Letter

Solitonic gravastars in a U(1) gauge-Higgs model

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(Received 16 March 2023; accepted 18 May 2023; published 16 June 2023)

We numerically obtain gravastar solutions as nontopological solitons in a system that consists of a U(1) gauge Higgs model with a complex scalar field and Einstein gravity. The solitonic gravastar solutions are compact enough to have a photon sphere.

DOI: 10.1103/PhysRevD.107.L121501

I. INTRODUCTION

Black holes, mathematical solutions to the Einstein equations, are widely accepted as astrophysical objects. Recently, two important observations concerning black holes in our Universe have been reported. One is the detection of gravitational waves from black hole binaries [1,2], and the other is photographic evidence of black holes at the center of M87 and Sgr A* in our Galaxy [3,4]. However, observable phenomena for distant observers should occur outside the event horizon, or before the formation of event horizons. Therefore, verification of astrophysical objects with an event horizon is still an open question.

As an alternative to black holes, a compact nonsingular object, the so-called ‘gravastar’, has been proposed to take quantum effects into account [5,6] (see also updated versions [7,8]). The interior geometry of the gravastar is described by a de Sitter metric and the exterior is described by a Schwarzshild metric, and these two regions are joined by a spherical shell with a finite thickness. The radius of the shell is smaller than the de Sitter horizon and larger than the Schwarzschild radius. Then, the gravastar has no horizon and no central singularity.

Nontopological solitons, on the other hand, have been studied as interesting astrophysical objects [9–16]. In a U(1) gauge-Higgs model coupled to a complex scalar field, various types of nontopological solitons are obtained [17–21]. In a type of nontopological solitons called ‘potential balls’ in Ref. [19], the vacuum energy of the Higgs scalar field is surrounded by a spherical shell, and the energy vanishes outside the shell. If the potential ball couples to the Einstein gravity, we would expect to obtain a solitonic gravastar solution.[1]

II. MODEL

We consider the theory described by the action

$$S = \int \sqrt{-g}d^4x\bigg(\frac{R}{16\pi G} - g^{\mu\nu}(D_\mu\psi)^*(D_\nu\psi) - g^{\mu\nu}(D_\mu\phi)^*(D_\nu\phi) - \frac{\lambda}{4}(|\phi|^2-\eta^2)^2 - \mu|\psi|^2|\phi|^2 - \frac{1}{4}g^{\mu\nu}g^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta}\bigg), \tag{1}$$

where R is the scalar curvature of a metric $g_{\mu\nu}$, g denotes $\det(g_{\mu\nu})$, G is the gravitational constant, ψ and ϕ are complex scalar fields, and $F_{\mu\nu} \coloneqq \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength of a U(1) gauge field A_μ, respectively. The scalar field ϕ has the self-coupling term characterized by a constant λ and a symmetry breaking scale η, and the interaction term with ψ characterized by a constant μ. Both scalar fields interact with the gauge field through the gauge-covariant derivative, $D_\mu \coloneqq \partial_\mu - ieA_\mu$ with a coupling constant e.

By varying the action (1), we obtain a coupled system of the field equations,

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[1]Nontopological soliton solutions with dustlike energy that couple to the Einstein gravity were obtained [22].

$$\frac{1}{\sqrt{-g}}D_\mu(\sqrt{-g}g^{\mu\nu}D_\nu\psi) - \mu\psi|\phi|^2 = 0, \tag{2}$$

$$\frac{1}{\sqrt{-g}}D_\mu(\sqrt{-g}g^{\mu\nu}D_\nu\phi) - \frac{\lambda}{2}\phi(|\phi|^2 - \eta^2) - \mu|\psi|^2\phi = 0, \tag{3}$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}F^{\mu\nu}) = j_\psi^\nu + j_\phi^\nu, \tag{4}$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{5}$$

where j_ψ^μ and j_ϕ^μ are four-currents defined by

$$\begin{aligned} j_\psi^\mu &\coloneqq ie(\psi^*(D^\mu\psi) - (D^\mu\psi)^*\psi), \\ j_\phi^\mu &\coloneqq ie(\phi^*(D^\mu\phi) - (D^\mu\phi)^*\phi), \end{aligned} \tag{6}$$

$R_{\mu\nu}$ is the Ricci tensor, and $T_{\mu\nu}$ is the energy-momentum tensor defined by

$$\begin{aligned} T_{\mu\nu} = {} & 2(D_\mu\psi)^*(D_\nu\psi) - g_{\mu\nu}(D_\alpha\psi)^*(D^\alpha\psi) \\ & + 2(D_\mu\phi)^*(D_\nu\phi) - g_{\mu\nu}(D_\alpha\phi)^*(D^\alpha\phi) \\ & - g_{\mu\nu}\left(\frac{\lambda}{4}(|\phi|^2 - \eta^2)^2 + \mu|\psi|^2|\phi|^2\right) \\ & + \left(F_{\mu\alpha}F_\nu{}^\alpha - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right). \end{aligned} \tag{7}$$

The action (1) is invariant under the transformations

$$\psi(x) \to \psi'(x) = e^{i(\chi(x)-\gamma)}\psi(x), \tag{8}$$

$$\phi(x) \to \phi'(x) = e^{i(\chi(x)+\gamma)}\phi(x), \tag{9}$$

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) + e^{-1}\partial_\mu\chi(x), \tag{10}$$

and we have conservation equations of the currents,

$$\partial_\mu(\sqrt{-g}j_\psi^\mu) = 0 \quad \text{and} \quad \partial_\mu(\sqrt{-g}j_\phi^\mu) = 0. \tag{11}$$

We assume a static and spherically symmetric metric,

$$\begin{aligned} ds^2 = {} & -\sigma(r)^2\left(1 - \frac{2m(r)}{r}\right)dt^2 + \left(1 - \frac{2m(r)}{r}\right)^{-1}dr^2 \\ & + r^2 d\theta^2 + r^2\sin^2\theta d\varphi^2, \end{aligned} \tag{12}$$

and spherically symmetric fields in the form

$$\psi = e^{-i\omega t}u(r), \qquad \phi = e^{-i\bar{\omega}t}f(r), \qquad A_\mu dx^\mu = A_t(r)dt, \tag{13}$$

where $\sigma(r)$, $m(r)$, $u(r)$, $f(r)$, and $A_t(r)$, are functions of r, and parameters ω and $\bar{\omega}$ are constants. Total charges of ψ and ϕ on the $t = \text{const}$ slices defined by

$$Q_\psi \coloneqq \int d^3x\sqrt{-g}\rho_\psi, \qquad Q_\phi \coloneqq \int d^3x\sqrt{-g}\rho_\phi, \tag{14}$$

are conserved, respectively, where $\rho_\psi \coloneqq j_\psi^t$ and $\rho_\phi \coloneqq j_\phi^t$.

By using the gauge transformation (8)–(10), we can fix the variables as

$$\begin{aligned} & \psi(t, r) \to e^{i\Omega t}u(r), \qquad \phi(t, r) \to f(r), \\ & A_t(r) \to \alpha(r) \coloneqq A_t(r) + e^{-1}\bar{\omega}, \end{aligned} \tag{15}$$

where $\Omega \coloneqq \bar{\omega} - \omega$. Substituting (12) and (15) into (2)–(5), we obtain a system of coupled ordinary differential equations to be solved in the form:

$$u'' + \left(\frac{2}{r}\left(1 + \frac{m - rm'}{r - 2m}\right) + \frac{\sigma'}{\sigma}\right)u' + \left(1 - \frac{2m}{r}\right)^{-1}\left(\frac{(e\alpha - \Omega)^2 u}{\sigma^2(1 - 2m/r)} - \mu f^2 u\right) = 0, \tag{16}$$

$$f'' + \left(\frac{2}{r}\left(1 + \frac{m - rm'}{r - 2m}\right) + \frac{\sigma'}{\sigma}\right)f' + \left(1 - \frac{2m}{r}\right)^{-1}\left(\frac{e^2 f\alpha^2}{\sigma^2(1 - 2m/r)} - \frac{\lambda}{2}f(f^2 - \eta^2) - \mu f u^2\right) = 0, \tag{17}$$

$$\alpha'' + \left(\frac{2}{r} - \frac{\sigma'}{\sigma}\right)\alpha' + \left(1 - \frac{2m}{r}\right)^{-1}(-2e^2 f^2\alpha - 2e(e\alpha - \Omega)u^2) = 0, \tag{18}$$

$$\frac{2m'}{r^2} - 8\pi G\left[\frac{e^2 f^2\alpha^2 + (e\alpha - \Omega)^2 u^2}{\sigma^2(1 - 2m/r)} + \left(1 - \frac{2m}{r}\right)\left(\left(\frac{df}{dr}\right)^2 + \left(\frac{du}{dr}\right)^2\right) + \frac{\lambda}{4}(f^2 - \eta^2)^2 + \mu f^2 u^2 + \frac{1}{2\sigma^2}\left(\frac{d\alpha}{dr}\right)^2\right] = 0, \tag{19}$$

$$\frac{(1 - 2m/r)\sigma'}{r\sigma} - 8\pi G\left[\frac{e^2 f^2\alpha^2 + (e\alpha - \Omega)^2 u^2}{\sigma^2(1 - 2m/r)} + \left(1 - \frac{2m}{r}\right)\left(\left(\frac{df}{dr}\right)^2 + \left(\frac{du}{dr}\right)^2\right)\right] = 0, \tag{20}$$

where prime denotes the derivative with respect to r.

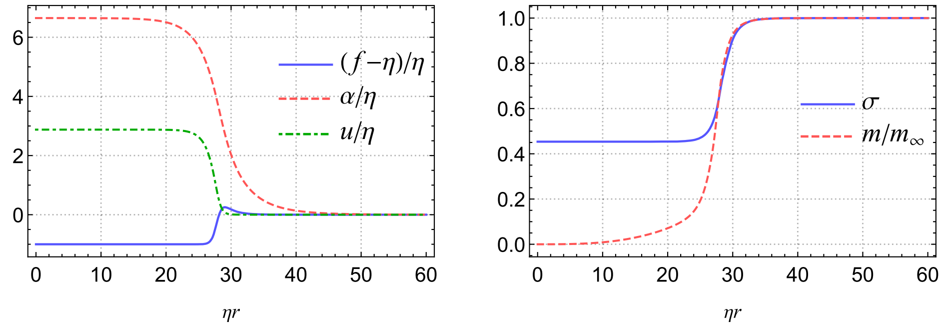

FIG. 1. Field configurations of the numerical solution for the parameter $\Omega/\eta = 0.665$. The scalar fields u, f and the gauge field α are plotted in the left panel, and the metric components σ and m are plotted in the right panel. At the origin, $r = 0$, it is found that $\alpha = \Omega/e$ and $f = 0$. The mass at infinity is obtained numerically as $m_\infty = 11.94\eta^{-1}$.

We require that all fields at the origin be regular, and that the scalar fields and the gauge field be localized in a finite region; then we impose

$$\frac{d\sigma}{dr} = 0, \qquad m = 0, \qquad \frac{du}{dr} = 0, \qquad \frac{df}{dr} = 0, \\ \frac{d\alpha}{dr} = 0, \quad \text{at } r = 0, \tag{21}$$

and

$$u = 0, \qquad f = \eta, \qquad \alpha = 0, \qquad \text{at spatial infinity.} \tag{22}$$

On these assumptions, the geometry should be described by a Schwarzschild metric in a far region, namely, we can impose

$$\sigma = 1, \qquad m = m_\infty = \text{const}, \qquad \text{at spatial infinity.} \tag{23}$$

III. SOLITONIC GRAVASTAR SOLUTIONS

We fix the coupling constants as $e = 0.1$, $\mu = 1.4$, and $\lambda = 1.0$, and we set the symmetry breaking scale $\eta = 10^{-2} M_P$, for an example.[2] In Fig. 1, the field variables of a numerical solution are shown as functions of r. The matter variables u, f, and α change quickly in a layer of thickness $\Delta r \sim 10\eta^{-1}$ around radius $r = r_{\text{sl}} \sim 28\eta^{-1}$. We call the layer the surface layer.

Outside the radius r_{sl}, matter variables decay to the values for the symmetry-breaking vacuum, namely the fields are excited in the compact region inside the radius. The fact that the metric functions $\sigma = 1$ and $m = m_\infty =$ const means the metric exhibits the Schwarzschild metric,

$$ds^2 = -\left(1 - \frac{2m_\infty}{r}\right)dt^2 + \left(1 - \frac{2m_\infty}{r}\right)^{-1} dr^2 \\ + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{24}$$

where the value of m_∞ is obtained numerically as $11.94\eta^{-1}$.

Inside the radius r_{sl}, we see that $u = \text{const}$, $f = 0$, $\alpha = \Omega/e$, then only the potential term of ϕ contributes to the energy-momentum tensor as

$$T^t_t = T^r_r = T^\theta_\theta = T^\varphi_\varphi = -\frac{\lambda}{4}\eta^4. \tag{25}$$

Using the log-log plot of $m(r)$ in Fig. 2, we see that

$$m(r) = \frac{\Lambda}{6} r^3, \tag{26}$$

where the value of Λ is given by

$$\Lambda = 8\pi G \frac{\lambda}{4}\eta^4 \sim 6.3 \times 10^{-4}\eta^2. \tag{27}$$

Furthermore, since σ takes a constant, say σ_0, the geometry is described by the de Sitter metric given by

$$ds^2 = -\left(1 - \frac{\Lambda}{3}r^2\right)d\tilde{t}^2 + \left(1 - \frac{\Lambda}{3}r^2\right)^{-1} dr^2 \\ + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{28}$$

where $\tilde{t} \coloneqq \sigma_0 t$.

The surface layer connects the de Sitter inner region and the Schwarzschild outer region. The de Sitter horizon radius, r_{dS}, and the Schwarzschild radius, r_{Sch}, of the numerical solution is estimated as

$$r_{\text{dS}} = \sqrt{\frac{3}{\Lambda}} \sim 0.7 \times 10^2 \eta^{-1}, \qquad r_{\text{Sch}} = 2m_\infty \sim 24\eta^{-1}, \tag{29}$$

[2]For the set of parameters, the potential balls are found as solutions in the case that the gravity is decoupled [19].

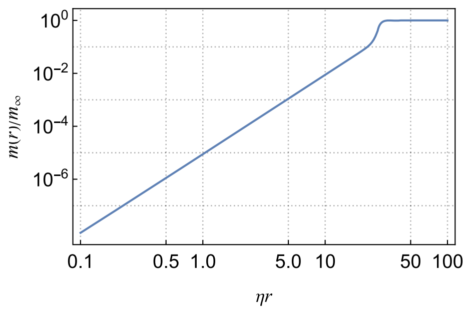

FIG. 2. The mass variable m/m_∞ is plotted as a function of r on a log-log scale.

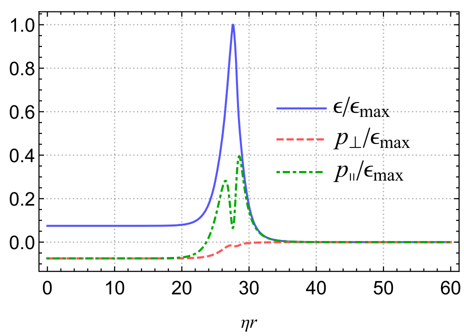

FIG. 3. Energy density ϵ, tangential pressure $p_\parallel$, and radial pressure $p_\perp$ are shown as functions of r. The pressure components are normalized by the maximum value of ϵ.

and therefore we have $r_{\rm Sch} < r_{\rm sl} < r_{\rm dS}$. The nontopological soliton solution describes the gravastar.

In Fig. 3 we show energy density $\epsilon = -T^t_t$, radial pressure $p_\perp = T^r_r$, and tangential pressure $p_\parallel = T^\theta_\theta = T^\varphi_\varphi$ for the numerical solution as functions of r. The surface layer has the structure within its thickness given by the Compton length of the gauge field $\sim(e\eta)^{-1}$. The energy density ϵ has a peak, and $p_\parallel$ has two peaks with almost $1/3 \sim 2/5$ of the peak height of ϵ, while $p_\perp$ is almost zero.

We show the charge densities ρ_ψ and ρ_ϕ in Fig. 4 as functions of r. The positive ρ_ψ is induced on the inner-side surface of the surface layer, and the negative ρ_ϕ on the outer side (see the left panel). Namely, an electric double layer emerges at the surface layer. The total charge contained inside a radius r, shown in the right panel, decays quickly outside of the surface radius, namely, the charge is screened for a distant observer. Therefore, the radial electric field appears in the electric double layer. Owing to this charge screening effect, the geometry of outside is given by the Schwarzschild metric instead of Reissner-Nordström one.

For a numerical solution, we define the surface radius of the solitonic gravastar, say $r_{\rm gs}$, by

$$m(r_{\rm gs}) \coloneqq 0.99\, m_\infty, \tag{30}$$

namely, 99% of total mass of the solitonic gravastar is included within the radius $r_{\rm gs}$. For the numerical solution shown in Fig. 1, we estimate $r_{\rm gs} \sim 33.2\eta^{-1}$. By the numerical values of m_∞ and $r_{\rm gs}$, we estimate the compactness as

$$C \coloneqq \frac{2m_\infty}{r_{\rm gs}} \sim 0.718 \geq 2/3, \tag{31}$$

then the solitonic gravastar is compact so that it has the photon sphere.

IV. SUMMARY

We have studied numerically the coupled system of a U(1) gauge-Higgs model with a matter complex scalar field and Einstein gravity, which is characterized by a set of parameters; coupling constants and a symmetry-breaking scale. For a choice of the parameters, we have found the solitonic gravastar solutions. Each solution has an internal de Sitter geometry in the symmetric vacuum with the potential energy of the Higgs scalar field, and an external Schwarzschild geometry in the symmetry-breaking vacuum. These regions are joined by a spherical surface layer with a finite thickness that has nonvanishing tangential pressure. Within the thickness of the surface layer, an electric double layer is produced by the two complex scalar fields, and the total charge is screened for a distant observer. For the set of parameters used in this paper, the solitonic gravastar obtained is compact enough to have a photon

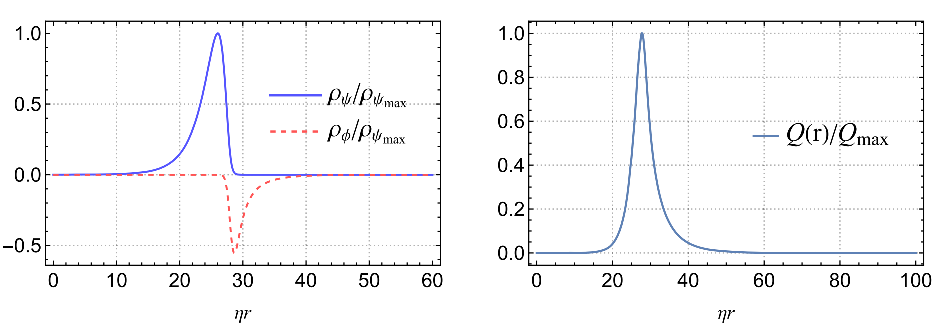

FIG. 4. The charge densities of the complex scalar fields, ρ_ψ and ρ_ϕ, normalized by the maximum value of ρ_ψ, $\rho_{\psi_{\rm max}} = 0.851\eta^{-3}$, are shown in the left panel. Total charge included within radius r, $Q(r)$, is shown in the right panel, where $Q_{\rm max} = 1.44 \times 10^4$.

sphere. Then, it is a compact regular object without the event horizon as an alternative to a black hole.

For the numerical solutions, the total gravitational mass $M_G = m_\infty/G$ is of the order of 10^3 times the Planck mass, which is much smaller than the astrophysical scale. The surface layer with the thickness about $1/3$ times the radius of the solitonic gravastar has the internal structure. These are different properties from original gravastars, a final state of gravitational collapsing astrophysical objects, where solutions are constructed by using a thin shell approximation [5,6]. However, as seen in the previous work [22], the total mass, surface radius, and thickness of the surface layer of the numerical solutions would depend on the model parameters. Therefore, it is interesting to clarify whether the solitonic gravastar can have astrophysical mass scale, and thickness of the surface layer becomes much smaller than its radius. It should be noted that for a fixed set of parameters in the Lagrangian, we expect the mass of the solitonic gravaster to be restricted to a finite range. Then, it would be impossible to replace a black hole with arbitrary mass by a solitonic gravastar.

There are important and interesting works on gravastar solutions; the stability of the solutions [23], the behavior of null geodesics around the photon sphere [24], gravitational-wave emission [25], and Hawking radiation [26]. These issues are addressed using thin shell approximations. We aim to study these problems using solutions in U(1) gauge Higgs models in future works. Furthermore, it would be interesting to investigate whether the solitonic gravastar solutions are a possible final state for the gravitational collapse of the system.

ACKNOWLEDGMENTS

We would like to thank K.-i. Nakao and H. Yoshino for valuable discussions and comments. This work was partly supported by Osaka Central Advanced Mathematical Institute: MEXT Joint Usage/Research Center on Mathematics and Theoretical Physics No. JPMXP0619217849.

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