Theoretical investigation of the molecular nature of $D_{s0}^{*}(2317)$ and $D_{s1}(2460)$ and the possibility of observing the $D\bar{D}K$ bound state $K_{c\bar{c}}(4180)$ in inclusive $e^{+}e^{-} \to c\bar{c}$ collisions

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Searching for exotic multiquark states and elucidating their nature remains a central topic in understanding quantum chromodynamics–the underlying theory of the strong interaction. Two of the most studied such states are the charm-strange states $D_{s0}^{*}(2317)$ and $D_{s1}(2460)$. In this paper, we show for the first time that their prompt production yields in inclusive $e^{+}e^{-} \to c\bar{c}$ collisions near $\sqrt{s} = 10.6$ GeV measured by the *BABAR* collaboration, $Y(D_{s0}^{*}(2317))$ and $Y(D_{s1}(2460))$, in particular the ratio $R = Y(D_{s0}^{*}(2317))/Y(D_{s1}(2460))$, can be well explained in the molecular picture, which provide a highly nontrivial verification of their nature being $DK/D^{*}K$ molecules. On the contrary, treating them as pure $c\bar{s}$ P-wave states, the statistical model predicts a ratio R smaller than unity, in contrast with the experimental central value, though in agreement with it considering its relatively large uncertainty. In addition, we predict the production yield of the $D\bar{D}K$ three-body bound state, $K_{c\bar{c}}(4180)$, in $e^{+}e^{-} \to c\bar{c}$ collisions and find that it is within the reach of the ongoing Belle II experiment. The present study demonstrates the feasibility of a novel method to unravel the nature of exotic hadrons and the potential of electron-positron collisions in this regard.

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I. INTRODUCTION

In 2003, two exotic hadrons were discovered, i.e., $X(3872)$ [1] and $D_{s0}^{*}(2317)$ [2], which cannot be easily accommodated in the conventional quark model of Gell-Mann and Zweig [3,4] where baryons and mesons are viewed as three quark and quark-antiquark color singlets, respectively. In the following years, more such hadrons were discovered [5–10], some of which, because of their proximity to the mass thresholds of pairs of conventional hadrons, are often interpreted as hadronic molecules [5], i.e., multihadron states bound by the residual strong nuclear force instead of the electromagnetic interaction. In particular, the $D_{s0}^{*}(2317)$ [2,11,12] and its heavy-spin symmetry partner $D_{s1}(2460)$ [11,12], with masses lower by 160 and 70 MeV than their counterparts in the Godfrey-Isgur (GI) quark model [13], are shown to qualify as bound states of DK and $D^{*}K$ [14–26]. It should be mentioned although the molecular picture for $D_{s0}^{*}(2317)/D_{s1}(2460)$ is the prevailing one, alternative interpretations do exist, such as conventional $q\bar{q}$ states [27,28] or compact tetraquark states [29] or mixtures of them [30].

By analogy with the only well-established "hardronic molecules" in nature, i.e., atomic nuclei and hypernuclei, it was realized recently that one can build heavier three-body even four-body systems starting from the fact that the $DK/D^{*}K$ interactions are attractive and strong enough to form the $D_{s0}^{*}(2317)$ and $D_{s1}(2460)$ states [31–33]. Note that this picture has attracted considerable attention and many similar studies have been performed and several good candidates have been identified [34–42]. The next step forward is to search for their existence experimentally. Recently, the Belle Collaboration has performed the first dedicated search for the predicted DDK bound state in

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e^+e^- collisions [43]. Further theoretical studies are in urgent need to verify the above conjecture [31–42] and guide future experiments.

The nucleus(molecule)-like nature of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ suggests that one should be able to explain their prompt production yields using the coalescence mechanism, which is known to work well in reproducing the productions of light (anti)nuclei and hyper nuclei in heavy-ion and hadron-hadron collisions [44–49]. In the coalescence mechanism, composite particles are formed by coalescence of their constituents that satisfy the constraints in phase space at kinetic freeze-out. From such a perspective, the production yields of exotic hadrons in hadron-hadron and heavy-ion collisions have been studied [50–57], but similar studies in electron-positron collisions are much rare. In particular, a dedicated study of the yields of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ in e^+e^- collisions, where they were first observed, is still missing. In this work, we fill this gap and study the prompt productions of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ in e^+e^- collisions. We find that the results are in nice agreement with the *BABAR* data [58] and therefore provide a highly nontrivial support for their molecular nature. Built on this success, we further predict the production yield of the three-body $D\bar{D}K$ bound state, which is formed by the same $DK(\bar{D}K)$ interaction that binds $D_{s0}^*(2317)$, and find that it is within the reach of the ongoing Belle II experiment, which, if found in the future, will not only help further confirm the molecular nature of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ but also open a new chapter in studies of the nonperturbative strong interaction.

II. THEORETICAL FRAMEWORK

We adopt the time-honored coalescence model [59] to study the production of $DK/D^*K/D\bar{D}K$ molecules. There are two essential ingredients in such a study, i.e., the production of primary hadrons and the coalescence process. For the former, a transport model is used to provide the phase space information of the particle source containing the primary hadrons (D, D^*, $\bar{D}$, and K) of our interest at the kinetic freeze-out. For the latter, the Wigner function method is adopted.

For the transport process, we adopt the PACIAE model [52,60,61], which is a transport model based on the event generator PYTHIA [62]. Once the primary hadrons are produced, we apply the coalescence model to study the formation of hadronic molecules. The coalescence model is widely used to calculate the production rates of composite particles such as nuclear clusters and hadronic molecules [44,45,50,63–66]. The basic idea is that the constituents of a shallow bound composite particle, whose binding energy is small compared to the evolution temperature, only combine together until the whole system reaches the kinetic freeze-out.

The formation of clusters can be described in the final-state interaction approximation [67] (see the Supplemental Material for details [68]), indicating that it only occurs when the interactions between their constituents almost cease and the formation time is short compared to their interaction time. Then the production yield is the overlap integral between the cluster density $\hat{\rho}_C$ and the final-state source density $\hat{\rho}_S$. Both densities need to be transformed into the Wigner densities $\hat{\rho}_S^W$ and $\hat{\rho}_C^W$ since the source density obtained from the transport model is semiclassical. The source Wigner density $\hat{\rho}_S^W$ can be constructed from the positions $\tilde{\boldsymbol{x}}_n$ and momenta $\tilde{\boldsymbol{p}}_n$ of the primary particles after the kinetic freeze-out [45], to which each particle n contributes a product of delta functions $\delta^3(\boldsymbol{x}_n-\tilde{\boldsymbol{x}}_n)\delta^3(\boldsymbol{p}_n-\tilde{\boldsymbol{p}}_n)$. The cluster Wigner density $\hat{\rho}_C^W$ can be obtained by the following Wigner transformation

$$\begin{aligned}\hat{\rho}_C^W(\boldsymbol{r}_1,\boldsymbol{q}_1,\ldots,\boldsymbol{r}_{n-1},\boldsymbol{q}_{n-1}) &= \int \Psi_C\left(\boldsymbol{r}_1+\frac{1}{2}\boldsymbol{y}_1,\ldots,\boldsymbol{r}_{n-1}+\frac{1}{2}\boldsymbol{y}_{n-1}\right)\\ &\quad\times\Psi_C^*\left(\boldsymbol{r}_1-\frac{1}{2}\boldsymbol{y}_1,\ldots,\boldsymbol{r}_{n-1}-\frac{1}{2}\boldsymbol{y}_{n-1}\right)\\ &\quad\times e^{-i\boldsymbol{q}_1\cdot\boldsymbol{y}_1}\cdots e^{-i\boldsymbol{q}_{n-1}\cdot\boldsymbol{y}_{n-1}}d^3\boldsymbol{y}_1\cdots d^3\boldsymbol{y}_{n-1},\end{aligned}\tag{1}$$

where Ψ_C is the relative wave function of the n-body cluster defined in its c.m. system, $\boldsymbol{r}_{n-1}$ and $\boldsymbol{q}_{n-1}$ are the $n-1$ relative coordinates calculated from space and momentum coordinates $\boldsymbol{x}_n$ and $\boldsymbol{p}_n$. Then one obtains the yield of the n-body cluster [44,45]

$$\begin{aligned}\frac{dN}{d\boldsymbol{P}} &= g\int\hat{\rho}_S^W(\boldsymbol{x}_1,\boldsymbol{p}_1,\ldots,\boldsymbol{x}_n,\boldsymbol{p}_n)\hat{\rho}_C^W(\boldsymbol{r}_1,\boldsymbol{q}_1,\ldots,\boldsymbol{r}_{n-1},\boldsymbol{q}_{n-1})\\ &\quad\times\delta^3(\boldsymbol{P}-(\boldsymbol{p}_1+\cdots\boldsymbol{p}_n))\frac{d\boldsymbol{x}_1d\boldsymbol{p}_1}{(2\pi)^3}\cdots\frac{d\boldsymbol{x}_nd\boldsymbol{p}_n}{(2\pi)^3},\\ N &= g\left\langle\sum_{(\mathrm{c})}\hat{\rho}_C^W(\tilde{\boldsymbol{r}}_1,\tilde{\boldsymbol{q}}_1,\ldots,\tilde{\boldsymbol{r}}_{n-1},\tilde{\boldsymbol{q}}_{n-1})\right\rangle,\end{aligned}\tag{2}$$

where $\boldsymbol{P}$ is the total momentum of the cluster, $\tilde{\boldsymbol{r}}_n$ and $\tilde{\boldsymbol{q}}_n$ are the relative space and momentum coordinates calculated with the space coordinate $\tilde{\boldsymbol{x}}_n$ and momentum coordinate $\tilde{\boldsymbol{p}}_n$ of the primary hadrons in each combination (c) from the transport model, and $\langle\cdots\rangle$ denotes that the result is averaged over all the event runs. We note that the spin statistical factor g for DK, D^*K, and $D\bar{D}K$ is 1.

To compute the cluster Wigner density $\hat{\rho}_C^W$, we need the wave functions of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ as DK and D^*K bound states, which have been studied thoroughly in the chiral unitary approaches at leading order [73–75], next-to-leading order [22,76,77] and next-to-next-to-leading order [78–80]. Here we follow Refs. [31,34] and adopt the wave functions in coordinate space obtained in the Gaussian expansion method. The radial part of the relative wave function can be expanded as a sum of Gaussian functions,

$$\Psi_{D^{(*)}K}(\boldsymbol{r}) = \sum_{i=1}^{N} c_i \left(\frac{2\omega_i}{\pi}\right)^{3/4} e^{-\omega_i r^2}, \qquad (3)$$

where ω_i is the width parameter of each basis, c_i is the coefficient, and N is the number of Gaussian functions. With these wave functions, one can easily obtain the Wigner densities and yields for the $D^{(*)}K$ bound states. We stress that the binding energies of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ as DK and D^*K bound states alone cannot uniquely determine their wave functions. Therefore, following Refs. [31], we employ three representative wave functions from differently regularized potentials (see the Supplemental Material [68] for more details) and study the corresponding impact on the production yields.

Following the same technique, one can calculate the production yield of the $D\bar{D}K$ state predicted in Ref. [34], later confirmed in Ref. [81]. In Ref. [34], the relative wave function of $D\bar{D}K$ is written in terms of three Jacobi coordinates, as shown in Fig. 1, which reads [34]

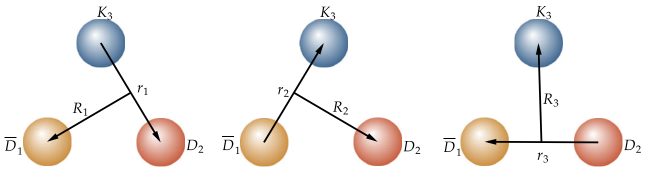
$$\Psi_{D\bar{D}K} = \sum_{\mathrm{ch_i}=1}^{3} \sum_{i,j=1}^{N} c_{i,j}^{(\mathrm{ch_i})} \left(\frac{4\omega_i\omega_j}{\pi^2}\right)^{3/4} \times \exp(-\omega_i \boldsymbol{r}_{\mathrm{ch_i}}^2) \exp(-\omega_j \boldsymbol{R}_{\mathrm{ch_i}}^2), \qquad (4)$$

where $\mathrm{ch_i}$ is the label of the three Jacobi channels shown in Fig. 1. Using this wave function, with a bit of algebra, one can obtain the corresponding Wigner density and yield.

It is also instructive to study the production yields of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ assuming that they are conventional $c\bar{s}$ P-wave states. This can be done in the statistical model [82]. The details of the statistical model can be found in Ref. [82] and those relevant to the present work are given in the Supplemental Material [68].

III. RESULTS AND DISCUSSION

First, we estimate the production yields of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ treated as hadronic molecules of DK and D^*K in our transport plus coalescence model. Since $D_{s0}^*(2317)$ and $D_{s1}(2460)$ were observed in inclusive $e^+e^- \to c\bar{c}$ collisions at a c.m. energy around $\Upsilon(4S)$ [58], we use the $e^+e^- \to c\bar{c}$ mode in PACIAE to simulate this process. All the parameters in this mode are fixed at their default values, except for parj(13), the probability that a charm or heavier meson has spin 1. It is set at 0.54 according to the measured ratios of D^+ and D^0, D^{*+} and D^{*0}, D^+ and D^{*+}, D^0 and D^{*0} [83], instead of its default value of 0.75. The details of the simulation can be found in the Supplemental Material [68]. The resulting yields of primary hadrons are found in reasonable agreement with the experimental data as shown in Table I. We stress that this level of agreement with the data (about 10%) is enough for our purpose of estimating the production yields of $D_{s0}(2317)$, $D_{s1}(2460)$, and $K_{c\bar{c}}(4180)$ and therefore we do not further fine-tune the PACIAE parameters.



channel 1 channel 2 channel 3

FIG. 1. Jacobi coordinates of the $D\bar{D}K$ system.

TABLE I. Experimental and simulated yields (per event) of primary hadrons in e^+e^- annihilations. D mesons are measured near $\sqrt{s} = 10.5$ GeV and simulated at $\sqrt{s} = 10.52$ GeV in the $e^+e^- \to c\bar{c}$ process.

Particle	Data [83]	PACIAE results
D^+	0.2639 ± 0.0139	0.2386
D^0	0.5772 ± 0.0241	0.5276
D^{*+}	0.2470 ± 0.0137	0.2100
D^{*0}	0.2241 ± 0.0304	0.2026

From the produced primary hadrons, we can calculate the production yields of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ using the wave functions given in Eq. (3) with the Wigner function approach. The predicted yields are given in Table II in comparison with the *BABAR* measurements, where Cases 1, 2, and 3 correspond to the results obtained with the three wave functions for different interaction ranges of 1, 2, and 3 fm (see the Supplemental Material [68] for details). The uncertainties in Table II are obtained by varying the simulation parameter $\mathrm{r_p}$ from its reference value of 1.16[1] by 20%. This parameter sets the radius of the sphere centered at the position of a parent particle, where the daughter particles are located, and which affects the dispersion of final states in phase space and controls the hadron rescattering effect considered in the PACIAE model. As a result, it can affect the production yields of composite particles, such as those studied here. We find again very reasonable agreement between the theoretical yields and the experimental measurements. In particular, the ratio $R = Y_{D_{s0}^*(2317)}/Y_{D_{s1}(2460)}$ is found to be about 1.5, also in reasonable agreement with data. The agreements in terms of both the absolute production yields and relative ratio provide a highly nontrivial support for the molecular nature of $D_{s0}^*(2317)$ and $D_{s1}(2460)$. We stress that it is the first time that the experimental measurements have been reproduced. In addition, we note that the production yields

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[1] This value reflects the size of a typical hadron, such as that of the nucleon, which is about 1 fm, and it was determined by reproducing the yields of light (anti)nuclei in pp collisions [61].

TABLE II. Yields (per $e^+e^- \to c\bar{c}$ event, containing charge conjugated states) of $D_{s0}(2317)$ and $D_{s1}(2460)$ in e^+e^- collisions and their ratio at $\sqrt{s} = 10.58$ GeV obtained in the Wigner function approach for c.m. momentum $p^* > 3.2$ GeV/c. The experimental data are estimated from the *BABAR* data in inclusive $c\bar{c}$ productions near 10.6 GeV [58]. The uncertainty of the ratio is the standard deviation of the three ratios calculated from the central and two boundary values of $\mathrm{r_p}$.

	Case 1	Case 2	Case 3	Data [58]
$\mathrm{Y}_{D_{s0}(2317)}$	$5.87^{+3.54}_{-2.27} \times 10^{-3}$	$4.72^{+1.60}_{-1.44} \times 10^{-3}$	$3.43^{+0.75}_{-0.84} \times 10^{-3}$	$5.37^{+1.39}_{-1.75} \times 10^{-3}$
$\mathrm{Y}_{D_{s1}(2460)}$	$3.68^{+2.43}_{-1.32} \times 10^{-3}$	$2.98^{+1.10}_{-0.73} \times 10^{-3}$	$2.15^{+0.56}_{-0.33} \times 10^{-3}$	$3.86^{+1.00}_{-1.00} \times 10^{-3}$
$\mathrm{Y}_{D_{s0}(2317)}/\mathrm{Y}_{D_{s1}(2460)}$	1.56 ± 0.04	1.53 ± 0.07	1.53 ± 0.09	$1.39^{+0.29\mathrm{a}}_{-0.40}$

[a] The uncertainty of the ratio is calculated from the cross sections directly, not from the yields.

TABLE III. Charm fragmentation production of $D^*_{s0}(2317)$, $D_{s1}(2460)$, and $D_{s1}(2536)$ in the statistical model. The masses of the $D^*_{s0}(2317)$, $D_{s1}(2460)$, and $D_{s1}(2536)$ are taken from the review of particle physics [85], and the results for $D^*_{s0}(2317)$ and $D_{s1}(2460)$ are obtained for the momentum range $p^* > 3.2$ GeV/c.

$f(c \to D_s)$	Statistical model	*BABAR* [58]	ALEPH [86]	ZEUS [87]
$D^*_{s0}(2317)$	$3.98 \pm 0.26 \times 10^{-3}$	$5.37^{+1.39}_{-1.75} \times 10^{-3}$	...	...
$D_{s1}(2460)$	$5.38 \pm 0.35 \times 10^{-3}$	$3.86^{+1.00}_{-1.00} \times 10^{-3}$	...	...
$D_{s1}(2536)$	$8.77 \pm 0.57 \times 10^{-3}$	...	$(9.4 \pm 2.2 \pm 0.7) \times 10^{-3}$	$(11.1 \pm 1.6^{+0.8}_{-1.0}) \times 10^{-3}$

of $D^*_{s0}(2317)/D_{s1}(2460)$ decrease with the increasing size of the molecules from Cases 1 to 3, while the ratio stays almost constant, which is a manifestation of the underlying heavy-quark spin symmetry relating the DK and D^*K interactions. We stress that the production of hadrons in electron-positron collisions is a very involved process, therefore we think that the level of agreement obtained in this work, taking into account the theoretical and experimental uncertainties, is reasonable. Clearly, more accurate data will undoubtedly further refine our knowledge on these enigmatic mesons.

It is interesting to check whether the conventional $c\bar{s}$ picture for $D^*_{s0}(2317)$ and $D_{s1}(2460)$ can explain the *BABAR* data. For this, we turn to the statistical model [82], the details of which can be found in the Supplemental Material [68]. The corresponding results are shown in Table III. We note that although the absolute production yields are in the ballpark of 10^{-3} (consistent with the data), the ratio $R = Y_{D^*_{s0}(2317)}/Y_{D_{s1}(2460)}$ is about 0.7, which is much different from the experimental central value but marginally consistent with the lower bound considering the large experimental uncertainty.[2] We stress that the ratio is a very robust prediction of the statistical model, where the production yield of a vector D_s meson is larger than its pseudoscalar cousin mainly by the spin factor, as is the case for D^* and D mesons (see the Supplemental Material [68] for details). As a result, the ratio of the production yields of $D^*_{s0}(2317)$ and $D_{s1}(2460)$ provides strong and nontrivial support for their molecular nature as DK and D^*K bound states. For reference, we also calculated the production yield of $D_{s1}(2536)$, which is a typical excited $c\bar{s}$ state [24], and the result is consistent with the experimental measurement at the level of $20 \sim 30\%$ as expected.

Having verified the validity of our transport plus coalescence model, we now study the $D\bar{D}K$ molecule in the same framework. Considering all the three Jacobi channels of the $D\bar{D}K$ system [34], the yield of $D\bar{D}K$ per $e^+e^- \to c\bar{c}$ event (containing charge conjugated states) is found to be $1.75^{+2.66}_{-1.11} \times 10^{-6}$, which is three orders of magnitude lower than the yields of $D^*_{s0}(2317)$ and $D_{s1}(2460)$. Such a reduction in the production yield of a three-body bound state in comparison with that of a two-body bound state is consistent with those observed for deuteron and triton [88,89]. To facilitate experimental searches, we show the transverse momentum and rapidity distribution of $D\bar{D}K$ in Figs. 2 and 3. We note that the spectra are similar to those of normal hadrons.

We can now estimate the number of $K_{c\bar{c}}(4180)$ expected in e^+e^- collisions. In Ref. [58], the number of $D^*_{s0}(2317)$ observed in the $D_s\pi$ mode is 26290 ± 650, therefore, the number of $K_{c\bar{c}}(4180)$ produced can be estimated as $N_{D^*_{s0}(2317)} \times Y_{K_{c\bar{c}}(4180)}/Y_{D^*_{s0}(2317)} \approx 10$. In Ref. [38], it was shown that $K_{c\bar{c}}(4180)$ decays dominantly to $J/\psi K$. Both final states can be easily measured in e^+e^- collisions [90]. Considering further that the Belle experiment accumulated a data sample of 980 fb^{-1}, which is about four times larger than that of *BABAR*, 232 fb^{-1} [58], we estimate that the Belle data might contain as much as one hundred $K_{c\bar{c}}(4180)$. Belle II will

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[2] If we replace the $D^*_{s0}(2317)$ and $D_{s1}(2460)$ masses with those of the GI model [84], the ratio will become 0.5, much smaller than the experimental value, which shows again the inadequacy of the GI model for these two states.

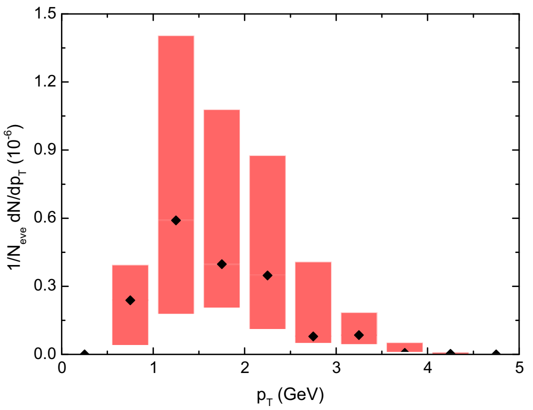

FIG. 2. Transverse momentum distribution of the $D\bar{D}K$ yield in bins of 0.5 GeV, where the uncertainty bands in red are generated by varying r_p from its default value by 20%.

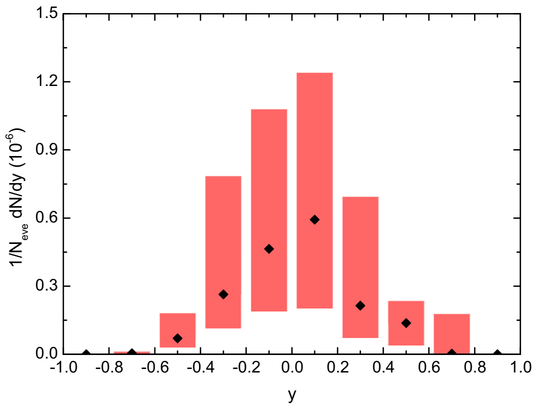

FIG. 3. Rapidity distribution of the $D\bar{D}K$ yield in bins of 0.2, where the uncertainty bands in red are generated by varying r_p from its default value by 20%.

collect 50 ab^{-1} data [91] and therefore the number of $K_{c\bar{c}}(4180)$ can reach the order of a few thousands.

IV. SUMMARY AND OUTLOOK

In this work, assuming $D_{s0}^*(2317)$ and $D_{s1}(2460)$ as DK and D^*K molecules, respectively, we investigated the prompt production yields of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ in inclusive $e^+e^- \to c\bar{c}$ collisions. The productions of primary hadrons (D^+, D^{+*}, and K^-) were simulated in the transport model PACIAE, and the formations of DK, D^*K, and $D\bar{D}K$ bound states were estimated in the Wigner function approach with the wave functions from the accurate Gaussian expansion method. We find that the prompt production yields of $D_{s0}^*(2317)$ and $D_{s1}(2460)$, and their ratio, which is less sensitive to theoretical uncertainties, are in nice agreement with the available data. These results provide non-trivial evidence that $D_{s0}^*(2317)$ and $D_{s1}(2460)$ are largely DK and D^*K molecules. We further calculated the production yield of the $D\bar{D}K$ state, $K_{c\bar{c}}(4180)$, and found that it is of the order of 10^{-6}. This is within the reach of the ongoing Belle II experiment. As a result we encourage dedicated searches for this exotic state in the near future. We stress that our method can be applied to reveal the nature of other enigmatic hadrons, such as the $\Lambda(1405)$ [92], $X(3872)$ [1], $\bar{K}NN$ [93], and $D\bar{D}^*K$ [33] states. Works along this line are in progress.

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