Letter

Constructing on-shell operator basis for all masses and spins

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We propose a theory to systematically construct a complete set of on-shell effective operator bases involving massive particles with any spins. The amplitude bases involving massive fields can be factorized into two charged and neutral parts under the little groups of massive particles, respectively. The complete bases of these two parts can be constructed by the Young diagrams of Lorentz subgroup $SU(2)_r$ and global symmetry $U(N)$ (N is the number of external particles), respectively, without any redundancies. The corresponding effective field theory bases with the lowest dimension can be obtained by eliminating the linear correlation bases from a complete but redundant set of bases with all possible polarization tensors. Based on this theory, the amplitude bases involving identical particles can be constructed by a matrix projection method. A generic massive effective field theory can thus be constructed automatically by computer programs.

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I. INTRODUCTION

The standard model (SM) successfully describes particle physics up to the TeV scale, but it cannot explain some puzzles such as Higgs naturalness [1,2], dark matter [3,4], neutrino mass [5,6], etc. These puzzles suggest that SM is incomplete and that new physics (NP) should be introduced. Nevertheless, NP is not observed, indicating that the scale of NP may be too high to be within the direct experimental search. So detecting its indirect low-energy effects may be the only available way to search for NP currently, which motivates the study of effective field theory (EFT).

A complete set of EFT bases is essential for fully parametrizing the infrared effects of any ultraviolet theory. But constructing the EFT bases is difficult in traditional field theory because of EFT operator redundancy from the equations of motion (EOMs) and integration by parts (IBPs). Recently it was found that scattering amplitudes are efficient in dealing with some problems in EFT, such as calculating SM EFT running [7–10] and constructing scalar EFTs with nontrivial soft limits [11–14]. Especially it can efficiently construct EFT bases of massless particles, called amplitude bases [15–18], without EOM and IBP redundancy through the semistandard Young tableaux (SSYTs) of the global symmetry of massless spinors [19].

However, this method only applies to constructing amplitude bases for massless fields. On the other hand, EFTs involving massive particles are also widely applied in particle physics, such as Higgs EFT (HEFT) [20,21], dark matter EFT [22–26], and high spin particles [27,28]. Previous attempts have been focused on constructing the EFT bases of massive particles with spin $\leq 1/2$ [29,30], which can be directly mapped from massless bases since the number of physical degrees of freedom is the same. For the case of generic massive EFT, there is no efficient method to construct their EFT bases involving massive higher spin particles.

In this paper, we propose a novel theory to solve this problem. We first split the massive amplitude basis into two parts: the massive little group (LG) tensor structure (MLGTS), which is required to be the holomorphic function of massive right-handed spinors, and the massive LG neutral structure (MLGNS), which is only charged

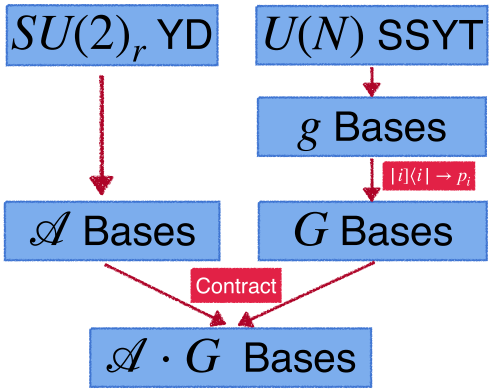


FIG. 1. Procedure for constructing $\mathcal{A} \cdot G$ bases.

under LGs of massless particles. The complete bases of MLGTS and MLGNS can be constructed by the representations of Lorentz subgroup $SU(2)_r$ ($SO(3,1) \simeq SU(2)_l \otimes SU(2)_r$) and the $U(N)$ global symmetry, respectively. Then the complete massive amplitude bases can be obtained by contracting the MLGTS bases with their corresponding MLGNS bases (the procedure is shown in Fig. 1).

Based on these complete bases, using the group representation theorem and some algebraic methods (details can be found in [31]), a set of lowest-dimensional scattering amplitude bases that can be directly mapped to operator bases can be constructed systematically. The amplitude bases involving identical particles can be projected by acting the Young operators on the lowest dimensional amplitude bases [31]. Within our framework, the complete bases of any massive EFT, such as HEFT and dark matter EFT, can be constructed automatically by computer programs.

II. CONSTRUCTING AMPLITUDE BASES

The modern on-shell method is to directly construct scattering amplitudes based on some physical principles, such as Lorentz invariance and locality. Lorentz symmetry requires that amplitudes should be covariant under the LG, thus being the functions of spinor-helicity variables charged under the LG. Massive spinors can be decomposed from the momentum of a massive particle-i,

$$(p_i)_{\dot{\alpha}\alpha} \equiv (p_i)_\mu (\sigma^\mu)_{\dot{\alpha}\alpha} = \epsilon_{IJ} |i^I]_{\dot{\alpha}} \langle i^J|_\alpha, \tag{1}$$

where the massive right/left-handed spinor $|i^I]_{\dot{\alpha}}/|i^J\rangle_\alpha$ is in the fundamental representation of Lorentz subgroup $SU(2)_r$ ($SU(2)_l$) and LG $SU(2)_i$ [the massless spinor $|i]_{\dot{\alpha}}/|i\rangle_\alpha$ is similar, except that its LG is $U(1)_i$]. Two spinors with the same chirality can contract together to form the Lorentz invariant building block of scattering amplitudes. The massive left- and right-handed spinors can be related to each other through EOMs, $p_i|i^I] = m_i|i^I\rangle$. The amplitudes for a massive (massless) particle with its spin s (helicity h) should be in the $(2s+1)$-dimensional representation of LG $SU(2)$ [take $2h$ unit charge of LG $U(1)$] [32,33]. The locality principle requires that the amplitude bases of EFT should be the independent polynomials of scalar products of the spinors.

Since the left-handed spinor can be transferred into the right-handed spinor through EOMs, the polarization tensor of a massive particle can always be expressed as a holomorphic function of right-handed spinors. Therefore each monomial term of the scattering amplitude of m massive and n massless particles can be factorized into MLGTS $\mathcal{A}$, which is the linear function of holomorphic polarization tensors of massive particles, and MLGNS G. The amplitude can be written as the combination of the terms with different $\mathcal{A}$ or G structures,

$$\mathcal{M}^{\{I\}}_{m,n} = \sum_{\{\mathcal{A},G\}} \sum_{\{\dot{\alpha}\}} \mathcal{A}^{\{I\}}_{\{\dot{\alpha}\}}(\{\epsilon_i\}) G^{\{\dot{\alpha}\}}(|j], |j\rangle, p_i), \tag{2}$$

where $\epsilon_i \equiv |i]^{\{I_1}_{\dot{\alpha}_1}, \ldots, |i]^{I_{2s_i}\}}_{\dot{\alpha}_{2s_i}}$ is the holomorphic polarization tensor of massive particle-i with its spin s_i. The bracket $\{I^i_1, \ldots, I^i_{2s_i}\}$ in the ϵ_{s_i} expression means that these $2s_i$ indices of LG $SU(2)_i$ are totally symmetric. Therefore, the EFT amplitude bases can be chosen as the polynomials of spinors which are the product of structures $\mathcal{A}$ and G.

Since $\mathcal{A}$ is the linear function of holomorphic ϵ_i, it is free of EOM and IBP redundancy because it does not contain any momentum factor. For the MLGNS G, it is only charged under the massless LG $U(1)_j$ associated with massless particle-j, so it is the function of massless spinors ($|j]$ and $|j\rangle$) and massive momentums (p_i).

A. $\mathcal{A}$ bases

Since $\mathcal{A}$ is linear in ϵ_i, it must belong to the outer product of all the massive polarization tensors' $SU(2)_r$ representations, $\mathcal{A} \subset \otimes^m_{i=1} (\mathbf{2s_i+1})$. So all the bases of MLGTS $\mathcal{A}$ one-to-one correspond to all the $SU(2)_r$ irreducible representations decomposed from this $\otimes^m_{i=1} (\mathbf{2s_i+1})$ representation. The complete set of $\mathcal{A}$ bases can be constructed as follows: Based on the Littlewood-Richardson rule, we first find all the irreducible $SU(2)_r$ representations decomposed from the outer product of the m massive polarization tensors' $SU(2)_r$ representations. Then all the MLGTS bases can be read off from the Young diagrams (YDs) of these irreducible representations according to the $SU(2)_r$ index permutation symmetry.

Next, we will demonstrate how to use YDs to construct MLGTS bases, taking four-point vertices of massive fermion-fermion-vector-scalar $\psi\psi' V h$ as an example. The polarization tensors are in the $(\mathbf{2s_i+1})$ representations of $SU(2)_r$, and their YDs are

$$\psi \sim \boxed{1} \quad \psi' \sim \boxed{2} \quad Z \sim \boxed{3}\boxed{3} \quad h \sim \bullet, \tag{3}$$

where the number in the box is used to label the $SU(2)_r$ indices of different polarization tensors, and the bullet • represents the $SU(2)_r$ singlet. Then we can reduce the outer product of these four YDs to the irreducible representations by Littlewood-Richardson rule and get four representations [34],

$$\begin{aligned}&\begin{array}{|c|}\hline 1\\\hline\end{array}\otimes\begin{array}{|c|}\hline 2\\\hline\end{array}\otimes\begin{array}{|c|c|}\hline 3&3\\\hline\end{array}\otimes\bullet\\ &=\begin{array}{|c|c|}\hline 1&2\\\hline 3&3\\\hline\end{array}\oplus\begin{array}{|c|c|c|}\hline 1&2&3\\\hline 3\\\cline{1-1}\end{array}\oplus\begin{array}{|c|c|c|}\hline 1&3&3\\\hline 2\\\cline{1-1}\end{array}\oplus\begin{array}{|c|c|c|c|}\hline 1&2&3&3\\\hline\end{array}\\ &\equiv\mathcal{A}^I_{[2,2]}\oplus\mathcal{A}^I_{[(3,1)^1]}\oplus\mathcal{A}^I_{[(3,1)^2]}\oplus\mathcal{A}^I_{[4]},\end{aligned}\tag{4}$$

where the subscript $[r_1, r_2, \ldots, r_n]$ denotes the shape of YD, having n rows and r_j boxes at the jth row, and the superscripts in $[(3,1)^{1,2}]$ denote two different SSYTs with the same shape of [3, 1]. Then the complete set of MLGTS bases can be read off from the above YDs filled with numbers. For example, according to the $SU(2)_r$ index permutation symmetry of these polarizations in the YD, the first base $\mathcal{A}_{[2,2]}$ is given by

$$\begin{aligned}&\mathcal{A}^{\{I\}}_{[2,2]}\equiv\begin{array}{|c|c|}\hline 1&2\\\hline 3&3\\\hline\end{array}\\ &=(|1^I]_{\dot\alpha}|2^J]_{\dot\beta}|3^{K_1}]_{\dot\gamma_1}|3^{K_2}]_{\dot\gamma_2}+\text{perms in}\,SU(2)_r\,\text{indices})\\ &=2[1^I3^{\{K_1}][2^J3^{K_2\}}].\end{aligned}\tag{5}$$

Since $\mathcal{A}$ is the linear function of polarization ϵ_i, the above expression contains one $|1^I]$, one $|2^J]$, and two $|3^K]$ s.

B. *G* bases

The massless amplitude bases can automatically get rid of EOM redundancy because the left-handed and right-handed spinor bases cannot be related by the EOM of massless spinor ($\not{p}_j|j]=0$). So they only have IBP redundancy (corresponding to momentum conservation), which can be systematically eliminated by the SSYT method [19]. Different from the massless case, MLGNS G bases can suffer from both EOM ($p_i^{\alpha\dot\alpha}|i^I]_{\dot\alpha}=m_i|i^I\rangle^\alpha$) and IBP redundancy.

We find that EOM redundancy in G bases can be eliminated by first constructing the massless limits of MLGNS G bases and then one-to-one mapping them into massive G bases. Since $G(|j],|j\rangle,p_i)$ is massive LG singlet, it has a definite massless limit, equal to the value as all the massive momentums in G go to the massless limit,

$$g(|j],|j\rangle,|i]\langle i|)=G(|j],|j\rangle,p_i)|_{p_i\to|i]\langle i|},\tag{6}$$

where $|i]_{\dot\alpha}\langle i|_\alpha$ is the massless limit of massive momentum $p_{i,\alpha\dot\alpha}$, and g is the massless limit of G. We know that the difference between two G bases related to each other through EOM must be proportional to the terms with the mass factors, which can only be generated through EOMs of massive spinors, so their massless limits must be the same. Therefore a set of $\{G\}$ bases with different massless limits must be free of EOM redundancy.

To construct a complete set of $\{G\}$ bases without EOM and IBP redundancy, we should first construct the complete set of massless $\{g\}$ bases without IBP redundancy via the SSYT method proposed in [19], and then one-to-one map $\{g\}$ into $\{G\}$ through recovering massive spinors from their massless limits, $|i],|i\rangle\to|i^I],|i_{I'}\rangle$, and choosing any kind of LG index contractions between $|i^I]$ s and $|i_{I'}\rangle$ s, equivalent to the momentum replacement $|i]_{\dot\alpha}\langle i|_\alpha\to p_{i,\dot\alpha\alpha}$ (two Gs generated from different contractions of LG indices are related by EOMs).

In order to form Lorentz singlet amplitudes, $\mathcal{A}$'s partner $\{G\}$ bases should be in the same representation as the $\mathcal{A}$ basis. The G bases should also be $SU(2)_l$ singlet, requiring that the total number of left-handed spinors should be even $\sum_{k=1}^{m+n}n_k=\text{even}\equiv L$, where n_k is the number of massive or massless spinor $|k^I\rangle$ or $|k\rangle$ in G bases. To be neutral under massive LGs, the number of massive spinors $|i^I]$ and $|i_{I'}\rangle$ should be equal, and massless LG symmetry requires that the difference between the number of massless spinors $|j]$ and $|j\rangle$ in G should be equal to twice the helicity h_j of massless particle-j,

$$\begin{aligned}&\tilde n_i-n_i=0,\quad\text{with}\quad i=1,\ldots,m\\ &\tilde n_j-n_j=2h_j,\quad\text{with}\quad j=m+1,\ldots,m+n,\end{aligned}\tag{7}$$

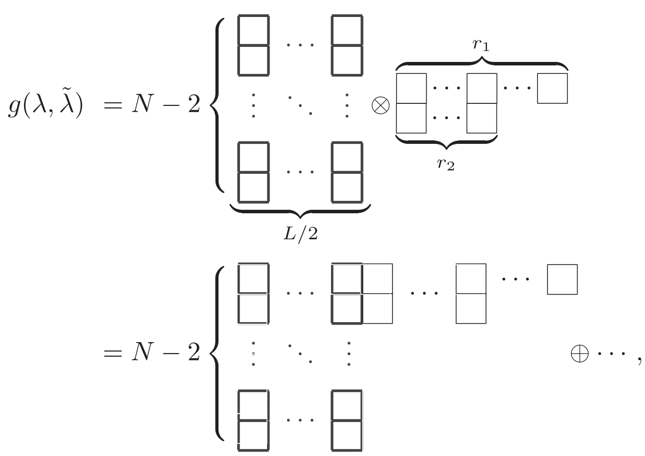
where $\tilde n_i$ ($\tilde n_j$) is the number of spinor $|i^I]$ ($|j]$). The corresponding $\{g\}$ bases should also satisfy these constraints.

Next, we will briefly discuss how to systematically construct the complete set of $\{g\}$ bases without IBP redundancy. The massless spinors of N external momentums $\tilde\lambda^k_{\dot\alpha}\equiv|k]$ ($\lambda_{k\alpha}\equiv|k\rangle$) are embedded into the (anti)fundamental representation of $U(N)$ symmetry with $k=1,\ldots,N$. So one basis of the $U(N)$ representation [i.e., a $U(N)$ SSYT] corresponds to a polynomial of massless spinors. Conversely, this polynomial can also be written down through the SSYT according to the permutation symmetry of the $U(N)$ indices. For example, the scalar product of a right/left-handed spinor pair can be obtained from the $U(N)$ SSYT with shape $[1^2]/[1^{N-2}]$ (the $[1^2]$ is the short notation of [1, 1] and so is $[1^{N-2}]$):

$$\begin{aligned}&\begin{array}{|c|}\hline i\\\hline j\\\hline\end{array}=(\tilde\lambda^i_{\dot\alpha}\tilde\lambda^j_{\dot\beta}-\tilde\lambda^j_{\dot\alpha}\tilde\lambda^i_{\dot\beta})=[ij]\\ &N-2\left\{\begin{array}{|c|}\hline k_1\\\hline k_2\\\hline \cdot\\\hline \cdot\\\hline\end{array}\right.=\varepsilon^{ijk_1\ldots k_{N-2}}\lambda_{i\alpha}\lambda_{j\beta}=\frac{\langle ij\rangle}{2}\varepsilon^{ijk_1\ldots k_{N-2}},\end{aligned}\tag{8}$$

where $\varepsilon^{ijk_1\cdots k_{N-2}}$ is the epsilon tensor. Notice that the columns in the SSYT associated with the $U(N)$ indices of λ are bold to distinguish them from $\tilde{\lambda}$ indices.

If some $g(\tilde{\lambda})$ $[g(\lambda)]$ polynomials, which are holomorphic functions of right-handed (left-handed) spinors, furnish a $U(N)$ representation, they must be free of IBP redundancy because there is no momentum factor in their expressions. Since $\tilde{\lambda}$ (λ) only has the $U(N)$ and $SU(2)_r$ $[SU(2)_l]$ indices, the Lorentz and $U(N)$ YDs of g are correlated. So the expression of the g basis can be determined by its $U(N)$ SSYT. For example, if $g(\tilde{\lambda})$ is the holomorphic function of (r_1+r_2) $\tilde{\lambda}$s and in the $(\mathbf{r_1}-\mathbf{r_2}+\mathbf{1})$ symmetric representation of $SU(2)_r$, its $U(N)$ YD is in the shape of $[r_1, r_2]$ [see the white YD in Eq. (9)]. While, if $g(\lambda)$ is the holomorphic function of L λs and is Lorentz scalar, its $U(N)$ YD has $N-2$ rows and $L/2$ columns [see the tall YD on the left in Eq. (9)]. For the nonholomorphic case, if the spinors λs ($\tilde{\lambda}$s) of $g(\lambda,\tilde{\lambda})$ are in the same $U(N)$ representation as $g(\lambda)$ $[g(\tilde{\lambda})]$, $g(\lambda,\tilde{\lambda})$ equals the outer product of $U(N)$ representations of $g(\lambda)$ and $g(\tilde{\lambda})$. It can be decomposed into irreducible representations via Littlewood-Richardson rules,



(9)

where $\oplus\cdots$ represents the other irreducible representations. Only the g bases in the first irreducible YD representation, obtained by gluing the tall (bold) YD and the short YD without shifting around the short YD, are independent [19]. Then the MLGNS G bases can be obtained from the g bases in the above $U(N)$ representations by restoring massive momentums from their massless limits,

$$G(|j],|j\rangle,p_i) = g(|j],|j\rangle,|i]\langle i|)|_{|i]_\alpha\langle i|_{\dot{\alpha}}\to p_{i\alpha\dot{\alpha}}}. \tag{10}$$

Finally, a complete set of massive amplitude bases can be constructed as in Fig. 1. We can prove that the amplitude bases constructed in this way are independent because of the independence of $\{\mathcal{A}\}$ and $\{g\}$ bases (rigorous proof is presented in Supplemental Material [35]).

Based on the constraints on G bases, if $\mathcal{A}$ is in the $(\mathbf{\Delta r}+\mathbf{1})$ $SU(2)_r$ representation, its partner G bases should have the following SSYTs:

(i) $U(N)$ YD $[(L/2+r_1),(L/2+r_2),(L/2)^{N-4}]$ filled with $L/2$ number-i for massive particle-i and $(L/2+2h_j)$ number-j for massless particle-j.

(ii) The shape of the YD is determined by

$$\begin{aligned} L &= D-N-\sum s_i-\sum h_j,\\ r_1 &= \frac{1}{2}\Big(D-N-\sum s_i+\sum h_j+\Delta r\Big),\\ r_2 &= \frac{1}{2}\Big(D-N-\sum s_i+\sum h_j-\Delta r\Big), \end{aligned} \tag{11}$$

where D is the dimension of the operator mapped from amplitude basis $\mathcal{A}\cdot G$, and $\sum s_i$ and $\sum h_j$ are the sums of all massive particle spin and massless particle helicity, respectively.

III. EXAMPLES

Next, we demonstrate how to construct four-point massive amplitude bases of $\psi\psi' Vh$ at dimensions $D=6$ and 8, corresponding to the operators $\psi\psi'(\partial V)h$ and $\psi\psi'(\partial V)h\partial^2$. The complete set of MLGTS bases $\mathcal{A}$ is shown in Eq. (4). Since the first basis $\mathcal{A}_{[2,2]}$ is a Lorentz scalar and its operator dimension is $D=6$, the basis at $D=6$ is just $\mathcal{A}_{[2,2]}$ (the corresponding MLGNS G is a constant). Since the dimension of the other MLGTS bases is the same as $\mathcal{A}_{[2,2]}$, and they are not Lorentz singlet (their G bases are nontrivial), the amplitude bases with these MLGTS bases must be at a dimension larger than 6.

The $D=8$ bases $\psi\psi'(\partial V)h\partial^2$ should take the structures $\mathcal{A}_{[2,2]}$, $\mathcal{A}_{[(3,1)^1]}$, and $\mathcal{A}_{[(3,1)^2]}$ [36]. According to the conditions in Eq. (11), we can get the $U(4)$ SSYTs of their G bases as follows:

$$\begin{aligned} G^{[(2,2)^{1,2}]} &\equiv \begin{array}{|c|c|}\hline 1&2\\\hline 3&4\\\hline\end{array}\,,\ \begin{array}{|c|c|}\hline 1&3\\\hline 2&4\\\hline\end{array}\,,\\ G^{[(3,1)^{1,2,3}]} &\equiv \begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4\\\cline{1-1}\end{array}\,,\ \begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3\\\cline{1-1}\end{array}\,,\ \begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2\\\cline{1-1}\end{array}\,, \end{aligned} \tag{12}$$

where the first (second) line is the G bases of $\mathcal{A}_{[2,2]}$ ($\mathcal{A}_{[(3,1)^{1,2}]}$), and the superscript $[(r_1,r_2)^i]$ of G basis denotes the ith $U(N)$ SSYT with the shape of $[r_1,r_2]$. Focusing on $\mathcal{A}_{[2,2]}$'s partner bases $G^{[(2,2)^{1,2}]}$, we can read off their expressions from their SSYTs,

$$\begin{aligned} G^{[(2,2)^1]} &= (\langle i_1 i_2\rangle\varepsilon^{13i_1i_2}[24]+\langle i_1 i_2\rangle\varepsilon^{23i_1i_2}[14]\\ &\quad+\langle i_1 i_2\rangle\varepsilon^{14i_1i_2}[23]+\langle i_1 i_2\rangle\varepsilon^{24i_1i_2}[13])|_{|i]\langle i|\to p_i}\\ &= \langle 2_J 4_I\rangle[4^I 2^J]-\langle 1_J 4_I\rangle[4^I 1^J]\\ &\quad-\langle 2_J 3_I\rangle[3^I 2^J]+\langle 1_J 3_I\rangle[3^I 1^J]. \end{aligned} \tag{13}$$

In the last identity, we add the massive LG indices to these massless limit spinors and choose one contraction pattern of these LG indices to restore massive momentums. $G^{[(2,2)^2]}$ can be obtained in the same procedures. Combining them with tensor structure $\mathcal{A}_{[2,2]}$, we get the explicit expressions of amplitude bases $\mathcal{A}_{[2,2]} \cdot G^{[2,2]}$ at $D = 8$,

$$\begin{aligned}\mathcal{A}^I_{[2,2]} \cdot G^{[(2,2)^1]} &= 8[1^I 3^{K_1}][2^J 3^{K_2}](s_{12} + 2s_{13}) + \mathcal{O}(m_i^2),\\ \mathcal{A}^I_{[2,2]} \cdot G^{[(2,2)^2]} &= 8[1^I 3^{K_1}][2^J 3^{K_2}](2s_{12} + s_{13}) + \mathcal{O}(m_i^2),\end{aligned} \tag{14}$$

where $s_{ij} = (p_i + p_j)^2$. It is apparent that these two bases are independent (the other amplitude bases at $D = 8$ are presented in Supplemental Material [35]).

IV. LOWEST DIMENSIONAL AMPLITUDE BASES

Since $\mathcal{A}$ is the holomorphic function of right-handed spinors, some spinors $|i^I]$ in $\mathcal{A}$ may contract with momentums p_i in G bases to produce an overall mass m_i factor, which makes $\{\mathcal{A} \cdot G\}$ bases cannot be directly mapped into EFT operator bases. To overcome this issue, we should reduce the dimension of $\{\mathcal{A} \cdot G\}$ bases to the lowest dimension, which means that any basis in this set cannot be decomposed into a combination of lower-dimensional bases.

A complete set of amplitude bases with the lowest dimension can be constructed in the following way [31]. First, we find that all the possible unfactorizable amplitudes can be classified by the massive particle-i's polarization tensor configuration (PTC),

$$\epsilon_i^{l_i} \equiv (|i^{\{I}\rangle)^{l_i}(|i^{I\}}])^{2s_i - l_i}, \tag{15}$$

where $l_i \in [0, 2s_i]$ is the number of left-handed spinors in the polarization tensor, parametrizing different PTCs, and the LG indices should be totally symmetric. Any basis with the lowest dimension must have one kind of PTCs, so a complete set of amplitude bases with this PTC must contain it. Then all the lowest dimensional amplitude bases must belong to all the complete basis sets, each with a different PTC. Meanwhile, each complete basis set with one kind of PTC can still be constructed by $SU(2)_r$ and $U(N)$ SSYT, similar to $\{\mathcal{A} \cdot G\}$ bases construction. Thus a complete but redundant basis set containing the complete bases with the lowest dimension can be systematically constructed. Then we can always decompose these redundant bases from low to high dimension to $\{\mathcal{A} \cdot G\}$ bases, and eliminate the linear correlation bases according to their coordinates in $\{\mathcal{A} \cdot G\}$ space. Based on the independence and completeness of $\{\mathcal{A} \cdot G\}$ bases, a complete set of amplitude bases with the lowest dimension can be picked up.

V. IDENTICAL PARTICLES

If the amplitude bases involve n identical bosons (fermions), they should be in the permutation group S_n's totally (anti)symmetric representation [the corresponding S_n YD is $[n]$ ($[1^n]$)]. Within the framework of our theory, the lowest dimensional amplitude bases involving identical particles can be systematically constructed by the Plethysm operation [37,38]. However, we find a much simpler way to construct such amplitude bases [31]. Based on group theory, the Young operator $\mathcal{Y}_{[R]}$ of $[R] \equiv [n]$ ($[1^n]$) is the permutation operation that makes the wave function of the n identical particles totally (anti)symmetric. So if an amplitude basis is in the $[R]$ representation, it should be the eigenstate of the Young operator $\mathcal{Y}_{[R]}$. To find these eigenstates, we can use $\mathcal{Y}_{[R]}$ to act on the complete amplitude bases and then get the representation matrix of $\mathcal{Y}_{[R]}$ in the basis space. Finally, the amplitude bases in $[R]$ representation correspond to the eigenvectors with nonzero eigenvalue.

VI. CONCLUSION AND OUTLOOK

The EFT of massive particles is widely applied in various fields of physics. How to construct massive EFT bases is still a problem. This work proposes a theory that can systematically build a complete set of on-shell massive amplitude bases without EOM and IBP redundancies. Some examples are given to demonstrate how to construct massive amplitude bases. Based on the independence and completeness of these bases, a complete set of amplitude bases with the lowest dimension that can be directly mapped into EFT operators could be obtained [31,39]. For the massive amplitude bases involving identical particles, it can be built through the Young operator of the permutation symmetry representation required by the spin statistic. Based on our theory, the generic EFTs involving massive fields with any spin can be automatically constructed by computer programs [31,41], such as HEFT and various dark matter EFTs, etc. The massive EFT could have many advantages in various physics research, and a lot of its exciting applications deserve to be explored in the future.

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[35] See Supplemental Materials at http://link.aps.org/supplemental/10.1103/PhysRevD.107.L111901 for rigorous proof of amplitude basis independence and an example for constructing $\{\mathcal{A}\cdot G\}$ bases.
[36] The MLGNS bases of $\mathcal{A}^I_{[4]}$ should contain at least four massive right-handed spinors to contract the bare $SU(2)_r$ indices of $\mathcal{A}^I_{[4]}$ and four massive left-handed spinors to be massive LG neutral. So the lowest dimension of the bases with $\mathcal{A}^I_{[4]}$ should be $D = 10$.
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