Vacuum energy density and gravitational entropy

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The failure to calculate the vacuum energy is a central problem in theoretical physics. Presumably the problem arises from the insistent use of effective field theory reasoning in a context that is well beyond its intended scope. If one follows this path, then one is led inevitably to statistical or anthropic reasoning for observations. It appears that a more palatable resolution of the vacuum energy problem requires some form of UV/IR feedback. In this paper we take the point of view that such feedback can be thought of as arising by defining a notion of quantum space-time. We reformulate the regularized computation of vacuum energy in such a way that it can be interpreted in terms of a sum over elementary phase space volumes, that we identify with a ground state degeneracy. This observation yields a precise notion of UV/IR feedback, while leaving a scale unfixed. Here we argue that holography can be thought to provide a key piece of information: we show that equating this microscopic ground state degeneracy with macroscopic gravitational entropy yields a prediction for the vacuum energy that can easily be consistent with observations. Essentially, the smallness of the vacuum energy is tied to the large size of the Universe. We discuss how within this scenario notions of effective field theory can go so wrong.

The success of the standard models of particle physics and of cosmology are unquestionable. And yet, vexing problems remain, often associated with apparent hierarchies of scales. In the absence of gravity in our description, effective field theory (EFT) methods are expected to apply and in fact are foundational in the way that we think of quantum field theoretic physics [1]. In recent years however, it has become increasingly clear that the nature of the vacuum in gauge theories and gravity is much richer than previously thought [2,3] (see also [4,5]). In addition, it is without a doubt that holography and related quantum information theoretic ideas are important ingredients in our understanding of gravity and are expected to play a central role in a quantum theory [6]. It is natural to ask if each of these concepts are important for the central gravitational hierarchy problem, that of the vacuum energy density, and if they complicate the application of EFT methods [7,8].

There are many extant reviews on the vacuum energy density problem [9–13]. The basic prediction (of effective field theory) is that this scales as the fourth power of a momentum space cutoff. This prediction differs exponentially from the observed value unless that cutoff is significantly smaller than expected. It is also notable that perturbative string theory seemingly ameliorates the problem through its ultraviolet finiteness (certainly a non-EFT property) and yet offers no solution as an arbitrary cutoff is simply replaced by the string mass scale, as pointed out by Polchinski long ago [14,15]. Much effort has gone into explaining away the problem by anthropic or statistical reasoning but to most these ideas [10,13], if they could be convincingly implemented, would be disappointing. Clearly, a resolution of the problem within traditional

EFT is impossible. As mentioned briefly by Polchinski in his review [13] and by others, the vacuum energy density problem seems to point to some feedback mechanism between the ultraviolet (UV) and infrared (IR). An implementation of this mechanism, as far as we are aware, has never been offered.

In this paper, we will introduce such a mechanism. In a few words, using just prime principles of quantum theory, we will introduce a way to enumerate the vacua of a microscopic theory. Essentially this constitutes a novel regulator for observables (such as the vacuum energy density) that directly address vacua and it can in particular be employed to calculate a regulated value of the vacuum energy density. This calculation involves an undetermined length scale; this scale can be fixed by equating the microscopic count of vacua with the macroscopic value of gravitational entropy. Remarkably, using this information for the regulated vacuum energy density yields a prediction that is consistent with observation. Heuristically one can say that the holographic bound on entropy precludes the size of momentum space (and thus effectively its cutoff) being as large as EFT assumes. These ideas originate in the realization that the trace involved in the computation of the vacuum energy can be understood in terms of phase space geometry. The vacua referred to above correspond to elementary quantum areas in phase space. A prime principle of quantum theory is the basis independence of a trace; in the present context, a judicious choice of basis (called a modular basis in [16,17]) makes direct contact with minimal phase space areas. Remarkably, this quantum area constraint translates to a link between UV and IR scales.

The standard calculation of the vacuum energy density proceeds by computing the circle amplitude. This can be thought of directly in Feynman diagram terms but since the diagram has no external legs and involves no interactions at one loop, is equally well expressible using the worldline formalism, yielding[1]

$$Z_{S^1} = \int_0^\infty \frac{d\tau}{2\tau} \mathrm{Tr} e^{i\hat{\mathcal{H}}\tau}, \tag{1}$$

where in this expression $\hat{\mathcal{H}}$ is the Hamiltonian constraint and τ the intrinsic length of the worldline. As a trace, we expect the value of Z_{S^1} to be basis independent. A choice of basis is equivalent to a choice of polarization of phase space. In particle theory, it is natural to choose the momentum basis, and we would write[2]

$$Z_{S^1} = \int_0^\infty \frac{d\tau}{2\tau} \int \frac{d^4 p}{(2\pi)^4} \langle p_\mu | e^{i\hat{\mathcal{H}}(\hat{p})\tau} | p_\mu \rangle, \tag{2}$$

$$= \delta^{(4)}(0) \int_0^\infty \frac{d\tau}{2\tau} \int \frac{d^4 p}{(2\pi)^4} e^{i\mathcal{H}(p)\tau}. \tag{3}$$

We see that this choice of basis yields a singular result. The standard way to deal with this is well known: one interprets the momentum space δ function as the space-time volume,

$$Z_{S^1} = \int_0^\infty \frac{d\tau}{2\tau} \int \frac{d^4 q d^4 p}{(2\pi\hbar)^4} e^{i\mathcal{H}(p)\tau}. \tag{4}$$

We note that this casts the trace as an integration over the classical phase space. The usual treatment is to take the space-time volume $V_q = \int d^4 q$ as fixed but finite and treat the momentum and τ integrals separately (which require further regularization). It should be emphasized that this step explicitly uses a standard but singular polarization of phase space: the Schrödinger polarization in which the fundamental phase space cell is collapsed onto a line [17].

Integrating over the modular parameter τ reduces the trace to a sum over *on-shell* particle states and gives the familiar result

$$\int_0^\infty \frac{d\tau}{2\tau} V_q \int \frac{d^4 p}{(2\pi\hbar)^4} e^{i\mathcal{H}(p)\tau} \rightarrow V_q \int \frac{d^3 p}{(2\pi\hbar)^3} \frac{1}{2} \omega_{\vec{p}}, \tag{5}$$

where we have taken $\mathcal{H}(p) = \frac{1}{2}(p_\mu^2 + m^2)$ and $\hbar\omega_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$. Recall that the one-loop vacuum amplitude is related to the circle amplitude by exponentiation, $Z_{\mathrm{vac}} = \exp(Z_{S^1}) = \langle e^{-i\hat{H}T/\hbar} \rangle$, yielding

$$Z_{S^1} = \rho V_q / \hbar, \tag{6}$$

where ρ is the vacuum energy density [14].

As usual, this integral is UV divergent, and implementing a simple cutoff yields a scaling as the cutoff to the fourth power [8]. We emphasize here that in this computation, there is an implicit assumption of a unique spatially translation-invariant ground state, with the momentum $\vec{p}$ thought of as a label on the physical particle states that propagate in a fixed space. This is a possible interpretation once the τ integration has been done, reducing the trace to a sum over on-shell particle states. If we had several species of particles, we would continue with this interpretation, simply summing over the species.

In perturbative string theory [14,15], the integral over p comes from the zero modes of the string in flat space-time; in the limit of a long thin string, the torus amplitude has a traditional interpretation as an (infinite) sum over particle states, again with p_μ interpreted as the momentum of each off-shell particle state. In string theory, the τ integral is also replaced by an integral over the moduli of the torus, which can be thought of as a complexification of τ.

Having reviewed the standard approach to the vacuum energy problem, let us return to the formula (4). In the analogous calculation in string theory, one leaves the integral over moduli to the end. It is convenient to do

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[1]See for example Sec. 7.3 of [14], pages 216–222.
[2]By $\delta^{(4)}(0)$, we mean the limit $\lim_{q\to 0}\delta^{(4)}(q)$ of a delta function in momentum space.

the same here in the field theory computation, focusing on the integral

$$Z(\tau) = \int \frac{d^4q\, d^4p}{(2\pi\hbar)^4} e^{-p_\mu^2\tau/2} = \text{Tr}\, e^{-\hat{p}_\mu^2\tau/2}, \qquad (7)$$

where we have Wick rotated to Euclidean signature and similarly redefined τ, and

$$Z_{S^1} = \int_0^\infty \frac{d\tau}{2\tau} e^{-m^2\tau} Z(\tau). \qquad (8)$$

By doing so, we are free to regard this trace directly, without an assumed interpretation in terms of particle states —it is simply a trace in (four copies of) the Heisenberg group Hilbert space. In what follows, we will make use of the expectation that the volume in phase space is related to a count of the number of degrees of freedom (that is, an entropy), and indeed we will suggest that the vacuum energy is bounded by this count.

To proceed, note that we may write

$$Z(\tau) = \prod_{j=1}^4 \left[\frac{1}{2\pi\hbar} \int_{-\infty}^\infty dq_j \int_{-\infty}^\infty dp_j e^{-p_j^2\tau/2} \right], \qquad (9)$$

$$= \left[\frac{\lambda\varepsilon}{2\pi\hbar} \sum_{k,\tilde{k}\in\mathbb{Z}} \int_0^1 dx \int_0^1 d\tilde{x}\, e^{-(\tilde{x}+k)^2\varepsilon^2\tau/2} \right]^4. \qquad (10)$$

Here we have split the integral over phase space into a sum over integrals in a finite cell, via $p \to \varepsilon\tilde{x}$, $q \to \lambda x$. The dimensionful scales λ, ε are arbitrary here. While the manipulations done above are trivial rearrangements of the integral, the result on the second line has an interpretation of having done the trace in another basis, a so-called *modular polarization* [17–19], which is unitarily equivalent (via Zak transform) to the momentum basis. One refers to $(x, \tilde{x}) \in [0,1]^2$ as a *modular cell* [17]. The sums over $k, \tilde{k}$ can then be interpreted as counting such modular cells. In fact it is possible to carry out the quantization procedure directly in the modular polarization [17], and one finds that there is one state per cell, characterized by a function possessing one zero per modular cell. Rewriting $Z(\tau)$ in the form (10) suggests that the modular cell should be interpreted as the fundamental minimal area cell in phase space. That is, we identify

$$\lambda\varepsilon = 2\pi\hbar, \qquad (11)$$

which we refer to as the quantum area constraint. Whereas the area of the modular cell is fixed by this quantum constraint, its shape is not.[3] One might expect the shape to be determined contextually.

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[3] More generally one could consider an arbitrary tiling of phase space, with local values of λ, ε. In this paper, we make a simplifying choice of homogeneity.

Now of course $Z(\tau)$ is divergent and so we must introduce a regulator to deal with it further. Here, we will simply restrict the sums over $k, \tilde{k}$ to a finite range, and write

$$Z(\tau)_{m.r.} := \left[\sum_{k=0}^{N_q-1} \sum_{\tilde{k}=0}^{N_p-1} \int_0^1 dx\, d\tilde{x}\, e^{-(\tilde{x}+k)^2\varepsilon^2\tau/2} \right]^4 \qquad (12)$$

where N_p and N_q are finite integers. We refer to this as a *modular regularization*, but we note that given λ, ε, then N_q and N_p determine the total spatial and momentum size of the regulated phase space

$$L := N_q\lambda, \qquad M := N_p\varepsilon. \qquad (13)$$

From this point of view, we have not done anything nonstandard and have merely cut off the size of both space and momentum. However, given the interpretation of the modular cell as a minimal area cell in phase space, and assigning one quantum degree of freedom to each cell, we can associate N_q, N_p with the total number of degrees of freedom

$$N = (N_q N_p)^4 \qquad (14)$$

in four space-time dimensions. This allows us to rewrite $(\lambda, \varepsilon, N_q, N_p)$ in terms of $(L, N, \lambda, \hbar)$. We note in particular that combining (14) with (11) and (13), we obtain

$$\frac{LM}{2\pi\hbar} = N^{1/4}. \qquad (15)$$

This result is unusual: we interpret it to mean that if we regard N as fixed, then the cutoffs on space and momentum are not separately arbitrary but are inversely related. This clearly can be interpreted as a UV/IR mixing phenomenon. In EFT, there is no such relation, because there is no notion of finite N.

Let us expand on this point. As we have seen above, the vacuum partition function Z_{S^1} for a particle scales as the product of the volume of spacetime and the volume of momentum space, i.e., as the covariant *phase space volume*. It is natural to associate this volume with the counting of a number of states. In the classical particle picture, there is one state for each value of the position in spacetime and each value of 4-momentum. Naively, the number of states is, therefore, infinite. Both UV and IR regulators are needed to make the phase space volume finite, where it becomes the product of two factors, V_q and $V_p \sim M^4$. In quantum theory, there is a bound on how many states we can put in a given finite size phase space, since $\hbar$ sets the scale for a minimal phase space volume. *Therefore, the finite phase space volume corresponds to a finite number of quantum states.* That is, with an IR and UV regulator in place, the phase space is compact and the

Hilbert space is finite dimensional [20]. Here N is the dimension of the Hilbert space.

Returning to the regulated circle amplitude (12), we then find the corresponding regulated energy density from (6),

$$\rho(\tau)_{m.r.} = \hbar \left[\frac{\varepsilon N_p}{2\pi\hbar} \frac{1}{N_q} \sum_{k=0}^{N_q-1} \int_0^1 d\tilde{x}\, e^{-(\tilde{x}+k)^2 \varepsilon^2 \tau/2} \right]^4. \quad (16)$$

We note that all states that make an appreciable contribution are such that the argument of the exponential in (16) is small. The τ integration is not expected to change the scaling, and so we arrive at an upper bound on the vacuum energy density

$$\rho_{m.r.} \lesssim \hbar \left[\frac{\varepsilon N_p}{2\pi\hbar} \right]^4 = \hbar \left[\frac{M}{2\pi\hbar} \right]^4. \quad (17)$$

This result is no surprise, as M is the total size of regulated momentum space; if it is identified as a large mass scale such as m_P, then the usual conundrum pertains.

On the other hand, given the quantum area constraint, the bound on the energy density can be rewritten as

$$\rho_{m.r.} \lesssim \hbar \frac{N}{V_q}, \quad (18)$$

where $V_q \simeq L^4$. As far as we are aware, this formula is new. We note that it relates the vacuum energy density times space-time volume, ρV_q, to N. Given the form of this relation, if we interpret N as a count of microscopic degrees of freedom, it is natural to say that N should be interpreted as an entropy, in which case the bound on the vacuum energy density resembles an "equation of state," $\rho V \sim S$. Indeed, in what follows we will equate this entropy with a macroscopic notion of gravitational entropy. This makes a direct connection between the microscopic calculation of the vacuum energy density and (quantum) gravitational physics and as we will see yields a bound on the vacuum energy density that is remarkably consistent with observations.

We reiterate that if we were to identify λ with the Planck length $\ell_P = \sqrt{\hbar G_N}$, and correspondingly $\varepsilon \sim m_P = \sqrt{\hbar/G_N}$, then this bound corresponds to the usual EFT result $\rho \sim m_P^4$. However, there is nothing in this construction that forces us to take $\lambda \sim \ell_P$; λ and ε are constrained only by the quantum phase space area constraint $\lambda\varepsilon = 2\pi\hbar$. In ordinary quantum mechanics similarly, the shape of a phase space cell is determined contextually by some experimental setup. For example, in the context of a double slit experiment the length scale λ is the distance between the slits [16]. Often in quantum gravity discussions, some notion of minimal volumes or areas in space-time is introduced. All such notions are *ad hoc*, while here

we are introducing minimal volumes in phase space which have a natural interpretation in quantum theory.

Nevertheless, it is well known that in gravity there is a concept of entropy which scales as area rather than the volume of a spatial subregion [21,22]. Furthermore in any theory of quantum gravity, it is commonly suggested that this should be thought of as entanglement entropy [6]. Below, we will argue that the missing ingredient in EFT is this nonextensivity of entropy. We suggest that the holographic bound on entropy can be thought of as providing a contextual scale that determines the shape of elementary phase space cells.

To do so, we will identify N with the gravitational entropy

$$S_{\text{grav}} = \ell_P^{-2} \text{Area} \sim (\ell/\ell_P)^2, \quad (19)$$

where ℓ is some characteristic length scale. This identification can also be phrased in terms of a Bekenstein bound [23–26], but here we will need only a simple implementation: we are interested in a space-time with a positive vacuum energy density, in other words, an asymptotically de Sitter space-time with its cosmological horizon [22]. In this case the Bekenstein bound gives the entropy S_{grav} of the cosmological horizon in four space-time dimensions.

Thus we identify the length scale L with the size of the whole observable Universe ℓ, the cosmological horizon, $\ell \equiv L = N_q\lambda$. Equating S_{grav} with N, Eq. (18) gives

$$N \sim \ell^2/\ell_P^2, \quad (20)$$

which gives us a bound on energy density given by

$$\rho \lesssim \hbar \left(\frac{\ell_P}{\ell} \right)^2 \ell_P^{-4} = \frac{\hbar}{\ell^2 \ell_P^2} \quad (21)$$

or equivalently (by looking at the Einstein-Hilbert action on shell), a value for the cosmological constant

$$\Lambda_{\text{c.c.}} \lesssim 8\pi G_N \rho \sim \frac{1}{\ell^2}. \quad (22)$$

Note that this result does not depend on the characteristic UV scale represented by ℓ_P, but it only depends on the characteristic IR scale ℓ. This result is also technically natural, because the value of the cosmological constant goes to zero as the IR size increases to infinity. The factor $(\ell_P/\ell)^2$ provides the needed hierarchy between the observed energy density and the naive prediction of EFT.

This result is vastly smaller than the naive effective field theory computation. What has happened? We interpret this result to imply that the holographic bound on entropy implies a corresponding bound on the vacuum energy density. Apparently, the naive effective field theory reasoning overestimates the available phase space by a large amount, assuming implicitly that entropy is extensive.

This can be stated in terms of an effective momentum space size which is set by εN_p. Note that whereas N_q is very large, being associated with the size of the cosmological horizon, N_p is unconstrained and for simplicity can be taken to be order one. Thus the effective momentum space size is set approximately by $M \sim \varepsilon$ and given the above result, we see that

$$M \sim \hbar \ell_P^{-1} (\ell_P / \ell)^{1/2} = \frac{\hbar}{\sqrt{\ell \ell_P}}. \qquad (23)$$

Interestingly, this is a macroscopic scale. It implies that the typical phase space cell is much wider in the space direction and much thinner in the momentum direction than one would guess by identifying them with the Planck scales. One might regard the gravitational entropy as providing a context for setting these scales. Indeed, if we had an actual theory of quantum geometry to which we could couple a quantum field theory, we should expect that this matching of entropies would be automatic.

To complete the discussion, let us provide some realistic numbers. The Hubble scale of the visible Universe is $\ell \sim 10^{27} m$, the Planck length is $\ell_P \sim 10^{-35} m$, and thus $N \sim \ell^2 / \ell_P^2 \sim 10^{124}$. This formula summarizes our main point, which is that the size of the Universe is simply proportional to the number of degrees of freedom in the Universe and that the Universe is large because it contains a lot of degrees of freedom. Indeed if the Universe contains N degrees of freedom then the fluctuations scale as $1/\sqrt{N}$, and the stability of the Universe condition that the fluctuations are relatively small translates to the condition that N is large [27]. The observed vacuum energy density [28,29] is $[10^{-3} \text{ eV}]^4$, and this value is indeed 10^{124} smaller than the naive EFT estimate corresponding to the Planckian vacuum energy density $[10^{19} \text{ GeV}]^4$. The above geometric mean of the Hubble scale and the Planck length is $\lambda \sim (\ell \ell_P)^{1/2} \sim 10^{-4} m$, which corresponds to the observed vacuum energy scale of $M \sim 10^{-3}$ eV.

In this paper we have introduced the concept of modular regularization (12). In a quantum theory, the trace that yields the vacuum energy density will be basis-independent; any two choices of basis are unitarily related (for example, the momentum polarization and the modular polarization are related by Zak transform [17]). In EFT though, we are tied to a preferred basis, having assumed a *fixed classical space-time*. Regulating and performing the trace in terms of a modular polarization might then be interpreted as introducing a quantum notion of space-time geometry. The reader may find it confusing that we have identified the modular cells with ground states, as in EFT the ground state

is unique. This is not inconsistent though: if the momentum polarization is thought of as a singular limit in which the modular cell is squashed, $\varepsilon \to 0$ and $\lambda \to \infty$, then the information about multiple cells is simply lost in the limit. Thus by interpreting the trace as a phase space integral in a modular polarization, we introduce a regulator that retains the short-distance quantum structure of the phase space. On the other hand, most EFT calculations (such as of correlation functions) are unaffected by such considerations, where information about the vacuum energy simply factors out. But for computations that do directly address vacua, EFT simply has no mechanism to access a ground state degeneracy. In standard treatments where we have classical space-time geometry, one would attempt to calculate an entropy by referring to a quantum field theory Hilbert space; it would be interesting to connect our notion of ground state degeneracy with a regularization of such a calculation.

In this short paper, we have chosen to present the concepts in the most straightforward terms. There however is no obstruction to implementing them in string theory or other quantum gravitational contexts [30]. For example, we can implement the same construction in the vacuum energy computation in perturbative string theory [14,15]. It seems possible that this can be interpreted in terms of duality-covariant constructions of string theory [17,19,31–35]. In such a formulation, a four-dimensional space-time emerges through a process of extensification [17], with compact directions remaining of Planckian size and not affecting the vacuum energy.

Predicting a suitably small vacuum energy is only part of the cosmological constant problem. It is well known that there is another aspect, the coincidence problem [10]. Having tied the vacuum energy density to the size of the cosmological horizon, this problem may be resolved as well. It will also be interesting to consider if other hierarchy problems might be treated by similar methods; in the context of string theory, we note that there is a relation between the cosmological constant and the gauge hierarchy problem [36].

[1] M. Baumgart *et al.*, arXiv:2210.03199.

[2] A. Strominger, arXiv:1703.05448.

[3] W. Donnelly and L. Freidel, J. High Energy Phys. 09 (2016) 102; L. Ciambelli, R. G. Leigh, and P.-C. Pai, Phys. Rev. Lett. **128,** 171302 (2022); L. Freidel, arXiv:hep-th/2111.14747.

[4] L. Freidel, J. Kowalski-Glikman, R. G. Leigh, and D. Minic, Int. J. Mod. Phys. D **30,** 2141002 (2021).

[5] P. Berglund, L. Freidel, T. Hubsch, J. Kowalski-Glikman, R. G. Leigh, D. Mattingly, and D. Minic, Infrared properties of quantum gravity: UV/IR mixing, gravitizing the quantum—theory and observation, Contribution to Snowmass (2021), arXiv:2202.06890.

[6] D. Harlow *et al.*, arXiv:2210.01737.

[7] A. G. Cohen, D. B. Kaplan, and A. E. Nelson, Phys. Rev. Lett. **82,** 4971 (1999).

[8] J. F. Donoghue, Phys. Rev. D **104,** 045005 (2021).

[9] S. Weinberg, Rev. Mod. Phys. **61,** 1 (1989).

[10] S. Weinberg, arXiv:astro-ph/0005265.

[11] E. Witten, arXiv:hep-ph/0002297.

[12] T. Padmanabhan, Phys. Rep. **380,** 235 (2003).

[13] J. Polchinski, arXiv:hep-th/0603249.

[14] J. Polchinski, *String Theory. Vol. 1: An Introduction to the Bosonic String* (Cambridge University Press, Cambridge, England, 1998).

[15] J. Polchinski, Commun. Math. Phys. **104,** 37 (1986).

[16] Y. Aharonov and D. Rohrlich, *Quantum Paradoxes: Quantum Theory for the Perplexed* (Wiley, New York, 2005).

[17] L. Freidel, R. G. Leigh, and D. Minic, Phys. Rev. D **94,** 104052 (2016).

[18] L. Freidel, R. G. Leigh, and D. Minic, Int. J. Mod. Phys. D **24,** 1544028 (2015).

[19] L. Freidel, R. G. Leigh, and D. Minic, J. High Energy Phys. 06 (2015) 006.

[20] S. Bates and A Weinstein, *Lectures on the Geometry of Quantization* (American Mathematical Society, Providence, 1996).

[21] J. D. Bekenstein, Phys. Rev. D **7,** 2333 (1973).

[22] *Euclidean Quantum Gravity*, edited by G. W. Gibbons and S. W. Hawking (World Scientific, Singapore, 1993).

[23] J. D. Bekenstein, Phys. Rev. D **23,** 287 (1981).

[24] R. Bousso, Rev. Mod. Phys. **74,** 825 (2002).

[25] D. Marolf, D. Minic, and S. F. Ross, Phys. Rev. D **69,** 064006 (2004).

[26] H. Casini, Classical Quantum Gravity **25,** 205021 (2008).

[27] E. Schrödinger, *What is Life?* (Cambridge University Press, Cambridge, England, 1992).

[28] A. G. Riess *et al.* (Supernova Search Team), Astron. J. **116,** 1009 (1998).

[29] S. Perlmutter *et al.* (Supernova Cosmology Project), Astrophys. J. **517,** 565 (1999).

[30] J. de Boer *et al.*, Frontiers of quantum gravity: Shared challenges, converging directions, Contribution to Snowmass (2021), arXiv:2207.10618.

[31] L. Freidel, R. G. Leigh, and D. Minic, Phys. Lett. B **730,** 302 (2014).

[32] L. Freidel, R. G. Leigh, and D. Minic, Int. J. Mod. Phys. D **23,** 1442006 (2014).

[33] L. Freidel, R. G. Leigh, and D. Minic, J. High Energy Phys. 09 (2017) 060.

[34] L. Freidel, R. G. Leigh, and D. Minic, Phys. Rev. D **96,** 066003 (2017).

[35] L. Freidel, J. Kowalski-Glikman, R. G. Leigh, and D. Minic, Phys. Rev. D **99,** 066011 (2019).

[36] S. Abel and K. R. Dienes, Phys. Rev. D **104,** 126032 (2021).