Letter

4D dS vacua from AdS vacua of type IIB string theory and the AdS distance conjecture

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In order for string theory to be made compatible with the low-energy observations of a positive cosmological constant, there have been attempts to construct de Sitter (dS) vacua in string theory which are particularly difficult to realize. Instead of attempting to find dS vacuum solutions, we point to a new way to make string theory consistent with low-energy dS cosmology. In this way, string theory lives in an anti–de Sitter (AdS) vacuum (which is simple to construct) that exists only in the high-energy regime; however, as going to the low-energy scales where the heavy string excitations and Kaluza-Klein modes are integrated out, we show that the effective picture of string theory in lower dimensions would exhibit a 4D dS vacuum without needing to add additional structures such as anti-D3 branes. Additionally, we point to evidence from bottom-up physics for the strong version of the AdS distance conjecture realized from AdS vacua in string theory. This evidence, hence, supports the sharpening of the AdS distance conjecture as one of the universal features of quantum gravity.

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I. MOTIVATIONS

String theory has been considered as one of the candidates for a correct theory of quantum gravity. However, it has been faced with the challenges of matching with the experimental observations at low-energy scales. One of them is to construct de Sitter (dS) vacua motivated by the observation of positive cosmological constant [1]. However, it has been known that dS vacua in string theory are particularly difficult to realize. No-go theorems imply that there are no dS vacua in supergravity and string theory if the internal space is static, is compact, and has no singularities [2,3]. The difficulties of the dS vacuum construction come also from explicitly computing string loop, higher-derivative, and nonperturbative corrections. The Kachru-Kallosh-Linde-Trivedi construction allows anti–de Sitter (AdS) vacua uplifted to metastable dS vacua by adding anti-D3 branes [4–7]. But, there have been recent results in the literature which point to the problems with this dS vacuum construction regarding the back-reaction of anti-D3 branes on the internal geometry [8–13] and on the 4D moduli [14] and nonsupersymmetric (non-SUSY) Giddings-Kachru-Polchinski solutions (derived in Ref. [15]) [16]. In addition, there have been some attempts at embedding dS cosmology within string theory [17,18]. The technical difficulties have, hence, suggested the possibility that string theory admits no dS vacua which do not suffer from instabilities at all [19]. Hence, the dS conjecture [20] was proposed as well as the dS instability [21] was studied to realize no dS vacua in string theory. For reviews about the status of the dS vacuum construction in string theory, see [22,23].

On the contrary, AdS vacua in string theory are understood very well and simple to construct. And, another fact is that the dS vacuum that we observe has been realized in the low-energy regime so far. This means that it is not known whether the vacuum is still dS in the high-energy regime; in other words, it is possible that the vacuum would be AdS at the high-energy scales. These facts imply an ideal that makes string theory compatible with the low-energy observations of positive vacuum energy without needing to find its dS vacua as follows. We start from the well-known AdS vacua of type IIB string theory with the compactification geometry given by $M_5 \times X_5$, where solving the stringy 10D equations of motion would lead to M_5 being an AdS_5 factor, which exists only in the high-energy regime. In addition, we consider the compactification of M_5 on a circle in order to obtain the observed 4D world where the 4D tensor component of the M_5 metric is, in general, dependent on the fifth coordinate. We point to that the nontrivial dynamics of the 4D tensor component of the M_5 metric along the fifth dimension leads to a positive contribution to energy in the 4D effective theory. As a result, a 4D dS vacuum can emerge in the low-energy regime from an AdS vacuum of the higher-dimensional theory existing at the high-energy scales.

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A feature of AdS vacua in string theory is that an infinite tower of states becomes light in the limit of the AdS curvature radius going to infinity due to no scale separation between the AdS curvature radius and the radius of the internal space. This implies the proposal of the AdS distance conjecture [24] which is a generalization of the swampland distance conjecture [25] to the metric configuration space and is stated as follows: The near-flat limit of any AdS vacuum is accompanied by an infinite tower of states whose mass scale behaves as $m \sim |\Lambda|^\alpha$ with Λ to be a cosmological constant and $\alpha \geq \frac{1}{2}$ required by the strong version. This conjecture together with other swampland conjectures have been used to place the constraints on the effective theories which can be completed consistent with quantum gravity in the ultraviolet [26–28], and interesting implications for the neutrino masses, cosmological constant, and electroweak vacuum have been found [29–36]. However, evidence coming from the bottom-up physics for the strong version of the AdS distance conjecture, which is necessary to test this conjecture and sharpens it (and, thus, the swampland distance conjecture) as one of the universal features of quantum gravity, is still missing. We will show that the mass spectrum of the Kaluza-Klein (KK) tower for the 5D bulk fields represents bottom-up evidence for the strong version of the AdS distance conjecture. This result, hence, supports that the AdS distance conjecture (and, hence, the swampland distance conjecture) realized from the AdS vacuum construction in string theory can be applied, in general, for quantum gravity.

II. AdS VACUA IN TYPE IIB STRING THEORY

Our starting point is the 10D action for the massless string excitations of type IIB string theory, which is given by

$$S = \frac{1}{\kappa_{10}^2}\int d^{10}X\sqrt{-g}\left[\mathcal{R} - \frac{1}{2}(\nabla\Phi)^2 - \frac{g_s^2}{2}e^{a_p\Phi}F_{p+2}^2\right], \quad (1)$$

where $\kappa_{10}^2 \equiv (2\pi)^7 g_s^2 l_s^8$ with g_s and l_s the string coupling and the string length, respectively, the dilaton coupling parameter a_p for the Ramond-Ramond sector is $a_p = (3-p)/2$, and $F_{p+2}^2 \equiv F_{M_1M_2\ldots M_{p+2}}F^{M_1M_2\ldots M_{p+2}}/(p+2)!$ with $F_{M_1M_2\ldots M_{p+2}}$ being the field strength tensor $(p+1)$-form gauge fields. We consider the solution of Eq. (1) with the following geometry:

$$\begin{aligned} ds_{10}^2 &= ds_{M_5}^2 + L^2 ds_{X_5}^2, \\ e^\Phi &= g_s, \\ g_s F_5 &= \alpha(1+\star)\mathrm{vol}_{X_5}, \end{aligned} \quad (2)$$

where X_5 is a 5D Sasaki-Einstein manifold [37] which has the curvature satisfying $\mathcal{R}_{X_5} = 20$ and is stabilized by N units of flux, vol_{X_5} refers to the five-form volume of X_5, $\alpha = 16\pi g_s N l_s^4(\pi^3/\mathrm{Vol}_{M_5})$ which is determined by the flux quantization constraint $\int_{X_5}\star F_5 = (2\pi l_s)^{-3}N\kappa_{10}^2/(g_s l_s)$ with Vol_{X_5} being the volume of X_5, and other $(p+1)$-form fields are trivial.

The equations of motion $\Box\Phi = g_s^2 a_p e^{a_p\Phi}F_{p+2}^2/[2(p+2)!]$ and $\nabla_M(e^{a_p\Phi}F^{MN_1\ldots N_{p+1}}) = 0$ are satisfied for the constant dilaton and self-dual five-form field [and other $(p+1)$-form fields which vanish], respectively, whereas Einstein field equations $\mathcal{R}_M{}^N = F_{MM_1M_2M_3M_4}F^{NM_1M_2M_3M_4}/96$ lead to

$$L^4 = 4\pi g_s N l_s^4 \frac{\pi^3}{\mathrm{Vol}_{X_5}}, \quad (3)$$

$$\mathcal{R}_{M_5} = -\frac{20}{L^2}, \quad (4)$$

where $\mathcal{R}_{M_5}$ denotes the scalar curvature of M_5. Equations (3) and (4) together with the geometry (2) imply that an exact background of type IIB string theory which is obtained from solving the stringy 10D equations of motion is a factor AdS_5 times a 5D internal manifold, i.e., $\mathrm{AdS}_5 \times X_5$.[1]

III. DIMENSIONALLY REDUCED ACTION OF STRING THEORY

With the background geometry (2), reducing the 10D action (1) of type IIB string theory on X_5, we get the following 5D effective action:

$$S_{5D} = \frac{M_5^3}{2}\int d^5X\sqrt{-g_5}[\mathcal{R}_{M_5} - 2\Lambda], \quad (5)$$

where $M_5^3 = 2\mathrm{Vol}_{M_5}L^5/\kappa_{10}^2$ and $\Lambda = -6/L^2$ with L as given by Eq. (3). This action means that the 5D effective theory of string theory would be in the AdS vacuum.

In addition, in order to obtain the 4D observed world, we consider the compactification of M_5 on a circle S^1 where the most general setting of this compactification is given by a principal bundle with the typical fiber to be $U(1)$ [39–41], which adopts the local coordinates as (x^μ, θ) with $\{x^\mu\} \in \mathbb{R}^4$ and θ being an angle parametrizing the fifth dimension of M_5 corresponding to the coordinate transformation as $x^\mu \to x'^\mu = x'^\mu(x)$ and $\theta \to \theta' = \theta + \alpha(x)$. Hence, the metric equipped on M_5 takes the following general form:

$$ds_{M_5}^2 = g_{\mu\nu}dx^\mu dx^\nu + R^2[d\theta + g_A A_\mu dx^\mu]^2, \quad (6)$$

where $g_{\mu\nu}$, A_μ, and R are the 4D tensor, 4D vector, and 4D scalar component fields of the bulk metric on M_5, respectively, and g_A is the corresponding gauge coupling. With this ansatz, we can explicitly expand $\mathcal{R}_{M_5}$ given in the

[1]The well-known case is that X_5 is a five-sphere S^5 related to AdS/CFT correspondence [38].

action (5) in terms of the 4D component fields (see Appendix A in Supplemental Material [42] for the detailed computation) as follows:

$$\mathcal{R}_{M_5} = \hat{\mathcal{R}} + \frac{1}{4R^2}(\partial_\theta g^{\mu\nu}\partial_\theta g_{\mu\nu} + g^{\mu\nu}g^{\rho\lambda}\partial_\theta g_{\mu\nu}\partial_\theta g_{\rho\lambda}) - \frac{g_A^2 R^2}{4}F_{\mu\nu}F^{\mu\nu}, \tag{7}$$

where $\hat{\mathcal{R}} \equiv g^{\mu\nu}(\hat{\partial}_\lambda\Gamma^\lambda_{\nu\mu} - \hat{\partial}_\nu\Gamma^\lambda_{\lambda\mu} + \Gamma^\rho_{\nu\mu}\Gamma^\lambda_{\lambda\rho} - \Gamma^\rho_{\lambda\mu}\Gamma^\lambda_{\nu\rho})$ with $\Gamma^\rho_{\mu\nu} \equiv \frac{g^{\rho\lambda}}{2}(\hat{\partial}_\mu g_{\lambda\nu} + \hat{\partial}_\nu g_{\lambda\mu} - \hat{\partial}_\lambda g_{\mu\nu})$, $\hat{\partial}_\mu \equiv \partial_\mu - g_A A_\mu \partial_\theta$, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

IV. THE EMERGENCE OF DS VACUUM IN 4D WORLD

In the previous section, we have seen that type IIB string theory exists at the AdS vacuum. However, in the following we will show that this AdS vacuum exhibits only in the high-energy regime. But, when approaching the low-energy regime, the effective theory of type IIB string theory in lower dimensions would exhibit a 4D dS vacuum which is consistent with the low-energy observations. The emergence of dS vacuum here is essentially due to the presence of the second term in Eq. (7) which has been ignored in the literature, because the θ dependence of the 4D tensor component of the bulk metric equipped on M_5 is usually not considered.

Let us first obtain the bulk profile of the 4D tensor component which describes its dynamics along the fifth dimension of M_5. By varying the action (5) in the 4D tensor component of the bulk metric, we find the following equation:

$$\bar{\mathcal{R}}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\bar{\mathcal{R}} + \Lambda g_{\mu\nu} + \frac{1}{4R^2}\Big[g_{\mu\rho}g_{\nu\lambda}\partial_\theta^2 g^{\rho\lambda} - \partial_\theta^2 g_{\mu\nu} + g^{\rho\lambda}\partial_\theta g_{\rho\lambda}\partial_\theta g_{\mu\nu} + 2\partial_\theta(g_{\mu\nu}g^{\rho\lambda}\partial_\theta g_{\rho\lambda}) - \frac{1}{2}g_{\mu\nu}\{\partial_\theta g^{\rho\lambda}\partial_\theta g_{\rho\lambda} - (g^{\rho\lambda}\partial_\theta g_{\rho\lambda})^2\}\Big] = 0, \tag{8}$$

where $\bar{\mathcal{R}}_{\mu\nu} \equiv (\partial_\lambda\bar{\Gamma}^\lambda_{\nu\mu} - \partial_\nu\bar{\Gamma}^\lambda_{\lambda\mu} + \bar{\Gamma}^\rho_{\nu\mu}\bar{\Gamma}^\lambda_{\lambda\rho} - \bar{\Gamma}^\rho_{\lambda\mu}\bar{\Gamma}^\lambda_{\nu\rho})$ with $\bar{\Gamma}^\rho_{\mu\nu} \equiv \frac{g^{\rho\lambda}}{2}(\partial_\mu g_{\lambda\nu} + \partial_\nu g_{\lambda\mu} - \partial_\lambda g_{\mu\nu})$ and $\bar{\mathcal{R}} \equiv g^{\mu\nu}\bar{\mathcal{R}}_{\mu\nu}$. We have here obtained Eq. (8) in the vacuum $R = \text{const}$ and $A_\mu = 0$, which are the solution of their equations of motion as seen later. We separate the variables as $g_{\mu\nu}(x,\theta) = \chi(\theta)g^{(4)}_{\mu\nu}(x)$, where $g^{(4)}_{\mu\nu}(x)$ is identified as the usual metric in the 4D effective theory and $\chi(\theta)$ is its profile. Then, we find

$$\mathcal{R}^{(4)}_{\mu\nu} - \frac{1}{2}g^{(4)}_{\mu\nu}\mathcal{R}^{(4)} + \Lambda_4 g^{(4)}_{\mu\nu} = 0, \tag{9}$$

$$3\chi''(\theta) + 8\frac{\chi'(\theta)^2}{\chi(\theta)} + \frac{2\Lambda}{R^{-2}}\chi(\theta) = \frac{2\Lambda_4}{R^{-2}}, \tag{10}$$

where $\mathcal{R}^{(4)}_{\mu\nu}$ and $\mathcal{R}^{(4)}$ are the usual Ricci and scalar curvatures of the 4D effective geometry of spacetime written in terms of $g^{(4)}_{\mu\nu}(x)$, respectively, and Λ_4 is a constant. It is important to remark that Eq. (10) is a nonlinear differential equation, and, hence, the solution of $g_{\mu\nu}(x,\theta)$ should not be given as the linear combination of partial solutions, whereas Eq. (9) determines the 4D effective geometry of spacetime sourced by a cosmological constant Λ_4 which is originated from the dynamics of the 4D tensor component of the bulk metric along the fifth dimension of M_5.

To find an analytical solution for Eq. (10) with the boundary condition $\chi(-\pi) = \chi(\pi)$ [43] for the general value of Λ_4 is a difficult task. However, with $\Lambda_4 = 0$ corresponding to the situation of small Λ_4, it is easy to find an analytical solution as follows:

$$\chi(\theta) = \cosh^{\frac{3}{11}}\left(\frac{\sqrt{22}}{3}\kappa\theta\right), \tag{11}$$

where $\kappa \equiv \sqrt{-\Lambda/R^{-2}}$. For the $\Lambda_4 \neq 0$, a particularly analytical solution is found as

$$\chi(\theta) = \frac{11\Lambda_4}{19|\Lambda|}\left[\cosh\left(\sqrt{\frac{2}{11}}\kappa\theta\right) - 1\right]. \tag{12}$$

Note that, due to the topology of S^1, the 4D metric component and, thus, its bulk profile $\chi(\theta)$ must be periodic with the period 2π, i.e., $\chi(\theta) = \chi(\theta + 2\pi)$. One can make the solution $\chi(\theta)$ periodic with the period 2π by reflecting it at the boundary as discussed in Appendix B [42].[2]

The fact that $\chi(\theta)$ is non-negative implies $\Lambda_4 \geq 0$, which means that the nontrivial profile given by Eqs. (11) and (12) for the 4D tensor component of the bulk metric along the fifth dimension of M_5 should give a non-negative contribution to the energy in the 4D effective theory as seen in Eq. (9). In this sense, the 4D effective theory of type IIB string theory would be at the dS vacuum.

A key question that here arises is, what would lead to the nontrivial profile for the 4D tensor component? We can see that the essential point which leads to this nontrivial profile is due to the presence of the second term on the left-hand side of Eq. (10). This term comes from the nonlinear property of the gravitational field: Gravity is itself a source that creates gravity. If this term is absent, then Eq. (9) becomes linear, and, hence, the solution would be a sum of all possible modes with the different values of Λ_4. However,

[2] In the case of $\Lambda > 0$, the solution for $\chi(\theta)$ is related to the cosine function, which is periodic and, thus, itself is compatible with the topology of S^1 [36,44].

the excitation modes with $\Lambda_4 > 0$ would decay to the lower modes with $\Lambda_4 < 0$. As a result, the 4D effective theory would exist at the negative energy state or the AdS vacuum. Therefore, we can realize that the nontrivial profile for the 4D tensor component leading to the emergence of the dS vacuum in the 4D effective theory is essentially due to its nonlinear property.

In order to show the 4D dS vacuum actually emerged in the effective picture of string theory in lower dimensions, we need to demonstrate that the ansatz (6) with the vacuum configuration $g_{\mu\nu} = \chi(\theta) g^{(4)}_{\mu\nu}(x)$ (with the 4D metric $g^{(4)}_{\mu\nu}$ corresponding to the 4D dS geometry), $R = \text{const}$, and $A_\mu = 0$ which has just been found above satisfies the 4D stringy equations of motion. The ansatz (6) and Eq. (9) suggest that the dimensional reduction of the 5D action (5) on S^1 leads to the 4D effective action given in Einstein frame as follows:

$$S_{4D} = \int d^4x \sqrt{|g_4|} \left[\frac{M^2_{\mathrm{Pl}}}{2} \left\{ \mathcal{R}^{(4)} - \frac{3}{2} \left(\frac{\partial_\mu R}{R} \right)^2 \right\} - V(R) - \frac{g_A^2 \pi M_5^3 R^3}{4} F_{\mu\nu} F^{\mu\nu} - \frac{m_A^2}{2} A^\mu A_\mu \right], \tag{13}$$

where $M^2_{\mathrm{Pl}} = M_5^3 R_0 \int_{-\pi}^{\pi} d\theta \chi$, $m_A^2 = 3 M_5^3 R_0 g_A^2 \int_{-\pi}^{\pi} d\theta [\chi'' - \chi'^2/(2\chi)]$, and $V(R)$ refers to the potential of the radion field R. (In order to change to Einstein frame, we have rescaled $g^{(4)}_{\mu\nu} \to \Omega^{-2} g^{(4)}_{\mu\nu}$ where $\Omega^2 = R/R_0$ with R_0 being an arbitrary scale.) The radion potential $V(R)$ is given as follows:

$$V(R) = V_{\mathrm{tr}}(R) + V_{1\mathrm{L}}(R), \tag{14}$$

where $V_{\mathrm{tr}}(R)$ is the tree-level term which is generated by the dynamics of 4D tensor component along the fifth dimension of M_5 and is given by

$$V_{\mathrm{tr}}(R) = M^2_{\mathrm{Pl}} \frac{R_0}{R} \Lambda_4 \tag{15}$$

and $V_{1\mathrm{L}}(R)$ is the quantum level term which is generated by the (one-loop) Casimir energy contribution and reads

$$V_{1\mathrm{L}}(R) = \sum_i (-1)^{s_i} n_i R \left(\frac{R_0}{R} \right)^2 \rho_i(R) \int_{-\pi}^{\pi} d\theta \chi^2(\theta), \tag{16}$$

where $s_i = 0$ (1) for the fermions (bosons), n_i refers to the number of degrees of freedom with respect to the ith particle, and the Casimir energy density is given by [45]

$$\rho_i(R) = \sum_{n=1}^{\infty} \frac{2 m_i^5}{(2\pi)^{5/2}} \frac{K_{5/2}(2\pi n m_i R)}{(2\pi n m_i R)^{5/2}}, \tag{17}$$

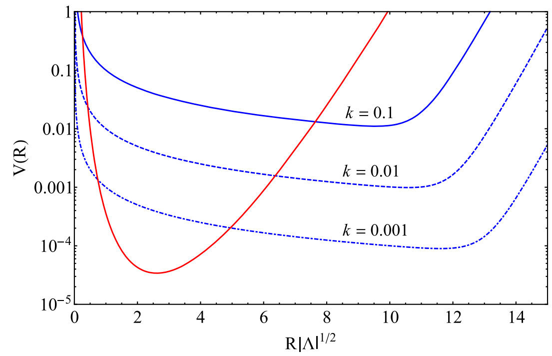

FIG. 1. The radion potential for two cases: The red and blue curves correspond to the solutions (11) and (12), respectively. Here, the radion potential corresponding to the red curve is rescaled by $|\Lambda|^3 R_0^2$, $k \equiv |\Lambda|^{1/2} \Lambda_4 R_0$, and we have considered $\sum_f n_f - \sum_b n_b = 2$ and $m_{f,b} = 0$ as a benchmark case.

with m_i and $K_{5/2}(z)$ being the mass of the ith particle and the modified Bessel function, respectively.[3] We can easily see that the radion potential behaves as $(\sum_f n_f - \sum_b n_b)/R^6$ and $(\sum_f n_f - \sum_b n_b)|_{m=0} \int_{-\pi}^{\pi} d\theta \chi^2(\theta)/R^6$ in the regions of $R \to 0$ and $R \to \infty$, respectively, where $\sum_f n_f - \sum_b n_b$ $[(\sum_f n_f - \sum_b n_b)|_{m=0}]$ is the net number of (massless) fermionic and bosonic degrees of freedom. This means that for $\sum_f n_f - \sum_b n_b > 0$ and $(\sum_f n_f - \sum_b n_b)|_{m=0} > 0$ the radion potential would approach positive infinity for both $R \to 0$ and $R \to \infty$, and, hence, there is always a stable minimum. Additionally, with the proper parameters, this minimum has positive energy corresponding to the dS vacuum, as depicted in Fig. 1. The radion potential $V(R)$, thus, allows us to fix physically the size of the fifth dimension of M_5 or the vacuum expectation value of the radion field R.

The 4D stringy equations of motion associated with the 4D effective action (13) are derived in Appendix C [42]. It is easy to see that the equations of motion for the radion field R and the graviphoton A_μ lead to the vacuum solution $R = \text{const}$ and $A_\mu = 0$ where the constant corresponds to the stable minimum of the radion potential $V(R)$, whereas the equation of motion for the 4D metric $g^{(4)}_{\mu\nu}$ is $\mathcal{R}^{(4)}_{\mu\nu} = \Lambda'_4 g^{(4)}_{\mu\nu}$, where $\Lambda'_4 \equiv M^{-2}_{\mathrm{Pl}} V_{\mathrm{sm}}$ with V_{sm} referring to the stable minimum of the radion potential $V(R)$. Because V_{sm} is positive with the proper parameters of the radion potential, the vacuum geometry of 4D effective theory is dS, but the vacuum value is now shifted compared to Eq. (9) due to the radion stabilization.

Because the present construction is done relying on the 5D effective action obtained from the dimensional

[3]Here, the matter fields such as the Standard Model which are not contained in the field content of AdS_5 supergravity can be embedded by adding probe D7-branes wrapping around an internal cycle which is a submanifold of M_5 [46,47].

reduction of string theory on X_5, it is necessary to have a separation of the scales between the radius of S^1 and the scale L of X_5. On the other hand, because of Eqs. (3) and (4), the radius of S^1 must be much larger than the curvature radius of AdS$_5$, i.e., $R|\Lambda|^{1/2} \gg 1$. We observe that the solution (12) can lead to the minimum of the radion potential fixing physically the radius of S^1 to satisfy this separation of the scales. Indeed, as seen in Fig. 1, the minimum of the radion potential in the $R|\Lambda|^{1/2}$ direction would get larger when $k \equiv |\Lambda|^{1/2}\Lambda_4 R_0$ decreases. This means that at a sufficiently small value of k or Λ_4 it would yield the separation of the scales between the radius of S^1 and the scale of X_5.

Finally, let us emphasize that the transition from the AdS vacuum in the high-energy regime to the dS vacuum in the low-energy regime allows the effective theory to avoid the constraint of non-SUSY AdS conjecture [48] because of the fact that supersymmetry is broken at the low-energy scales. If non-SUSY AdS vacuum is stable, then the effective theory would be in the swampland. On the contrary, non-SUSY AdS vacua would develop nonperturbative instabilities and, thus, decay into SUSY vacua via bubble nucleation [49,50].

V. AdS DISTANCE CONJECTURE

We point to the first evidence from the bottom-up physics for the strong version of the AdS distance conjecture [24]. In order to do this, we consider the 5D action (5) where the AdS radius L is, in general, arbitrary instead of being given by Eq. (3). This means that the 5D action (5) in this situation is not originated from the string theory compactification.

Let us first obtain the profile $Y_n(\theta)$ of the 5D bulk fields along the fifth dimension of M_5 corresponding to the solution (11). The equations for the bulk profile of the 5D bulk fields are given in Appendix D [42]. In general, it is not easy to obtain the analytical solutions to these equations. However, for small κ, we can find the analytical solutions to these equations by expanding $\chi(\theta)$ in κ. Up to the order κ^2, the solution form of these equations satisfying the boundary condition $Y(-\pi) = Y(\pi)$ [43] is found as

$$Y_n(\theta) = \left[N_n H_{2n}\left(\sqrt{\frac{b_n}{3}}\theta\right) + {}_1F_1\left(-n; \frac{1}{2}; \frac{b_n}{3}\theta^2\right)\right] \times \exp\{-a_n\theta^2\}, \tag{18}$$

where $n = 0, 1, 2, \ldots$, a_n and b_n are parameters depending on n, κ, R, and the bulk mass, $H_{2n}(z)$ and ${}_1F_1(a; b; z)$ are the Hermite polynomial and the confluent hypergeometric function, respectively, and N_n are the normalization constants.

The degree of Hermite polynomial in the expression of $Y_n(\theta)$ must be even as a result of the boundary condition $Y_n(-\pi) = Y_n(\pi)$. From this, we can obtain the mass spectrum of the KK tower as follows:

$$m_n^2 = \frac{|\Lambda|}{3} \times \begin{cases} [\frac{5}{2} + 4c_n + \frac{1+4n}{2}\sqrt{25 + 8c_n}], & \text{scalar;} \\ [1 + 4c_n + (1+4n)\sqrt{7 + 2c_n}], & \text{fermion;} \\ [\frac{3}{2} + 4c_n + \frac{1+4n}{2}\sqrt{9 + 8c_n}], & \text{vector,} \end{cases} \tag{19}$$

which correspond to the scalar, fermion, and vector fields, respectively, where $c_n \equiv n(1 + 2n)$. It should be noted here that, first, we have considered the scalar field with zero bulk mass. Second, the mass of the bulk fermion must be zero due to the boundary condition on the bulk profile, which implies that the left- and right-handed components of the bulk fermion decouple, and this is consistent with the fact that the compactification of spacetime on the circle S^1 breaks $SO(1, 4)$ to $SO(1, 3) \times U(1)$. Equation (19) suggests that the mass of KK tower behaves in the power law in the cosmological constant as $m_{KK} \sim |\Lambda|^\alpha$ with $\alpha = \frac{1}{2}$, which becomes light in the limit $\Lambda \to 0$. This is consistent with the strong version of the AdS distance conjecture.

The above conclusion is still true in the general case. We observe from Fig. 1 that the minimum of the radion potential $V(R)$ would fix $R|\Lambda|^{1/2}$. This implies $R^{-1} \propto |\Lambda|^{1/2}$, which means that the mass of the KK tower would be proportional to $|\Lambda|^{1/2}$. Hence, taking the cosmological constant Λ to be zero would encounter an infinite tower of light states.

VI. CONCLUSIONS

To achieve dS vacua in string theory motivated by the low-energy observations of a positive cosmological constant has proved to be a particularly difficult endeavor. Contrary to this, AdS vacua in string theory are common and simple to construct. Motivated by this fact and low-energy dS cosmology, we have indicated a new approach for the embedding of the observed dS vacuum into string theory, which is one of the candidates for a unitary theory of quantum gravity. We do not attempt to find a dS vacuum solution in string theory, but we start from a well-known AdS vacuum with the compactification geometry given by $M_5 \times X_5$. Then, we show that this AdS vacuum of string theory exists only in the high-energy regime. In other words, as approaching the low-energy scales where the heavy string excitations and KK modes are integrated out, the effective picture of string theory in lower dimensions would exhibit a 4D dS vacuum (which has so far been observed in the low-energy regime) due to the nontrivial dynamics of the 4D tensor component of the M_5 metric along the fifth dimension which contributes positive energy in the 4D effective theory. This result clearly provides a new path in the construction of realistic models on AdS vacua (rather than dS vacua) in type IIB string theory, but it still leads to a dS vacuum in the 4D effective theory consistent with the low-energy observation of positive vacuum energy.

In addition, due to the nontrivial bulk profile of the 4D tensor component of the bulk metric, we revisited the mass spectrum of the KK tower for the 5D bulk fields. We showed that this mass spectrum satisfies the strong version of the AdS distance conjecture, which is a generalization of the swampland distance conjecture and whose proposal was motivated by the realizations in the AdS vacuum construction in string theory. This offers evidence that comes from bottom-up physics and, hence, supports the AdS distance conjecture (and, thus, the swampland distance conjecture) as one of the universal features of quantum gravity.

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[43] Because the point with $\theta = -\pi$ and $\theta = \pi$ is the same, any 5D field $\Psi(x, \theta)$ should satisfy the following boundary condition:

$$\Psi(x, -\pi) = \Psi(x, \pi).$$

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