Synchrotron radiation by slowly rotating fermions

Matteo Buzzegoli[®], Jonathan D. Kroth, Kirill Tuchin[®], and Nandagopal Vijayakumar[®] Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011, USA

(Received 12 September 2022; revised 4 November 2022; accepted 9 March 2023;

published 21 March 2023; corrected 18 October 2023)

We study the synchrotron radiation emitted by a charged fermion, rotating as a part of a larger system, in a constant magnetic field *B* parallel to the axis of rotation. The rotation is classical and independent of the magnetic field. The angular velocity of rotation Ω is assumed to be much smaller than the inverse magnetic length \sqrt{qB} , which allows us to ignore the boundary effects at $r = 1/\Omega$. We refer to such rotation as slow, even though in absolute value it may be an extremely rapid rotation. Using the exact solution of the Dirac equation, we derived the intensity of electromagnetic radiation, its spectrum, and its chirality. We demonstrate by explicit numerical calculation that the effect of rotation on the radiation intensity increases with the particle energy. Depending on the relative orientation of the vectors Ω and B and the sign of the electric charge, the rotation can either strongly enhance or strongly suppress the radiation.

DOI: 10.1103/PhysRevD.107.L051901

Synchrotron radiation is emitted by charged particles moving in magnetic fields. It has numerous applications in many areas of physics. In some systems, the charged particles are a part of a larger rotating system that is subject to an external magnetic field. A computation of the combined effect of rotation and the magnetic field on the intensity of the electromagnetic radiation is the subject of this paper. Our motivation derives from the recent discovery that the quark-gluon plasma produced in relativistic heavy ion collisions possesses high vorticity [1–6] and is subject to an intense magnetic field [7–14]. However, our results certainly apply to any rotating terrestrial or astrophysical system.

Consider a medium rotating in the laboratory frame with the constant angular velocity $\Omega = \Omega \hat{z}$. Let a fermion of electric charge q and mass M be embedded into the medium such that it is dragged by the medium to rotate with the same angular velocity. In particular, the medium exerts a radial force on the particle that balances the centrifugal force and prevents it from moving to infinity in the (xy) plane. In the rotating frame, the only unbalanced force exerted on the particle is the Lorentz force due to the constant magnetic field $B = B\hat{z}$. Thus, a classical particle performs a rotating motion about the z axis with the synchrotron frequency. The particle trajectory in the laboratory frame can be obtained by rotating it through the angle $-\Omega t$. The quantum state of the fermion is described by the Dirac equation. In the frame rotating with the angular velocity $-\Omega$, we use the symmetric gauge $A^{\mu} = (0, -By/2, Bx/2, 0)$, to cast it in the Schrödinger form $i\partial_t \psi = H\psi$ with the Hamiltonian

$$H = \gamma^0 \boldsymbol{\gamma} \cdot (\boldsymbol{p} - q\boldsymbol{A}) + \gamma^0 + \Omega J_z, \qquad (1)$$

where $p = -i\nabla$ and $J_z = -i\partial_{\phi} + \frac{i}{2}\gamma^x\gamma^y$ are the operators of momentum and longitudinal total angular momentum correspondingly and we use the units $\hbar = c = M = 1$. The Ω -independent part H_0 of the Hamiltonian (1) describes a fermion in the magnetic field and its spectrum E_0 , and the corresponding eigenfunctions ψ_0 are well known [15,16].

The leading-order formula for the synchrotron radiation by a nonrotating¹ fermion was obtained by Sokolov and Ternov [17]. In view of the axial symmetry, it is convenient to represent it in cylindrical coordinates $\{t, \phi, r, z\}$. The functions ψ_0 , corresponding to the nonrotating fermion, are eigenstates of H_0 , p_z , and J_z , whose eigenvalues we denote as E_0 , p_z , and *m* correspondingly. While p_z is continuous, the magnetic quantum number *m* is a half-integer. Additionally, each eigenfunction ψ_0 is labeled by a nonnegative integer radial quantum number *a*. In a nonrotating system, energy levels E_0 depend only on p_z and the principal quantum number $n = m + \frac{1}{2} + a$. The energy eigenfunctions in the rotating frame can be obtained by replacing $E_0 = E - m\Omega$,

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

¹Throughout this paper, by "nonrotating," we mean that the system containing the fermion performs no rotation, i.e., $\Omega = 0$.

$$\psi = e^{-iEt} \frac{e^{ip_z z}}{\sqrt{L}} \frac{e^{im\phi}}{\sqrt{2\pi}} \sqrt{|qB|} \begin{pmatrix} C_1 I_{n-1,a}(\rho) e^{-i\frac{\phi}{2}} \\ iC_2 I_{n,a}(\rho) e^{i\frac{\phi}{2}} \\ C_3 I_{n-1,a}(\rho) e^{-i\frac{\phi}{2}} \\ iC_4 I_{n,a}(\rho) e^{i\frac{\phi}{2}} \end{pmatrix}, \quad (2)$$

where $C_{1,3} = \frac{1}{2\sqrt{2}}B_+(A_+ \pm \zeta A_-)$, $C_{2,4} = \frac{1}{2\sqrt{2}}B_-(A_- \mp \zeta A_+)$ with $\zeta = \pm 1$ the fermion polarization, and

$$A_{\pm} = \left(\frac{E - \Omega m \pm p_z}{E - \Omega m}\right)^{\frac{1}{2}},\tag{3a}$$

$$B_{\pm} = \left(1 \pm \frac{\zeta}{\sqrt{(E - \Omega m)^2 - p_z^2}}\right)^{\frac{1}{2}}.$$
 (3b)

We also defined $\rho = \frac{|qB|}{2}r^2$ and

$$I_{n,a}(\rho) = \sqrt{\frac{a!}{n!}} e^{-\rho/2} \rho^{\frac{n-a}{2}} L_a^{n-a}(\rho),$$
(4)

where $L_n^{\alpha}(z)$ are the generalized Laguerre polynomials. In Refs. [18,19], the functions ψ were obtained in a different form. The energy spectrum reads

$$(E - \Omega m)^2 = 2n|qB| + p_z^2 + 1.$$
 (5)

Note that the energy levels explicitly depend on the magnetic quantum number m.

The support of the wave functions ψ_0 is the entire Minkowski space. As a function of the radial distance r from the rotation axis, they increase as a power law followed by exponential suppression. The typical size of the orbit can be gleaned from the expectation value $\langle r^2 \rangle = (2n + 2a + 1)/|qB|$. Unlike ψ_0 , the wave functions ψ belong to a rotating noninertial frame. Causality demands these functions vanish at the radial distance $r = 1/\Omega$ from the origin.² Nevertheless, if the magnetic field is strong and the rotation is relatively slow, the wave function ψ is always exponentially small in the causality violating region and can be ignored there. More precisely, our derivation is valid as long as

$$n, a \ll N_{\text{caus}} \equiv \frac{|qB|}{2\Omega^2}.$$
 (6)

Consider now the photon wave function with given energy ω , transverse momentum k_{\perp} , the longitudinal momentum k_z , and the magnetic quantum number l in cylindrical coordinates:

$$A = \frac{1}{\sqrt{2\omega V}} \Phi e^{-i\omega t}.$$
 (7)

We assume that photons are not interacting with the medium and, in particular, completely unaffected by the medium rotation. It is convenient to choose Φ to be an eigenstate of the curl operator. The corresponding eigenfunctions are states with definite chirality,

$$\boldsymbol{\Phi} = \frac{\omega}{k_{\perp}} \frac{1}{\sqrt{2}} (h\boldsymbol{T} + \boldsymbol{P}) e^{i(k_z z + l\phi)}, \qquad (8)$$

where $h = \pm 1$ labels right- or left-handed photon states. The toroidal and poloidal functions appearing in (8) read

$$\boldsymbol{T} = \frac{il}{kr} J_l(k_{\perp}r) \hat{\boldsymbol{r}} - \frac{k_{\perp}}{k} J_l'(k_{\perp}r) \hat{\boldsymbol{\phi}}$$
(9)

$$\mathbf{P} = \frac{ik_z k_\perp}{k^2} J_l'(k_\perp r) \hat{\mathbf{r}} - \frac{lk_z}{k^2 r} J_l(k_\perp r) \hat{\boldsymbol{\phi}} + \frac{k_\perp^2}{k^2} J_l(k_\perp r) \hat{\boldsymbol{z}}.$$
 (10)

For a photon emitted at the polar angle θ , $k_z = \omega \cos \theta$ and $k_{\perp} = \omega \sin \theta$.

The photon emission amplitude by a fermion of charge q transitioning between the two energy levels is given by the S-matrix element

$$S = (2\pi)\delta(E' + \omega - E)\frac{(-iq)}{\sqrt{2\omega V}}$$
$$\times \int \bar{\psi}_{n',a',p'_{z},\zeta'}(\mathbf{x})\Phi^{*}_{h,l,k_{\perp},k_{z}}(\mathbf{x}) \cdot \gamma \psi_{n,a,p_{z},\zeta}(\mathbf{x})d^{3}x, \quad (11)$$

where primed quantities refer to the final energy level. Integrating and summing $|S|^2$ over the phase space of the final particles, dividing it by the observation time, and multiplying by the photon energy yields the differential radiation intensity for a photon with the circular polarization h,

$$\frac{dW_{n,a,p_{z},\zeta}^{h}}{d\omega} = \frac{q^{2}}{4\pi} \sum_{n',a',\zeta'} \delta_{m,m'+l} \int \omega^{2} \sin\theta d\theta \delta(\omega - E + E') I_{a,a'}^{2}(x)$$

$$\times |\sin\theta[K_{4}I_{n-1,n'-1}(x) - K_{3}I_{n,n'}(x)]$$

$$+ K_{1}(h - \cos\theta)I_{n,n'-1}(x)$$

$$- K_{2}(h + \cos\theta)I_{n-1,n'}(x)|^{2}, \qquad (12)$$

where

$$K_1 = C'_1 C_4 + C'_3 C_2, \qquad K_2 = C'_4 C_1 + C'_2 C_3,$$

$$K_3 = C'_4 C_2 + C'_2 C_4, \qquad K_4 = C'_1 C_3 + C'_3 C_1, \qquad (13)$$

and we introduced a dimensionless variable $x = k_{\perp}^2/2|qB|$. Conservation of the *z* component of the angular momentum requires that m = m' + l. Energy conservation is expressed by the delta function in (12), which can be written as

 $^{^{2}}$ The importance of the causal boundary is discussed in Refs. [20–23].

$$\delta(\omega - E + E') = \frac{\delta(\omega - \omega_0)}{1 + \frac{\omega \cos^2 \theta}{E' - m'\Omega}}.$$
 (14)

The characteristic frequency ω_0 takes the simplest form in the frame where $p_z = 0$, which we can always choose by virtue of the translation symmetry along the rotation axis,

$$\omega_0 = \frac{E - m'\Omega}{\sin^2\theta} \left\{ 1 - \left[1 - \frac{\mathcal{B}\sin^2\theta}{(E - m'\Omega)^2} \right]^{1/2} \right\}, \quad (15)$$

with

$$\mathcal{B} = 2(n - n')|qB| - \Omega^2 (m - m')^2 + 2(E - m'\Omega)\Omega(m - m').$$
(16)

The radiation intensity for any initial and final fermion polarization states is obtained by summing over ζ' and averaging over ζ :

$$\overline{K_1^2} \equiv \frac{1}{2} \sum_{\zeta,\zeta'} K_1^2 = \overline{K_2^2} = \overline{K_3^2} = \overline{K_4^2}$$
$$= \frac{(E - m\Omega)(E' - m'\Omega) - 1}{4(E - m\Omega)(E' - m'\Omega)}, \qquad (17a)$$

$$\overline{K_1 K_2} = \overline{K_3 K_4} = \frac{\sqrt{2n|qB|}\sqrt{2n'|qB|}}{4(E - m\Omega)(E' - m'\Omega)},$$
(17b)

$$\overline{K_1 K_4} = -\overline{K_2 K_3} = -\frac{\sqrt{2n|qB|}\omega\cos\theta}{4(E - m\Omega)(E' - m'\Omega)}, \quad (17c)$$

$$\overline{K_1 K_3} = \overline{K_2 K_4} = 0. \tag{17d}$$

The total radiation intensity additionally requires integration over ω , which is trivial thanks to the delta function (14):

$$W_{\text{tot}} \equiv \frac{1}{2} \sum_{\zeta} W^{h}_{n,a,p_{z}=0,\zeta}$$

= $\frac{q^{2}}{4\pi} \sum_{n',a'} \int_{0}^{\pi} d\theta \frac{\omega_{0}^{2} \sin \theta}{1 + \frac{\omega_{0} \cos^{2} \theta}{E' - m' \Omega}} \frac{1}{2} (\Gamma^{(0)}_{n,a} + h \Gamma^{(1)}_{n,a}), \quad (18)$

where

$$\Gamma_{n,a}^{(0)} = I_{a,a'}^2(x) \{ 2\overline{K_1^2} [I_{n,n'-1}^2(x) + I_{n-1,n'}^2(x)] + \overline{K_1^2} \sin^2 \theta [I_{n,n'}^2(x) + I_{n-1,n'-1}^2(x) - I_{n,n'-1}^2(x) - I_{n-1,n'}^2(x)] \\
- 2\overline{K_1 K_2} \sin^2 \theta [I_{n,n'}(x) I_{n-1,n'-1}(x) + I_{n-1,n'}(x) I_{n,n'-1}(x)] \\
- 2\overline{K_1 K_4} \sin \theta \cos \theta [I_{n-1,n'-1}(x) I_{n,n'-1}(x) + I_{n-1,n'}(x) I_{n,n'}(x)] \}$$
(19)

$$\Gamma_{n,a}^{(1)} = I_{a,a'}^2(x) \{ 2\overline{K_1^2} \cos \theta (I_{n-1,n'}^2(x) - I_{n,n'-1}^2(x)) + 2\overline{K_1 K_4} \sin \theta (I_{n-1,n'-1}(x) I_{n,n'-1}(x)) - I_{n-1,n'}(x) I_{n,n'}(x) \}.$$
(20)

Integration over θ as well as summation over n' and a'can only be done numerically. The summations are restricted by energy and longitudinal momentum conservation and by the causality constraint (6) applied to the quantum numbers n' and a'. A similar inequality was found in Ref. [18]. Without the causality constraint (6), which itself follows from $\langle r^2 \rangle \Omega^2 < 1$, the sum over the phase space would be divergent. However, it can be shown that as long as $\Omega \ll \sqrt{|qB|}$ the intensity is independent of the cutoff N_{caus} . In the no-rotation limit $\Omega \to 0$, one can show that the sum $\sum_{n',a'}$ reduces to $\sum_{n'=0}^{n} \sum_{a'=0}^{\infty}$. Moreover, in this limit, the photon energy ω_0 depends only on n and n', but not on a and a'. This allows explicit summation over a'in (18), which can be performed using the identity $\sum_{a'} I_{a,a'}^2(x) = 1$ [17] and yields the well-known result for the synchrotron radiation intensity by a nonrotating fermion. We numerically verified that our results are not sensitive to the cutoff N_{caus} .

A typical synchrotron radiation spectrum is shown in Fig. 1. For comparison, we also plotted the spectrum emitted by the nonrotating fermion. It is seen that, while the spectrum of the nonrotating fermion depends only on the principal quantum number n', the spectrum of the rotating fermion is split in many lines having different a', as expected from the energy shift caused by rotation. Moreover, the positions of the spectral lines are shifted toward smaller values of ω , and their heights are diminished in comparison with the nonrotating spectrum. This indicates that the radiation intensity is suppressed when **B** is antiparallel to Ω and q < 0.

The total intensity is conventionally represented with respect to the corresponding classical expression

$$W_{\rm cl} = \frac{q^2}{4\pi} \frac{2(qB)^2 E^2}{3}.$$
 (21)

The result is shown in Figs. 2 and 3. The main observation is that the effect of rotation increases with energy. One can

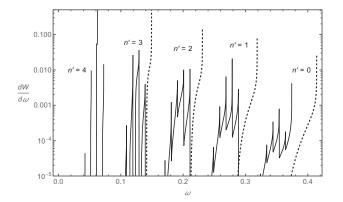


FIG. 1. The spectrum of synchrotron radiation (12) at qB = -0.1 emitted by a fermion with initial quantum numbers n = 5, a = 1, m = 7/2, and $p_z = 0$, summed over $a' \in [0, 16]$ and averaged over photon helicity h. Solid lines: $\Omega = -0.01$ (corresponding to E = 1.379); dashed lines: $\Omega = 0$ (corresponding to E = 1.414). Our units: $\hbar = c = M = 1$.

qualitatively understand this dependence by noting that the classical trajectory of the fermion is a combination of two circular motions: one with angular velocity Ω due to the system rotation and another one with angular velocity $\omega_B = qB/E$ due to the Lorentz force exerted by the magnetic field. The former is independent of the fermion energy *E*, whereas the latter decreases as E^{-1} . One can also notice that when the direction of rotation due to the magnetic field coincides with the direction of the system rotation of the fermion (e.g., qB > 0 and $\Omega < 0$) the result

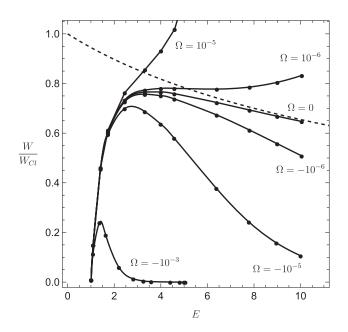


FIG. 2. The total intensity of the synchrotron radiation in units of the classical intensity (21) as a function of the initial energy *E* at qB = -0.01. Solid lines correspond to various angular velocities Ω , and the dashed line is the quasiclassical approximation at $\Omega = 0$. The dependence of the intensity on the initial value of *a* is weak and not noticeable in the figure. Our units: $\hbar = c = M = 1$.

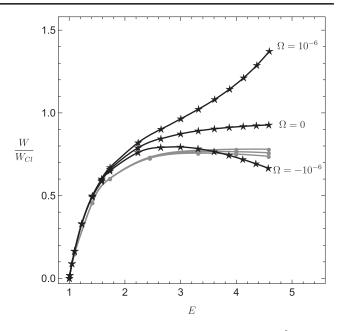


FIG. 3. The same as in Fig. 2 but with $\Omega = 0, \pm 10^{-6}$ and two values of *qB*. Gray lines with circles are for $qB = -10^{-2}$. Black lines with stars are for $qB = -10^{-3}$. Note the effect of rotation is greater at lower energies for the smaller field. Our units: $\hbar = c = M = 1$.

is enhancement of radiation. This happens because the rotating fermion experiences larger effective ω_B , hence a larger effective magnetic field [24]. Conversely, when the two rotations are in the opposite direction (e.g., qB > 0 and $\Omega > 0$), we observe suppression of the radiation.

At $\Omega = 0$, the quasiclassical formula (dashed line) approaches our exact result (dot-dashed line) at high energy *E*. This is because the quasiclassical approximation neglects the discreteness of the fermion spectrum, which is a good approximation only in the ultrarelativistic case. It is remarkable that at high energy the intensity of radiation by the rotating system deviates from that at $\Omega = 0$ even for very small Ω 's.

It seems from our numerical results that the maximum of the ratio W/W_{cl} for $\Omega < 0$, or the inflection point for $\Omega > 0$, depends on angular velocity roughly as $E_{max} \sim -\log_{10} |\Omega| - 2$. If this trend persists at even lower $|\Omega|$'s, then the effect of rotation on the synchrotron radiation may be essential even in nonextreme astrophysical systems that rotate with typical angular velocities. This observation is a strong motivation to investigate the synchrotron radiation in a variety of magnetic fields and angular velocities and will be a subject of a further study.

The effect of rotation on the synchrotron radiation that we have reported in this paper is mostly classical, as it stems from the peculiar form of the metric in the rotating coordinates. We expect that the quantum effects induced by rotation become prominent when the angular velocity becomes comparable or larger than the inverse magnetic length. In heavy-ion collisions, the direction of the magnetic field and the direction of rotation coincide. This implies that the synchrotron radiation by the negative charges must be significantly stronger than by the positive charges. As a result, we expect that rotation significantly enhances the contribution of the synchrotron radiation to the total photon spectrum, as compared to the nonrotating case [25]. The formalism developed in this paper lays the foundation

for the phenomenological applications that should be addressed in a dedicated work.

In summary, we computed the effect of rotation on the synchrotron radiation in the limit of relatively slow rotation. We argued that the effect of rotation is surprisingly strong, which makes it amenable to experimental study.

This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-87ER40371.

- L. Adamczyk *et al.* (STAR Collaboration), Global Λ hyperon polarization in nuclear collisions: Evidence for the most vortical fluid, Nature (London) 548, 62 (2017).
- [2] L. P. Csernai, V. K. Magas, and D. J. Wang, Flow vorticity in peripheral high energy heavy ion collisions, Phys. Rev. C 87, 034906 (2013).
- [3] W. T. Deng and X. G. Huang, Vorticity in heavy-ion collisions, Phys. Rev. C 93, 064907 (2016).
- [4] Y. Jiang, Z. W. Lin, and J. Liao, Rotating quark-gluon plasma in relativistic heavy ion collisions, Phys. Rev. C 94, 044910 (2016); 95, 049904(E) (2017).
- [5] X. L. Xia, H. Li, Z. B. Tang, and Q. Wang, Probing vorticity structure in heavy-ion collisions by local Λ polarization, Phys. Rev. C 98, 024905 (2018).
- [6] F. Becattini, G. Inghirami, V. Rolando, A. Beraudo, L. Del Zanna, A. De Pace, M. Nardi, G. Pagliara, and V. Chandra, A study of vorticity formation in high energy nuclear collisions, Eur. Phys. J. C 75, 406 (2015); 78, 354(E) (2018).
- [7] D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, The effects of topological charge change in heavy ion collisions: 'Event by event P and *CP* violation', Nucl. Phys. A803, 227 (2008).
- [8] V. Skokov, A. Y. Illarionov, and V. Toneev, Estimate of the magnetic field strength in heavy-ion collisions, Int. J. Mod. Phys. A 24, 5925 (2009).
- [9] V. Voronyuk, V. D. Toneev, W. Cassing, E. L. Bratkovskaya, V. P. Konchakovski, and S. A. Voloshin, (Electro-)Magnetic field evolution in relativistic heavy-ion collisions, Phys. Rev. C 83, 054911 (2011).
- [10] A. Bzdak and V. Skokov, Event-by-event fluctuations of magnetic and electric fields in heavy ion collisions, Phys. Lett. B 710, 171 (2012).
- [11] J. Bloczynski, X. G. Huang, X. Zhang, and J. Liao, Azimuthally fluctuating magnetic field and its impacts on observables in heavy-ion collisions, Phys. Lett. B 718, 1529 (2013).
- [12] W. T. Deng and X. G. Huang, Event-by-event generation of electromagnetic fields in heavy-ion collisions, Phys. Rev. C 85, 044907 (2012).
- [13] K. Tuchin, Time and space dependence of the electromagnetic field in relativistic heavy-ion collisions, Phys. Rev. C 88, 024911 (2013).

- [14] B. G. Zakharov, Electromagnetic response of quark-gluon plasma in heavy-ion collisions, Phys. Lett. B 737, 262 (2014).
- [15] C. G. de Oliveira and J. Tiomno, Representations of Dirac equation in general relativity, Nuovo Cimento 24, 672 (1962).
- [16] F. W. Hehl and W. T. Ni, Inertial effects of a Dirac particle, Phys. Rev. D 42, 2045 (1990).
- [17] A. A. Sokolov, I. M. Ternov, and C. W. Kilmister, *Radiation from relativistic electrons*, revised ed. (American Institute of Physics, New York, 1986).
- [18] H. L. Chen, K. Fukushima, X. G. Huang, and K. Mameda, Analogy between rotation and density for Dirac fermions in a magnetic field, Phys. Rev. D 93, 104052 (2016).
- [19] K. Mameda and A. Yamamoto, Magnetism and rotation in relativistic field theory, Prog. Theor. Exp. Phys. 2016, 093B05 (2016).
- [20] G. Duffy and A. C. Ottewill, The rotating quantum thermal distribution, Phys. Rev. D 67, 044002 (2003).
- [21] V. E. Ambrus and E. Winstanley, Rotating fermions inside a cylindrical boundary, Phys. Rev. D 93, 104014 (2016).
- [22] S. Ebihara, K. Fukushima, and K. Mameda, Boundary effects and gapped dispersion in rotating fermionic matter, Phys. Lett. B 764, 94 (2017).
- [23] M. N. Chernodub and S. Gongyo, Effects of rotation and boundaries on chiral symmetry breaking of relativistic fermions, Phys. Rev. D 95, 096006 (2017).
- [24] K. Tuchin, Magneto-rotational dissociation of heavy hadrons in relativistic heavy-ion collisions, Phys. Lett. B 820, 136582 (2021).
- [25] K. Tuchin, Role of magnetic field in photon excess in heavy ion collisions, Phys. Rev. C 91, 014902 (2015).

Correction: A typographical error in the value in the last sentence in the 11th paragraph has been fixed. Errors in the qB values presented in the captions of Figs. 1–3 have been set right. In the sixth, eighth, and ninth sentences of the 12th paragraph, the original wording "suppression" was replaced by "enhancement," "smaller" was replaced by "larger," and "enhancement" was replaced by "suppression," respectively. In the second sentence of the 16th paragraph, the original wording "positive" was replaced by "negative" and "negative" was replaced by "positive."