\bar{B}\langle f_{+\mu}{}^{\mu\nu} \rangle f_{-\mu}{}^\lambda \sigma_{\mu\nu} D_\nu T_\rho + \text{H.c.}$	373		750	
$\bar{T}^\mu u^\nu u^\lambda h_{\nu}{}^\rho \gamma_5 \gamma_\lambda D_\rho T_\mu + \text{H.c.}$			318		$i\bar{B}\langle f_{+\mu}{}^{\mu\nu} \rangle h_{\mu}{}^\lambda D_\nu T_\lambda + \text{H.c.}$	374		751	
$\bar{T}^\mu u^\nu u^\lambda h_{\nu}{}^\rho \gamma_5 \gamma_\rho D_\lambda T_\mu + \text{H.c.}$			319		$i\bar{B}\langle f_{+\mu}{}^{\mu\nu} \rangle h_{\mu}{}^\lambda D_\lambda T_\nu + \text{H.c.}$	375		752	
$\bar{T}^\mu u^\nu u^\lambda h^{\rho\sigma} \gamma_5 \gamma_\nu D_{\lambda\rho\sigma} T_\mu + \text{H.c.}$			320		$i\bar{B}\langle f_{+\mu}{}^{\mu\nu} \rangle h^{\lambda\rho} D_{\mu\lambda\rho} T_\nu + \text{H.c.}$	376		753	
$\bar{T}^\mu u_\mu f_{-\nu}{}^\lambda u^\nu \gamma_5 \gamma_\nu D_\rho T_\lambda + \text{H.c.}$			321		$\bar{B}\langle f_{+\mu}{}^{\mu\nu} \rangle h^{\lambda\rho} \sigma_{\mu\nu} D_\lambda T_\rho + \text{H.c.}$	377		754	
$\bar{T}^\mu \langle u_\mu \nabla^\nu f_{-\nu}{}^\lambda \rangle T_\lambda + \text{H.c.}$	152		322		$\bar{B}\langle f_{+\mu}{}^{\mu\nu} \rangle h^{\lambda\rho} \sigma_{\mu\lambda} D_\nu T_\rho + \text{H.c.}$	378		755	
$\bar{T}^\mu \langle u^\lambda \nabla_\mu f_{-\nu}{}^\lambda \rangle T_\lambda + \text{H.c.}$	153		323		$i\bar{B}\langle f_{+\mu}{}^{\mu\nu} f_{-\mu}{}^\lambda \rangle D_\nu T_\lambda + \text{H.c.}$	379		756	
$\bar{T}^\mu \langle u^\lambda \nabla^\lambda f_{-\nu}{}^\lambda \rangle T_\mu$	154		324		$i\bar{B}\langle f_{+\mu}{}^{\mu\nu} f_{-\mu}{}^\lambda \rangle D_\lambda T_\nu + \text{H.c.}$	380		757	
$\bar{T}^\mu \langle u_\mu \nabla^\nu f_{-\nu}{}^\lambda \rangle D_{\nu\lambda} T_\rho + \text{H.c.}$	155		325		$\bar{B}\langle f_{+\mu}{}^{\mu\nu} f_{-\mu}{}^\lambda \rangle \sigma_{\mu\nu} D_\lambda T_\rho + \text{H.c.}$	381		758	
$\bar{T}^\mu \langle u^\lambda \nabla^\lambda f_{-\nu}{}^\rho \rangle D_{\lambda\rho} T_\mu$	156		326		$\bar{B}\langle f_{+\mu}{}^{\mu\nu} f_{-\mu}{}^\lambda \rangle \sigma_{\mu\lambda} D_\nu T_\rho + \text{H.c.}$	382		759	
$\bar{T}^\mu \langle u^\lambda \nabla^\lambda f_{-\lambda}{}^\rho \rangle D_{\nu\rho} T_\mu$	157		327		$i\bar{B}\langle f_{+\mu}{}^{\mu\nu} h_{\mu}{}^\lambda \rangle D_\nu T_\lambda + \text{H.c.}$	383		760	
$\bar{T}^\mu \langle u^\nu \nabla_\mu h_{\nu}{}^\lambda \rangle T_\lambda + \text{H.c.}$	158		328		$i\bar{B}\langle f_{+\mu}{}^{\mu\nu} h_{\mu}{}^\lambda \rangle D_\lambda T_\nu + \text{H.c.}$	384		761	
$\bar{T}^\mu \langle u_\mu \nabla^\nu h^{\lambda\rho} \rangle D_{\nu\lambda} T_\rho + \text{H.c.}$	159		329		$i\bar{B}\langle f_{+\mu}{}^{\mu\nu} h^{\lambda\rho} \rangle D_{\mu\lambda\rho} T_\nu + \text{H.c.}$	385		762	
$\bar{T}^\mu \langle u^\nu \nabla_\nu h^{\lambda\rho} \rangle D_{\lambda\rho} T_\mu$	160		330		$\bar{B}\langle f_{+\mu}{}^{\mu\nu} h^{\lambda\rho} \rangle \sigma_{\mu\nu} D_\lambda T_\rho + \text{H.c.}$	386		763	
$\bar{T}^\mu \langle u^\nu \nabla^\lambda h^{\rho\sigma} \rangle D_{\nu\lambda\rho\sigma} T_\mu$	161		331		$\bar{B}\langle f_{+\mu}{}^{\mu\nu} h^{\lambda\rho} \rangle \sigma_{\mu\lambda} D_\nu T_\rho + \text{H.c.}$	387		764	
$\bar{T}^\mu \langle h^{\nu\lambda} h_{\nu\lambda} \rangle T_\mu$	162		332		$i\bar{B}\langle \nabla^\mu f_{+\mu}{}^\nu \rangle u^\lambda D_\nu T_\lambda + \text{H.c.}$	388		765	
$\bar{T}^\mu u_\mu \nabla^\nu f_{-\nu}{}^\lambda T_\lambda + \text{H.c.}$	163		333		$i\bar{B}\langle \nabla^\mu f_{+\mu}{}^\nu \rangle u^\lambda D_\lambda T_\nu + \text{H.c.}$	389		766	
$\bar{T}^\mu u^\nu \nabla_\mu f_{-\nu}{}^\lambda T_\lambda + \text{H.c.}$	164		334		$i\bar{B}\langle \nabla^\mu f_{+\mu}{}^\nu u^\lambda \rangle D_\nu T_\lambda + \text{H.c.}$	390		767	
$\bar{T}^\mu u^\nu \nabla_\nu f_{-\mu}{}^\lambda T_\lambda + \text{H.c.}$			335		$i\bar{B}\langle \nabla^\mu f_{+\mu}{}^\nu u^\lambda \rangle D_\lambda T_\nu + \text{H.c.}$	391		768	
$\bar{T}^\mu u^\nu \nabla^\lambda f_{-\mu\lambda} T_\nu + \text{H.c.}$			336		$i\bar{B}f_{+\mu}{}^\nu f_{-\mu}{}^\lambda D_\nu T_\lambda + \text{H.c.}$	392		769	
$\bar{T}^\mu u^\nu \nabla^\lambda f_{-\nu\lambda} T_\mu + \text{H.c.}$			337		$i\bar{B}f_{+\mu}{}^\nu f_{-\mu}{}^\lambda D_\lambda T_\nu + \text{H.c.}$	393		770	
$\bar{T}^\mu u_\mu \nabla^\nu f_{-\nu}{}^\lambda D_{\nu\lambda} T_\rho + \text{H.c.}$	165		338		$\bar{B}f_{+\mu}{}^\nu f_{-\mu}{}^\lambda \sigma_{\mu\nu} D_\lambda T_\rho + \text{H.c.}$	394		771	
$\bar{T}^\mu u^\nu \nabla_\mu f_{-\nu}{}^\lambda D_{\nu\lambda} T_\rho + \text{H.c.}$	166		339		$\bar{B}f_{+\mu}{}^\nu f_{-\mu}{}^\lambda \sigma_{\mu\lambda} D_\nu T_\rho + \text{H.c.}$			772	
$\bar{T}^\mu u^\nu \nabla^\lambda f_{-\mu}{}^\rho D_{\nu\lambda} T_\rho + \text{H.c.}$			340		$i\bar{B}f_{+\mu}{}^\nu h_{\mu}{}^\lambda D_\nu T_\lambda + \text{H.c.}$	395		773	
$\bar{T}^\mu u^\nu \nabla^\lambda f_{-\mu}{}^\rho D_{\lambda\rho} T_\nu + \text{H.c.}$			341		$i\bar{B}f_{+\mu}{}^\nu h_{\mu}{}^\lambda D_\lambda T_\nu + \text{H.c.}$	396		774	
$\bar{T}^\mu u^\nu \nabla^\lambda f_{-\nu}{}^\rho D_{\lambda\rho} T_\mu + \text{H.c.}$			342		$i\bar{B}f_{+\mu}{}^\nu h^{\lambda\rho} D_{\mu\lambda\rho} T_\nu + \text{H.c.}$			775	
$\bar{T}^\mu u^\nu \nabla^\lambda f_{-\lambda}{}^\rho D_{\nu\rho} T_\mu + \text{H.c.}$			343		$\bar{B}f_{+\mu}{}^\nu h^{\lambda\rho} \sigma_{\mu\nu} D_\lambda T_\rho + \text{H.c.}$	397		776	
$i\bar{T}^\mu u_\mu \nabla^\nu f_{-\nu}{}^\lambda \sigma_{\nu\lambda} T_\rho + \text{H.c.}$	167		344		$\bar{B}f_{+\mu}{}^\nu h^{\lambda\rho} \sigma_{\mu\lambda} D_\nu T_\rho + \text{H.c.}$	398		777	
$i\bar{T}^\mu u^\nu \nabla_\mu f_{-\nu}{}^\lambda \sigma_{\nu\lambda} T_\rho + \text{H.c.}$			345		$i\bar{B}\nabla^\mu f_{+\mu}{}^\nu u^\lambda D_\nu T_\lambda + \text{H.c.}$	399		778	
$\bar{T}^\mu u^\nu \nabla_\mu h_{\nu}{}^\lambda T_\lambda + \text{H.c.}$			346		$i\bar{B}\nabla^\mu f_{+\mu}{}^\nu u^\lambda D_\lambda T_\nu + \text{H.c.}$	400		779	
$\bar{T}^\mu u_\mu \nabla^\nu h^{\lambda\rho} D_{\nu\lambda} T_\rho + \text{H.c.}$			347		$i\bar{B}u^\mu \nabla_\mu f_{+\mu}{}^\nu D_\nu T_\lambda + \text{H.c.}$			780	
$\bar{T}^\mu u^\nu \nabla_\nu h^{\lambda\rho} D_{\lambda\rho} T_\mu + \text{H.c.}$			348		$i\bar{B}u^\mu \nabla^\nu f_{+\mu}{}^\lambda D_\nu T_\lambda + \text{H.c.}$			781	
$\bar{T}^\mu u^\nu \nabla^\lambda h^{\rho\sigma} D_{\nu\lambda\rho\sigma} T_\mu + \text{H.c.}$			349		$i\bar{B}u^\mu \nabla^\nu f_{+\nu}{}^\lambda D_\mu T_\lambda + \text{H.c.}$			782	
$i\bar{T}^\mu u^\nu \nabla_\mu h^{\lambda\rho} \sigma_{\nu\lambda} T_\rho + \text{H.c.}$	168		350		$i\bar{B}u^\mu \nabla^\nu f_{+\nu}{}^\lambda D_\lambda T_\mu + \text{H.c.}$			783	
$\bar{T}^\mu f_{-\mu}{}^\nu f_{-\nu}{}^\lambda T_\lambda$	169		351		$\bar{B}u^\mu \nabla^\nu f_{+\mu}{}^\lambda \sigma_{\mu\nu} D_\lambda T_\rho + \text{H.c.}$			784	
$\bar{T}^\mu f_{-\nu}{}^\lambda f_{-\mu\nu} T_\lambda$			352		$\bar{B}u^\mu \nabla^\nu f_{+\mu}{}^\lambda \sigma_{\mu\lambda} D_\nu T_\rho + \text{H.c.}$			785	
$\bar{T}^\mu f_{-\nu}{}^\lambda f_{-\nu\lambda} T_\mu$			353		$\bar{B}u^\mu \nabla^\nu f_{+\mu}{}^\lambda \sigma_{\nu\lambda} D_\mu T_\rho + \text{H.c.}$			786	
$\bar{T}^\mu f_{-\nu}{}^\lambda f_{-\nu}{}^\rho D_{\lambda\rho} T_\mu$			354		$i\bar{B}f_{-\mu}{}^\nu f_{+\mu}{}^\lambda D_\nu T_\lambda + \text{H.c.}$			787	
$\bar{T}^\mu h^{\nu\lambda} h_{\nu\lambda} T_\mu$			355		$i\bar{B}f_{-\mu}{}^\nu f_{+\mu}{}^\lambda D_\lambda T_\nu + \text{H.c.}$			788	
$i\bar{T}^\mu \langle f_{+\mu}{}^\nu \rangle \langle u_\nu u^\lambda \rangle T_\lambda + \text{H.c.}$	170		356		$i\bar{B}\langle \nabla^\mu \nabla_\mu f_{+\mu}{}^\nu \rangle \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$			789	
$i\bar{T}^\mu \langle f_{+\mu}{}^\nu \rangle \langle u^\lambda u_\lambda \rangle T_\nu$	171		357		$i\bar{B}\nabla^\mu \nabla_\mu f_{+\mu}{}^\nu \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$	401		790	
$i\bar{T}^\mu \langle f_{+\mu}{}^\nu \rangle \langle u^\lambda u^\rho \rangle D_{\nu\lambda} T_\rho + \text{H.c.}$	172		358		$\bar{B}\langle f_{+\mu}{}^{\mu\nu} \rangle \langle f_{+\mu}{}^\lambda \rangle \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$			791	

(Table continued)

TABLE VIII. (Continued)

O_n/o_n	$SU(2)$	$\tilde{e}_n^{(4)}$	$SU(3)$	$\tilde{E}_n^{(4)}$	O_n	$SU(2)$	$\tilde{e}_n^{(4)}$	$SU(3)$	$\tilde{E}_n^{(4)}$
$i\bar{T}^\mu \langle f_{+\mu}^\nu \rangle \langle u^\lambda u^\rho \rangle D_{\lambda\rho} T_\nu$	173		359		$\bar{B} \langle f_{+\mu}^\nu \rangle \langle f_{+\mu}^{\lambda\rho} \rangle \gamma_5 \gamma_\mu D_{\nu\lambda} T_\rho + \text{H.c.}$				792
$\bar{T}^\mu \langle f_{+\mu}^\nu \rangle \langle u^\lambda u^\rho \rangle \sigma_{\nu\lambda} T_\rho + \text{H.c.}$	174		360		$\bar{B} \langle f_{+\mu}^\nu \rangle f_{+\mu}^\lambda \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$	402			793
$i\bar{T}^\mu \langle f_{+\mu}^\nu \rangle u_\nu u^\lambda T_\lambda + \text{H.c.}$	175		361		$\bar{B} \langle f_{+\mu}^\nu \rangle f_{+\mu}^\lambda \gamma_5 \gamma_\lambda T_\nu + \text{H.c.}$	403			794
$i\bar{T}^\mu \langle f_{+\mu}^\nu \rangle u^\lambda u_\nu T_\lambda + \text{H.c.}$			362		$\bar{B} \langle f_{+\mu}^\nu \rangle f_{+\mu}^{\lambda\rho} \gamma_5 \gamma_\mu D_{\nu\lambda} T_\rho + \text{H.c.}$	404			795
$i\bar{T}^\mu \langle f_{+\mu}^\nu \rangle u^\lambda u_\lambda T_\nu$			363		$\bar{B} \langle f_{+\mu}^\nu \rangle f_{+\mu}^{\lambda\rho} \gamma_5 \gamma_\lambda D_{\mu\rho} T_\nu + \text{H.c.}$	405			796
$i\bar{T}^\mu \langle f_{+\mu}^{\nu\lambda} \rangle u_\nu u_\lambda T_\mu$	176		364		$\bar{B} \langle f_{+\mu}^\nu f_{+\mu}^{\lambda\lambda} \rangle \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$	406			797
$i\bar{T}^\mu \langle f_{+\mu}^\nu \rangle u^\lambda u^\rho D_{\nu\lambda} T_\rho + \text{H.c.}$	177		365		$\bar{B} \langle f_{+\mu}^\nu f_{+\mu}^{\lambda\rho} \rangle \gamma_5 \gamma_\mu D_{\nu\lambda} T_\rho + \text{H.c.}$	407			798
$i\bar{T}^\mu \langle f_{+\mu}^\nu \rangle u^\lambda u^\rho D_{\nu\rho} T_\lambda + \text{H.c.}$			366		$\bar{B} f_{+\mu}^{\mu\nu} f_{+\mu}^{\lambda\lambda} \gamma_5 \gamma_\nu T_\lambda + \text{H.c.}$	408			799
$i\bar{T}^\mu \langle f_{+\mu}^\nu \rangle u^\lambda u^\rho D_{\lambda\rho} T_\nu$			367		$\bar{B} f_{+\mu}^{\mu\nu} f_{+\mu}^{\lambda\lambda} \gamma_5 \gamma_\lambda T_\nu + \text{H.c.}$				800
$i\bar{T}^\mu \langle f_{+\mu}^{\nu\lambda} \rangle u_\nu u^\rho D_{\lambda\rho} T_\mu + \text{H.c.}$	178		368		$\bar{B} f_{+\mu}^{\mu\nu} f_{+\mu}^{\lambda\rho} \gamma_5 \gamma_\mu D_{\nu\lambda} T_\rho + \text{H.c.}$				801
$\bar{T}^\mu \langle f_{+\mu}^\nu \rangle u^\lambda u^\rho \sigma_{\nu\lambda} T_\rho + \text{H.c.}$	179		369		$\bar{B} \langle u^\mu u^\nu \rangle \chi_+ \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$	409			802
$\bar{T}^\mu \langle f_{+\mu}^\nu \rangle u^\lambda u^\rho \sigma_{\nu\rho} T_\lambda + \text{H.c.}$			370		$\bar{B} \langle u^\mu u^\nu \chi_+ \rangle \gamma_5 \gamma_\nu T_\mu + \text{H.c.}$	410			803
$i\bar{T}^\mu \langle f_{+\mu}^\nu u_\nu \rangle u^\lambda T_\lambda + \text{H.c.}$	180		371		$\bar{B} \langle u^\mu u^\nu \chi_+ \rangle \gamma_5 \gamma_\nu T_\mu + \text{H.c.}$	411			804
$i\bar{T}^\mu \langle f_{+\mu}^\nu u^\lambda \rangle u_\nu T_\lambda + \text{H.c.}$	181		372		$\bar{B} \langle u^\mu \chi_+ \rangle u^\nu \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$	412			805
$i\bar{T}^\mu \langle f_{+\mu}^\nu u^\lambda \rangle u_\lambda T_\nu$	182		373		$\bar{B} \langle u^\mu \chi_+ \rangle u^\nu \gamma_5 \gamma_\nu T_\mu + \text{H.c.}$	413			806
$i\bar{T}^\mu \langle f_{+\mu}^\nu u^\lambda \rangle u^\rho D_{\nu\lambda} T_\rho + \text{H.c.}$	183		374		$\bar{B} \langle \chi_+ \rangle \langle u^\mu u^\nu \rangle \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$				807
$i\bar{T}^\mu \langle f_{+\mu}^\nu u^\lambda \rangle u^\rho D_{\nu\rho} T_\lambda + \text{H.c.}$			375		$\bar{B} \langle \chi_+ \rangle u^\mu u^\nu \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$	414			808
$i\bar{T}^\mu \langle f_{+\mu}^\nu u^\lambda \rangle u^\rho D_{\lambda\rho} T_\nu$	184		376		$\bar{B} \langle \chi_+ \rangle u^\mu u^\nu \gamma_5 \gamma_\nu T_\mu + \text{H.c.}$				809
$\bar{T}^\mu \langle f_{+\mu}^\nu u^\lambda \rangle u^\rho \sigma_{\nu\lambda} T_\rho + \text{H.c.}$	185		377		$\bar{B} u^\mu u^\nu \chi_+ \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$				810
$i\bar{T}^\mu \langle f_{+\mu}^\nu u_\nu u^\lambda \rangle T_\lambda + \text{H.c.}$	186		378		$\bar{B} u^\mu u^\nu \chi_+ \gamma_5 \gamma_\nu T_\mu + \text{H.c.}$				811
$i\bar{T}^\mu \langle f_{+\mu}^\nu u^\lambda u_\nu \rangle T_\lambda + \text{H.c.}$			379		$\bar{B} u^\mu \chi_+ u^\nu \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$				812
$i\bar{T}^\mu \langle f_{+\mu}^\nu u^\lambda u_\lambda \rangle T_\nu$			380		$\bar{B} u^\mu \chi_+ u^\nu \gamma_5 \gamma_\nu T_\mu + \text{H.c.}$				813
$i\bar{T}^\mu \langle f_{+\mu}^{\nu\lambda} u_\nu u_\lambda \rangle T_\mu$	187		381		$\bar{B} \chi_+ u^\mu u^\nu \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$				814
$i\bar{T}^\mu \langle f_{+\mu}^\nu u^\lambda u^\rho \rangle D_{\nu\lambda} T_\rho + \text{H.c.}$	188		382		$\bar{B} \langle u^\mu \nabla^\nu \chi_+ \rangle D_\mu T_\nu + \text{H.c.}$	415			815
$i\bar{T}^\mu \langle f_{+\mu}^\nu u^\lambda u^\rho \rangle D_{\nu\rho} T_\lambda + \text{H.c.}$			383		$\bar{B} \langle u^\mu \nabla^\nu \chi_+ \rangle D_\nu T_\mu + \text{H.c.}$	416			816
$i\bar{T}^\mu \langle f_{+\mu}^\nu u^\lambda u^\rho \rangle D_{\lambda\rho} T_\nu$			384		$\bar{B} \langle \chi_+ \rangle f_{-\mu}^{\mu\nu} D_\mu T_\nu + \text{H.c.}$	417			817
$i\bar{T}^\mu \langle f_{+\mu}^{\nu\lambda} u_\nu u^\rho \rangle D_{\lambda\rho} T_\mu + \text{H.c.}$	189		385		$\bar{B} \langle \chi_+ \rangle h^{\mu\nu} D_\mu T_\nu + \text{H.c.}$	418			818
$\bar{T}^\mu \langle f_{+\mu}^\nu u^\lambda u^\rho \rangle \sigma_{\nu\lambda} T_\rho + \text{H.c.}$	190		386		$\bar{B} u^\mu \nabla^\nu \chi_+ D_\mu T_\nu + \text{H.c.}$	419			819
$\bar{T}^\mu \langle f_{+\mu}^\nu u^\lambda u^\rho \rangle \sigma_{\nu\rho} T_\lambda + \text{H.c.}$			387		$\bar{B} u^\mu \nabla^\nu \chi_+ D_\nu T_\mu + \text{H.c.}$	420			820
$i\bar{T}^\mu \langle u_\mu u^\nu \rangle f_{+\nu}^\lambda T_\lambda + \text{H.c.}$	191		388		$\bar{B} \chi_+ f_{-\mu}^{\mu\nu} D_\mu T_\nu + \text{H.c.}$				821
$i\bar{T}^\mu \langle u^\nu u_\nu \rangle f_{+\mu}^\lambda T_\lambda$	192		389		$\bar{B} \chi_+ h^{\mu\nu} D_\mu T_\nu + \text{H.c.}$				822
$i\bar{T}^\mu \langle u_\mu u^\nu \rangle f_{+\mu}^{\lambda\rho} D_{\nu\lambda} T_\rho + \text{H.c.}$	193		390		$i\bar{B} \langle f_{+\mu}^{\mu\nu} \rangle \langle \chi_+ \rangle \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$	421			823
$i\bar{T}^\mu \langle u^\nu u^\lambda \rangle f_{+\mu}^\rho D_{\nu\lambda} T_\rho$	194		391		$i\bar{B} \langle f_{+\mu}^{\mu\nu} \rangle \chi_+ \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$	422			824
$\bar{T}^\mu \langle u_\mu u^\nu \rangle f_{+\mu}^{\lambda\rho} \sigma_{\nu\lambda} T_\rho + \text{H.c.}$	195		392		$i\bar{B} \langle f_{+\mu}^{\mu\nu} \chi_+ \rangle \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$	423			825
$i\bar{T}^\mu f_{+\mu}^\nu u_\nu u^\lambda T_\lambda + \text{H.c.}$			393		$i\bar{B} \langle \chi_+ \rangle f_{+\mu}^{\mu\nu} \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$	424			826
$i\bar{T}^\mu f_{+\mu}^\nu u^\lambda u_\nu T_\lambda + \text{H.c.}$			394		$i\bar{B} f_{+\mu}^{\mu\nu} \chi_+ \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$	425			827
$i\bar{T}^\mu f_{+\mu}^\nu u^\lambda u_\lambda T_\nu + \text{H.c.}$			395		$i\bar{B} \chi_+ f_{+\mu}^{\mu\nu} \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$				828
$i\bar{T}^\mu f_{+\mu}^{\nu\lambda} u_\mu u_\nu T_\lambda + \text{H.c.}$			396		$i\bar{B} \langle u^\mu u^\nu \rangle \chi_- D_\mu T_\nu + \text{H.c.}$	426			829
$i\bar{T}^\mu f_{+\mu}^{\nu\lambda} u_\nu u_\mu T_\lambda + \text{H.c.}$			397		$i\bar{B} \langle u^\mu u^\nu \chi_- \rangle D_\mu T_\nu + \text{H.c.}$	427			830
$i\bar{T}^\mu f_{+\mu}^{\nu\lambda} u_\nu u_\lambda T_\mu + \text{H.c.}$			398		$i\bar{B} \langle u^\mu u^\nu \chi_- \rangle D_\nu T_\mu + \text{H.c.}$	428			831
$i\bar{T}^\mu f_{+\mu}^\nu u^\lambda u^\rho D_{\nu\lambda} T_\rho + \text{H.c.}$			399		$i\bar{B} \langle u^\mu \chi_- \rangle u^\nu D_\mu T_\nu + \text{H.c.}$	429			832
$i\bar{T}^\mu f_{+\mu}^\nu u^\lambda u^\rho D_{\nu\rho} T_\lambda + \text{H.c.}$			400		$i\bar{B} \langle u^\mu \chi_- \rangle u^\nu D_\nu T_\mu + \text{H.c.}$	430			833
$i\bar{T}^\mu f_{+\mu}^\nu u^\lambda u^\rho D_{\lambda\rho} T_\nu + \text{H.c.}$			401		$i\bar{B} \langle \chi_- \rangle \langle u^\mu u^\nu \rangle D_\mu T_\nu + \text{H.c.}$				834
$i\bar{T}^\mu f_{+\mu}^{\nu\lambda} u_\nu u^\rho D_{\lambda\rho} T_\mu + \text{H.c.}$			402		$i\bar{B} \langle \chi_- \rangle u^\mu u^\nu D_\mu T_\nu + \text{H.c.}$	431			835
$i\bar{T}^\mu f_{+\mu}^{\nu\lambda} u_\nu u_\rho D_{\lambda\rho} T_\mu + \text{H.c.}$			403		$i\bar{B} \langle \chi_- \rangle u^\mu u^\nu D_\nu T_\mu + \text{H.c.}$				836
$\bar{T}^\mu f_{+\mu}^\nu u^\lambda u^\rho \sigma_{\nu\lambda} T_\rho + \text{H.c.}$			404		$i\bar{B} u^\mu u^\nu \chi_- D_\mu T_\nu + \text{H.c.}$				837
$\bar{T}^\mu f_{+\mu}^\nu u^\lambda u^\rho \sigma_{\nu\rho} T_\lambda + \text{H.c.}$			405		$i\bar{B} u^\mu u^\nu \chi_- D_\nu T_\mu + \text{H.c.}$				838
$i\bar{T}^\mu u_\mu f_{+\mu}^{\nu\lambda} u_\nu T_\lambda + \text{H.c.}$			406		$i\bar{B} u^\mu \chi_- u^\nu D_\mu T_\nu + \text{H.c.}$				839

(Table continued)

TABLE VIII. (Continued)

O_n/o_n	$SU(2)$	$\tilde{e}_n^{(4)}$	$SU(3)$	$\tilde{E}_n^{(4)}$	O_n	$SU(2)$	$\tilde{e}_n^{(4)}$	$SU(3)$	$\tilde{E}_n^{(4)}$
$i\bar{T}^\mu u^\nu f_{+\nu}{}^\lambda u_\lambda T_\mu$			407		$i\bar{B}u^\mu \chi_- u^\nu D_\nu T_\mu + \text{H.c.}$			840	
$i\bar{T}^\mu u_\mu f_{+}{}^{\nu\lambda} u^\rho D_{\nu\rho} T_\lambda + \text{H.c.}$			408		$i\bar{B}\chi_- u^\mu u^\nu D_\mu T_\nu + \text{H.c.}$			841	
$i\bar{T}^\mu u^\nu f_{+\nu}{}^\lambda u^\rho D_{\lambda\rho} T_\mu + \text{H.c.}$			409		$i\bar{B}\langle u^\mu \nabla^\nu \chi_- \rangle \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$	432		842	
$\bar{T}^\mu u_\mu f_{+}{}^{\nu\lambda} u^\rho \sigma_{\nu\lambda} T_\rho$			410		$i\bar{B}\langle u^\mu \nabla^\nu \chi_- \rangle \gamma_5 \gamma_\mu T_\mu + \text{H.c.}$	433		843	
$i\bar{T}^\mu \langle f_{+\mu}{}^\nu \rangle f_{-\lambda\rho} \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$	196		411		$i\bar{B}\langle f_{-\mu}{}^\nu \chi_- \rangle \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$	434		844	
$i\bar{T}^\mu \langle f_{+\mu}{}^\nu \rangle f_{-\lambda\rho} \gamma_5 \gamma_\lambda D_\nu T_\rho + \text{H.c.}$	197		412		$i\bar{B}\langle h^{\mu\nu} \chi_- \rangle \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$	435		845	
$i\bar{T}^\mu \langle f_{+\mu}{}^\nu \rangle h^{\lambda\rho} \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$	198		413		$i\bar{B}\langle \chi_- \rangle f_{-\mu}{}^\nu \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$	436		846	
$i\bar{T}^\mu \langle f_{+\mu}{}^\nu \rangle h^{\lambda\rho} \gamma_5 \gamma_\lambda D_\nu T_\rho + \text{H.c.}$	199		414		$i\bar{B}\langle \chi_- \rangle h^{\mu\nu} \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$	437		847	
$i\bar{T}^\mu \langle f_{+}{}^{\nu\lambda} \rangle h_\mu{}^\rho \gamma_5 \gamma_\nu D_\lambda T_\rho$	200		415		$i\bar{B}\langle \nabla^\mu \chi_- \rangle u^\nu \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$	438		848	
$i\bar{T}^\mu \langle f_{+}{}^{\nu\lambda} \rangle h_\nu{}^\rho \gamma_5 \gamma_\lambda D_\rho T_\mu$	201		416		$i\bar{B}\langle \nabla^\mu \chi_- \rangle u^\nu \gamma_5 \gamma_\nu T_\mu + \text{H.c.}$	439		849	
$i\bar{T}^\mu \langle f_{+\mu}{}^\nu f_{-\lambda\rho} \rangle \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$	202		417		$i\bar{B}u^\mu \nabla^\nu \chi_- \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$	440		850	
$i\bar{T}^\mu \langle f_{+\mu}{}^\nu f_{-\lambda\rho} \rangle \gamma_5 \gamma_\lambda D_\nu T_\rho + \text{H.c.}$	203		418		$i\bar{B}u^\mu \nabla^\nu \chi_- \gamma_5 \gamma_\mu T_\mu + \text{H.c.}$	441		851	
$i\bar{T}^\mu \langle f_{+\mu}{}^\nu h^{\lambda\rho} \rangle \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$	204		419		$i\bar{B}f_{-\mu}{}^\nu \chi_- \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$	442		852	
$i\bar{T}^\mu \langle f_{+\mu}{}^\nu h^{\lambda\rho} \rangle \gamma_5 \gamma_\lambda D_\nu T_\rho + \text{H.c.}$	205		420		$i\bar{B}h^{\mu\nu} \chi_- \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$	443		853	
$i\bar{T}^\mu \langle f_{+}{}^{\nu\lambda} h_\mu{}^\rho \rangle \gamma_5 \gamma_\nu D_\lambda T_\rho$	206		421		$i\bar{B}\chi_- f_{-\mu}{}^\nu \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$			854	
$i\bar{T}^\mu \langle f_{+}{}^{\nu\lambda} h_\nu{}^\rho \rangle \gamma_5 \gamma_\lambda D_\rho T_\mu$	207		422		$i\bar{B}\chi_- h^{\mu\nu} \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$			855	
$i\bar{T}^\mu \langle \nabla^\nu f_{+\nu}{}^\lambda \rangle u^\rho \gamma_5 \gamma_\rho D_\lambda T_\mu$	208		423		$i\bar{B}\nabla^\mu \chi_- u^\nu \gamma_5 \gamma_\mu T_\nu + \text{H.c.}$			856	
$i\bar{T}^\mu \langle \nabla^\nu f_{+\nu}{}^\lambda u^\rho \rangle \gamma_5 \gamma_\rho D_\lambda T_\mu$	209		424		$i\bar{B}\nabla^\mu \chi_- u^\nu \gamma_5 \gamma_\nu T_\mu + \text{H.c.}$			857	
$i\bar{T}^\mu f_{+\mu}{}^\nu f_{-\lambda\rho} \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$	210		425		$\bar{B}\langle f_{+}{}^{\mu\nu} \rangle \langle \chi_- \rangle D_\mu T_\nu + \text{H.c.}$	444		858	
$i\bar{T}^\mu f_{+\mu}{}^\nu f_{-\lambda\rho} \gamma_5 \gamma_\lambda D_\nu T_\rho + \text{H.c.}$			426		$\bar{B}\langle f_{+}{}^{\mu\nu} \rangle \chi_- D_\mu T_\nu + \text{H.c.}$	445		859	
$i\bar{T}^\mu f_{+}{}^{\nu\lambda} f_{-\mu}{}^\rho \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$	211		427		$\bar{B}\langle f_{+}{}^{\mu\nu} \chi_- \rangle D_\mu T_\nu + \text{H.c.}$	446		860	
$i\bar{T}^\mu f_{+}{}^{\nu\lambda} f_{-\mu}{}^\rho \gamma_5 \gamma_\nu D_\rho T_\lambda + \text{H.c.}$			428		$\bar{B}\langle \chi_- \rangle f_{+}{}^{\mu\nu} D_\mu T_\nu + \text{H.c.}$	447		861	
$i\bar{T}^\mu f_{+\mu}{}^\nu h^{\lambda\rho} \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$	212		429		$\bar{B}f_{+}{}^{\mu\nu} \chi_- D_\mu T_\nu + \text{H.c.}$			862	
$i\bar{T}^\mu f_{+\mu}{}^\nu h^{\lambda\rho} \gamma_5 \gamma_\lambda D_\nu T_\rho + \text{H.c.}$			430		$i\bar{B}\langle D^\mu D_\mu F_L^{\nu\lambda} \rangle \gamma_5 \gamma_\nu T_\lambda + P + C + \text{H.c.}$	448			
$i\bar{T}^\mu f_{+}{}^{\nu\lambda} h_\mu{}^\rho \gamma_5 \gamma_\nu D_\lambda T_\rho + \text{H.c.}$			431		$\bar{B}\langle F_L^{\mu\nu} F_L^{\lambda\rho} \rangle \gamma_5 \gamma_\nu T_\lambda + P + C + \text{H.c.}$	449		863	
$i\bar{T}^\mu f_{+}{}^{\nu\lambda} h_\mu{}^\rho \gamma_5 \gamma_\nu D_\rho T_\lambda + \text{H.c.}$			432		$\bar{B}\langle F_L^{\mu\nu} F_L^{\lambda\rho} \rangle \gamma_5 \gamma_\mu D_{\nu\lambda} T_\rho + P + C + \text{H.c.}$	450		864	
$i\bar{T}^\mu f_{+}{}^{\nu\lambda} h_\nu{}^\rho \gamma_5 \gamma_\lambda D_\rho T_\mu + \text{H.c.}$			433		$\bar{B}\langle F_L^{\mu\nu} \rangle \langle F_L^{\lambda\rho} \rangle \gamma_5 \gamma_\nu T_\lambda + P + C + \text{H.c.}$	451			
$i\bar{T}^\mu \nabla^\nu f_{+\nu}{}^\lambda u^\rho \gamma_5 \gamma_\rho D_\lambda T_\mu + \text{H.c.}$			434		$\bar{B}\langle F_L^{\mu\nu} \rangle \langle F_L^{\lambda\rho} \rangle \gamma_5 \gamma_\mu D_{\nu\lambda} T_\rho + P + C + \text{H.c.}$	452			
$\bar{T}^\mu \langle f_{+\mu}{}^\nu \rangle \langle f_{+\nu}{}^\lambda \rangle T_\lambda$			435						

TABLE IX. Independent Lagrangian terms in the heavy diquark limit at the order $\mathcal{O}(p^4)$. Columns 2 and 5 (3 and 6) label the number for each term in the two-flavor (three-flavor) case. The terms without a number are not independent.

P_n/p_n	$SU(2)$	$SU(3)$	P_n/p_n	$SU(2)$	$SU(3)$
$\bar{\psi}^\mu \langle u^\nu u_\nu \rangle \langle u^\lambda u_\lambda \rangle \psi_\mu$	1	1	$\epsilon^{\mu\nu\lambda\rho} \bar{\psi}^\sigma \langle u_\mu f_{-\nu\lambda} \rangle u_\rho \psi_\sigma$	46	99
$\bar{\psi}^\mu \langle u^\nu u^\lambda \rangle \langle u_\nu u_\lambda \rangle \psi_\mu$	2	2	$\epsilon^{\mu\nu\lambda\rho} \bar{\psi}^\sigma \langle u_\mu f_{-\nu\lambda} \rangle u^\delta v_\rho v_\delta \psi_\sigma$	47	100
$\bar{\psi}^\mu \langle u^\nu u_\nu \rangle \langle u^\lambda u^\rho \rangle v_\lambda v_\rho \psi_\mu$	3	3	$\epsilon^{\mu\nu\lambda\rho} \bar{\psi}^\sigma \langle u_\mu f_{-\nu}{}^\delta \rangle u_\lambda v_\rho v_\delta \psi_\sigma$	48	101
$\bar{\psi}^\mu \langle u^\nu u^\lambda \rangle \langle u_\nu u^\rho \rangle v_\lambda v_\rho \psi_\mu$	4	4	$\bar{\psi}^\mu \langle f_{+}{}^{\nu\lambda} \rangle f_{-\nu}{}^\rho \gamma_5 \gamma_\lambda v_\rho \psi_\mu$	49	
$\bar{\psi}^\mu \langle u^\nu u^\lambda \rangle \langle u^\rho u^\sigma \rangle v_\nu v_\lambda v_\rho v_\sigma \psi_\mu$	5	5	$\bar{\psi}^\mu \langle f_{+}{}^{\nu\lambda} \rangle f_{-\nu}{}^\rho \gamma_5 \gamma_\rho v_\lambda \psi_\mu$	50	
$\bar{\psi}^\mu \langle u^\nu u_\nu \rangle u^\lambda u_\lambda \psi_\mu$		6	$\epsilon^{\mu\nu\lambda\rho} \bar{\psi}^\sigma u_\mu u_\nu f_{-\lambda\rho} \psi_\sigma + \text{H.c.}$		102
$\bar{\psi}^\mu \langle u^\nu u^\lambda \rangle u_\nu u_\lambda \psi_\mu$		7	$\epsilon^{\mu\nu\lambda\rho} \bar{\psi}^\sigma u_\mu u_\nu f_{-\lambda}{}^\delta v_\rho v_\delta \psi_\sigma + \text{H.c.}$		103
$\bar{\psi}^\mu \langle u^\nu u_\nu \rangle u^\lambda u^\rho v_\lambda v_\rho \psi_\mu$		8	$\epsilon^{\mu\nu\lambda\rho} \bar{\psi}^\sigma u_\mu u_\nu f_{-\nu\lambda} v_\rho v_\delta \psi_\sigma + \text{H.c.}$		104
$\bar{\psi}^\mu \langle u^\nu u^\lambda \rangle u_\nu u^\rho v_\lambda v_\rho \psi_\mu + \text{H.c.}$		9	$\bar{\psi}^\mu \langle f_{+}{}^{\nu\lambda} f_{-\nu}{}^\rho \rangle \gamma_5 \gamma_\lambda v_\rho \psi_\mu$	51	105
$\bar{\psi}^\mu \langle u^\nu u^\lambda \rangle u^\rho u_\rho v_\nu v_\lambda \psi_\mu$		10	$\bar{\psi}^\mu \langle f_{+}{}^{\nu\lambda} f_{-\nu}{}^\rho \rangle \gamma_5 \gamma_\rho v_\lambda \psi_\mu$	52	106
$\bar{\psi}^\mu \langle u^\nu u^\lambda \rangle u^\rho u^\sigma v_\nu v_\lambda v_\rho v_\sigma \psi_\mu$		11	$\bar{\psi}^\mu \langle \nabla^\nu f_{+}{}^\lambda \rangle u^\rho \gamma_5 \gamma_\lambda v_\rho \psi_\mu$	53	
$i\bar{\psi}^\mu \langle u^\nu u_\nu \rangle u^\lambda u^\rho \sigma_{\lambda\rho} \psi_\mu$	6	12	$\bar{\psi}^\mu \langle \nabla^\nu f_{+}{}^\lambda \rangle u^\rho \gamma_5 \gamma_\rho v_\lambda \psi_\mu$	54	
$i\bar{\psi}^\mu \langle u^\nu u^\lambda \rangle u_\nu u^\rho \sigma_{\lambda\rho} \psi_\mu + \text{H.c.}$	7	13	$\bar{\psi}^\mu \langle \nabla^\nu f_{+}{}^{\lambda\rho} \rangle u_\nu \gamma_5 \gamma_\lambda v_\rho \psi_\mu$	55	
$i\bar{\psi}^\mu \langle u^\nu u^\lambda \rangle u^\rho u^\sigma \sigma_{\nu\rho} v_\lambda v_\sigma \psi_\mu + \text{H.c.}$	8	14	$\bar{\psi}^\mu \langle \nabla^\nu f_{+}{}^{\lambda\rho} \rangle u_\lambda \gamma_5 \gamma_\nu v_\rho \psi_\mu$	56	
$i\bar{\psi}^\mu \langle u^\nu u^\lambda \rangle u^\rho u^\sigma \sigma_{\rho\sigma} v_\nu v_\lambda \psi_\mu$	9	15	$\bar{\psi}^\mu \langle \nabla^\nu f_{+}{}^{\lambda\rho} \rangle u^\sigma \gamma_5 \gamma_\lambda v_\nu v_\rho v_\sigma \psi_\mu$	57	
$\bar{\psi}^\mu \langle f_{+}{}^{\nu\lambda} \rangle \langle u_\nu u^\rho \rangle \sigma_{\lambda\rho} \psi_\mu$	10		$\bar{\psi}^\mu \langle \nabla^\nu f_{+}{}^{\lambda\rho} \rangle \gamma_5 \gamma_\lambda v_\rho \psi_\mu$	58	107
$\bar{\psi}^\mu \langle f_{+}{}^{\nu\lambda} \rangle \langle u^\rho u_\rho \rangle \sigma_{\nu\lambda} \psi_\mu$	11		$\bar{\psi}^\mu \langle \nabla^\nu f_{+}{}^{\lambda\rho} \rangle \gamma_5 \gamma_\rho v_\lambda \psi_\mu$	59	108
$\bar{\psi}^\mu \langle f_{+}{}^{\nu\lambda} \rangle \langle u^\rho u^\sigma \rangle \sigma_{\nu\lambda} v_\rho v_\sigma \psi_\mu$	12		$\bar{\psi}^\mu \langle \nabla^\nu f_{+}{}^{\lambda\rho} u_\nu \rangle \gamma_5 \gamma_\lambda v_\rho \psi_\mu$	60	109
$\bar{\psi}^\mu \langle f_{+}{}^{\nu\lambda} \rangle \langle u^\rho u^\sigma \rangle \sigma_{\nu\rho} v_\lambda v_\sigma \psi_\mu$	13		$\bar{\psi}^\mu \langle \nabla^\nu f_{+}{}^{\lambda\rho} u_\lambda \rangle \gamma_5 \gamma_\nu v_\rho \psi_\mu$	61	110
$i\bar{\psi}^\mu \langle f_{+}{}^{\nu\lambda} \rangle u_\nu u_\lambda \psi_\mu$	14		$\bar{\psi}^\mu \langle \nabla^\nu f_{+}{}^{\lambda\rho} u^\sigma \rangle \gamma_5 \gamma_\lambda v_\nu v_\rho v_\sigma \psi_\mu$	62	111
$i\bar{\psi}^\mu \langle f_{+}{}^{\nu\lambda} \rangle u_\nu u^\rho v_\lambda v_\rho \psi_\mu + \text{H.c.}$	15		$\bar{\psi}^\mu \langle f_{-}{}^{\nu\lambda} h_{\nu}{}^\rho \rangle v_\lambda v_\rho \psi_\mu$	63	112
$\bar{\psi}^\mu \langle u^\nu u_\nu u^\lambda \rangle u_\lambda \psi_\mu$		16	$\bar{\psi}^\mu f_{+}{}^{\nu\lambda} f_{-\nu}{}^\rho \gamma_5 \gamma_\lambda v_\rho \psi_\mu + \text{H.c.}$		113
$\bar{\psi}^\mu \langle u^\nu u_\nu u^\lambda \rangle u^\rho v_\lambda v_\rho \psi_\mu$		17	$\bar{\psi}^\mu f_{+}{}^{\nu\lambda} f_{-\nu}{}^\rho \gamma_5 \gamma_\rho v_\lambda \psi_\mu + \text{H.c.}$		114
$\bar{\psi}^\mu \langle u^\nu u^\lambda u^\rho \rangle u_\nu v_\lambda v_\rho \psi_\mu$		18	$\bar{\psi}^\mu \nabla^\nu f_{+}{}^{\lambda\rho} u^\rho \gamma_5 \gamma_\lambda v_\rho \psi_\mu + \text{H.c.}$		115
$\bar{\psi}^\mu \langle u^\nu u^\lambda u^\rho \rangle u^\sigma v_\nu v_\lambda v_\rho v_\sigma \psi_\mu$		19	$\bar{\psi}^\mu \nabla^\nu f_{+}{}^{\lambda\rho} u^\rho \gamma_5 \gamma_\rho v_\lambda \psi_\mu + \text{H.c.}$		116
$i\bar{\psi}^\mu \langle u^\nu u^\lambda u^\rho \rangle u_\nu \sigma_{\lambda\rho} \psi_\mu$		20	$\bar{\psi}^\mu \nabla^\nu f_{+}{}^{\lambda\rho} u_\nu \gamma_5 \gamma_\lambda v_\rho \psi_\mu + \text{H.c.}$		117
$i\bar{\psi}^\mu \langle u^\nu u^\lambda u^\rho \rangle u^\sigma \sigma_{\nu\lambda} v_\rho v_\sigma \psi_\mu$		21	$\bar{\psi}^\mu \nabla^\nu f_{+}{}^{\lambda\rho} u_\lambda \gamma_5 \gamma_\nu v_\rho \psi_\mu + \text{H.c.}$		118
$\bar{\psi}^\mu \langle u^\nu u_\nu u^\lambda u^\rho \rangle \psi_\mu$		22	$\bar{\psi}^\mu \nabla^\nu f_{+}{}^{\lambda\rho} u^\sigma \gamma_5 \gamma_\lambda v_\nu v_\rho v_\sigma \psi_\mu + \text{H.c.}$		119
$\bar{\psi}^\mu \langle u^\nu u_\nu u^\lambda u^\rho \rangle v_\lambda v_\rho \psi_\mu$		23	$\bar{\psi}^\mu f_{-}{}^{\nu\lambda} h_{\nu}{}^\rho v_\lambda v_\rho \psi_\mu + \text{H.c.}$		120
$i\bar{\psi}^\mu \langle u^\nu u_\nu u^\lambda u^\rho \rangle \sigma_{\lambda\rho} \psi_\mu$		24	$i\bar{\psi}^\mu f_{-}{}^{\nu\lambda} h_{\nu}{}^\rho \sigma_{\lambda\rho} \psi_\mu + \text{H.c.}$	64	121
$i\bar{\psi}^\mu \langle u^\nu u^\lambda u^\rho u^\sigma \rangle \sigma_{\nu\lambda} v_\rho v_\sigma \psi_\mu$		25	$i\bar{\psi}^\mu f_{-}{}^{\nu\lambda} h^{\rho\sigma} \sigma_{\nu\lambda} v_\rho v_\sigma \psi_\mu + \text{H.c.}$	65	122
$\bar{\psi}^\mu u^\nu u_\nu u^\lambda u_\lambda \psi_\mu$		26	$i\bar{\psi}^\mu f_{-}{}^{\nu\lambda} h^{\rho\sigma} \sigma_{\nu\rho} v_\lambda v_\sigma \psi_\mu + \text{H.c.}$	66	123
$\bar{\psi}^\mu u^\nu u_\nu u^\lambda u^\rho v_\lambda v_\rho \psi_\mu + \text{H.c.}$		27	$i\epsilon^{\mu\nu\lambda\rho} \bar{\psi}^\sigma f_{+\mu\nu} f_{-\lambda\rho} \psi_\sigma + \text{H.c.}$	67	124
$i\bar{\psi}^\mu u^\nu u_\nu u^\lambda u^\rho \sigma_{\lambda\rho} \psi_\mu + \text{H.c.}$		28	$i\epsilon^{\mu\nu\lambda\rho} \bar{\psi}^\sigma f_{+\mu\nu} f_{-\lambda}{}^\delta v_\rho v_\delta \psi_\sigma + \text{H.c.}$	68	125
$i\bar{\psi}^\mu u^\nu u^\lambda u_\nu u^\rho \sigma_{\lambda\rho} \psi_\mu + \text{H.c.}$		29	$i\epsilon^{\mu\nu\lambda\rho} \bar{\psi}^\sigma \nabla_\mu f_{+\nu}{}^\delta u_\lambda v_\rho v_\delta \psi_\sigma + \text{H.c.}$	69	126
$i\bar{\psi}^\mu u^\nu u^\lambda u^\rho u^\sigma \sigma_{\nu\lambda} v_\rho v_\sigma \psi_\mu + \text{H.c.}$		30	$\bar{\psi}^\mu \nabla^\nu \nabla_\nu f_{+}{}^{\lambda\rho} \sigma_{\lambda\rho} \psi_\mu$	70	127
$i\bar{\psi}^\mu u^\nu u^\lambda u^\rho u^\sigma \sigma_{\nu\rho} v_\lambda v_\sigma \psi_\mu + \text{H.c.}$		31	$\bar{\psi}^\mu \nabla^\nu \nabla^\lambda f_{+}{}^\rho \sigma_{\lambda\rho} \psi_\mu$	71	128
$\bar{\psi}^\mu \langle f_{+}{}^{\nu\lambda} u_\nu \rangle u^\rho \sigma_{\lambda\rho} \psi_\mu$	16	32	$\bar{\psi}^\mu \langle u^\nu \nabla^\lambda f_{-\nu\lambda} \rangle \psi_\mu$	72	129
$\bar{\psi}^\mu \langle f_{+}{}^{\nu\lambda} u^\rho \rangle u_\nu \sigma_{\lambda\rho} \psi_\mu$	17	33	$\bar{\psi}^\mu \langle u^\nu \nabla^\lambda f_{-\lambda}{}^\rho \rangle v_\nu v_\rho \psi_\mu$	73	130
$\bar{\psi}^\mu \langle f_{+}{}^{\nu\lambda} u^\rho \rangle u_\rho \sigma_{\nu\lambda} \psi_\mu$	18	34	$\bar{\psi}^\mu u^\nu \nabla^\lambda f_{-\nu\lambda} \psi_\mu + \text{H.c.}$		131
$\bar{\psi}^\mu \langle f_{+}{}^{\nu\lambda} u^\rho \rangle u^\sigma \sigma_{\nu\lambda} v_\rho v_\sigma \psi_\mu$	19	35	$\bar{\psi}^\mu u^\nu \nabla^\lambda f_{-\lambda}{}^\rho v_\nu v_\rho \psi_\mu + \text{H.c.}$		132
$\bar{\psi}^\mu \langle f_{+}{}^{\nu\lambda} u^\rho \rangle u^\sigma \sigma_{\nu\rho} v_\lambda v_\sigma \psi_\mu$	20	36	$i\bar{\psi}^\mu u^\nu \nabla_\nu f_{-}{}^{\lambda\rho} \sigma_{\lambda\rho} \psi_\mu + \text{H.c.}$	74	133
$\bar{\psi}^\mu \langle f_{+}{}^{\nu\lambda} u^\rho \rangle u^\sigma \sigma_{\nu\sigma} v_\lambda v_\rho \psi_\mu$	21	37	$\bar{\psi}^\mu \langle f_{-}{}^{\nu\lambda} f_{-\nu\lambda} \rangle \psi_\mu$	75	134
$i\bar{\psi}^\mu \langle f_{+}{}^{\nu\lambda} u_\nu u_\lambda \rangle \psi_\mu$	22	38	$\bar{\psi}^\mu \langle f_{-}{}^{\nu\lambda} f_{-\nu}{}^\rho \rangle v_\lambda v_\rho \psi_\mu$	76	135
$i\bar{\psi}^\mu \langle f_{+}{}^{\nu\lambda} u_\nu u^\rho \rangle v_\lambda v_\rho \psi_\mu + \text{H.c.}$	23	39	$\bar{\psi}^\mu f_{-}{}^{\nu\lambda} f_{-\nu\lambda} \psi_\mu$		136
$\bar{\psi}^\mu \langle f_{+}{}^{\nu\lambda} u_\nu u^\rho \rangle \sigma_{\lambda\rho} \psi_\mu + \text{H.c.}$		40	$\bar{\psi}^\mu f_{-}{}^{\nu\lambda} f_{-\nu}{}^\rho v_\lambda v_\rho \psi_\mu$		137
$\bar{\psi}^\mu \langle f_{+}{}^{\nu\lambda} u^\rho \rangle \sigma_{\nu\lambda} \psi_\mu$		41	$i\bar{\psi}^\mu f_{-}{}^{\nu\lambda} f_{-\nu}{}^\rho \sigma_{\lambda\rho} \psi_\mu$	77	138
$\bar{\psi}^\mu \langle f_{+}{}^{\nu\lambda} u^\rho u^\sigma \rangle \sigma_{\nu\lambda} v_\rho v_\sigma \psi_\mu$		42	$i\bar{\psi}^\mu f_{-}{}^{\nu\lambda} f_{-\nu}{}^\rho \sigma_{\nu\rho} v_\lambda v_\sigma \psi_\mu$	78	139

(Table continued)

TABLE IX. (Continued)

P_n/p_n	$SU(2)$	$SU(3)$	P_n/p_n	$SU(2)$	$SU(3)$
$\bar{\psi}^\mu \langle f_+^{\nu\lambda} u^\rho u^\sigma \rangle \sigma_{\nu\rho} v_\lambda v_\sigma \psi_\mu + \text{H.c.}$		43	$\bar{\psi}^\mu \langle u^\nu u_\nu \rangle \chi_+ \psi_\mu$	79	140
$\bar{\psi}^\mu \langle u^\nu u_\nu \rangle f_+^{\lambda\rho} \sigma_{\lambda\rho} \psi_\mu$	24	44	$\bar{\psi}^\mu \langle u^\nu u^\lambda \rangle \chi_+ v_\nu v_\lambda \psi_\mu$	80	141
$\bar{\psi}^\mu \langle u^\nu u^\lambda \rangle f_{+\nu}^{\rho\sigma} \sigma_{\lambda\rho} \psi_\mu$	25	45	$\bar{\psi}^\mu \langle u^\nu u_\nu \chi_+ \rangle \psi_\mu$	81	142
$\bar{\psi}^\mu \langle u^\nu u^\lambda \rangle f_+^{\rho\sigma} \sigma_{\nu\rho} v_\lambda v_\sigma \psi_\mu$	26	46	$\bar{\psi}^\mu \langle u^\nu u^\lambda \chi_+ \rangle v_\nu v_\lambda \psi_\mu$	82	143
$\bar{\psi}^\mu \langle u^\nu u^\lambda \rangle f_+^{\rho\sigma} \sigma_{\rho\sigma} v_\nu v_\lambda \psi_\mu$	27	47	$i\bar{\psi}^\mu \langle u^\nu u^\lambda \chi_+ \rangle \sigma_{\nu\lambda} \psi_\mu$	83	144
$i\bar{\psi}^\mu \langle u^\nu u^\lambda h_\nu^\rho \rangle \gamma_5 \gamma_\lambda v_\rho \psi_\mu + \text{H.c.}$	28	48	$\bar{\psi}^\mu \langle u^\nu \chi_+ \rangle u_\nu \psi_\mu$	84	145
$i\bar{\psi}^\mu \langle u^\nu u^\lambda h_\nu^\rho \rangle \gamma_5 \gamma_\rho v_\lambda \psi_\mu + \text{H.c.}$	29	49	$\bar{\psi}^\mu \langle u^\nu \chi_+ \rangle u^\lambda v_\nu v_\lambda \psi_\mu$	85	146
$i\bar{\psi}^\mu \langle u^\nu u^\lambda h^{\rho\sigma} \rangle \gamma_5 \gamma_\nu v_\lambda v_\rho v_\sigma \psi_\mu + \text{H.c.}$	30	50	$\bar{\psi}^\mu \langle \chi_+ \rangle \langle u^\nu u_\nu \rangle \psi_\mu$		147
$i\bar{\psi}^\mu f_+^{\nu\lambda} u_\nu u_\lambda \psi_\mu + \text{H.c.}$		51	$\bar{\psi}^\mu \langle \chi_+ \rangle \langle u^\nu u^\lambda \rangle v_\nu v_\lambda \psi_\mu$		148
$i\bar{\psi}^\mu f_+^{\nu\lambda} u_\nu u^\rho v_\lambda v_\rho \psi_\mu + \text{H.c.}$		52	$\bar{\psi}^\mu \langle \chi_+ \rangle u^\nu u_\nu \psi_\mu$		149
$i\bar{\psi}^\mu f_+^{\nu\lambda} u^\rho u_\nu v_\lambda v_\rho \psi_\mu + \text{H.c.}$		53	$\bar{\psi}^\mu \langle \chi_+ \rangle u^\nu u^\lambda v_\nu v_\lambda \psi_\mu$		150
$\bar{\psi}^\mu f_+^{\nu\lambda} u_\nu u^\rho \sigma_{\lambda\rho} \psi_\mu + \text{H.c.}$		54	$i\bar{\psi}^\mu \langle \chi_+ \rangle u^\nu u^\lambda \sigma_{\nu\lambda} \psi_\mu$	86	151
$\bar{\psi}^\mu f_+^{\nu\lambda} u^\rho u_\nu \sigma_{\lambda\rho} \psi_\mu + \text{H.c.}$		55	$\bar{\psi}^\mu \langle f_+^{\nu\lambda} \rangle \langle \chi_+ \rangle \sigma_{\nu\lambda} \psi_\mu$	87	
$\bar{\psi}^\mu f_+^{\nu\lambda} u^\rho u_\rho \sigma_{\nu\lambda} \psi_\mu + \text{H.c.}$		56	$\bar{\psi}^\mu \langle f_+^{\nu\lambda} \rangle \chi_+ \sigma_{\nu\lambda} \psi_\mu$	88	
$\bar{\psi}^\mu f_+^{\nu\lambda} u^\rho u^\sigma \sigma_{\nu\lambda} v_\rho v_\sigma \psi_\mu + \text{H.c.}$		57	$\bar{\psi}^\mu u^\nu u_\nu \chi_+ \psi_\mu + \text{H.c.}$		152
$\bar{\psi}^\mu f_+^{\nu\lambda} u^\rho u^\sigma \sigma_{\nu\rho} v_\lambda v_\sigma \psi_\mu + \text{H.c.}$		58	$\bar{\psi}^\mu u^\nu u^\lambda \chi_+ v_\nu v_\lambda \psi_\mu + \text{H.c.}$		153
$\bar{\psi}^\mu f_+^{\nu\lambda} u^\rho u^\sigma \sigma_{\nu\sigma} v_\lambda v_\rho \psi_\mu + \text{H.c.}$		59	$i\bar{\psi}^\mu u^\nu u^\lambda \chi_+ \sigma_{\nu\lambda} \psi_\mu + \text{H.c.}$		154
$i\bar{\psi}^\mu u^\nu f_{+\nu}^{\lambda\rho} u_\lambda \psi_\mu$		60	$i\bar{\psi}^\mu u^\nu \chi_+ u^\lambda \sigma_{\nu\lambda} \psi_\mu$		155
$i\bar{\psi}^\mu u^\nu f_{+\nu}^{\lambda\rho} v_\lambda v_\rho \psi_\mu + \text{H.c.}$		61	$\bar{\psi}^\mu \langle f_+^{\nu\lambda} \chi_+ \rangle \sigma_{\nu\lambda} \psi_\mu$	89	156
$i\bar{\psi}^\mu u^\nu u_\nu h^{\lambda\rho} \gamma_5 \gamma_\lambda v_\rho \psi_\mu + \text{H.c.}$		62	$\bar{\psi}^\mu \langle \chi_+ \rangle f_+^{\nu\lambda} \sigma_{\nu\lambda} \psi_\mu$	90	157
$i\bar{\psi}^\mu u^\nu u^\lambda h_\nu^\rho \gamma_5 \gamma_\lambda v_\rho \psi_\mu + \text{H.c.}$		63	$\bar{\psi}^\mu f_+^{\nu\lambda} \chi_+ \sigma_{\nu\lambda} \psi_\mu + \text{H.c.}$		158
$i\bar{\psi}^\mu u^\nu u^\lambda h_\nu^\rho \gamma_5 \gamma_\rho v_\lambda \psi_\mu + \text{H.c.}$		64	$i\bar{\psi}^\mu u^\nu \nabla^\lambda \chi_+ \gamma_5 \gamma_\nu v_\lambda \psi_\mu + \text{H.c.}$	91	159
$i\bar{\psi}^\mu u^\nu u^\lambda h_\lambda^\rho \gamma_5 \gamma_\nu v_\rho \psi_\mu + \text{H.c.}$		65	$i\bar{\psi}^\mu u^\nu \nabla^\lambda \chi_+ \gamma_5 \gamma_\lambda v_\nu \psi_\mu + \text{H.c.}$	92	160
$i\bar{\psi}^\mu u^\nu u^\lambda h_\lambda^\rho \gamma_5 \gamma_\rho v_\nu \psi_\mu + \text{H.c.}$		66	$i\bar{\psi}^\mu f_-^{\nu\lambda} \chi_+ \gamma_5 \gamma_\nu v_\lambda \psi_\mu + \text{H.c.}$	93	161
$i\bar{\psi}^\mu u^\nu u^\lambda h^{\rho\sigma} \gamma_5 \gamma_\nu v_\lambda v_\rho v_\sigma \psi_\mu + \text{H.c.}$		67	$\bar{\psi}^\mu \langle \nabla^\nu \nabla_\nu \chi_+ \rangle \psi_\mu$	94	162
$i\bar{\psi}^\mu u^\nu u^\lambda h^{\rho\sigma} \gamma_5 \gamma_\lambda v_\nu v_\rho v_\sigma \psi_\mu + \text{H.c.}$		68	$\bar{\psi}^\mu \nabla^\nu \nabla_\nu \chi_+ \psi_\mu$	95	163
$i\bar{\psi}^\mu u^\nu u^\lambda h^{\rho\sigma} \gamma_5 \gamma_\rho v_\nu v_\lambda v_\sigma \psi_\mu + \text{H.c.}$		69	$\bar{\psi}^\mu \langle \chi_+ \rangle \langle \chi_+ \rangle \psi_\mu$	96	164
$i\bar{\psi}^\mu u^\nu h_\nu^\lambda u^\rho \gamma_5 \gamma_\lambda v_\rho \psi_\mu + \text{H.c.}$		70	$\bar{\psi}^\mu \langle \chi_+ \rangle \chi_+ \psi_\mu$	97	165
$\epsilon^{\mu\nu\lambda\rho} \bar{\psi}^\sigma \langle u_\mu h_\nu^\delta \rangle u_\lambda v_\rho v_\delta \psi_\sigma$	31	71	$\bar{\psi}^\mu \langle \chi_+^2 \rangle \psi_\mu$		166
$\bar{\psi}^\mu \langle f_+^{\nu\lambda} \rangle f_{+\nu}^{\rho\sigma} \psi_\mu$	32		$\bar{\psi}^\mu \chi_+^2 \psi_\mu$		167
$\bar{\psi}^\mu \langle f_+^{\nu\lambda} \rangle f_{+\nu}^{\rho\sigma} v_\lambda v_\rho \psi_\mu$	33		$\bar{\psi}^\mu \langle u^\nu u^\lambda \chi_- \rangle \gamma_5 \gamma_\nu v_\lambda \psi_\mu + \text{H.c.}$	98	168
$\bar{\psi}^\mu \langle f_+^{\nu\lambda} \rangle h_\nu^\rho \gamma_5 \gamma_\rho v_\lambda \psi_\mu$	34		$\bar{\psi}^\mu \langle \chi_- \rangle u^\nu u^\lambda \gamma_5 \gamma_\nu v_\lambda \psi_\mu + \text{H.c.}$	99	169
$\epsilon^{\mu\nu\lambda\rho} \bar{\psi}^\sigma u_\mu u_\nu h_\lambda^\delta v_\rho v_\delta \psi_\sigma + \text{H.c.}$		72	$i\bar{\psi}^\mu \langle f_+^{\nu\lambda} \rangle \langle \chi_- \rangle \gamma_5 \gamma_\nu v_\lambda \psi_\mu$	100	
$\bar{\psi}^\mu \langle f_+^{\nu\lambda} h_\nu^\rho \rangle \gamma_5 \gamma_\rho v_\lambda \psi_\mu$	35	73	$i\bar{\psi}^\mu \langle f_+^{\nu\lambda} \rangle \chi_- \gamma_5 \gamma_\nu v_\lambda \psi_\mu$	101	
$\bar{\psi}^\mu \langle h^{\nu\lambda} h_{\nu\lambda} \rangle \psi_\mu$	36	74	$\bar{\psi}^\mu u^\nu u^\lambda \chi_- \gamma_5 \gamma_\nu v_\lambda \psi_\mu + \text{H.c.}$		170
$\bar{\psi}^\mu \langle h^{\nu\lambda} h_\nu^\rho \rangle v_\lambda v_\rho \psi_\mu$	37	75	$\bar{\psi}^\mu u^\nu u^\lambda \chi_- \gamma_5 \gamma_\lambda v_\nu \psi_\mu + \text{H.c.}$		171
$\bar{\psi}^\mu \langle h^{\nu\lambda} h^{\rho\sigma} \rangle v_\nu v_\lambda v_\rho v_\sigma \psi_\mu$	38	76	$\bar{\psi}^\mu u^\nu \chi_- u^\lambda \gamma_5 \gamma_\nu v_\lambda \psi_\mu + \text{H.c.}$		172
$\bar{\psi}^\mu f_+^{\nu\lambda} f_{+\nu}^{\rho\sigma} \psi_\mu$		77	$i\bar{\psi}^\mu \langle f_+^{\nu\lambda} \chi_- \rangle \gamma_5 \gamma_\nu v_\lambda \psi_\mu$	102	173
$\bar{\psi}^\mu f_+^{\nu\lambda} f_{+\nu}^{\rho\sigma} v_\lambda v_\rho \psi_\mu$		78	$i\bar{\psi}^\mu \langle u^\nu \nabla_\nu \chi_- \rangle \psi_\mu$	103	174
$i\bar{\psi}^\mu f_+^{\nu\lambda} f_{+\nu}^{\rho\sigma} \sigma_{\lambda\rho} \psi_\mu$	39	79	$i\bar{\psi}^\mu \langle u^\nu \nabla^\lambda \chi_- \rangle v_\nu v_\lambda \psi_\mu$	104	175
$i\bar{\psi}^\mu f_+^{\nu\lambda} f_{+\nu}^{\rho\sigma} \sigma_{\nu\rho} v_\lambda v_\sigma \psi_\mu$	40	80	$i\bar{\psi}^\mu \langle \chi_- \rangle f_+^{\nu\lambda} \gamma_5 \gamma_\nu v_\lambda \psi_\mu$	105	176
$\bar{\psi}^\mu f_+^{\nu\lambda} h_\nu^\rho \gamma_5 \gamma_\rho v_\lambda \psi_\mu + \text{H.c.}$		81	$i\bar{\psi}^\mu \langle \chi_- \rangle h^{\nu\lambda} v_\nu v_\lambda \psi_\mu$	106	177
$\bar{\psi}^\mu h^{\nu\lambda} h_{\nu\lambda} \psi_\mu$		82	$i\bar{\psi}^\mu \langle \nabla^\nu \chi_- \rangle u_\nu \psi_\mu$	107	178
$\bar{\psi}^\mu h^{\nu\lambda} h_\nu^\rho v_\lambda v_\rho \psi_\mu$		83	$i\bar{\psi}^\mu f_+^{\nu\lambda} \chi_- \gamma_5 \gamma_\nu v_\lambda \psi_\mu + \text{H.c.}$		179
$\bar{\psi}^\mu h^{\nu\lambda} h^{\rho\sigma} v_\nu v_\lambda v_\rho v_\sigma \psi_\mu$		84	$i\bar{\psi}^\mu u^\nu \nabla_\nu \chi_- \psi_\mu + \text{H.c.}$		180
$i\bar{\psi}^\mu h^{\nu\lambda} h_\nu^\rho \sigma_{\lambda\rho} \psi_\mu$	41	85	$i\bar{\psi}^\mu u^\nu \nabla^\lambda \chi_- v_\nu v_\lambda \psi_\mu + \text{H.c.}$		181
$i\bar{\psi}^\mu h^{\nu\lambda} h^{\rho\sigma} \sigma_{\nu\rho} v_\lambda v_\sigma \psi_\mu$	42	86	$\bar{\psi}^\mu u^\nu \nabla^\lambda \chi_- \sigma_{\nu\lambda} \psi_\mu + \text{H.c.}$	108	182
$i\bar{\psi}^\mu \langle u^\nu u^\lambda f_{-\nu}^\rho \rangle \gamma_5 \gamma_\lambda v_\rho \psi_\mu + \text{H.c.}$	43	87	$\bar{\psi}^\mu f_-^{\nu\lambda} \chi_- \sigma_{\nu\lambda} \psi_\mu + \text{H.c.}$	109	183
$i\bar{\psi}^\mu \langle u^\nu u^\lambda f_{-\nu}^\rho \rangle \gamma_5 \gamma_\rho v_\lambda \psi_\mu + \text{H.c.}$	44	88	$\bar{\psi}^\mu \langle \chi_- \rangle \langle \chi_- \rangle \psi_\mu$	110	184

(Table continued)

TABLE IX. (Continued)

P_n/p_n	$SU(2)$	$SU(3)$	P_n/p_n	$SU(2)$	$SU(3)$
$i\bar{\psi}^\mu u^\nu u_\nu f_{-\lambda}^{\rho} \gamma_5 \gamma_\lambda v_\rho \psi_\mu + \text{H.c.}$		89	$\bar{\psi}^\mu \langle \chi_- \rangle \chi_- \psi_\mu$	111	185
$i\bar{\psi}^\mu u^\nu u^\lambda f_{-\nu}^{\rho} \gamma_5 \gamma_\lambda v_\rho \psi_\mu + \text{H.c.}$		90	$\bar{\psi}^\mu \langle D^\nu D_\nu F_L^{\lambda\rho} \rangle \sigma_{\lambda\rho} \psi_\mu + P$	112	
$i\bar{\psi}^\mu u^\nu u^\lambda f_{-\nu}^{\rho} \gamma_5 \gamma_\rho v_\lambda \psi_\mu + \text{H.c.}$		91	$\bar{\psi}^\mu \chi^2 \psi_\mu$		186
$i\bar{\psi}^\mu u^\nu u^\lambda f_{-\lambda}^{\rho} \gamma_5 \gamma_\nu v_\rho \psi_\mu + \text{H.c.}$		92	$\bar{\psi}^\mu \langle F_L^{\nu\lambda} F_{L\nu\lambda} \rangle \psi_\mu + P$	113	187
$i\bar{\psi}^\mu u^\nu u^\lambda f_{-\lambda}^{\rho} \gamma_5 \gamma_\rho v_\nu \psi_\mu + \text{H.c.}$		93	$\bar{\psi}^\mu \langle F_L^{\nu\lambda} F_{L\nu}^{\rho} \rangle v_\lambda v_\rho \psi_\mu + P$	114	188
$i\bar{\psi}^\mu u^\nu u^\lambda f_{-\rho}^{\sigma} \gamma_5 \gamma_\rho v_\nu v_\lambda v_\sigma \psi_\mu + \text{H.c.}$		94	$\bar{\psi}^\mu \langle F_L^{\nu\lambda} \rangle \langle F_{L\nu\lambda} \rangle \psi_\mu + P$	115	
$i\bar{\psi}^\mu u^\nu f_{-\nu}^{\lambda} u^\rho \gamma_5 \gamma_\lambda v_\rho \psi_\mu + \text{H.c.}$		95	$\bar{\psi}^\mu \langle F_L^{\nu\lambda} \rangle \langle F_{L\nu}^{\rho} \rangle v_\lambda v_\rho \psi_\mu + P$	116	
$i\bar{\psi}^\mu u^\nu f_{-\nu}^{\lambda} u^\rho \gamma_5 \gamma_\rho v_\lambda \psi_\mu + \text{H.c.}$		96	$\bar{\psi}^\mu \langle \chi \chi^\dagger \rangle \psi_\mu$	117	189
$\epsilon^{\mu\nu\lambda\rho} \bar{\psi}^\sigma \langle u_\mu u^\delta \rangle f_{-\nu\lambda}^{\rho} v_\rho v_\delta \psi_\sigma$	45	97	$\bar{\psi}^\mu \det \chi \psi_\mu + \text{H.c.}$	118	
$\epsilon^{\mu\nu\lambda\rho} \bar{\psi}^\sigma \langle u_\mu u^\delta f_{-\nu\lambda}^{\rho} \rangle v_\rho v_\delta \psi_\sigma + \text{H.c.}$		98			

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