Pauli blocking effects on pair creation in strong electric field

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The process of electron-positron pair creation and oscillation in a uniform electric field is studied, taking into account the Pauli exclusion principle. Generally, we find that pair creation is suppressed; hence, coherent oscillations occur on longer timescales. Considering pair creation in already existing electron-positron plasma, we find that the dynamics depends on pair distribution function. We considered Fermi-Dirac distribution of pairs and found that for small temperatures pair creation is suppressed, while for small chemical potentials it increases: heating leads to enhancement of pair creation.

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I. INTRODUCTION

Quantum electrodynamics predicts the creation of electron-positron pairs in the strong electric field out of vacuum as a nonperturbative process with the field strength exceeding the critical value [1] $E_c = m^2c^3/e\hbar \sim 10^{18}$ V/m, where m and e are electron mass and charge, respectively, c is the speed of light, and $\hbar$ is reduced Planck's constant. It was predicted more than ninety years ago [2], right after the invention of the positron by Paul Dirac [3,4]. So far this process is not observed in the laboratory, despite strong efforts in increasing electric field strength, in particular, by focusing ultraintense optical laser beams; see, e.g., [5,6]. Because of copious amount of pairs created in such electric fields it is believed that the process of pair creation cannot be considered on a fixed background, so that accounting for the backreaction of newly created particles on the external field is mandatory in this problem.

The study of the process of electron-positron pair creation and oscillations induced by backreaction in the homogeneous time dependent electric field has a long history; for reviews see Refs. [7,8]. A comparison of solutions of quantum Vlasov equations with classical kinetic Vlasov-Boltzmann equations performed in [9–11] showed that the classical description is in surprisingly good agreement with the full quantum treatment even for the field strengths $E > E_c$. It was also shown that for largely overcritical fields quantum treatment leads to non-Markovian kinetic equations [11,12]. Numerical solutions of these equations were obtained in [13], showing that for overcritical fields memory effects become important. Finally, effects of quantum statistics were analyzed in [14] and shown to be important when quantum interference occurs [15]. A hydrodynamic approach developed in [16,17] following [18] allowed us to establish that plasma oscillates with a frequency comparable to the plasma frequency. The study of this problem with Boltzmann-Vlasov equations describing, in addition to pair creation, also interaction with photons [19] showed that plasma thermalization occurs on much longer timescales than oscillations do; see also [20,21].

Pair production in strong electric fields has been also discussed in the context of the early Universe, in particular during inflation [22]. The role of the gravitational field in the process of pair creation is considered in [23]. Of course, one of the key directions of these efforts is its verification in laboratory experiments [24].

Very recently [25] quantum Vlasov equations were derived from the nonequilibrium quantum field theory. Backreaction of electron-positron that pairs onto rapidly oscillating electric fields was studied in [26] using quantum Vlasov equations [27] confirming that only for undercritical fields can the backreaction be safely neglected. The validity of the locally constant field approximation is discussed in [28].

Recently, Pauli blocking effects in the thermalization of relativistic plasma were studied in [29–31]. So far no systematic analysis of the influence of quantum degeneracy on pair creation and plasma oscillations was carried out. Our previous works did not include the effects of Pauli blocking. In this work we close this gap.

The paper is organized as follows. In Sec. II the framework is presented: Boltzmann-Vlasov equations pairs together with the Maxwell equation for the electric field.

In Sec. III, the main results for a vacuum initial state are reported. In Sec. IV a nonvacuum initial state is considered and the role of the inverse Schwinger process is emphasized. Conclusions follow in the last section.

II. FRAMEWORK

There are two main assumptions in this work. First, following the results in [9–11], we assume that classical kinetic equations provide good approximation to quantum dynamics of pairs created in overcritical electric fields. Second, we assume that the pair creation rate computed for the vacuum state does not change when electron-positron pairs are present.

In this section we present a kinetic description based on the relativistic Boltzmann-Maxwell equation with the source term accounting for pair creation in strong electric fields [18], modified to include the Pauli blocking effect. In what follows, the system of units $\hbar = c = 1$ is adopted, then $e = \sqrt{\alpha}$, where α is the fine structure constant.

As uniform electric field $E(t)$ is considered, the problem has axial symmetry. We introduce cylindrical coordinates in momentum space $\mathbf{p} = \{p_\perp, \phi, p_{||}\}$ with the $p_{||}$ axis parallel to electric field E. Particle energy is then $p^0 = [p_\perp^2 + p_{||}^2 + m^2]^{1/2}$.

Particle evolution is described by one-particle electron-positron distribution function $f(t, p_\perp, p_{||})$, which is normalized on particle density $n = \int \frac{d^3p}{(2\pi)^3} f$. Energy density (of electrons and positrons) and energy per particle are defined as follows: $\rho = 2\int d^3p(2\pi)^{-3}p^0 f$ and $\epsilon = n^{-1}\int \frac{d^3p}{(2\pi)^3} p^0 f$. Since for this work there is no difference between electrons and positrons (apart from the opposite direction of acceleration by the electric field) in the following for definiteness we denote f the positron distribution function. Particle collisions are neglected, as they occur on much larger timescales than what is considered in this work, leading eventually to plasma thermalization [19].

The collisionless Boltzmann equation governing the evolution of $f(t, p_\perp, p_{||})$ is (see e.g. [9,10])

$$\frac{\partial f}{\partial t} + eE\frac{\partial f}{\partial p_{||}} = S(E, p_\perp, p_{||}), \tag{1}$$

where S is the source term for the Schwinger process:

$$S = -(1-2f)e|E| \ln\left[1 - \exp\left(-\frac{\pi(p_\perp^2 + m^2)}{e|E|}\right)\right]\delta(p_{||}), \tag{2}$$

with the factor $(1-2f)$ being the Pauli blocking accounting for both electrons and positrons.

The time evolution of the electric field is defined from the Maxwell equation $\frac{dE}{dt} = -j_{\mathrm{cond}} - j_{\mathrm{pol}}$ containing conductive current j_{cond} generated by the motion of pairs and polarization current j_{pol} generated by the pair creation process. These currents are defined as follows:

$$j_{\mathrm{cond}} = 2e\int \frac{d^3p}{(2\pi)^3}\frac{p_{||}}{p^0} f, \tag{3}$$

$$j_{\mathrm{pol}} = 2e|E|E^{-1}\int \frac{d^3p}{(2\pi)^3} p^0 S, \tag{4}$$

where factor 2 is included to account for both electrons and positrons. Then the Maxwell equation becomes

$$\frac{dE}{dt} = -e\int \frac{d^3p}{(2\pi)^3}\frac{p_{||}}{p^0} f + \frac{e|E|}{E}\int \frac{d^3p}{(2\pi)^3}(1-2f) \times \ln\left[1 - \exp\left(-\frac{\pi(p_\perp^2 + m^2)}{|qE|}\right)\right] p^0\delta(p_{||}). \tag{5}$$

It is useful to introduce dimensionless quantities $\tilde{t} = tm$, $\tilde{p} = p/m$, $\tilde{E} = Ee/m^2$ and rewrite Eqs. (1) and (5) in the dimensionless form:

$$\frac{\partial f}{\partial \tilde{t}} + \tilde{E}\frac{\partial f}{\partial \tilde{p}_{||}} = -(1-2f)|\tilde{E}| \times \ln\left[1 - \exp\left(-\frac{\pi(\tilde{p}_\perp^2 + 1)}{|\tilde{E}|}\right)\right]\delta(\tilde{p}_{||}), \tag{6}$$

$$\frac{\partial \tilde{E}}{\partial \tilde{t}} = -2e^2\int \frac{d^3\tilde{p}}{(2\pi)^3}\frac{\tilde{p}_{||}}{\tilde{p}^0} f + \frac{2e^2|E|}{E}(1-2f) \times \int \frac{d^3\tilde{p}}{(2\pi)^3}\ln\left[1 - \exp\left(-\frac{\pi(\tilde{p}_\perp^2 + 1)}{|\tilde{E}|}\right)\right]\tilde{p}^0\delta(\tilde{p}_{||}), \tag{7}$$

here $\tilde{p}^0 = \sqrt{\tilde{p}_\perp^2 + \tilde{p}_{||}^2 + 1}$ and $d^3\tilde{p} = 2\pi\tilde{p}_\perp d\tilde{p}_\perp d\tilde{p}_{||}$ is the phase space element.

Equations (6) and (7) are solved numerically using the finite difference scheme. For this goal we define a grid in the $\{\tilde{p}_\perp, \tilde{p}_{||}\}$ space as follows. The $\tilde{p}_\perp$ grid is logarithmic containing 10 nodes covering the interval (0.001, 50). The $\tilde{p}_{||}$ grid is uniform containing 500 nodes covering the interval $(-1000, 1000)$. Replacing momentum derivatives by finite differences in the Boltzmann equation (6) we use the upwinding scheme for both positive and negative values of electric field $\tilde{E}$. Then on a finite grid the Boltzmann equation (6) transforms into the system of ordinary differential equations for time variable $\tilde{t}$. The integral in the rhs of Eq. (7) transforms to a finite sum. We use the implicit Gear's method to solve the system of ordinary differential equations.

III. VACUUM INITIAL STATE

We explore the process of pair creation in the overcritical electric field with different initial conditions for electric fields and pairs. First we focus on the overcritical electric field with the vacuum initial state. Then we turn to the initial state with electron-positron pairs distributed according to the Fermi-Dirac statistics in an overcritical electric field.

In Fig. 1 we demonstrate the effect of Pauli blocking on the pair creation process by comparing the time evolution of dimensionless quantities: electric field $\tilde{E} = E/E_c$, positron number density $\tilde{n} = n/m^3$, and average energy per particle $\tilde{\epsilon} = \epsilon/m$ in two cases: when Pauli blocking is accounted for (solid curves) and when it is neglected (dashed curves). In both cases plasma oscillations develop due to the backreaction of the pairs: an electric current is induced due to charged particle acceleration in the electric field; then particles overshoot thanks to their inertia and change the direction of the field. This process repeats as damped oscillations, due to creation of new pairs. It is clear that when Pauli blocking is taken into account the rate of pair creation is strongly suppressed and consequently oscillation frequency is smaller and particle average energy is higher. It implies that oscillations are damped on a longer timescale. We also illustrate in Fig. 2 the phase space evolution for selected initial electric field strengths. Comparing the upper and lower figures with different initial electric fields we find that pairs are produced with zero parallel momentum and with orthogonal momentum up to the value $p_\perp \sim \sqrt{E}$. Once pairs are created they are accelerated in the direction parallel to the electric field and hence are distributed over the parallel momentum $p_{||}$ rather uniformly. The degree of degeneracy is different: the larger electric field, the higher it is. Comparing the left and the right columns in Fig. 2 we find that before the electric field vanishes particles are gaining positive $p_{||}$, while after this moment particles are directed towards negative $p_{||}$. In the process of particle motion, the electric field continues to create new pairs: particle density is higher with negative $p_{||}$ than it is with positive $p_{||}$ on the right column.

Next, we study the pair creation process as a function of initial electric field $E_{\rm in}$. Dimensionless energy density for the electric field is $\tilde{\rho}_E = \tilde{E}^2/(2\alpha)$ and for pairs it is $\tilde{\rho}_{\rm pair} = 2\tilde{\rho}$. For the vacuum initial state the energy conservation gives $\tilde{\rho}_E + \tilde{\rho}_{\rm pair} = \tilde{\rho}_{E_{\rm in}}$. The amount of energy transferred into pairs can be determined as $\tilde{\rho}_{\rm pair}/\tilde{\rho}_{E_{\rm in}} = 1 - \tilde{\rho}_E/\tilde{\rho}_{E_{\rm in}}$. In Table I we summarize some characteristic quantities for the pair creation process from vacuum depending on initial electric field $E_{\rm in}$: $\tilde{t}_0$ is the time moment when the electric field vanishes for the first time; $\tilde{n}(\tilde{t}_0)$ and $\tilde{\epsilon}(\tilde{t}_0)$ are positron density and average energy in this moment; $\tilde{t}_{1/2}$ is time moment

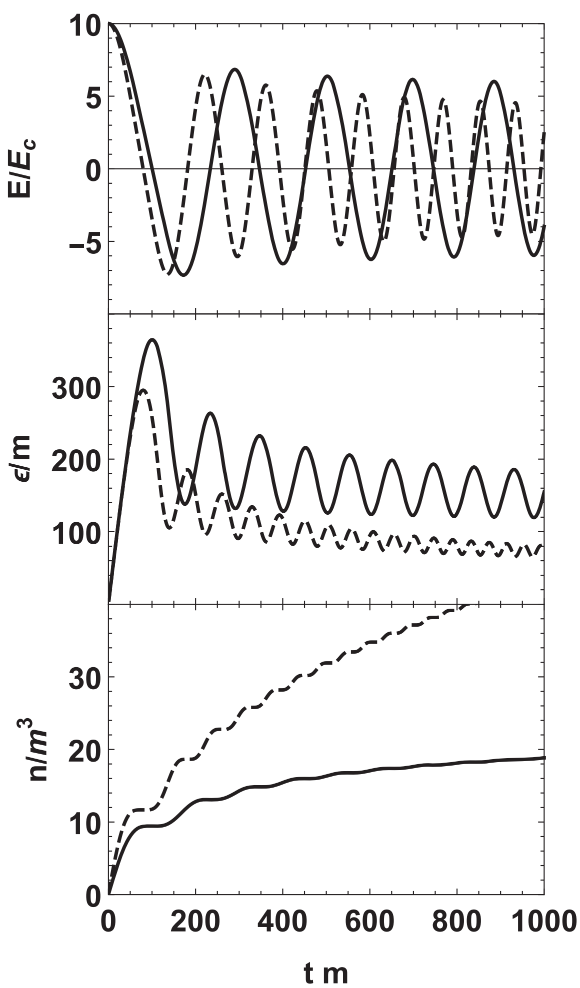

FIG. 1. Time evolution of electric field E, positron number density n, and average energy per particle ϵ for $E_{\rm in} = 10E_c$. Pauli blocking factor included on solid curve and excluded on dashed curve.

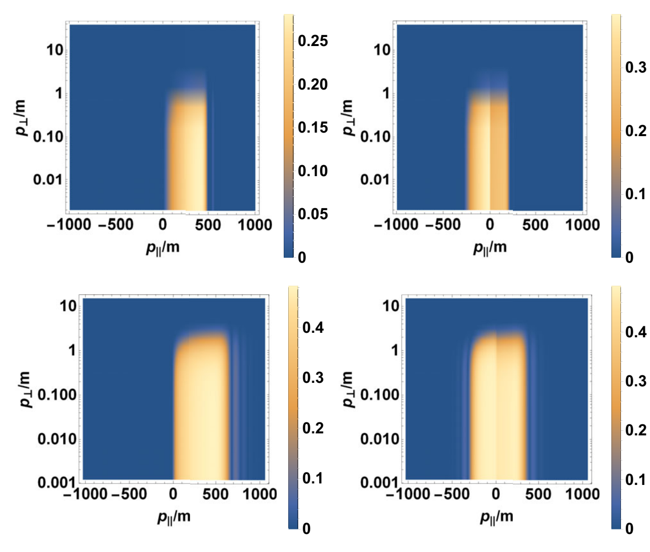

FIG. 2. Positron distribution function f for the first oscillation at two time moments: When the electric field equals zero (left column) and when electric field acquires local minimum (right column). The upper row corresponds to $E_{\rm in}/E_c = 3$. The lower row corresponds to $E_{\rm in}/E_c = 11$. For more details see Supplementary Material [32].

TABLE I. Results for the pairless initial state.

$\tilde{E}_{\rm in}$	$\tilde{t}_0$	$\tilde{n}(\tilde{t}_0)$	$\tilde{\epsilon}(\tilde{t}_0)$	$\tilde{t}_{1/2}$	$\frac{\tilde{\rho}_{\rm pair}(\tilde{t}_{1/2})}{\tilde{\rho}_{E_{\rm in}}}$	$\tilde{n}(\tilde{t}_{1/2})$	$\tilde{\epsilon}(\tilde{t}_{1/2})$
1	1431	0.057	600	88070	0.64	0.31	72
3	264	0.99	311	1010	0.55	1.76	99
6	136	4.04	306	204	0.6	4.35	171
10	100	9.43	365	130	0.84	9.59	300
10.1	99.8	9.58	366	1.53			
30	49	65.23	631	0.25			
60	33.7	198.15	839	0.11			
100	27.6	353.47	971	0.10			
1000	8.7	11302	3048	0.01			

when f increases to the value $1/2$; $\tilde{n}(\tilde{t}_{1/2})$ and $\tilde{\epsilon}(\tilde{t}_{1/2})$ are positron density and average energy in this moment; and $\tilde{\rho}_{\rm pair}(\tilde{t}_{1/2})/\tilde{\rho}_{E_{\rm in}}$ is the relative fraction of the energy transferred into pairs at this moment. The main results are as follows: with increasing the initial electric field both the frequency of oscillations and pair density monotonically increase. The average energy per particle first decreases then, starting at about $6E_c$ it increases; see Refs. [17,19]. For $E_{\rm in} > 10E_c$ the distribution function of pairs overcomes the value $1/2$ before the moment $\tilde{t}_0$ and we do not report the quantities at $\tilde{t}_{1/2}$. Pauli blocking operates in such a way that $f = 1/2$ is reached. The average energy per particle at the moment $\tilde{t}_{1/2}$ monotonically increases as particles occupy more and more orthogonal momentum space.

IV. NONVACUUM INITIAL STATE

Below we present the results of simulations with the nonvacuum initial state. The statistical factor $(1-2f)$ in Eqs. (6) and (7) plays a crucial role in the interaction between the electric field and pairs. Initial conditions with $f < 1/2$ pair creation from the electric field leads to an increase in particle number density and damping of oscillations. This effect is well described in the literature; see, e.g., [9,17,19].

On the contrary, for $f > 1/2$ the statistical factor becomes negative, which implies the negative source term in Eq. (6) and also the opposite sign of the polarization current in Eq. (7). Under these conditions the inverse Schwinger process, namely *pair annihilation in the external electric field* takes place. Quantum electrodynamics predicts that the rate of the inverse Schwinger process is equal to the rate of the direct one. This effect is clearly absent in vacuum and to our knowledge it is not discussed in the literature so far.

As we are interested in the effects of quantum degeneracy, we explore the influence of particle distribution in the phase space on the dynamics of pairs and electric fields. Electron-positron pairs in the initial state are assumed to obey the Fermi-Dirac statistics

$$f = \left[1 + e^{(\sqrt{\tilde{p}^2+1}-\tilde{\mu})/\tilde{T}}\right]^{-1}, \tag{8}$$

where $\tilde{T} = T/m$ is dimensionless temperature and $\tilde{\mu} = \mu/m$ is dimensionless chemical potential. Note that equilibrium distribution with relativistic temperature and $\mu = 0$ corresponds to $f < 1/2$, while a fully degenerate distribution with $T = 0$ corresponds to $f > 1/2$.

A. Nonvacuum initial state with $f < 1/2$

In this section we consider the initial distribution function of pairs with $f < 1/2$. First we treat the case of distribution (8) with $\mu = 0$. In Fig. 3 we show the relative number of pairs produced after three oscillations depending on the initial electric field and pair temperature. The dynamics in this case is qualitatively similar to the case with the vacuum initial state discussed above. From Fig. 3 it is clear that in the electric energy domination region (above black line) pair production is efficient. Conversely, in the pair energy domination region (below black line) pair production is suppressed. This is expected because in the pair dominated region the Schwinger process is suppressed and plasma keeps oscillating with relativistic plasma frequency $\tilde{\omega}_p = \sqrt{\alpha \tilde{n}_{\rm pair}/\tilde{\epsilon}}$. When initial conditions are in the electric field dominated region (black line in Fig. 3), the field accelerates particles much stronger. As the electric field

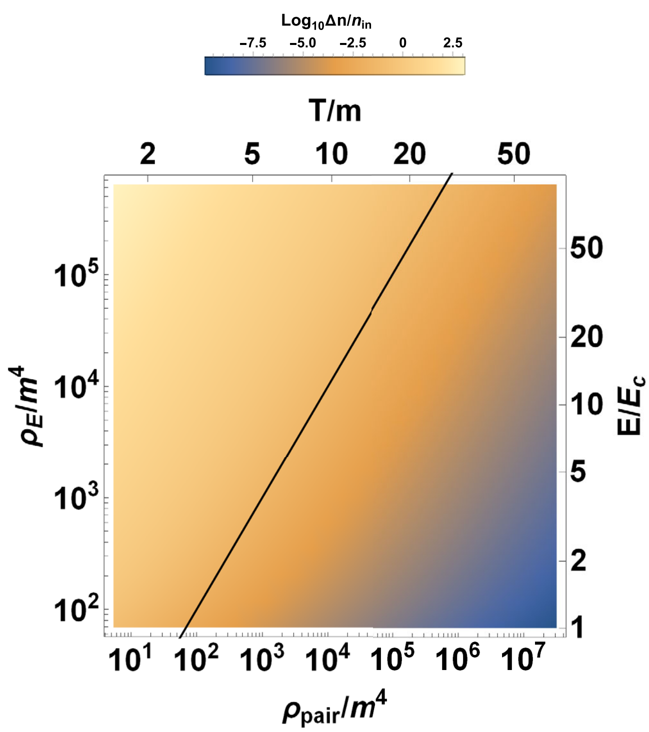

FIG. 3. Relative change of pair number after three oscillation periods depending on the initial electric field strength and plasma temperature. We also indicate energy density of the electric field and pairs. The black line corresponds to equality $\rho_{\rm pair} = \rho_E$. The chemical potential is zero.

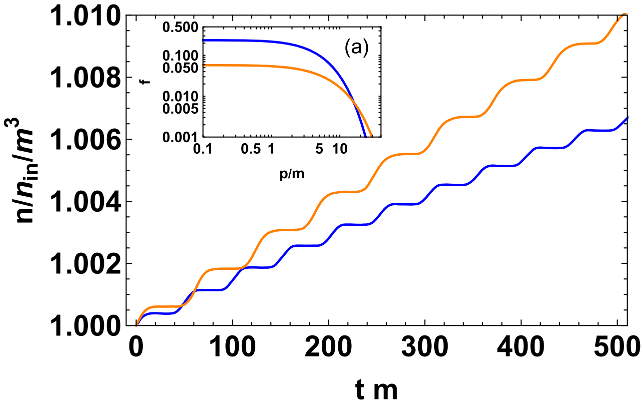

FIG. 4. Time evolution of pair number density with initial electric field $E_{\rm in} = E_c$ and two different initial pair states. Blue: $\tilde{\mu} = -3.71$, $\tilde{T} = 4$; orange: $\tilde{\mu} = -18.57$, $\tilde{T} = 7$; inset (a): the corresponding initial distribution functions.

drags particles out from the region $p_{||} = 0$ the phase space opens up and pair creation becomes possible.

In general, both the temperature and chemical potential in (8) can be nonzero. As we are interested in the influence of the particle distribution function on the Schwinger process we consider two types of initial conditions: those with the same energy density and those with the same number density. The first choice corresponds to the same point on the diagram in Fig. 3 but with different parameters of the distribution function (8). The second choice allows exploration of the role of heating. In fact, when number density is kept constant and the temperature increases, the average energy per particle increases. In the diagram in Fig. 3 it represents a shift to the right.

In Fig. 4 we show the relative number of pairs with the same initial number density $n/m^3 = 9.66$, but different initial energy density $\rho/m^4 = 119.7$ (blue curve) and $\rho/m^4 = 204.4$ (orange curve), evolving in the initial electric field $E_{\rm in} = E_c$. Pair initial distributions are shown on the inset. While pair creation is small in both cases, it clearly increases for initial conditions with higher energy density. This demonstrates the effect of heating of initially present plasma onto the pair creation process. The result is that despite the energy density of pairs increasing due to heating, and initial conditions shift onto the pair dominated region in Fig. 3 disfavoring pair creation, the effect of the opening up of the phase space due to the change of the distribution function prevails and pair creation becomes enhanced.

B. Nonvacuum initial state with $f > 1/2$

In this section we consider the initial distribution function of pairs (8) with $T = 0$. In Fig. 5 we show the absolute value of the change in the relative number of pairs after three oscillations depending on the initial electric field and pair chemical potential or equivalently Fermi momentum p_F. In contrast with the previous case, here $f > 1/2$ and the statistical factor in Eqs. (6) and (7) become negative, which implies particle annihilation in the external electric field. During time evolution the number density of pairs increases above the red line and decrease below it. Naively, one could expect that pair annihilation would result in the amplification of the electric field as the process that is opposite to pair creation and field depletion, shown in Fig. 1. However, there is no direct analogy in this process. A diminishing of the number of pairs (and hence their rest mass energy) leads not to an increase of the energy density of the electric field, but to an increase of internal energy of pairs. This is because the electric field accelerates particles, redistributing them in momentum space. There is no possibility to use the inverse Schwinger process to enhance the electric field.

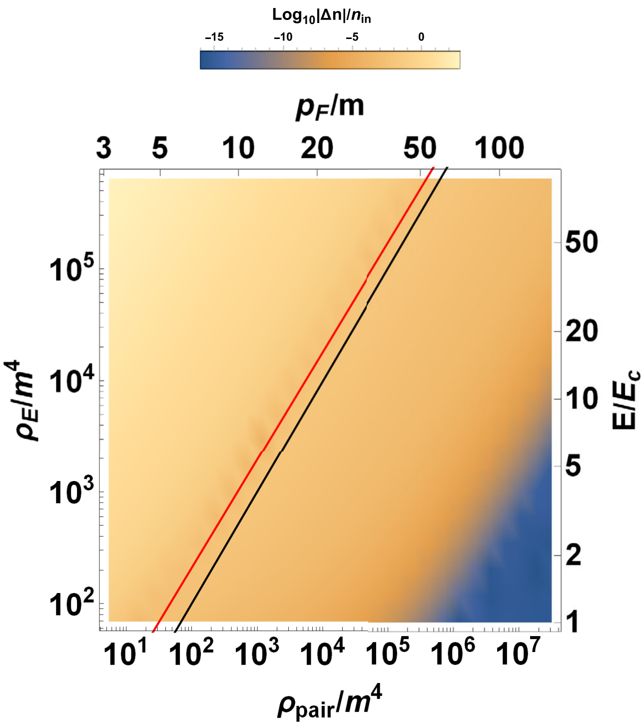

FIG. 5. The absolute value of the relative change of the pair number after three oscillation periods depending on the initial electric field strength and plasma chemical potential. We also indicate the energy density of the electric field and pairs. The black line corresponds to equality $\rho_{\rm pair} = \rho_E$. The red line corresponds to the transition from pair creation (above this line) to pair annihilation (below this line). For convenience we plot here Fermi momentum p_F instead of the chemical potential. The temperature is zero.

In Fig. 6 we present the distribution function $f(p_\perp, p_{||}, t)$ after 16 oscillations. Note the different scale in momentum axes. It is evident that pair annihilation leads to depletion of the distribution function only for small $p_\perp$, where the source term is significant. For larger $p_\perp$ the source term is negligible. As the source term is largest in absolute value

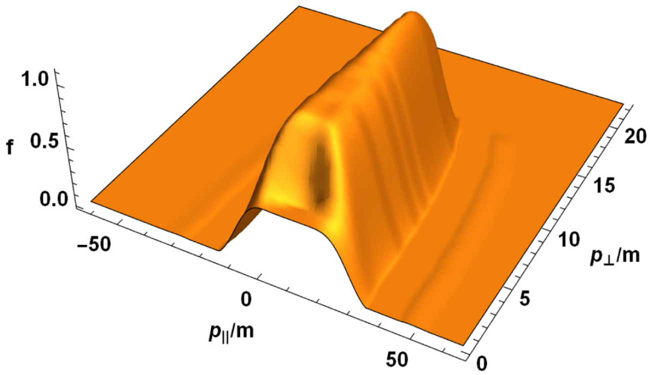

FIG. 6. Distribution function of electron-positron pairs after 16 oscillations with $E = 4E_c$ and initial distribution (8) with $T = 0$ and $\mu = 16$.

just below the red line in Fig. 5, this imposes a limit on the effect of pair annihilation because the energy density of the electric field cannot exceed much of the energy density of pairs. In other words, the effect can be enhanced by increasing the initial electric field, but this leads also to an increase of the Fermi momentum in the initial distribution function, thus reducing the part of the distribution affected by pair annihilation.

In Fig. 7 we show the change in the relative number of pairs after three oscillations as a function of the initial electric field for two cases with the same initial energy density of pairs: pairs with zero chemical potential and relativistic temperature (orange curve) and fully degenerate pairs with zero temperature (blue curve). Clearly the first case corresponds to $f < 1/2$ and leads to pair creation, while the second case represents initial conditions with $f > 1/2$ and leads to pair annihilation. Both cases show saturation at large electric fields. All initial conditions represented in Fig. 5 are located on the black line in Fig. 3.

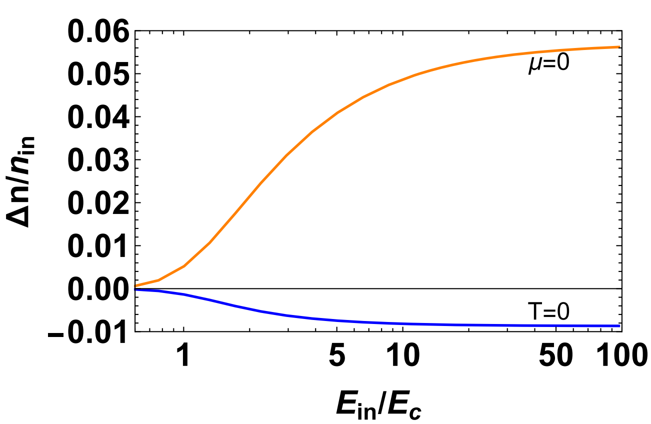

FIG. 7. Relative change of the pair number after three oscillations as a function of the electric field strength when the initial electric field energy equals initial pair energy. The blue curve corresponds to fully degenerate pairs with $T = 0$ and the orange curve corresponds to $\mu = 0$.

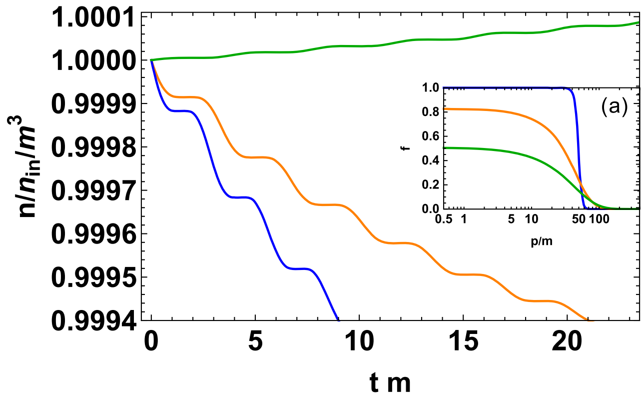

FIG. 8. Time evolution of pair number density with initial electric field $E_{in} = 10E_c$ and three different initial pair states. Blue: $\tilde{\mu} = 49.41$, $\tilde{T} = 3$; orange: $\tilde{\mu} = 29.21$, $\tilde{T} = 18$; green: $\tilde{\mu} = 1.58$, $\tilde{T} = 28$. Inset (a): the corresponding initial distribution functions.

In Fig. 8 we present time evolution of pair number density with $E_{in} = 10E_c$ and with initial pair number density $\tilde{n}_{in} = 4.2 \times 10^3$ for three different initial pair states: $\tilde{\mu} = 49.41$, $\tilde{T} = 3$ ($\tilde{\rho} = 1.6 \times 10^5$); $\tilde{\mu} = 29.21$, $\tilde{T} = 18$ ($\tilde{\rho} = 2.6 \times 10^5$), and $\tilde{\mu} = 1.58, \tilde{T} = 28$ ($\tilde{\rho} = 3.7 \times 10^5$). The inset in Fig. 7 illustrates the distribution function for these initial conditions. As can be seen, only the case with smallest chemical potential corresponds to $f < 1/2$ and pairs are created; in the other two cases pairs annihilate with time.

V. CONCLUSIONS

Our main result in this work is the demonstration of how the quantum exclusion principle suppresses pair creation in the overcritical uniform electric field, which in turn modifies the backreaction dynamics. We studied electron-positron pair creation and oscillations with the initial vacuum state as well as with the electron-positron plasma initially present. Two cases can be distinguished. (1) When the energy in the electric field dominates that in pairs, oscillations are induced, which leads to the opening up of the phase space and consequent prolific pair creation. (2) In the opposite case, when pairs dominate energetically over the electric field, plasma oscillations do occur with much higher frequency, since the electric field is unable to displace them significantly in momentum space: as a consequence pair creation remains strongly suppressed.

We also considered the effect of the inverse Schwinger process, namely, annihilation of pairs in external electric fields when the statistical factor becomes negative. Despite the naive expectation that pair annihilation could lead to the amplification of the electric field, we found that this is not the case, even for the limiting case of the completely degenerate initial distribution function with $T = 0$. Despite the fact that the number of pairs may significantly decrease, the backreaction of pairs on the electric field leads to the transformation of their rest mass energy into their internal energy, and not the energy of the electric field.

We found that plasma heating leads to the enhancement of pair creation. This effect may be relevant for astrophysical models of quark stars or neutron stars with a strong electric field on their surface [33,34].

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Note added.—Recently, we learned about very similar work [35] just published. Using a different formalism, the authors of this publication reached similar conclusions and their results are consistent with ours. However, it appears they did not look into the evidence for the inverse Schwinger process.

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[32] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevD.108.013002, for a time evolution of electric field (top left), number density (top right), particle distribution in the phase space (bottom right) and particle distribution at zero orthogonal component of the momentum (bottom left) for vacuum initial state and with 4 times critical electric field. Pair creation occurs in the region of the phase space near zero parallel momentum. Acceleration in electric field results in shifting the distribution of particles toward higher parallel momenta. It can be seen that after several oscillations the distribution function reaches the limit of 1/2. Subsequent evolution results in creating pairs just on the wings around the top, where the phase space is still available.
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