Letter

Critical fermions with spontaneously broken scale symmetry

Charlie Cresswell-Hogg[*] and Daniel F. Litim[†]
Department of Physics and Astronomy, University of Sussex, Brighton, BN1 9QH, United Kingdom

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We study relativistic fermions in three euclidean dimensions with four- and six-fermion interactions of the Gross-Neveu type. In the limit of many fermion flavors, and besides the isolated free fixed point, the theory displays a line of interacting ultraviolet fixed points. At the endpoint of the critical line, we establish that mass is generated through the spontaneous breaking of quantum scale invariance. Curiously, broken parity symmetry is a prerequisite for the spontaneous generation of mass rather than a consequence thereof. We also calculate critical exponents and find that hyperscaling relations are violated. Further similarities with critical scalar theories, and implications for conformal field theories and higher spin theories are discussed.

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I. INTRODUCTION

Fixed points of the renormalization group play a fundamental role in particle and statistical physics. At fixed points, theories become scale and possibly conformally invariant [1], and correlation functions are characterized by universal numbers [2]. An intriguing scenario arises if quantum scale invariance is broken spontaneously, leading to the appearance of a mass which is not determined by fundamental parameters. It has been speculated that this type of mechanism may explain the Higgs particle as a light dilaton in Standard Model extensions [3–5].

The spontaneous breaking of scale invariance was first observed by Bardeen, Moshe, and Bander (BMB) in strongly coupled $3d$ $O(N)$ or $U(N)$ symmetric scalar field theories [6–8]. Besides the Wilson-Fisher fixed point, the theory displays a line of UV fixed points at large N, curtesy of an exactly marginal sextic scalar self-interaction [7–11]. At the endpoint, scale symmetry is broken spontaneously and hyperscaling relations are violated [9], owing to a nonanalyticity of the effective potential. Subsequently, the phenomenon has been observed in multicritical bosonic theories [12], in finite N extensions [10], and away from integer dimensionality [13]. Further examples with spontaneously broken scale symmetry include Wess-Zumino models [14–16], and models with a topological Chern-Simons term [17–20].

[*]c.cresswell-hogg@sussex.ac.uk
[†]d.litim@sussex.ac.uk

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In this Letter, we study the spontaneous breaking of scale symmetry in a purely fermionic theory. This is motivated by the recent discovery that Gross-Neveu-type $(\bar\psi\psi)^3_{3\mathrm{d}}$ theories at large N display a line of interacting UV fixed points and an isolated IR fixed point [21], very much like bosonic $(\phi^2)^3_{3\mathrm{d}}$ theories [6–8] and with identical critical points and scaling dimensions. It is conceivable that this equivalence is rooted in a deeper connection between critical fermions and critical bosons. If so, we expect that the fermionic theory equally displays a version of spontaneous scale symmetry breaking. Here, we demonstrate that this is indeed the case. Implications of our results for 3d fermion-boson equivalences, conformal field theory, and higher spin theories are indicated.

II. GROSS-NEVEU THEORY

We recall fermionic quantum field theories in three euclidean dimensions with fundamental four- and six-fermion interactions of the Gross Neveu type [22]. The classical action takes the form

$$S_{\mathrm{f}} = \int_x \left\{ \bar\psi_a (\partial\!\!\!/ + M)\psi_a + \frac{G}{2}(\bar\psi_a\psi_a)^2 + \frac{H}{3!}(\bar\psi_a\psi_a)^3 \right\}, \tag{1}$$

where ψ_a are four-component Dirac spinors, and summation over the index $a \in \{1, \ldots, N\}$ is understood. The theory has a global $U(N)$ flavor symmetry. The theory (1) is nonperturbatively renormalizable, and characterized by a line of UV fixed points in the limit of many fermion flavors $1/N \to 0$ [21]. In terms of the couplings (m, g, h), which are the dimensionless counterparts of (M, G, H) in the action, the line of fixed points reads

$$m_* = 0, \qquad g_* = -\frac{1}{2}, \qquad |h_*| \le h_*^{\mathrm{crit}}. \tag{2}$$

At short distances, the mass $\propto m$ and the four fermion (4F) interaction $\propto g$ are relevant operators, while the 6F coupling h is exactly marginal. Consequently, small deviations $\delta m(\Lambda)$ and $\delta g(\Lambda)$ at the high scale Λ and the value of the 6F coupling h_* characterize UV-complete renormalization group (RG) trajectories running from the UV to the IR. If the 6F coupling h_* is taken to vanish, the theory is invariant under a discrete parity symmetry [23,24],

$$\psi \to \gamma^5\psi, \qquad \bar{\psi} \to -\bar{\psi}\gamma^5. \tag{3}$$

It follows that theories are either strictly massless ($\delta g > 0$), or massive ($\delta g < 0$ or $\delta m \neq 0$) owing to the dynamical or explicit breaking of parity symmetry.

If the 6F coupling is nonzero, parity symmetry is absent and explicit mass terms or parity-odd $(4n+2)$F interactions are permitted. For $\delta m \neq 0$, mass is generated explicitly. For $\delta m = 0$ and $\delta g < 0$ with $|h_*| < h_*^{\rm crit}$, mass is generated dynamically through strong interactions, while for $\delta m = 0$ and $\delta g > 0$ with $|h_*| < h_*^{\rm crit}$, theories remain strictly massless, and parity symmetry "emerges" in the infrared [21]. In this work, we investigate the critical endpoint $|h_*| = h_*^{\rm crit}$ to show that scale symmetry is broken spontaneously leading to a fermion mass without the breaking of any other symmetry.

III. RENORMALIZATION GROUP

To uncover the phenomenon in question, we employ functional renormalization [25–27]. Briefly, the method proceeds by adding a Wilsonian cutoff term to the path integral representation of a quantum field theory, which acts to integrate out momentum modes of the fields. By a Legendre transform, this defines an effective action Γ_k, dependent on the RG scale k, which interpolates between a classical action S at the high scale Λ and the full quantum effective action Γ in the IR limit $k \to 0$. The scale dependence of Γ_k is governed by an exact functional identity

$$\partial_t \Gamma_k = \frac{1}{2}{\rm STr}\Big\{[\Gamma_k^{(2)} + R_k]^{-1} \cdot \partial_t R_k\Big\}, \tag{4}$$

and $t = \ln(k/\Lambda)$. The right-hand side features a functional trace over all momenta and a sum over all internal indices. The bilinear cutoff term added to the action $\bar{\psi} R_k \psi$ respects the discrete symmetry (3) if we take the regulator proportional to $\not{q}$ in momentum space, $R_k(q) = \not{q} \cdot r(q^2/k^2)$. The cutoff shape function r vanishes as q^2/k^2 grows large, allowing UV modes to propagate in the trace, and becomes large as q^2/k^2 goes to zero, suppressing IR modes [28]. We adopt the optimized cutoff $r(x) = (1/\sqrt{x} - 1) \cdot \Theta(1-x)$ [29,30], involving the Heaviside step function. It allows for an analytical evaluation of functional traces and improves the stability and convergence of approximations in a wide range of quantum and statistical field theories [31–33]. We have checked that our central results are independent of the choice.

In the large-N limit, we solve the flow equation (4) exactly using the ansatz

$$\Gamma_k[\bar{\psi}, \psi] = \int {\rm d}^d x \{\bar{\psi}_a \not{\partial} \psi_a + V_k(\bar{\psi}_a\psi_a)\}, \tag{5}$$

where we kept space-time dimension d as a free parameter for now. It consists of a classical kinetic term and an "effective potential" V_k, parametrizing interactions built from the scalar combination $\bar{\psi}_a\psi_a$. A virtue of the large-N limit is that tensor structures other than those already present in (5) are not generated by fluctuations [34–36]. Higher derivative interactions will likewise remain absent from the action (5), and the kinetic term remains unrenormalized and anomalous dimensions vanish [35,37]. Hence, the closure of the ansatz (5) under (4) ensures that the theory can be solved exactly by solving the flow for V_k.

The flow equation for the function V_k is obtained by inserting the ansatz (5) into (4) and projecting onto constant fields. It takes the form

$$\partial_t v = -dv + (d-1)zv' - \frac{1}{1+(v')^2}, \tag{6}$$

which is written in terms of the dimensionless variables

$$z = k^{1-d}\bar{\psi}_a\psi_a, \qquad v_k(z) = k^{-d}V_k(\bar{\psi}_a\psi_a) \tag{7}$$

with primes denoting partial differentiation with respect to the field variable z. Prior to taking the large-N limit, each variable is additionally rescaled with a factor $4NA_d$, where $A_d = S_{d-1}/[d(2\pi)^d]$ with S_n the surface of a unit n sphere. The first two terms on the right-hand side of (6) represent the classical scaling of v and z, and are made explicit due to our choice of variables, while the final term originates from integrating out quantum fluctuations, i.e. the right-hand side of (4). Using the method of characteristics, the most general solution to (6) is found to be

$$z \cdot (v')^{1-d} - F_d(v') = G(v'e^t), \tag{8}$$

where the function $G(x) = x^{1-d}z_\Lambda(x) - F_d(x)$ is determined by the boundary condition $v'_\Lambda(z)$ at $k = \Lambda$, and with $z_\Lambda(v')$ the inverse function of $v'_\Lambda(z)$. The function $F_d(x)$ can be expressed in terms of a Gaussian hypergeometric integral for arbitrary dimension, $F_d(x) = \frac{2}{d-2}x^{2-d}{}_2F_1(2, 1-\frac{d}{2}; 2-\frac{d}{2}; -x^2)$. For $d = 3$ it can be expressed in terms of more elementary functions as

$$F_3(x) = -\frac{1}{x}\Big(2 + \frac{x^2}{1+x^2} + 3x \arctan x\Big). \tag{9}$$

Then, (8) provides implicit solutions $z(v', t)$ which can be converted into explicit functions $v'(z, t)$ for any scale k and all fields.

IV. CRITICAL POINTS

At a fixed point of the renormalization group, the flow (6) vanishes identically, for all fields, and the theory becomes scale invariant. To find all possible fixed points of the theory, the right-hand side of (8) must be t independent, meaning that the function G can at best be a constant. We therefore find a one-parameter family of integral curves $z(v')$ with

$$z \cdot (v')^{1-d} - F_d(v') = c, \qquad c \in \mathbb{R}, \tag{10}$$

which implicitly define all fixed points [21]. A heat map of the solutions (10) in three dimensions for all c is shown in Fig. 1. Series expansions about vanishing field values give

$$v'(z) = m + gz + \frac{1}{2}hz^2 + \mathcal{O}(z^3), \tag{11}$$

where m, g, and h are the dimensionless mass, four-fermion, and six-fermion couplings, respectively. On the other hand, from the fixed-point solution (10) we find

$$v' = -\frac{z}{2} + \frac{c}{8}z^2 - \frac{4-c^2}{16}z^3 + \frac{5c(8-c^2)}{128}z^4 + \mathcal{O}(z^5). \tag{12}$$

From the solution (12) we confirm that the couplings in (11) are given by the fixed point couplings (2), with

$$h_* = c/4. \tag{13}$$

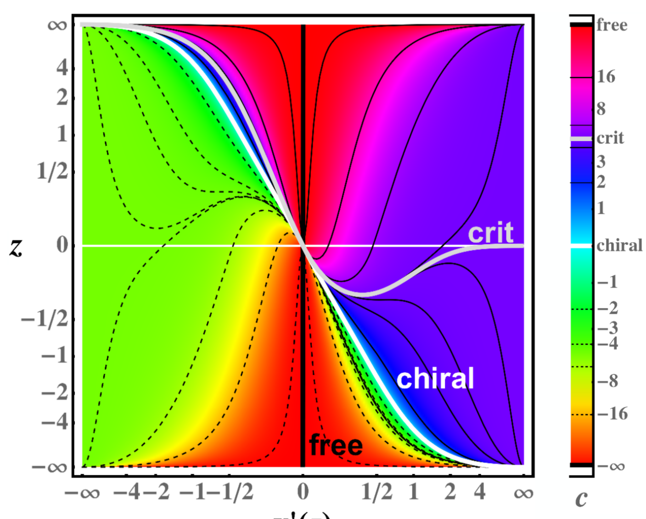

FIG. 1. All fixed point $v'(z)$ for all fields z of the large N $(\bar{\psi}\psi)^3_{3\mathrm{d}}$ theory, characterized and color coded by the parameter c. The chirally symmetric fixed point ($c_{\mathrm{chiral}} = 0$), the critical solution ($c_{\mathrm{crit}} = \frac{3}{2}\pi$), and the free fixed point ($1/c_{\mathrm{free}} = 0$) are highlighted by a thick white, gray, and black line, respectively. A further selection of solutions is shown by full ($c > 0$) or dashed ($c < 0$) black lines to guide the eye. Global fixed points arise for $1/c = 0$ (free) and for $|c| \in [0, c_{\mathrm{crit}}]$ (interacting). Axes are rescaled as $X \to X/(1+|X|)$ for better display.

All fixed points have vanishing mass parameter, identical 4F couplings, and a 6F coupling proportional to the free parameter c. Hence, the family of solutions corresponds to a line of fixed points, continuously connected to the parity-even Gross-Neveu theory by the exactly marginal 6F coupling h.

The line of fixed points does not extend indefinitely [21], although the local field expansion (11) is blind to this fact. In fact, the range is limited to within

$$0 \le |c| \le c_{\mathrm{crit}}, \tag{14}$$

where the critical value $c_{\mathrm{crit}} \equiv \frac{3}{2}\pi$ relates to solutions (10) which become singular at the origin of field space. This result can be appreciated by considering the large $|v'|$ limit. From (9) we have $F_3(x) = -c_{\mathrm{crit}} - \frac{2}{5}x^{-5} +$ subleading ($x \gg 1$), which with (10) implies that almost all solutions have the asymptotic behaviour $v' \sim \sqrt{z}$. In other words, the large v' limit corresponds to the large field limit. For the critical value c_{crit}, however, the leading terms cancel out and the subleading term dictates that large $v' \gg 1$ now relates to small fields ($z \ll 1$) as

$$v'_{\mathrm{crit}} = \left(-\frac{5}{2}z\right)^{-1/3} + \text{subleading}, \tag{15}$$

instead of (12). The singular and nonanalytic behaviour, which can be seen in Fig. 1 from the line marked "crit" indicates their borderline nature beyond which the effective potential becomes ill defined globally [21] (see also [9–11,15,16]).

We close with a comment on parity symmetry, which is realized whenever v' is an odd function of z. Since F_3 is an odd function, it follows from (10) that only the $c = 0$ and the $1/c = 0$ solutions respect parity. In Fig. 1, the former is marked "chiral" (white line), while the latter is given by the vertical line marked "free" (black). For any other value of c, the fundamental theory is not parity symmetric.

V. SPONTANEOUSLY BROKEN SCALE INVARIANCE

Quantum field theories at free or interacting fixed points of the RG are scale invariant by definition. Ordinarily, this implies the absence of a mass scale. Following [10,11,16], we explain how scale symmetry may nevertheless be broken spontaneously even though the theory is at an exact RG fixed point. To that end, we consider the quantity

$$M_k = V'_k(\bar{\psi}\psi)|_{\bar{\psi}\psi=0}, \tag{16}$$

which is extracted from the function V_k at vanishing field, also using (8). It relates to the physical fermion mass M in the limit $k \to 0$, irrespective of whether the theory is critical or not. At a fixed point, then, we have

$$M = \lim_{k\to 0} k \cdot m_*, \tag{17}$$

where the dimensionless fixed point value m_* is independent of k. Clearly, for finite m_* masslessness follows trivially from (17). Here, $m_* = 0$ for any $|c| < c_{\rm crit}$. However, this conclusion can be upset provided that m_* diverges, which happens precisely at $|c| = c_{\rm crit}$.

To see the implications more explicitly, we express the family of fixed point solutions (10) in terms of the dimensionful mass (16). Taking the limit $k \to 0$ for theories at a fixed point, we find a gap equation for the physical mass (17),

$$\left[c - \frac{3\pi}{2}\,\mathrm{sgn}(M)\right] M^2 = 0. \tag{18}$$

The gap equation depends on the parameter c which is proportional to the critical six-fermion coupling. We emphasize that the term $\frac{3\pi}{2}\mathrm{sgn}(M)$ in (18) originates from the nonanalytical behavior (15). For any value of c within the range (14), the prefactor is nonzero and the only solution to the gap equation (18) is that of a vanishing mass,

$$M = 0, \tag{19}$$

in accord with (12) and the vanishing of (17). Consequently, scale invariance at the fixed point remains intact. However, at the boundary of (14) where $c = \pm c_{\rm crit}$, the prefactor in (18) vanishes identically, and the fermion mass is unconstrained and free to take any value,

$$M = \text{free parameter}, \tag{20}$$

with $\mathrm{sgn}(M) = \mathrm{sgn}(c)$. In consequence, scale symmetry is broken spontaneously, and precisely because of (15). The result can also be interpreted as a version of dimensional transmutation [38], in that the role of a dimensionless parameter, c, is taken over by a dimensionful one, M, as a consequence of quantum fluctuations. The value of the spontaneously generated fermion mass is not determined by fundamental parameters and cannot be deduced from the nonanalytic critical potential. In accordance with Goldstone's theorem, a single massless scalar mode should appear in the spectrum, the dilaton, related to the generator of scale transformations. We conclude that the critical endpoints provide explicit examples of critical fermionic quantum field theories where mass is generated spontaneously.

VI. VIOLATION OF HYPERSCALING RELATIONS

Critical exponents measure the response of macroscopic observables to changes in the microscopic parameters of a system close to criticality. Many fluids, magnets, or models in particle and condensed matter physics share the same behavior as criticality described by universal numbers, such as the scaling exponent for the correlation length, ν, or the anomalous dimension of the order parameter. Further critical exponents are linked to these two by scaling relations. We begin by introducing the scaling exponent ν as

$$\xi \propto |r|^{-\nu}, \qquad r \to 0, \tag{21}$$

where ξ is the correlation length and r is a control parameter measuring the distance from the critical point. In the fermionic theory studied here, the correlation length is set by the physical fermion mass, $\xi \sim 1/M$, which diverges at a critical point. We also introduce the specific heat exponent α, defined by

$$\chi \propto |r|^{-\alpha}, \qquad r \to 0. \tag{22}$$

In a thermal phase transition, χ relates to the specific heat, while its zero-temperature analog, a "control parameter susceptibility," relates to the second derivative of the free energy with respect to r [39].

The correlation length exponent ν, the specific heat exponent α, and the space-time dimensionality d are linked to each other by the hyperscaling relation

$$d\nu = 2 - \alpha, \tag{23}$$

which generally holds true for all d below the upper critical dimension [39]. Here, we demonstrate that the hyperscaling relation (23) is violated.

Since the violation of (23) originates from the nonanalytic behavior (15), it invalidates the extraction of critical exponents as eigenvalues of RG beta functions. Therefore, we adopt more elementary ideas [9] to identify the BMB scaling exponents at the critical endpoint. We solve the RG equation (6) using the initial condition

$$v(z; t = 0) = mz + \frac{1}{2} g z^2 + \frac{1}{3!} h z^3 \tag{24}$$

at $k = \Lambda$, see (8), and evolve the solution to the IR by taking $k \to 0$. We then obtain a gap relation at zero field, whose solution determines the physical mass M for a given choice of the microscopic parameters m, g, h. Fixing g and h to their critical values allows the scaling of M with m near a fixed point to be determined. In this manner we extract the value for ν from (21) as

$$M \propto m^\nu, \qquad m \to 0^+ \tag{25}$$

in the different parameter regions.

In our setup, the gap relation takes the form of a transcendental equation $m = G(g, h, \tilde{M})$, where G is a function of the initial parameters at the scale Λ and the physical mass in units of this scale $\tilde{M} = M/\Lambda$. Without loss of generality, we take h positive. An expansion in powers of $\tilde{M}$ yields

$$m = \left(1 - \frac{g}{g_*}\right)\tilde{M} + \frac{1}{2g_*^2}\left(h - \frac{g}{g_*} h_*^{\rm crit}\right)\tilde{M}^2 + \left(\frac{h_*^{\rm crit} h}{2g_*^4} - 2\frac{g}{g_*}\right)\tilde{M}^3 + \mathcal{O}(\tilde{M}^4), \tag{26}$$

where $(g_*, h_*^{\rm crit}) = (-\frac{1}{2}, \frac{3}{8}\pi)$ denote the critical four- and six-fermion coupling, respectively.

We are interested in three regions of parameter space, each giving rise to distinct critical behaviors. The first region is $g > g_*$, which leads to the free theory in the IR limit. In this regime (26) dictates $m \propto M$ for $m \to 0$, reproducing classical scaling

$$m \propto M: \qquad \nu = 1. \tag{27}$$

The second region are the points on the critical line with $g = g_*$ and $h < h_*^{\rm crit}$. Upon setting g to its critical value the linear term in (26) drops out, leading to $m \propto M^2$ in the limit $m \to 0^+$, with nonclassical exponent

$$m \propto M^2: \qquad \nu = \frac{1}{2}. \tag{28}$$

Lastly, we consider the critical endpoint point where $g = g_*$ and $h = h_*^{\rm crit}$. In this case both the linear and quadratic terms in (26) vanish while the cubic term remains nonzero, resulting in $m \propto M^3$ with nonclassical exponent

$$m \propto M^3: \qquad \nu = \frac{1}{3}. \tag{29}$$

A linearization of local beta functions also gives the correct results at the free and the interacting fixed points (27) and (28), respectively. However, at the critical endpoint, the result from the local RG flows would have been (28) instead of (29). The failure of this standard method to capture the scaling at the endpoints correctly is due to the fact that the local flow is unaware of the nonanalyticity (15), which influences the scaling globally [9]. Finally, we note that it is not possible to make the first three coefficients in the gap equation (26) vanish simultaneously, meaning that the three cases cover all possibilities.

We now proceed to calculate the specific heat exponent α. The ordered phase of the system is characterized by a fermion condensate $Q = \langle \bar\psi\psi \rangle$, which is proportional to the physical mass M and is the conjugate variable to the bare mass m in an expansion of the free energy. This latter point implies that the control susceptibility χ is proportional to $\partial Q/\partial m$, up to an irrelevant prefactor. We then have $\alpha = 1 - \nu$, provided $\alpha > 0$. On the line of interacting fixed points, but away from the endpoints, we have $\nu = \frac{1}{2}$, giving

$$\alpha = \frac{1}{2}. \tag{30}$$

This result is in accord with the hyperscaling relation (23). At the critical endpoints, however, we have $\nu = \frac{1}{3}$ instead, leading to

$$\alpha = \frac{2}{3}. \tag{31}$$

We conclude that the hyperscaling relation (23) is violated at the endpoints, where scale invariance is broken spontaneously.

As a final remark, we note that our result is in quantitative agreement with the observed breaking of hyperscaling relations at the critical endpoint in scalar $(\phi^2)^3_{\rm 3d}$ models [9]. As such, our work adds the new result that scaling exponents between the fermionic $(\bar\psi\psi)^3_{\rm 3d}$ and the bosonic $(\phi^2)^3_{\rm 3d}$ theories agree at *all* UV or IR critical points, including at tricritical endpoints with spontaneously broken scale symmetry.

VII. DISCUSSION AND CONCLUSIONS

In this Letter, we have established for the first time that purely fermionic quantum field theories exhibit quantum critical points with spontaneously broken scale symmetry. The fingerprint for scale symmetry breaking are nonanalyticities in the effective action at a critical point (Fig. 1). The spontaneously generated mass breaks the conformal symmetry and becomes a new free parameter. Additionally, we have demonstrated that this nonperturbative effect violates hyperscaling relations.

The underlying trigger for scale symmetry breaking in fermionic theories is similar to what has been observed in critical scalar [6–11] or supersymmetric models [14–16], even though these theories appear to be otherwise rather different. The common denominator is the existence of a finite line of exactly marginal deformations. Whether the marginal direction arises out of an asymptotically free or asymptotically safe fixed point is irrelevant from the viewpoint of scale symmetry breaking.

The marginality of $(\bar\psi\psi)^3$ interactions at large N was previously noticed in [40], where it was also found that the 6F fixed point at the next-to-leading order in $1/N$ is located in a region of instability. This is consistent with our work in that the Gat-Kovner-Rosenstein fixed point [40] relates to one of the solutions with $|c| > c_{\rm crit}$, which are unphysical nonperturbatively (see Fig. 1). This pattern is structurally mirrored in critical scalar theories where a remnant of the Pisarski fixed point [41] is similarly located in the unstable region at the next-to-leading order, and thus unphysical nonperturbatively [6,7].

We also comment on our results from the viewpoint of fermion mass generation [21]. *A priori*, fermion mass in the theory (1) can be generated both explicitly and dynamically, with or without underlying parity symmetry. What is new here is that a fermion mass can also arise spontaneously at an interacting fixed point. Curiously, for the theory (1), it turns out that broken parity symmetry is a prerequisite for the spontaneous generation of mass, and not a consequence thereof.

At their critical points, our models become conformal field theories (CFT). It would then be useful to confirm results using CFT techniques and to understand whether

CFT three-point functions are modified at critical points where scale symmetry and hyperscaling relations are broken [42,43]. This is also of interest for higher spin gauge theories on AdS_4, which relate through the AdS/CFT conjecture to critical bosonic [44] or critical fermionic theories [45] on the AdS boundary. While the duality is well understood for parity-even boundary CFTs, it would be interesting to understand whether higher spin duals can also be found for parity-odd boundary CFTs, with or without spontaneously broken scale symmetry.

Finally, the parallels between fermionic $(\bar{\psi}\psi)^3_{3d}$ theories and bosonic $(\phi^2)^3_{3d}$ theories at large N continue to be striking. Despite of their elementary differences on the level of the path integral, both display equivalent fixed points and scaling dimensions, and equivalent scenarios with spontaneously broken scale symmetry. The evidence is highly suggestive of a deeper link, perhaps similar to bosonization dualities and holographic correspondences observed in Chern-Simons-matter theories [17–19,42,43,46], or in the spirit of large N equivalences and orbifold reductions [47], which we take as natural directions for future work.

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