Quantum trans-Planckian physics inside black holes and its spectrum

Norma G. Sanchez [*]
International School of Astrophysics Daniel Chalonge - Hector de Vega, CNRS, INSU, Sorbonne University, 75014 Paris, France

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We provide a quantum unifying picture for black holes of all masses and their main properties covering classical, semiclassical, Planckian, and trans-Planckian gravity domains: space-time, size, mass, vacuum ("zero-point") energy, temperature, partition function, density of states, and entropy. Novel results of this paper are that black hole interiors are always quantum, trans-Planckian, and of constant curvature. This is so for *all* black holes, including the most macroscopic and astrophysical ones. The black hole interior trans-Planckian vacuum is similar to the earliest cosmological vacuum, where the classical gravity dual is the low energy gravity vacuum—today, dark energy. There is *no* singularity boundary at $r = 0$; the quantum space-time is regular. We display the quantum Penrose diagram of the Schwarzschild-Kruskal black hole. The complete black hole instanton (imaginary time) covers the known classical Gibbons-Hawking instanton plus a new, central, highly dense quantum core of Planck length radius and constant curvature. The complete partition function, entropy, temperature, decay rate, discrete levels, and density of states *all* include the trans-Planckian domain. The semiclassical black hole entropy (the Bekenstein-Hawking entropy) $(\sqrt{n})^2$ "interpolates" between the quantum point particle entropy (n) and the quantum string entropy $\sqrt{n}$, while the quantum trans-Planckian entropy is $1/(\sqrt{n})^2$. Black hole evaporation finishes in a pure (nonmixed) quantum state of particles, gravitons, and radiation.

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I. INTRODUCTION AND RESULTS

Quantum theory is more complete than classical theory; it tells us what values physical observables should have. Planckian and trans-Planckian domains are theoretically allowed and physically motivated in the very early stages of the universe, as well as in the last stages of black hole evaporation and the black hole interiors, as we show here. Quantum eras in the far past universe are trans-Planckian, and they determine the post-Planckian eras, e.g., inflation and the cosmological vacuum energy up to dark energy; see Refs. [1–4].

Using quantum theory to reach the Planck scale and the trans-Planckian domain (instead of starting from classical gravity by quantizing general relativity) reveals novel results, "quantum relativity" and quantum space-time structure [2–4]. The space-time coordinates can be promoted to quantum noncommuting operators: Comparison to the harmonic oscillator and global phase space structure is enlightening; the hyperbolic quantum space-time structure generates a quantum light cone due to the relevant $[X, T]$ nonzero commutator, and a new quantum vacuum region beyond the Planck scale emerges.

The space-time coordinates in the Planckian and trans-Planckian domains are no longer commuting, but they obey nonzero commutation relations: The concept of space-time is replaced by quantum algebra. Classical space-time is recovered when the quantum operators are the classical space-time continuum coordinates (c-numbers) with all commutators vanishing.

In this paper we investigate the black hole interior, its structure and physical properties, with Planckian and trans-Planckian physics, classical-quantum gravity duality, and quantum space-time in this context.

One of the novel results of this paper is that quantum physics is an inherent constituent of all black hole interiors, from the horizon to the center, in particular, inside the largest and most astrophysical black holes. Thus, the results of this paper have implications for both quantum theory and gravity, as well as for searching for quantum gravitational signals, for e-LISA [5], for instance, after the success of LIGO [6,7]. For quantum black holes, black hole evaporation and its last stage produce huge emissions; this is also the case for macroscopic astrophysical black holes.

As we discuss in Sec. II, a complete quantum theory of gravity should be a finite theory (which is more than a renormalizable theory): The renormalization procedure applies for the noncomplete theories in the Wilsonian sense [8] because they are valid in a limited range of validity, and such known theories are not complete at the Planck scale and the trans-Planckian domain.

[*]Norma.Sanchez@obspm.fr; https://chalonge-devega.fr/sanchez

This framework provides, in particular, the gravitational entropy and temperature in the quantum trans-Planckian domain, that is, the extension to this domain of Bekenstein-Hawking entropy and Hawking temperature, which are semiclassical gravity magnitudes. Interestingly, this approach also applies to cosmology and allows a clarification of the cosmic vacuum energy or cosmological constant; see Refs. [1–3]: The quantum (Λ_Q) and classical (Λ) cosmological vacuum energy values' duals of each other correspond precisely to the early and late universe state values, respectively [1–3].

In this paper we analyze the new quantum vacuum region inside the Planck scale hyperbolae which delimitate the quantum light cone in the Schwarzschild-Kruskal space-time. The zero-point (vacuum) quantum energy bends the space-time and produces a constant curvature central region. We find the quantum discrete levels of the black hole space-time and the vacuum trans-Planckian region. In Sec. IV we describe the global quantum space-time structure of the Schwarzschild-Kruskal black hole and extend the Penrose diagram [9] to the quantum domain. In Fig. 1 we display the new quantum Penrose diagram.

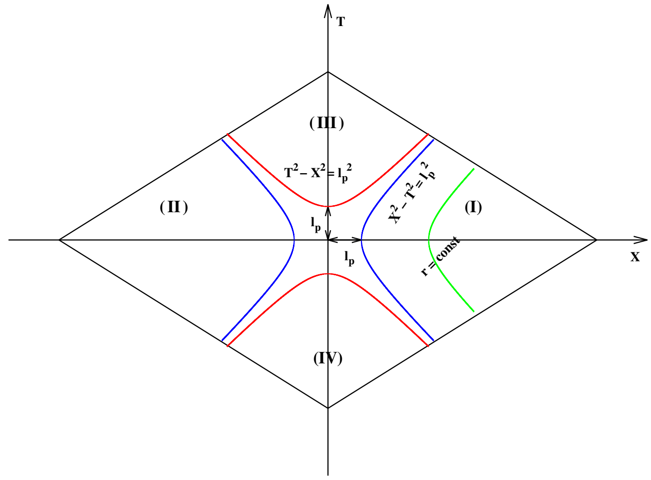

FIG. 1. Quantum Penrose diagram of the Schwarzschild-Kruskal black hole. The quantum hyperbolae $X^2 - T^2 = \pm l_P^2$ replace the classical null horizons $X = \pm T$. Their internal region is purely quantum and trans-Planckian. The difference between the four classical Kruskal regions (I–IV) disappears in the quantum domain, and they become a single central region. The exterior regions are semiclassical/classical asymptotically flat space-times. There is no curvature singularity at $r = 0$ or any other place. The quantum space-time is totally regular. Regions extend regularly without any finite boundary or curvature singularity. The central quantum region is of constant finite curvature. Moreover, the discrete spectrum confirms this picture: The quantum hyperbolae $(T^2 - X^2) = \pm\sqrt{2}$, which replace the classical singularity $(T^2 - X^2)_{\text{classical}}(r = 0) = \pm 1$, lie outside the allowed quantum levels $(T^2 - X^2)_n = (2n + 1), n = 0, 1, 2, \ldots$, and therefore, the $(r = 0)$ hyperbolae singularities are ruled out.

The quantum space-time structure is discretized into quantum hyperbolic levels. For times and lengths larger than the Planck scale, the global space-time levels are $(X_n, T_n) = \sqrt{2n + 1}$, $n = 0, 1, 2\ldots$ (in Planck units), as well as the mass levels M_n. The allowed levels cover the whole domain from the Planck scale $(X_n, T_n) = 1$ $(n = 0)$ and the quantum (low and intermediate n) levels to the quasiclassical and classical ones, and tend asymptotically (very large n) to a continuum classical space-time. In the trans-Planckian domain (lengths and masses smaller than the Planck scale), in the black hole central region, (X_n, T_n) are $(1/\sqrt{2n + 1})$, with the highest n being the most quantum, excited, and trans-Planckian ones.

The size of the black hole is gravitational length L_G in the classical-semiclassical regime; it is quantum length $L_Q = l_P^2/L_G$ in the full quantum gravity regime. Similarly, for the quantum mass, $M_Q = m_P^2/M$, and for the quantum surface gravity, $K_Q = \kappa_P^2/K_G$. Gravitational thermal features such as Hawking radiation are typical of the semiclassical gravity regime. The end of evaporation is purely quantum and nonthermal. For masses smaller than the Planck mass, the final state is not a black hole anymore but a composite particlelike (or stringlike) state. Moreover, the quantum mass spectrum for *all* masses we found (Sec. IV here) and the decay rates (Sec. VII) confirm this picture.

In Sec. V, we describe the imaginary time manifold (quantum instanton): The quantum trans-Planckian central core allows us to complete the classical gravity Gibbons-Hawking instanton, which is cut at the horizon. The classical black hole instanton is regular but *not* complete, and the black hole quantum instanton is regular and complete. In Fig. 2 we depict the new quantum instanton black hole picture.

These results allow us to describe (in Sec. VI) the complete partition function covering all (classical and quantum) gravity regimes, as well as the trans-Planckian entropy. We discuss the comparison between the point particle QFT entropy (without gravity), the black hole entropy, and quantum strings in terms of ordered and nonordered partition numbers.

The discrete levels in the trans-Planckian central core of the black hole extend with decreasing n from the most quantum highly excited levels (very large n), with smaller entropy $S_{Qn} = 1/(2n + 1)$ and higher vacuum density $\Lambda_{Qn} = (2n + 1)$, to the Planck scale level $(n = 0)$. In the external black hole space-time, the discrete levels extend from the Planck scale $(n = 0)$ and low n to the quasiclassical and classical levels, tending (very large n) to a continuum space-time. Consistently, these levels have larger gravitational (Gibbons-Hawking) entropy $S_{Gn} = (2n + 1), n = 0, 1, 2, \ldots$ and lower vacuum energy $\Lambda_{Gn} = 1/(2n + 1)$.

There is no singularity at the black hole origin for the following reasons. (i) The $r = 0$ mathematical singularity is not physical, but the result of the extrapolation of the purely

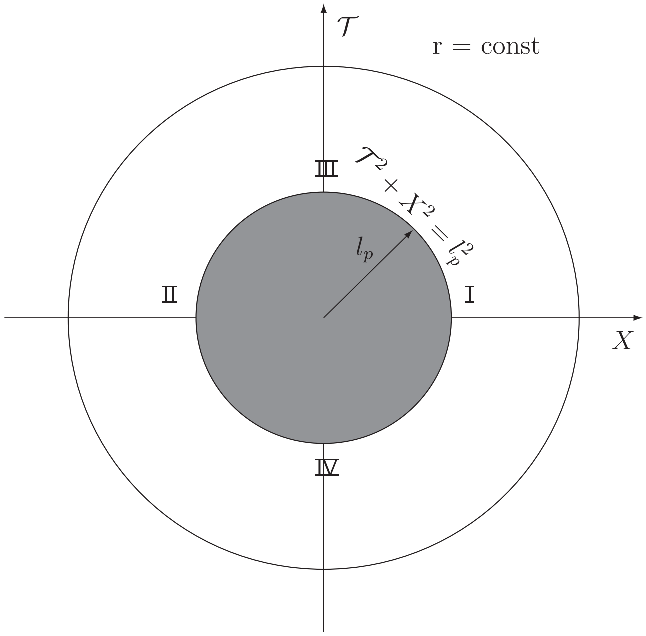

FIG. 2. Quantum gravitational instanton of the Schwarzschild-Kruskal black hole (imaginary time: $T = i\mathcal{T}, t = i\tau$). The classical null horizons corresponding to the origin $X = \pm\mathcal{T} = 0$ in the classical gravitational instanton of the Schwarzschild-Kruskal black hole (Gibbons-Hawking instanton) are quantum mechanically replaced by the circle of Planck length radius $(X^2 + \mathcal{T}^2) = [X, \mathcal{T}] = 1$ (in Planck units). Quantum theory consistently extends the instanton manifold: Classically, the instanton is regular but is not complete because it is cut at the horizon $r = 2M$, while at the quantum level, it is both regular and complete: The quantum gravitational black hole instanton is the usual classical instanton for radius larger than the Planck length plus a central highly dense quantum core of Planck length radius and of high finite curvature, which is absent classically. The difference between the four Kruskal regions disappears in the Euclidean manifold, and they are identified. (We just indicate their locations to remind us about hyperbolic manifold.) The imaginary time τ in the classical instanton is periodic with period $\beta = 2L_G = 1/\kappa$, with $1/\beta$ being the intrinsic (Hawking) temperature. In the complete quantum instanton, the imaginary time is periodic, too, but with the complete $L_{QG} = (L_G + L_Q)$, which includes the quantum Planckian and trans-Planckian magnitudes. The complete temperature T_{QG}, entropy S_{QG}, and density of states all include the trans-Planckian domain, as shown in Secs. V and VI.

classical (nonquantum) general relativity theory is out of its domain of physical validity. The Planck scale and the quantum uncertainty principle in quantum gravity preclude the extrapolation to the zero length or time, which is precisely what is expected from quantum trans-Planckian physics: the smoothness of the classical gravitational singularities. (ii) The vacuum interior of the black hole is a small region of high but bounded trans-Planckian constant curvature, and therefore, it has no singularity. There are no singularity boundaries in the quantum space-time, not at $r = 0$ or any other place. The quantum Schwarzschild-Kruskal space-time is totally regular. Moreover, the quantum hyperbolae $(T^2 - X^2) = \pm\sqrt{2}$, which replace the classical singularity $(T^2 - X^2)_{\text{classical}}(r = 0) = \pm 1$, lie outside the allowed quantum levels $(T^2 - X^2)_n = (2n + 1), n = 0, 1, \ldots$; therefore, they are excluded at the quantum level: The singularity is removed from the quantum space-time.

This paper is organized as follows: In Sec. II we discuss why a quantum theory of gravity must be finite. In Sec. III we describe the classical, semiclassical, and quantum Planckian and trans-Planckian black hole regions and regimes, their properties, and the gravitational entropy in these three regimes. In Sec. IV we describe the quantum global Schwarzschild-Kruskal space-time structure, its quantum Penrose diagram, and the new results obtained with it. Section IV deals with the black hole mass spectrum in the whole mass range, from astrophysical black holes to masses smaller than the Planck mass, passing through the Planck mass (the crossing scale). Sections V and VI describe the new imaginary time black hole instanton, including the trans-Planckian region, the partition function, and the trans-Planckian entropy. In Sec. VII we discuss the implications of these results for the early and last phases of black hole evaporation and the quantum pure (non-mixed) decay rate. In Sec. VIII we describe the black hole interior and its quantum (trans-Planckian) de Sitter vacuum. Sections IX and X contain our remarks and conclusions.

II. QUANTUM THEORY OF GRAVITY MUST BE FINITE

The construction of a complete, consistent, quantum theory of gravitation continues to be the greatest challenge in physics today. This is a problem of fundamental relevance for the quantum unification of all interactions and particle physics, theoretical physics and cosmology, the physical origin of the universe and its earliest phases, as well as black hole interiors, quantum origin and end of black holes, multiverse possibilities, and several other physical implications.

In addition, there is a possibility of "low energy" $(E \ll M_{\text{Planck}})$ physical effects that could be experimentally tested. One example is dark energy [10–13], described as the low energy (classical, dilute, large scale) cosmological vacuum, a remanent of the high energy (quantum trans-Planckian, highly dense, small scale) cosmological vacuum at the origins; see Refs. [1,3].

A problem mostly discussed in connection with gravity quantization is renormalizability of Einstein theory (or its various generalizations) when quantized as a local quantum field theory (QFT). A complete quantum theory at the Planckian and trans-Planckian domains must have today's general relativity, quantum mechanics, and quantum field theory as limiting cases. Physical effects combining gravitation and quantum mechanics are relevant at energies of the order of $M_{\rm Planck} = \sqrt{\hbar/G} = 1.22\,10^{16}$ TeV and beyond, namely, the trans-Planckian domain

$$E_{\rm Planck} \leq E < \infty, \quad 0 < L \leq l_{\rm Planck} = 10^{-33}\ {\rm cm}. \qquad (2.1)$$

Such energies were available in the Universe at times $0 < t \leq t_{\rm Planck} = 5.4 \times 10^{-44}$ sec.

Nevertheless, low energy ($E \ll M_{\rm Planck}$) physical effects could be experimentally tested, including today's cosmological vacuum (see Refs. [10–20]). In addition, one may speculate about effects analogous to the presence of magnetic monopoles in some grand unified theories (monopoles can be detected by low energy experiments in spite of their large mass).

A theory valid at the Planck scale and beyond—that is, in the trans-Planckian domain $E > E_{\rm Planck}$, $L < l_{\rm Planck}$, necessarily involving quantum gravitation—will also be valid at any lower energy scale. One may ignore higher energy phenomena in a low energy theory, but the opposite is not true. In other words, a theory of quantum gravity will be a theory of everything. This conclusion is totally independent of whether or not string models are used. It may not make physical sense to quantize pure gravity. A physically sensible quantum theory cannot contain only gravitons. For example, a theoretical prediction for the graviton-graviton scattering at energies of the order of $M_{\rm Planck}$ must include all particles produced in a real experiment—that is, in practice, all existing particles in nature since gravity couples to all matter.

From a conceptual point of view, we discuss the renormalizability problem of gravity. As is clear from the preceding discussion, we have $M_{\rm Planck} \leq \Lambda_0 < \infty$ for gravity. There cannot be any quantum field theory of particles beyond it. Therefore, if ultraviolet divergences appear in a quantum theory of gravitation, there is no way to interpret them as coming from a higher energy scale as is usually done in QFT. In other words, no physical understanding can be given to such ultraviolet infinities. The only logically consistent possibility would be to find a *finite* theory of quantum gravitation that is a theory of everything (TOE).

These simple arguments based on the renormalization group lead us to the conclusion that a consistent quantum theory of gravitation must be a finite theory and must include all other interactions. That is, it must be a TOE (theory of everything). In particular, we need to understand the present desert between 1 TeV and 10^{16} TeV.

There is an additional dimensional argument about the inference of a quantum theory of gravitation → TOE: There are only three dimensional physical magnitudes in nature (length, energy, and time) and, correspondingly, only three dimensional constants in nature (c, h, G). All other physical constants such as $\alpha = 1/137,04\ldots$, $M_{\rm proton}/m_{\rm electron}, \theta_{WS}, \ldots$ are pure numbers, and they must be calculable in a TOE.

The constants (c, G, h) help one recognize the different relevant scales and physical regimes. Even if a hypothetical underlying theory of everything only required pure numbers (option three in Ref. [21]), physical touch, at some level, requires the use of fundamental constants [22–25]. Here, we use three fundamental constants (with tension being c^2/G). From our study here and in Refs. [1–4], it appears that a complete quantum theory of gravity is a theory of pure numbers.

III. CLASSICAL, SEMICLASSICAL, AND QUANTUM BLACK HOLES

The physical classical, semiclassical, and quantum Planckian and trans-Planckian gravity regimes are particularly important for several reasons: e.g., the different stages of the universe and black hole evolution (origin, evaporation, and end) and the different regions of the global complete (Kruskal-like completion) black hole space-times.

The classical gravity regimes are those of classical space-time with very low energies ($E \ll E_{\rm Planck}$ and large sizes $L_G \gg l_{\rm Planck}$); semiclassical gravity is that of curved space-times with QFT for matter, backreaction included, as the cosmic inflation quasi–de Sitter stage of the universe (with the typical energy scale being the grand unification scale and no larger), and black hole evaporation in its early and middle stages. The quantum gravity regime includes Planckian and trans-Planckian energies, in the early universe stage at and before the Planck time, the last black hole evaporation stages, the quantum space-time black hole regions inside the event horizon, and more generally, the quantum space-time region inside the quantum light cone.

(i) The classical or semiclassical gravity regime corresponds to any of the external space-time regions outside the black hole horizon until the asymptotic far regions, as well as the early (semiclassical or semiquantum) gravity phases of black hole evaporation.

(ii) The quantum black hole regimes refer to the very small quantum trans-Planckian interior of the black hole, as well as to the last quantum gravity phases of black hole evaporation.

(iii) For any black hole, the classical or semiclassical gravity regimes and the quantum (Planckian and trans-Planckian) gravity regimes are classical-quantum duals of each other in the precise sense of classical-quantum duality.

(iv) The classical or semiclassical black hole $(BH)_G$ (that is, large size and mass, external black hole regions) is clearly characterized by the set of physical gravitational magnitudes or observables (size, mass, classical temperature or surface gravity, entropy) $\equiv (L_G, M, T_G, S_G)$:

$$(BH)_G = (L_G, M_G, T_G, S_G). \qquad (3.1)$$

(v) The highly dense, very quantum, black hole regime $(BH)_Q$ is characterized by the corresponding set of quantum dual physical quantities (L_Q, M_Q, T_Q, S_Q) in the precise meaning of classical-quantum duality:

$$(BH)_Q = (L_Q, M_Q, T_Q, S_Q), \tag{3.2}$$

$$(BH)_Q = \frac{(bh)_P^2}{(BH)_G}, \qquad (bh)_P = (l_P, m_P, t_P, s_P), \tag{3.3}$$

where $(bh)_P$ stands for the corresponding quantities at the fundamental constant Planck scale, the crossing scale between the two main (classical and quantum) gravity domains.

The black hole horizon separates the interior region, which is quantum and trans-Planckian, from the external space-time regions, which are classical and semiclassical with energies lower than the Planck energy. The classical $(BH)_G$ and quantum $(BH)_Q$ black hole regimes (classical or semiclassical phases of black holes and their quantum Planckian and trans-Planckian interiors, or their very late phases of evaporation) satisfy Eqs. (3.1)–(3.3).

Total or complete black holes $(BH)_{QG}$ are composed of their classical or semiclassical external regions and their quantum interior:

$$(BH)_{QG} = BH[(bh)_P, (BH)_Q, (BH)_G]. \tag{3.4}$$

The subscript G stands for the classical gravitation magnitudes or domain, Q stands for the quantum ones, and P for their fundamental Planck scale constant values. Details are given in Sec. IV for the black hole regions and different regimes, and for the QG black hole properties and physical magnitudes: surface gravity, black hole instantons, temperature, partition function, density of states, entropy, and decay rates.

The quantum black hole $(BH)_Q$ is generated from the classical black hole $(BH)_G$ through Eqs. (3.1)–(3.4): classical-quantum black hole duality. The complete (classical plus quantum) black hole $(BH)_{QG}$ endows classical-quantum black hole CPT symmetry. This includes, in particular, the classical, quantum, and total black hole temperatures and entropies and allows us to characterize, in a precise way, the different classical, semiclassical, Planckian, and trans-Planckian black hole domains.

The black hole size is the gravitational length L_G in the classical regime and the quantum length $L_Q = l_P^2/L_G$ in the quantum dual regime (which includes the full quantum Planckian and trans-Planckian regimes). The complete size L_{QG} endows the symmetry $Q \longleftrightarrow G\colon (L_G/l_P) \longleftrightarrow (l_P/L_G)$. The complete (QG) (classical and quantum) variables—in particular, the length $L_{QG}(l_P, L_G)$—cover the complete black hole manifold, including the quantum trans-Planckian interior and the semiclassical and classical black hole exteriors. (i) For $m_P < M \leq \infty\colon L_{QG} \simeq L_G, L_G > L_Q$, which is the classical or semiclassical gravity domain. (ii) For $0 \leq M < m_P\colon L_{QG} \simeq L_Q, L_Q > L_G$, which is the standard elementary particle physics domain. (iii) For $M = m_P\colon L_{QG} = 1 = L_Q = L_G = l_P$, which is the Planck scale (the crossing scale).

Similarly, the horizon acceleration (surface gravity) $K_G = c^2/L_G$ of the black hole in the classical gravity regime becomes the quantum acceleration $K_Q = k_P^2/K_G$ in the quantum dual gravity regime. The classical temperature T_G of the classical gravitational length or mass (in units of κ_B) becomes the quantum temperature T_Q (quantum size or Compton length) in the quantum regime. Consistently, the Gibbons-Hawking temperature is precisely the quantum temperature T_Q.

Similarly, the classical or semiclassical gravitational area or entropy S_G (Bekenstein-Hawking entropy) has a quantum dual $S_Q = s_P^2/S_G$ in the quantum gravity (Planckian and trans-Planckian) regime, $s_P = \pi\kappa_B$ being the Planck entropy:

$$S_G = \frac{s_P}{4}\left(\frac{A_G}{a_P}\right) = s_P\left(\frac{M}{m_P}\right)^2, \tag{3.5}$$

$$S_Q = \frac{s_P}{4}\left(\frac{a_P}{A_G}\right) = s_P\left(\frac{m_P}{M}\right)^2. \tag{3.6}$$

The total QG (classical and quantum) gravitational entropy S_{QG} derives from the general expression

$$S_{QG} = k_B \frac{A_{QG}}{4l_P^2}$$

where $A_{QG} = 4\pi L_{QG}^2 = 4\pi(L_Q + L_G)^2$ is the total area, which is expressed as$A_{QG} = A_Q + A_G + 2a_P$. Recall that $L_Q = l_P^2/L_G$ and $a_P = 4\pi l_P^2$. As a consequence,

$$S_{QG} = 2s_P + S_G + S_Q = 2s_P\left[1 + \frac{1}{2}\left(\frac{S_G}{s_P} + \frac{s_P}{S_G}\right)\right]. \tag{3.7}$$

The total (QG) gravitational entropy is the sum of the three components: classical (subscript G), quantum (subscript Q), and Planck (subscript P) values, corresponding to the three gravity regimes. The term s_P arises from the duality between the quantum and classical black hole sizes L_Q and L_G across the Planck scale. It reflects the complete QG covering: the Planck scale being the bordering or crossing scale common to the two (classical and quantum) Q and G domains, and to the classical (exterior) and quantum (interior) black hole regions.

The gravitational entropy S_G of large (classical) astrophysical black holes is a huge number, consistent with the fact that classical black holes contain a very large amount of information. Moreover, to reach such a large entropy, the black hole in its late collapse state should have been in a highly energetic vacuum state S_G.

The gravitational (Gibbons-Hawking [26] and Bekenstein [27]) entropy covers classical or semiclassical gravity but not the fully quantum gravity domain. In this domain the relevant appropriate size of the quantum system is the Compton or quantum length L_Q and not the gravitational size. The gravitational entropies in the two different domains are classical-quantum gravity duals of each other. The total gravitational entropy is the sum of the entropies in the three main gravity regimes: classical or semiclassical gravity, Planckian, and trans-Planckian. The complete (QG) variables entail precisely those three regimes and also provide the additive constant, that is, the pure Planckian scale term (a constant). The total or complete (QG) entropy here refers to the inclusion of the quantum gravity entropy, which is trans-Planckian and corresponds to the central quantum interior region of the black hole. The imaginary time quantum gravitational instanton treatment and the Euclidean partition function we present here (in Secs. V and VI) provide further support of this entropy.

The complete (classical plus quantum) physical quantities are invariant under the classical-quantum duality: $G \leftrightarrow Q$. As the basis of quantum physics, the wave-particle-gravity duality is reflected in all black hole regions and its associated physical quantities, temperature and entropy. The classical-quantum or wave-particle-gravity duality between the different gravity regimes can be viewed as a mapping between the asymptotic (in and out) states, characterized by the sets BH_Q and BH_G, and thus as a scattering-matrix description. Recall that wave-particle-gravity duality also manifests in the different cosmological eras and associated gravity quantities, temperature and entropy [1–3]: Cosmological evolution goes from a very early or precursor quantum trans-Planckian phase to a semiclassical gravity accelerated era (de Sitter inflation) and then to the known classical gravity eras until the present classical de Sitter phase.

IV. QUANTUM SPACE-TIME STRUCTURE OF BLACK HOLES

The complete QG variables show that there is an underlying classical-quantum duality structure in the complete analytic extension or global structure of the Kruskal space-time: The external or visible region and its mirror copy are the classical or semiclassical gravitational domains while the internal region is a quantum gravitational trans-Planckian scale domain. A duality symmetry between the two external regions and between the internal and external parts appears as a classical-quantum duality through the Planck scale. External and internal regions can be seen with respect to the Planck scale hyperbolae $X^2 - T^2 = \pm 1$, which delimitate the different black hole regions. In fact, the terms "interior" and "exterior" lose their meaning in this region because the classical $X = \pm T$ disappear at the quantum level and become $X^2 - T^2 = \pm 1$, (in Planck units).

Quantum space-time can be described as a quantum oscillator with its quantum algebra. From the classical-quantum duality and quantum oscillator (X, P) variables in global phase space, the space-time coordinates are promoted to quantum noncommuting operators. In classical phase space, the mapping between the Schwarzschild $(x*, p*)$ and Kruskal (X, P) coordinates is given by

$$X = \exp(\kappa x*) \cos(\kappa p*), \qquad P = \exp(\kappa x*) \sin(\kappa p*), \tag{4.1}$$

$$(X^2 + P^2) = \exp(2\kappa x*) = 2H_{\mathrm{osc}},$$
$$(X^2 - P^2) = \exp(2\kappa x*) \cos(2\kappa p*). \tag{4.2}$$

As is known, the classical Kruskal coordinates (X, T) in terms of the Schwarzschild representation $(x*, t*)$ are given by

$$X = \exp(\kappa x*) \cosh(\kappa t*), \qquad T = \exp(\kappa x*) \sinh(\kappa t*), \tag{4.3}$$

$$(X^2 - T^2) = \exp(2\kappa x*) = 2H,$$
$$(X^2 + T^2) = \exp(2\kappa x*) \cos(2\kappa t*), \tag{4.4}$$

with the Schwarzschild star coordinate $x*$:

$$\exp(\kappa x*) = \sqrt{2\kappa r - 1}\exp(\kappa r), \qquad 2\kappa r > 1, \tag{4.5}$$

with $t*$ being the usual Schwarzschild time and κ the dimensionless (in Planck units) gravity acceleration or surface gravity. A similar case, but with X and T exchanged and $x*$ defined by $\exp(\kappa x*) = \sqrt{1 - 2\kappa r}\exp(\kappa r)$, holds for $2\kappa r < 1$.

For (X, T) quantum coordinates, i.e., noncommuting operators, and similarly for $(x*, t*)$, the transformation is given by

$$X = \exp(\kappa x*) \cosh(\kappa t*), \qquad T = \exp(\kappa x*) \sinh(\kappa t*), \tag{4.6}$$

$$(X^2 - T^2) = \exp(2\kappa x*) \cosh(\kappa [x*, t*]), \tag{4.7}$$

$$(X^2 + T^2) = \exp(2\kappa x*) \cosh(2\kappa t*), \tag{4.8}$$

$$[X, T] = \exp(2\kappa x*) \sinh(\kappa [x*, t*]), \tag{4.9}$$

where we used the usual exponential operator product $\exp(A)\exp(B) = \exp(B)\exp(A)\exp([A,B])$.

New terms appear due to the quantum commutators. At the classical level,

$$[X,T] = 0, \qquad [x*, t*] = 0 \quad \text{(classically)},$$

and the known classical Schwarzschild-Kruskal equations are recovered.

Equations (4.6)–(4.9) describe the quantum Schwarzschild-Kruskal space-time structure and its properties. The equation for the quantum hyperbolic "trajectories" is

$$\begin{aligned}(X^2 - T^2) &= \pm\sqrt{\exp(4\kappa x*) + [X,T]^2} \\ &= \pm\sqrt{(1-2\kappa r)^2 \exp(4\kappa r) + [X,T]^2}. \end{aligned} \tag{4.10}$$

The characteristic lines and the classically light-cone generating horizons $X = \pm T$ (at $2\kappa r = 1$ or $x* = -\infty$) become

$$X = \pm\sqrt{T^2 + [X,T]^2} \quad \text{at } 2\kappa r = 1\text{: } X \neq \pm T, \text{no horizons.} \tag{4.11}$$

Here, $X \neq \pm T$ at $2\kappa r = 1$, and the null horizons are erased. Similarly, in the interior regions, the classical hyperbolae $(T^2 - X^2)_{\text{classical}} = \pm 1$, which describe the known past and future classical singularity $r = 0$ ($x* = 0$), become

$$\begin{aligned}(T^2 - X^2) &= \pm\sqrt{1 + [X,T]^2} = \pm\sqrt{2} \\ &\quad \text{at } r = 0\text{: } (T^2 - X^2) \neq \pm 1 \text{ no singularity} \\ (T^2 - X^2)_{\text{classical}} &= \pm 1 \quad \text{at } r = 0 \text{ classically}\end{aligned} \tag{4.12}$$

at the quantum level. Moreover, the quantum Kruskal light-cone variables in hyperbolic space,

$$U = \frac{1}{\sqrt{2}}(X - T), \qquad V = \frac{1}{\sqrt{2}}(X + T), \tag{4.13}$$

are, upon the identification $P = iT$, the (a, a^+) operators in phase space: The creation and annihilation operators (a, a^+) are the light-cone type quantum coordinates of the phase space (X, P):

$$a = \frac{1}{\sqrt{2}}(X + iP), \qquad a^+ = \frac{1}{\sqrt{2}}(X - iP). \tag{4.14}$$

The temporal variable T in the space-time configuration (X, T) is like the (imaginary) momentum in phase space (X, P). The identification $P = iT$ yields

$$X = \frac{1}{\sqrt{2}}(a^+ + a), \;\; T = \frac{1}{\sqrt{2}}(a^+ - a), \;\; [a, a^+] = 1, \tag{4.15}$$

satisfying the algebra

$$\begin{aligned}2H &= (X^2 - T^2) = (2a^+a + 1), \\ (X^2 + T^2) &= (a^2 + a^{+2}), \\ [2H, X] &= T, \quad [2H, T] = X, \quad [X, T] = 1,\end{aligned} \tag{4.16}$$

with $a^+a = N$ being the number operator.

The quantum space-time coordinates (X, T) can therefore be considered quantum oscillator coordinates $(X, T = iP)$, including quantum space-time fluctuations with length and mass within the Planck scale domain and quantized levels. The quadratic form (symmetric order of operators),

$$\begin{aligned}2H &= UV + VU = X^2 - T^2 = (2VU + 1), \\ VU &= N \equiv \text{number operator},\end{aligned}$$

yields the quantum hyperbolic structure and the discrete hyperbolic space-time levels:

$$X_n^2 - T_n^2 = (2n + 1), \quad n = 0, 1, \ldots. \tag{4.17}$$

The amplitudes (X_n, T_n) are

$$X_n = \sqrt{2n + 1}, \qquad T_n = \sqrt{2n + 1}. \tag{4.18}$$

With the identification $T = -iP$, the quantum coordinates (U, V) for hyperbolic space-time are precisely the (a, a^+) operators, and as a consequence, VU is the number operator. The expectation value $(2n + 1)$ has a minimal nonzero value for $n = 0$, which is the zero-point energy or Planck scale vacuum.

(i) The future and past regions of the quantum Planck hyperbolae, $(T^2 - X^2)_{n=0} = \pm 1$, all contain totally allowed levels and behaviors. There is *no* singularity boundary in the quantum space-time, not at $r = 0 = x*$ or any other place. The quantum Schwarzschild-Kruskal space-time is totally regular.

(ii) There are *no* singularity boundaries at the quantum level, not at $(T^2 - X^2)(2\kappa r = 1) = \pm 1$ nor at $(T^2 - X^2)(r = 0) = \pm\sqrt{2}$. The quantum space-time extends without boundary beyond the Planck hyperbolae $(T^2 - X^2)(n = 0) = \pm 1$ towards *all* levels: from the more quantum (low n) levels to the classical (large n) ones. The black hole interior is quantum and trans-Planckian. The internal region to the four quantum Planck hyperbolae $(T^2 - X^2)(n = 0) = \pm 1$ is totally quantum and within the Planck scale: This is the quantum vacuum or zero-point energy region of the quantum interior of the black hole.

(iii) The null horizons disappear at the quantum level. Because of the quantum $[X, T]$ commutator, quantum (X, T) dispersions, and fluctuations, the difference between the four classical Kruskal regions (I–IV) disappears in the trans-Planckian domain, and they become one single central region. This provides support for the quantum identification at the Planck scale of the Kruskal regions, which translates into the CPT symmetry at the quantum level; see Refs. [1–3,28].

(iv) In terms of the local Schwarzschild variables $(x*_{n\pm}, t*_{n\pm})$ or $(x_{n\pm}, t_{n\pm})$, with $x = \exp(\kappa x*)$ and $t = \exp(\kappa t*)$, the levels are

$$x_{n\pm} = \left[\sqrt{2\kappa r_{n\pm} - 1}\right] \exp(\kappa r_{n\pm}) = \left[\sqrt{2n+1} \pm \sqrt{2n}\right], \tag{4.19}$$

$$t_{n\pm} = \left[\sqrt{2n+1} \pm \sqrt{(2n+1) + 1/2}\right],$$
$$x_{n=0}(+) = x_{n=0}(-) = 1: \text{ Planck scale}, \tag{4.20}$$

which complete all the levels. The low n, intermediate, and large n levels describe, respectively, the quantum, semiclassical, and classical behaviors, and their $(\pm)$ branches consistently reflect the classical-quantum duality properties, as shown explicitly for similar branches of the mass spectrum in this section below.

The classical singularity $r = 0 = x*$ is quantum mechanically smeared or erased, which is what is expected in a quantum space-time description. The diagram of the global quantum Schwarzschild-Kruskal space-time, which we name the quantum Penrose diagram, is shown in Fig. 1.

In Eqs. (4.18) and (4.19), X_n and x_n are given in Planck units. In terms of the mass global variables $X = M/m_P$, or the local ones $x = m/m_P$, they translate into the mass levels

$$M_n = m_P\sqrt{(2n+1)}, \quad \text{all } n = 0, 1, 2, \ldots, \tag{4.21}$$

$$M_n n_{\gg 1} = m_P\left[\sqrt{2n} + \frac{1}{2\sqrt{2n}} + O(1/n^{3/2})\right], \tag{4.22}$$

$$m_{n\pm} = \left[M_n \pm \sqrt{M_n^2 - m_P^2}\right]. \tag{4.23}$$

The condition $M_n^2 \geq m_P^2$ simply corresponds to the whole spectrum $n \geq 0$:

$$m_{n\pm} = m_P\left[\sqrt{2n+1} \pm \sqrt{2n}\right]. \tag{4.24}$$

(i) The quantum mass levels here hold for all masses and not only for black holes. Namely, the quantum mass levels are associated with the quantum space-time structure. Space-time can be parametrized by masses (“mass coordinates”), related only to length and time, as the QG variables, on the same footing as space and time variables.

(ii) The branch $(+)$ covers all macroscopic and astrophysical black holes as well as semiclassical black hole quantization $\sqrt{n}$, up to masses nearby the Planck mass; the branch $(-)$ covers quantum masses $1/\sqrt{n}$ in the Planckian and trans-Planckian domains.

(iii) The black hole $m_P\sqrt{n}$ mass quantization is like the string mass quantization $M_n = m_s\sqrt{n}$, $n = 0, 1, \ldots$, with the Planck mass m_P instead of the fundamental string mass m_s, i.e., with G/c^2 instead of the string constant α'.

V. IMAGINARY TIME: THE NEW TRANS-PLANCKIAN BLACK HOLE INSTANTON

In classical (nonquantum) Schwarzschild-Kruskal space-time, taking imaginary time $T = i\mathcal{T}$, $t = i\tau$, transforms the hyperbolic space-time structure into a circular structure: The characteristic lines $X^2 + T^2 = 0$ collapse to $X = \pm\mathcal{T} = 0$. Therefore, the classical horizon $X = \pm T(2\kappa r = 1)$ collapses to the origin, and in the classical (nonquantum) black hole instanton, the black hole interior is cut, with no horizon or curvature $r = 0$ singularity. Therefore, the classical black hole instanton is regular but not complete: The interior black hole region is not covered by the imaginary time, classical (nonquantum) black hole manifold.

In the quantum Schwarzschild imaginary time manifold, the quantum trans-Planckian region corresponds to the black-hole interior (Fig. 2). Moreover, without any singularity (not at $r = 0$ or any other place), the quantum manifold consistently and regularly covers both the external and internal black hole regions. This is the case for both the hyperbolic (real time) and Euclidean (imaginary time) manifolds because of the quantum nonzero commutators $[X, T]$ and $[X, \mathcal{T}]$, respectively.

The complete quantum black hole instanton includes the usual classical or semiclassical black hole instanton for a radius larger than the Planck length, plus a new, central, highly dense quantum core of Planck length radius and high constant and finite curvature at $r = 0$, corresponding to the black-hole interior, which is absent in the noncomplete (classical) black hole instanton.

In the quantum instanton Schwarzschild-Kruskal manifold, Eqs. (4.6) hold but for $T = i\mathcal{T}$, $t* = i\tau*$, and the same star coordinate $x*$:

$$\exp(\kappa x*) = \sqrt{2\kappa r - 1}\,\exp(\kappa r), \qquad 2\kappa r > 1 \tag{5.1}$$

with $\kappa = (c^2/2L_G) = \kappa_P(m_P/4M)$ the gravity acceleration or surface gravity. Another similar patch holds for $2\kappa r < 1$ but with X and $\mathcal{T}$ exchanged (similarly for $x*$ and $\tau*$)

and with $x*$ defined by $\exp(\kappa x*) = \sqrt{1-2\kappa r}\,\exp(\kappa r)$. Therefore,

$$X = \exp(\kappa x*)\cos(\kappa\tau*), \qquad \mathcal{T} = \exp(\kappa x*)\,\sin(\kappa\tau*), \tag{5.2}$$

$$(X^2 + \mathcal{T}^2) = \exp\,(2\kappa x*)\,\cos(\kappa[x*, \tau*]), \tag{5.3}$$

$$(X^2 - \mathcal{T}^2) = \exp(2\kappa x*)\,\cos(2\kappa\tau*), \tag{5.4}$$

$$[X, \mathcal{T}] = \exp(2\kappa x*)\,\sin(\kappa[x*, \tau*]), \tag{5.5}$$

where we used the usual exponential operator product $\exp(A)\exp(B) = \exp(B)\exp(A)\exp([A, B])$.

The Euclidean (imaginary time) quantum instanton clearly shows the new trans-Planckian region because, for $2\kappa r = 1$, $(X^2 + \mathcal{T}^2)$ is *not* zero and has a Planckian radius. The equation for the quantum instanton "trajectories" is

$$\begin{aligned}(X^2 + \mathcal{T}^2) &= \pm\sqrt{\exp(4\kappa x*) + [X, \mathcal{T}]^2} \\ &= \pm\sqrt{(1-2\kappa r)^2\exp(4\kappa r) + [X, \mathcal{T}]}. \end{aligned} \tag{5.6}$$

What was classically the zero radius $X = \pm\mathcal{T} = 0$ at $2\kappa r = 1$ or $x* = -\infty$ is now

$$(X^2+\mathcal{T}^2) = [X, \mathcal{T}]^2 \quad \text{at}\, 2\kappa r = 1: X \neq \pm\mathcal{T} = 0, \quad \text{no horizons}. \tag{5.7}$$

We see that

$$X \neq \pm\mathcal{T} \neq 0 \quad \text{at } 2\kappa r = 1.$$

The classical null horizons corresponding to the origin $X = \pm\mathcal{T} = 0$ in the Euclidean signature space-time (instanton) are quantum mechanically replaced by the Planck circle

$$(X^2 + \mathcal{T}^2) = [X, \mathcal{T}] = 1.$$

Figure 2 clearly displays this picture. In other words, quantum theory consistently extends the instanton manifold. Classically, the instanton is cut at the horizon $r = 1/(2\kappa)$, while at the quantum level, it extends beyond it; it contains the quantum region of Planck length radius l_P, which is necessarily trans-Planckian and is absent at the classical level.

Thus, the quantum and regular imaginary time manifolds (quantum gravitational instantons) are the usual classical or semiclassical instantons for radius larger than the Planck length plus a central, highly dense quantum core of Planck length radius and of high finite curvature, which is absent classically.

The imaginary time τ in the classical instanton is periodic with period $\beta = 2L_G = 1/\kappa_G$:

$$0 \leq \tau \leq \beta = 2L_G = 1/\kappa_G \qquad \text{(classically)}, \tag{5.8}$$

with $1/\beta$ being the intrinsic manifold semiclassical temperature: the Hawking temperature

$$T_Q = t_P\left(\frac{l_P}{2L_G}\right), \tag{5.9}$$

where t_P is the Planck temperature. In the complete or total quantum instanton, the imaginary time is periodic as in Eq. (5.8) but with the complete L_{QG}, which includes the quantum Planckian and trans-Planckian magnitudes:

$$0 \leq \tau \leq \beta = 2L_{QG} = 2(L_G + L_Q) = 1/\kappa_{QG}, \tag{5.10}$$

$$\kappa_{QG} = \kappa_P(l_P/L_{QG}), \quad \kappa_Q = \kappa_P^2/\kappa_G, \quad \kappa_P = c^2/2l_P, \tag{5.11}$$

$$\kappa_{QG} = \frac{\kappa_G}{[1 + (\kappa_G/\kappa_P)^2]} = \frac{\kappa_Q}{[1 + (\kappa_Q/\kappa_P)^2]}. \tag{5.12}$$

In the classical or semiclassical gravity domain, $\kappa_G \ll \kappa_P$, which yields the usual classical surface gravity κ_G of massive bodies with masses $M > m_P$. For $\kappa_Q \ll \kappa_P$, in the quantum domain of masses $M < m_P$ (elementary particle domain), we obtain the quantum $\kappa_Q = \kappa_P(4M/m_P)$. The corresponding complete temperature is

$$T_{QG} = t_P\kappa_{QG}/(2\pi\kappa_P), \quad T_Q = t_P^2/T_G, \quad t_P = m_Pc^2/(8\pi\kappa_B), \tag{5.13}$$

$$T_{QG} = \frac{T_G}{[1 + (T_G/t_P)^2]} = \frac{T_Q}{[1 + (T_Q/t_P)^2]}. \tag{5.14}$$

For large masses, in the astrophysical domain, $T_Q \ll t_P$, we obtain the quantum temperature T_Q, which is the Hawking temperature, as expected. For small masses $0 < M < m_P$, $T_G \ll t_P$, we obtain the usual temperature T_G proportional to the mass, as expected in the elementary particle domain. This is also manifest in the partition function (Sec. VI below) and the corresponding complete entropy. The temperature is a measure of the length (in units of κ_B), $T_G = t_P(L_G/l_P)$, $T_Q = t_P(L_Q/l_P)$, while the gravitational entropy is a measure of the area. In this respect, it is interesting to notice that

$$S_{QFT} = s_P(L/l_P)^3 => n, \tag{5.15}$$

$$S_G = s_P(L/l_P)^2 = M^2 => (\sqrt{n})^2, \tag{5.16}$$

$$S_{\text{string}} = s_P(L/l_P) = M => \sqrt{n}, \tag{5.17}$$

$$S_Q = s_P(l_P/L)^2 = M^{-2} => 1/(\sqrt{n})^2. \tag{5.18}$$

In pure QFT without gravity, the number of modes of the fields is proportional to the volume of the system (i.e., a box), and a short-distance external cutoff is necessary, naturally placed at the Planck length l_P, because of QFT ultraviolet divergences. The string entropy $S_{\rm string}$ is proportional to the length. The black hole gravitational entropy is proportional to the area (S_G or S_Q) and thus interpolates between the nongravitational entropy S_{QFT} and the string entropy $S_{\rm string}$. The known Bekenstein-Hawking entropy S_G exhibits classical or semiclassical nature, i.e., $L \gg l_P$ (equivalently, $M_G \gg m_P, \kappa_G \ll \kappa_P, T_Q \ll t_P$):

$$S_G = s_P(T_G/T_Q) = (Mc^2/T_Q).$$

VI. PARTITION FUNCTION: TRANS-PLANCKIAN ENTROPY

As is known, $D+1$ dimensional quantum field theory with imaginary periodic time $0 \le \tau \le \beta$ corresponds to classical statistical mechanics or field theory with temperature $1/\beta$, which is also used in the Euclidean path integral of gravity (Ref. [26]),

$$\mathcal{Z} = Tr \exp(-\beta\mathcal{H}), \tag{6.1}$$

with $\mathcal{H}$ being the Euclidean Hamiltonian (the "evolution" generator in imaginary time, with the trace implying periodic evolution $0 \le \tau \le \beta$).

The complete (including both classical and quantum) black hole radius and temperature are L_{QG} and T_{QG} and are discussed in Sec. V. The complete (whole range) discrete levels are discussed in Secs. IV and VI. We stress the following about the partitions or the density of levels:

(i) The different types of discrete partitions depend on the physical nature of quantum elements considered (point particles, composite or extended quantum objects).

(ii) The number of partitions depends on whether one considers ordered or unordered partitions, that is, counting or not counting the permutations.

(iii) The degeneracy, the number of states corresponding to the same quantum number (energy, mass, spin, or other), depends on items (i) and (ii) above.

(iv) The ensemble of all partitions considered as a Gibbs ensemble yields a thermodynamical partition.

Let us recall that the number $P_o(n)$ of *ordered* partitions of an integer n into integers grows exponentially with n:

$$P_o(n) = 2^{n-1} = \frac{1}{2}\exp(n\, ln\, 2). \tag{6.2}$$

The number $P_{no}(n)$ of nonordered partitions of n [29] (i.e., without counting permutations), asymptotically for large n, grows exponentially with $\sqrt{n}$:

$$P_{no}(n) = \frac{1}{4\sqrt{3}n}\exp(\pi\sqrt{2n/3})\left[1 + O\left(\frac{\log\, n}{n^{1/4}}\right)\right]. \tag{6.3}$$

(i) Nonordered partitions grow more slowly than the ordered ones. Naturally, the density of states and its degeneracy are smaller when the permutations are not accounted for than when including the permutations.

(ii) The nonordered case corresponds to the density $P_{no}(n)$ of quantum composite elements (with internal structure, extended objects, strings, and hadronic matter). The ordered case corresponds to point particles or quantum point oscillators. Moreover, $\sqrt{n}$ characterizes the mass spectra of composite or extended oscillating objects, while n is typical of the spectra of the punctual objects.

(iii) The existence of a limiting temperature in the corresponding ensembles is determined by a pure number combinatorial structure: that is, by whether permutations are included or not, by whether partitions are ordered or unordered, and by whether the elements are point particles or extended objects with internal composite structure as hadrons, strings, or other higher dimensional objects.

The total gravitational entropy S_{QG} of the total or complete (classical and quantum) black hole Euclidean manifold is the sum of the classical, quantum, and Planck scale entropies:

$$P_{QG} = e^{S_{QG}}, \tag{6.4}$$

$$S_{QG} = 2\left[s_P + \frac{1}{2}(S_G + S_Q)\right], \tag{6.5}$$

$$S_G = \frac{\kappa_B}{4}\frac{A_G}{l_P^2}, \quad S_Q = \frac{\kappa_B}{4}\frac{A_Q}{l_P^2}, \quad s_P = \frac{\kappa_B}{4}\frac{a_P}{l_P^2} = \pi\kappa_B, \tag{6.6}$$

The concept of gravitational entropy is the same for any of the gravity regimes: $Area/4l_P^2$ in units of k_B. For a classical object of size L_G, this is the classical area $A_G = 4\pi L_G^2$. For a quantum object of quantum size L_Q, this is the quantum area $A_Q = 4\pi L_Q^2$ (recall $L_Q = l_P^2/L_G$). For the Planck length, this is the Planck area a_P, and $s_P = \pi\kappa_B$ is the Planck entropy:

$$A_G = a_P\left(\frac{L_G}{l_P}\right)^2, \quad A_Q = a_P\left(\frac{l_P}{L_G}\right)^2 = \frac{a_P^2}{A_G}, \quad a_P = 4\pi l_P^2, \tag{6.7}$$

$$S_G = s_P\frac{\rho_Q}{\rho_P} = s_P\left(\frac{M}{m_P}\right)^2, \tag{6.8}$$

$$S_Q = s_P\frac{\rho_G}{\rho_P} = s_P\left(\frac{m_P}{M}\right)^2. \tag{6.9}$$

The complete entropy is

$$S_{QG} == 2s_P\left[1+\frac{1}{2}(A_G+A_Q)\right], \tag{6.10}$$

and consistently, the complete partition function is

$$\mathcal{Z}_{QG} = e^{S_{QG}} = z_P \mathcal{Z}_Q \mathcal{Z}_G. \tag{6.11}$$

In the quantum space-time region, which classically corresponds to the interior region, the total black hole entropy S_{QG} is dominated by the Planck entropy s_P, the quantum entropy S_Q being extremely low (minimal). The total entropy S_{QG} is very high in the external (semiclassical/classical) regions and dominated by the Bekenstein-Hawking entropy S_G, which is a classical or semiclassical gravity entropy.

The discrete levels $n = 0, 1, 2, \ldots.$ cover all gravity regimes: from the quantum gravity (trans-Planckian and Planckian) central black hole region to the semiclassical and classical exterior black hole regions.

In the non-trans-Planckian domain, black hole space-time levels (in Planck units) for the distances L_{Gn}, vacuum energy Λ_{Gn}, and gravitational (Gibbons-Hawking) entropy S_{Gn} are

$$L_{Gn} = \sqrt{(2n+1)}, \qquad \Lambda_{Gn} = 1/(2n+1),$$
$$S_{Gn} = (2n+1), \qquad n = 0, 1, 2, \ldots.. \tag{6.12}$$

In the trans-Planckian phase $0 < r \leq l_P$, the quantum trans-Planckian levels (Q denoting quantum) are

$$L_{Qn} = 1/\sqrt{(2n+1)}, \qquad \Lambda_{Qn} = (2n+1),$$
$$S_{Qn} = 1/(2n+1), \quad n = 0, 1, 2, \ldots.. \tag{6.13}$$

The respective associated mass levels are

$$M_n = \sqrt{(2n+1)}, \qquad M_{Qn} = 1/\sqrt{(2n+1)}. \tag{6.14}$$

The density of states in the classical and quantum gravity phases is thus

$$d_{Gn} = \exp{(2n+1)} = \exp(M_n)^2,$$
$$d_{Qn} = \exp{[1/(2n+1)]} = \exp(M_{Qn})^2, \tag{6.15}$$

$$d_{QGn} = \exp{[(2n+1) + 1/(2n+1)]} = \exp{[M_n^2 + M_{Qn}^2]}. \tag{6.16}$$

The complete (QG) density of states has both the classical or semiclassical gravity density with the known (Bekenstein-Hawking) entropy S_{Gn} and the quantum gravity density with the new trans-Planckian entropy S_{Qn}. As n increases, the distances increase, S_{Gn} increases, and, consistently, the black hole space-time classicalizes. In the central quantum region, n decreases from the most highly excited central trans-Planckian levels—increasing S_{Qn}, decreasing n until $n = 0$, and then increasing in the semiclassical and classical space-time. As described in Sec. V, the n levels range over *all* scales from the lowest excited levels to the highest excited ones covering the twofold dual branches, classical and quantum, and passing through the Planck scale ($n = 0$), or the crossing scale.

VII. EARLY AND LAST STAGES OF BLACK HOLE EVAPORATION

Our results here—mainly the quantum mass spectrum in Sec. IV—have implications for black hole evaporation in its entire range. Note that (X_n, T_n) are given in Planck (length and time) units. In terms of the global quantum gravity dimensionless length $\mathcal{L} = L_{QH}/l_P$ and mass $\mathcal{M} = M_{QH}/m_P$, Eqs. (4.18) and (4.21) translate into the discrete mass levels:

$$\mathcal{L}_n = \sqrt{(2n+1)} = \mathcal{M}_n,$$
$$n = 0, 1, 2, \ldots. \tag{7.1}$$

The black hole mass and radius have discrete levels $M_{n\pm}, L_{n\pm}$, from the most fundamental one ($n = 0$), to the semiclassical (intermediate n), to the classical ones (large n), which yield a continuum classical space-time, radius, and mass, as expected. This is clearly seen from the mass levels $M_{n\pm}$ [Eqs. (4.21) and (4.22)] (and similarly for the radius levels):

$$M_{(n=0)+} = M_{(n=0)-} = M_{Q(n=0)} = m_P,$$
$$n = 0\text{: Planck mass}, \tag{7.2}$$

$$M_{n+} = m_P\left[2\sqrt{2n} - \frac{1}{2\sqrt{2n}} + O(1/n^{3/2})\right],$$
$$\text{branch}(+)\text{: masses} > m_P, \tag{7.3}$$

$$M_{n-} = \frac{m_P}{2\sqrt{2n}} + O(1/n^{3/2}), \quad \text{branch}(-)\text{: masses} < m_P. \tag{7.4}$$

(i) Large n levels are semiclassical, tending towards a classical continuum space-time. Low n are quantum, the lowest mode ($n = 0$) being the Planck scale. Two dual ($\pm$) branches are present in the local variables $(\sqrt{2n+1} \pm \sqrt{2n})$ reflecting the duality of the large and small n behaviors and covering the *whole* spectrum: from the largest astrophysical masses and scales in branch (+) to the smallest quantum masses and scales in branch (−) passing by the Planck mass and length.

The last stage of black hole evaporation and its quantum decay belong to the quantum mass branch (-), with Planck scale masses and smaller until zero mass.

(i) Black hole masses belong to both branches (+) and (−): Branch (+) covers all macroscopic and astrophysical black holes as well as the semiclassical black hole quantization $\sqrt{2n+1}$ until masses are nearby the Planck mass ($n = 0$).

(ii) The microscopic quantum black holes (with masses near the Planck mass and smaller until zero mass, i.e., originating as a consequence of black hole evaporation, or from Planckian and trans-Planckian primordial fluctuations) belong to the branch (−).

(iii) The branches (+) and (−) cover *all* the black hole masses. The black hole masses in the process of black hole evaporation go from branches (+) to (−). Black hole evaporation ends in branch (−), decaying as a pure (nonmixed) quantum state.

(iv) Black hole evaporation is thermal in its semiclassical gravity phase (Hawking radiation), and it is nonthermal in its last quantum stage, with a pure (nonmixed) quantum decay rate.

(v) In its last phase (mass of the order of and smaller than the Planck mass m_P), the state is no longer a black hole state but a pure (nonmixed) quantum state, decaying like a quantum heavy particle. The quantum black holes decay in discrete levels, into elementary particle states, that is, pure (nonmixed) quantum states with the decay rate

$$\Gamma = \frac{g^2 m}{\text{num. factor}} \tag{7.5}$$

where g is the (dimensionless) coupling constant, m is the typical mass in the theory considered (the mass of the unstable particle or object), and the numerical factor often contains the relevant mass ratios in the decay process.

The unifying formula, Eq. (7.5), for quantum heavy particles [30] nicely encompasses all the particle width decays in the standard model (muons, Higgs, etc.), as well as the decay width of topological and nontopological solitons, cosmic defects, and fundamental quantum strings [30].

For the last stages of quantum black holes, in terms of the discrete mass levels, the decay levels are

$$\Gamma_n = G\sqrt{2n+1},$$

which is the same $\sqrt{n}$ dependence as for the decay Γ_{string} of quantum strings.

A quantum closed string in an nth excited state decays into lower excited states (including the dilaton, graviton, and massless antisymmetric tensor fields) [31] with a total width, given to the dominant order (one string loop), by $\Gamma_{\text{string}} = G\,T_s^3/n^0 \approx G l_s^3$, which can also be written as

$$\Gamma_{\text{string}} = g^2 m_s/n^0 = G m_s/\alpha' n^0, \tag{7.6}$$

with n^0 being a numerical factor, and l_s, m_s, and T_s being the string length, mass, and string temperature (α' playing the role of G/c^2). In other words, the string decay Γ_s has the same structure as Eq. (7.5) with $g \equiv \sqrt{G}/\alpha'$.

A semiclassical black hole decays thermally, except in the last evaporation phases, as a graybody at the Hawking temperature T_Q, the graybody factor being the classical black hole absorption cross section σ_G, e.g., the black hole area A_G, the mass loss rate being $(dM/dt) = -\sigma L_G^2 T_Q^4 \approx 1/L_G^2$ (with σ the Stefan constant). Therefore, the semiclassical black hole decay rate is given by

$$\Gamma_{BH} = \left|\frac{d\ln M}{dt}\right| = G T_Q^3/n^0 \approx G/L_G^3. \tag{7.7}$$

As evaporation proceeds, the black hole temperature increases until it reaches the string temperature $T_s = \hbar c/(2\pi\kappa_B l_s)$, $l_s = \sqrt{\hbar\alpha'/c}$ (Refs. [32–35]), undergoing a phase transition into a quantum string or a quantum composite state regime $T_G \to, T_s$, $L_G \to l_s$. The black hole becomes a quantum string or quantum composite state and decays with a width

$$\Gamma_{BH} \to G T_s^3 \approx G/l_s^3 \to \Gamma_{\text{string}}.$$

The semiclassical black hole decay rate Γ_{BH} tends to the string decay rate Γ_s. The similarity between the black hole decay and the elementary particle decay rate is achieved for quantum black holes, when the black hole enters its quantum gravity regime, e.g., the Planck mass at the ending phase of evaporation.

Here, we compare with the string case because the computations of black hole radiation in string theory (Refs. [32–35]) explicitly support this picture. On the other hand, without using string theory, we find that the mass quantum discrete spectrum of black holes is similar to the mass quantum string spectrum. A similar picture holds for a quantum Planckian decaying state (instead of a quantum decaying string state): a quantum state at the typical Planck (or trans-Planckian) temperature T_P, with Planck mass and length (m_P, l_P) instead of the string ones:

$$\Gamma_{BH} \to G T_P^3 = G/l_P^3 \to \Gamma_P.$$

There are no quantum objects at heavy masses such as the Planck mass which would remain stable. They naturally decay quantum mechanically in all particles, mainly gravitons and radiation. Therefore, the remnant states of the black holes are the last emitted particles—gravitons, radiation, and other elementary particles—but not stable Planck mass objects.

Finally, let us point out that the whole process of black hole formation and evaporation can be considered in terms of a scattering matrix between the asymptotic states.

Black hole formation through the gravitational collapse of a star can be described as an S-matrix evolution ($\mathcal{S}_{BH}$):

$$|\Psi_{BH}(t)\rangle = \mathcal{S}_{BH}(t)|\Psi_{\rm star}(t=t_{\rm in})\rangle. \qquad (7.8)$$

It can be expressed in terms of the final star state at $t = t_{\rm final}$, that is, the black hole state. In general,

$$|\Psi_{\rm star}(t)\rangle = \mathcal{S}_{\rm star}(t)|\Psi_{\rm star}(t_{\rm in})\rangle. \qquad (7.9)$$

In addition, black holes evaporate and, after a long enough time, asymptotically end in a gas of particles and radiation, which eventually, under gravity and pressure evolution, forms a star. In other words, the initial gravitating gas state forming a star can be the final gravitating gas state emitted by the evaporating quantum black hole (or at least part of it):

$$|\Psi_{\rm star}(t_{\rm in})\rangle = \mathcal{S}_{\rm star}(t_{\rm in})|\Psi_{QBH}(t_{\rm final})\rangle. \qquad (7.10)$$

Therefore,

$$|\Psi_{\rm star}(t)\rangle = \mathcal{S}_{\rm star}(t)\mathcal{S}_{\rm star}(t_{\rm in})|\Psi_{QBH}(t_{\rm final})\rangle. \qquad (7.11)$$

It can also be expressed in terms of the initial state $|\Psi_{BH}(t_{\rm in})\rangle$ instead of the final state $|\Psi_{QBH}(t_{\rm final})\rangle$. Therefore,

$$|\Psi_{\rm star}(t)\rangle = \mathcal{S}_{\rm star}(t)\mathcal{S}_{BH}(t)^{-1}|\Psi_{BH}(t)\rangle. \qquad (7.12)$$

This is another example of unitarity in a complete quantum evolution; the S-matrix in the whole process is unitary, $SS^{+} = 1 = S^{+}S$. “In nature nothing is lost, all is transformed” [36].

VIII. BLACK HOLE INTERIOR: THE QUANTUM TRANS-PLANCKIAN DE SITTER VACUUM

In Sec. IV, we described the quantum space-time structure of black holes in terms of a quantum oscillator algebra with discrete hyperbolic levels $(X^2 - T^2)_n = (2n+1)$, $n = 0, 1, 2, \ldots$. The zero-point energy ($n = 0$) is the quantum and trans-Planckian vacuum in the central region delimitated by the four hyperbolae $X^2 - T^2 = \pm 1$ of the Planck scale ($n = 0$) level. This is precisely a constant curvature de Sitter vacuum: The de Sitter vacuum can be described as a (inverted, i.e., with imaginary frequency) harmonic oscillator, the oscillator constant being [1,3]

$$\kappa_{\rm osc} = H^2, \qquad H = \sqrt{(8\pi G\Lambda)/3} = c/l_{\rm osc}. \qquad (8.1)$$

The oscillator length $l_{\rm osc}$ is the Hubble radius, the Hubble constant $H = \kappa$ being the surface gravity, as the black hole surface gravity is the inverse of (twice) the black hole radius. The description of de Sitter space-time as an (inverted) harmonic oscillator derives from the Einstein equations on the one hand [1,37,38]; on the other hand, it stems more generally from the de Sitter geometrical description—as a hyperboloid embedded in a flat Minkowski space-time with one more spatial dimension,

$$-T^2 + X^2 + X_i^2 + Z^2 = L_{QG}^2, \qquad (8.2)$$

$$L_{QG} = (L_Q + L_G) = l_P(H/h_P + h_P/H), \quad h_P = c/l_P. \qquad (8.3)$$

In the anti–de Sitter case, the description is the same but with $-T^2 + X^2 + X_i^2 + Z^2 = -L_{QG}^2$, and therefore the anti–de Sitter background is associated with a real frequency (noninverted) harmonic oscillator. In addition, the propagation of fields and linearized perturbations in the de Sitter vacuum all satisfy equations which are like the inverted oscillator equations (see Refs. [39–41]) or normal oscillators in anti–de Sitter.

In the black hole case, the physical magnitudes such as the oscillator constant H^2 and typical length $(c/l_{\rm osc})$ are related to the black hole mass M:

$$H = c/l_{\rm osc} = h_P\left(\frac{m_P}{M}\right)\Lambda = \lambda_P\left(\frac{m_P}{M}\right)^2,$$
$$h_P = c/l_P, \qquad \lambda_P = 3h_P^2/c^4. \qquad (8.4)$$

Note that $L_{QG} = (L_G + L_Q)$ in Eq. (8.2) is the complete length, allowing us to describe the classical, semiclassical, and quantum (trans-Planckian) gravity domains. The complete vacuum density ρ_{QG} in the quantum gravity regime, including the classical and quantum ones (ρ_G, ρ_Q) (ρ_P being the Planck density scale), is

$$\rho_{QG} = \frac{\rho_G}{[1+\rho_G/\rho_P]^2} = \frac{\rho_Q}{[1+\rho_Q/\rho_P]^2},$$
$$\rho_{QG}(\rho_G) = \rho_{QG}(\rho_Q) = \rho_{QG}(\rho_P^2/\rho_G), \qquad (8.5)$$

$$\rho_G = \rho_P(H/h_P)^2 = \rho_P(\Lambda/\lambda_P), \quad \rho_P = 3h_P^2/8\pi G, \qquad (8.6)$$

$$\rho_Q = \rho_P(H_Q/h_P)^2 = \rho_P(\Lambda_Q/\lambda_P) = \rho_P^2/\rho_G. \qquad (8.7)$$

The QG magnitudes are complete variables covering classical and quantum, as well as Planckian and trans-Planckian domains. The high density ρ_Q and Λ_Q describe the quantum trans-Planckian vacuum. This is precisely expressed by Eqs. (3.1) and (3.2) applied to this case:

$$\frac{\rho_G}{\rho_P} = \left(\frac{l_P}{L_G}\right)^2 = \left(\frac{m_P}{M}\right)^2 = \left(\frac{S_Q}{s_P}\right), \qquad (8.8)$$

$$\frac{\rho_Q}{\rho_P} = \left(\frac{l_P}{\Lambda}\right) = \left(\frac{M}{m_P}\right)^2 = \left(\frac{S_G}{s_P}\right). \qquad (8.9)$$

The right-hand side of Eqs. (8.8) and (8.9) show the link to the gravitational entropy: quantum gravitational S_Q and classical or semiclassical S_G entropy.

Here, (Λ, ρ_G) describe a classical gravitational vacuum: an empty or dilute gravitational vacuum state of large classical sizes $L_G = l_P\sqrt{\lambda_P/\Lambda} = l_P(M/m_P)$, very small density, and very low Λ values. Consistently, the *small* value of the quantum gravitational entropy S_Q is equal to such a small Λ value.

Note that (Λ_Q, ρ_Q) describe a quantum gravitational vacuum, in the trans-Planckian domain of very small sub-Planckian sizes $L_Q = l_P\sqrt{\Lambda/\lambda_P} = l_P(m_P/M)$, very high density, and very high Λ_Q values. Consistently, the *high* value of the classical gravitational entropy S_G is equal (in Planck units) to such a high Λ_Q value.

The external black hole region is precisely a classical gravity dilute vacuum, which in the present universe cannot be larger than the observed very low values of the classical cosmic vacuum density and cosmic vacuum energy (Λ, ρ_G) [10–14,19,20]. The quantum duals of the present classical universe cosmic vacuum values provide an upper bound to the high values (Λ_Q, ρ_Q) in the quantum central vacuum black hole region, as determined by Eqs. (8.6)–(8.9).

We quantize the (X, T) dimensions which are relevant to the quantum space-time structure. The remaining spatial transverse dimensions $X_\perp$ are considered here as noncommuting coordinates. This corresponds to quantizing the two-dimensional surface (X, T) relevant for the light-cone structure. Notice that although the transverse spatial dimensions $\perp$ have zero commutators, they could fluctuate. It is enough to consider the novel features arising in the quantum space-time structure and the quantum light cone.

IX. CONCLUDING REMARKS

This approach is a first step to globally and nonperturbatively cover the classical, semiclassical, and quantum gravity domains of black holes. This framework supports and is consistent with the idea that a quantum theory must be finite. Here, the global *QG* variables and quantum discrete space-time include the highly quantum trans-Planckian domain and go well beyond other approaches.

The trans-Planckian domain in black holes is found in the central interior region, and this is so for *all* black hole masses, including astrophysical and macroscopical black holes whose exterior space-times are classical and semiclassical regions. The highly excited vacuum central region is a constant curvature de Sitter vacuum without any singularity. The most central quantum trans-Planckian black hole regions have the most higher excited levels, with $\Lambda_{Qn} = M_n = \sqrt{2n+1}$ (in Planck units), and the smallest quantum gravitational entropies $S_{Qn} = 1/(2n+1)$.

De-excitation of the levels starts from the central quantum trans-Planckian core of the black hole with high n until $n = 0$ (the Planck scale) and then enters the semiclassical/classical gravity exterior space-time region and becomes more and more de-excited and classical for increasing n (the classical branch), with decreasing vacuum energy and a continuum spectrum reaching asymptotically flat space-time. In the process of classicalization, n increases from the Planck level ($n = 0$), and $X_n = \sqrt{(2n+1)}$ increases; the huge and finite values of the central black hole vacuum energy and curvature diminish as $1/(2n+1)$ and vanish asymptotically for huge n. This is coherently accomplished by the increasing distances $L_n = \sqrt{(2n+1)}$, and the increasing levels $S_{Gn} = (2n+1)$ of the Bekenstein-Hawking entropy, which is a classical or semiclassical gravitational entropy, and it is always an upper bound to the other entropies.

Recall that quantum backreaction effects, gravitational scattering near an event horizon structure, produces a quantum shift too (the shifted horizon) [35,42–47]. This approach consistently describes the cosmological phases from the pre-Planckian or trans-Planckian quantum phases to the Planck scale and then to the post-Planckian universe (Refs. [1,3]).

The identification of space-time (IST) has been investigated in the past and recent years at the level of semiclassical gravity [28,48–52]. In our framework, we have not used IST, but as already pointed out in [1,2], our results support *CPT* and IST in the full quantum theory. In semiclassical gravity, the symmetric (or antisymmetric) IST QFT provides a *CPT* symmetry of the theory. In the Euclidean (imaginary time) manifold, the differences between the four Kruskal space-time regions disappear, and they are automatically identified. In the central trans-Planckian region of the hyperbolic (real time) quantum space-time, the four Kruskal regions merge into one single region and are automatically identified.

Other approaches to the black hole interiors have been considered recently; see, for example, Refs. [28,53,54]. In Refs. [55,56], a regular black hole interior is described classically with a classical space-time geometry sourced by a maximal negative radial pressure. Interestingly (e.g., in Ref. [53] and references therein), the black hole interior model is also regular, with a de Sitter–like geometry. These are effective models that could help us to disentangle the properties of the black hole interiors through different observational gravitational signals.

In our work, the black hole interior appears as a fully quantum gravity region. Interestingly enough, this feature also appears in a different approach using scaling arguments in maximal entropic states, e.g., Ref. [57], which shows the consistency of the results. In this paper, such a feature is a direct consequence of the classical-quantum gravity duality, which shows, in addition, that the black hole interior is necessarily trans-Planckian. From a fully quantum space-time description (a quantum algebra of nonconmutative space-time instead of a space-time metric), we find that the interior is totally regular and of constant curvature. Here, the black hole interior is a truly quantum trans-Planckian vacuum, totally regular and of constant curvature. In addition, the quantum Penrose diagram is new and has not been considered before, as well as the quantum completion of the Gibbons-Hawking instanton, with the quantum trans-Planckian core at the black hole center. These results allow us to better describe and understand the total regularity of the quantum black hole space-time, e.g., the nonsingularity at the center, the description of such interior and exterior regions, and their connection to the constant curvature vacuum describing dark energy. The complete partition function is new and allows us to understand the discrete spectrum of the different black hole regions, accomplished by the complete entropy and black hole evaporation stages.

It is not our aim here to review the black hole interior literature. Our work here is in the context of trans-Planckian physics, which appears necessary to describe the black hole interiors and determine which classical gravitational dual provides the black hole exteriors; thus, a global unifying description of the space-time is provided, and the same approach allows the description of the very early cosmological phase before inflation, with its classical gravitational dual (dark energy).

X. CONCLUSIONS

(i) Overall, a consistent quantum picture of the black hole space-time appears from the internal central black hole regions, which are the most quantum and trans-Planckian, to the semiclassical and classical external regions until the asymptotically flat far regions from the black hole, together with their physical magnitudes and spectrum: size, mass, partition function, gravitational entropies, and temperatures covering all mass ranges and gravity domains—quantum (trans-Planckian) gravity and semiclassical/classical gravity.

(ii) The quantum vacuum energy bends space-time and produces a constant curvature background in the central black hole region of Planck length radius l_P. We find the quantum discrete levels: length, mass vacuum energy, gravitational entropy, and temperature from the black hole central trans-Planckian vacuum, passing through the Planck scale, to the external semiclassical and classical exterior vacuum regions. The gravitational entropy of the Universe today, $S_{\mathrm{today}} = (2n+1) = 10^{122}$, is the absolute upper bound to all entropies, in particular, to all black hole entropies.

(iii) The quantum space-time structure allows a new quantum region, which is purely quantum vacuum, or zero-point Planckian and trans-Planckian energy and constant curvature. This central quantum vacuum core is a de Sitter quantum trans-Planckian vacuum described by the relevant quantum noncommutative coordinates and the quantum hyperbolic structure.

(iv) In the external black hole space-time, the discrete levels extend from the Planck scale level ($n = 0$) and low n to the quasiclassical and classical levels (intermediate and large n), tending asymptotically (very large n) to a classical continuum space-time. Consistently, these levels have larger gravitational (Gibbons-Hawking) entropy $S_{Gn} = (2n+1)$, $n = 0, 1, 2, \ldots$, and lower vacuum energy $\Lambda_n = 1/(2n+1)$. In the central quantum trans-Planckian core of the black hole, the levels extend from the Planck scale ($n = 0$) to lengths smaller than the Planck scale, until the quantum highly excited trans-Planckian levels (very large n), which are those of smaller entropy $S_{Qn} = 1/(2n+1)$ and higher vacuum density $\Lambda_{Qn} = (2n+1)$.

(v) There is no singularity at the black hole origin. First, the $r = 0$ mathematical singularity is not physical: It is the result of extrapolation of the purely classical (nonquantum) general relativity theory, out of its domain of physical validity. The Planck scale is not merely a useful system of units but a physically meaningful scale, the onset of quantum gravity; this scale precludes the extrapolation until zero time or length. This is precisely what is expected from quantum trans-Planckian physics in gravity: the smoothness of the classical gravitational singularities. Second, the de Sitter vacuum, which is the vacuum interior region of the black hole, is a smooth constant curvature vacuum without any curvature singularity. Third, the small and trans-Planckian vacua have a high but bounded trans-Planckian constant curvature and are therefore without singularity.

(vi) There are *no* singularity boundaries at the quantum level at $(T^2 - X^2)(r = 0) = \pm 1$ nor at $(T^2 - X^2) = \pm\sqrt{2}$. The quantum space-time extends without boundary beyond the Planck hyperbolae $(T^2 - X^2)$ $(n = 0) = \pm 1$ towards *all* levels. Note that $(T^2 - X^2) = \pm\sqrt{2}$ are the quantum hyperbolae which replace the classical singularity: $(T^2 - X^2)_{\mathrm{classical}}$ $(r = 0) = \pm 1$. Moreover, the quantum hyperbolae $(T^2 - X^2) = \pm\sqrt{2}$ lie outside the allowed quantum hyperbolic levels $(T^2 - X^2)_n = (2n+1)$, $n = 0, 1, 2, \ldots$, and therefore, they are excluded at the quantum level: The singularity is removed from the quantum space-time. There is *no* singularity boundary in the quantum space-time, not at $r = 0 = x*$, not at any other place. The quantum Schwarzschild-Kruskal space-time is totally regular.

(vii) The quantum trans-Planckian core is present in *all* black holes, macroscopic and astrophysical ones.

It also appears in the imaginary time manifold (instanton), and it allows us to complete the classical gravity Gibbons-Hawking instanton, which is cut at the horizon: The classical black hole instanton is thus regular but *not* complete. The black hole quantum instanton is regular and complete. The complete partition function, temperature, and entropy all reflect this feature and clearly include the highly excited and dense trans-Planckian central region of radius l_P, as well as the discrete levels, the density of black hole states, and the black hole decay rate.

(viii) States with the Planck mass m_P are *not* black holes; they are entirely quantum gravity states, decaying in the way heavy particles or quantum strings do—in this case, in gravitons, other elementary particles, and radiation. Black holes reaching the Planck mass in the process of their evaporation undergo a phase transition into a pure (nonmixed) quantum state which decays in gravitons, particles, and radiation.

(ix) The results of this paper could provide insights for research directions and new understanding in quantum theory and gravity and for the search of quantum gravitational signals, for e-LISA [5], for instance, after the success of LIGO [6,7], as well as for other quantum signals in space-time [17–20, 58–64], black holes in particular, for astrophysical black holes and for quantum black holes, or the last stages and remnants of black hole evaporation and black hole explosions. One of the novel results of this paper is that quantum physics is an inherent constituent of all black hole interiors, from the horizon to the center, in particular, in the largest and astrophysical black holes. Our results also show that the black hole interior trans-Planckian vacuum is of the same nature as the very early cosmological vacuum: quantum, trans-Planckian, and of constant curvature, whose classical gravity dual is a very dilute, very low energy gravitational vacuum (dark energy).

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Correction: Some change requests made on the second set of proofs were overlooked and have been set right (primarily renumbering of section number citations).