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Letter

Fiducial Drell-Yan production at the LHC improved by transverse-momentum resummation at $\mathrm{N^4LL_p + N^3LO}$

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Drell-Yan production is one of the precision cornerstones of the LHC, serving as calibration for measurements such as the W-boson mass. Its extreme precision at the level of 1% challenges theory predictions at the highest level. We present the first independent calculation of Drell-Yan production at order α_s^3 in transverse-momentum (q_T) resummation improved perturbation theory. Our calculation reaches the state-of-the-art through inclusion of the recently published four loop rapidity anomalous dimension and three loop massive axial-vector contributions. We compare to the most recent data from CMS with fiducial and differential cross-section predictions and find excellent agreement at the percent level. Our resummed calculation including the matching to $Z+\text{jet}$ production at NNLO is publicly available in the upcoming CuTe-MCFM 10.3 release and allows for theory-data comparison at an unprecedented level.

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I. INTRODUCTION

Drell-Yan (Z-boson) production is among the most important standard candles of the high-energy LHC physics program due to its very precise measurement at the level of one percent [1–4]. It is used for the extraction of the strong coupling [5,6], fitting of parton distribution functions [7,8] that further constrain and determine Standard Model (SM) input parameters, and is also a crucial ingredient of the W-boson mass determination [9–11].

The current precision in QCD for Drell-Yan predictions is at the level of α_s^3 both fully differentially [12–15] and more inclusively [16,17]. Calculations at this order have been performed at fixed order (N^{3}LO) and including the effects of transverse momentum (q_T) resummation up to N^{3}LL logarithmic accuracy. Currently all fully differential calculations at the level of α_s^3 employ transverse momentum subtractions or transverse momentum resummation. They have been enabled by the recent availability of the three-loop beam-functions [18–20], complete three-loop hard function [21–25] and the existence of a NNLO calculation of $Z+\text{jet}$ production [26–30]. Beyond pure QCD corrections, the full set of two-loop mixed QCD $\otimes$ EW corrections have been calculated very recently [31–33].

Traditionally there has been a focus on fixed-order calculations for total fiducial cross sections, but now that relatively high perturbative orders have been reached, convergence issues of the perturbative series due to fiducial cuts have been identified [34–36]. These issues trace back to a linear sensitivity of acceptance cuts to small transverse momenta, where fixed-order predictions are unreliable, leading to factorially divergent contributions [35]. It has shifted the focus toward resummation-improved results even for total fiducial cross sections, which can cure such problems without requiring any modification of analysis cuts.

All calculations matched to NNLO $Z+\text{jet}$ fixed-order at large q_T have so far been based on the NNLOjet results [27]. Different implementations of q_T resummation and subtractions are built on top of this calculation. Results for a matching to the resummation in DYTurbo [37] have been presented in Ref. [13] where only nonsinglet and vector singlet[1] contributions are included and truncation uncertainties are estimated by considering differences between successive orders. A matching to the RadISH resummation approach [14,38] has been presented in Refs. [12,14], also neglecting axial singlet contributions. Axial singlet contributions in the $m_t \to \infty$ EFT have been included in the resummed calculation of Ref. [39] but without the matching to α_s^3 fixed-order. The NNLOjet setup

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[1] In singlet contributions the Z boson does not directly couple to the incoming quarks, but is separated through loops involving gluons. These contributions therefore only enter at higher orders.

has subsequently been extended to calculate fiducial cross sections also at fixed-order N^{3}LO, comparing the impact of power corrections through studying the difference between symmetric and product cuts [15] and comparing with 13 TeV ATLAS data [4]. The RadISH based calculations provide uncertainty estimates for differential and fiducial results for the first time. Despite these studies, it is crucial to have an independent calculation of both the fixed-order components and the resummation implementation. While the NNLOjet calculation is tested by the correct approach of the triple singular limits through an implementation of (differential) q_T subtractions, it is important to also probe the finite contributions. As well as acting as a cross-check, an additional calculation also provides an independent estimate of uncertainties.

In this paper we present both a publicly available calculation of Z-boson production as well as differential and fiducial cross sections at the state-of-the-art level $\mathrm{N^4LL_p + N^3LO}$. The “p” subscript denotes that we are α_s^3 accurate in fixed-order and RG-improved perturbation theory up to missing effects from exact N^{3}LO PDFs that contribute both to fixed-order and logarithmic accuracy at α_s^3. We include the four loop rapidity anomalous dimension [40,41], pushing the accuracy to this level for the first time. We also include the massive three-loop axial singlet contributions [25] without the need for approximations. We compare at α_s^3 accuracy with the CMS 13 TeV precision measurement. All parts, both resummation and fixed-order are publicly available in the next CuTe-MCFM release 10.3. Public codes are crucial to ensure reproducibility, allow the community to perform independent checks, to calculate predictions with different parameters, and provide the basis for future theoretical improvements as strongly advocated by our community [42].

II. CALCULATION

Our calculation in CuTe-MCFM [43,44] matches q_T-resummation in the SCET formalism of Refs. [45–47] at the level of $\mathrm{N^4LL_p}$ to α_s^3 fixed-order $Z + \mathrm{jet}$ production. Apart from missing N^{3}LO PDF effects we achieve full α_s^3 fixed-order and transverse momentum renormalization-group-improved (RG-improved) logarithmic accuracy by counting $\log(q_T^2/Q^2) \sim 1/\alpha_s$. These logarithms are resummed through RG evolution of hard- and beam functions in the small-q_T factorization theorem. Rapidity logarithms are directly exponentiated through the collinear-anomaly formalism.

We switch between the fixed-order region of large q_T and resummation region of small q_T using a transition function [43]. The overlap between fixed-order and resummation has to be subtracted by expanding the resummation to a fixed-order. This difference is referred to as matching corrections. For Z boson production they quickly approach zero for $q_T \to 0$ and remain at the few percent level up to $\sim$30 GeV, while becoming negligible below a certain value, see our Supplemental Material [48].

Crucial ingredients in our calculation on the resummation side include the three loop transverse momentum dependent beam functions [18–20] and the four loop rapidity anomalous dimension [40,41]. The five loop cusp anomalous dimension only enters through the hard function evolution and we have checked that it is numerically completely negligible. Already at a lower order the hard function evolution is precise at the level of one per-mille. We nevertheless include the hard function evolution taking four loop collinear anomalous dimensions from Ref. [49] and a five loop cusp estimate from Ref. [50] that agrees with our own Padé approximant estimate. The five loop beta function is taken from Ref. [51].

We include the resummation of linear power corrections [34] through a recoil prescription [52]. This is crucial to improve the resummation itself as well as the numerical stability by allowing a larger matching cutoff (the value of q_T below which matching corrections are set to zero). It is also crucial for the stability of our fixed-order results in the presence of symmetric lepton cuts, see our Supplemental Material [48].

Transverse momentum Fourier conjugate logarithms $L_\perp \sim \log(x_T^2\mu^2)$ appearing in the factorization theorem would traditionally be integrated over the full range of x_T. This requires the introduction of a prescription to avoid the Landau pole. Following the SCET resummation formalism of Refs. [45,46] this is not necessary as scales are always set in the perturbative regime. The formalism further employs an improved power counting $L_\perp \sim 1/\sqrt{\alpha_s}$ that is crucial to improve the resummation at small q_T [46]. At $\mathrm{N^4LL_p}$ the α_s^3 beam functions [18–20] are then not sufficient for *improved* α_s^3 accuracy. Using the beam function RGEs we reconstructed the logarithmic beam function terms up to order $\alpha_s^6 L_\perp^6$, $\alpha_s^4 L_\perp^4$, and $\alpha_s^4 L_\perp^2$. We performed the Mellin convolutions of beam function kernels and splitting functions up to three loops [53,54] using the MT package [55].

The hard function entering the factorization formula consists of $\overline{\mathrm{MS}}$-renormalized virtual corrections. We naturally include the three-loop corrections to the vector part [21–23], but also the three-loop corrections to the axial singlet part with full top-quark mass dependence [25] (see also Ref. [24] for massless QCD).

Our fixed-order NNLO $Z + \mathrm{jet}$ calculation is based on Ref. [28], employing 1-jettiness slicing subtractions [26,56,57] and relies on the NNLO 1-jettiness soft function [58,59]. The $Z + \mathrm{jet}$ and $Z + 2$ jet calculations that enter at a lower order include top-quark loop corrections at the one-loop level [60]. Two-loop axial singlet contributions in the $Z + \mathrm{jet}$ hard function are unknown so far and have been neglected in our calculation.

We performed extensive cross-checks of all elements of our calculation by comparing amplitudes with automatized codes [61] and the literature [62–64]. A further strong check is the correct asymptotic approach of the triple singular limits through the limit $q_T \to 0$, see below. As a final check, we compared with fiducial NNLOjet results presented in Ref. [65] for different partonic channels, as well as Ref. [15], and find agreement.

Since our $Z+\mathrm{jet}$ NNLO calculation is based on 1-jettiness slicing subtractions we have to pay attention to the 1-jettiness slicing cutoff. For our resummed calculation the jettiness slicing cutoff needs to be sufficiently small compared to the q_T matching cutoff, and for fixed-order q_T-subtraction predictions compared to the q_T slicing cutoff. We verified this through checking the correct asymptotic cancellation of large q_T logarithms for $q_T \to 0$ differentially in q_T and integrated with a slicing cutoff, see our Supplemental Material [48].

The q_T matching cutoff must be small enough that residual matching corrections can be neglected. The impact of this on fiducial results can be estimated by multiplying the resummed cross section integrated up to the matching cutoff with the relative size of the neglected matching corrections. At α_s and α_s^2 matching corrections can be safely neglected below 1 GeV. For the α_s^3 coefficient we find that they can be neglected below 5 GeV with residual per-mille level effects at the order of the numerical integration uncertainty. This larger value is possible due to the inclusion of linear power corrections in our resummation formalism. This results in an error that is below the quoted numerical precision of our fiducial results (one pb). Similarly, the effect on all shown differential distributions is at the per-mille level.

III. RESULTS

We present results at $\sqrt{s} = 13$ TeV using the NNPDF4.0 PDF set at NNLO with $\alpha_s(m_Z) = 0.118$ [66]. Electroweak input parameters are chosen in the G_μ scheme with $m_Z = 91.1876\mathrm{GeV}$, $m_W = 80.385\mathrm{GeV}$, $\Gamma_Z = 2.4952$ GeV and $G_F = 1.16639 \times 10^{-5}$ GeV^{-2}. We denote the matched resummation accuracy with α_s for $\mathrm{N^2LL+NLO}$, α_s^2 for $\mathrm{N^3LL+NNLO}$ and α_s^3 for $\mathrm{N^4LL_p+N^3LO}$.

Our fiducial selection cuts in Table I are chosen to compare with the most recent Z-boson precision measurement by CMS in Ref. [3]. The symmetric lepton cuts used in this analysis cause a poor perturbative convergence for fixed-order calculations and can also lead to numerical issues. However, the use of resummation resolves such issues [34–36].

TABLE I. Fiducial cuts for $Z \to l^+ l^-$ used in the CMS 13 TeV analysis [3].

Lepton cuts	$q_T^l > 25$ GeV, $\lvert\eta^l\rvert < 2.4$
Separation cuts	76.2 GeV $< m^{l^+l^-} < 106.2$ GeV, $\lvert y^{l^+l^-}\rvert < 2.4$

In our calculation we distinguish between three scales for estimating uncertainties. We use a low (resummation) scale $\sim q_T$ (see Ref. [43] for details) to which RGEs are evolved down from the hard scale chosen as $\sqrt{m_Z^2 + p_{T,Z}^2}$. The CuTe-MCFM resummation formalism [45–47] is originally derived using an analytic regulator to regulate rapidity divergences in the transverse position dependent PDFs (collinear anomaly formalism). This is opposed to using a rapidity regulator that introduces a rapidity scale [67]. We have reintroduced a scale estimating the effect of a different rapidity scale as suggested in Ref. [68]. We vary hard and low scale by a factor of two, and rapidity scale by a factor of six, tuned on the truncation of the improved power counting, to obtain a robust estimate of truncation uncertainties. Most importantly our formalism allows for the variation of the low scale, which dominates uncertainties at small q_T. Last, in our uncertainty bands we include the effect of varying the transition function in the region of about 40 GeV to 60 GeV where matching corrections become significant, following the same procedure as in Ref. [43] at a lower order.

While for Drell-Yan production our resummation formalism does not set the central low scale below ~2 GeV [43], a downwards variation would probe close toward the nonperturbative regime. We therefore set a minimum of 2 GeV and symmetrize the uncertainty bands since the variation becomes ineffective at small scales. Note that about 2% of the total fiducial cross section comes from the region $q_T < 1$ GeV where one might expect additional nonperturbative effects of an unknown size.

A. Differential results

In Fig. 1 we present the Z boson transverse momentum distribution at different orders and compare it to the CMS 13 TeV measurement [3] with cuts as in Table I.

Overall there is an excellent agreement between theory and data at the highest order. Going from α_s^2 to α_s^3 decreases uncertainties and improves agreement with data noticeably at both large and small q_T. In the first bin $0\ \mathrm{GeV} < q_T < 1\ \mathrm{GeV}$ we notice a relatively large difference to the data, but this is also where one would expect a non-negligible contribution from nonperturbative effects. For the Φ^* distribution shown in Fig. 2 results are overall very similar.

Since our resummation implementation is fully differential in the electroweak final state we can naturally also present the transverse momentum distribution of the final state lepton, see Fig. 3. This is plagued by a Jacobian peak at fixed-order and crucially requires resummation. The higher-order α_s^3 corrections further stabilize the results with smaller uncertainties.

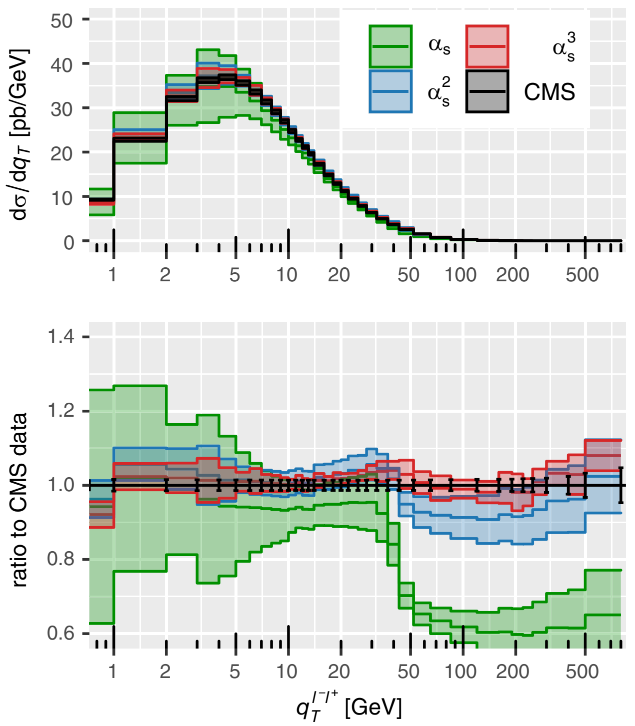

FIG. 1. Differential transverse-momentum resummation improved predictions for the $q_T^{l^-l^+}$ distribution at order α_s, α_s^2, and α_s^3.

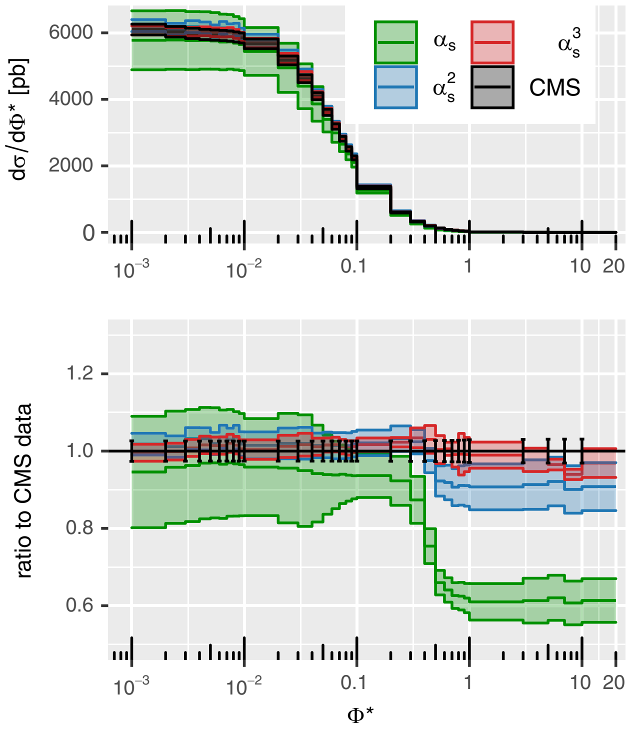

FIG. 2. Differential transverse-momentum resummation improved predictions for the Φ^* distribution at order α_s, α_s^2, and α_s^3.

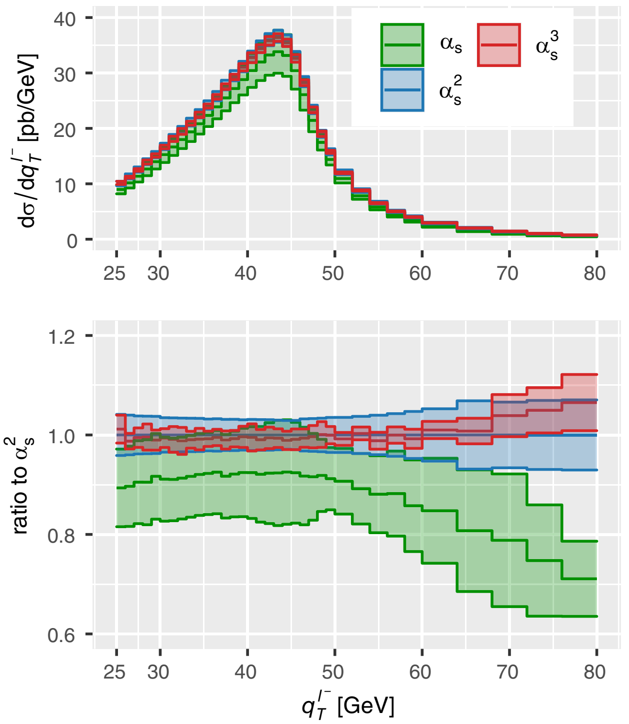

FIG. 3. Differential transverse-momentum resummation improved predictions for the lepton transverse momentum distribution at order α_s, α_s^2, and α_s^3.

B. Total fiducial cross section

In Table II we present total fiducial cross sections. Uncertainties of the fixed-order NNLO (α_s^2) result, obtained by taking the envelope of a variation of renormalization and factorization scales by a factor of two, are particularly small at the level of 0.5% and do not improve toward N^{3}LO with large corrections. The resummation improved results are obtained by integrating over the matched q_T spectrum shown in Fig. 1. Uncertainties of the resummation improved predictions are obtained by taking the envelope of the variation of hard, low, and rapidity scales in the

TABLE II. Fiducial cross sections in pb for the cuts in Table I and input parameters as in the text. Uncertainties for the resummation-improved results include matching to fixed-order (mat.), neglected matching corrections (m.c.), and by scale variation (sc.). The fixed-order result at N^{3}LO has an additional slicing-cutoff uncertainty. For comparison, the final row shows the CMS measurement (for electron and muon channels combined) [3].

Order k	Fixed-order α_s^k	Resummation improved α_s^k
0	694^{+85}_{-92}	—
1	732^{+19}_{-30}	$637 \pm 8_{\text{mat.}} \pm 70_{\text{sc.}}$
2	720^{+4}_{-3}	$707 \pm 3_{\text{mat.}} \pm 29_{\text{sc.}}$
3	$700^{+4}_{-6} \pm 1_{\text{slicing}}$	$702 \pm 1_{\text{mat.}} \pm 1_{\text{m.c.}} \pm 17_{\text{sc.}}$
	$699 \pm 5(\text{syst}) \pm 17(\text{lumi})$ (e, μ combined) [3]	

fixed-order and resummation region. The matching uncertainty from the transition function variation is quoted separately. We estimate the effect of neglecting matching corrections at α_s^3 below $q_T \leq 5$ GeV to be less than 1 pb.

The resummation improved result at α_s has large uncertainties that stem from an insufficient order of the resummation (N^2LL), which still has substantial uncertainties in the Sudakov peak region (cf. Fig. 1). The results quickly stabilize, with less than a percent difference between the central α_s^2 and α_s^3 predictions. Nevertheless, the uncertainties we obtain are noticeably larger than the fixed-order uncertainties. We further observe that going from $\mathrm{N}^3\mathrm{LL}/\alpha_s^2$ to $\mathrm{N}^4\mathrm{LL}_\mathrm{p}/\alpha_s^3$ does not reduce uncertainties as substantially as when going from α_s to α_s^2. This is because the resummation uncertainties around the Sudakov peak region at small $q_T \sim 5$ GeV do not improve dramatically.

While this behavior, of only moderately decreasing uncertainties going from α_s^2 to α_s^3, is consistent with the findings of Ref. [15] using RadISH resummation, our uncertainties of the resummation improved fiducial cross section are larger than the uncertainties presented there. Our α_s^3 prediction has uncertainties of about 2.5%, while using RadISH for the resummation results in uncertainties of about 1%. Given that differentially in Fig. 1 we see still some variation in the low q_T region between the central α_s^2 and α_s^3 results, we are confident in our more conservative uncertainty estimate.

Indeed, theory uncertainties have become an important topic within recent years [69]. First, they cannot be interpreted statistically and second, perturbative predictions are limited to the level presented here for the foreseeable future. It is therefore important to study them with as much scrutiny as possible. An approach followed in Ref. [13] has been to take half the difference between the two highest order results as an uncertainty. This would bring our uncertainties closer in line with the uncertainties presented in Ref. [15], less than one percent.

IV. CONCLUSIONS AND OUTLOOK

In this paper we presented the first transverse-momentum (q_T) resummation improved calculation at the level of $\mathrm{N}^4\mathrm{LL}_\mathrm{p} + \mathrm{N}^3\mathrm{LO}$, which broadly reduces theory uncertainties to the few percent level. Our results show excellent agreement with the 13 TeV CMS measurements within a few percent both at the differential level from $q_T^Z = 1$ GeV to ~500 GeV and for Φ^* over the whole spectrum, as well as for the total fiducial cross section. As a consequence of the resummation (and inclusion of linear power corrections), our calculation can provide reliable predictions also for past experimental analyses using symmetric lepton cuts.

All previous calculations of order $\mathrm{N}^3\mathrm{LL} + \mathrm{N}^3\mathrm{LO}$ rely on a single $Z +$ jet NNLO calculation [27]. Further, uncertainties (via scale variation) for resummation improved results were only estimated by using the RadISH resummation framework [14,38]. Due to the utmost importance of this process, it is crucial to provide an independent calculation using completely different methods to reliably estimate uncertainties. It allows future (experimental) studies to assess the validity of their input theory predictions through independent results. This becomes increasingly important with the advent of very precise collider measurements that might indicate tension with the SM [11]. The public availability of our calculation as part of the upcoming CuTe-MCFM release allows for a much larger audience to make use of this state-of-the-art precision, to implement modification of cuts and input parameters, and also to reuse parts and to validate other calculations [42].

Although the theoretical precision of the calculation discussed in this paper is now at an impressive level, there are two important aspects that require further work. Statistical PDF uncertainties have reached the level of one percent [66,70] and systematic effects can no longer be neglected. Since these uncertainties are at the same level as perturbative truncation uncertainties, a careful study of PDF effects at this order will be an important future direction. Indeed, while finalizing this manuscript, approximate N^3LO PDFs have been introduced by the MSHT group [71].[2]

In addition, in order to better match with data at very small q_T, it is possible to include a parametrization of nonperturbative effects [72,73]. This can then inform the modeling of the related process of W-boson production and thus have implications for the extraction of the W-boson mass. Extending W-boson production in CuTe-MCFM to α_s^3 accuracy will thus allow for very precise W/Z boson ratio predictions [39].

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