Letter

Searching for *CP* violation through two-dimensional angular distributions in four-body decays of bottom and charmed baryons

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We propose to search for CP violation through the twofold angular distributions in the four-body decays of bottom and charmed hadrons. The two polar angles in the twofold angular distributions are correlated, to which the interferences of intermediate resonances are one important origin. These interferences will leave tracks in the twofold angular distributions, with which the CP violation can be studied. Special attention is paid to the case when all the intermediate resonances are different, which is unique to four-body decays. It is suggested to look for CP violation in four-body decays such as $\Lambda_b^0 \to p\pi^-\pi^+\pi^-$ through the analysis of the twofold angular distributions. The method proposed in this paper is also widely applicable to other four-body decays processes.

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I. INTRODUCTION

CP violation (CP-Vs), as one of the cornerstones for the explanation of the matter-antimatter asymmetry of the Universe through baryogenesis [1], is of great importance in the baryon sector both theoretically and experimentally. Searches for CP-Vs have been carried out in the baryonic decay channels of CP-conjugate pairs, through direct measurements of the differences between the full partial decay widths [2–5], the decay widths corresponding to part of the phase space [6,7], the triple-product asymmetries (TPAs) [8–10], and the decay asymmetry parameters [11–13], or through other indirect techniques such as the amplitude analysis [14] and the energy test method [10], etc. Puzzling enough, although it was first discovered almost 60 years ago in the neutral-kaon decays [15], CP-Vs has never been observed in the baryon sector, despite so many efforts having been paid.

Contrary to the baryonic cases, CP-Vs has been observed in the decays of K and D mesons [15,16], and in plenty of decay channels of B mesons [17–20], some of which have quite large CP asymmetries (CP-As). One such kind of examples are the large regional CP-As observed in parts of the phase space for some three-body decays of B meson [21–24], in which the interfering effect between the nearby resonances (or the interference of resonance with the nonresonant part) is one important mechanism for the generating of large regional CP-As [25–29]. Similar to the B meson cases, multibody decays of bottom and charmed baryons are usually dominated by various intermediate resonances. The prevalent interferences between the amplitudes corresponding to various intermediate resonances are expected to leave tracks in the angular distributions of the final particles, in which the CP-Vs is potentially hidden.

Based on the Cabibbo-Kobayashi-Maskawa mechanism of the Standard Model [30,31], the CP-As in some of the bottom and charmed baryon decays are expected to have similar magnitudes with those in the bottom and charmed meson decays, respectively [32–34]. However, the baryon decays usually suffer from substantial lower statistics comparing with the meson cases. Hence one of the best strategies for the CP-Vs searching in the baryon sector is to start with the decay channels with the largest statistics. The four-body decays such as $\Lambda_b^0 \to pK^-\pi^+\pi^-$ and $\Lambda_b^0 \to p\pi^+\pi^-\pi^-$ are one important type that fulfill this criteria.

Although CP-As corresponding to angular distributions of final particles have been extensively investigated for the heavy baryon decays in the literature [34–47], for the four-body decay cases, however, this area is largely unexplored. As one kind of angular distribution correlated CP-As, the TPA induced CP-As have played an unique role in heavy meson and baryon decays [36,37,48–53]. In fact, the TPA induced CP-As are especially suitable be studied in four-body decays of heavy baryons [34,41]. Although it is in principle possible to study TPA and corresponding CP-As in two- or three-body decays of heavy baryons [36,37,51,52], this requires, however, that the mother baryons to be polarized, which is not possible in the current stage [54–56]. In this paper, CP-Vs corresponding

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to angular distributions, which complements to that corresponding to TPA, is investigated for the aforementioned four-body decays with the largest statistics.

II. THE TWOFOLD ANGULAR DISTRIBUTIONS FOR FOUR-BODY DECAYS

To put on a more general ground, let us first consider a four-body decay process of the branching form $H \to a(\to 12)b(\to 34)$, where H is the mother hadron, either baryon or meson, bottom or charmed, a and b are the intermediate resonances, and 1234 label the four final particles. The kinematical variables are illustrated in Fig. 1, where the Jackson convention is adopted for the reference frames [57]. In the c.m. frame of H, the z axis is conveniently chosen as the normal to the production plane of H. The polar and azimuthal angles of the momentum of a in this frame is denoted as θ and ϕ. In the c.m. frame of $a(b)$, the $z_{a(b)}$ axis is chosen along the direction of the momentum of H. The x_a and x_b (y_a and y_b) axes will be chosen (anti)aligned with each other. The polar and the azimuthal angles of the particle 1(3) in the c.m. frame of $a(b)$ will be denoted as $\theta_{a(b)}$ and $\phi_{a(b)}$, respectively. The angle from the decay plane of the $a \to 12$ to that of the $b \to 34$, which is related to TPA, will be denoted as φ.[1] One can find from Fig. 1 the relation $\varphi = 2\pi - \phi_a - \phi_b$.

The spin-averaged decay amplitude squared for unpolarized H, after integrating out the azimuthal angles ϕ_a and ϕ_b, will take the form,[2]

$$\int \overline{|\mathcal{A}|^2} d\varphi \propto \sum_{jl} \Gamma_{jl}(s_{12}, s_{34}) P_j(c_{\theta_a}) P_l(c_{\theta_b}), \tag{1}$$

where $P_{j(l)}$ are the Legendre polynomials with $c_{\theta_{a(b)}} \equiv \cos\theta_{a(b)}$, and

$$\Gamma_{jl} = \sum_{a,a',b,b'} \frac{\mathcal{W}_{jl}^{(ab,a'b')} \mathcal{G}_j^{(aa')} \mathcal{G}_l^{(bb')}}{\mathcal{I}_a \mathcal{I}_{a'}^* \mathcal{I}_b \mathcal{I}_{b'}^*}, \tag{2}$$

$\mathcal{I}$s are the reciprocals of the Breit-Wigner propagators, taking the form $\mathcal{I}_{a^{(\prime)}} = s_{12} - m_{a^{(\prime)}}^2 + i m_{a^{(\prime)}} \Gamma_{a^{(\prime)}}$ and $\mathcal{I}_{b^{(\prime)}} = s_{34} - m_{b^{(\prime)}}^2 + i m_{b^{(\prime)}} \Gamma_{b^{(\prime)}}$ with s_{12} and s_{34} the invariant mass squared of the 12 and the 34 systems, and

$$\mathcal{W}_{jl}^{(ab,a'b')} = \sum_{\sigma\rho} (-)^{\sigma - s_a + \rho - s_b} \langle s_a - \sigma s_{a'} \sigma | j0 \rangle \times \langle s_b - \rho s_{b'} \rho | l0 \rangle \mathcal{F}_{\sigma\rho}^{H\to ab} \mathcal{F}_{\sigma\rho}^{H\to a'b'*}, \tag{3}$$

$$\mathcal{G}_j^{(aa')} = \sum_{\lambda_1\lambda_2} (-)^{s_a - \lambda_{12}} \langle s_a - \lambda_{12} s_{a'} \lambda_{12} | j0 \rangle \mathcal{F}_{\lambda_1\lambda_2}^{a\to 12} \mathcal{F}_{\lambda_1\lambda_2}^{a'\to 12*}, \tag{4}$$

$$\mathcal{G}_l^{(bb')} = \sum_{\lambda_3\lambda_4} (-)^{s_b - \lambda_{34}} \langle s_b - \lambda_{34} s_{b'} \lambda_{34} | l0 \rangle \mathcal{F}_{\lambda_3\lambda_4}^{b\to 34} \mathcal{F}_{\lambda_3\lambda_4}^{b'\to 34*}, \tag{5}$$

with all the $\mathcal{F}$s being the decay amplitudes in the helicity form, "$\langle \cdots | \cdots \rangle$" being the Clebsh-Gordan coefficients, $s_{a^{(\prime)}}$ and $s_{b^{(\prime)}}$ being the spins of $a^{(\prime)}$ and $b^{(\prime)}$, σ and ρ being the helicities of $a^{(\prime)}$ and $b^{(\prime)}$ in the c.m. frame of H, and λ_i ($i = 1, \ldots, 4$) being the helicity index of particle i (defined in the c.m. frame of a or b), $\lambda_{ii'} = \lambda_i - \lambda_{i'}$. The summations over a, a', b, and b' indicate that there may be more than one resonances either for a or b. Note that there are four kinematic variables left, which can be chosen as the invariant mass squared of 12 and 34, s_{12} and s_{34}, the polar angles θ_a and θ_b. When s_{12} and s_{34} are set fixed in certain ranges, we get a two-dimensional phase space (2DPS) expanded by c_{θ_a} and c_{θ_b}, which is illustrated in Fig. 2.

The following remarks to the twofold angular distributions in Eq. (1) are in order:

(i) The two angles θ_a and θ_b are correlated, in the sense that Γ_{jl} can not be factorized into the form $\Gamma_{jl} = \xi_j \eta_l$. If Γ_{jl} were factorized, then that would mean that one can simply integrate out either θ_a or

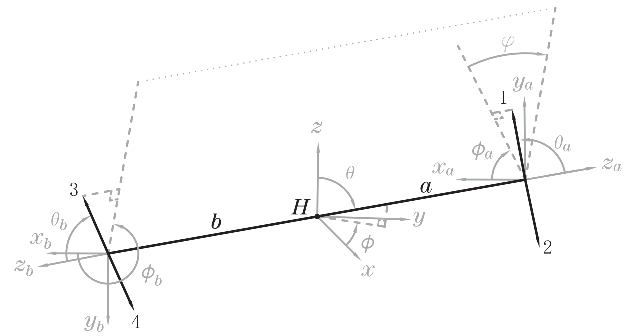

FIG. 1. Illustration of the kinematic variables for the four-body decay $H \to a(\to 12)b(\to 34)$. The reference frames is defined according to the Jackson convention. Note that θ and ϕ are defined in the c.m. frame of H, while $\theta_{a(b)}$ and $\phi_{a(b)}$ are defined in the c.m. frame of $a(b)$. The angle φ can be defined in any of the three frames. The invariant mass squared of 12 and 34, s_{12} and s_{34}, which are Lorentz invariant, are not shown in the figure.

[1]For the current situation, the TPAs constructed from the momenta of the final particles can be defined as $A_T \equiv \frac{N(C_T>0)-N(C_T<0)}{N(C_T>0)+N(C_T<0)}$, with $C_T \equiv (\mathbf{p}_1 \times \mathbf{p}_2) \cdot \mathbf{p}_3$, where $\mathbf{p}_j$ represents the momentum of particle j in the c.m. of H. From Fig. 1 one can see that the above definition is equivalent to $A_T = \frac{N(\sin\varphi>0)-N(\sin\varphi<0)}{N(\sin\varphi>0)+N(\sin\varphi<0)}$. Consequently, the aforementioned TPA A_T is sometimes also called as the up-down asymmetry is some references as the decay plane of the 12 system divide the space into the "up" part ($\sin\varphi > 0$) and the "down" part ($\sin\varphi < 0$) [53,58].

[2]The expression of the spin-averaged decay amplitude squared for polarized H without integrating out the azimuthal angles are presented in Appendix A.

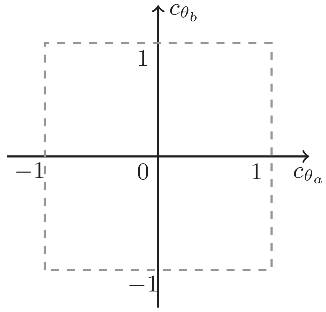

FIG. 2. Illustration of the 2DPS expanded by c_{θ_a} and c_{θ_b}.

θ_b without losing any information, so that the four-body decay will reduced effectively to three-body decays. However, this is not true, because the amplitudes are entangled in Eq. (2). Among the origins for the entanglement of j and l in Eq. (2), the genuine interferences (GIs) between the amplitudes of $H \to a(\to 12)b(\to 34)$ and $H \to a'(\to 12)b'(\to 34)$, where $a \neq a'$ and $b \neq b'$, are the most special ones. The intermediate resonances of the two amplitudes are all different, hence they are unique for the four-body decays and cannot be accounted for by any two- or three-body decays. In other word, GI is a twofold interference, because it contains both the $a \leftrightarrow a'$ and $b \leftrightarrow b'$ interferences. Anyway, it would be interesting to study the correlation between θ_a and θ_b both theoretically and experimentally.

(ii) From the Clebsh-Gordan coefficients in Eq. (4) one can see that j takes integer values from 0 to $2\max_a(s_a)$, where a runs over all the possible resonances. Similarly, l takes integer values from 0 to $2\max_b(s_b)$. Note that the allowed values of j and l are determined by the spin of the resonances through the Clebsh-Gordan coefficients, not directly by the angular momentum between the final particles 1 and 2 (or 3 and 4). Though they equal with each other (the spin of the resonances and the angular momentum of 12 or 34) only when all the final particles are spin-0 ones.

(iii) In principle, *CP*-Vs can show up in any of the Γ_{jl}s through the difference between Γ_{jl} and its *CP* conjugate $\overline{\Gamma_{jl}}$. However, it is more likely that *CP*-Vs show in those Γ_{jl}s that contain the resonant-interfering terms. This is because there may be, between the amplitudes corresponding to different resonances, nonperturbative strong phase differences, which are crucial for the generating of large *CP*-As. Consequently, it is important to find the rules whether there are resonant-interfering terms in each Γ_{jl} or not. To this end, notice that the parity conservation, as well as the properties of the Clebsh-Gordan coefficients, imply from Eqs. (4) and (5) that $\Pi_a\Pi_{a'}(-)^j = 1$ and $\Pi_b\Pi_{b'}(-)^l = 1$, where Πs represent the parities of the corresponding particles. It follows that there are selection rules (SRs) for the presence of (non)interference terms in Γ_{jl}, which fall into four situations: (1) The noninterfering terms ($a = a'$, $b = b'$) for $H \to ab$ will present in Γ_{jl} if $0 \le j \le 2s_a$ and $0 \le l \le 2s_b$, and j and l are even. (2) The singly interfering terms between $H \to ab$ and $H \to a'b$ will present in Γ_{jl} if $|s_a - s_{a'}| \le j \le s_a + s_{a'}$, $\Pi_a\Pi_{a'}(-)^j$ is positive, $0 \le l \le 2s_b$, and l is even. (3) The singly interfering terms between $H \to ab$ and $H \to ab'$ will present in Γ_{jl} if $|s_b - s_{b'}| \le l \le s_b + s_{b'}$, $\Pi_b\Pi_{b'}(-)^l$ is positive, $0 \le j \le s_a$, and j is even. (4) The GI terms will present in Γ_{jl} if $|s_a - s_{a'}| \le j \le s_a + s_{a'}$, and $|s_b - s_{b'}| \le l \le s_b + s_{b'}$, and $\Pi_a\Pi_{a'}(-)^j$ and $\Pi_b\Pi_{b'}(-)^l$ are positive.

(iv) One special case is when the intermediate resonances have opposite parities in each of the two decay branches. For example, suppose that there are two resonances at each decay branch of a and b, which are $R_a^{(+)}$, $R_a^{(-)}$ and $R_b^{(+)}$, $R_b^{(-)}$, respectively, with opposite parities indicated in the superscripts. According to the SRs, the noninterfering terms, the singly interfering terms, and the GI terms will show up in Γ_{jl} when both j and l are even, j is even and l is odd or j is odd and l is even, and both j and l odd, respectively. Hence they are all well separated in Γ_{jl}s.

(v) Each of the dynamical parameters Γ_{jl}s in Eq. (1) represents a degree of freedom for angular distributions, which can be expressed as

$$\Gamma_{jl} \propto \int \overline{|A|^2} P_j(c_{\theta_a}) P_l(c_{\theta_b}) dc_{\theta_a} dc_{\theta_b}. \tag{6}$$

Consequently, Γ_{jl} can be determined experimentally according to

$$\Gamma_{jl} \propto \sum_{i=1}^{N} P_j(c_{\theta_{ai}}) P_l(c_{\theta_{bi}}), \tag{7}$$

where N is the total event yields, and i labels each event. The relative jlth moment for the expansion of Eq. (1) is then defined as

$$A^{jl} \equiv \frac{\Gamma_{jl}}{\Gamma_{00}}. \tag{8}$$

The *CP*-As corresponding to A^{jl} can then be obtained according to

$$A^{jl}_{CP} \equiv \frac{1}{2}(A^{jl} - \overline{A^{jl}}), \quad (9)$$

for $j \neq 0$ and/or $l \neq 0$, where $\overline{A^{jl}}$ is the same with A^{jl}_{CP} but for the CP conjugate process.[3]

(vi) Although very clean from the theoretical side, the disadvantage of the CP-A defined in Eq. (9) is also obvious: the events located in different positions of the 2DPS are weighted differently, which makes the experimental analysis of uncertainties complex. To avoid this, we use an alternative definition for CP-A, which weights all the events equally throughout the whole 2DPS (up to a signature). To achieve this, one first notice that the contributions of each event to Γ_{jl} are different in signature because of the factor $P_j P_l$ in Eq. (7). Hence one can introduce a twisted Γ_{jl} according to

$$\tilde{\Gamma}_{jl} \propto \sum_{i=1}^{N} \mathrm{sgn}(P_J(\cos_{\theta_{ai}})P_l(\cos_{\theta_{bi}})). \quad (10)$$

This is equivalent to dividing the 2DPS into $(j+1)\times(l+1)$ bins that are boarded by the zero lines of the Legendre polynomials P_j and P_l, and assign the event yields of each bin by a proper signature. Equation (10) is then equivalent to

$$\tilde{\Gamma}_{jl} \propto \sum_{i_j=0}^{j}\sum_{i_l=0}^{l}(-)^{i_j+i_l-l}N_{i_j i_l}, \quad (11)$$

where $N_{i_j i_l}$ is the event yields of the bin $i_j i_l$. The twisted jlth relative moment takes the form

$$\tilde{A}^{jl} = \frac{\tilde{\Gamma}_{jl}}{\Gamma_{00}}. \quad (12)$$

The corresponding CP-A is then defined as[4]

$$\tilde{A}^{jl}_{CP} = \frac{1}{2}(\tilde{A}^{jl} - \overline{\tilde{A}^{jl}}). \quad (13)$$

(vii) Similar analysis is also applicable to the four-body sequential decays $H \to 1a$, $a \to 2b$, $b \to 34$. It turns out that the SRs for the presence of different type of terms for the sequential decays are exactly the same with those for the branching ones. A detailed analysis of this type of decays is presented in the Appendix.

III. APPLICATIONS

As an application, we propose to look for CP-Vs in $\Lambda_b^0 \to p\pi^+\pi^-\pi^-$ through the analysis of the twofold angular distributions in the 2DPS. Previous experiments indicate that this decay channel is dominated by quasi-two-body decay $\Lambda_b^0 \to N(1520)^0\rho(770)^0$. Hence there potentially are the $f_0(500)-\rho(770)^0$ and $N(1440)^0 - N(1520)^0$ interferences.[5] Moreover, there can also be the GI terms such as the interference between $\Lambda_b^0 \to N(1520)^0 f_0(500)$ and $\Lambda_b^0 \to N(1440)^0\rho(770)^0$. The spin-parity of these two pairs of resonances are $0^+ - 1^-$ and $(\frac{1}{2})^+ - (\frac{3}{2})^-$, respectively. According to the SRs, both j and l can take values 0, 1, 2. The noninterfering terms, the singly interfering terms, and the GI terms are all perfectly separated in Γ_{jl} because of the opposite parities of the two pairs of resonances. The noninterfering terms will show up in Γ_{jl} for $jl = 00$, 02, 20, 22; the singly interfering terms will show up in Γ_{jl} for $jl = 01$, 10, 12, 21; while the GI terms will show up in and only in Γ_{11}. This can be represented in a matrix form as

$$(\Gamma_{jl}) \sim \left(\begin{array}{c|c|c} \text{Nonint} & \begin{array}{c}(N_{1440}N_{1520})|f|^2,\\ (N_{1440}N_{1520})|\rho|^2\end{array} & \text{Nonint} \\ \hline \begin{array}{c}(f\rho)|N_{1440}|^2,\\ (f\rho)|N_{1520}|^2\end{array} & (N_{1440}N_{1520}f\rho)_{GI} & (f\rho)|N_{1520}|^2 \\ \hline \text{Nonint} & (N_{1440}N_{1520})|\rho|^2 & \text{Nonint} \end{array}\right). \quad (14)$$

[3] CP-As are alternatively defined as $A^{(jl)}_{CP} \equiv \frac{\Gamma_{jl}-\overline{\Gamma_{jl}}}{\Gamma_{00}+\overline{\Gamma_{00}}}$. For $j = 0 = l$, it reduces to the direct CP-A defined by the decay width: $A^{(00)}_{CP} = A^{\mathrm{dir}}_{CP}$, as one can see that $\Gamma_{00} = N$, and $\overline{\Gamma_{00}} = \bar{N}$.

[4] Alternatively, CP-As can be defined as $\tilde{A}^{(jl)}_{CP} \equiv \frac{\tilde{\Gamma}_{jl}-\overline{\tilde{\Gamma}_{jl}}}{\Gamma_{00}+\overline{\Gamma_{00}}}$.

[5] The contribution of $f_0(500)$ is suppressed in the factorization approach. Despite of this, similar analysis will apply as long as there are other contributions to the amplitudes interfering with $\rho(770)^0$, resonant or nonresonant.

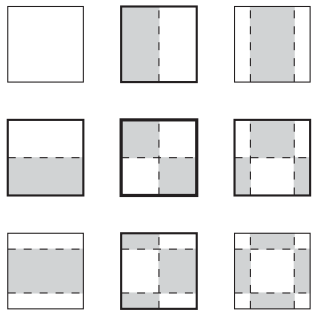

FIG. 3. Bin divisions of the 2DPS for the measurements of $\tilde{\Gamma}^{jl}$s, $\tilde{A}^{jl}$s, and $\tilde{A}^{jl}_{CP}$s in $\Lambda_b^0 \to p\pi^+\pi^-\pi^-$. There are nine ways of bin dividing in total, each of which corresponds to the measurements of the aforementioned observables for j and l equals to the row and the columns in this figure. The bin divisions corresponding to the singly interfering terms are emphasized with thick squares (for $jl = 01, 10, 12, 21$), while the one corresponding to the GI terms is emphasized with even thicker one in the center of this figure (for $jl = 11$). The padding in each bin indicates the signature in front of the corresponding event yields in Eq. (11) (white for positive and gray for negative).

CP-Vs induced by the interference of the intermediate resonances can be embedded in any of the aforementioned five Γ_{jl}s for $jl = 01, 10, 12, 21$, and 11, which can be measured according to Eqs. (8) and (9), or alternatively, according to Eqs. (12) and (13).

The bin divisions for the measurements of all the nine $\tilde{A}^{jl}$s are illustrated in Fig. 3. Since only Γ_{11} contains the GI terms for the four-body decays, it deserves special attentions. To measure it, one first needs to divide the 2DPS into four bins, which is illustrated as the one in the center of Fig. 3. The 11th relative moment $\tilde{A}^{11}$, which can also be called as the twofold forward-backward asymmetry, is then measured according to

$$\tilde{A}^{11} = \frac{(N_I - N_{II} + N_{III} - N_{IV})}{N}, \tag{15}$$

where the subscripts in the event yields denote the four quadrants according to the bin division shown in the center of Fig. 3. One immediately obtains the *CP*-A $\tilde{A}^{11}_{CP}$ according to Eq. (13) with $\tilde{A}^{11}$ and $\overline{\tilde{A}^{11}}$ at hand.

Similar 2DPS analysis can be performed in sequential decays. For example, the four-body decay $\Lambda_b^0 \to p\pi^+\pi^-\pi^-$ through the decay chains $\Lambda_b^0 \to p a_1(1260)^-$, $a_1(1260)^- \to \rho(770)^0\pi^-$, $\rho(770)^0 \to \pi^+\pi^-$, and $\Lambda_b^0 \to N^{*+}\pi^-$, $N^{*+} \to \Delta^{++}\pi^-$, $\Delta^{++} \to p\pi^+$, which are also dominant, are suitable for the 2DPS analysis. Take the former one as an example, potential interferences include those of $a_1(1260)^- - \pi(1300)^-$ and $\rho(770)^0 - f(500)^0$, the spin-parities of which are, respectively, $1^+ - 0^-$ and $1^- - 0^+$. Hence both of j and l take values 0, 1, and 2. Again, the singly interfering terms will show up in $jl = 01, 10, 02, 20$, while the GI terms will show up in $jl = 11$. Note that there are no apparent suppressions for both $\pi(1300)^-$ and $f(500)^0$. In this sense, this sequential decay is even more suitable for the *CP*-Vs searching through the analysis of the twofold angular distributions, though there are potentially contaminations from other resonances such as $a_2(1320)^-$.

With a marginal generalization, the method proposed in this paper is also applicable to other cases. One such example is the decay channel $\Lambda_c^+ \to \Lambda^0\overline{K^0}\pi^+$, where the $\overline{K^0}\pi^+$ can decay from $K^*(892)^+$, and Λ^0 is reconstructed according to $\Lambda^0 \to p\pi^-$. Since $\Lambda^0 \to p\pi$ is a weak process, SRs are no longer applicable. However, one can equivalently view the parity-conserving and -violating parts of the amplitudes of $\Lambda^0 \to p\pi$ as amplitudes of two spin-half particles with opposite parities decaying into $p\pi$. The interference of the parity-conserving and -violating parts of the amplitudes, together with that of $K_0^*(700) - K^*(892)$, make up three angular distribution and corresponding *CP*-A observables (corresponding to $jl = 01, 10, 11$) defined according to Eqs. (12) or (13), where $\tilde{A}^{10}$ is in fact the decay asymmetry parameter, $\tilde{A}^{01}$ represents the forward-backward asymmetry in the $K^*(892)$ branch, while $\tilde{A}_{11}$ contains the GI terms. It would be interesting to measure the twofold forward-backward asymmetry $\tilde{A}_{11}$ and the corresponding *CP*-A $\tilde{A}^{11}_{CP}$.

IV. SUMMARY

CP-Vs corresponding to the twofold angular distributions in four-body decays is analyzed. The interferences of intermediate resonances may generate large *CP*-As corresponding to the twofold angular distributions, which have never been studied experimentally. We propose to search for *CP*-Vs through the analysis of the twofold angular distributions in the decay channels such as $\Lambda_b^0 \to p\pi^+\pi^-\pi^-$ and $\Lambda_c^+ \to \Lambda^0(\to p\pi)\overline{K^0}\pi^+$. This method is quite general, which can be used in a wide class of four-body decays of bottom and charmed hadrons, and are not limited to *CP*-Vs studies.

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APPENDIX A: DECAY AMPLITUDE SQUARED WITHOUT INTEGRATING OUT φ

We present here the decay amplitude squared without integrating out φ, and without the assumption of unpolarized H, which reads

$$\overline{|\mathcal{A}|^2} \propto \sum \gamma^{(aa',bb')jl}_{\sigma_a\sigma_{a'}\sigma_b\sigma_{b'}} d^j_{\sigma_{a'a},0}(\theta_a) d^l_{\sigma_{b'b},0}(\theta_b) e^{i\sigma_{aa'}\phi_a} e^{i\sigma_{bb'}\phi_b}, \tag{A1}$$

where $\sum \equiv \sum_{aa'bb'} \sum_{\sigma_a\sigma_{a'}\sigma_b\sigma_{b'}} \sum_{jl}$, $\sigma_{a'a} = \sigma_{a'} - \sigma_a$, $\sigma_{b'b} = \sigma_{b'} - \sigma_b$, and

$$\gamma^{(aa',bb')jl}_{\sigma_a\sigma_{a'}\sigma_b\sigma_{b'}} = \frac{w^{(aa',bb')jl}_{\sigma_a\sigma_{a'}\sigma_b\sigma_{b'}} \mathcal{G}^{(aa')}_j \mathcal{G}^{(bb')}_l}{\mathcal{I}_a \mathcal{I}^*_{a'} \mathcal{I}_b \mathcal{I}^*_{b'}}, \tag{A2}$$

with

$$w^{(aa',bb')jl}_{\sigma_a\sigma_{a'}\sigma_b\sigma_{b'}} = \langle s_a - \sigma_a s_{a'} \sigma_{a'} | j\sigma_{a'a}\rangle \langle s_b - \sigma_b s_{b'} \sigma_{b'} | l\sigma_{b'b}\rangle \times (-)^{\sigma_a - s_a + \sigma_b - s_b} \mathcal{F}^{H\to ab}_{\sigma_a\sigma_b} \mathcal{F}^{H\to a'b'*}_{\sigma_{a'}\sigma_{b'}} P_{\sigma_{ab},\sigma_{a'b'}}(\theta), \tag{A3}$$

where $P_{\sigma_{ab},\sigma_{a'b'}}(\theta)$ describes the polarization of H with $\sigma_{a^{(\prime)}b^{(\prime)}} = \sigma_{a^{(\prime)}} - \sigma_{b^{(\prime)}}$.

Concerning the polarization of H, the only relevant H in practice are the spin-half baryons such as Λ_b. It is well known that H can only polarize along the normal to the production plane due to the constraint of the parity symmetry in the producing process. The factor $P_{\sigma_{ab},\sigma_{a'b'}}(\theta)$ then takes the form

$$P(\theta) = \frac{1}{2}\left(1 + \begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{pmatrix} P_z\right), \tag{A4}$$

with P_z the polarization of H along the z axis. For unpolarized H, $P_z = 0$, as observed for the Λ_b case in pp collision by LHCb and CMS at a few percents level [54–56], $P_{\sigma_{ab},\sigma_{a'b'}}(\theta)$ reduce to $\delta_{\sigma_{ab},\sigma_{a'b'}}/2$, so that $\sigma_{aa'} = \sigma_{bb'}$. The last two factors in Eq. (A1) then reduce to $e^{-i\sigma_{aa'}\varphi}$.

A simultaneous analysis of the correlation of θ_a, θ_b, and φ may give us deeper insights in CP-V. Take again the decay $\Lambda_b^0 \to p\pi^+\pi^-\pi^-$ as an example, according to the SRs, the presence of the GI will also generate terms (with $j = 1 = l$), which are proportional to $\sin\theta_a \sin\theta_b \sin\varphi$ and $\sin\theta_a \sin\theta_b \cos\varphi$, respectively. The simultaneous analysis can be performed by fitting the aforementioned two factors.

Besides a simultaneous analysis, an analysis of solely the φ dependence is simpler. For the two aforementioned GI terms, the former has to do with TPA, and the related CP-V can be studied accordingly. While the latter can be described by the left-right asymmetry (LRA):

$$A^{LR} = \frac{N_L - N_R}{N_L + N_R}, \tag{A5}$$

where $N_{R/L}$ are the event yields defined by $N_{L/R} \equiv N(\cos\phi \gtrless 0)$. The corresponding CP-V can be studied through the LRA induced CP asymmetry

$$A^{LR}_{CP} = \frac{1}{2}(A^{LR} - \bar{A}^{LR}), \tag{A6}$$

where $\bar{A}^{LR}$ is the LRA of the CP conjugate process.

APPENDIX B: TWO-DIMENSIONAL ANGULAR DISTRIBUTIONS FOR $H \to 1a(\to 2b(\to 34))$

For completeness, we present here a brief discussion on four-body sequential decays of the form $H \to 1a$, $a \to 2b$, $b \to 34$. The spin-averaged decay amplitude squared can be expressed as

$$\int \overline{|\mathcal{A}|^2} d\varphi \propto \sum_{jl} \hat{\Gamma}_{jl}(s_{234}, s_{34}) P_j(c_{\hat{\theta}_a}) P_l(c_{\hat{\theta}_b}), \tag{B1}$$

where s_{234} are the invariant mass squared of the 234 system, the angle $\hat{\theta}_{a(b)}$ is the polar angle of the momentum of 2(3) in the c.m. frame of $a(b)$, where z_a and z_b are defined, respectively, in the c.m. frame of a and b in a similar manner with those in the main text, and

$$\hat{\Gamma}_{jl} = \sum_{a,a'} \sum_{b,b'} \frac{\hat{\mathcal{W}}^{(a,a')}_j \hat{\mathcal{G}}^{(ab,a'b')}_{jl} \mathcal{G}^{(bb')}_l}{\mathcal{I}_a \mathcal{I}^*_{a'} \mathcal{I}_b \mathcal{I}^*_{b'}}, \tag{B2}$$

where $\mathcal{I}_{a^{(\prime)}} = s_{234} - m^2_{a^{(\prime)}} + i m_{a^{(\prime)}} \Gamma_{a^{(\prime)}}$ for now, and

$$\hat{\mathcal{W}}^{(a,a')}_j = \sum_{\lambda_1\sigma} (-)^{\sigma_a - s_a} \langle s_a - \sigma s_{a'} \sigma | j0\rangle \mathcal{F}^{H\to 1a}_{\lambda_1\sigma} \mathcal{F}^{H\to 1a'*}_{\lambda_1\sigma}, \tag{B3}$$

$$\hat{\mathcal{G}}^{(ab,a'b')}_{jl} = \sum_{\lambda_2\rho} (-)^{s_a + s_b - \lambda_2} \langle s_a(\rho - \lambda_2) s_{a'}(\lambda_2 - \rho) | j0\rangle \times \langle s_b - \rho s_{b'} \rho | l0\rangle \mathcal{F}^{a\to 2b}_{\lambda_2\rho} \mathcal{F}^{a'\to 2b'*}_{\lambda_2\rho}. \tag{B4}$$

Note that the azimuthal angles have also been integrated out.

Suppose that $a \to 2b$ and $b \to 34$ are strong processes, there comes the nonzeroness conditions for $\hat{\mathcal{G}}_{jl}^{(ab,a'b')}$ and $\mathcal{G}_{l}^{(bb')}$ followed by the parity symmetry and the properties of the Clebsh-Gordan coefficients, which can be organized as (i) $0 \le j \le 2\max_a(s_a)$, $0 \le l \le 2\max_b(s_b)$, (ii) $\Pi_b\Pi_{b'}(-)^l$ is positive, and (iii) $\Pi_a\Pi_{a'}(-)^j\Pi_b\Pi_{b'}(-)^l$ is positive. One see that the conditions are in fact the same with the branching decay $H \to a(\to 12)b(\to 34)$. Consequently, SRs for the presence of different kinds of terms in $\hat{\Gamma}_{jl}$ will also be exactly the same.

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