Letter

Holographic dissipation prefers the Landau over the Keldysh form

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Although holographic duality has been regarded as a complementary tool in helping understand the nonequilibrium dynamics of strongly coupled many-body systems, it still remains a remarkable challenge how to confront its predictions quantitatively with real experimental scenarios. By matching the holographic vortex dynamics with the phenomenological dissipative Gross-Pitaevskii models, we find that the holographic dissipation mechanism can be well captured by the Landau form rather than the Keldysh one, although the latter is much more widely used in numerical simulations. Our finding is expected to open up novel avenues for facilitating the quantitative test of holographic predictions against upcoming experimental data. Our result also provides a prime example how holographic duality can help select proper phenomenological models to describe far-from-equilibrium nonlinear dynamics beyond the hydrodynamic regime.

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I. INTRODUCTION

For the nonequilibrium dynamics of strongly interacting quantum systems where the quasiparticle picture does not apply and the perturbation method fails, developing its theoretical description remains an important task [1,2]. Gratefully, holographic duality [3–5], also known as anti–de Sitter space/conformal field theory correspondence, has provided a powerful insight into the universal behaviors of strongly coupled dynamics through the classical theory of gravity with one additional dimension. In particular, a variety of bottom-up gravitational models have been proposed to address the strongly correlated condensed matter systems [6–12]. But, nevertheless, associated with these bottom-up holographic models, there exists a significant deficiency; namely, the effective dual boundary descriptions are generically unknown, which makes it a notoriously difficult challenge to compare the holographic prediction with the experimental data.

Among others, the dynamics of the quantized vortices in superfluids, which plays a vital role in the fascinating nonequilibrium quantum turbulence, have recently become amenable to being engineered at finite temperature in a controllable manner due to the great experimental advances in cold atom gases [13,14]. In contrast to classical turbulence in normal fluids, which can be well described by dissipative hydrodynamics, quantum turbulence in superfluids exits the hydrodynamic regime due to the very presence of the quantized vortices. It is, thus, urgent to construct an effective boundary description of holographic superfluids, which provides a complete description—valid at all scales—of the superfluid dynamics, including the vortex dynamics.

On the other hand, different from the holographic duality, which provides a universal first principles description of the irreversible finite temperature dissipation in terms of the excitations absorbed by the bulk black holes, the conventional approach to incorporate dissipation in superfluids is essentially phenomenological. The dissipation terms in the different phenomenological models will give rise to different predictions as should be the case. Therefore, it is desirable to resort to a first principles calculation to help select which phenomenological model is the proper one.

This paper intends to serve as one such stone which attempts to kill the above two birds by matching the two available phenomenological dissipative Gross-Pitaevskii equations with the holographic vortex dynamics. As a

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result, we find that the dissipation mechanism in our holographic superfluid can be well described by the dissipative Gross-Pitaevskii equation with the dissipation given by the Landau form rather than the Keldysh one, although the latter is much more commonly used in numerical simulations of superfluid dynamics. Compared to the previous progress such as made in Refs. [15,16], which nonetheless restricts mainly within the equilibrium state or the near-equilibrium hydrodynamic regime, our work presents a prime example of how holographic duality can also help select proper phenomenological models to describe the far-from-equilibrium nonlinear dynamics beyond the hydrodynamic regime. On the other hand, with our finding, the holographic superfluid model with four bulk dynamical variables and one adjustable boundary value can be described effectively by only one dynamical variable with three adjusted parameters in one less dimension, which will make the quantitative comparison of holographic predictions with real upcoming experimental data much easier and much more efficient.

II. HOLOGRAPHIC SUPERFLUIDS MODEL AND DISSIPATIVE GROSS-PITAEVSKII MODELS

In the probe limit, where the holographic superfluid is implemented by the Abelian Higgs model with the Lagrangian density given by [6,8]

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - |D_\mu\Phi|^2 - m^2|\Phi|^2, \qquad (1)$$

on top of the $(3+1)$-dimensional planar Schwarzschild AdS black hole in the Eddington-Finkelstein coordinates

$$ds^2 = \frac{L^2_{\mathrm{AdS}}}{z^2}(-f(z)dt^2 + dx^2 + dy^2 - 2dtdz), \qquad (2)$$

where $D_\mu = \nabla_\mu - iA_\mu$ and $f(z) = 1 - (z/z_h)^3$ with z_h the location of the black hole horizon. The corresponding dynamics is governed by the following equations of motion:

$$\nabla_\mu F^{\mu\nu} = i(\Phi^* D^\nu\Phi - \Phi(D^\nu\Phi)^*), \qquad D_\mu D^\mu\Phi - m^2\Phi = 0, \qquad (3)$$

where the asterisk denotes the complex conjugation.

By holography, the temperature of the dual boundary system is given by the Hawking temperature $\tilde{T} = 3/(4\pi z_h)$, and the chemical potential is related to the boundary data of the bulk field A_t as $\tilde{\mu} = A_t|_{z=0}$. Because of the scaling symmetry, one can set $z_h = 1$ once and for all. Accordingly, it turns out that, when the chemical potential is higher than the critical value $\tilde{\mu}_c = 4.064$, the bulk complex scalar field will spontaneously condense, which signals the transition to the superfluid phase on the boundary. The corresponding order parameter ψ can be read off from the boundary data of Φ according to the holographic dictionary. It is noteworthy that holography provides a natural built-in mechanism to account for the irreversible finite temperature dissipation by geometrizing the excitations absorbed by the black hole.

Different from the above holographic model of superfluids, conventional phenomenological models have significant limitations and shortcomings, where the dissipation is essentially put in by hand. As to the Bose-Einstein condensates (BECs) in dilute cold atom gases at nearly zero temperature, the behavior of order parameter ψ can be successfully described by the Gross-Pitaevskii equation (GPE) [17]. However, GPE cannot describe BECs at finite temperature. In order to account for the finite temperature effect, one is required to introduce dissipative terms. For our purpose, we consider two such dissipative Gross-Pitaevskii equations (DGPEs), which can be written in the dimensionless form as follows:

$$\partial_t\psi = -\frac{(i+\gamma)}{2\tau}(-\nabla^2\psi + 2\mu(|\psi|^2 - 1)\psi), \qquad (4)$$

$$\partial_t\psi + i\lambda\psi\partial_t|\psi|^2 = -\frac{i}{2\tau}[(-\nabla^2\psi + 2\mu(|\psi|^2 - 1)\psi)]. \qquad (5)$$

Here, the parameter τ controls the characteristic timescale of dynamics, and μ is the chemical potential, from which the dimensionless healing length is given by $\xi = (2\mu)^{-1/2}$. The dissipative parameter γ in Eq. (4) is suspected to be determined by the Keldysh self-energy through the fluctuation-dissipation theorem [18–20]. So we call this equation as KGPE. On the other hand, we denote Eq. (5) with λ the dissipative parameter as LGPE, as it was phenomenologically motivated by Landau's requirement that the second law of thermodynamics hold in his two-fluid model for superfluidity [21,22].

III. MATCHING PROCEDURE AND RELEVANT RESULTS

In order to quantify how well the above two models serve as a phenomenological description of the holographic vortex dynamics, we need a matching procedure. In Ref. [23], the authors proposed such a procedure but used an invalid evolution scheme in holography (as explained in Supplemental Material [24]) and then made an unreliable claim that the holographic vortex dynamics was well described by KGPE. Here, we use the correct evolution scheme while still employing a similar procedure. Namely, we first determine the healing length in both models by fitting the order parameter profile for the holographic vortex of winding number 1 with the form $|\psi|^2 \propto \frac{\mathbf{x}^2}{2\xi^2+\mathbf{x}^2}$. Then, we intend to fit the holographic vortex dipole trajectory by adjusting the corresponding dissipation parameter. Finally, the parameter τ is fixed by tracking the real time evolution of the vortex dipole. We demonstrate our relevant results by focusing on a typical example,

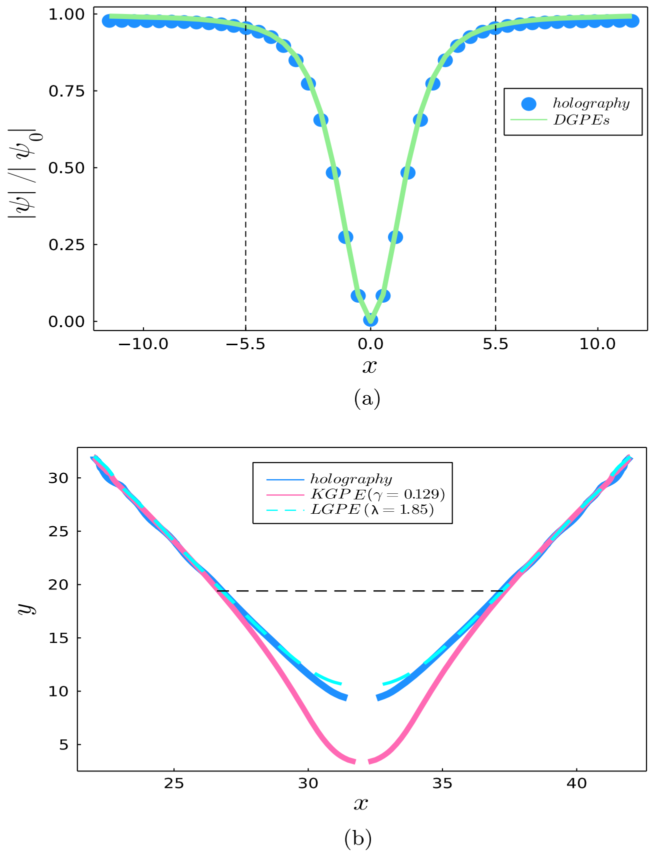

FIG. 1. The matching results between the holographic superfluid at $\tilde{\mu} = 4.5$ and DGPEs. In (a), the normalized condensate profile of a single static holographic vortex is well fitted by both DGPEs, where the black dotted line is used to identify the vortex size. In (b), the holographic vortex dipole trajectory is fitted by both KGPE and LGPE, where the black dotted line indicates the location where the vortex dipoles get contacted with each other.

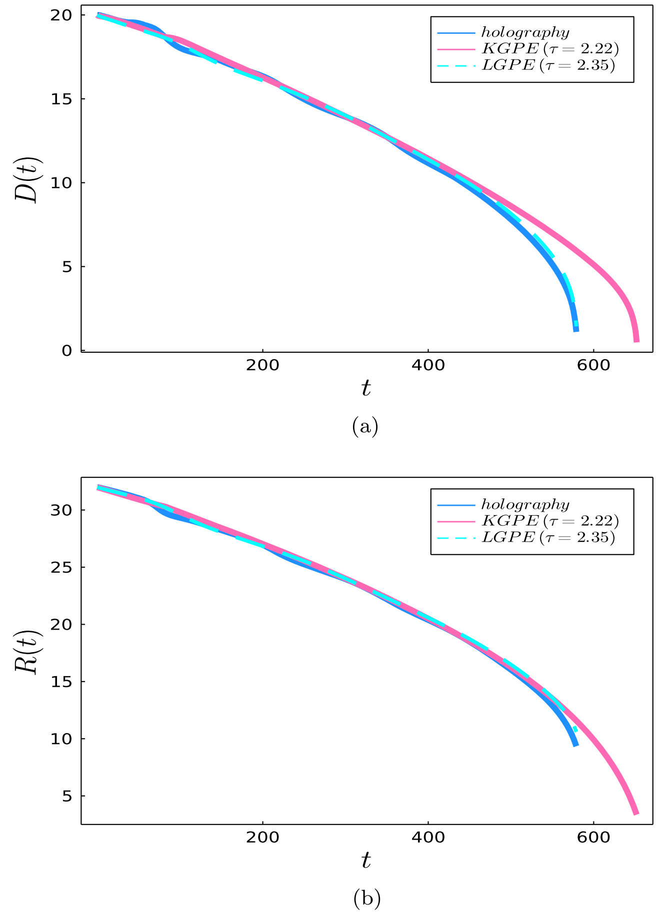

FIG. 2. The matching results for the temporal evolution of the relative distance and the center position of the holographic vortex dipole at $\tilde{\mu} = 4.5$ by DGPEs in (a) and (b), respectively.

namely, the holographic superfluid at $\tilde{\mu} = 4.5$. As illustrated in Fig. 1(a), the resulting holographic vortex can be well fitted by both models with the same healing length $\xi = 1.0$. On the other hand, as shown in Fig. 1(b), the corresponding holographic trajectory can be better modeled by LGPE with $\lambda = 1.85$ till the vortex dipole annihilation than KGPE, which starts to display an apparent deviation from the holographic behavior when the vortex dipoles get contacted with each other. Similarly, as one can see in Fig. 2, the real time evolution of the holographic vortex dipole can also be better captured by LGPE with $\tau = 2.35$ all the way to the annihilation stage than KGPE, which fails to describe the real time dynamics of the vortex dipole when close to each other. Similar matching results apply to the holographic superfluid at other chemical potentials. Here, we list only the resulting best fitting parameters in Table I for $\tilde{\mu} = 4.5$ and $\tilde{\mu} = 6$.

To substantiate the aforematched LGPE as the effective description of holographic vortex dynamics, we are left to check its generalization capability in other scenarios involving the vortex dynamics. As a demonstration, we examine the head-on collision of two vortex dipoles in light of the experimental setup prepared in Ref. [14]. To be more specific, with the matched parameters in Table I, we compare the numerical result from LGPE and that from our holographic simulation. We first present the four different stages for the head-on collision along the horizontal direction by density plot of the condensate in Fig. 3, where the vortices manifest themselves at the locations of zero density. After the collision, the vortex (antivortex) from the right-moving vortex dipole is seen to recombine with the antivortex (vortex) from the left-moving one, leading to the formation of new vortex dipoles. Then the

TABLE I. The best fitting parameters in DGPEs for the holographic superfluid at $\tilde{\mu} = 4.5$ and $\tilde{\mu} = 6$.

DGPEs	$\tilde{\mu}$	μ	λ	γ	τ
LGPE	4.5	0.50	1.85	⋯	2.35
LGPE	6	2.61	1.51	⋯	4.70
KGPE	4.5	0.50	⋯	0.129	2.22
KGPE	6	2.61	⋯	0.085	4.50

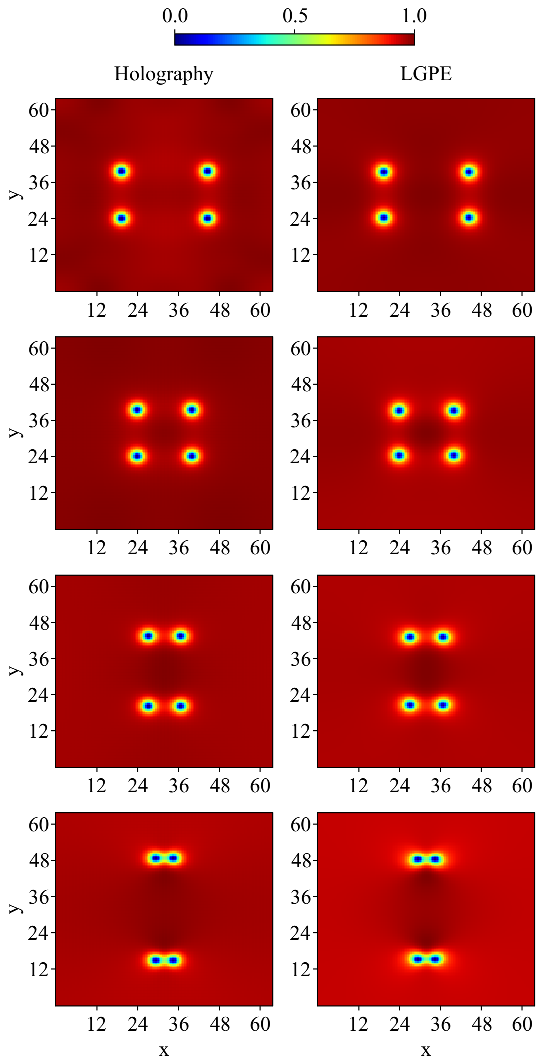

FIG. 3. The density plot of the normalized condensate $|\psi|^2/|\psi_0|^2$ for the head-on collision of the vortex dipoles in holographic superfluid at $\tilde{\mu} = 4.5$ (left) and the matched LGPE (right), which displays good agreement with each other. The top panel is for the initial stage, where the left- and right-moving vortex dipoles are prepared. The second panel is for the intermediate collision stage. The third panel denotes the newly formed vortex dipoles moving away from each other. The bottom panel is for the final annihilation stage.

new vortex dipoles move away from each other with one marching up and the other marching down. Eventually, both vortex dipoles get annihilated. As illustrated in Fig. 3, both results are in good agreement with each other. We further confirm this in Fig. 4 by tracking the motion of the involved four vortices. As one can see, the result from our holographic simulation still displays good agreement with that from LGPE till the annihilation of vortex dipoles. Actually, as demonstrated in Supplemental Material [24], such good agreement between the matched LGPE and the holographic superfluid is also confirmed in more

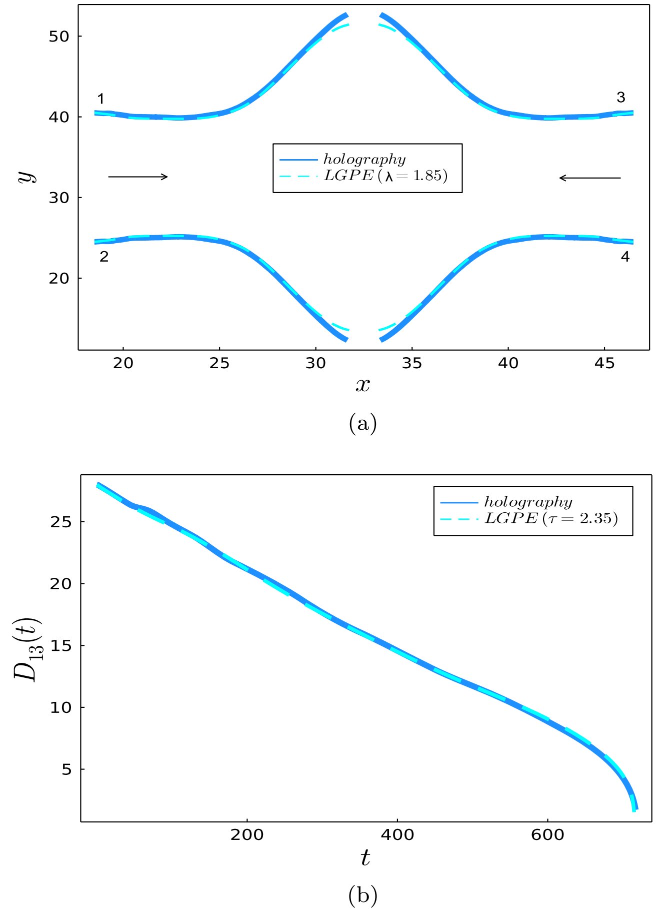

FIG. 4. The good agreement between the holographic simulation at $\tilde{\mu} = 4.5$ and the matched LGPE on the trajectories of vortices in (a) and the temporal evolution of the relative distance between vortex “1” from the right-moving vortex dipole and antivortex “3” from the left-moving vortex dipole in (b) during the head-on collision of the two vortex dipoles.

complicated scenarios such as the oblique collision of two vortex dipoles and the random motion of six vortices. This indicates that the matched LGPE can serve as an effective description of holographic vortex dynamics.

IV. DISCUSSIONS

By fitting the two available phenomenological DGPEs with the holographic superfluid model, we find that the holographic vortex dipole dynamics can be well matched by LGPE all the way down to the vortex dipole annihilation rather than KGPE, which matches up with our holographic data only when the vortex dipoles are far apart from each other and displays an apparent deviation when the vortex dipoles get close to each other. Although KGPE is much more widely used to attempt modeling the finite temperature BECs for decades than LGPE, actually the linear response theory of KGPE suffers from a serious defect, which, to our best knowledge, has not been noticed before (see Supplemental Material [24]). Together with the observation that LGPE displays a better consistence with holographic vortex dipole dynamics than KGPE, we are convinced that the reasonable phenomenological model for our holographic superfluid should be LGPE rather than KGPE. Our finding also invalidates the claim made recently by the authors in Ref. [23] that the holographic vortex dipole dynamics can be well fitted by KGPE. As detailed in Supplemental Material [24], their wrong result arises from the fact that a defective evolution scheme for the holographic numerical simulation is invoked therein. In this regard, our result presents a prime example of how a proper phenomenological dissipative model can be selected through the lens of holography to describe the far-from-equilibrium nonlinear dynamics beyond the hydrodynamic regime. We further consolidate LGPE as an effective description of holographic vortex dynamics by demonstrating its remarkable generalization capability in more complicated scenarios.

On the other hand, although the holographic superfluid model is superior to those phenomenological models such as DGPEs in the sense that it offers a first principles description of nonequilibrium dissipative dynamics at finite temperature, not only do DGPEs live in one less dimension, but also involve only one dynamical variable. Thus, it is much easier and much more efficient for one to perform a large scale of numerical simulations using DGPEs once the undetermined parameters are fixed. Now, according to our matching result, LGPE is selected by holography to serve as the appropriate phenomenological model for the vortex dynamics, so we can use it to greatly facilitate the quantitative confrontation of our holographic predictions with real experimental data. In particular, with the recent experimental progress in vortex dynamics [13,14], we expect our results can be verified by upcoming experiments in the future.

Last but not least, it is important to go beyond the probe limit taken in this paper. This is tantamount to including the backreaction of the matter fields onto the bulk metric. With this, one can explore the interaction between the stress tensor and the charge current and see how the superfluid component affects the dynamics of normal component. In particular, it is interesting to check whether the full dynamics can also be matched by the effective field theory approach to the superfluid dynamics at finite temperature [25].

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