Twisting and entangling gravitons in high-dimensional orbital angular momentum states via photon-graviton conversion

Haorong Wu and Lixiang Chen[*]
Department of Physics, Xiamen University, Xiamen 361005, China

(Received 29 August 2022; accepted 9 June 2023; published 30 June 2023)

The conflicts between quantum mechanics and general relativity have inspired people to search for quantum theories of gravity. If gravitons, as predicted by such theories, exist, it is natural to ask the following questions: Can gravitons carry quantized orbital angular momentum (OAM)? Can they be entangled with each other and with other particles in high-dimensional states? Here we answer these questions by studying the phenomenon of twisting and entangling gravitons in high-dimensional OAM states by possible photon-graviton conversion. The quantum aspect of this phenomenon indicates entanglement oscillation between photons, gravitons, and their hybrids and may lead to the potential development of high-dimensional gravitational wave (GW) communications. In addition, we investigate the phenomenon that photons in the early Universe may be imprinted with OAM information from plasma vortices and turned into OAM gravitons by the presence of possible strong magnetic fields. The resultant OAM GW may exist in the current Universe and provide a novel way of studying the plasma vortex structure in the early Universe.

DOI: 10.1103/PhysRevD.107.125027

I. INTRODUCTION

Quantum mechanics and general relativity are the two foundations of modern physics. However, they are still not unified in a coherent theory since a fully successful theory of quantum gravity has not yet been developed [1,2]. There are many candidate theories, among which are superstring theory and loop quantum gravity. Yet none of them have been tested in laboratories. Meanwhile, we could study the problem in other ways, e.g., by the similarities between electromagnetism and gravitation, and we will be inspired to ask the following questions: If gravitational waves (GWs) can be quantized in the form of gravitons, can they carry quantized orbital angular momentum (OAM)? Can they be entangled with each other and with other particles in the OAM space? Can they reflect some aspects of the early Universe? The answers will be of fundamental importance for understanding the nature of the early Universe as well as for future GW-based OAM quantum communications.

It is well known that the total photon angular momentum can be separated into two components: the spin angular momentum (SAM), which is connected to the polarization, and orbital angular momentum (OAM), which is related to the field spatial distribution. Since Allen *et al.* first proposed the OAM of light in 1992, it has received a significant amount of attention and the findings of investigations that followed have ushered in a new age of modern optics [3–14]. We will describe OAM beams by using the Laguerre-Gaussian (LG) modes, which in cylindrical coordinates are given by

$$\mathrm{LG}_{l,p}(\mathbf{x};k)=\frac{\mathcal{N}}{w(z)}\left(\frac{\sqrt{2}r}{w(z)}\right)^{|l|}\mathrm{L}_p^{|l|}\left(\frac{2r^2}{w^2(z)}\right)\times\exp\left(\frac{-r^2}{w^2(z)}+il\phi+\frac{ikr^2}{2R(z)}+i\Phi(z)\right),\qquad(1)$$

where $\mathcal{N}=\sqrt{\frac{2p!}{\pi(|l|+p)!}}$ is the normalization constant, $r=\sqrt{x^2+y^2}$, $\phi=\arctan(y/x)$, $w(z)=w_0\sqrt{1+(z/z_R)^2}$ with w_0 being the waist radius and $z_R=kw_0^2/2$ being the Rayleigh range, $R(z)=z[1+(z_R/z)^2]$ is the curvature radius of the wave fronts, $\Phi(z)=-(|l|+2p+1)$ $\arctan(z/z_R)$ is the Gouy phase, k is the wave number, and $\mathrm{L}_p^{|l|}(x)$ is the associated Laguerre polynomials. The radial index $p=0,1,2,\ldots$ indicates the number of radial nodes of the mode, while the azimuthal index $l=0,\pm1,\pm2,\ldots$ corresponds to the topological charge. It can be shown that the azimuthal index l represents the amount of orbital angular momentum carried by each photon in that mode. Some researchers have studied GWs with orbital angular momentum [15–17]. In this paper, we will investigate the OAM GWs from the perspective of photon-graviton conversion in a strong magnetic field, which is analogous to the photon-axion conversion [18–21]. Here, the process is generalized to include the OAM photon-graviton conversion.

[*]chenlx@xmu.edu.cn

If gravitational fields can be quantized, then they can support the entanglement transferred from photons and we can verify the entanglement once the gravitons are converted back to photons. This process results in the entanglement oscillation between photons, gravitons, and their hybrids, and holds promise for high-dimensional GW communications. In addition, the conversion may indicate the traces of plasma vortex structure in the early Universe. In the radiation-dominated era, the Universe is filled with a hot dense plasma composed of nuclei, electrons, and photons. Earlier epochs of the Universe may have caused enormous changes in plasma density and current, according to some studies [22,23]. Plasma vortices could exist in that period due to, for example, primordial black holes [24,25]. Meanwhile, there are studies showing the exchange of angular momentum between a light beam and a plasma vortex and demonstrating the possible excitation of photon angular momentum states in plasma [26,27]. If there exist strong magnetic fields in that epoch, photon beams with OAM might have been transformed into OAM gravitons and left a trace in the current Universe. We may be able to infer the plasma vortex structure in the early Universe by detecting those GWs.

Unless otherwise specified, geometrized units with $c = G = 1$ are used. The signature of metrics will be chosen as $(-, +, +, +)$. The following conventions apply to indices: all Greek indices run in $\{0, 1, 2, 3\}$, whereas Latin indices run in $\{1, 2, 3\}$. Throughout this paper, x or x^μ stand for the four position vector (x^0, x^1, x^2, x^3), $\mathbf{x}$ for its spatial components (x^1, x^2, x^3) and $\mathbf{x}_\perp$ for the spatial transverse components (x^1, x^2).

II. OAM PHOTON-GRAVITON CONVERSION AND TWISTING GRAVITONS

In this section, we review and generalize the theory that shows how photons and gravitons are coupled together in the presence of a strong magnetic field [18–21] and show that the gravitons can be twisted into the same OAM states as the OAM photons can. We choose the Minkowski spacetime as the background metric and start from the Einstein-Hilbert action,

$$S[g_{\rho\sigma}, A_\mu] = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa}R - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}\right), \qquad (2)$$

where g is the determinant of the metric tensor, R is the Ricci scalar, $\kappa = 8\pi$, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ are the electromagnetic fields. Under the weak gravitational field assumption, we can express the metric tensor as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, with $h_{\mu\nu}$ being the metric fluctuations and vary the action with respect to $h_{\mu\nu}$ to the first-order of fluctuations. Thus, we obtain the equations of motion (EOM) for gravitons, i.e., the linearized Einstein field equations,

$$\Box h_{\mu\nu} = (-\partial_t^2 + \nabla^2)h_{\mu\nu} = -2\kappa T_{\mu\nu}, \qquad (3)$$

where $T_{\mu\nu} = \frac{1}{2}\delta_{\mu\nu}\delta^{\rho\sigma}(E_\rho E_\sigma + B_\rho B_\sigma) - E_\mu E_\nu - B_\mu B_\nu$ is the energy-momentum tensor for the EM fields. Since there are gauge freedoms in $h_{\mu\nu}$ and $T_{\mu\nu}$, we will choose the transverse-traceless (TT) gauge, $h_{0\mu} = h^i{}_i = \partial^i h_{ij} = 0$, for the gravitational fields and the radiation gauge, $A_0 = \partial^i A_i = 0$, for the EM fields.

Now let us consider an OAM light beam propagating along the z axis and there is a strong background magnetic field which points at the x axis. The total four-potential can be written as $A_\mu = \bar{A}_\mu + \tilde{A}_\mu$, where $\bar{A}_\mu = (0, 0, 0, B_0 y)$ is the background magnetic field and $\tilde{A}_\mu = (0, B_y \mathrm{LG}_{l,p}(\mathbf{x})e^{ik(t-z)}/k, -B_x \mathrm{LG}_{l,p}(\mathbf{x})e^{ik(t-z)}/k, 0)$ is the OAM light field. By Fourier-transforming the energy-momentum tensor T_{ij} of the EM field to the momentum space, and applying the TT-projection operators $\Lambda_{ij}^{kl}(\mathbf{k})$ on $T_{kl}(\mathbf{k})$ to obtain its TT components $T_{ij}^{(\mathrm{TT})}(\mathbf{k})$, such that $T_{ij}^{(\mathrm{TT})}(\mathbf{k}) = \Lambda_{ij}^{kl}(\mathbf{k})T_{kl}(\mathbf{k})$, where $\Lambda_{ij}^{kl}(\mathbf{k}) = P_{ik}(\mathbf{k})P_{jl}(\mathbf{k}) - P_{ij}(\mathbf{k})P_{kl}(\mathbf{k})/2$, and $P_{ij}(\mathbf{k}) = \delta_{ij} - k_i k_j/|\mathbf{k}|^2$, we find that

$$\begin{aligned} T_{ij}^{(\mathrm{TT})} = {}& \mathrm{LG}_{l,p}(\mathbf{x})e^{ik(t-z)}\left[-i + \frac{1}{k}\partial_z \ln \mathrm{LG}_{l,p}(\mathbf{x})\right] \\ & \times \begin{pmatrix} -B_0B_x & -B_0B_y \\ -B_0B_y & B_0B_x \end{pmatrix} + \frac{1}{k}\mathrm{LG}_{l,p}^2(\mathbf{x})e^{2ik(t-z)} \\ & \times \partial_z(\ln \mathrm{LG}_{l,p}(\mathbf{x}))\left[-i + \frac{1}{2k}\partial_z(\ln \mathrm{LG}_{l,p}(\mathbf{x}))\right] \\ & \times \begin{pmatrix} B_y^2 - B_x^2 & -2B_xB_y \\ -2B_xB_y & B_x^2 - B_y^2 \end{pmatrix}, \end{aligned} \qquad (4)$$

where $i, j = 1, 2$.

Meanwhile, in the TT gauge, the metric fluctuations are given by

$$h_{\mu\nu}(x^\mu) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+(x^\mu) & h_\times(x^\mu) & 0 \\ 0 & h_\times(x^\mu) & -h_+(x^\mu) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad (5)$$

where the $+$ and $\times$ symbols in the subscripts are related to the $+$ and $\times$ polarizations of GWs, respectively. Given the large magnitude of k, the linearized Einstein field equations can be approximately solved by

$$h_+(x^\mu) = \kappa B_0 B_x z k^{-1}\mathrm{LG}_{l,p}(\mathbf{x})e^{ik(t-z)}, \qquad (6)$$

$$h_\times(x^\mu) = \kappa B_0 B_y z k^{-1}\mathrm{LG}_{l,p}(\mathbf{x})e^{ik(t-z)}. \qquad (7)$$

The results show that the linearized Einstein field equations guarantee that the GWs can be twisted with OAM similar to those of photons and that photons can only transform into gravitons with the same modes, i.e., frequencies k, OAM indices l and p, and polarizations as will be discussed later. We note that the graviton amplitude is regulated by a function $f(z) = \kappa B_0 z$. Since we have not considered the back action from gravitons, it will explode to infinity as $z \to \infty$.

Similarly, we can vary the Einstein-Hilbert action with respect to A_μ to derive the EOMs for photons,

$$\Box A_i = -\epsilon^{jl3} B_j \partial_3 h_{il}^{(\mathrm{TT})}, \tag{8}$$

where ϵ^{ijk} is the Levi-Civita symbol. Note we have used the TT gauge and the radiation gauge, and the fact that the strength of the background magnetic field B_0 is strong. A similar analysis gives the same result that gravitons can transform into photons with the same mode. Consequently, only photons and gravitons with the same mode are coupled together.

We now show that only photons and gravitons with the same polarization are coupled. Let the EM wave field be $\tilde{A} = (0, \tilde{A}_1, \tilde{A}_2, 0)$, and the static magnetic field be $\bar{B} = (0, \bar{B}_1, \bar{B}_2, 0)$. So the energy-momentum tensor is given by

$$T_{11} = -T_{22} = \bar{B}_2 \partial_3 \tilde{A}_1 + \bar{B}_1 \partial_3 \tilde{A}_2, \tag{9}$$

$$T_{12} = T_{21} = \bar{B}_2 \partial_3 \tilde{A}_2 - \bar{B}_1 \partial_3 \tilde{A}_1. \tag{10}$$

We can define $A_+ = (\bar{B}_1^2 + \bar{B}_2^2)^{-1/2} (\bar{B}_2 \tilde{A}_1 + \bar{B}_1 \tilde{A}_2)$, $A_\times = (\bar{B}_1^2 + \bar{B}_2^2)^{-1/2} (-\bar{B}_1 \tilde{A}_1 + \bar{B}_2 \tilde{A}_2)$. The EOMs for photons and gravitons will be written as

$$\Box A_\lambda = B \partial_3 h_\lambda, \qquad \Box h_\lambda = -2\kappa B \partial_3 A_\lambda, \tag{11}$$

with $B = \sqrt{\bar{B}_1^2 + \bar{B}_2^2}$ being the strength of the static background magnetic field, and $\lambda = +$ or $\times$ being the polarizations. Therefore, the gravitational fluctuations are only coupled to the EM fields with the same polarization.

Now we consider the EOMs for photons and gravitons together. Since we have shown that only the same mode photons and gravitons are coupled, we can choose a single mode and assume $A_\lambda = \alpha_\lambda(z) \mathrm{LG}_{l,p}(\mathbf{x}) e^{ik(t-z)}$ and $h_\lambda = \beta_\lambda(z) \mathrm{LG}_{l,p}(\mathbf{x}) e^{ik(t-z)}$ where $\alpha_\lambda(z)$ and $\beta_\lambda(z)$ are regulation functions and λ is the polarization. Under the paraxial approximation for LG mode functions and using the fact that the magnitude of k is large, we can derive

$$[(iBk\mathrm{LG}_{l,p}(\mathbf{x}) - B\partial_3 \mathrm{LG}_{l,p}(\mathbf{x}))I + M\partial_3] \begin{pmatrix} \alpha_\lambda(z) \\ \beta_\lambda(z) \end{pmatrix} = 0, \tag{12}$$

with I being an identity matrix, $M = \begin{pmatrix} M_1 & M_2 \\ -2M_2 & M_1 \end{pmatrix}$, and $M_1 = -B\mathrm{LG}_{l,p}(\mathbf{x})$, $M_2 = ik\mathrm{LG}_{l,p}(\mathbf{x})/\kappa - \partial_3 \mathrm{LG}_{l,p}(\mathbf{x})/\kappa$. Since only terms with LG mode functions dominate the equations, and we assume initially we only have photons, i.e., $\alpha_\lambda(0) = 1$ and $\beta_\lambda(0) = 0$, then $\alpha_\lambda(z)$ and $\beta_\lambda(z)$ can be solved to be $\alpha_\lambda(z) = \cos(\Gamma z)$, and $\beta_\lambda(z) = -\sqrt{2\kappa}\sin(\Gamma z)$, where $\Gamma = Bk(B + \sqrt{2/\kappa}k)^{-1}$ is the oscillating frequency. Therefore

$$A_\lambda(x^\mu) = \cos(\Gamma z) \mathrm{LG}_{l,p}(\mathbf{x}) e^{ik(t-z)}, \tag{13}$$

$$h_\lambda(x^\mu) = -\sqrt{2\kappa} \sin(\Gamma z) \mathrm{LG}_{l,p}(\mathbf{x}) e^{ik(t-z)}. \tag{14}$$

The results indicate that the OAM photon-graviton conversion oscillates at the frequency of Γ. In our analysis, we have not considered fluctuations in the background magnetic fields. However, from the definition of Γ, we know that the fluctuations of the background magnetic fields only affect the conversion rates in the process. Also, from Eqs. (13) and (14), the conservation of energy and orbital angular momentum is ensured, and the high-dimensional quantum states are merely transferred between photons and gravitons. Therefore, no decoherence and entanglement degradation will happen in our analysis. It appears that all OAM modes oscillate at the same frequency. In fact, there should be some minor differences between different modes, but we only keep the dominant terms. The curve for Γ is shown in Fig. 1 where the wavelength of the photon is set to 700 nm. If $B \gg \sqrt{2/\kappa}k$, then $\Gamma \approx k$. For $\lambda = 700$ nm, this will require $B \gg 10^{25} T$. Current studies show that heavy-ion collision experiments can produce very strong magnetic fields, which are estimated to be $B \sim \gamma Zeb/R^3$ for a collision of two ions of radius R with electric charge Ze, at impact parameter $\vec{b}$ and $\gamma = \sqrt{s_{NN}}/(2m_N)$ is the Lorentz factor [28–32]. It can reach 10^{15} T at Au-Au collisions at $\sqrt{s} = 200$ GeV and 10^{16} T at Pb-Pb collision at $\sqrt{s} = 2.76$ TeV. Still, it is a long step before we can verify the OAM photon-graviton conversion at the lab even if we extend the light wavelength and raise the collision energy. On the other hand, if $B \ll \sqrt{2/\kappa}k$, then

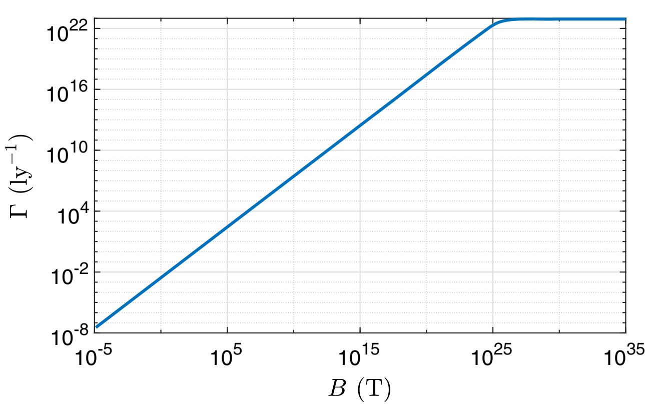

FIG. 1. The curve for Γ where the wavelength of the photon is set to 700 nm.

$\Gamma \approx \sqrt{\kappa/2}B$. This may signify some observable phenomena. For example, a light may be imprinted with OAM information when it passes close to a Kerr black hole, which drags and intermixes its surrounding space and time, deflecting and phase modifying the light in its vicinity [33,34]. A part of the OAM light will be turned into twisted GWs if it travels near a magnetar, which may have an extremely powerful magnetic field (about 10^9 to 10^{11} T). In some other cases where a magnetic field extends for a large range, the OAM photons may also be converted into OAM gravitons when passing through it. Numerically, a magnetic field with a strength of $B \sim 1$ T and a range of $D \sim 100$ ly can induce significant conversion.

III. ENTANGLING THE GRAVITONS AND HIGH-DIMENSIONAL ENTANGLEMENT OSCILLATIONS

The phenomenon of quantum entanglement is often regarded as the most nonclassical feature of quantum theory [35–39]. If gravitational fields can be quantized in the form of gravitons, as many physicists believe, then they should be able to convey quantum entanglements in the same way that other quantum field theories do. The OAM photon-graviton conversion might enable the creation of entanglements between photons, gravitons, and their hybrids.

Let us define $|l,p,z\rangle_\gamma = \int d^2\mathbf{x}_\perp \mathrm{LG}_{l,p}(\mathbf{x}) e^{ik(t-z)}|\mathbf{x}_\perp\rangle$ and $|l,p,z\rangle_g = \sqrt{2\kappa}\int d^2\mathbf{x}_\perp \mathrm{LG}_{l,p}(\mathbf{x}) e^{ik(t-z)}|\mathbf{x}_\perp\rangle$ being the spatial decomposition of OAM photon states and OAM graviton states, respectively. Suppose initially we send two entangled OAM photon beams, $|\psi(0)\rangle = \sum_{l,p} c_{l,p}|l,p,0\rangle_{\gamma_1}|-l,p,0\rangle_{\gamma_2}$, where in the subscript we will use γ_i and g_i to represent a photon state or a graviton state for the ith particle, respectively. Each beam will pass through a strong magnetic field to convert its OAM photons into OAM gravitons. Then at the distance z, the state will become

$$|\psi(z)\rangle = \omega_{\gamma\gamma}|\psi(z)\rangle_{\gamma\gamma} + \omega_{gg}|\psi(z)\rangle_{gg} + \omega_{\gamma g}|\psi(z)\rangle_{\gamma g} + \omega_{g\gamma}|\psi(z)\rangle_{g\gamma}, \tag{15}$$

with $\omega_{\gamma\gamma} = \cos(\Gamma_1 z)\cos(\Gamma_2 z)$, $\omega_{gg} = \sin(\Gamma_1 z)\sin(\Gamma_2 z)$, $\omega_{\gamma g} = -\cos(\Gamma_1 z)\sin(\Gamma_2 z)$, $\omega_{g\gamma} = -\sin(\Gamma_1 z)\cos(\Gamma_2 z)$ being the weight coefficients, and

$$|\psi(z)\rangle_{ab} = \sum_{l,p} c_{l,p}|l,p,z\rangle_{a_1}|-l,p,z\rangle_{b_2}, \tag{16}$$

where $a, b = \{\gamma, g\}$ and Γ_i being the oscillation frequency for each magnetic field. The states $|\psi(z)\rangle_{\gamma\gamma}$ and $|\psi(z)\rangle_{gg}$ describe the entanglements between photons and that between gravitons, respectively, while $|\psi(z)\rangle_{\gamma g}$

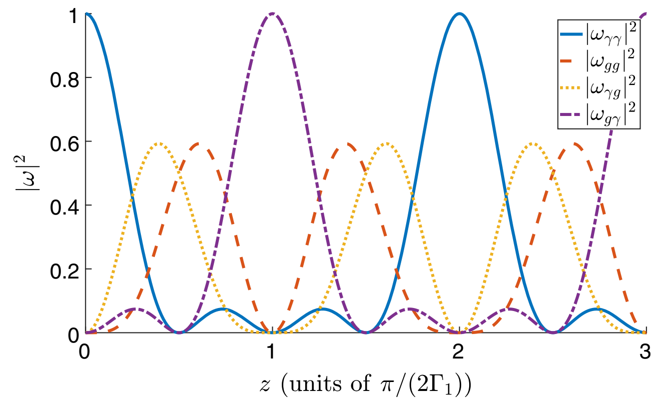

FIG. 2. Oscillation behaviors among entanglements shared by OAM photons, OAM gravitons, and hybrid entanglements of them.

and $|\psi(z)\rangle_{g\gamma}$ represent the hybrid entanglement between the photon γ_1 and the graviton g_2 and that between the photon γ_2 and the graviton g_1, respectively. We can see that the entanglements are oscillating between photons, gravitons, and their hybrids. Figure 2 depicts the intensities for different entanglements shared by OAM photons, OAM gravitons, and hybrid entanglements shared by them, with $\Gamma_2 = 2\Gamma_1$.

This quantum aspect of OAM photon-graviton conversion may lead to high-dimensional GW communications. Since OAM light is severely affected by turbulence [40–44], it is difficult to establish long-distance communications with OAM light. However, this deterioration may be avoided by first converting OAM light into OAM GWs, then transmitting the OAM gravitons to receivers, and then decoding the information by converting OAM gravitons back into OAM photons. This may be feasible with the development of future heavy-ion collision experiments.

IV. FOOTPRINTS OF PLASMA VORTEX STRUCTURE IN THE EARLY UNIVERSE

Since the interaction of GWs with matter is so minute, they directly carry information on their generation and the global evolution history of the Universe. As a result, the stochastic GW background provides us with invaluable opportunities to access various early Universe processes. There are many potential origins of GWs in the early Universe, including inflation, first-order phase transition, preheating after inflation, topological defects, etc. Also mentioned are several astrophysical sources, such as compact object binaries, stellar core collapse, r-mode instability of neutron stars, magnetars [45], etc. Here, we focus on the OAM photon-graviton conversion process in the early Universe. In the radiation-dominated era, the Universe was comprised of a hot dense plasma of nuclei, electrons, and photons. It has been hypothesized that the earlier epochs of the Universe had effects that could cause large fluctuations in plasma density and current [22,23]. Some studies show that plasma vortices could form as a result of, for example, primordial black holes [24,25]. We now study the traces of these plasma vortices from the perspective of OAM photon-graviton conversion.

We consider a nonrotating plasma vortex, where the electric fields are screened by the thermal plasma, and adopt the definitions of mean electron density and velocity by [26,27] $n = n_0 + \tilde{n}(\mathbf{r}, z)\cos(l_0\phi + q_0 z)$, $\mathbf{v} = \mathbf{v}_0 + \delta\mathbf{v}$, where n_0 and $\mathbf{v}_0$ are the background plasma conditions, $n(\mathbf{r}, z)\cos(l_0\phi + q_0 z)$ is the plasma helix vortex density perturbation, $\delta\mathbf{v}$ is the velocity perturbation induced by the EM waves. The current in the plasma, $\mathbf{J} = -en\mathbf{v}$, satisfies the electron fluid equations,

$$\partial_t n + \nabla \cdot n\mathbf{v} = 0, \tag{17}$$

$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla\mathbf{v} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})/m. \tag{18}$$

Since the timescale of the process is less than the Hubble time, we ignore the expansion of the Universe and work with the Minkowski background [18].

We generalize the Einstein-Hilbert action to include a current term,

$$S[g_{\rho\sigma}, A_\mu] = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa}R - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - j_\mu A^\mu\right), \tag{19}$$

where j^μ is the four-current. To keep the action gauge invariant, the current source should satisfy $\partial_\mu(\sqrt{|-g|}j^\mu) = 0$. We can vary the action with respect to A_μ to derive new EOMs for photons,

$$(\Box - \omega_p^2)A_i = -\epsilon^{jl3}B_j\partial_3 h_{il}^{(\mathrm{TT})}, \tag{20}$$

where $\omega_p^2 = \omega_{p0}^2(1 + \epsilon(\mathbf{r}, z))$ is the perturbed plasma frequency, $\omega_{p0}^2 = e^2 n_0/m_e$ is the frequency without fluctuations, and $\epsilon(\mathbf{r}, z) = \tilde{n}(\mathbf{r}, z)\cos(l_0\phi + q_0 z)/n_0$ represents the perturbations.

Assume initially, the photons have no OAM, and mode coupling is sufficiently weak such that the zero OAM mode is dominant over the entire interaction region. Then it has been shown that we can mainly consider the mode coupling between $l = 0$, l_0, and $-l_0$ [26,27]. Also, in previous sections, we have shown that photons are only coupled with gravitons in the same mode. Thus, we can assume $A_\lambda(x^\mu) = \sum_j \alpha_{\lambda,j}(z)\mathrm{LG}_j(\mathbf{x})e^{i(\omega t - kz)}$, and $h_\lambda(x^\mu) = \sum_j \beta_{\lambda,j}(z)\sqrt{2\kappa}\mathrm{LG}_j(\mathbf{x})e^{ik(t-z)}$, where $j = \{0, l_0, -l_0\}$, $\alpha_{\lambda,j}(z)$ and $\beta_{\lambda,j}(z)$ are functions to be determined, and ω satisfies the dispersion relation in plasma, $\omega^2 = k^2 + \omega_{p0}^2$. $\omega \gg \omega_{p0}$ is assumed. Since only the azimuthal indices are important, we ignore the radial indices here. Under the paraxial approximation, we obtain, in Dirac notation,

$$\sum_j [(-\omega_{p0}^2\epsilon(\mathbf{r}, z) - 2ik\partial_3)\alpha_{\lambda,j} + (ikB - B\partial_3)\beta_{\lambda,j}]|j\rangle = 0, \tag{21}$$

$$\sum_j [(2i\kappa kB - 2\kappa B\partial_3)\alpha_{\lambda,j} + 2ik\partial_3\beta_{\lambda,j}]|j\rangle = 0, \tag{22}$$

where $|j\rangle$ represents the OAM mode with an azimuthal index of j. In the simplest case where $\epsilon(\mathbf{r}, z)$ is only dependent on the azimuthal angle ϕ, $\langle k|\epsilon(\mathbf{r}, z)|j\rangle = \tilde{n}\pi[\delta(l_0 + j - k)e^{iq_0 z} + \delta(-l_0 + j - k)e^{-iq_0 z}]/n_0$, so by inserting the completeness relation, $1 = \sum_k |k\rangle\langle k|$, the first equation becomes

$$\sum_j [-\omega_{p0}^2\tilde{n}\pi(\alpha_{\lambda,j-l_0}e^{iq_0 z} + \alpha_{\lambda,j+l_0}e^{-iq_0 z})/n_0 - 2ik\partial_3\alpha_{\lambda,j} + ikB\beta_{\lambda,j} - B\partial_3\beta_{\lambda,j}]|j\rangle = 0. \tag{23}$$

Since $\{|j\rangle\}$ are orthogonal, then we obtain $(iBkI + N + M\partial_3)|\psi\rangle = 0$, where $|\psi\rangle = (\alpha_{\lambda,l_0}, \alpha_{\lambda,0}, \alpha_{\lambda,-l_0}, \beta_{\lambda,l_0}, \beta_{\lambda,0}, \beta_{\lambda,-l_0})^{\mathrm{T}}$ is the state vector, I is a 6-by-6 identity matrix, N and M are matrices determined by EOMs. So $|\psi\rangle = \exp(\int_0^z \mathcal{T}(z')dz')|c\rangle$ with $\mathcal{T}(z) = -M^{-1}(iBkI + N)$ and $|c\rangle$ being a 6-by-1 constant vector. As stated in Ref. [26], the transfer between different modes is inhibited after a distance of order $z_0 = 2\pi/q_0$, the helical path length. The favorable case is therefore that of an interaction distance shorter than this length, or in the limit, a plasma structure with no axial periodicity. Also, at the extremely high temperature of the early Universe, $1/\Gamma$ should be very small. Then $e^{\pm iq_0 z} \to 1$, and $|\psi\rangle = \sum_n c_n e^{\lambda_n z}|\psi_n\rangle$, where λ_n are the eigenvalues of $\mathcal{T}$, $|\psi_n\rangle$ are the corresponding eigenvectors, and c_n are constants. Given the initial state $|\psi(0)\rangle = (0, 1, 0, 0, 0, 0)^{\mathrm{T}}$, we obtain

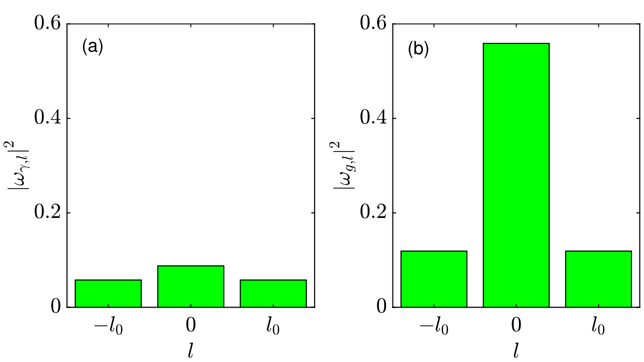

FIG. 3. The resultant OAM mode spectra for (a) photons, $\omega_{\gamma,l} = \alpha_{\lambda,l}(z)$, and (b) gravitons, $\omega_{g,l} = \beta_{\lambda,l}(z)$, at $z = 1.46 \times 10^{-11}$ m. We choose $T = 800$ keV, $\omega_{p0}^2 = e^2T^2$, $n_0 = 1 \times 10^{40}/\mathrm{m}^3$, $\tilde{n} = n_0/10$, $\omega = T$, and $B = 1 \times 10^{30}$ T.

$$\alpha_{\lambda,l_0}(z) = \alpha_{\lambda,-l_0}(z) = \frac{1}{2\sqrt{2}} e^{-i\omega_1 z} \left\{ e^{-i\omega_2 z} \left[-\cos(\omega_3 z) + \frac{i\omega_n^2}{\sigma_1} \sin(\omega_3 z) \right] + e^{i\omega_2 z} \left[\cos(\omega_4 z) + \frac{i\omega_n^2}{\sigma_2} \sin(\omega_4 z) \right] \right\}, \tag{24}$$

$$\alpha_{\lambda,0}(z) = \frac{1}{2} e^{-i\omega_1 z} \left\{ e^{-i\omega_2 z} \left[\cos(\omega_3 z) - \frac{i\omega_n^2}{\sigma_1} \sin(\omega_3 z) \right] + e^{i\omega_2 z} \left[\cos(\omega_4 z) + \frac{i\omega_n^2}{\sigma_2} \sin(\omega_4 z) \right] \right\}, \tag{25}$$

$$\beta_{\lambda,l_0}(z) = \beta_{\lambda,-l_0}(z) = e^{-i\omega_1 z} \left[e^{-i\omega_2 z} \frac{\sigma_3}{\sigma_1} \sin(\omega_3 z) + e^{i\omega_2 z} \frac{\sigma_4}{\sqrt{2}\sigma_2} \sin(\omega_4 z) \right], \tag{26}$$

$$\beta_{\lambda,0}(z) = e^{-i\omega_1 z} \left[e^{-i\omega_2 z} \frac{-\sqrt{2}\sigma_3}{\sigma_1} \sin(\omega_3 z) + e^{i\omega_2 z} \frac{\sigma_4}{\sigma_2} \sin(\omega_4 z) \right]. \tag{27}$$

with $\omega_n^2 = \omega_{p0}^2 \tilde{n}\pi/n_0$, $\omega_1 = 2\kappa B^2 k/(4k^2 - 2\kappa B^2)$, $\omega_2 = \sqrt{2}k\omega_n^2/(4k^2 - 2\kappa B^2)$, $\omega_3 = k\sigma_1/(\sqrt{2}(2k^2 - \kappa B^2))$, $\omega_4 = k\sigma_2/(\sqrt{2}(2k^2 - \kappa B^2))$, $\sigma_1 = [\omega_n^4 + 2\kappa B^2(2k^2 + \sqrt{2}\omega_n^2)]^{1/2}$, $\sigma_2 = [\omega_n^4 + 2\kappa B^2(2k^2 - \sqrt{2}\omega_n^2)]^{1/2}$, $\sigma_3 = \sqrt{\kappa}B(2k^4 + 2\sqrt{2}k^2\omega_n^2 + \omega_n^4)/(2\sqrt{2}k^3 + 2k\omega_n^2)$, $\sigma_4 = \sqrt{\kappa}B(-\sqrt{2}k^2 + \omega_n^2)/(\sqrt{2}k)$. At the radiation era with an extremely high temperature T, we have $\omega \lesssim T$, and $\omega_{p0}^2 \sim e^2 T^2$ with $e = \sqrt{4\pi\alpha_e} \approx 0.3$ and α_e being the fine structure constant. We consider a relatively large magnetic field $B \lesssim T^2$ [18] and plot the corresponding mode weights for photons and gravitons at an arbitrary distance, say, $z = 1.46 \times 10^{-11}$ m in Fig. 3. We notice that the total OAM of the photon-graviton system is conserved. It appears that the plasma vortex only plays the role of catalyzer to facilitate the corresponding conversion. We should mention that Ref. [18] states that the frequency of generated GWs is typically on the order of GHz with an amplitude of up to $\Omega_{\mathrm{GW}} h^2 \sim 10^{-10}$. However, the amplitude is too small to be observed by experiments unless the sensitivity of the experiments is improved.

V. CONCLUSION

In this paper, we address the problem of the quantum nature of gravitational fields in high-dimensional OAM space. By investigating the OAM photon-graviton conversion in strong magnetic fields, we show that the gravitons can be twisted into the same OAM states as the photons have. Also, they are capable of carrying quantum entanglements in the OAM space. Especially, in the presence of strong magnetic fields, the entanglements oscillate between photons, gravitons, and their hybrids, leading to the potential development of high-dimensional GW-based quantum communications, which can overcome the severe degradation faced by OAM light communications. Finally, by exploring the excitation of photon OAM states in plasma, we show that the plasma vortex structure in the early Universe may leave GW traces in the current Universe, i.e., a promising tool to probe the nature of the early Universe. Although the enigma of quantum gravity theory is still eluding us, our work may serve as a tool for detecting the existence of OAM gravitons, a key to future quantum communication, and a messenger from the early Universe.

ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China (No. 12034016), the Natural Science Foundation of Fujian Province of China (No. 2021J02002) and for Distinguished Young Scientists (No. 2015J06002), and the program for New Century Excellent Talents in University of China (No. NCET-13-0495).

[1] P. Xu, Y. Ma, J.-G. Ren, H.-L. Yong, T. C. Ralph, S.-K. Liao, J. Yin, W.-Y. Liu, W.-Q. Cai, X. Han *et al.*, Satellite testing of a gravitationally induced quantum decoherence model, Science **366**, 132 (2019).

[2] A. Ashtekar and T. A. Schilling, Geometrical formulation of quantum mechanics, in *On Einstein's Path* (Springer, New York, 1999), pp. 23–65.

[3] L. Allen, M. W. Beijersbergen, R. Spreeuw, and J. Woerdman, Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes, Phys. Rev. A **45**, 8185 (1992).

[4] G. F. Calvo, A. Picón, and E. Bagan, Quantum field theory of photons with orbital angular momentum, Phys. Rev. A **73**, 013805 (2006).

[5] A. Mair, A. Vaziri, G. Weihs, and A. Zeilinger, Entanglement of the orbital angular momentum states of photons, Nature (London) **412**, 313 (2001).

[6] G. Molina-Terriza, J. P. Torres, and L. Torner, Management of the Angular Momentum of Light: Preparation of Photons in Multidimensional Vector States of Angular Momentum, Phys. Rev. Lett. **88**, 013601 (2001).

[7] J. Leach, M. J. Padgett, S. M. Barnett, S. Franke-Arnold, and J. Courtial, Measuring the Orbital Angular Momentum of a Single Photon, Phys. Rev. Lett. **88**, 257901 (2002).

[8] A. Vaziri, G. Weihs, and A. Zeilinger, Experimental Two-Photon, Three-Dimensional Entanglement for Quantum Communication, Phys. Rev. Lett. **89**, 240401 (2002).

[9] A. Vaziri, J.-W. Pan, T. Jennewein, G. Weihs, and A. Zeilinger, Concentration of Higher Dimensional Entanglement: Qutrits of Photon Orbital Angular Momentum, Phys. Rev. Lett. **91**, 227902 (2003).

[10] L. Allen, M. Padgett, and M. Babiker, Iv the orbital angular momentum of light, Prog. Opt. **39**, 291 (1999).

[11] Y. S. Kivshar and E. A. Ostrovskaya, Optical vortices folding and twisting waves of light, Opt. Photonics News **12**, 24 (2001).

[12] G. Molina-Terriza, J. P. Torres, and L. Torner, Twisted photons, Nat. Phys. **3**, 305 (2007).

[13] A. M. Yao and M. J. Padgett, Orbital angular momentum: Origins, behavior and applications, Adv. Opt. Photonics **3**, 161 (2011).

[14] G. Molina-Terriza, E. M. Wright, and L. Torner, Propagation and control of noncanonical optical vortices, Opt. Lett. **26**, 163 (2001).

[15] I. Bialynicki-Birula and Z. Bialynicka-Birula, Gravitational waves carrying orbital angular momentum, New J. Phys. **18**, 023022 (2016).

[16] P. Baral, A. Ray, R. Koley, and P. Majumdar, Gravitational waves with orbital angular momentum, Eur. Phys. J. C **80**, 326 (2020).

[17] I. Bialynicki-Birula and S. Charzyński, Trapping and Guiding Bodies by Gravitational Waves Endowed with Angular Momentum, Phys. Rev. Lett. **121**, 171101 (2018).

[18] T. Fujita, K. Kamada, and Y. Nakai, Gravitational waves from primordial magnetic fields via photon-graviton conversion, Phys. Rev. D **102**, 103501 (2020).

[19] G. Raffelt and L. Stodolsky, Mixing of the photon with low-mass particles, Phys. Rev. D **37**, 1237 (1988).

[20] W. K. De Logi and A. R. Mickelson, Electrogravitational conversion cross sections in static electromagnetic fields, Phys. Rev. D **16**, 2915 (1977).

[21] A. N. Cillis and D. D. Harari, Photon-graviton conversion in a primordial magnetic field and the cosmic microwave background, Phys. Rev. D **54**, 4757 (1996).

[22] D. Lemoine, Fluctuations in the relativistic plasma and primordial magnetic fields, Phys. Rev. D **51**, 2677 (1995).

[23] K. A. Holcomb and T. Tajima, General-relativistic plasma physics in the early universe, Phys. Rev. D **40**, 3809 (1989).

[24] C. Bhattacharjee and D. J. Stark, Vortex generation in the early universe-the spacetime curvature drive in primordial black hole accretion disks, Astron. Astrophys. **642**, L6 (2020).

[25] S. Mahajan and Z. Yoshida, Relativistic generation of vortex and magnetic field, Phys. Plasmas **18**, 055701 (2011).

[26] J. Mendonça, B. Thidé, J. Bergman, S. Mohammadi, B. Eliasson, W. Baan, and H. Then, Photon orbital angular momentum in a plasma vortex, arXiv:0804.3221.

[27] F. Tamburini, A. Sponselli, B. Thidé, and J. Mendonça, Photon orbital angular momentum and mass in a plasma vortex, Europhys. Lett. **90**, 45001 (2010).

[28] K. Hattori and X.-G. Huang, Novel quantum phenomena induced by strong magnetic fields in heavy-ion collisions, Nucl. Sci. Tech. **28**, 1 (2017).

[29] L. McLerran and V. Skokov, Comments about the electromagnetic field in heavy-ion collisions, Nucl. Phys. **A929**, 184 (2014).

[30] K. Tuchin, Photon decay in a strong magnetic field in heavy-ion collisions, Phys. Rev. C **83**, 017901 (2011).

[31] G. Inghirami, M. Mace, Y. Hirono, L. Del Zanna, D. E. Kharzeev, and M. Bleicher, Magnetic fields in heavy ion collisions: Flow and charge transport, Eur. Phys. J. C **80**, 293 (2020).

[32] A. Das, S. S. Dave, P. Saumia, and A. M. Srivastava, Effects of magnetic field on plasma evolution in relativistic heavy-ion collisions, Phys. Rev. C **96**, 034902 (2017).

[33] F. Tamburini, B. Thidé, and M. Della Valle, Measurement of the spin of the m87 black hole from its observed twisted light, Mon. Not. R. Astron. Soc. **492**, L22 (2020).

[34] F. Tamburini, B. Thidé, G. Molina-Terriza, and G. Anzolin, Twisting of light around rotating black holes, Nat. Phys. **7**, 195 (2011).

[35] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Quantum entanglement, Rev. Mod. Phys. **81**, 865 (2009).

[36] P. Rungta, V. Bužek, C. M. Caves, M. Hillery, and G. J. Milburn, Universal state inversion and concurrence in arbitrary dimensions, Phys. Rev. A **64**, 042315 (2001).

[37] J. A. Anaya-Contreras, H. M. Moya-Cessa, and A. Zúñiga-Segundo, The von Neumann entropy for mixed states, Entropy **21**, 49 (2019).

[38] F. Mintert, A. R. Carvalho, M. Kuś, and A. Buchleitner, Measures and dynamics of entangled states, Phys. Rep. **415**, 207 (2005).

[39] O. Gühne and G. Tóth, Entanglement detection, Phys. Rep. **474**, 1 (2009).

[40] H. Wu, X. Fan, and L. Chen, How gravitational fluctuations degrade the high-dimensional spatial entanglement, Phys. Rev. D **106**, 045023 (2022).

[41] F. S. Roux, Infinitesimal-propagation equation for decoherence of an orbital-angular-momentum-entangled biphoton state in atmospheric turbulence, Phys. Rev. A **83**, 053822 (2011).

[42] F. S. Roux, T. Wellens, and V. N. Shatokhin, Entanglement evolution of twisted photons in strong atmospheric turbulence, Phys. Rev. A **92**, 012326 (2015).

[43] C. Paterson, Atmospheric Turbulence and Orbital Angular Momentum of Single Photons for Optical Communication, Phys. Rev. Lett. **94**, 153901 (2005).
[44] B. Rodenburg, M. P. Lavery, M. Malik, M. N. O'Sullivan, M. Mirhosseini, D. J. Robertson, M. Padgett, and R. W. Boyd, Influence of atmospheric turbulence on states of light carrying orbital angular momentum, Opt. Lett. **37**, 3735 (2012).
[45] S. Kuroyanagi, T. Chiba, and T. Takahashi, Probing the universe through the stochastic gravitational wave background, J. Cosmol. Astropart. Phys. 11 (2018) 038.